Bound states in perturbation theory

Proton Mass: ECT* April 3-7, 2017

Paul Hoyer
University of Helsinki

Aim: Address striking features of hadron data within QCD:

- $q\bar{q}$ and qqq quantum numbers, even for relativistic states (π, ρ, N,...)
- Freezing of gluon degrees of freedom at low scales (hybrids, glueballs)
- OZI rule: $\phi(1020) \to K\bar{K} \gg \phi(1020) \to \pi\,\pi\,\pi$
- Quark $\leftrightarrow$ hadron duality (DIS, e^+e^-, hh, ...)

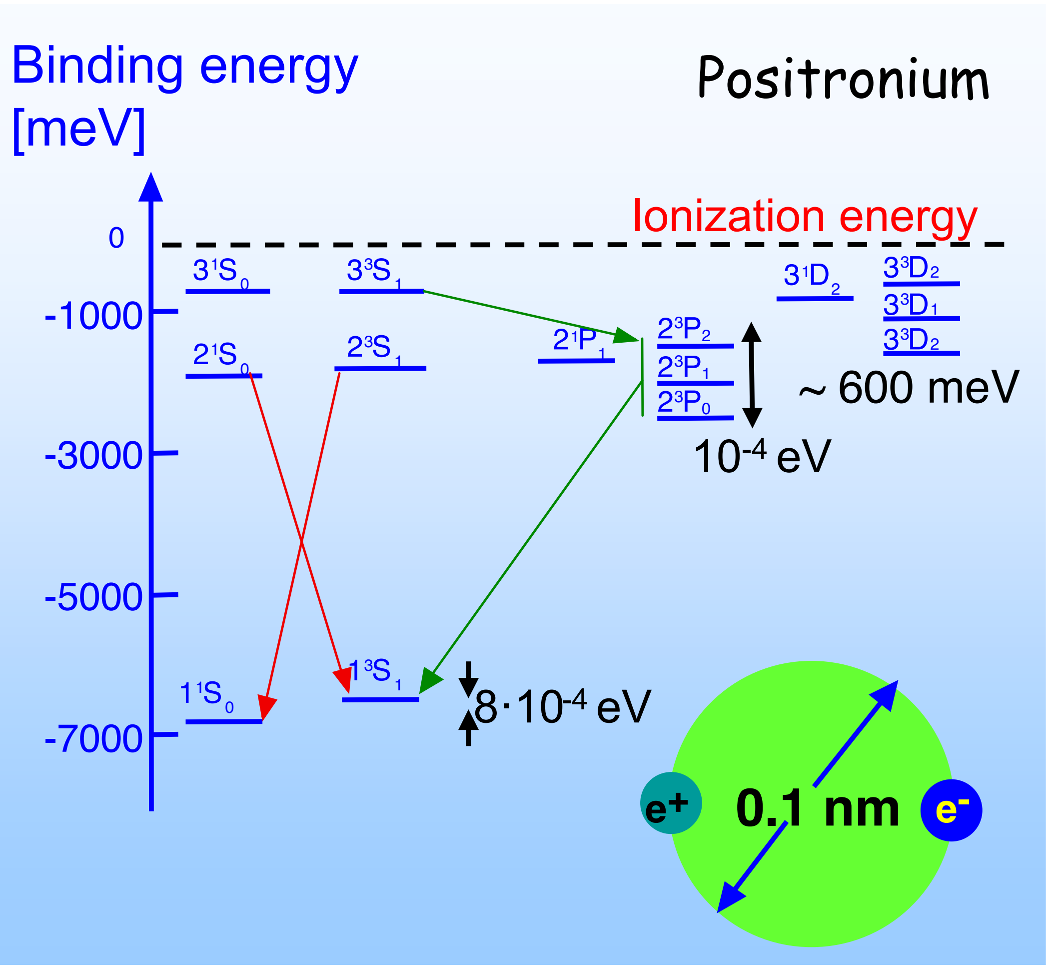
At face value: These phenomena indicate a weak coupling dynamics.

How is this consistent with relativistic binding and confinement?

How to proceed?

