

# Quark and spin content of the nucleon



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*2nd Workshop on The Proton Mass; At the Heart of Most Visible Matter*

# Outline

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## 1 Introduction and Motivation

- Current status of simulations
- Low-lying baryon masses
- Evaluation of matrix elements in lattice QCD

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## 2 Nucleon quark content

- Nucleon scalar charge
- The quark content of the nucleon ( $\sigma$ -terms)

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## 3 The nucleon spin decomposition

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## 4 Conclusions

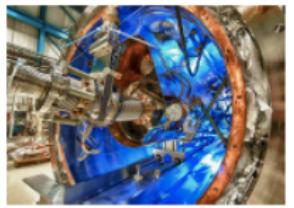
JLAB (12GeV Upgrade)



RHIC (BNL)



FERMILAB

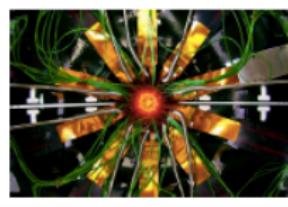


JPARC

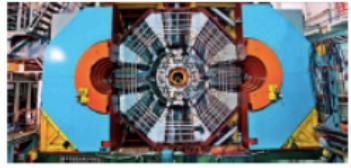


Rich experimental  
activities in  
major facilities

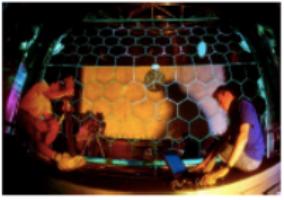
ALICE



BES III



COMPASS



PSI



MAMI



With simulations at the physical point lattice QCD can provide essential input for the experimental programs.

# Quantum ChromoDynamics (QCD)

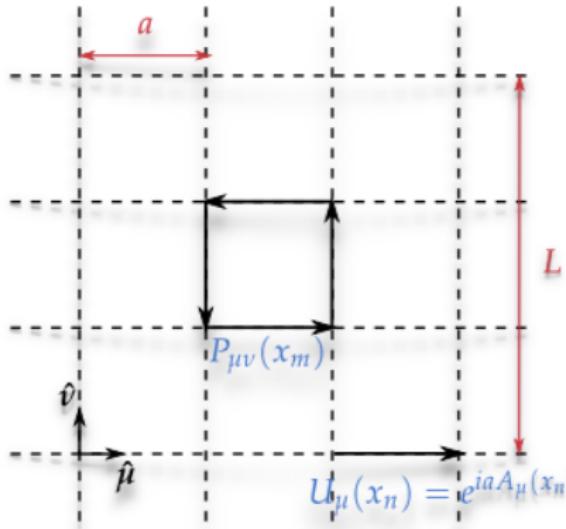
QCD-Gauge theory of the strong interaction

Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f$$

$$D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} A_\mu^a$$

Choice of fermion discretisation scheme e.g. Clover, Twisted Mass, Staggered, Domain Wall, Overlap  
Each has its advantages and disadvantages



Eventually,

- all discretization schemes must agree in the continuum limit  $a \rightarrow 0$
- observables extrapolated to the infinite volume limit  $L \rightarrow \infty$

## Questions we would like to address

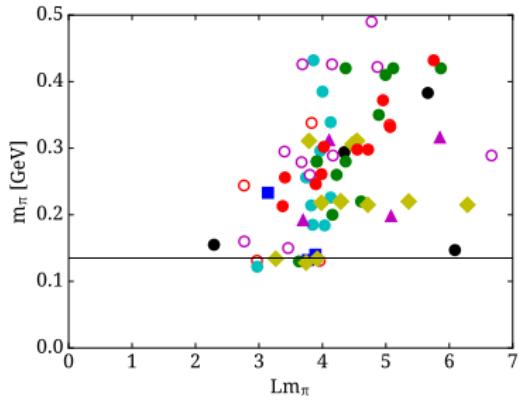
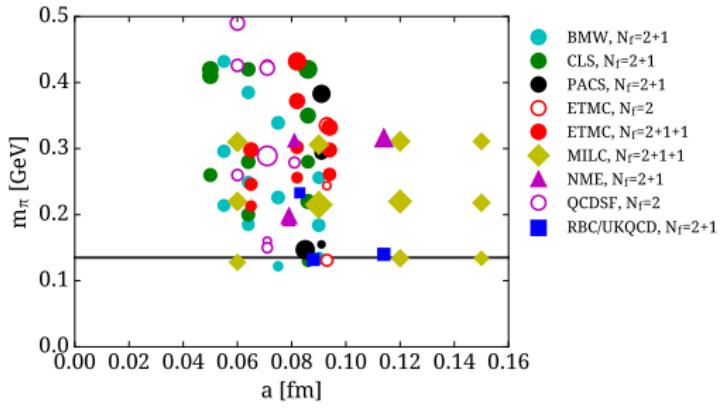
With simulations at the physical value of the pion mass there is a number of interesting questions we want to address:

- Can we reproduce known quantities including the excited spectrum of the nucleon and its associated resonances?
- Can we resolve the long-standing issue of the spin content of the nucleon?
- Can we determine accurately enough the charge radius of the proton?
- Can we provide input for experimental searches for new physics?

In this talk I will address two topics:

- The nucleon spin decomposition of the nucleon
- The nucleon scalar content or  $\sigma$ -terms as a probe of new physics

## Status of simulations



Size of the symbols according to the value of  $m_\pi L$ : smallest value  $m_\pi L \sim 3$  and largest  $m_\pi L \sim 6.7$ .

# Computational resources



## Juelich SuperComputing Centre, Germany

**Peak performance: 5.9 Petaflop/s**

**458 752 cores**

**Our time allocation: 65 Million core-h**

## Swiss National Supercomputing Centre, Switzerland

**Peak performance: 7.8 PFlops/s**

**42 176 cores**

**Tesla Graphic cards**

**Our time allocation: 2 Million node-h**

**(equiv. to 200 Million core-h)**



Europe's Fastest GPU SuperComputer

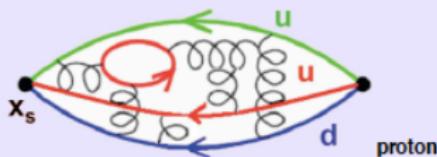


## Gauss Centre, Stuttgart, Germany

**Peak performance: 7.42 Petaflop/s**

**185 088 cores**

**Our time allocation: 48 Million core-h**



## Euclidean Correlation functions:

$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s \cdot \vec{q}} \langle J(\vec{x}_s, t_s) \bar{J}(0) \rangle = \sum_{n=0}^{\infty} A_n e^{-E_n(\vec{q}) t_s} \xrightarrow[t_s \rightarrow \infty]{} A_0 e^{-E_0(\vec{q}) t_s}$$

$J$  : interpolating operator with quantum numbers of the hadronic state under study

## Masses extracted from effective mass ( $m_{eff}$ )

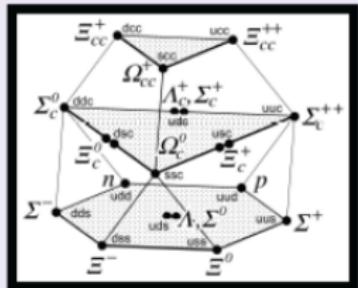
Large Euclidean time evolution: Ground state mass

$$a m_{eff}^X(t) = \log \left( \frac{G(0, t_s)}{G(0, t_s + 1)} \right) = a m^X + \log \left( \frac{1 + \sum_{i=1}^{\infty} c_i e^{\Delta_i t}}{1 + \sum_{i=1}^{\infty} c_i e^{\Delta_i (t+1)}} \right) \xrightarrow[t \rightarrow \infty]{} a m^X$$

$\Delta_i = m_i - m_X$  difference between ground state mass  $m_X$  and excited state mass  $m_i$

