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EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY

Institutional Member of the European Expert Committee NUPECC

TEMPLE UNIVERSITY

INFN Istituto Nazionale di Fisica Nucleare

How to test the mass decomposition?

Jianwei Qiu

Theory Center, Jefferson Lab

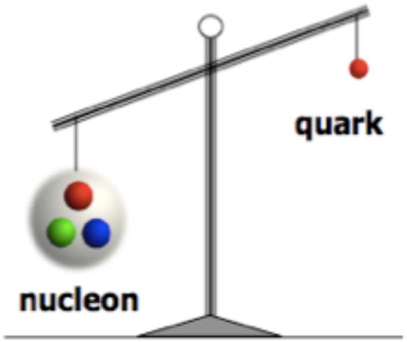
Castello di Trento (“Trin ... y back from Venice (1495). British Museum, London

The Proton Mass: At the Heart of Most Visible Matter

Trento, April 3 - 7, 2017

Proton Mass

- Nucleon mass – dominates the mass of visible world:



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$m_N \sim 1000$ MeV — $\sim 1\%$ proton's mass!

Higgs mechanism is not enough!!!

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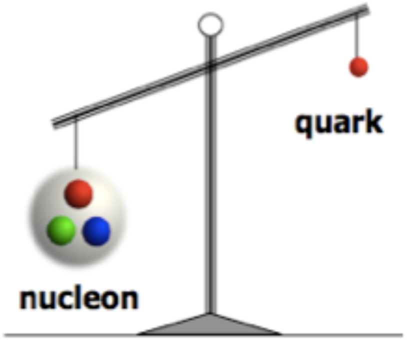

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"… The vast majority of the nucleon's mass is due to quantum fluctuations of quark-antiquark pairs, the gluons, and the energy associated with quarks moving around at close to the speed of light. …"

REACHING FOR THE HORIZON
The 2015 Long Range Plan for Nuclear Science

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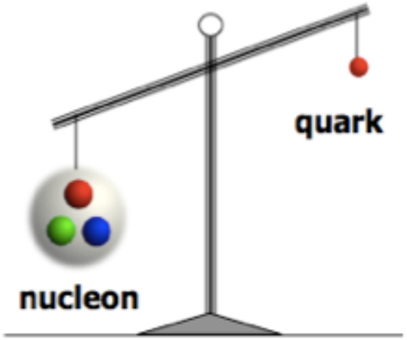

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How to quantify and verify this, theoretically and experimentally?

Mass vs. Spin

- **Mass – intrinsic to a particle:**

  = Energy of the particle when it is at the rest

  - **QCD energy-momentum tensor in terms of quarks and gluons**

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi}\,i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}\,g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

  - **Proton mass:**

X. Ji, PRL (1995)

$$m = \frac{\langle p|\int d^3x\, T^{00}\,|p\rangle}{\langle p|p\rangle} \sim \mathrm{GeV}$$

when proton is at rest!

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❑ Spin – intrinsic to a particle:

= Angular momentum of the particle when it is at the rest

✧ QCD angular momentum density in terms of energy-momentum tensor

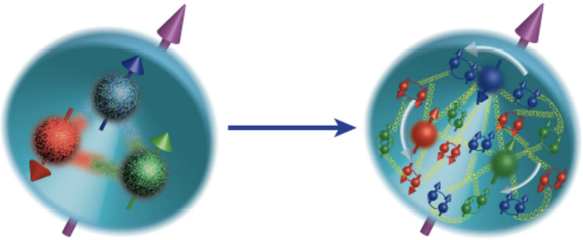
$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \qquad J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

✧ Proton spin:

$$S(\mu) = \sum_f \langle P,S|\hat{J}^z_f(\mu)|P,S\rangle = \frac{1}{2}$$

Hadron Spin

- **Proton's spin:**



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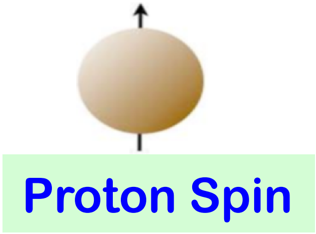

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

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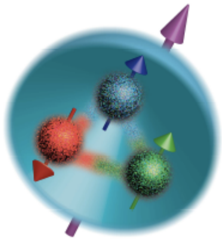

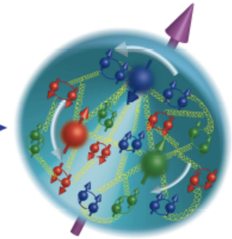

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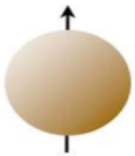

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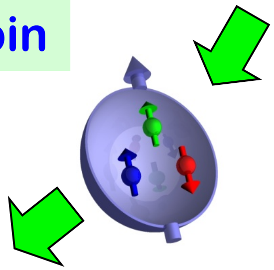

Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

$$\sim 30\%$$

Spin "puzzle"

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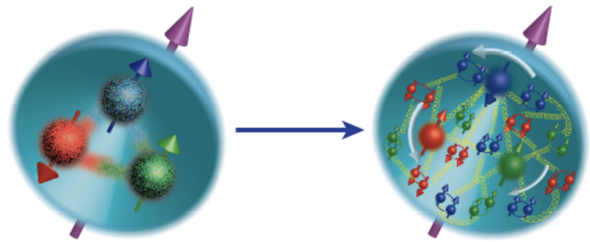

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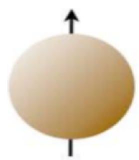

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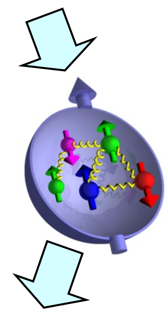

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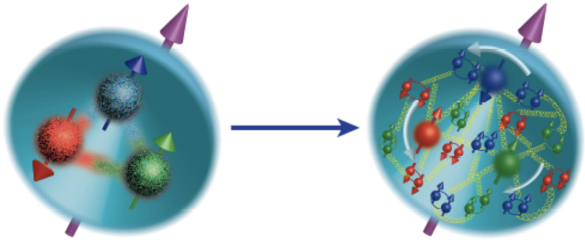

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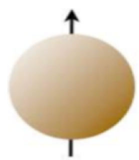