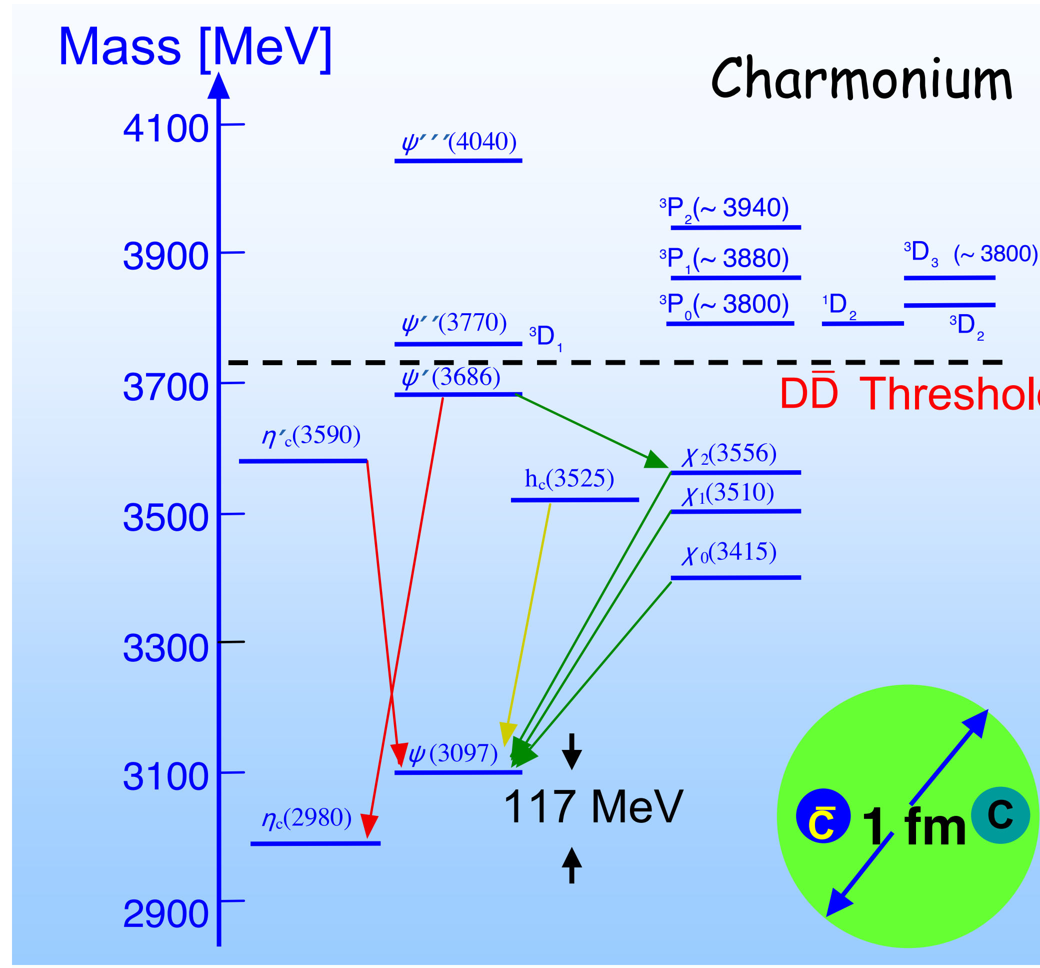
"The J/ψ is the Hydrogen atom of QCD"

QED



$$V(r) = -\frac{\alpha}{r}$$

QCD



$$V(r) = c\,r - \frac{4}{3}\frac{\alpha_s}{r}$$

PQED works for atoms

Example: Hyperfine splitting in Positronium

$$\Delta\nu_{QED} = m_e\alpha^4\left\{\frac{7}{12}-\frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right.$$
$$+\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha+\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2-\frac{53}{32}\zeta(3)\right]$$
$$\left.-\frac{7\alpha^3}{8\pi}\ln^2\alpha+\frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2-\frac{217}{90}\right)+\mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}$$

M. Baker et al, 1402.0876

A. Ishida et al, 1310.6923 : $\Delta\nu_{EXP} = 203.3941 \pm .003$ GHz

- Binding energy is perturbative in α and $\log(\alpha)$
- Wave function $\psi(r) \propto \exp(-m\alpha r)$ is of $\mathcal{O}(\alpha^\infty)$

How can all powers of α arise in a perturbative expansion?

Master formula for perturbative S-matrix

$$S_{fi} = {}_{\mathbf{out}}\langle f|\left\{\mathrm{T}\exp\left[-i\int_{-\infty}^{\infty} dt\, H_I(t)\right]\right\}|i\rangle_{\mathbf{in}}$$

Generates Feynman diagrams to arbitrary order for any scattering process

The free *in*- and *out*-states at $t = \pm\infty$ must overlap the physical i, f states.

$$\boxed{\textbf{Bound states have no overlap with free } \textit{in}\textbf{- and } \textit{out}\textbf{-states at } t = \pm\infty}$$

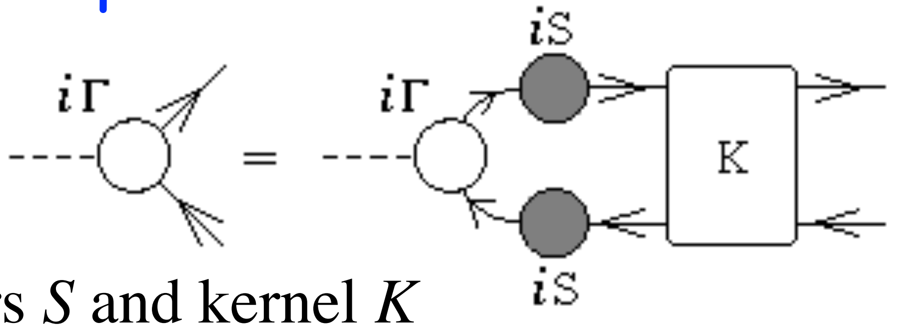
No finite order Feynman diagram for $e^+e^- \to e^+e^-$ has a positronium pole.

We need to perturbatively expand around a proto-bound state:

The first approximation is already of $\mathcal{O}(\alpha^\infty)$

- How do we choose the proto-atom in QED?
- Can we find a proto-hadron in QCD, with perturbative, $\mathcal{O}(\alpha_s)$ corrections?