## Hadron masses

## 20'-plet of spin-1/2 baryons

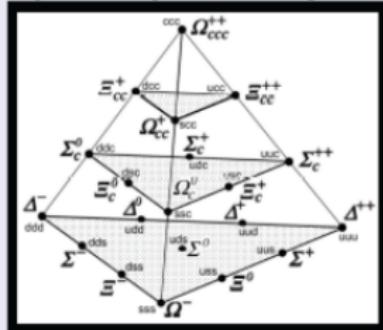


## ← Two charm quarks →

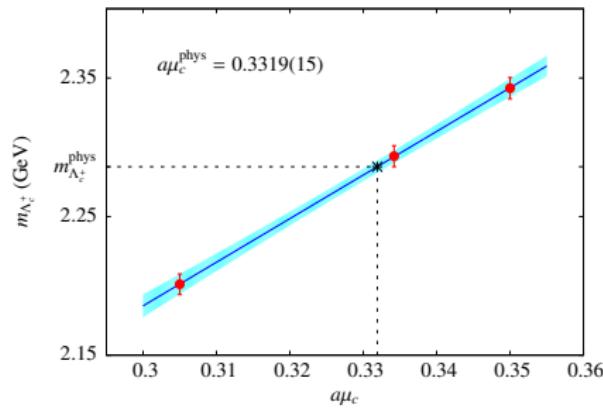
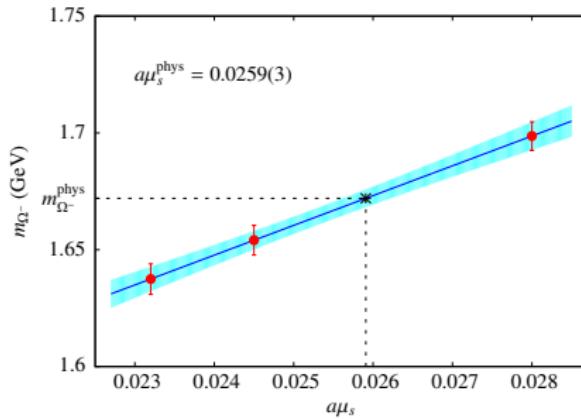
## ← One charm quarks →

← No charm quarks →

## 20-plet of spin-3/2 baryons



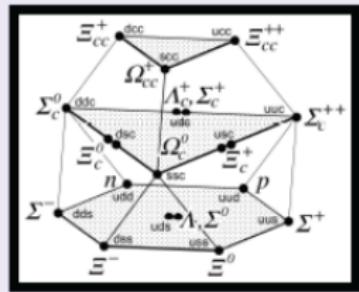
For our computation, the masses of the strange and charm quarks are fixed using the  $\Omega^-$  and  $\Lambda_c^+$ .



$m_s^R = 108.6(2.2)$  MeV and  $m_c^R = 1392.6(23.5)$  MeV, in the  $\overline{\text{MS}}$ -scheme at 2 GeV.

## Low-lying spectrum

## 20'-plet of spin-1/2 baryons

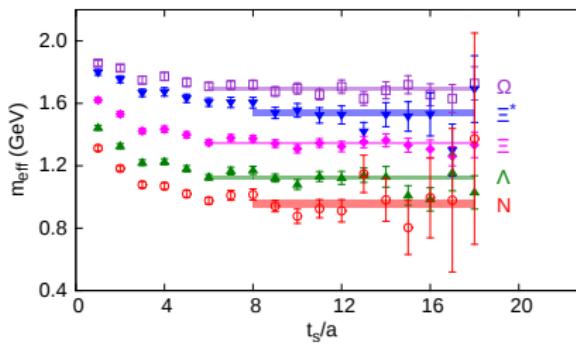
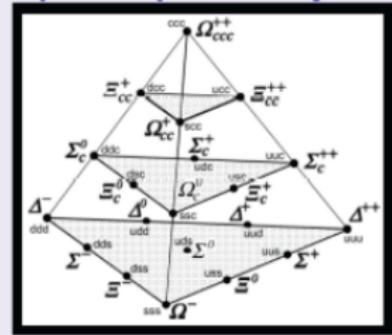


## ← Two charm quarks →

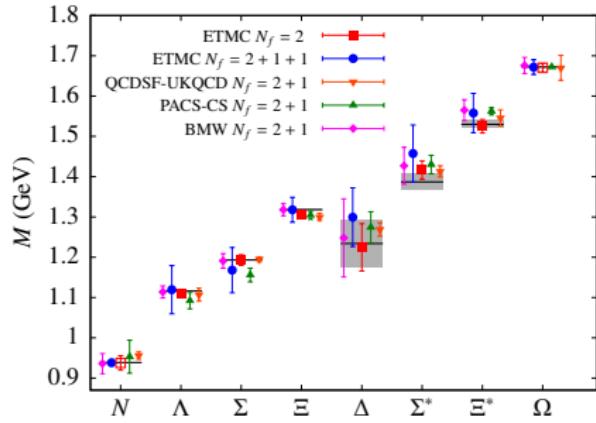
## ← One charm quarks →

$\longleftrightarrow$  No charm quarks  $\longleftrightarrow$

## 20-plet of spin-3/2 baryons

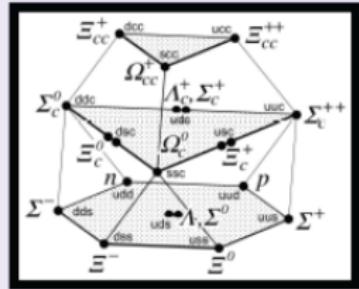


Using  $N_f = 2$  simulations at a physical value of the pion mass



## Low-lying spectrum

## 20'-plet of spin-1/2 baryons

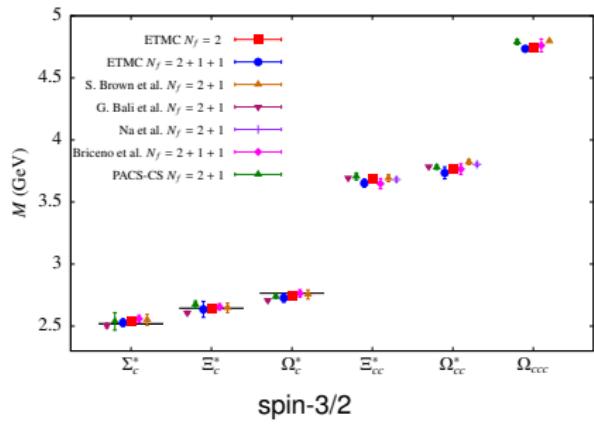
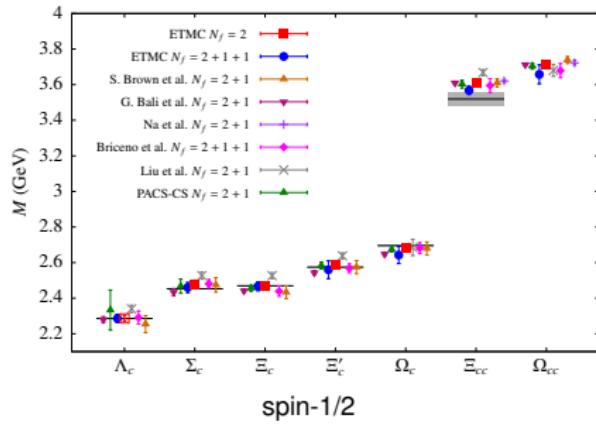
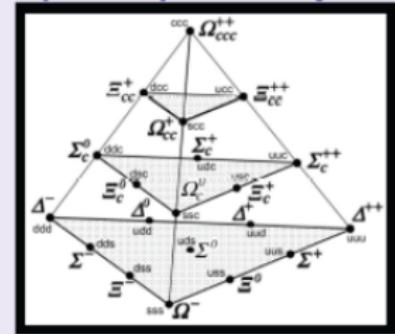


← Two charm quarks →

## ← One charm quarks →

$\longleftrightarrow$  No charm quarks  $\longleftrightarrow$

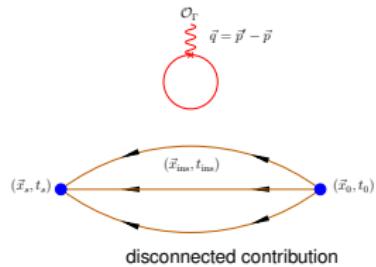
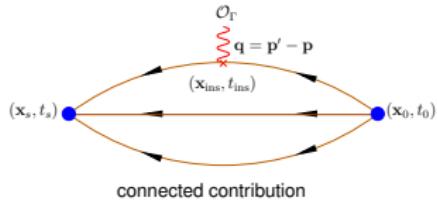
## 20-plet of spin-3/2 baryons