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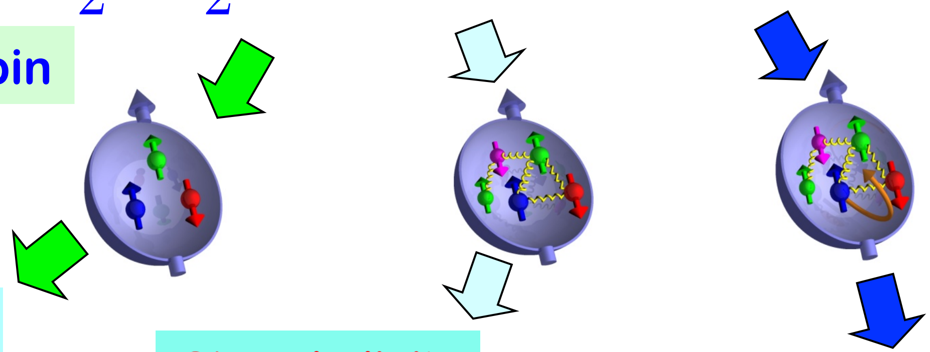

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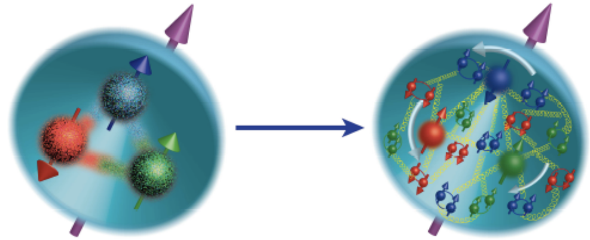

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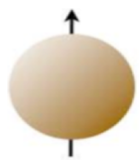

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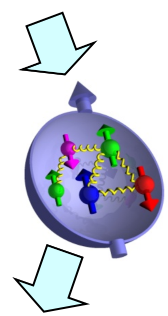

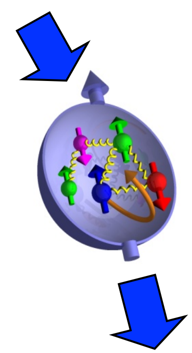

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If we do not understand proton spin, we do not understand QCD

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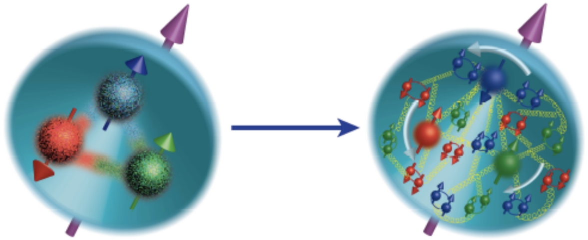
- **Proton's mass:**
  - ✧ QCD Lagrangian does not have mass dimension parameters, other than current quark masses
  - ✧ Asymptotic freedom ⟷ confinement:



➡ A dynamical scale, $\Lambda_{\rm QCD}$, consistent with $\dfrac{1}{R} \sim 200\ \mathrm{MeV}$

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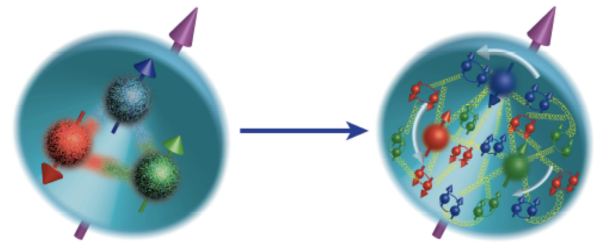

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❑ Bag model:

- ✧ Kinetic energy of three quarks: $K_q \sim 3/R$
- ✧ Bag energy (bag constant B): $T_b = \frac{4}{3}\pi R^3\, B$
- ✧ Minimize $K_q + T_b$: $M_p \sim \frac{4}{R} \sim \frac{4}{0.88 fm} \sim 912\mathrm{MeV}$

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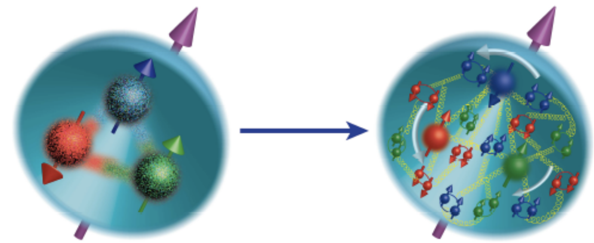

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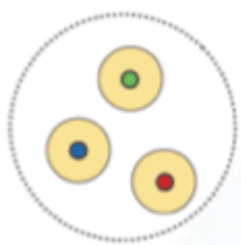

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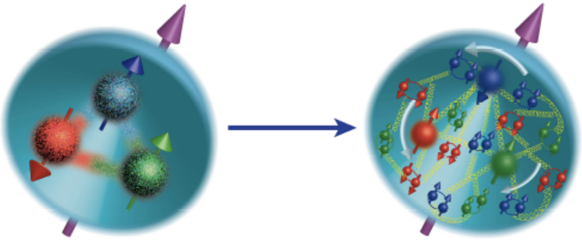

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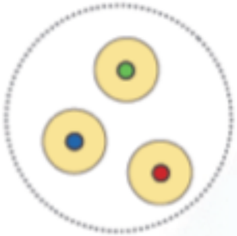

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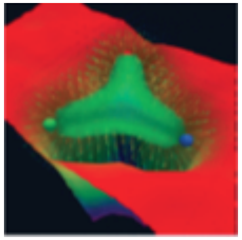


  - With "heavy" (or slow moving) quarks

    Energy concentrated in the gluon junction!

