Where does the proton mass come from?

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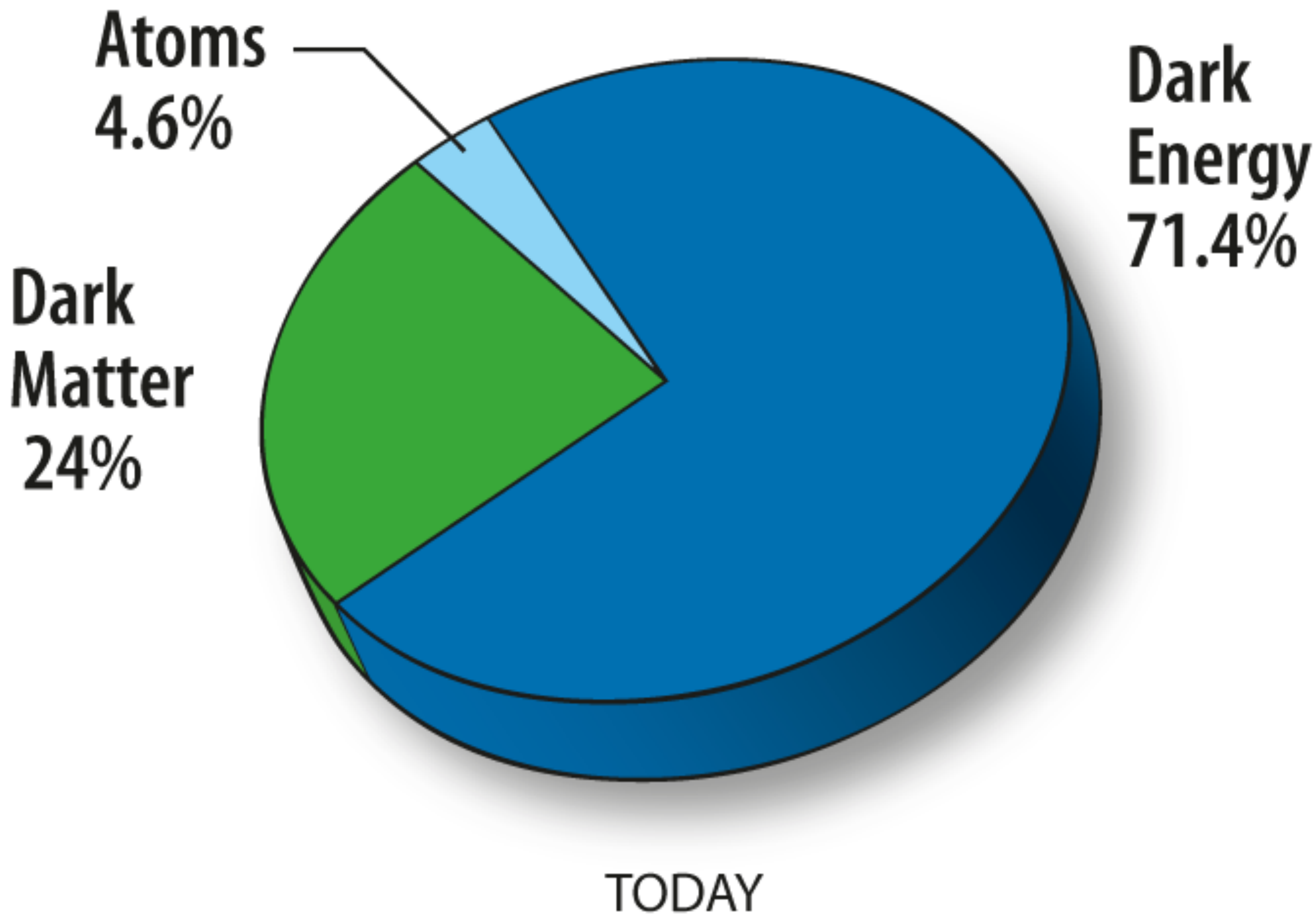
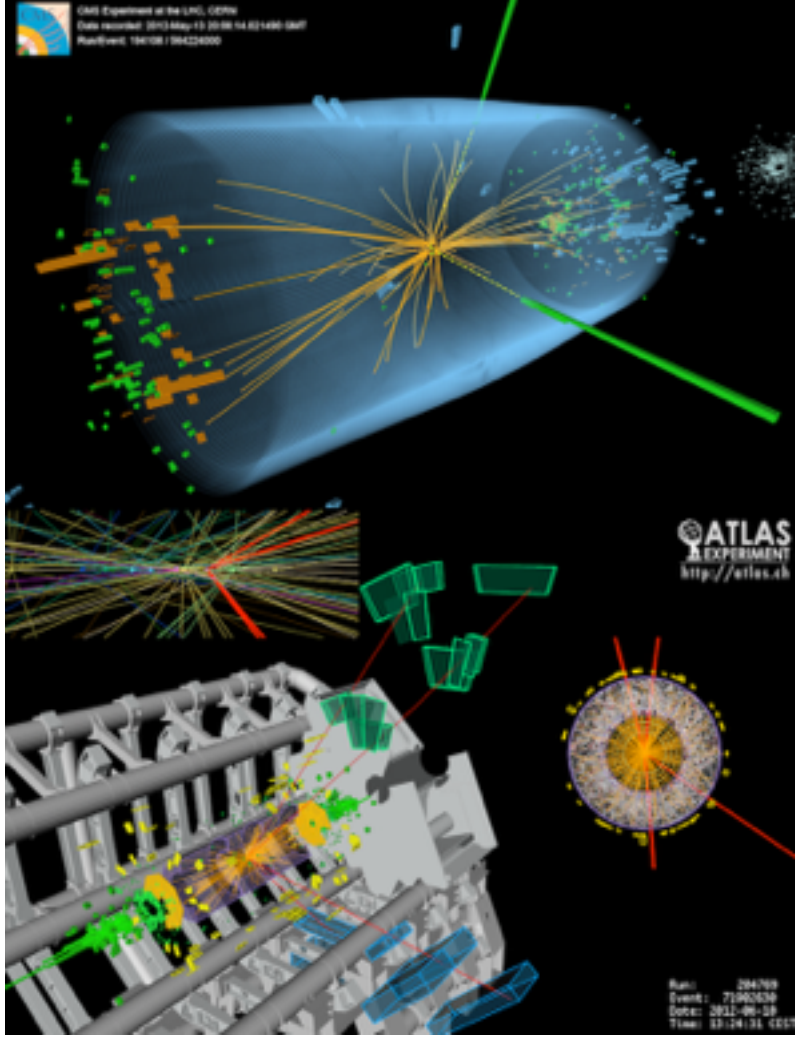
Motivation

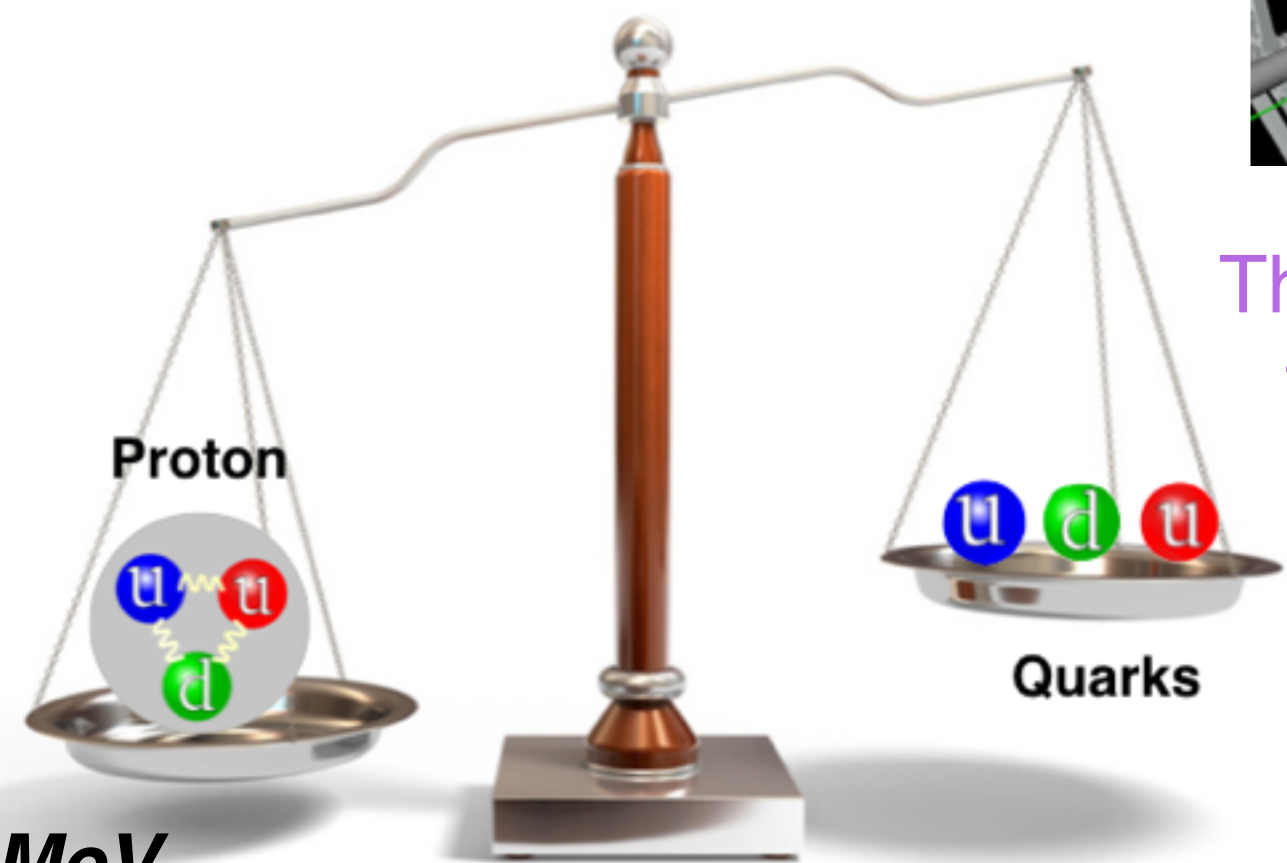
Where does this observable 4.6% come from, only due to Higgs?



Motivation



Where does the proton mass come from, and how ?



But the mass of the proton is

938.272046(21) MeV.

~100 times of the sum of the quark masses!