# Evaluation of matrix elements

Three-point functions:

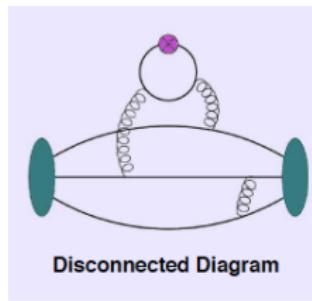
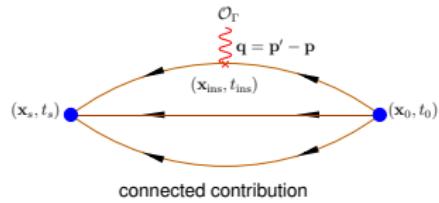
$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



## Evaluation of matrix elements

Three-point functions:

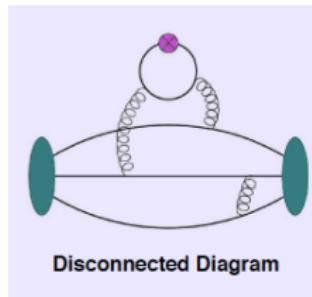
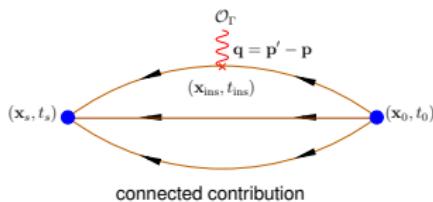
$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



# Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



- Plateau method:

$$R(t_s, t_{\text{ins}}, t_0) \xrightarrow[(t_s - t_{\text{ins}})\Delta \gg 1]{(t_{\text{ins}} - t_0)\Delta \gg 1} \mathcal{M}[1 + \dots e^{-\Delta(p)(t_{\text{ins}} - t_0)} + \dots e^{-\Delta(p')(t_s - t_{\text{ins}})}]$$

- Summation method: Summing over  $t_{\text{ins}}$ :

$$\sum_{t_{\text{ins}}=t_0}^{t_s} R(t_s, t_{\text{ins}}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(p)(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(p')(t_s - t_0)})].$$

Excited state contributions are suppressed by exponentials decaying with  $t_s - t_0$ , rather than  $t_s - t_{\text{ins}}$  and/or  $t_{\text{ins}} - t_0$

However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant

L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

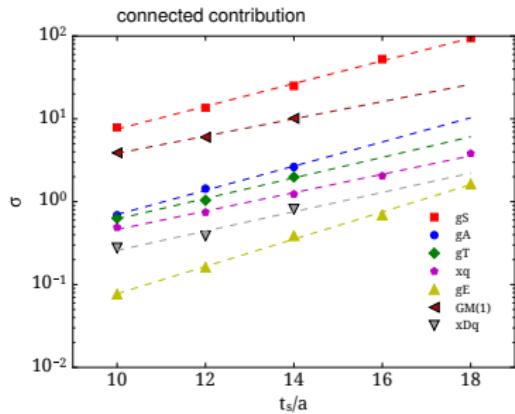
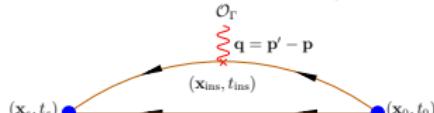
- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

## Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_s, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



- $\mathcal{M}$  the desired matrix element
- $t_s, t_{\text{ins}}, t_0$  the sink, insertion and source time-slices
- $\Delta(p)$  the energy gap with the first excited state

To ensure ground state dominance need multiple sink-source time separations ranging from 0.9 fm to 1.5 fm

## Nucleon isovector charges: $g_A$ , $g_S$ , $g_T$

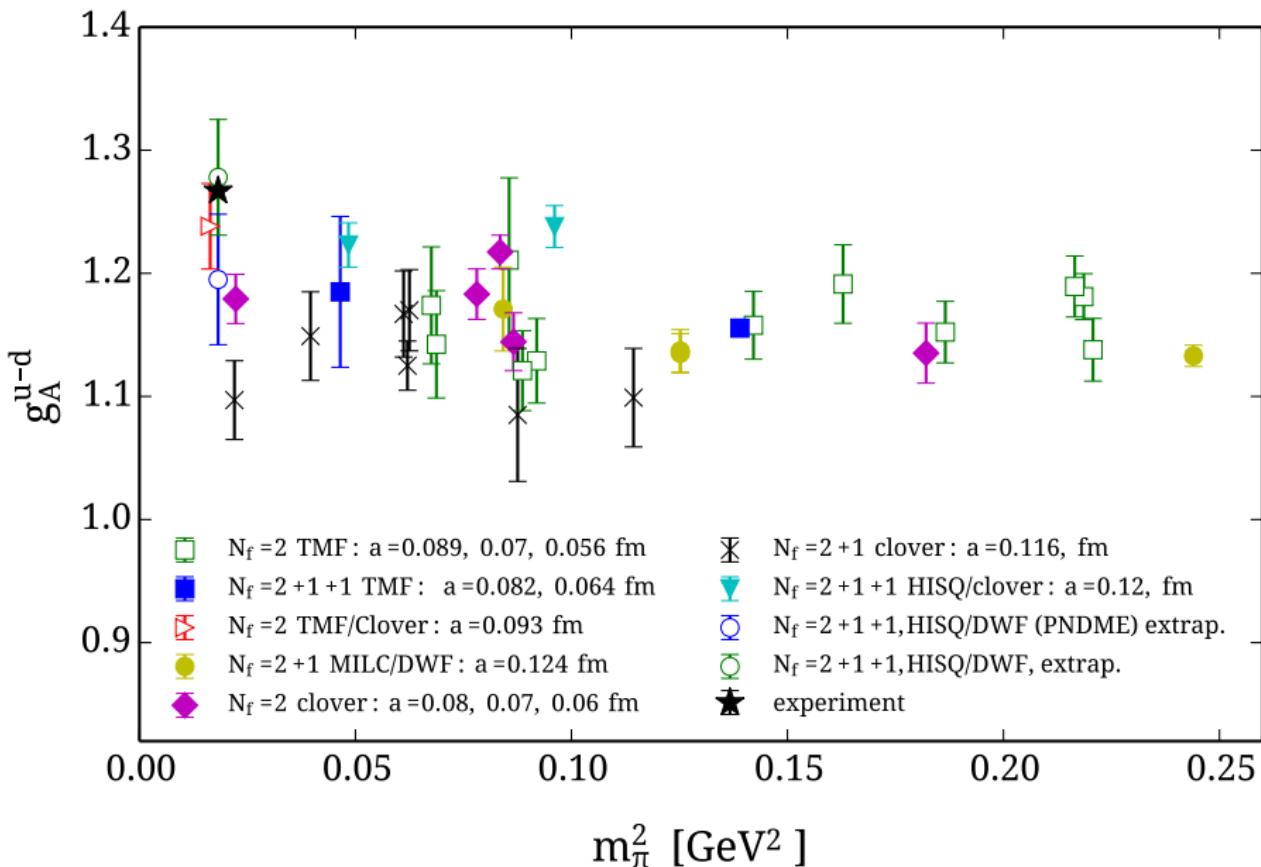
- axial-vector operator:  $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- scalar operator:  $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- pseudoscalar:  $\mathcal{O}_p^a = \bar{\psi}(x)\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator:  $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

⇒ extract from matrix element:  $\langle N(\vec{p}') \mathcal{O}_X N(\vec{p}) \rangle|_{q^2=0}$

- Axial charge  $g_A$
- Scalar charge  $g_S$
- Pseudoscalar charge  $g_p$ , • Tensor charge  $g_T$