Brief history of the proto-atom



1951: Salpeter & Bethe

Perturbatively expand propagators S and kernel K

Explicit Lorentz covariance ensured

1975: Caswell & Lepage: Not unique: ∞ # of equivalent equations, $S \leftrightarrow K$

1986: Caswell & Lepage NRQED: Effective NR field theory

Relativistic electrons are rare in atomic wave functions

Today: Accurate calculations of atomic properties use NRQED

Explicit Lorentz covariance is traded for physical arguments.
QED ensures validity of a rest frame calculation in any frame

The proto-atom (starting point of the expansion) is taken to be:

The solution of the Schrödinger equation with $V(r) = -\alpha/r$

Why choose the classical potential?

The $\hbar$ expansion

Recall functional integral formulation of QFT:

$$G(x_1,\ldots,x_n) = \int [d\varphi]\,\exp\left(iS[\varphi]/\hbar\right)\varphi(x_1)\ldots\varphi(x_n)$$

The limit $\hbar \to 0$ gives classical field eqs: $$\frac{\delta S[\varphi]}{\delta\varphi} = 0$$

Higher orders in $\hbar$ correspond to loop corrections in Feynman diagrams.

The S-atom with a classical potential is the Born term of the physical state:

$\mathcal{O}(\hbar^0)$ No photon loop corrections
No fermion loop corrections (NR) $\mathcal{O}(\alpha^\infty)$

The Born approximation applies also to relativistic dynamics. QCD?

Lattice QCD: Quenched approximation (1)

Neglecting quark loops gives the light hadron spectrum at 10% accuracy

Light hadron spectrum in quenched approximation

	Expt.	m_K input Mass (GeV)	Deviation
K	0.4977	$\cdots$	$\cdots$
K^*	0.8961	0.858(09)	-4.2% 4.3σ
ϕ	1.0194	0.957(13)	-6.1% 4.8σ
N	0.9396	0.878(25)	-6.6% 2.5σ
Λ	1.1157	1.019(20)	-8.6% 4.7σ
Σ	1.1926	1.117(19)	-6.4% 4.1σ
Ξ	1.3149	1.201(17)	-8.7% 6.8σ
Δ	1.2320	1.257(35)	2.0% 0.7σ
Σ^*	1.3837	1.359(29)	-1.8% 0.9σ
Ξ^*	1.5318	1.459(26)	-4.7% 2.8σ
Ω	1.6725	1.561(24)	-6.7% 4.7σ

S. Aoki, et al., Phys. Rev. Lett. 84 (2000) 238

Lattice QCD: Quenched approximation (2)

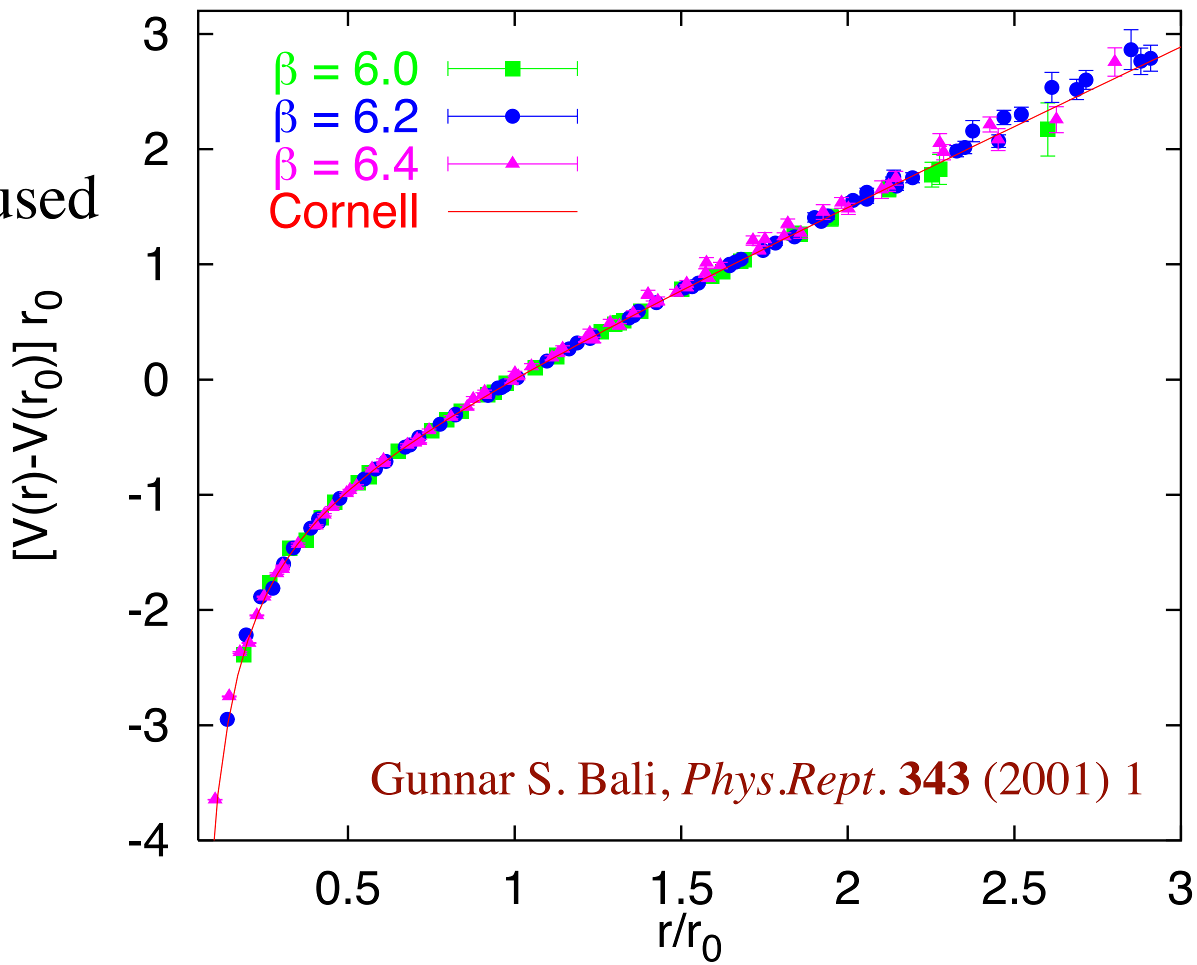
What about gluon loops?