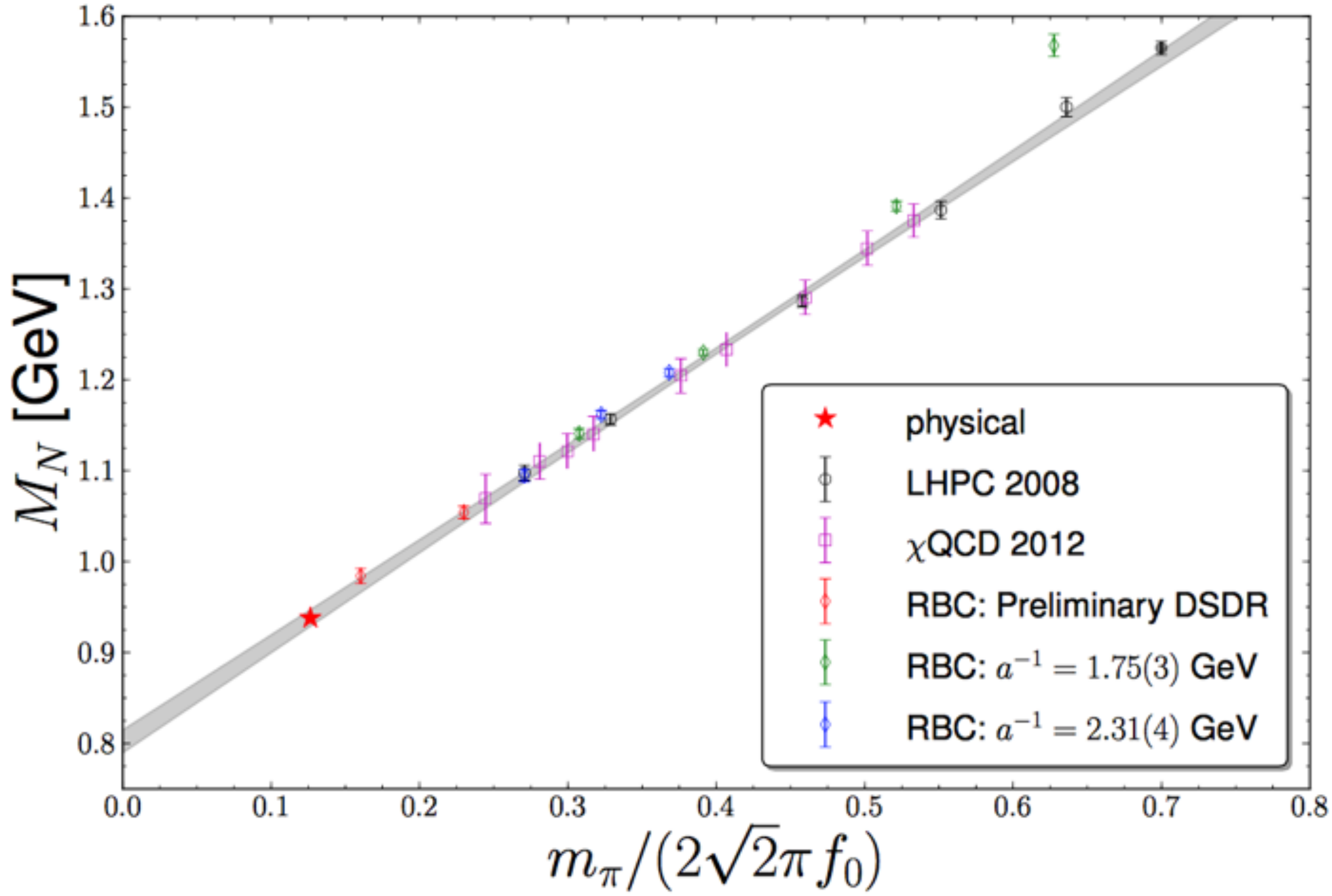
    ➡ Gluon radius < Charge Radius   EIC!

    Mass scale: Lattice space – "a"

Hadron Mass

Martin Savage @ Temple meeting

- Nucleon mass from lattice QCD:



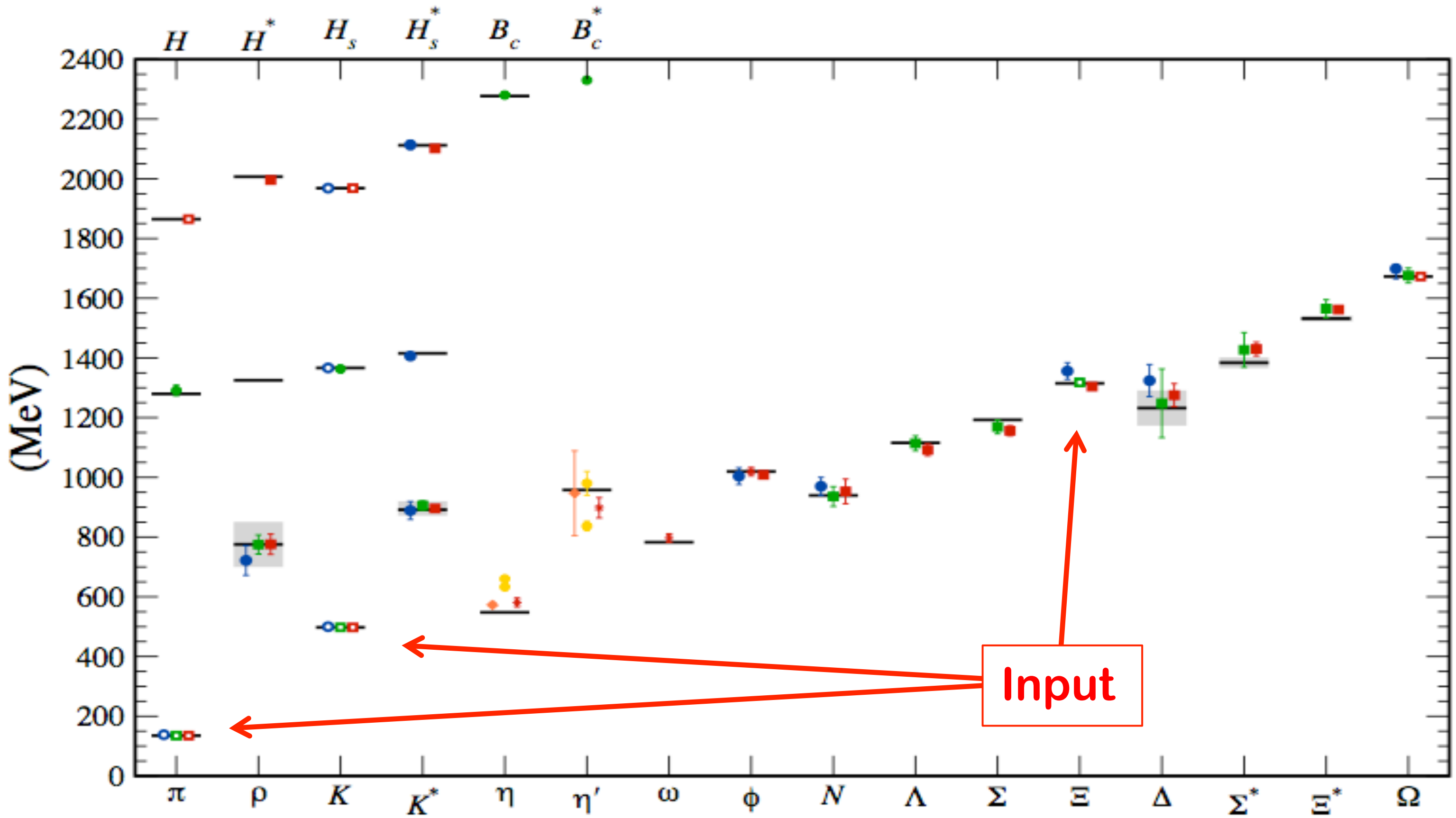
$$M_N = 800 \text{ MeV} + m_\pi$$

Unexpected behavior !!