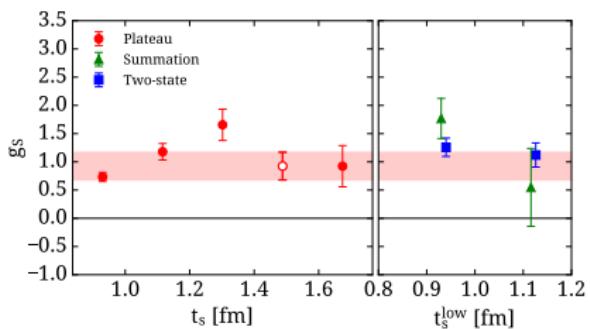
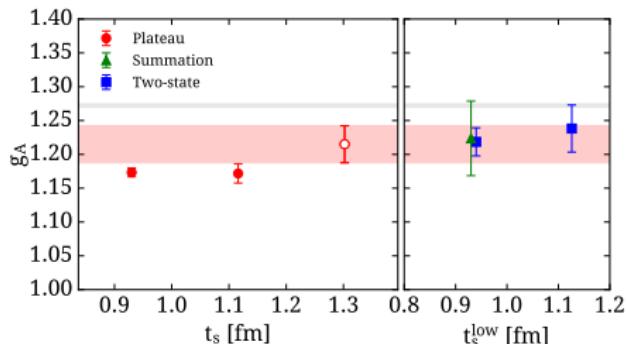
(i) isovector combination has no disconnect contributions; (ii)  $g_A$  well known experimentally, Goldberger-Treiman relation yields  $g_p$ ,  $g_T$  to be measured at JLab, Predict  $g_S$

## Nucleon axial charge $g_A$



# Nucleon isovector charges

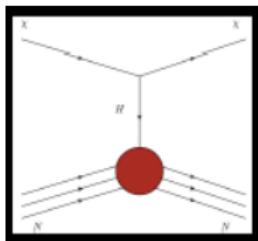
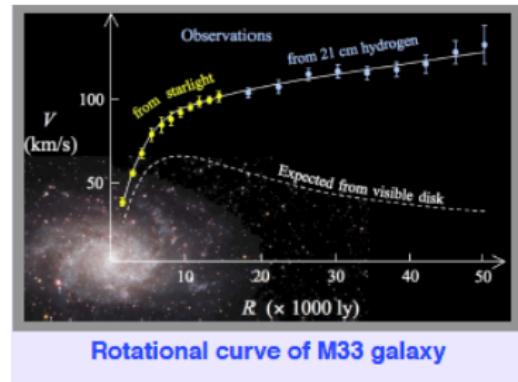
- $N_f = 2$  twisted mass plus clover,  $48^3 \times 96$ ,  $a = 0.093(1)$  fm,  $m_\pi = 131$  MeV
- $\sim 9260$  statistics for  $t_s/a = 10, 12, 14$ ,  $\sim 48000$  for  $t_s/a = 16$  and  $\sim 70000$  for  $t_s/a = 18$
- 5 sink-source time separations ranging from 0.9 fm to 1.7 fm



At the physical point we find from the plateau method:

- $g_A^{\text{isov}} = 1.21(3)(3)$ ,  $g_S^{\text{isov}} = 0.93(25)(33)$ ,  $g_T^{\text{isov}} = 1.00(2)(1)$
- $g_A$  further study for larger  $t_s$ . Important to keep constant error  
A. Abdel-Rehim *et al.* (ETMC): 1507.04936, 1507.05068, 1411.6842, 1311.4522
- New analysis of COMPASS and Belle data:  $g_T^{\mu-d} = 0.81(44)$ , M. R. A. Courtoy, A. Bacchettad, M. Guagnellia, arXiv: 1503.03495
- For  $g_S$  increasing the sink-source time separation to  $\sim 1.5$  fm is crucial

# The quark content of the nucleon



- $\sigma_f \equiv m_f \langle N | \bar{q}_f q_f | N \rangle$ : measures the explicit breaking of chiral symmetry  
Largest uncertainty in interpreting experiments for direct dark matter searches - Higgs-nucleon coupling depends on  $\sigma$ ,  
e.g. spin-independent cross-section can vary an order of magnitude if  $\sigma_{\pi N}$  changes from 35 MeV to 60 MeV, [J. Ellis, K. Olive, C. Savage, arXiv:0801.3656](#)
- In lattice QCD:

- ▶ Feynman-Hellmann theorem:  $\sigma_I = m_I \frac{\partial m_N}{\partial m_I}$

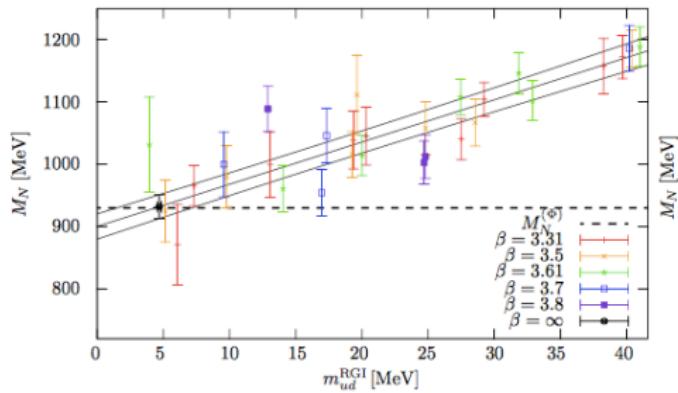
Similarly  $\sigma_S = m_S \frac{\partial m_N}{\partial m_S}$ , S. Dürre *et al.* (BMW<sub>C</sub>) *Phys.Rev.Lett.* 116 (2016) 172001

- ▶ Direct computation of the scalar matrix element

G. Bali, *et al.* (RQCD) *Phys.Rev. D93* (2016) 094504, [arXiv:1603.00827](#); Yi-Bo Yang *et al.* ( $\chi$ QCD) *Phys.Rev. D94* (2016) no.5, 054503;  
A. Abdel-Rehim *et al.* [arXiv:1601.3656](#), *PRL*116 (2016) 252001;

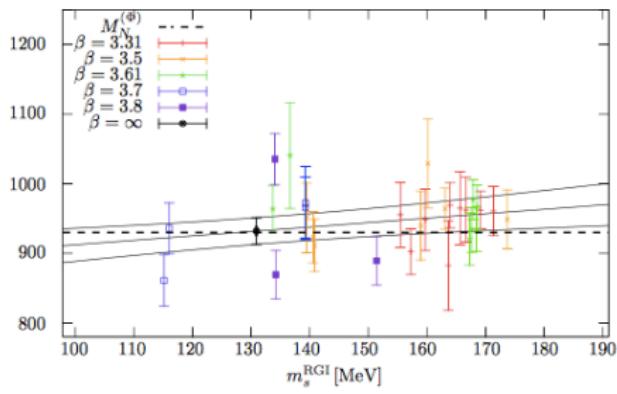
# The quark content of the nucleon via Feynman-Hellmann

BMW Collaboration: 47 lattice ensembles with  $N_f = 2 + 1$  clover fermions, 5 lattice spacings down to 0.054 fm, lattice sizes up to 6 fm and pion masses down to 120 MeV.



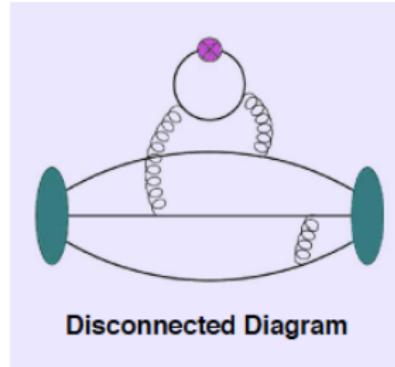
$$\sigma_{\pi N} = 38(3)(3) \text{ MeV}$$

$$\sigma_s = 105(41)(37) \text{ MeV}$$



## The quark content of the nucleon via direct determination

Need disconnected contributions



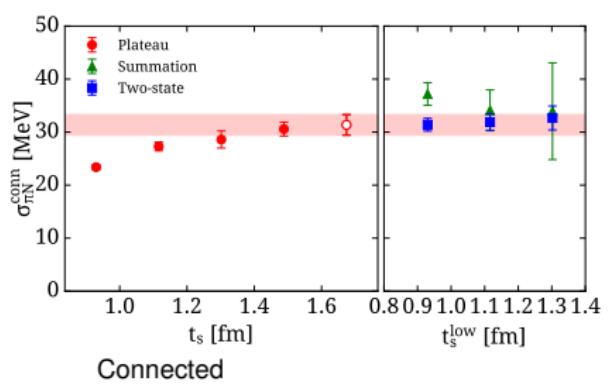
- RQCD:  $N_f = 2$  clover fermions with a range of pion masses down to  $m_\pi = 150$  MeV and  $a = 0.06 - 0.08$  fm G. Bali, *et al.*, Phys.Rev. D93 (2016) 094504, arXiv:1603.00827
- $\chi$ QCD: Valence overlap fermions on  $N_f = 2 + 1$  flavor domain-wall fermion (DWF) configurations, 3 ensembles of  $m_\pi = 330$  MeV,  $m_\pi = 300$  MeV and  $m_\pi = 139$  MeV Yi-Bo Yang *et al.*, Phys.Rev. D94 (2016) no.5, 054503; M/ Gong *et al.*, Phys. Rev. D 88 (2013) 014503 arXiv:1304.1194
- ETM Collaboration:  $N_f = 2$  twisted mass plus clover,  $48^3 \times 96$ ,  $a = 0.093(1)$  fm,  $m_\pi = 131$  MeV, A. Abdel-Rehim *et al.*, arXiv:1601.3656, PRL116 (2016) 252001