The static (heavy quark) potential agrees with the Cornell potential used in the Schrödinger equation.

Consistent with dominance of a *classical* gluon field

Neglecting quark and gluon loops gives a reasonable approximation of hadron physics at low scales.

The quenched Wilson action SU(3) potential.



⇒ The Born approximation of QCD maintains confinement and chiral symmetry breaking.

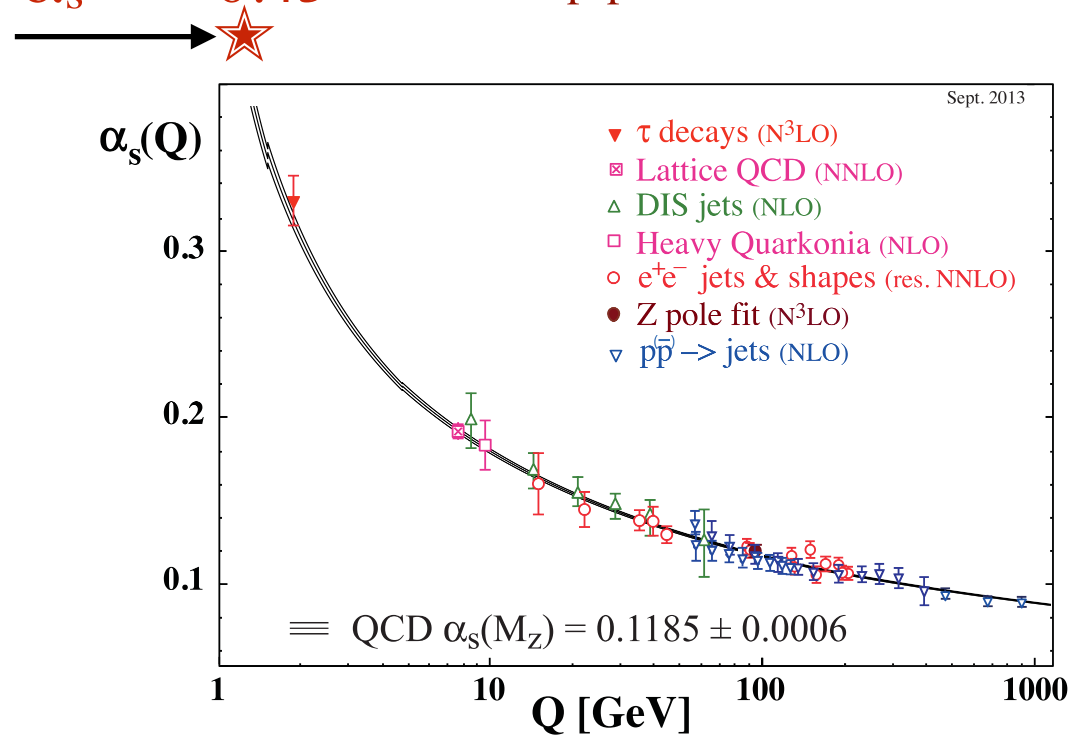
Two consequences of $\hbar \to 0$ in QCD

1. In the absence of loops,
   α_s stops running

   Gribov and others have argued that $\alpha_s(0)/\pi \approx 0.14$

   This makes PQCD corrections to proto-hadrons relevant.

$\alpha_s^{crit} \approx 0.43$ Gribov hep-ph/9902279



2. In the absence of loops, the QCD scale Λ_{QCD} cannot arise from loop renormalization.

Λ_{QCD} must come from a boundary condition to the classical field equations.

Advantages of the Born approximation

Systematic: Lowest term of first principles expansion in $\hbar$

Simplicity: Enables analytic approach to bound states

Relevance: Loop expansion works for scattering amplitudes and Positronium

Symmetries: Hold at each order of $\hbar$ (Poincaré, gauge invariance,…)

Unitarity: Valid at each order of $\hbar$ (hadron level!)