Hadron Mass

- ❑ **From Lattice QCD calculation:**

See Richards' talk on excite states …



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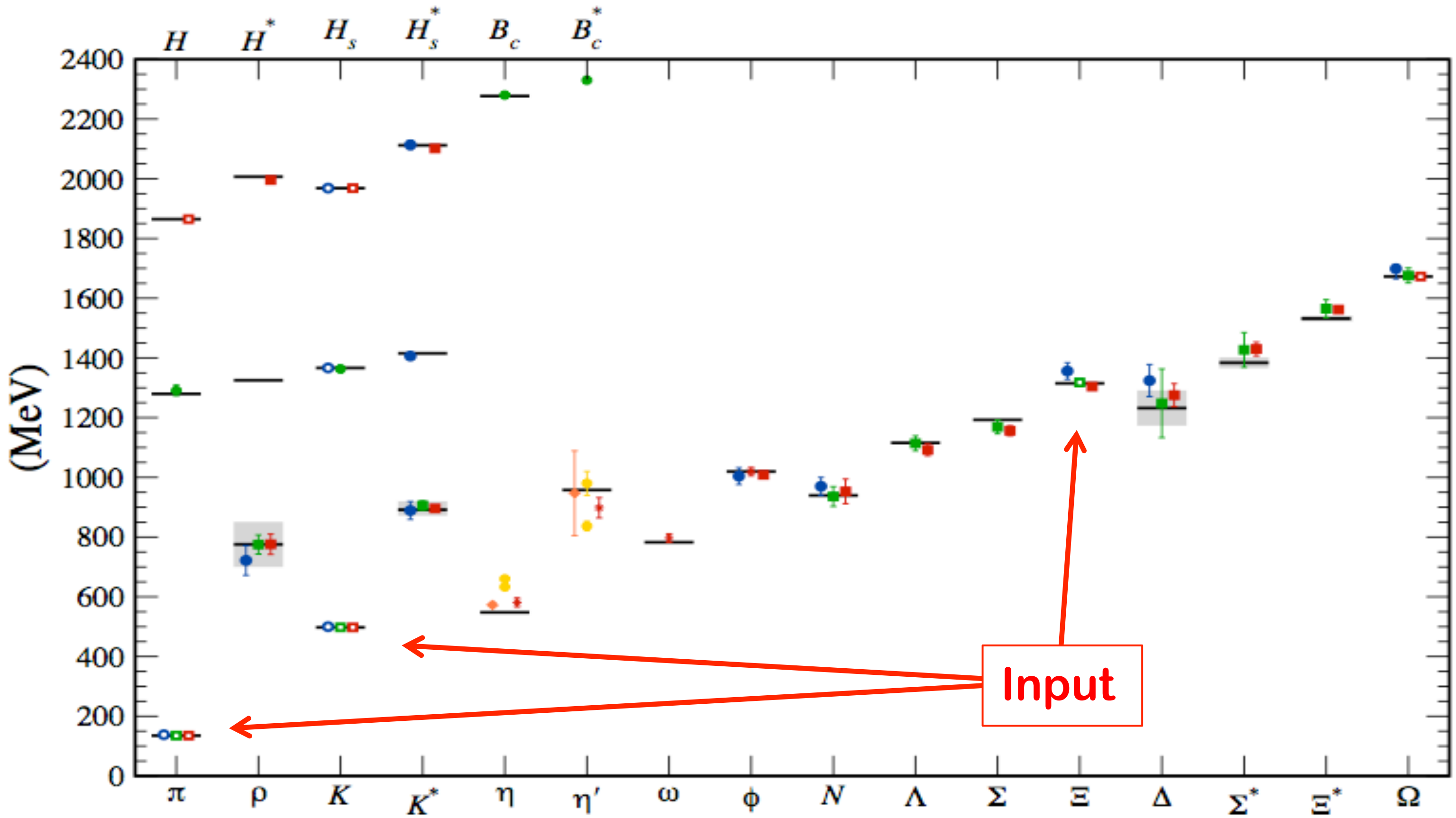