# The quark content of the nucleon from ETMC

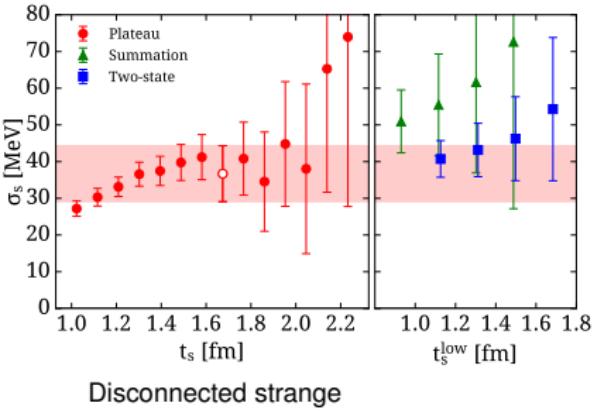
$N_f = 2$  twisted mass plus clover,  $48^3 \times 96$ ,  $a = 0.093(1)$  fm,  $m_\pi = 131$  MeV

- Connected:  $t/a = 10, 12, 14$  9264 statistics,  $t/a = 16 \sim 47,600$  statistics and  $t/a = 18 \sim 70,000$  statistics
- Disconnected:  $\sim 213,700$  statistics

A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



Connected



Disconnected strange

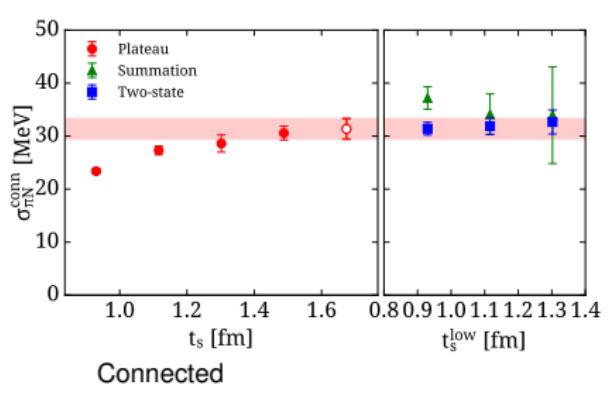
Our results are:  $\sigma_{\pi N} = 36(2)$  MeV

# The quark content of the nucleon from ETMC

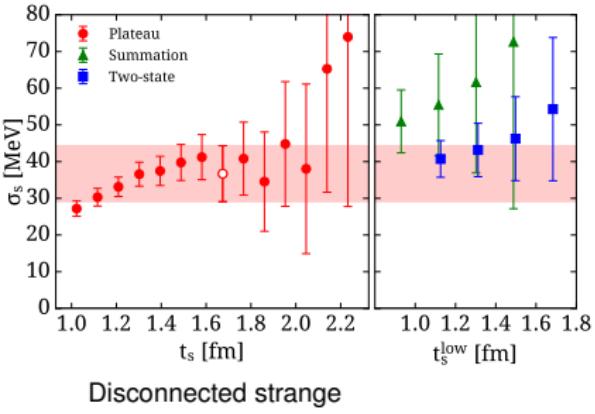
$N_f = 2$  twisted mass plus clover,  $48^3 \times 96$ ,  $a = 0.093(1)$  fm,  $m_\pi = 131$  MeV

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A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



Connected

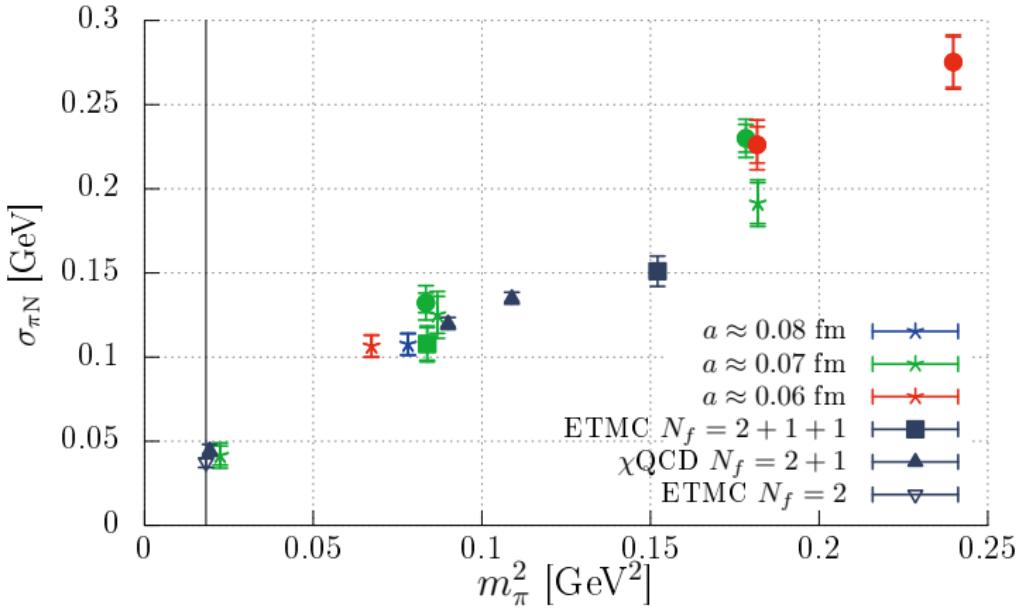


Disconnected strange

Our results are:  $\sigma_{\pi N} = 36(2)$  MeV |  $\sigma_s = 37(8)$  MeV |  $\sigma_c = 83(17)$  MeV

# The quark content of the nucleon

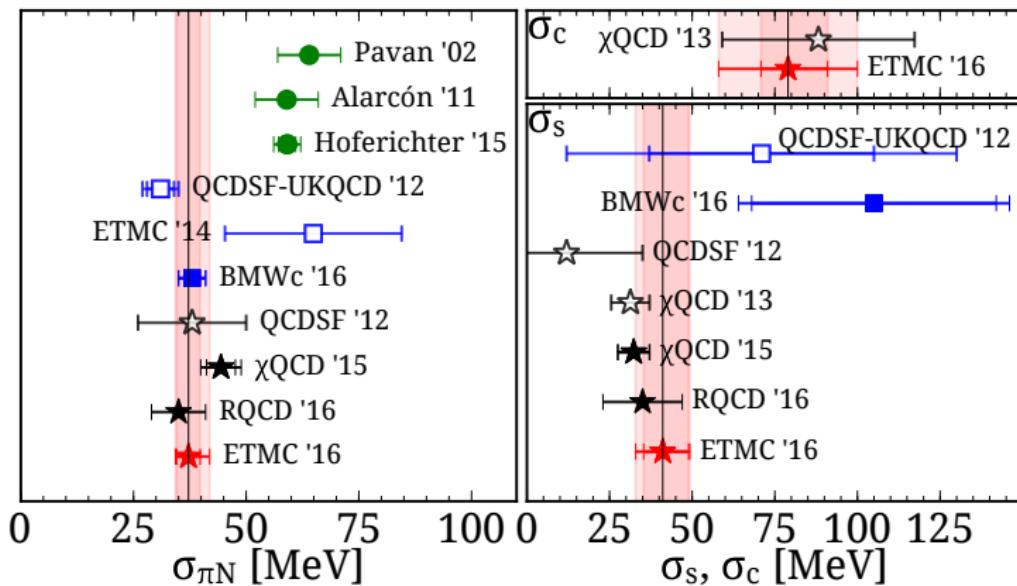
Comparison of results



G. Bali, et al., Phys. Rev. D93 (2016) 094504, arXiv:1603.00827

# The quark content of the nucleon

Comparison of results

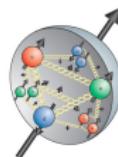


Recent results from lattice QCD at the physical point and from phenomenology. Filled symbols for lattice QCD results include simulations with pion mass close to its physical value, A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001