Implementation:

1. Positronium
2. Dirac *states*
3. Hadrons

Find unexpected features, including duality.

arXiv:1612.09463
arXiv:1605.01532

Positronium: Classical photon field

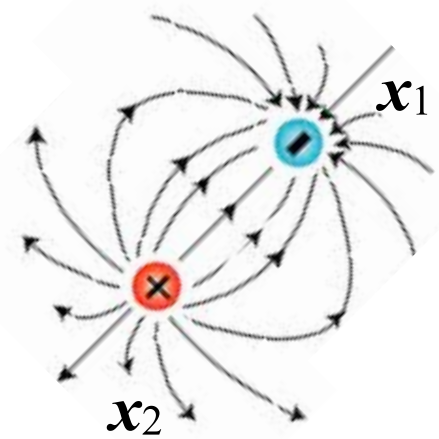
Non-relativistic dynamics: $A^j/A^0 = \mathcal{O}(\alpha)$: Transverse photons freeze

$$\frac{\delta \mathcal{S}_{QED}}{\delta \hat{A}^0(t,\boldsymbol{x})} = 0 \qquad \Rightarrow \qquad -\boldsymbol{\nabla}^2 \hat{A}^0(t,\boldsymbol{x}) = e\psi^\dagger(t,\boldsymbol{x})\psi(t,\boldsymbol{x})$$

The eigenvalue of the $\hat{A}^0$ field operator for $\big|e^-(\boldsymbol{x}_1)\, e^+(\boldsymbol{x}_2)\big\rangle$

is the classical field:

$$eA^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_2|}$$



Note: A^0 is determined instantaneously for all $\boldsymbol{x}$
It depends on $\boldsymbol{x}_1, \boldsymbol{x}_2$
No gauge fixing is necessary at this level

$\Longrightarrow$ **Only the electron field operator appears at $\mathcal{O}(\hbar^0)$**

Positronium state at rest

In terms of the Schrödinger wave function $\varphi_n(\boldsymbol{k})$:

$$|n;\boldsymbol{P}=0\rangle = \int \frac{d\boldsymbol{k}}{(2\pi)^3}\,\varphi_n(\boldsymbol{k})\,b_\lambda^\dagger(\boldsymbol{k})\,d_{\bar\lambda}^\dagger(-\boldsymbol{k})\;|0\rangle$$

$$= \int d\boldsymbol{x}_1\,d\boldsymbol{x}_2\,\bar\psi(t,\boldsymbol{x}_1)_\alpha\,\Phi_n^{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\,\psi(t,\boldsymbol{x}_2)_\beta\,|0\rangle$$

where

$$\Phi_n^{\alpha\beta}(\boldsymbol{k}) \equiv \big[\gamma^0 u(\boldsymbol{k},\lambda)\big]_\alpha v_\beta^\dagger(-\boldsymbol{k},\bar\lambda)\varphi_n(\boldsymbol{k})$$

Bound state condition

$$\mathcal{H}_{QED}\;|n;\boldsymbol{P}=0\rangle = (2m_e+E_b)\;|n;\boldsymbol{P}=0\rangle$$

$$\mathcal{H}_{QED}(t) = \int d\boldsymbol{x}\,\psi^\dagger(t,\boldsymbol{x})\left[-i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0 + \tfrac{1}{2}e\mathcal{A}^0(\boldsymbol{x})\right]\psi(t,\boldsymbol{x})$$

where $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$ and $e\mathcal{A}^0(\boldsymbol{x}) = \int d\boldsymbol{y}\,\frac{\alpha}{|\boldsymbol{x}-\boldsymbol{y}|}\,\psi^\dagger(t,\boldsymbol{y})\psi(t,\boldsymbol{y})$

The factor $\frac{1}{2}$ is due to the field energy,

$$\int d\boldsymbol{x}\,\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} = \tfrac{1}{2}\int d\boldsymbol{x}\,\mathcal{A}^0\cdot\boldsymbol{\nabla}^2\mathcal{A}^0 = -\tfrac{1}{2}\int d\boldsymbol{x}\,\bar{\psi}(\boldsymbol{x})e\mathcal{A}^0\psi(\boldsymbol{x})$$

Using $\left\{\psi_\alpha^\dagger(t,\boldsymbol{x}),\psi_\beta(t,\boldsymbol{y})\right\} = \delta_{\alpha,\beta}\,\delta^3(\boldsymbol{x}-\boldsymbol{y})$ and neglecting e^+e^- pair production/annihilation

the Schrödinger equation follows: $\left(-\frac{\boldsymbol{\nabla}^2}{m_e} - \frac{\alpha}{|\boldsymbol{x}|}\right)\varphi_n(\boldsymbol{x}) = E_b\,\varphi_n(\boldsymbol{x})$

Positronium in motion

The $\mathcal{O}(\hbar^0)$ contribution is not limited to non-relativistic dynamics.

Positronium states with CM momentum $\boldsymbol{P} = (0, 0, P)$ are obtained by boosts $U(\xi)$ along the z-axis, which transforms the $(t = 0)$ fields as

$$U(\xi)\psi(0,\boldsymbol{x})U^\dagger(\xi) = e^{-\xi\alpha_3/2}\psi(z\sinh\xi,\boldsymbol{x})$$

Note: The transformed time depends on z.
States of any momentum $\boldsymbol{P}$ are defined at equal time.
Boost covariance emerges via the QED dynamics.