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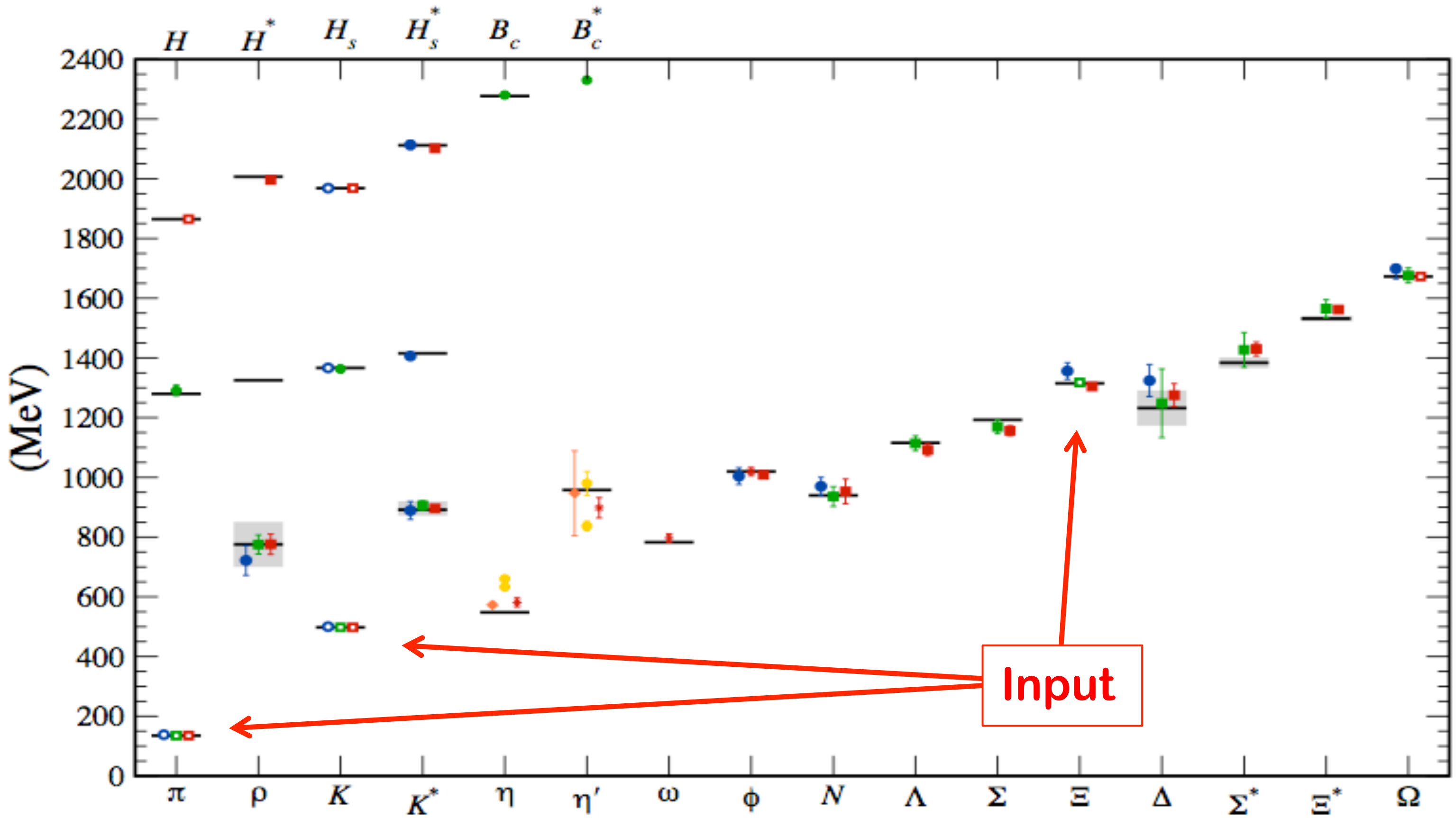

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Hadron Mass

- ❑ **Three-pronged approach to explore the origin of hadron mass**
  - ✧ **Lattice QCD**
  - ✧ **Mass decomposition – roles of the constituents**
  - ✧ **Model calculation – approximated analytical approach**

https://phys.cst.temple.edu/meziani/proton-mass-workshop-2016/

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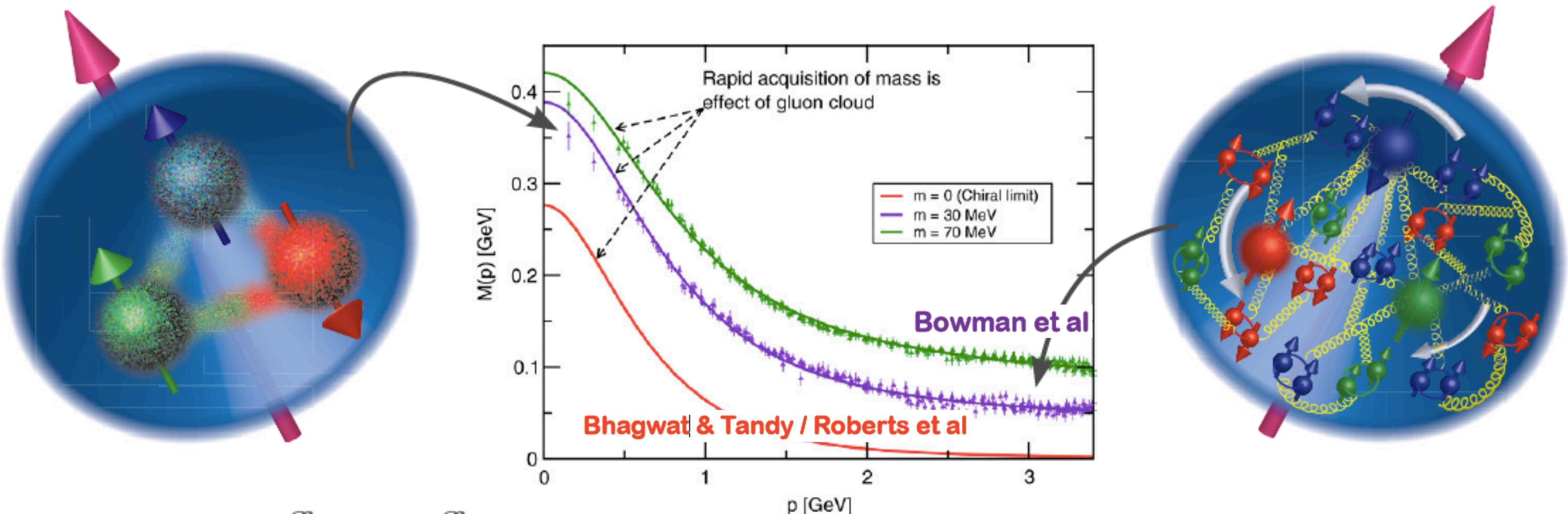