## Nucleon spin

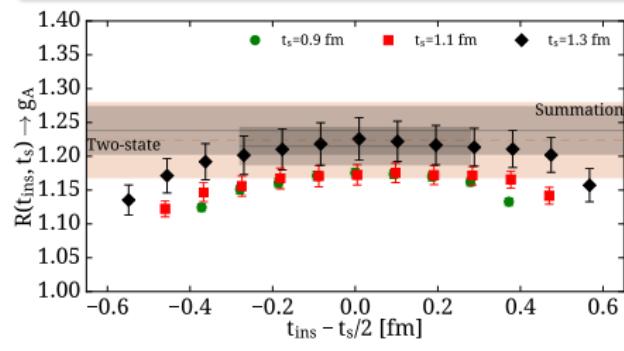
$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left( \frac{1}{2} \Delta \Sigma^q + L^q \right)}_{J^q} + J^g$$

$$J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \text{ and } \Delta \Sigma^q = g_A^q$$

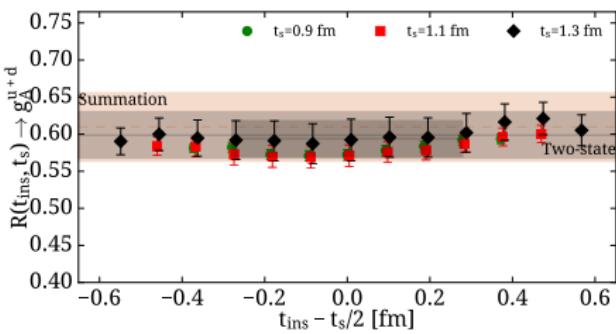


Need isoscalar  $g_A$ , which has disconnected contributions

- $N_f = 2$  twisted mass fermions with a clover term at a **physical value of the pion mass**,  $48^3 \times 96$  and  $a = 0.093(1)$  fm
- Intrinsic quark spin:  $\Delta \Sigma^q = g_A^q$



Isovector

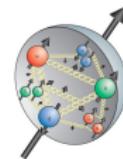


Connected isoscalar

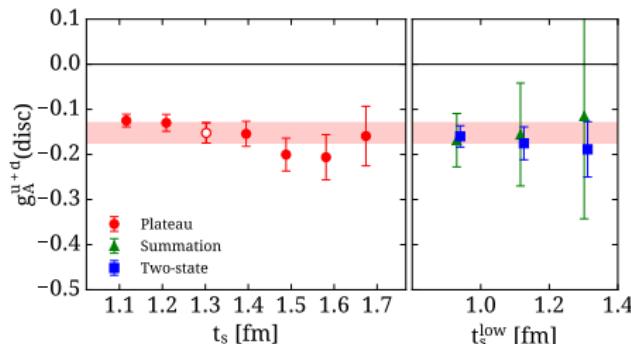
## Nucleon spin

$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left( \frac{1}{2} \Delta \Sigma^q + L^q \right)}_{J^q} + J^g$$

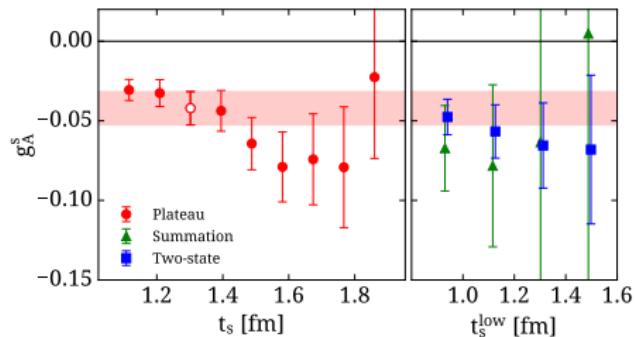
$$J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \text{ and } \Delta \Sigma^q = g_A^q$$



Need isoscalar  $g_A$ , which has disconnected contributions



Isoscalar disconnected



Strange

We find from the plateau method:

- $g_A^{u+d} = -0.15(2)$  (disconnected only) with 854,400 statistics
- Combining with the isovector we find:  $g_A^u = 0.828(21)$ ,  $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$  with 861,200 statistics

## Quark total spin $J^q$

Generalized parton distributions functions (GPDs) are matrix elements of light cone operators that cannot be computed directly → Factorization leads to matrix elements of local operators:

- vector operator

$$\mathcal{O}_{V^a}^{\mu_1 \dots \mu_n} = \bar{\psi}(x) \gamma^{\{\mu_1} i \overset{\leftrightarrow}{D}^{\mu_2} \dots i \overset{\leftrightarrow}{D}^{\mu_n\}} \frac{\tau^a}{2} \psi(x)$$

- axial-vector operator

$$\mathcal{O}_{A^a}^{\mu_1 \dots \mu_n} = \bar{\psi}(x) \gamma^{\{\mu_1} i \overset{\leftrightarrow}{D}^{\mu_2} \dots i \overset{\leftrightarrow}{D}^{\mu_n\}} \gamma_5 \frac{\tau^a}{2} \psi(x)$$

- tensor operator

$$\mathcal{O}_{T^a}^{\mu_1 \dots \mu_n} = \bar{\psi}(x) \sigma^{\{\mu_1, \mu_2} i \overset{\leftrightarrow}{D}^{\mu_3} \dots i \overset{\leftrightarrow}{D}^{\mu_n\}} \frac{\tau^a}{2} \psi(x)$$

Special cases:

- no-derivative → nucleon form factors

- For  $Q^2 = 0 \rightarrow$  parton distribution functions

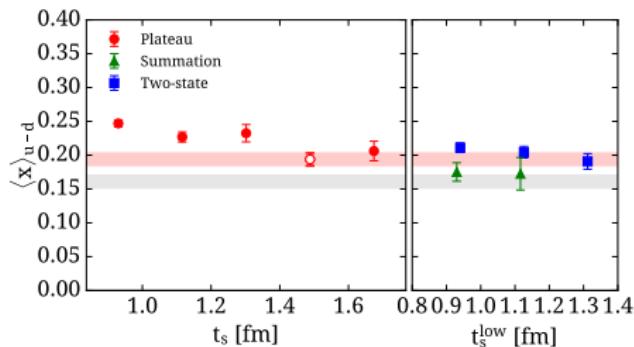
one-derivative → first moments e.g. average momentum fraction  $\langle x \rangle$

Generalized form factor decomposition:

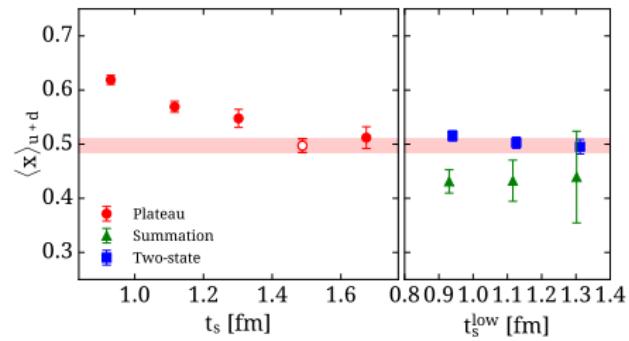
$$\langle N(p', s') | \mathcal{O}_{V^3}^{\mu\nu} | N(p, s) \rangle = \bar{u}_N(p', s') \left[ A_{20}(q^2) \gamma^{\{\mu} P^{\nu\}} + B_{20}(q^2) \frac{i \sigma^{\{\mu\alpha} q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2) \frac{q^{\{\mu} q^{\nu\}}}{m} \right] \frac{1}{2} u_N(p, s)$$

$$\text{Total quark spin } J^q = \frac{1}{2} \left[ A_{20}^q(0) + B_{20}^q(0) \right] \text{ and } \langle x \rangle_q = A_{20}^q(0)$$