The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)$ MeV

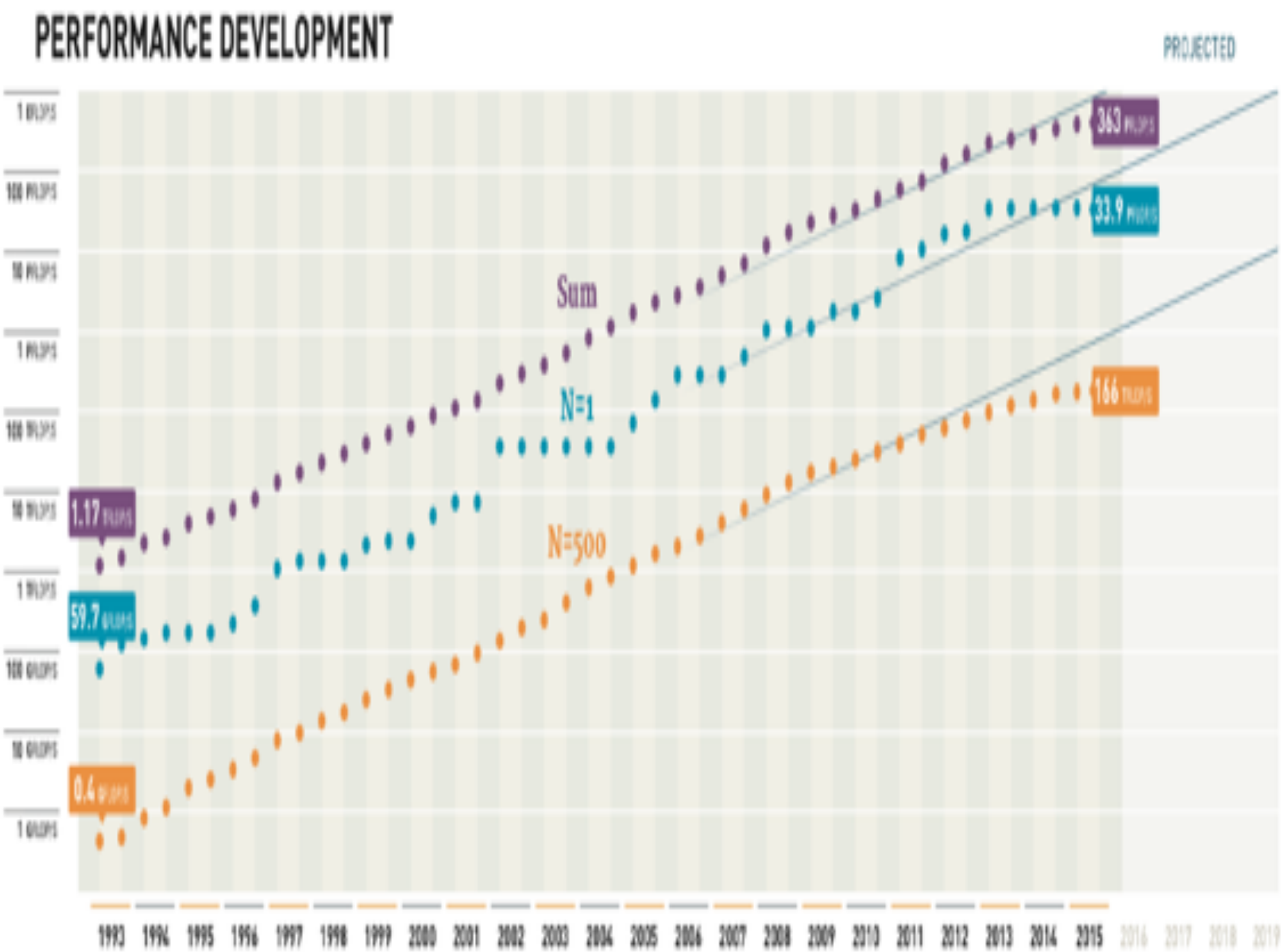
$m_d = 4.73(12)$ MeV

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

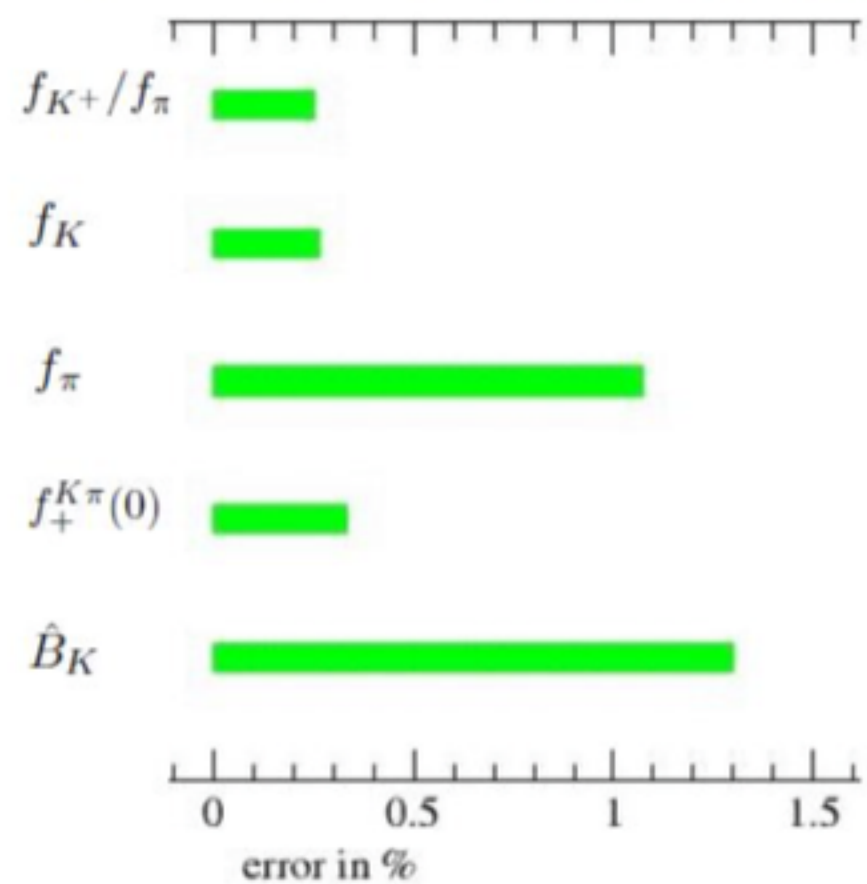
Lattice QCD

The power of supercomputers

…and hard works from Lattice QCD communities



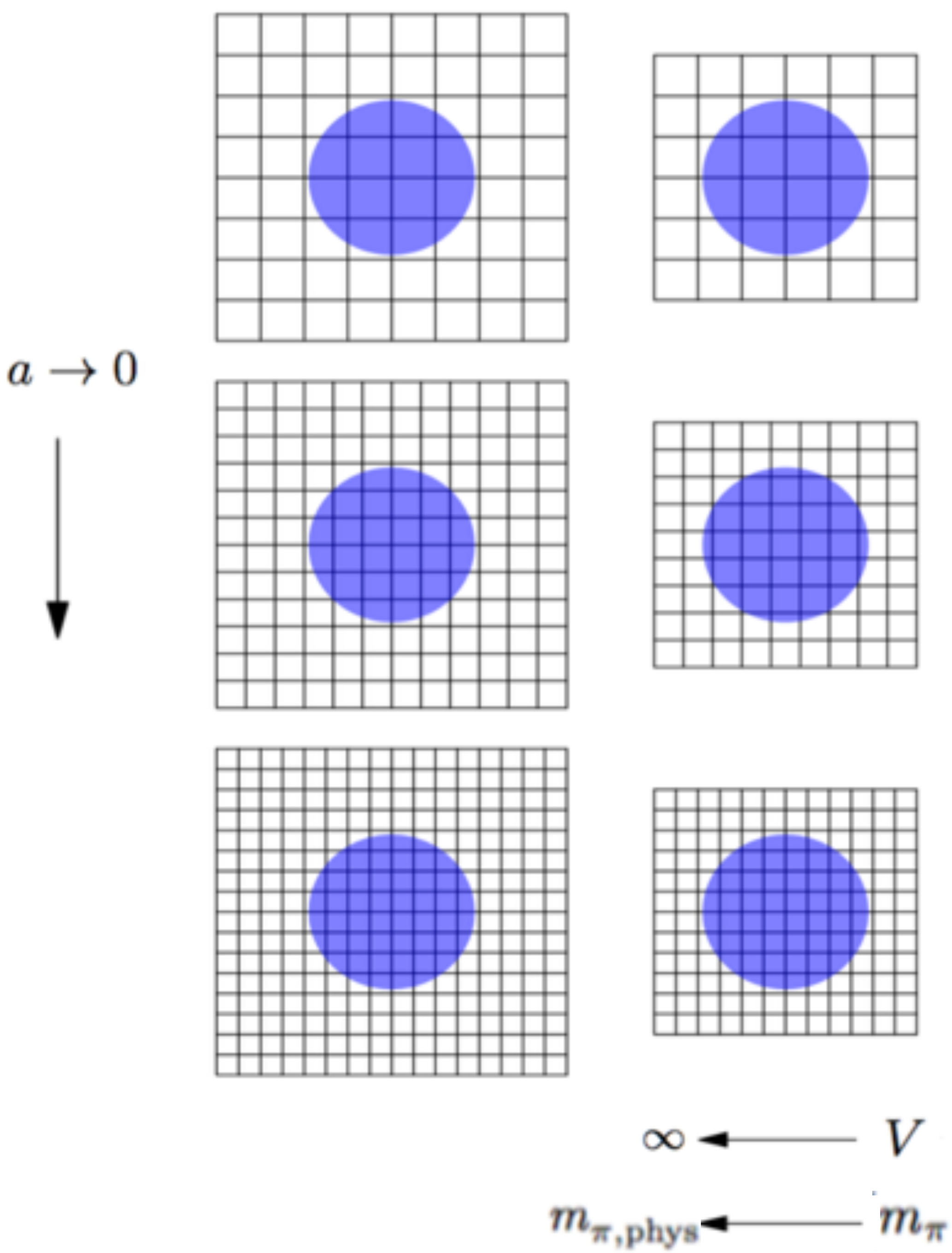
FLAG-3 averages



From A. El-Khadra, Sep. 2015, INT workshop
“QCD for New Physics at the Precision Frontier”

Lattice QCD

Continuum and infinite volume limit



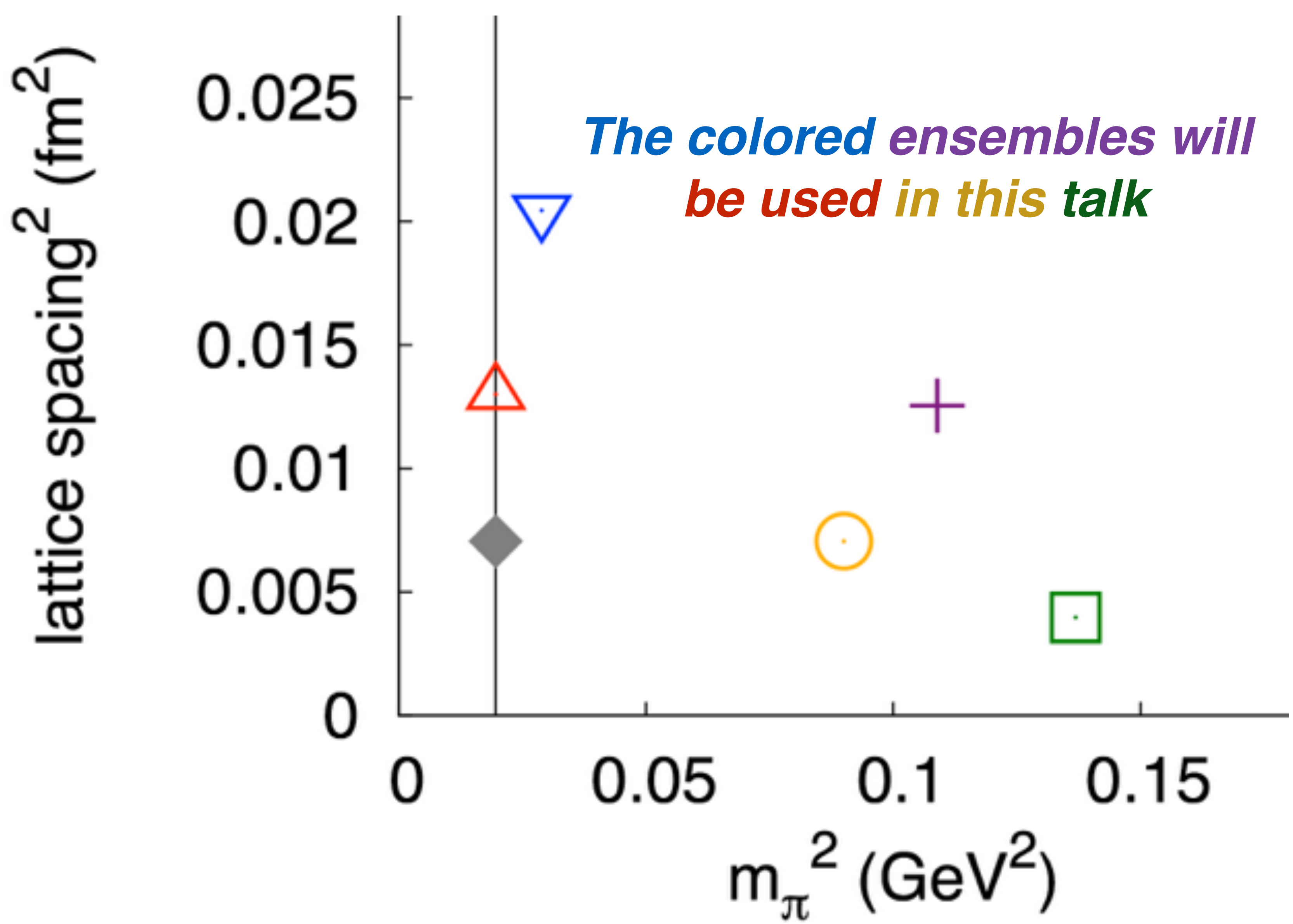
$$
\begin{aligned}
C(m_\pi, m_{\pi,sea}, a, L) = \\
& C_{\mathrm{phys}} + C_1(m_\pi^2 - m_{\pi,\mathrm{phys}}^2) \\
& + C_2(m_{\pi,sea}^2 - m_{\pi,\mathrm{phys}}^2) \\
& + C_3 a^2 + C_4 e^{-m_\pi L} \\
& + O(m_\pi^3, m_{\pi,sea}^3, a^4)\ldots
\end{aligned}
$$

The **larger** volume also allow us to simulate the quark corresponding to **lighter** pion mass correctly.

Lattice QCD

The joint fit

2+1 flavor DWF configurations (RBC-UKQCD)



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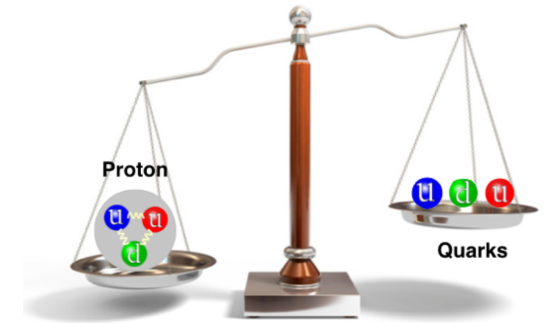
$$

The **larger** volume also allow us to simulate the quark corresponding to **lighter** pion mass correctly.

Proton mass decomposition



$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2,$$

The energy momentum tensor in the classic level

$$\overline{T}_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu}\psi - \frac{1}{16}g_{\mu\nu}\bar{\psi}\gamma_{(\rho}\overleftrightarrow{D}_{\rho)}\psi + F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$

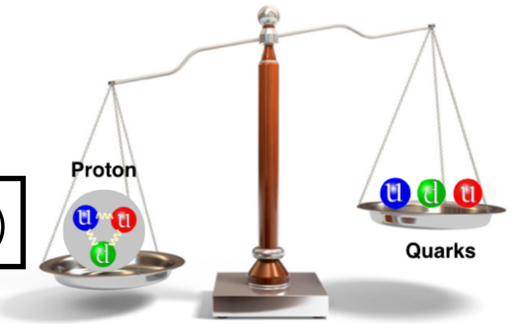
The traceless part of the energy momentum tensor

$$\langle T_{\mu\mu}\rangle = \langle -m\bar{\psi}\psi - \gamma_m m\bar{\psi}\psi + \frac{\beta(g)}{2g}F^2\rangle$$

The trace part of the energy momentum tensor with equation of motion (EOM) applied, plus the quantum trace anomalies.

Xiangdong Ji, PRL 74, 1071-1074 (1995)

Proton mass decomposition



Xiangdong Ji, PRL 74, 1071-1074 (1995)

YBY, et.al. χQCD Collaboration, Phys. Rev. D 91, 074516 (2015)

Then we have

$$M = \langle T_{00}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$
$$\frac{1}{4}M = \langle \hat{T}_{00}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

in the rest frame.

With

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

The quark mass

Gauge Invariant and scale independent combinations.

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$
$$H_g^a = \int d^3x\, \frac{\beta(g)}{4g}(E^2 - B^2),$$
$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x\, \frac{1}{4}\gamma_m m\bar{\psi}\psi.$$

The glue anomaly

The quark mass anomaly

The total energy

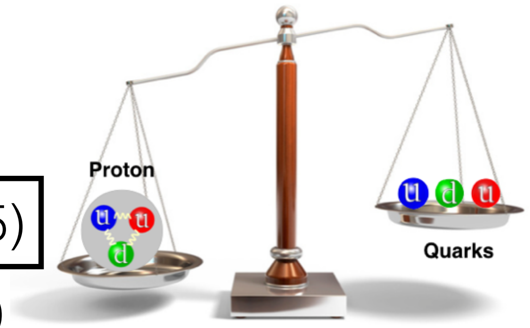
$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \bar{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(E^2 + B^2)$$

The glue field energy

Proton mass decomposition



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YBY, et.al. χQCD Collaboration, Phys. Rev. D 91, 074516 (2015)

Then we have

$$
\begin{aligned}
M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\
&= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\
\frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,
\end{aligned}
$$

in the rest frame.