An infinitesimal boost $U(d\xi) = 1 - id\xi\,\mathcal{K}$ generated by

$$\mathcal{K}(t=0) = -\int d\boldsymbol{x}\,\psi^\dagger(0,\boldsymbol{x})\left[z(-i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0 + \tfrac{1}{2}\gamma^0 e\not{A}) - \tfrac{1}{2}i\alpha_3\right]\psi(0,\boldsymbol{x})$$

$$U(d\xi)\,|n; P = 0\rangle = |n; P = M\,d\xi\rangle$$

Positronium state with finite momentum P

In a finite boost to $P = M\sinh\xi$ the rest frame state turns into:

$$|n;P\rangle \equiv U(\xi)\,|n;P=0\rangle =$$

$$= \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}(0,\boldsymbol{x}_1)\, e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\, e^{-\xi\alpha_3/2}\, \Phi_n^{(\xi)}(\boldsymbol{x}_1-\boldsymbol{x}_2)\, e^{\xi\alpha_3/2}\, \psi(0,\boldsymbol{x}_2)\,|0\rangle$$

where the wave function is Lorentz contracted: $\Phi_n^{(\xi)}(x,y,z/\cosh\xi) = \Phi_n^{(0)}(x,y,z)$

$|n;P\rangle$ is an eigenstate of the momentum $\mathcal{P}$ and energy $\mathcal{H}$ operators, with the appropriate (boosted) eigenvalues.

The A^0 field in QCD

The $\mathcal{A}_a^0$ gluon field is instantaneous: No $\partial_t \mathcal{A}_a^0$ term in $\mathcal{L}_{QCD}$.

Loophole in the Positronium treatment:

Gauss' law for $\mathcal{A}_a^0$ has also homogeneous solutions.

A non-vanishing boundary condition for $\mathcal{A}_a^0$ at $|\boldsymbol{x}| = \infty$ introduces Λ_{QCD} and makes $\mathcal{A}_a^0$ of $\mathcal{O}\left(\alpha_s^0\right)$, thus dominating $\mathcal{A}_a^j$, which is of $\mathcal{O}\left(g\right)$.

$A^j/A^0 = \mathcal{O}(g)$ holds even for relativistic dynamics: Transverse gluons freeze.

At $\mathcal{O}\left(\alpha_s^0\right)$ the non-abelian contributions vanish: $f_{abc}\,\mathcal{A}_a^0\,\mathcal{A}_b^0 = 0$

and Gauss' law allows to express $\mathcal{A}_a^0$ in terms of only the quark fields:

$$-\boldsymbol{\nabla}^2 \mathcal{A}_a^0(t,\boldsymbol{x}) = g\,\psi_A^\dagger(t,\boldsymbol{x}) T_a^{AB} \psi_B(t,\boldsymbol{x})$$

Homogeneous solutions: $\boldsymbol{\nabla}^2 \mathcal{A}_a^0(t,\boldsymbol{x}) = 0$

A confining gluon field for QCD

Translation invariance requires $\boldsymbol{E} = \nabla A^0$ to be $\boldsymbol{x}$-independent:

$$A_a^0(t,\boldsymbol{x}) = \kappa \int d\boldsymbol{y}\, \psi_A^\dagger(t,\boldsymbol{y}) T_a^{AB} \psi_B(t,\boldsymbol{y})\, \boldsymbol{x}\cdot\boldsymbol{y}$$

$$\kappa \neq \kappa(\boldsymbol{x})$$

$$\boldsymbol{\nabla}^2 \mathcal{A}_a^0(t,\boldsymbol{x}) = 0$$

The $q\bar{q}$ states are expressed as for Positronium:

$$|n, P=0\rangle = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}_A(\boldsymbol{x}_1)\, \Phi_n^{AB}(\boldsymbol{x}_1 - \boldsymbol{x}_2)\, \psi_B(\boldsymbol{x}_2)\, |0\rangle$$

with a (globally) color singlet wave function: $\Phi_n^{AB}(\boldsymbol{x}) = \dfrac{\delta^{AB}}{\sqrt{N_c}}\, \Phi_n(\boldsymbol{x})$

Color singlet states do not generate a color field: $\mathcal{A}_a^0(\boldsymbol{x})\, |n, P\rangle = 0$

$\boxed{\text{The color field is invisible to an external observer (unlike in QED!)}}$

Nevertheless, each $q\bar{q}$ color component is bound by a linear potential:

$$V(\boldsymbol{x}) = \tfrac{1}{2}\sqrt{C_F}\, g\Lambda^2 |\boldsymbol{x}|$$

Relativistic dynamics: consider Dirac states

$$\big(-i\boldsymbol{\nabla}\cdot\boldsymbol{\gamma}+m+e\not{A}\big)\phi_n(\boldsymbol{x})=E_n\gamma^0\phi_n(\boldsymbol{x}) \qquad E_n>0$$

$$\big(-i\boldsymbol{\nabla}\cdot\boldsymbol{\gamma}+m+e\not{A}\big)\bar\phi_n(\boldsymbol{x})=-\bar{E}_n\gamma^0\bar\phi_n(\boldsymbol{x}) \qquad \bar{E}_n>0$$

Dirac eq. in a *fixed* external field $A^\mu(\boldsymbol{x})$

What states do the Dirac wave functions ϕ, $\bar\phi$ describe?