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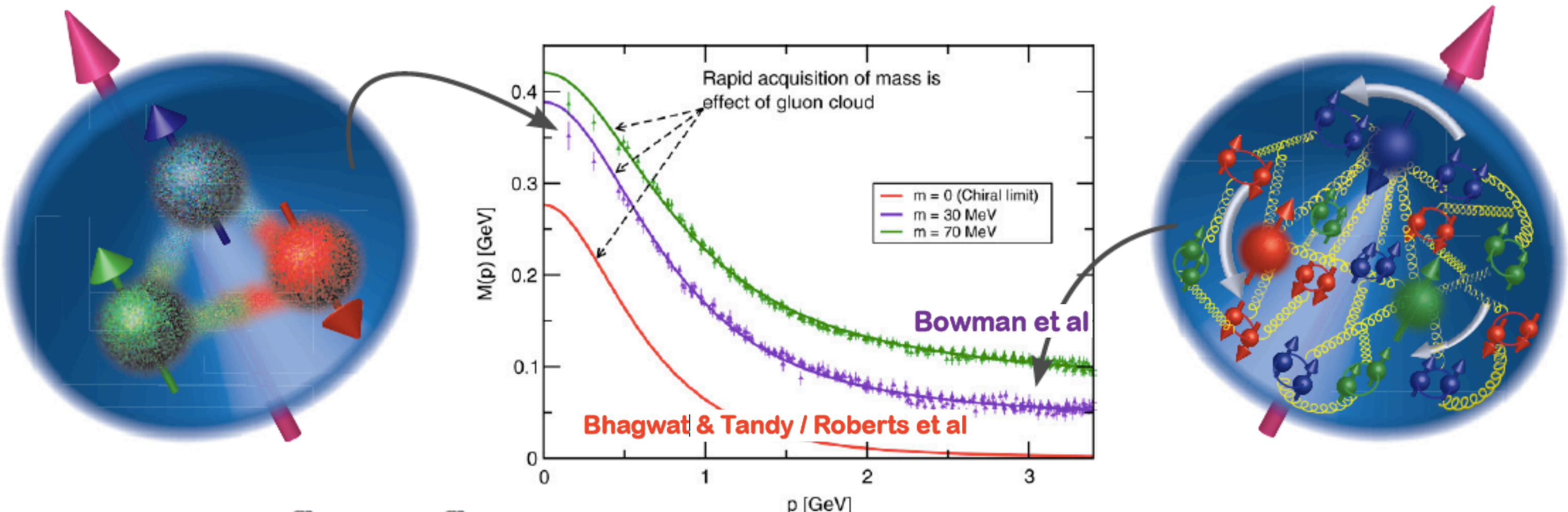

$$M_p = 2m_u^{\rm eff} + m_d^{\rm eff}$$

Relativistic motion

Quantum fluctuation

$$M_p = \left.\frac{\langle P|\int d^3x\, T^{00}|P\rangle}{\langle P|P\rangle}\right|_{\rm at\ rest} = M_q + M_g + M_m + M_a$$

Quark Energy

Gluon Energy

Quark Mass

Trace Anomaly

Decomposition – Sum Rules

- **Roles of quarks and gluons?**
  - **QCD energy-momentum tensor:**

$$T^{\mu\nu} = \overline{T^{\mu\nu}} + \widehat{T^{\mu\nu}}$$

Traceless term: $$\overline{T^{\mu\nu}} \equiv T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha}$$

Trace term: $$\widehat{T^{\mu\nu}} \equiv \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha}$$

Vacuum expectation breaks chiral symmetry

with $$T^{\alpha}_{\ \alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} m_q (1+\gamma_m) \overline{\psi}_q \psi_q$$

QCD trace anomaly $\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$

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✧ **Invariant hadron mass (in any frame):**

$$\langle p | T^{\mu\nu} | p \rangle \propto p^{\mu} p^{\nu} \quad \Rightarrow \quad \langle p | T^{\mu\nu} | p \rangle (g_{\mu\nu}) \propto p^{\mu} p^{\nu} (g_{\mu\nu}) = m^2$$

$$m^2 \propto \langle p | T^{\alpha}_{\ \alpha} | p \rangle \quad \Rightarrow \quad \frac{\beta(g)}{2g} \langle p | F^2 | p \rangle$$

➡ **At the chiral limit, the entire mass is from gluons!**

See Dima's talk on Wednesday

Decomposition – Sum Rules

- **Sum rules for Proton Mass:**

X. Ji, PRL (1995)
His talk this afternoon

Sum rules are only useful if individual terms can be measured independently

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Quark Energy Gluon Energy Quark Mass Trace Anomaly

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Quark Energy | Gluon Energy | Quark Mass | Trace Anomaly

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$|P\rangle$ **With the normalization:** $\langle P|P\rangle = (E/M)(2\pi)^3\delta^3(\mathbf{0})$

✧ **Hamiltonian:**

$$H_{\rm QCD} = \int d^3\vec{x}\, T^{00}(0,\vec{x}) \qquad \langle P|H_{\rm QCD}|P\rangle = (E^2/M_p)(2\pi)^3\delta^3(0)$$

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No $g^{\mu\nu}$ term!

$\Rightarrow$ $$\langle P|\overline{T}^{\mu\nu}|P\rangle = (P^\mu P^\nu - \frac{1}{4}M_p^2\, g^{\mu\nu})/M_p \qquad \langle P|\widehat{T}^{\mu\nu}|P\rangle = \frac{1}{4}M_p\, g^{\mu\nu}$$

$\Rightarrow$ $$\frac{\langle P| \int d^3x\, \overline{T}^{00} |P\rangle}{\langle P|P\rangle}\Big|_{\text{at rest}} = \frac{3}{4}M_p \qquad \frac{\langle P| \int d^3x\, \widehat{T}^{00} |P\rangle}{\langle P|P\rangle}\Big|_{\text{at rest}} = \frac{1}{4}M_p$$

"Traceless" term ***"Trace" term***

Decomposition – Sum Rules

❑ Identities:

$$\frac{\langle P|\int d^3x\, T^{\alpha}_{\ \alpha}|P\rangle}{\langle P|P\rangle} = 4\ \frac{\langle P|\int d^3x\, \widehat{T}^{00}|P\rangle}{\langle P|P\rangle}\Bigg|_{\text{at rest}} = \frac{4}{3}\ \frac{\langle P|\int d^3x\, \overline{T}^{00}|P\rangle}{\langle P|P\rangle}\Bigg|_{\text{at rest}}$$

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- **Traceless terms:**

$$\overline{T}^{\mu\nu} = \overline{T}^{\mu\nu}_{q} + \overline{T}^{\mu\nu}_{g} \qquad \bar{T}^{\mu\nu}_{q} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi - \frac{1}{4}g^{\mu\nu}\bar{\psi}m\psi, \qquad \bar{T}^{\mu\nu}_{g} = \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}_{\ \alpha}$$

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$$\langle P|\overline{T}^{\mu\nu}_{g}|P\rangle \equiv [1 - a(\mu^2)](P^\mu P^\nu - \frac{1}{4}M_p^2\, g^{\mu\nu})/M_P$$

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Let $\mu \to + \quad \nu \to +$

$$\Rightarrow \quad a(\mu^2) = \sum_f \int_0^1 x[q_f(x,\mu^2) + \overline{q}_f(x,\mu^2)]dx$$

Total momentum fraction carried by the quarks - reasonably known!

$$\frac{\langle P|\int d^3x\, \overline{T}^{00}_q|P\rangle}{\langle P|P\rangle}\bigg|_{\text{at rest}} = a(\mu^2)\, \frac{3}{4}\, M_p \qquad \frac{\langle P|\int d^3x\, \overline{T}^{00}_g|P\rangle}{\langle P|P\rangle}\bigg|_{\text{at rest}} = [1 - a(\mu^2)]\, \frac{3}{4}\, M_p$$

Decomposition – Sum Rules

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❑ Trace terms:

$$\widehat{T}^{\mu\nu} = \widehat{T}^{\mu\nu}_m + \widehat{T}^{\mu\nu}_a \quad \Rightarrow \quad \langle P|\widehat{T}^{\mu\nu}_m|P\rangle \equiv b\,\frac{1}{4}M_p\, g^{\mu\nu} \qquad \langle P|\widehat{T}^{\mu\nu}_a|P\rangle \equiv [1-b]\,\frac{1}{4}M_p\, g^{\mu\nu}$$

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Decomposition – Sum Rules

❑ Roles of quarks and gluons?