With

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

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Gauge Invariant and scale independent combinations.

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The glue anomaly

The quark mass anomaly

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The quark energy

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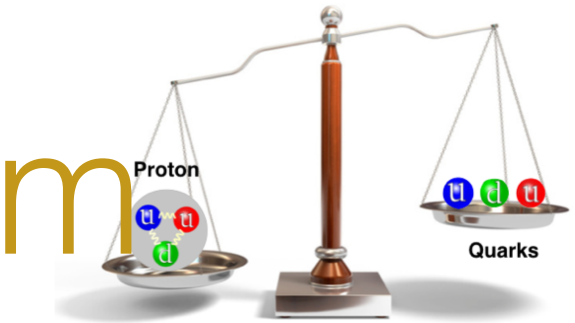
The glue field energy

Proton mass decomposition

The quark mass term



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$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

in the rest frame.

$$H_m = \sum_{u,d,s\cdots}\int d^3x\, m\bar{\psi}\psi,$$

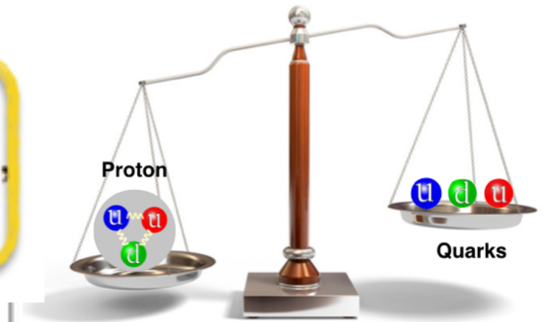
The quark mass

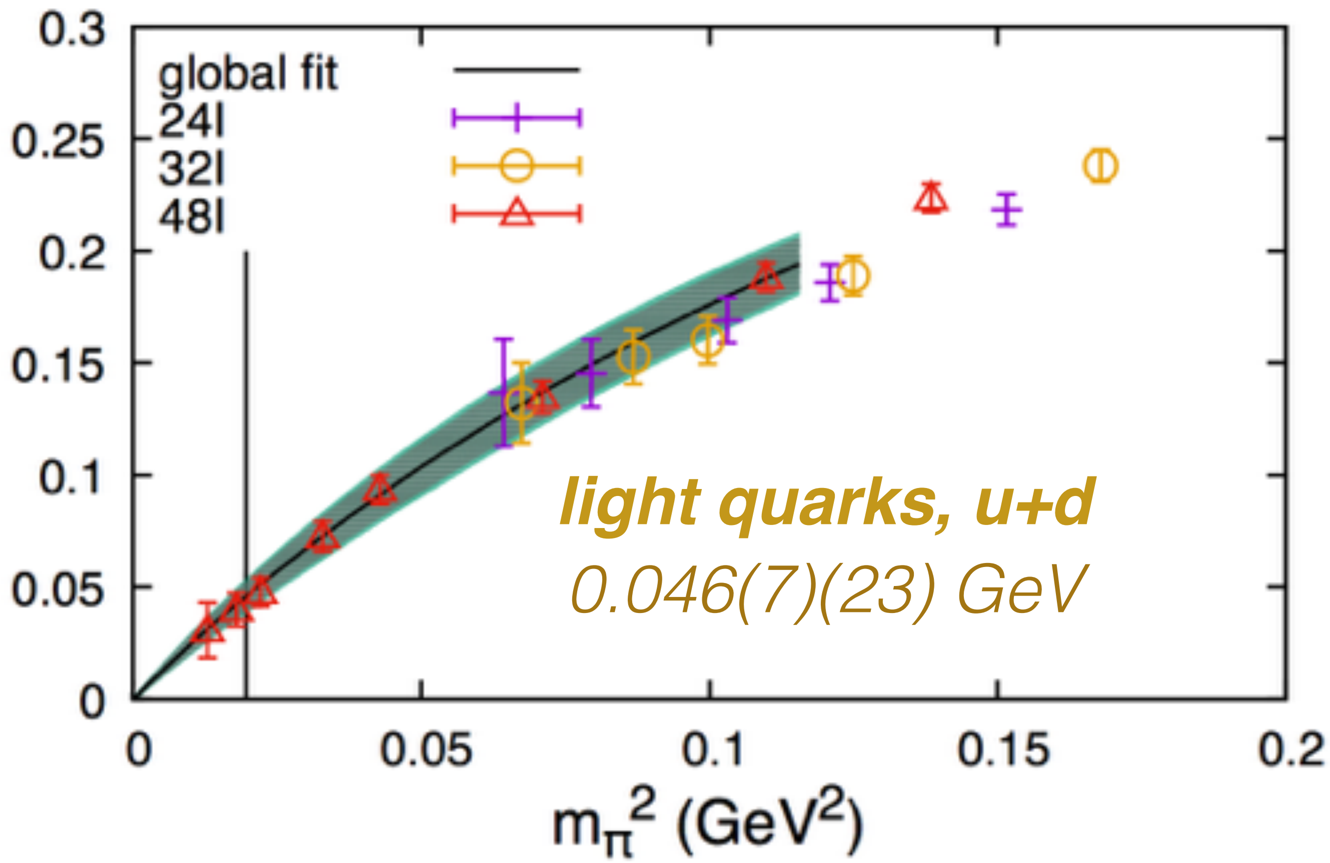
- Renormalization scheme/scale independent in continuum; also in discrete case when the chiral fermion is used.
- The term where the Higgs boson contributes.
- **Highly desired by the WIMP dark matter search.**
- Can be calculated directly in the lattice simulation while suffers from **the additive renormalization effect** for most of the lattice action**.**

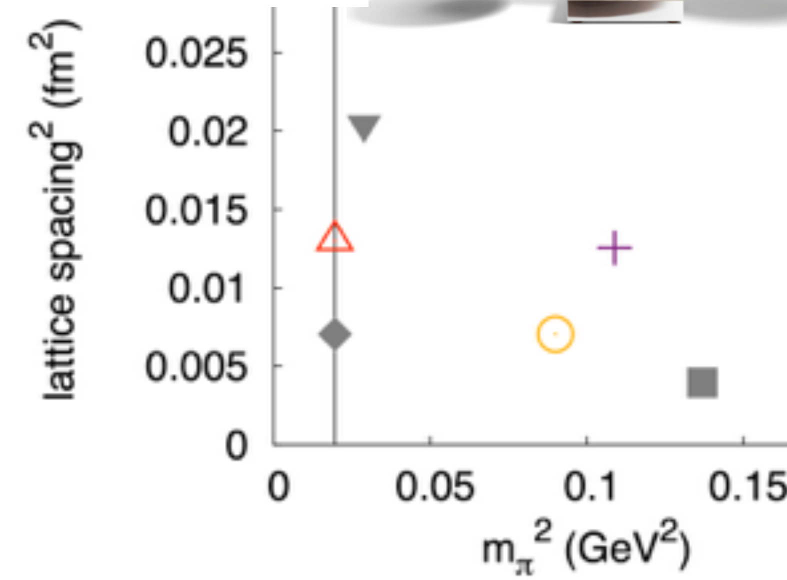
Proton mass decomposition

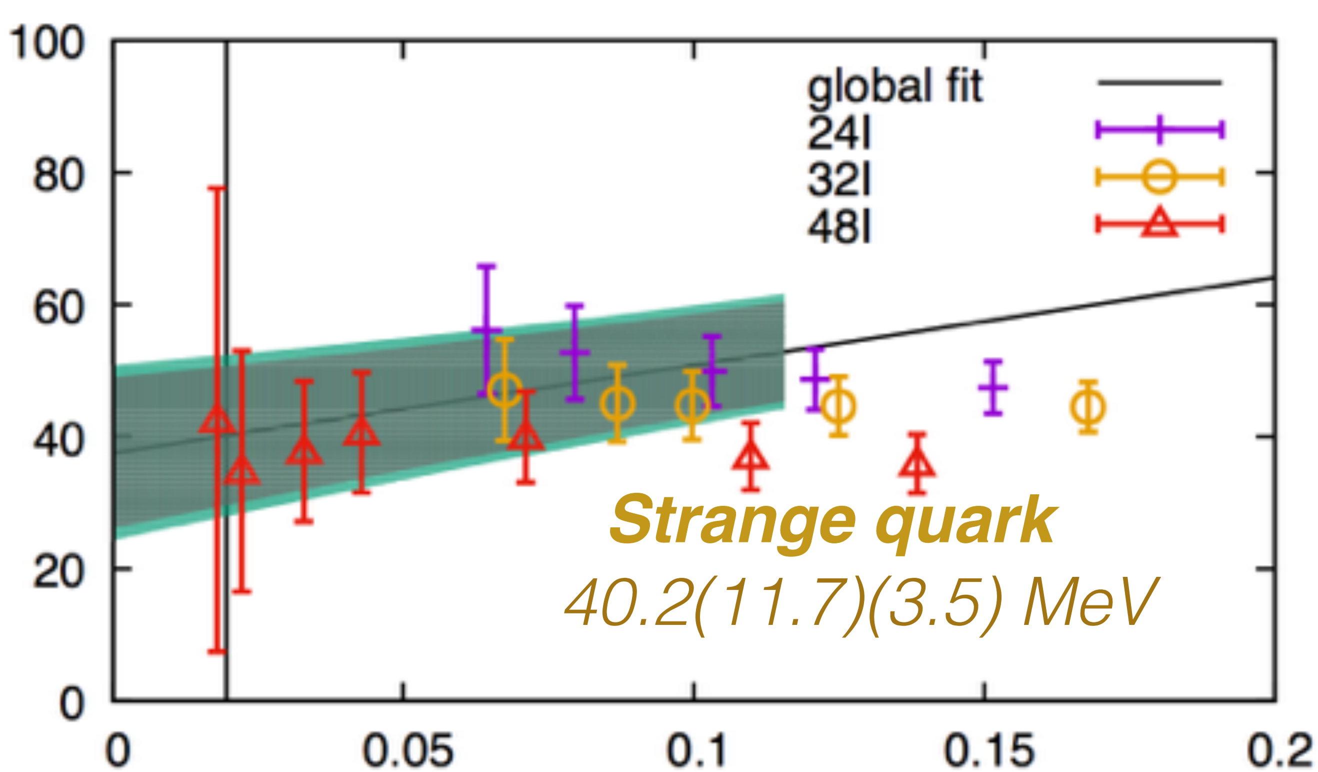
The quark mass term

$$\sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$









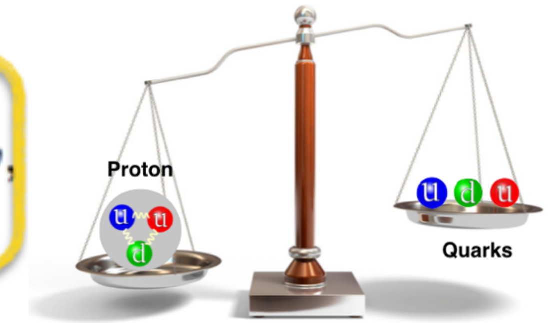
- The chiral fermion action for the valence quark (no additive renormalization);
- The joint fit with multiple valence quark masses on all the three ensembles to control the systematic uncertainties

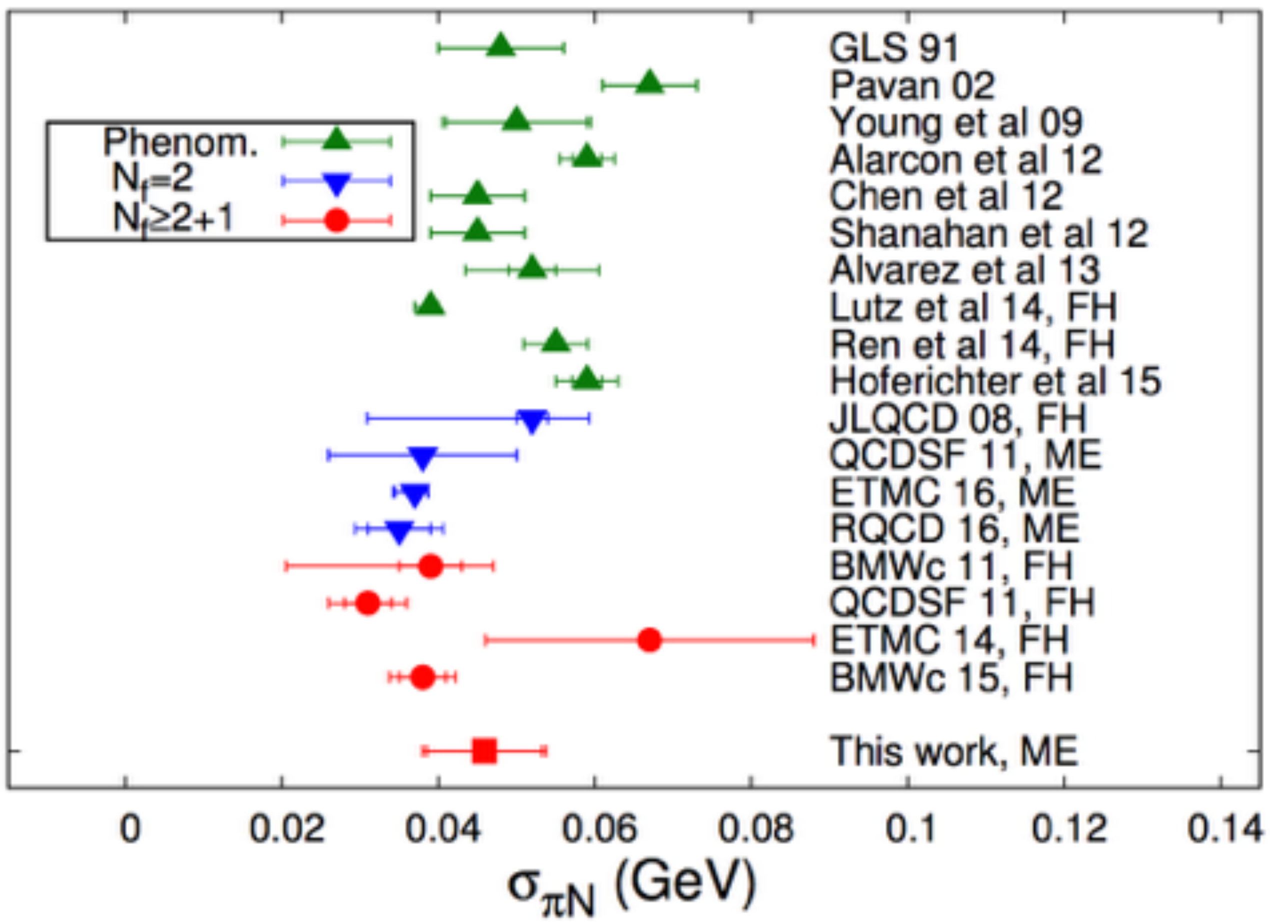
YBY, et.al. χQCD Collaboration, Phys. Rev. D 94, 054503 (2016)