Need to diagonalize the Dirac Hamiltonian,

J.-P. Blaizot, PH (2015)

$$H=\int d^3\boldsymbol{x}\bar\psi(\boldsymbol{x})\big[-i\boldsymbol{\nabla}\cdot\boldsymbol{\gamma}+m+e\not{A}(\boldsymbol{x})\big]\psi(\boldsymbol{x})$$

$$H\left|n\right\rangle=E_n\left|n\right\rangle \qquad \left|n\right\rangle=\int d\boldsymbol{x}\,\psi_\alpha^\dagger(\boldsymbol{x})\phi_{n\,\alpha}(\boldsymbol{x})\left|\Omega\right\rangle\equiv c_n^\dagger\left|\Omega\right\rangle$$

$$H\left|\bar n\right\rangle=\bar{E}_n\left|\bar n\right\rangle \qquad \left|\bar n\right\rangle=\int d\boldsymbol{x}\,\bar\phi_{n\,\alpha}^\dagger(\boldsymbol{x})\psi_\alpha(\boldsymbol{x})\left|\Omega\right\rangle\equiv \bar{c}_n^\dagger\left|\Omega\right\rangle$$

The "valence" e^- and e^+ determine the single particle quantum numbers.

The vacuum $\left|\Omega\right\rangle$ is a superposition of e^-e^+ pairs.

The Dirac ground state $|\Omega\rangle$

The state operators are Bogoliubov transforms of the free operators:

$$c_n = \sum_{\boldsymbol{p}} \phi_n^\dagger(\boldsymbol{p})\big[u(\boldsymbol{p})b_{\boldsymbol{p}} + v(-\boldsymbol{p})d_{-\boldsymbol{p}}^\dagger\big] \equiv B_{np}b_p + D_{np}d_p^\dagger$$

$$\bar{c}_n = \sum_{\boldsymbol{p}} \big[b_{\boldsymbol{p}}^\dagger u^\dagger(\boldsymbol{p}) + d_{-\boldsymbol{p}}v^\dagger(-\boldsymbol{p})\big]\bar{\phi}_n(\boldsymbol{p}) \equiv \bar{B}_{np}b_p^\dagger + \bar{D}_{np}d_p$$

They diagonalize the Dirac Hamiltonian: $H = \sum_n \big[E_n c_n^\dagger c_n + \bar{E}_n \bar{c}_n^\dagger \bar{c}_n\big]$

The ground state $\quad |\Omega\rangle = N_0 \exp\Big[-b_p^\dagger\big(B^{-1}\big)_{pm} D_{mq} d_q^\dagger\Big]|0\rangle$

is a superposition of e^+e^- pairs which satisfies

$$c_n\,|\Omega\rangle = \bar{c}_n\,|\Omega\rangle = H\,|\Omega\rangle = 0$$

Dirac states for a linear potential in D=1+1 dimensions

The linear potential confines e^-, repels e^+ : $V(e^+) = -V(e^-) = -\frac{1}{2}e^2|x|$

Positrons with kinetic energies $T \sim \frac{1}{2}e^2|x|$ are allowed at large $|x|$.

The accelerating/decelerating positrons have a continuous energy spectrum.