## Momentum fraction $\langle x \rangle_{u-d}$

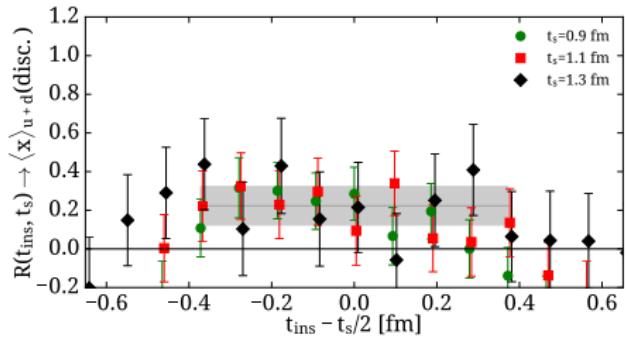


Results for the isovector in the  $\overline{\text{MS}}$  at 2 GeV



Results for the connected isoscalar in the  $\overline{\text{MS}}$  at 2 GeV

## Momentum fraction $\langle x \rangle_{u-d}$



Results for the disconnected isoscalar

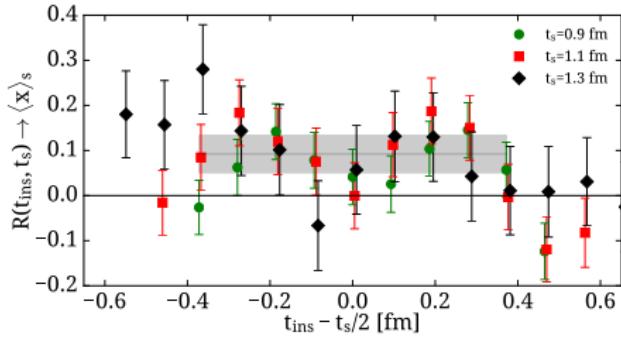
At the physical point we find in the MS at 2 GeV from the plateau method ( $\mathcal{O}(860,000)$  statistics):

- $\langle x \rangle_{u-d} = 0.194(9)(10)$

- $\langle x \rangle_{u+d+s} = 0.74(10)$

$\langle x \rangle_{u+d+s}$  is perturbatively renormalized to one-loop due to its mixing with the gluon operator.

A. Abdel-Rehim et al. (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

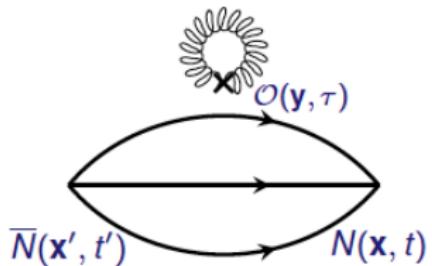


Results for the strange

At the physical point we find in the MS at 2 GeV from the plateau method ( $\mathcal{O}(860,000)$  statistics):

## Gluon content of the nucleon

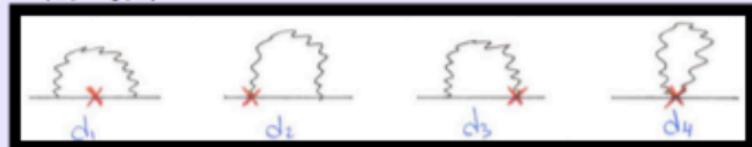
- Gluons carry a significant amount of momentum and spin in the nucleon
  - ▶ Compute gluon momentum fraction :  $\langle x \rangle_g = A_{20}^g$
  - ▶ Compute gluon spin:  $J^g = \frac{1}{2}(A_{20}^g + B_{20}^g)$
- Nucleon matrix of the gluon operator:  $O_{\mu\nu} = -G_{\mu\rho} G_{\nu\rho}$   
→ gluon momentum fraction extracted from  
 $\langle N(0) | O_{44} - \frac{1}{3} O_{jj} | N(0) \rangle = m_N < x >_g$
- Disconnected correlation function, known to be very noisy  
⇒ we employ several steps of **stout smearing** in order to remove fluctuations in the gauge field
- Results are computed on the  $N_f = 2$  ensemble at the physical point,  $m_\pi = 131$  MeV,  $a = 0.093$  fm,  
 $V = 48^3 \times 96$ , [A. Abdel-Rehim et al. \(ETMC\):1507.04936](#)
- The methodology was tested for  $N_f = 2 + 1 + 1$  twisted mass at  $m_\pi = 373$  MeV, [C. Alexandrou, V. Drach, K. Hadjyiannakou, K. Jansen, B. Kostrzewa, C. Wiese, PoS LATTICE2013 \(2014\) 289](#)



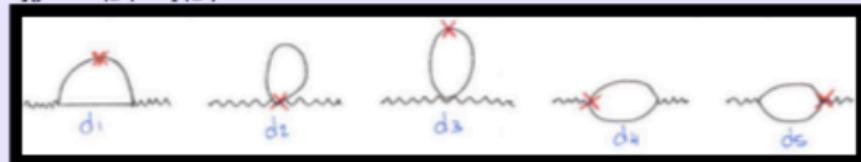
## Nucleon gluon moment-Renormalization

Mixing with  $\langle x \rangle_{u+d+s} \Rightarrow$  Perturbation theory - M. Constantinou and H. Panagopoulos

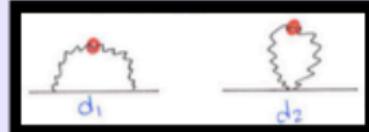
$$\times Z_{qq} : \quad \Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$$



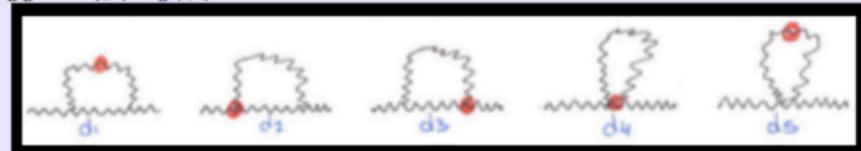
$$\times Z_{qg} : \quad \Lambda_{qg} = \langle g | \mathcal{O}_q | g \rangle$$



$$\bullet Z_{gq} : \quad \Lambda_{gq} = \langle q | \mathcal{O}_g | q \rangle$$



$$\bullet Z_{gg} : \quad \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$$



## Nucleon gluon moment-Renormalization

Mixing with  $\langle x \rangle_{u+d+s} \implies$  Perturbation theory - M. Constantinou and H. Panagopoulos

$\times Z_{qq} : \quad \Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$

$$Z_{gg} = 1 + \frac{g^2}{16\pi^2} \left( 1.0574 N_f + \frac{-13.5627}{N_c} - \frac{2 N_f}{3} \log(a^2 \bar{\mu}^2) \right)$$

$\times Z_{qg} : \quad \Lambda_{qg} = \langle g | \mathcal{O}_q | g \rangle$

$$Z_{gq} = 0 + \frac{g^2 C_f}{16\pi^2} (0.8114 + 0.4434 c_{SW} - 0.2074 c_{SW}^2 + \frac{4}{3} \log(a^2 \bar{\mu}^2))$$