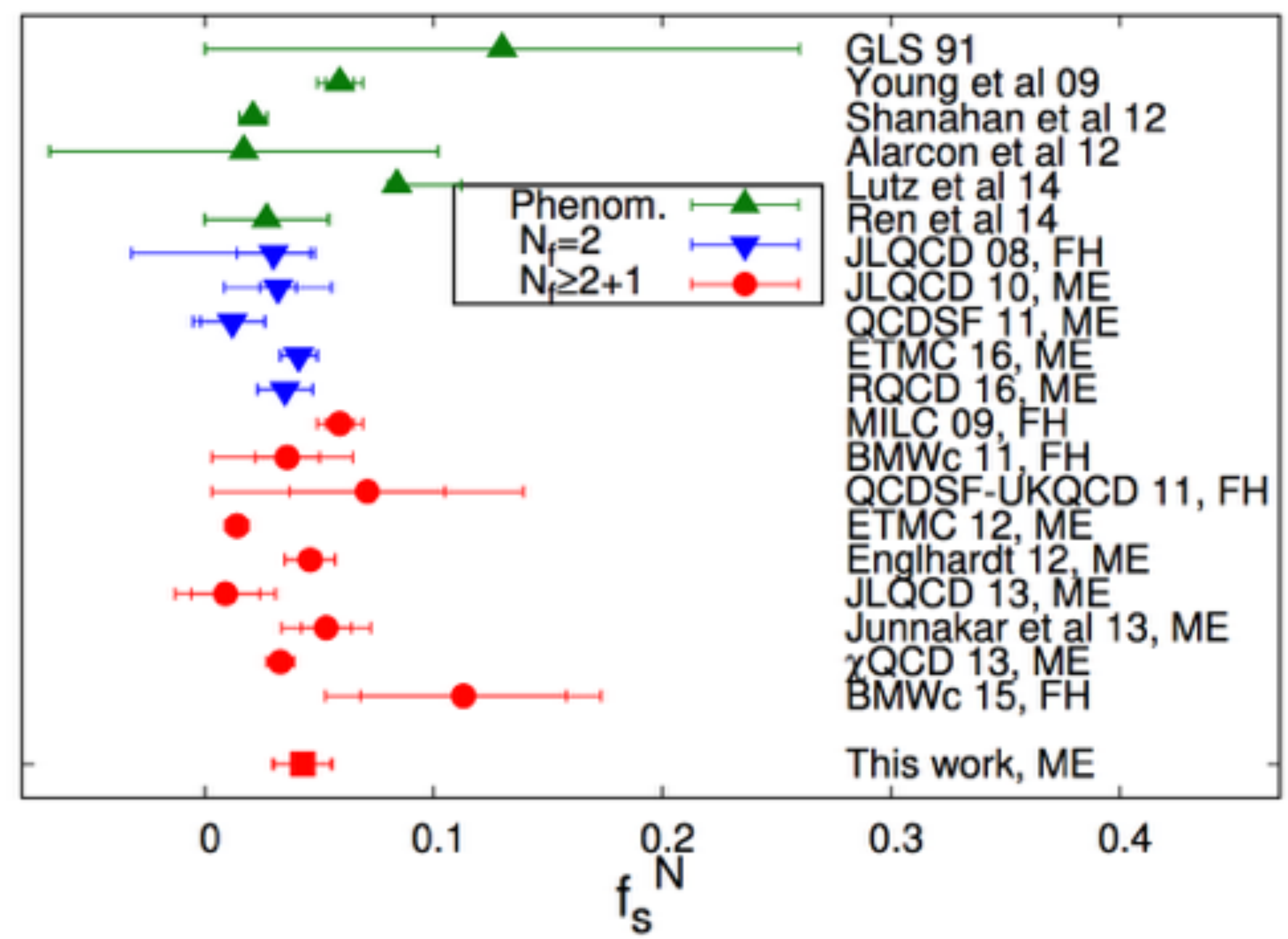
Proton mass decomposition

The quark mass term

$$\sum_{u,d,s\cdots} \int d^3x \, m\bar{\psi}\psi,$$







$\sigma_{\pi N} = \langle H_m(u)+H_m(d)\rangle = 45.9(7.4)(2.8)$ MeV $\quad f_s^N M_N = \langle H_m(s)\rangle = 40.2(11.7)(3.5)$ MeV

YBY, et.al. χQCD Collaboration, Phys. Rev. D 94, 054503 (2016)

with the quark masses:

$m_{ud}^{MS}(2\text{GeV}) = 3.41(5)$ MeV, $\quad m_s^{MS}(2\text{GeV}) = 94.4(1.1)$ MeV.

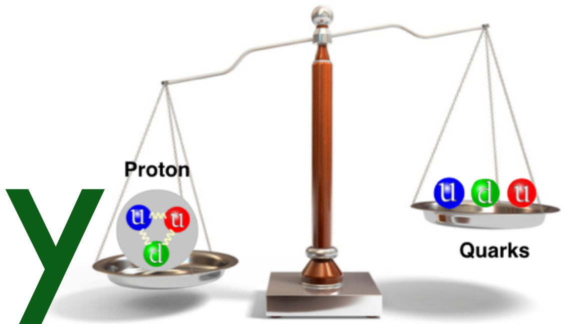
Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

The best result without the systematic uncertainty from the explicit breaking:

$$\langle H_m(u,d,s)\rangle / M_N = 9(2)\%$$

Proton mass decomposition

The QCD anomaly



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The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

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$$H_g^a = \int d^3x \, \frac{-\beta(g)}{4g}(E^2 + B^2),$$

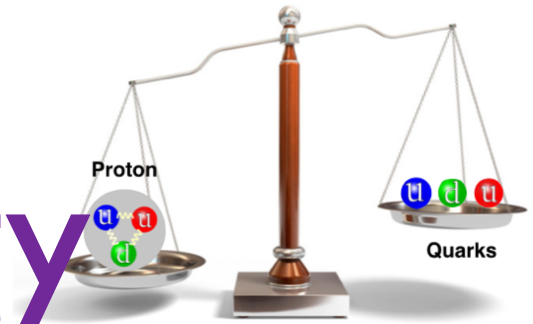
$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x \, \frac{1}{4}\gamma_m m \bar{\psi}\psi.$$

The quark mass anomaly

- **The joint contribution of the QCD anomaly can be deduced from the quark mass term, with the sum rule above.**
- The total QCD anomaly is renormalization scheme/scale independent.
- H_a / M_N **= 23(1)%**

Proton mass decomposition

The quark/gluon energy



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$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$
$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

- The quark/glue energy can be deduced from the momentum fraction,

$$\langle H_E\rangle = \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle$$
$$\langle H_q\rangle = \frac{3}{4}\langle x\rangle_q M + \frac{1}{4}\langle H_m\rangle$$
$$\langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M.$$

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.

- **It is more straightforward to obtain the quark/glue momentum fraction first, and convert it to the quark/glue energy.**

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

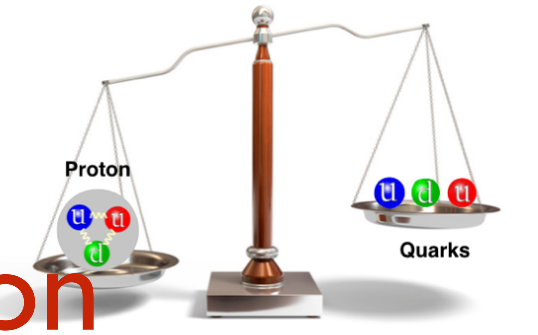
The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