- ✧ **Quark energy contribution:**

$$H_q = \int d^3\vec{x}\; \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi, \qquad M_q = \left.\frac{\langle P|H_q|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (a-b)\,\frac{3}{4}\,M_p$$

- ✧ **Gluon energy contribution:**

$$H_g = \int d^3\vec{x}\; \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2) \qquad M_g = \left.\frac{\langle P|H_g|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (1-a)\,\frac{3}{4}\,M_p$$

- ✧ **Quark mass contribution:**

$$H_m = \int d^3\vec{x}\; \bar{\psi} m \psi \qquad M_m = \left.\frac{\langle P|H_m|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = b\,M_p$$

- ✧ **Trace anomaly contribution:**

$$H_a = \int d^3\vec{x}\; \frac{9\alpha_s}{16\pi}\,(\mathbf{E}^2-\mathbf{B}^2) \qquad M_a = \left.\frac{\langle P|H_a|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (1-b)\,\frac{1}{4}\,M_p$$

Decomposition – Sum Rules

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➡

$$M_p = \left.\frac{\langle P|\int d^3x\,T^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \overbrace{\underbrace{M_q}_{\text{Quark Energy}} + \underbrace{M_g}_{\text{Gluon Energy}}}^{\text{Relativistic motion}} + \overbrace{\underbrace{M_m}_{\text{Quark Mass}} + \underbrace{M_a}_{\text{Trace Anomaly}}}^{\text{Quantum fluctuation}}$$

➡ **Need to find the "*b*" independently!**

Decomposition – Sum Rules

- **Quark mass contribution – the *b*-term:**

$$b\,M_p = \langle P|m_u\overline{u}u + m_d\overline{d}d|P\rangle + \langle P|m_s\overline{s}s|P\rangle + \ldots(\text{heavy flavors})$$

- **The first term – πN σ-term:**

$$\sigma_{\pi N} = \hat{m}\langle N|\bar{u}u + \bar{d}d|N\rangle$$ **with** $$\hat{m} = (m_u + m_d)/2$$

Both lattice QCD and phenomenological analyses give

$$\sigma_{\pi N} \sim 45 - 50 \text{ MeV}$$

See talks by Cloet,
Liu, Roberts, Yang, …

Decomposition – Sum Rules

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$$\sigma_{\pi N} \sim 45 - 50 \text{ MeV}$$

- **The second term – strange scalar charge:**

Light strange quark:

$$\sigma_{KN} = (\hat{m} + m_s)\langle N|\bar{u}u + \bar{d}d + 2\bar{s}s|N\rangle/4$$

Both lattice QCD and phenomenological analyses give

$$\sigma_{KN} \sim 360 - 400 \text{ MeV}$$

See talks by Cloet,
Liu, Roberts, Yang, …

Decomposition – Sum Rules

See talks by Cloet, Roberts, Liu, Yang, …

- **Key to proton mass sum rule:**
  - Chiral symmetry breaking
  - Trace anomaly

Decomposition – Sum Rules

- Key to proton mass sum rule:
  - Chiral symmetry breaking
  - Trace anomaly
- From Ji's original paper:

See talks by Cloet, Roberts, Liu, Yang, …

Mass type	H_i	M_i	$m_s \to 0$ (MeV)	$m_s \to \infty$ (MeV)
Quark energy	$\psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi$	$3(a-b)/4$	270	300
Quark mass	$\bar{\psi}m\psi$	b	160	110
Gluon energy	$\frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2)$	$3(1-a)/4$	320	320
Trace anomaly	$\frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2)$	$(1-b)/4$	190	210

$$a(\mu^2) = \sum_f \int_0^1 x[q_f(x,\mu^2) + \bar{q}_f(x,\mu^2)]dx$$

$$bM = \langle P|m_u\bar{u}u + m_d\bar{d}d|P\rangle + \langle P|m_s\bar{s}s|P\rangle$$

Decomposition – Sum Rules

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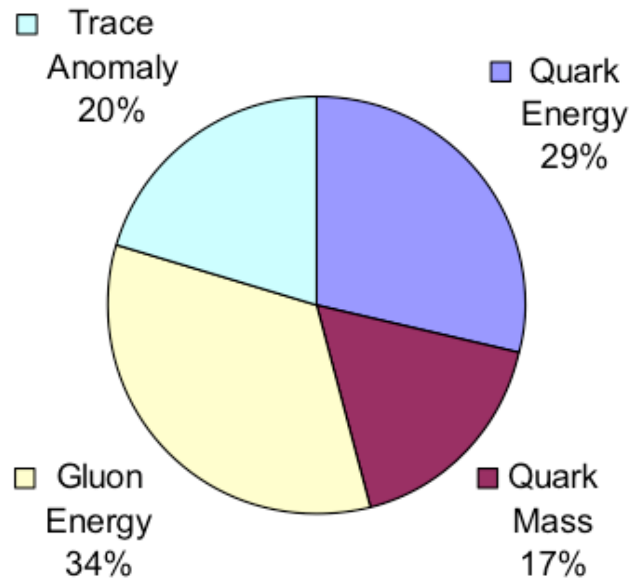

- **Lattice QCD calculation:**

Total quark fractional momentum

$$a(\mu^2) \approx 0.55$$

Test the scale dependence on lattice?

Measurement of Trace Anomaly?