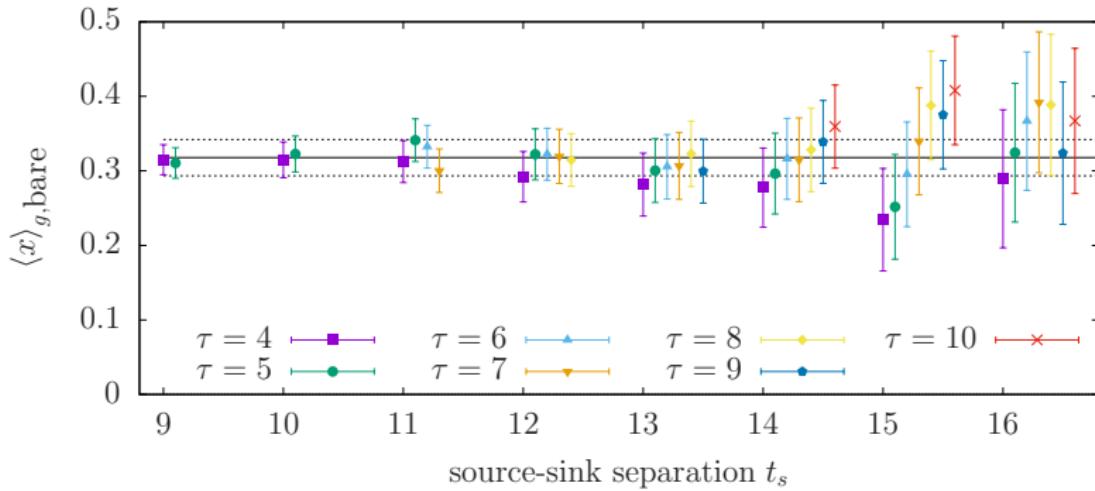
•  $Z_{gq} : \quad \Lambda_{gq} = \langle q | \mathcal{O}_g | q \rangle$

$$Z_{qq} = 1 + \frac{g^2}{16\pi^2} (-1.8557 + 2.9582 c_{SW} + 0.3984 c_{SW}^2 - \frac{8}{3} \log(a^2 \bar{\mu}^2))$$

•  $Z_{gg} : \quad \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$

$$Z_{qg} = 0 + \frac{g^2 N_f}{16\pi^2} (0.2164 + 0.4511 c_{SW} + 1.4917 c_{SW}^2 - \frac{4}{3} \log(a^2 \bar{\mu}^2))$$

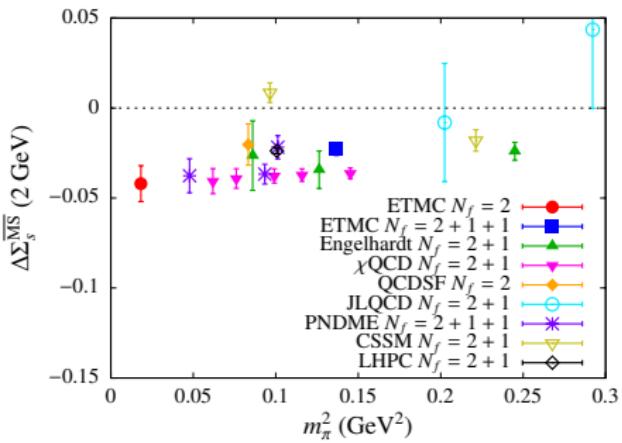
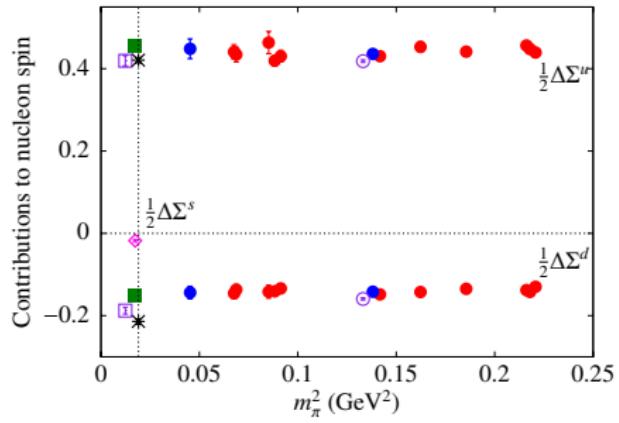
## Results for the gluon content



- 2094 gauge configurations with 100 different source positions each → more than 200 000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, M. Constantinou and H. Panagopoulos
- $\langle x \rangle_{g,\text{bare}} = 0.318(24) \xrightarrow{\text{Renormalization}} \langle x \rangle_g^R = Z_{gg} \langle x \rangle_g + Z_{gq} \langle x \rangle_{u+d+s} = 0.273(23)(24)$ . The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.
- Momentum sum is satisfied:  $\sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle_{u+d}^{CI} + \langle x \rangle_{u+d+s}^{DI} + \langle x \rangle_g = 1.01(10)(2)$

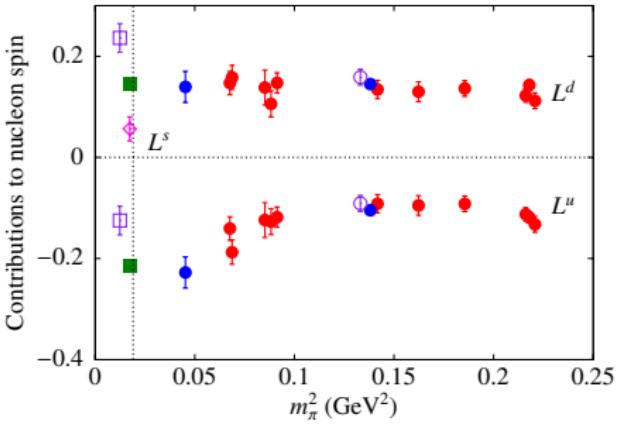
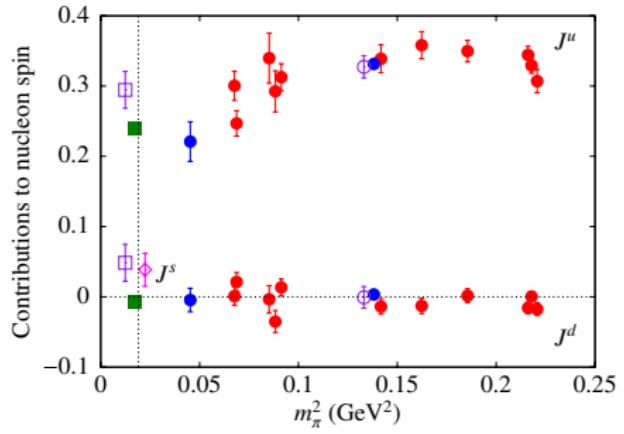
# Nucleon spin

Disconnected contribution using  $\mathcal{O}(860000)$  statistics



## Nucleon spin

Disconnected contribution using  $\mathcal{O}(860000)$  statistics



## Nucleon spin

$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left( \frac{1}{2} \Delta \Sigma^q + L^q \right)}_{J^q} + J^g$$

$$\begin{aligned} \frac{1}{2} \Delta \Sigma^u &= 0.413(13), & \frac{1}{2} \Delta \Sigma^d &= -0.193(7), & \frac{1}{2} \Delta \Sigma^s &= -0.021(5) \\ J^u &= 0.310(26), & J^d &= 0.056(26), & J^s &= 0.046(21) \\ L^u &= -0.104(29), & L^d &= 0.249(27), & L^s &= 0.067(21) \end{aligned} \tag{1}$$

We find that  $B_{20}^q(0) \sim 0 \longrightarrow$  taking  $B_{20}(0)^g \sim 0$  we can directly check the nucleon spin sum:

$$J_N = (0.310)_u + (0.056)_d + (0.046)_s + (0.136)_g = 0.51(5)(4)$$

## Conclusions and Future Perspectives

- Computation of  $g_A$ ,  $\langle x \rangle_{u-d}$ , etc, at the physical point allows direct comparison with experiment
- Provide predictions for  $g_s$ ,  $g_T$ , tensor moment,  $\sigma$ -terms, etc.
- Can address long-standing puzzles like the spin decomposition of the nucleon

On-going studies

- ETMC:
  - ▶ Assess volume effects using  $N_f = 2$  and lattice size  $64^3 \times 128$  at same pion mass
  - ▶ Analyse a new ensemble of  $N_f = 2 + 1 + 1$  twisted clover improved configurations with  $a \sim 0.08$  fm and lattice size  $64^3 \times 128$  and  $m_\pi \sim 135$  MeV
- A number of collaborations are now using simulations with close to physical values of the pion mass to:
  - ▶ Investigate the proton radius using new methods e.g. position methods
  - ▶ Compute gluonic observables
  - ▶ Study excited states and resonances
  - ▶ Scattering lengths and interactions
  - ▶ ...

# European Twisted Mass Collaboration

European Twisted Mass Collaboration (ETMC)



Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:

A. Abdel-Rehim, S. Bacchio, K. Cichy, M. Constantinou, J. Finkenrath, K. Hadjijian-nakou, K.Jansen, Ch. Kallidonis, G. Koutsou, K. Ott nad, M. Petschlies, F. Steffens, A. Vaquero, C. Wiese