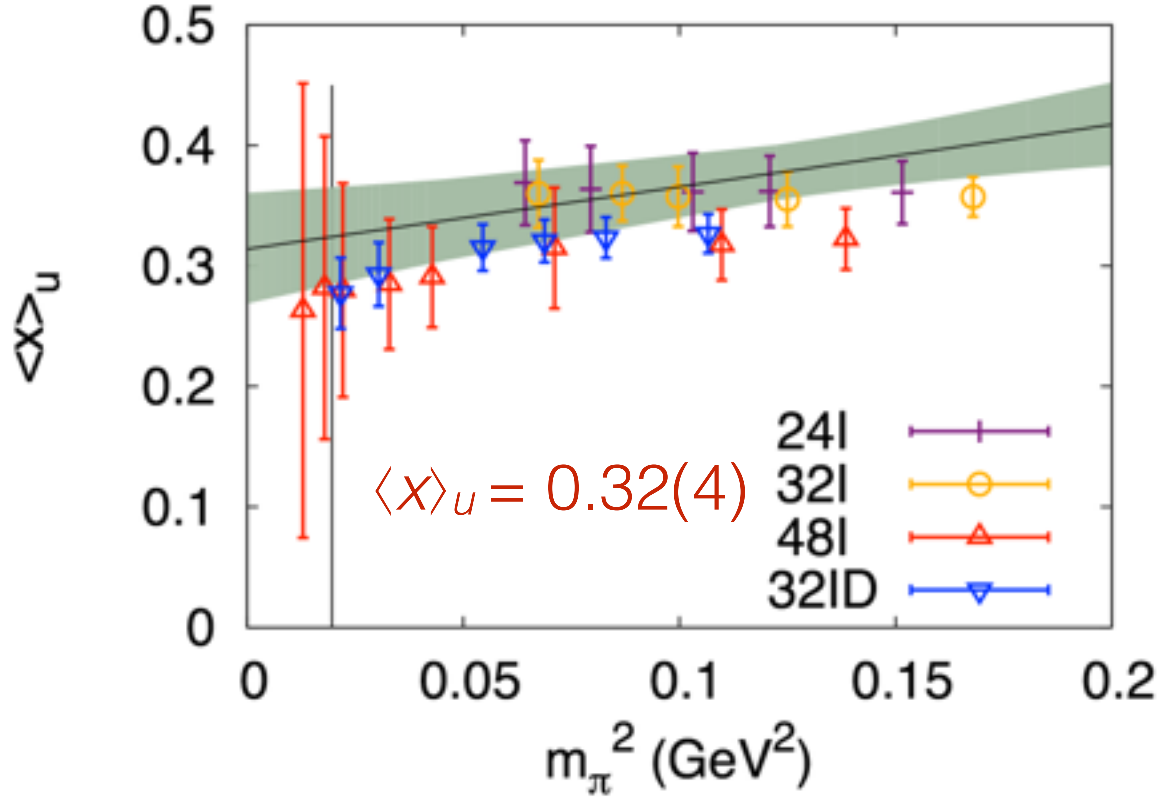
The glue field energy

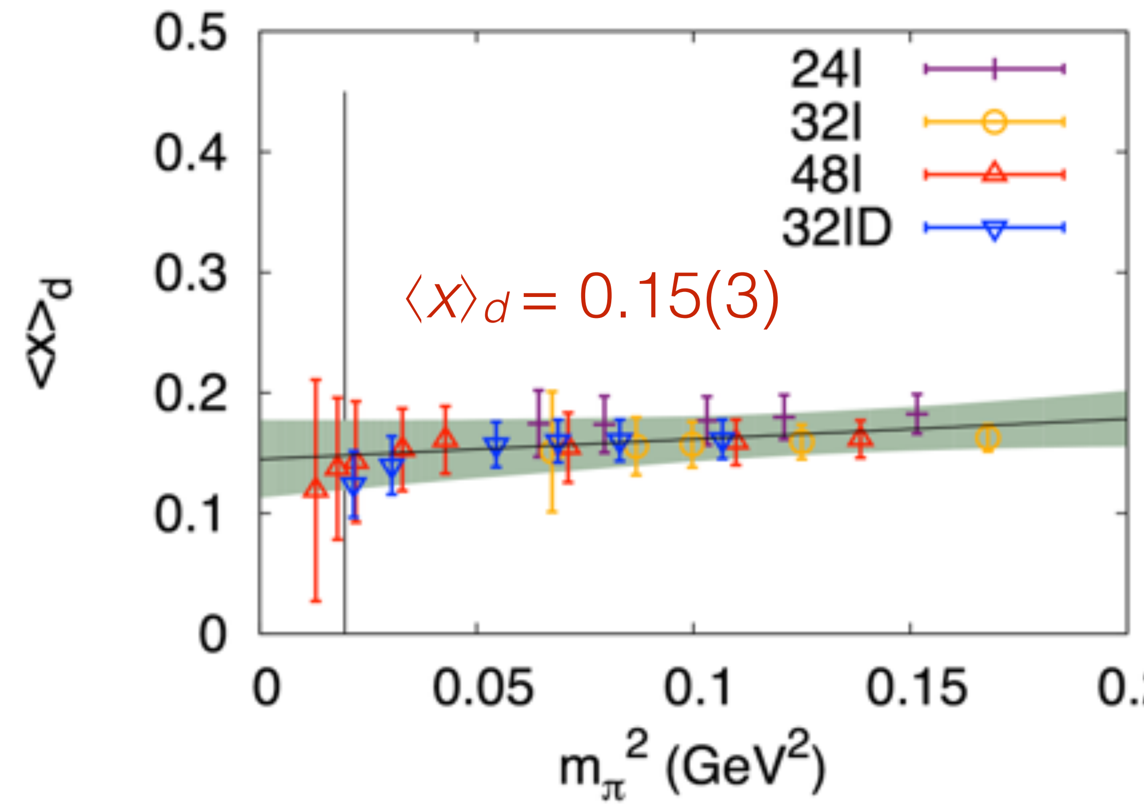
Proton mass decomposition

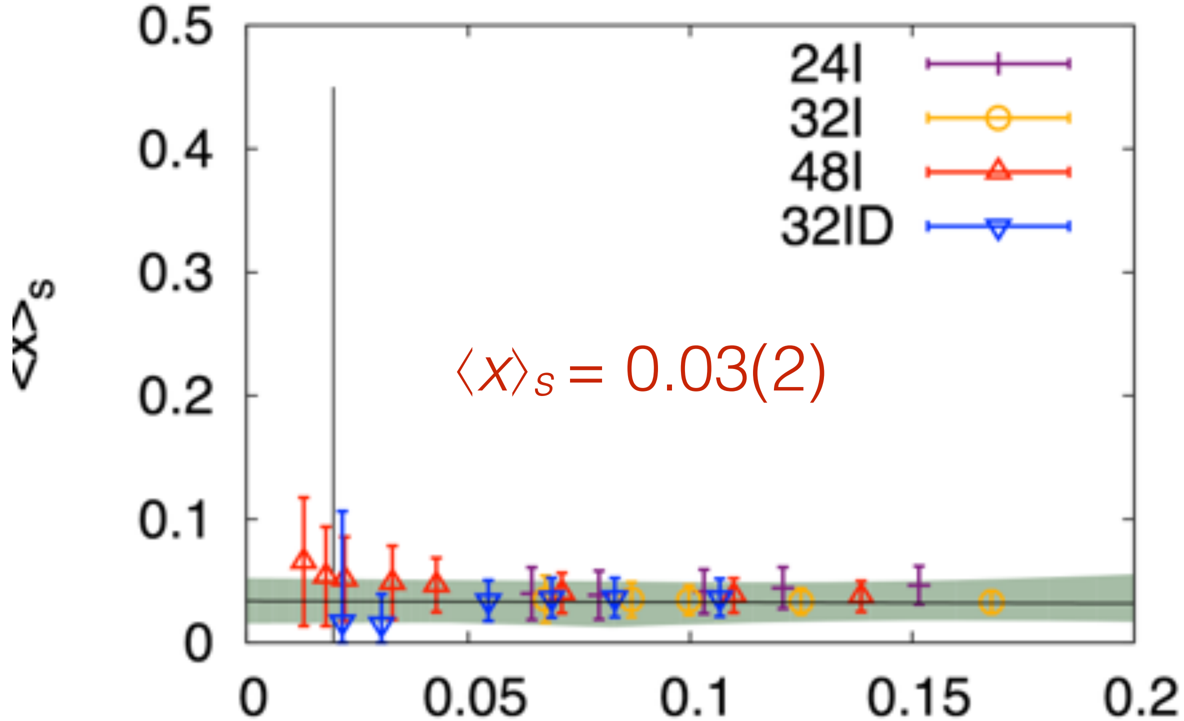
YBY, K. Liu, Y. Chen et al, χQCD Collaboration, in preparation



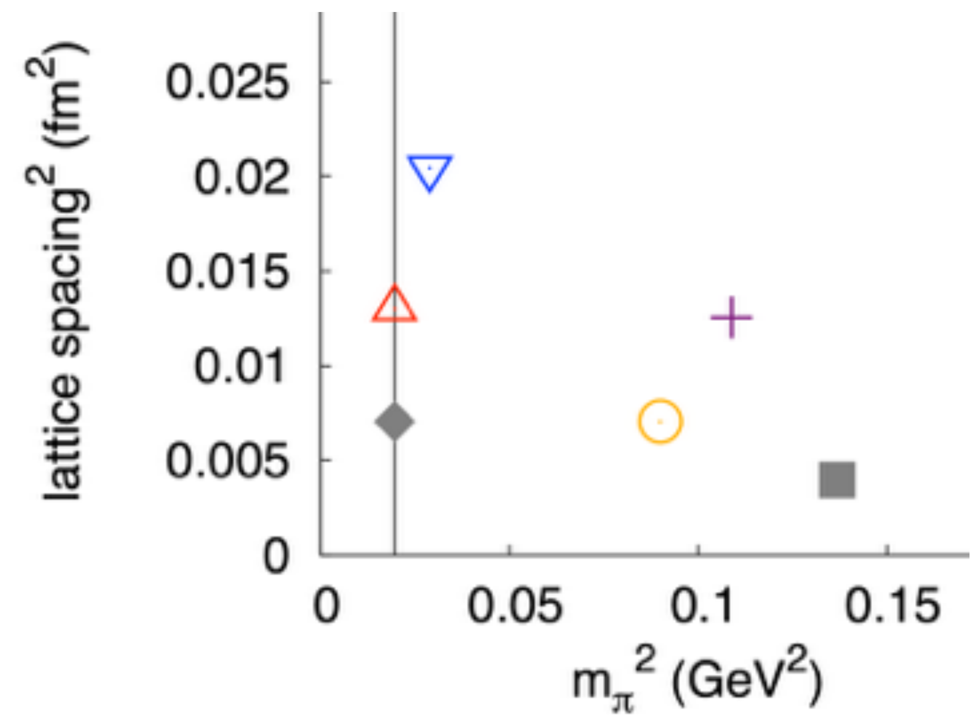
Different flavors of the Quark Momentum fraction







Lattice bare results running to 2GeV



Proton mass decomposition

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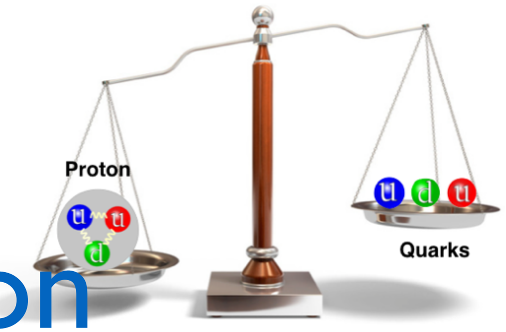
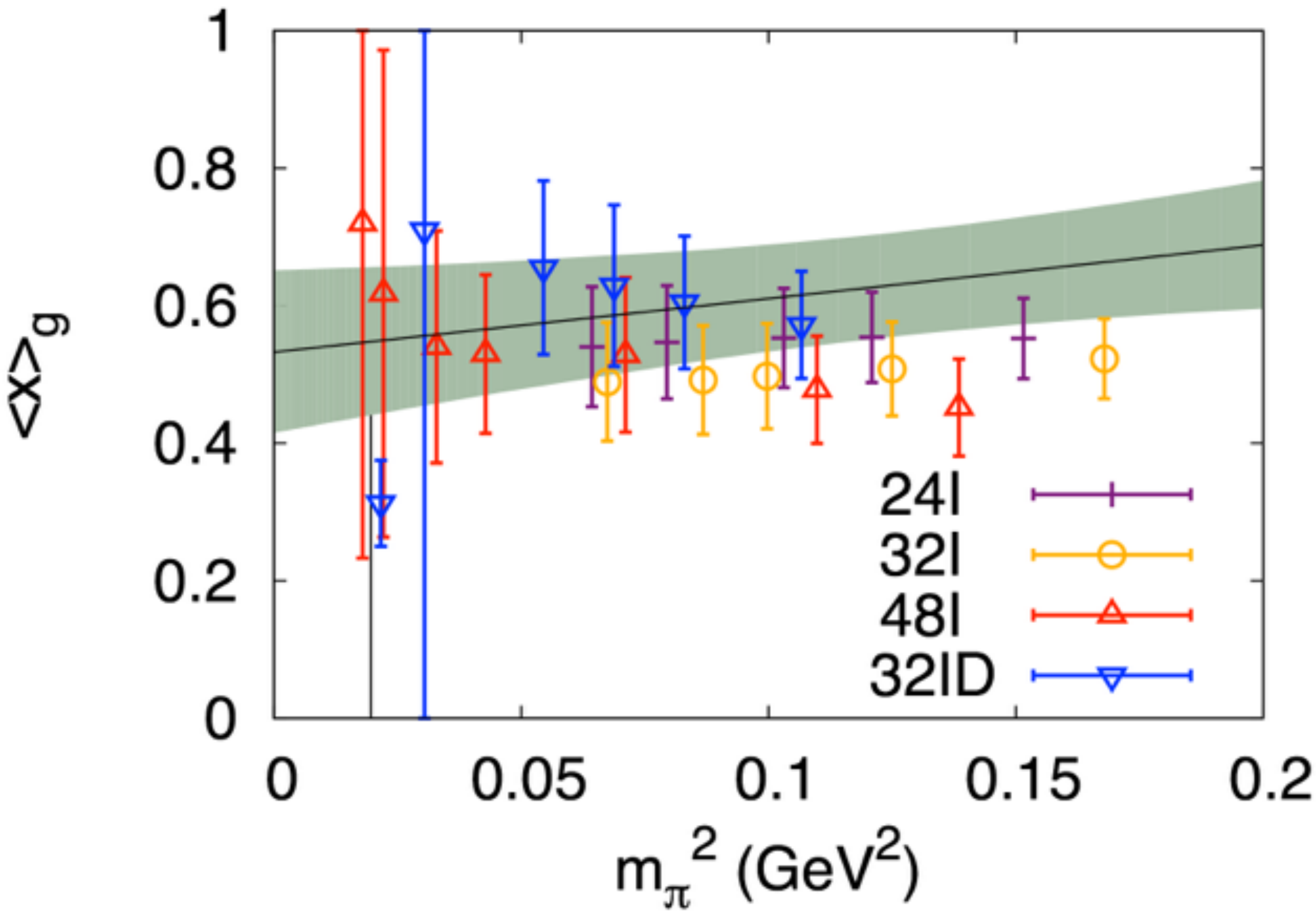
χQCD Collaboration,
in preparation

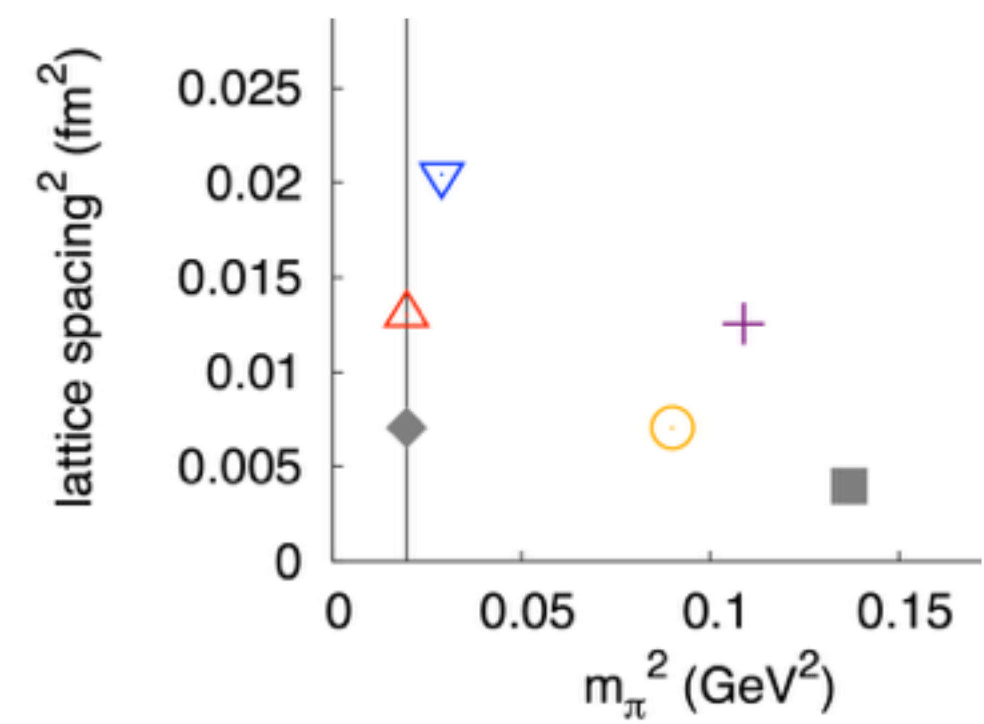


Gluon momentum fraction

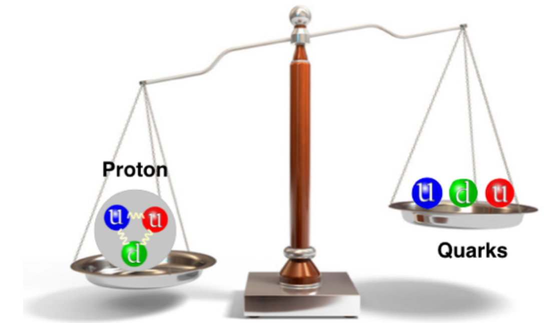


Lattice bare results
running to 2GeV,

$\langle x \rangle_g = 0.54(11)$



Proton mass decomposition



Renormalization

of the momentum fractions

From the lattice bare quantities with the chiral fermion and HYP smeared Iwasaki gluon to that under the MS-bar scheme, at a scale μ=1/a,

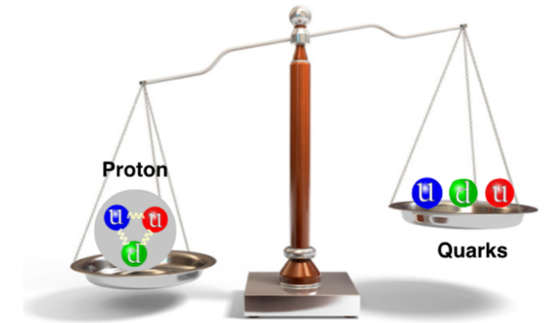
$$\begin{pmatrix} \overline{T}_Q^{\overline{\text{MS}}} \\ \overline{T}_G^{\overline{\text{MS}}} \end{pmatrix} = \begin{pmatrix} 1.0202 & 0.0123 N_f \\ 0.1565 & 2.08(25) - 0.0239 N_f \end{pmatrix} \begin{pmatrix} \overline{T}_Q^{lat} \\ \overline{T}_G^{lat} \end{pmatrix} + O(g^4),$$

YBY, et.al. [χQCD], arXiv: 1612.02855

- With the joint fit, $\langle x \rangle_q$ = **50(7)%** at MS-bar 2GeV.
- For the gluon operator renormalization at 1-loop level, the value and the uncertainty (from the estimate of the 4-gluon vertex tadpole contribution) are large and then indicate the convergence problem.
- The bare value of $\langle x \rangle_g$ is **54(11)%** and that deduced from the momentum fraction sum rule is $\langle x \rangle_g$ = **50(7)%**.