See Kharzeev's talk, …

- ❑ **Trace anomaly:**
  - ✧ **Recall:** $m^2 \propto \langle p|T^{\alpha}{}_{\alpha}|p\rangle \Rightarrow \frac{\beta(g)}{2g}\langle p|F^2|p\rangle$
- ❑ **Theory background:**

  Quarkonium-proton interaction

Brodsky, Schmit de Teramond '90
Luke, Monohar, Savage '92

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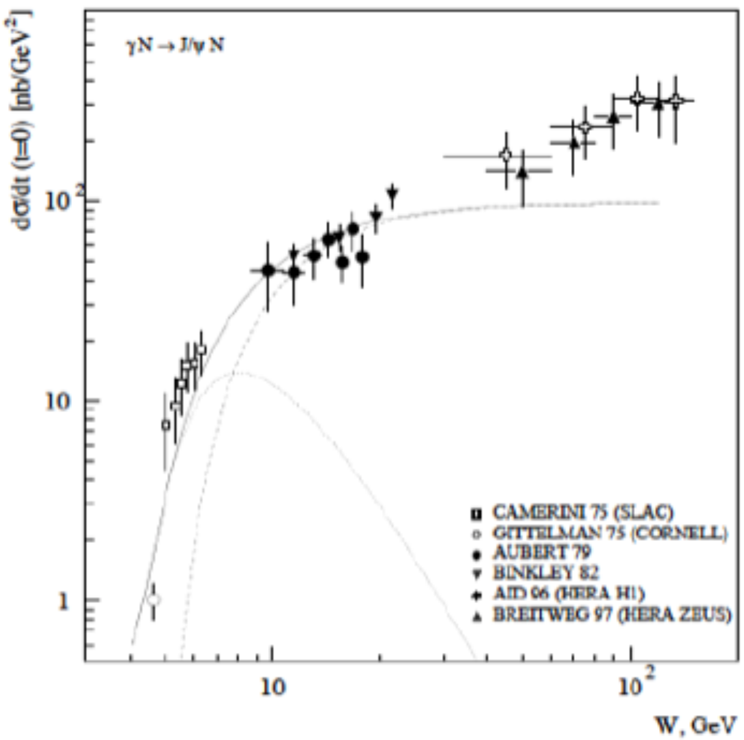
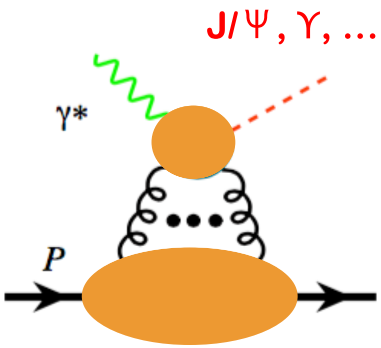
- Quarkonium production near threshold:



$$\gamma N \to J/\psi N$$



Kharzeev, Satz, Syamtomov
Zinovjev, EPJ '99

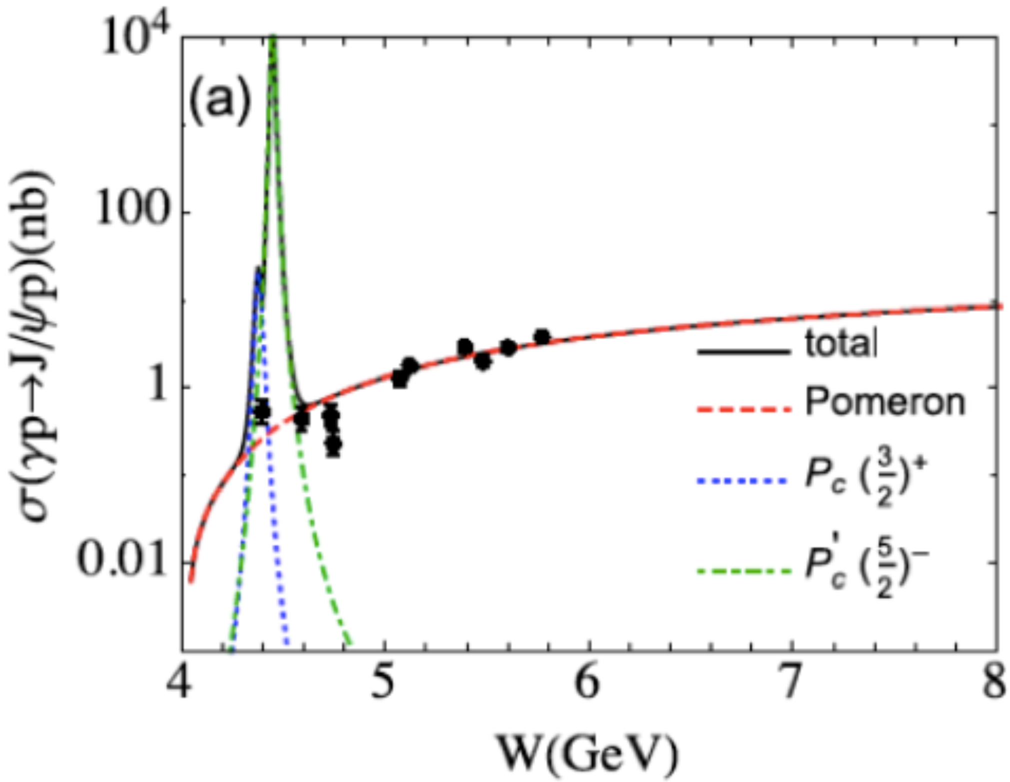
Measurement of Trace Anomaly?

See JP Chen's talk, …

❑ **Trace anomaly:**

✧ **Recall:** $m^2 \propto \langle p|T^{\alpha}{}_{\alpha}|p\rangle \Rightarrow \frac{\beta(g)}{2g}\langle p|F^2|p\rangle$

❑ **Charmonium production at Jlab near the threshold:**



Wang, Liu, Zhang
arxiv:1508.00339

Kubarovsky,
Voloshin
arxiv:1508.00888

But, the cross section may be much smaller – O(pb)
Gobbi, Boffi, DK '94

“Summary”

- The proton mass closely connected to quantum anomalies

  Non-perturbative QCD generates a new scale: $\langle 0 | F^2 | 0 \rangle \neq 0$

- Three-pronged approach to explore the origin hadron mass

  lattice QCD
  mass decomposition – roles of the constituents
  approximated analytical approach

- Questions:

  - What can lattice QCD do to explore the role of “individual” constituent in making up the proton mass?
  - What can the mass decomposition teach us?
  - How well can we control the approximation of the analytical or model approaches
  - …

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  ✧ …

Let's work together at this workshop