Proton mass decomposition

Comparing the momentum fractions



from the experiment

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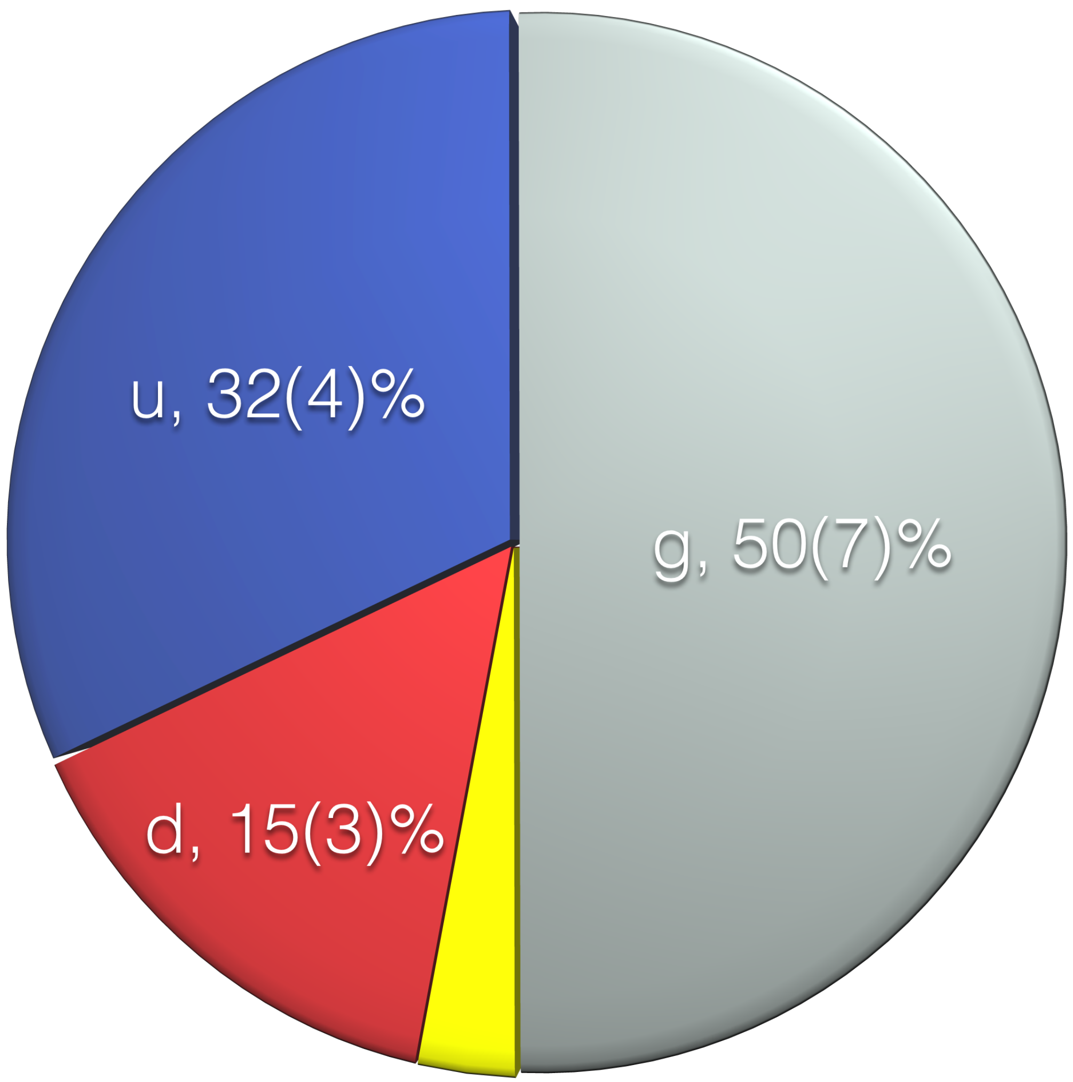
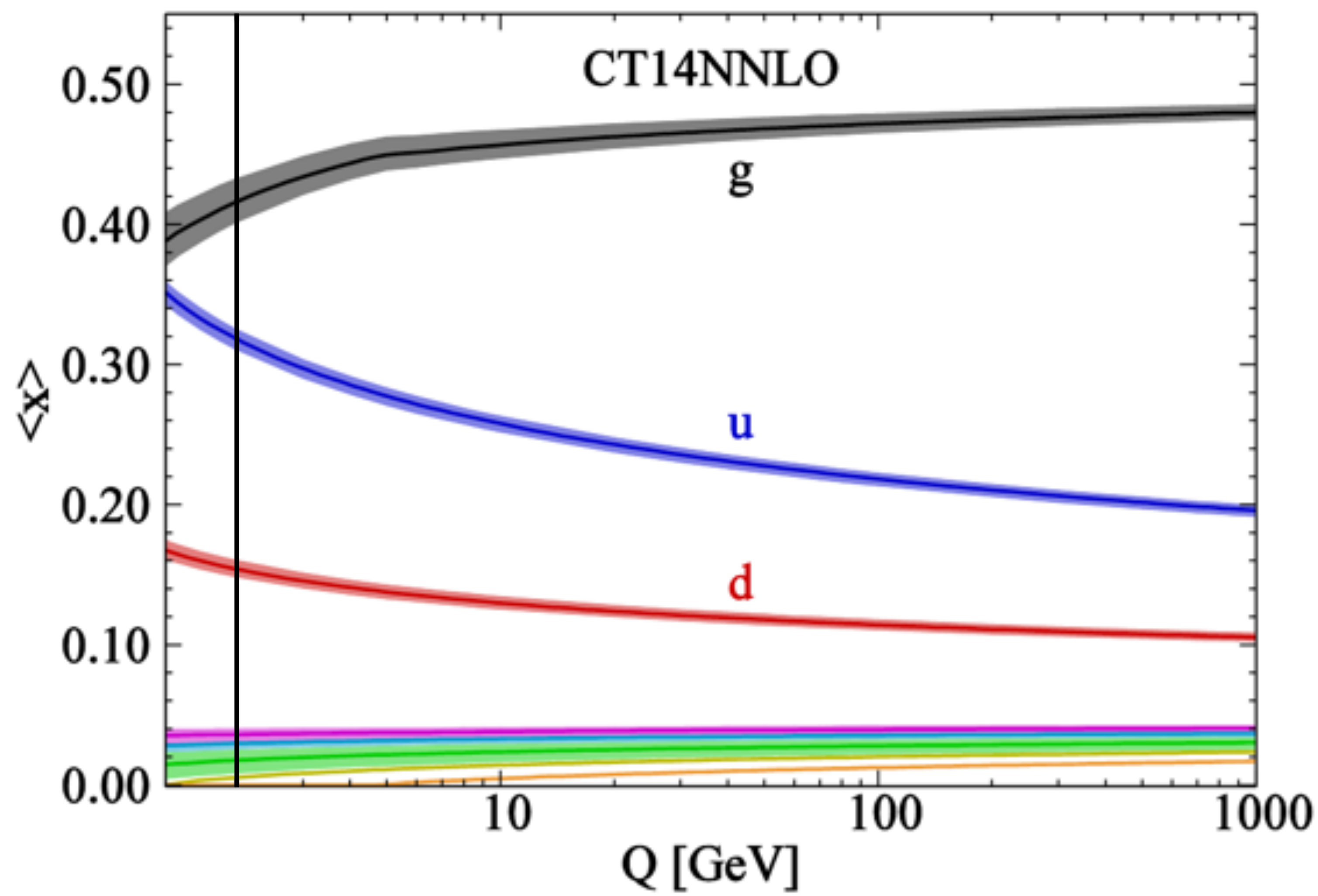
in preparation

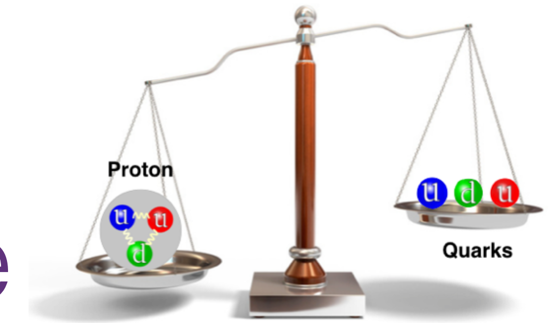




S. Dulat et al, Phys. Rev. D 93 (2016), 033006

Proton mass decomposition

by type

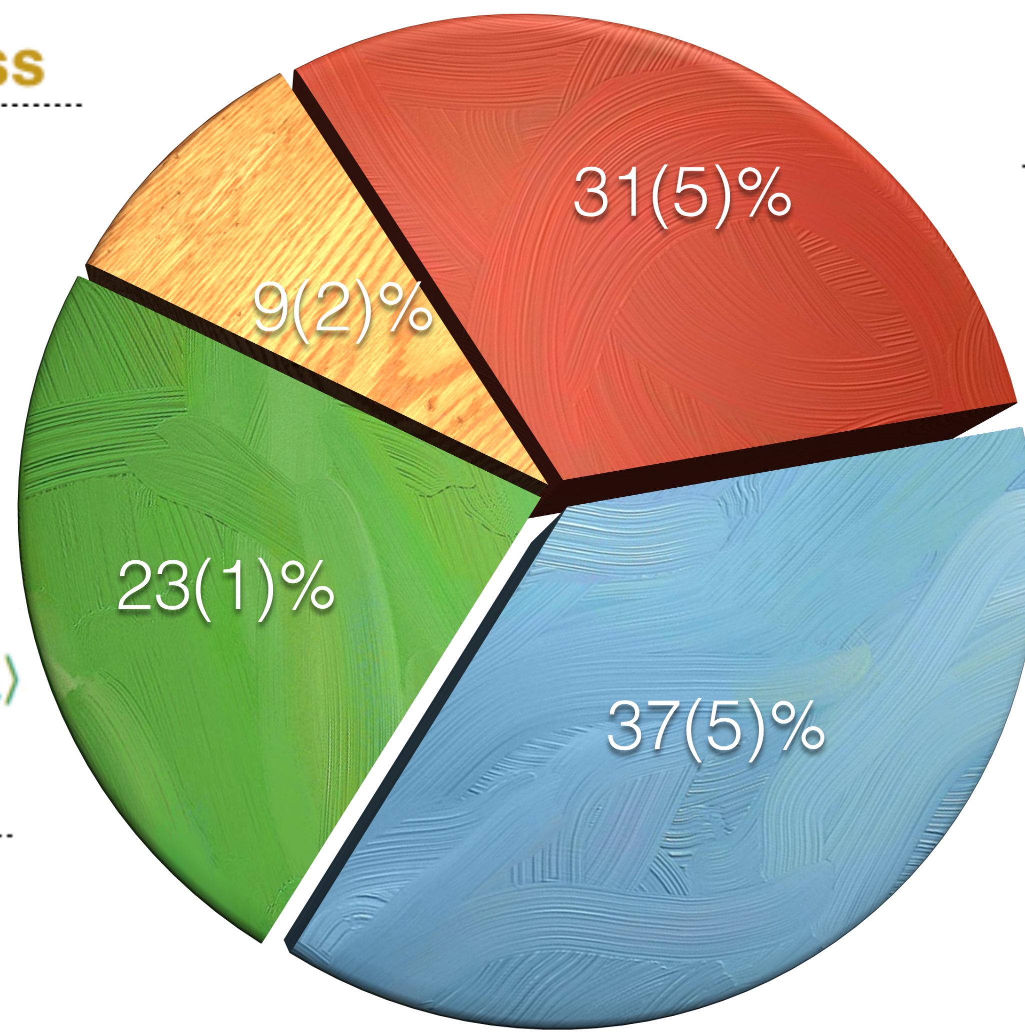


$\langle H_m \rangle / M_N =$ 9(2)%

quark mass

$\langle H_E \rangle = \frac{3}{4}\langle x \rangle_q M - \frac{3}{4}\langle H_m \rangle$

quark energy



$\langle x \rangle_q =$ 50(7)% and

$\langle x \rangle_g =$ 50(7)%

$\frac{1}{4}M = -\langle \hat{T}_{44} \rangle = \frac{1}{4}\langle H_m \rangle + \langle H_a \rangle$

QCD anomaly

$\langle H_g \rangle = \frac{3}{4}\langle x \rangle_g M$

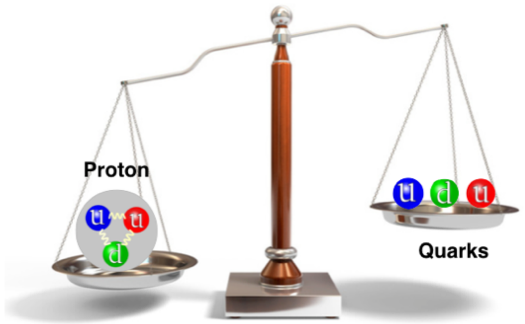
glue energy

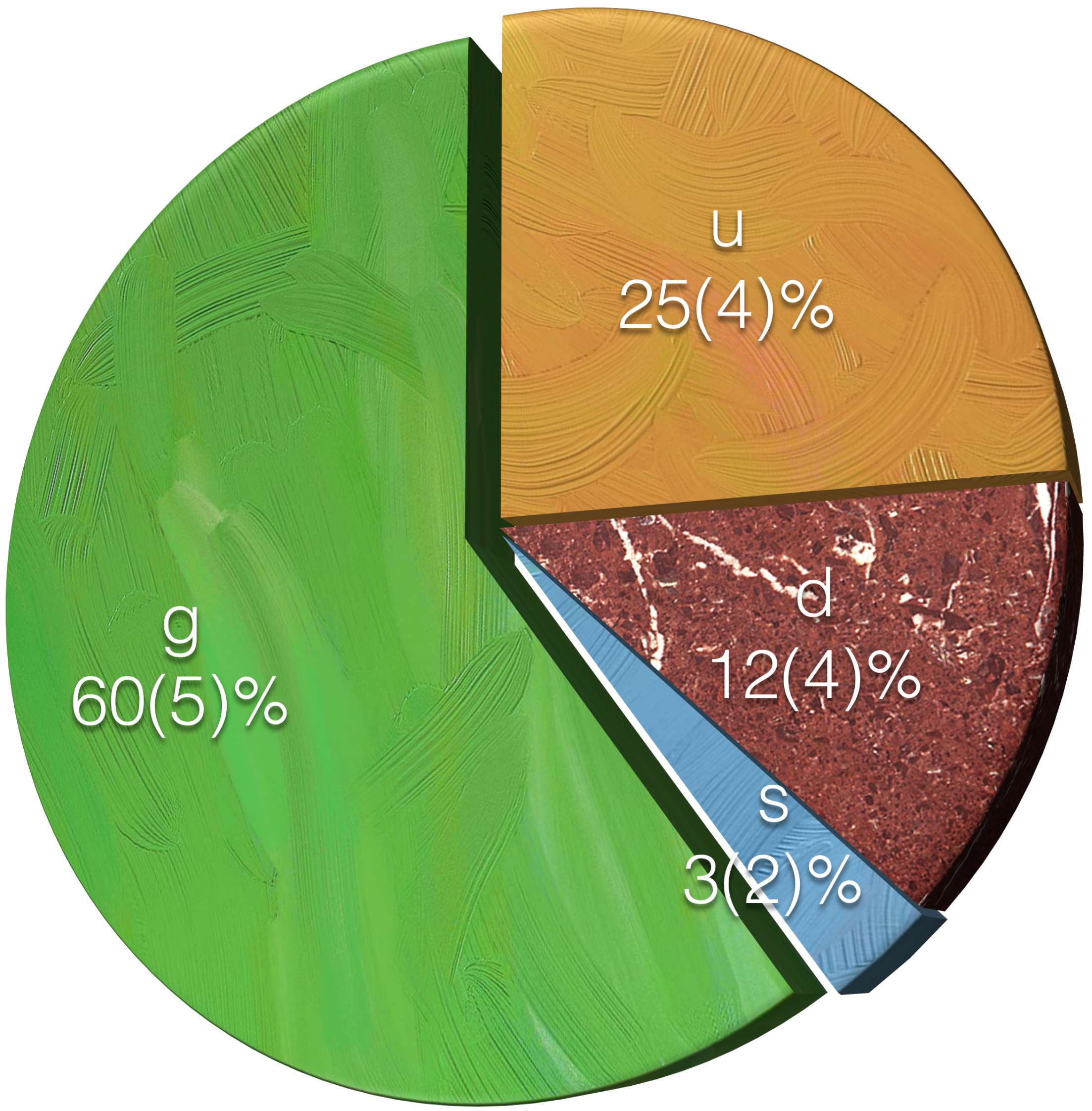
- *Renormalized momentum fraction at MS-bar 2GeV.*
- *QCD anomaly and gluon energy are deduced by the sum rule.*
- *The contribution from heavy quarks ignored since the simulation is based on 2+1 flavor ensembles.*

YBY, Y. Chen, K. Liu et al, χQCD Collaboration, in preparation

Proton mass decomposition

by u/d/s flavors+glue





- *Glue part: glue energy + QCD anomaly.*

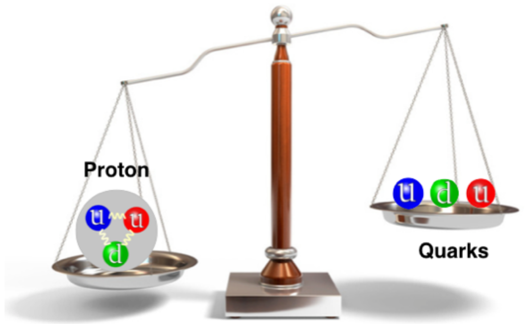
- *Quark part: quark mass term +quark energy term*

- *Renormalized momentum fraction at MS-bar 2GeV.*
- *QCD anomaly and gluon energy are deduced by the sum rule.*
- *The contribution from heavy quarks ignored since the simulation is based on 2+1 flavor ensembles.*

YBY, Y. Chen, K. Liu et al, χQCD Collaboration, in preparation

Proton mass decomposition

heavy quark contributions?

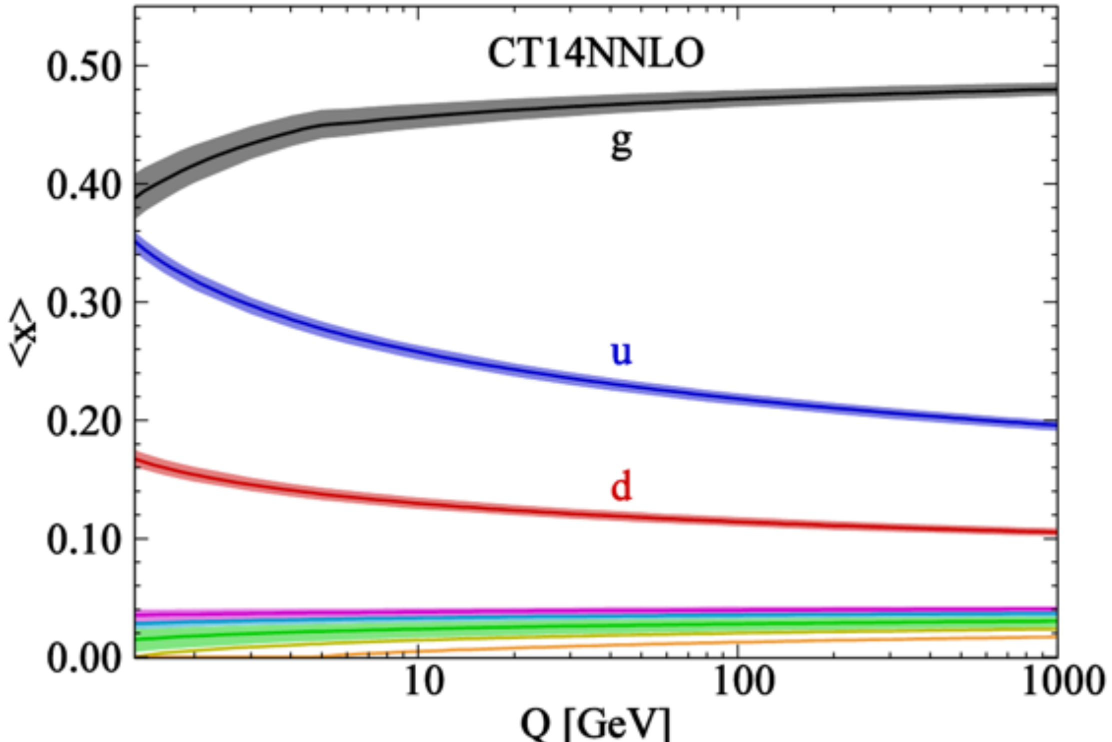


- *The contribution from heavy quarks ignored since the simulation is based on 2+1 flavor ensembles.*

- *The heavy quark contribution to the mass term is flavor independent and directly related to the QCD anomaly:*

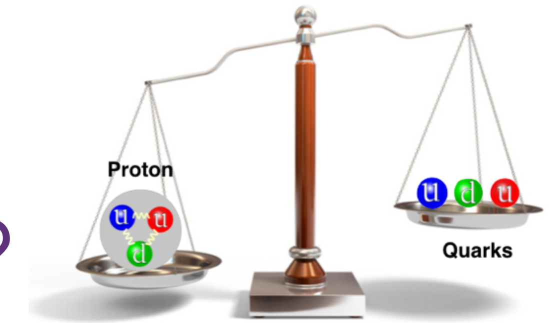
$$\sigma_Q \equiv m_Q \langle N|\bar{Q}Q|N\rangle \rightarrow -\frac{\alpha_s}{12\pi}\langle N|GG|N\rangle.$$

- *The intrinsic heavy quark momentum fraction at 2GeV is very small.*

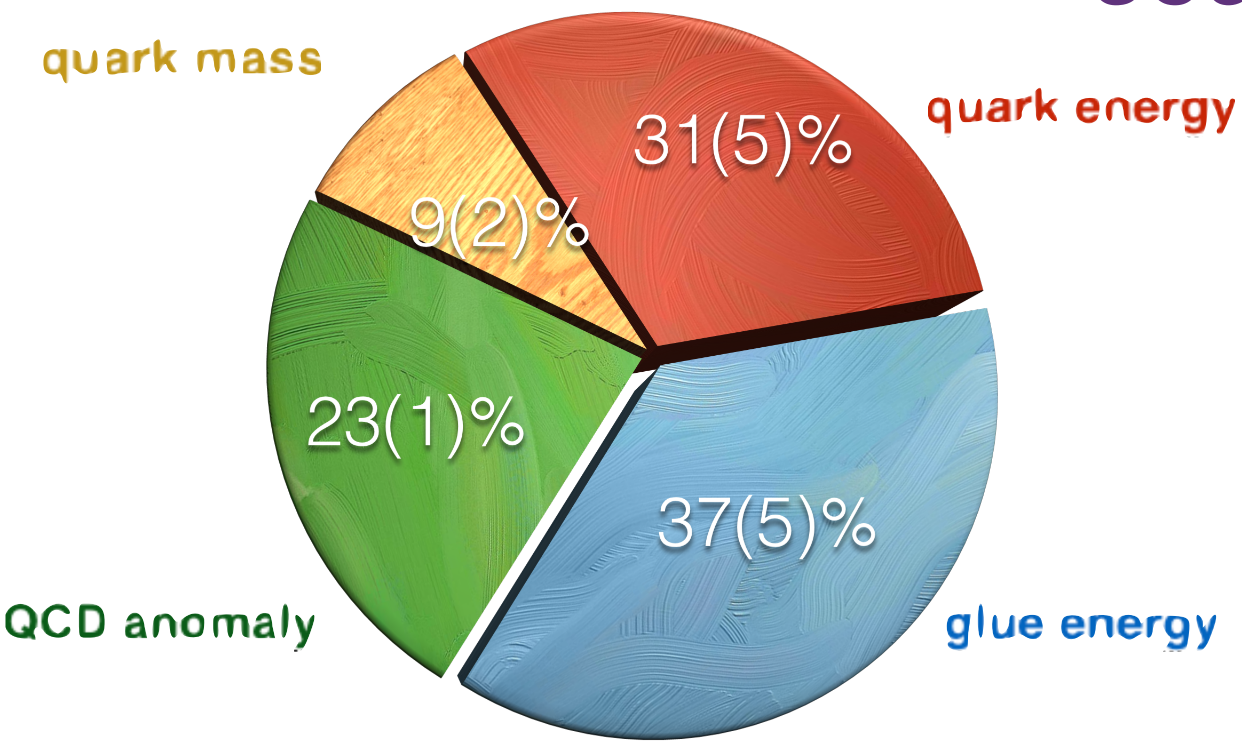


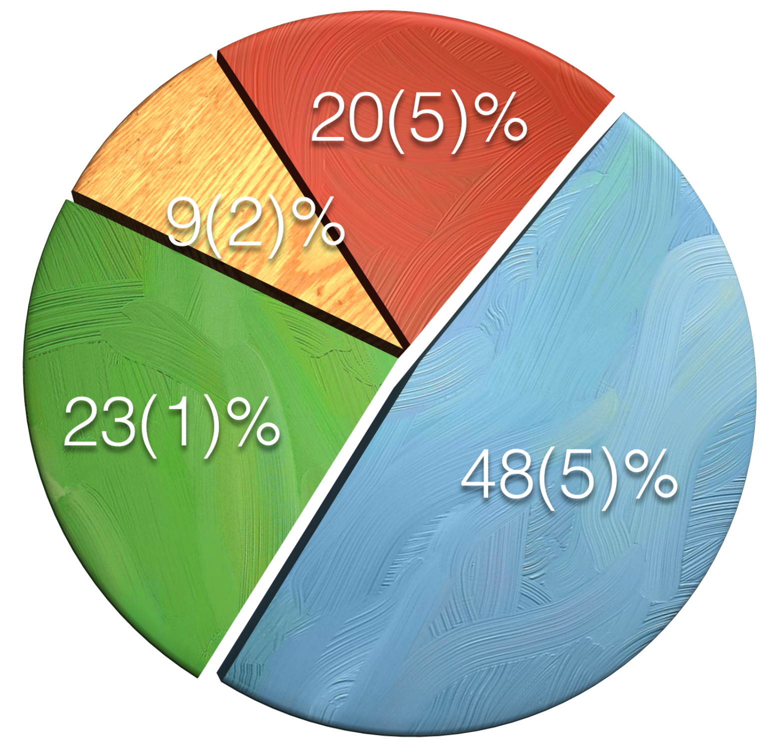
S. Dulat et al, Phys. Rev. D 93 (2016), 033006

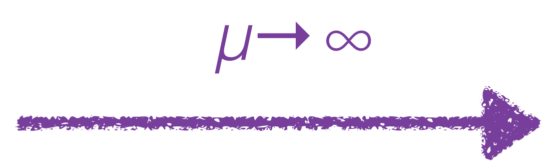
Proton mass decomposition

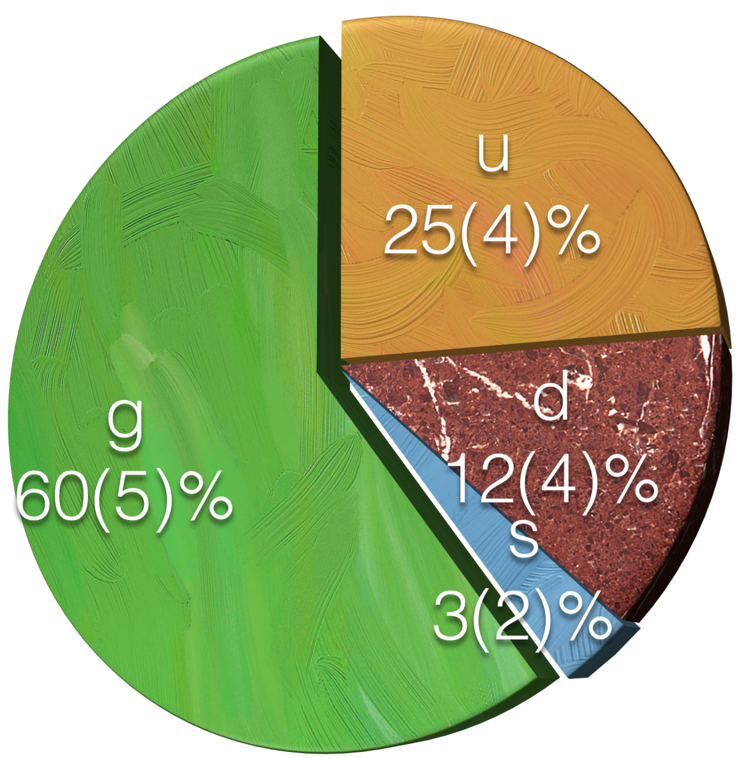


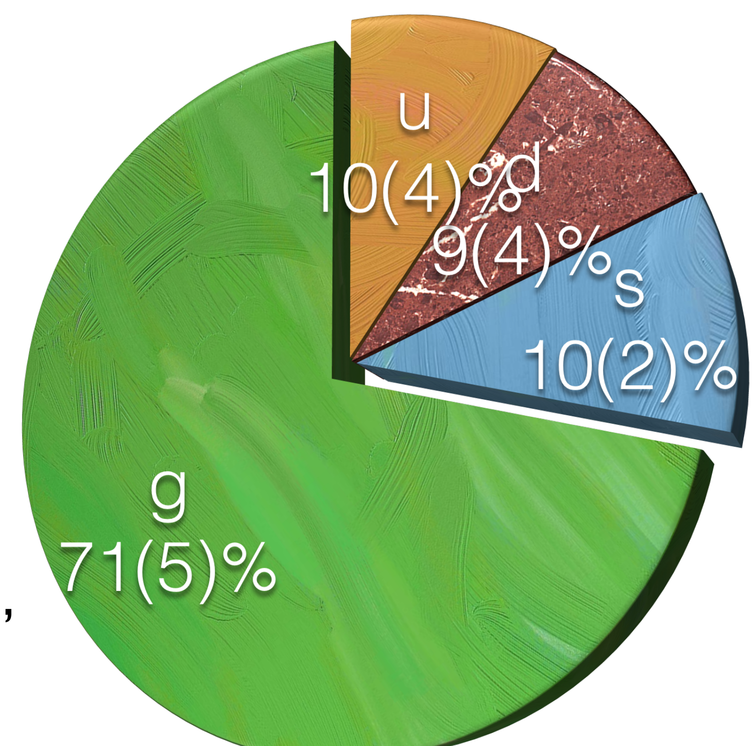
scale dependence?





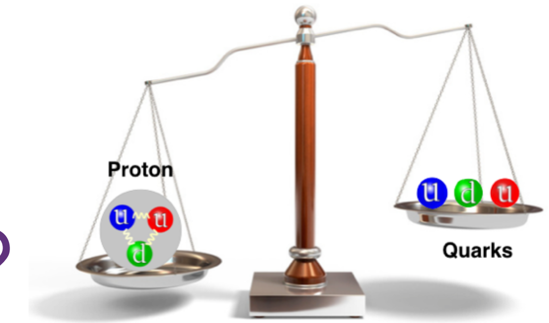




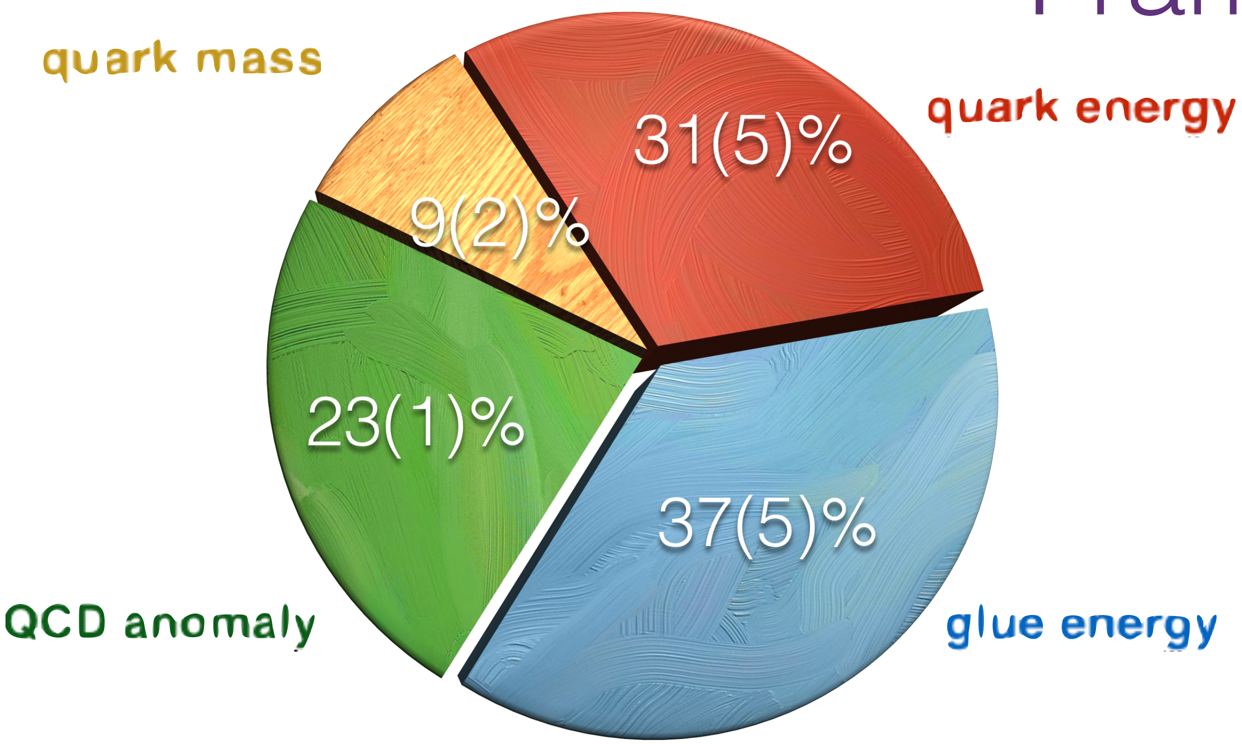


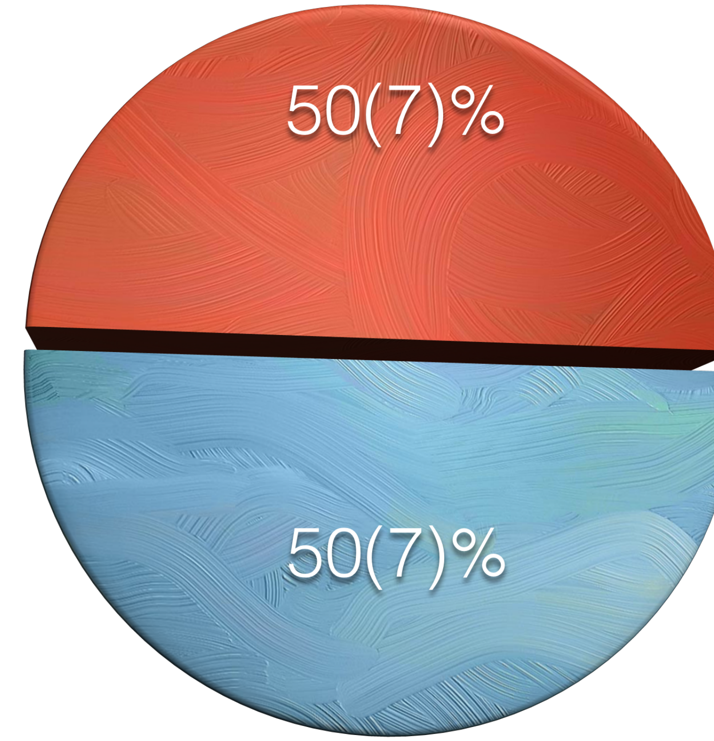
YBY, Y. Chen, K. Liu et al,
χQCD Collaboration,
in preparation

Proton *energy* decomposition

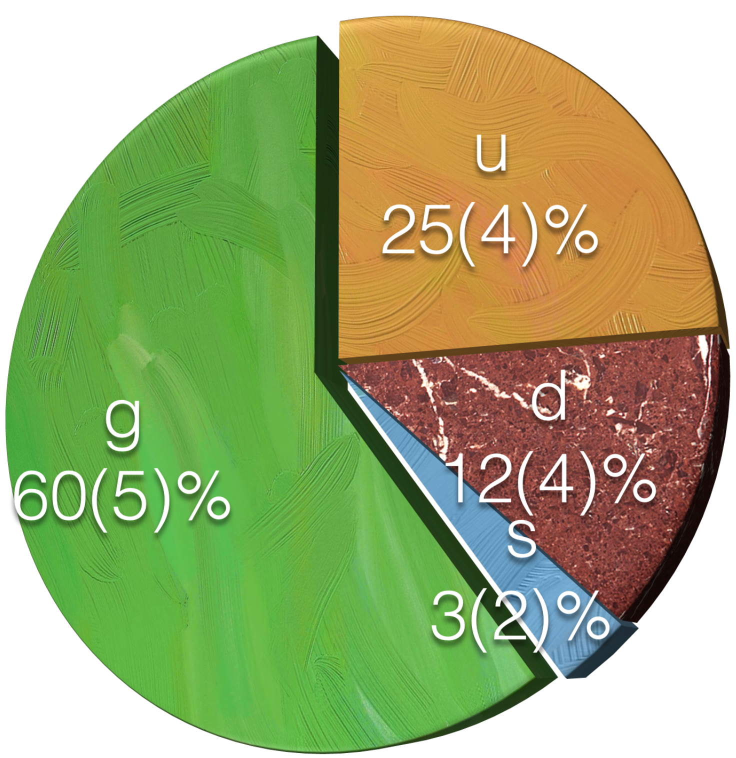


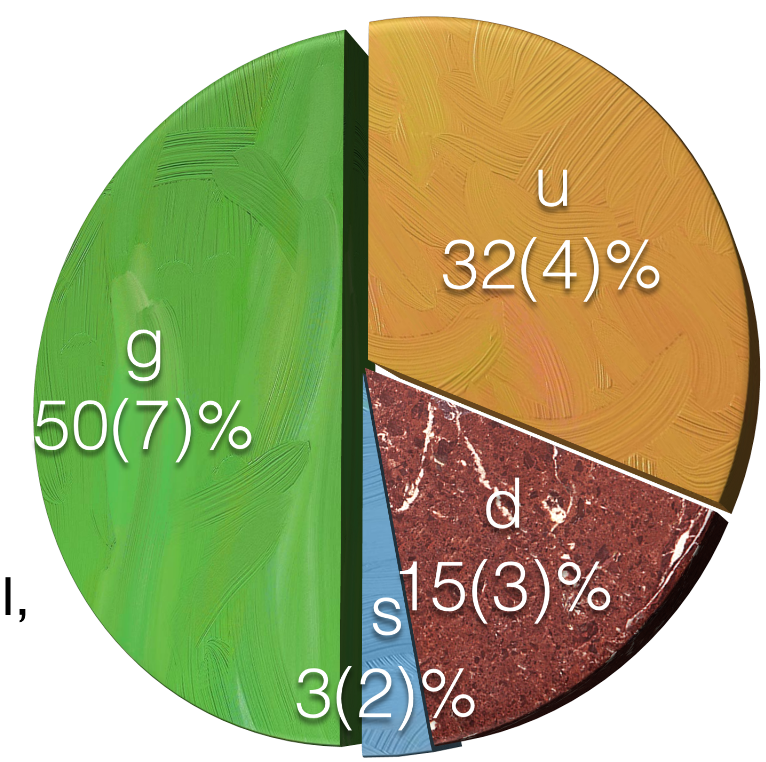
Frame dependence?





$P \to \infty$





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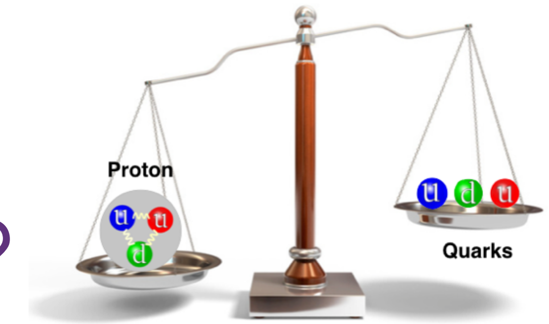
Proton mass decomposition

Another approach?



$$M^2 = M\langle T^\mu_\mu\rangle = M\langle H_m\rangle + 4M\langle H_a\rangle,$$

With

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

The quark mass

The QCD anomaly

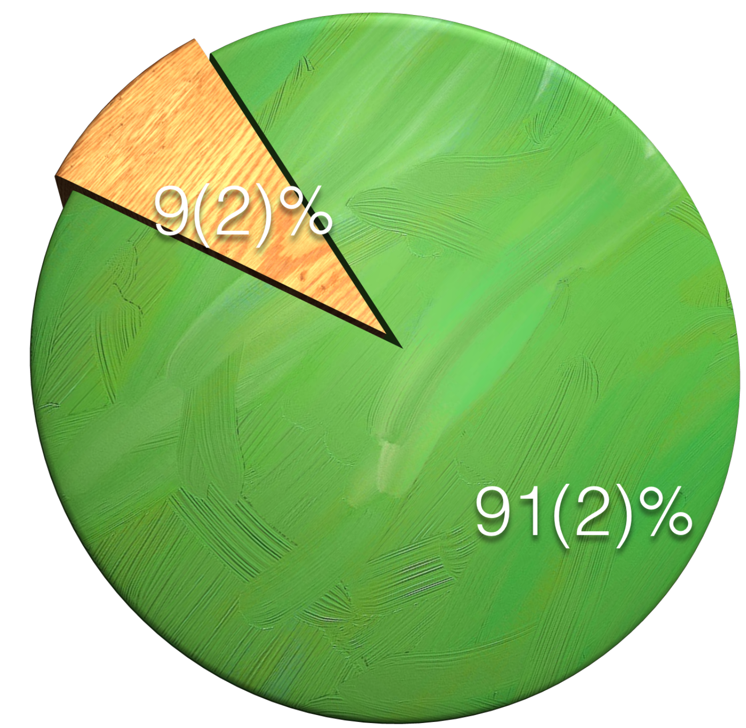
$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x\, \frac{\beta(g)}{4g}(E^2 - B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x\, \frac{1}{4}\gamma_m m\bar{\psi}\psi.$$

The quark mass anomaly



- Light quark mass contribution only.
- Scale and frame independent.
- No further decomposition can be done based on Lattice QCD.

Summary

- **The Lattice QCD is the unique tool to investigate QCD physics at the non-perturbative scale from the first principle theory.**

- **We decompose the proton mass into quark and gluon components in lattice simulation.**

1. **The joint u/d/s quark mass term contribute 9(2)%.**
2. **The joint quark/glue energy contributes 69(2)%.**
3. **The joint glue contributes half of the proton mass.**