$\boxed{\text{The Dirac states have a continuous energy spectrum.}}$

AUGUST 1, 1932 *PHYSICAL REVIEW* *VOLUME 41*

The Dirac Electron in Simple Fields*

By MILTON S. PLESSET
Sloane Physics Laboratory, Yale University

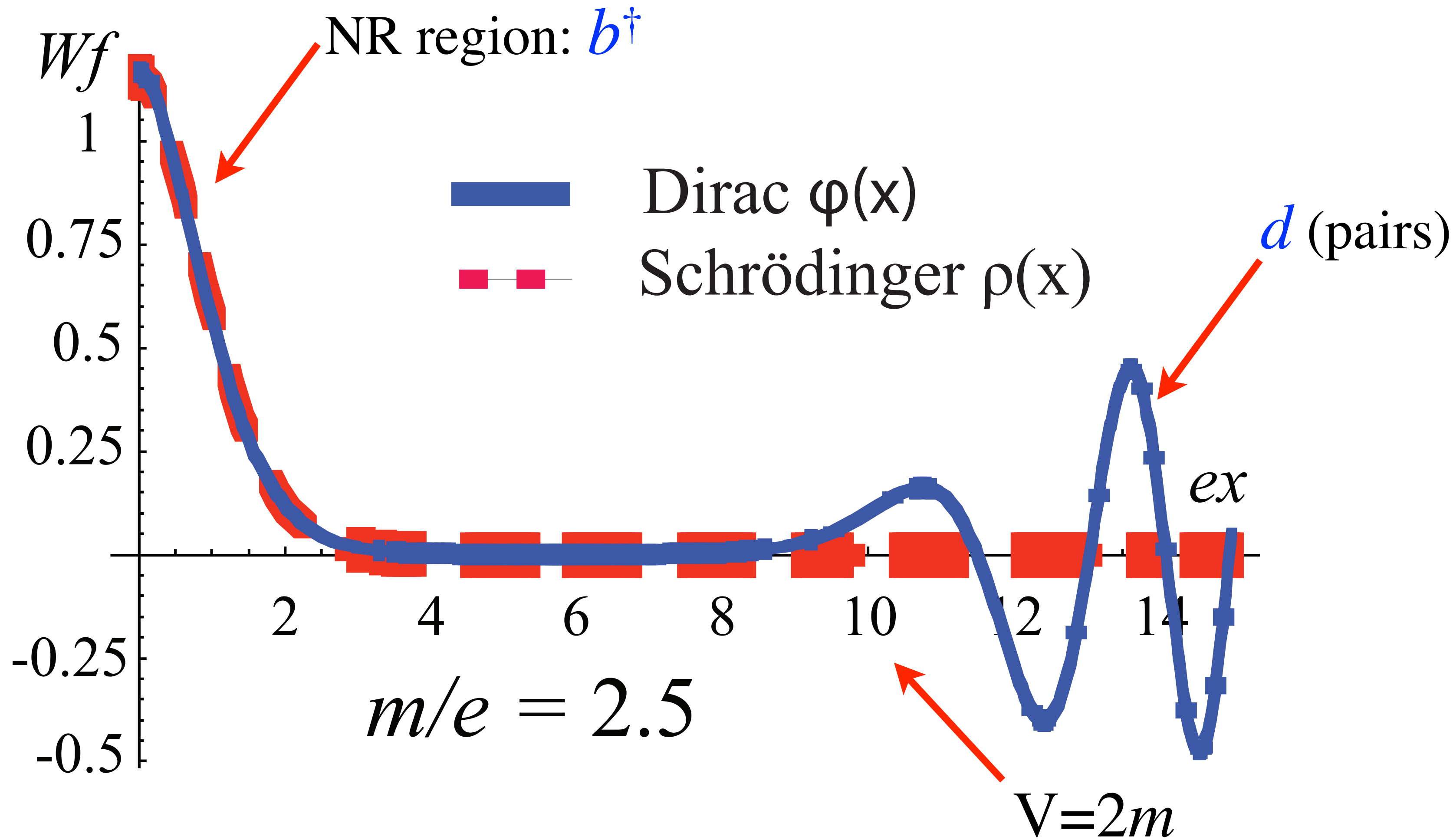
(Received June 6, 1932)

The relativity wave equations for the Dirac electron are transformed in a simple manner into a symmetric canonical form. This canonical form makes readily possible the investigation of the characteristics of the solutions of these relativity equations for simple potential fields. If the potential is a polynomial of any degree in x, a continuous energy spectrum characterizes the solutions. If the potential is a polynomial of any degree in $1/x$, the solutions possess a continuous energy spectrum when the energy is numerically greater than the rest-energy of the electron; values of the energy numerically less than the rest-energy are barred. When the potential is a polynomial of any degree in r, all values of the energy are allowed. For potentials which are polynomials in $1/r$ of degree higher than the first, the energy spectrum is again continuous. The quantization arising for the Coulomb potential is an exceptional case.

See also: E. C. Titchmarsh, Proc. London Math. Soc. (3) 11 (1961) 159 and 169; Quart. J. Math. Oxford (2), 12 (1961), 227.

Example of Dirac wave functions: $V(x) = \frac{1}{2}e^2|x|$ in D=1+1

$$|M \geq 0\rangle = \int \frac{dp}{2\pi 2E} \int dx \left[b_p^\dagger u^\dagger(p) e^{-ipx} + d_p v^\dagger(p) e^{ipx} \right] \begin{bmatrix} \varphi(x) \\ \chi(x) \end{bmatrix} |\Omega\rangle$$



The "single particle" Dirac wave function contains pair contributions (duality)

$q\bar{q}$ bound states

Analogously to Positronium, take

$$|q\bar{q};\boldsymbol{P}=0\rangle = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}_A(t,\boldsymbol{x}_1)\,\Phi^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_B(t,\boldsymbol{x}_2)\;|0\rangle$$

with $$\Phi^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2) = \frac{\delta^{AB}}{\sqrt{N_C}}\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$$

and $$A_a^0(t,\boldsymbol{x}) = \kappa\int d\boldsymbol{y}\,\psi_A^\dagger(t,\boldsymbol{y})T_a^{AB}\psi_B(t,\boldsymbol{y})\,\boldsymbol{x}\cdot\boldsymbol{y}$$

The bound state condition $\mathcal{H}_{QCD}\,|q\bar{q}\rangle = M\,|q\bar{q}\rangle$ requires

$$i\boldsymbol{\nabla}\cdot\left\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\right\} + m\left[\gamma^0,\Phi(\boldsymbol{x})\right] = \left[M - V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

with $V(\boldsymbol{x}) = \frac{1}{2}\sqrt{C_F}\,g\Lambda^2|\boldsymbol{x}|$

$q\bar{q}$ wave functions

The separation of angular and radial coordinates in the BSE

$$i\boldsymbol{\nabla}\cdot\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\}+m\left[\gamma^0,\Phi(\boldsymbol{x})\right]=\left[E-V(r)\right]\Phi(\boldsymbol{x})$$

for any radial potential $V = V(r)$ and
equal fermion masses $m_1 = m_2 = m$ is in: Geffen and Suura, PRD 16 (1977) 3305

The solutions of given spin j and j_z are classified according to their charge conjugation C and parity P quantum numbers:

pion trajectory: $P=(-1)^{j+1}$ $C=(-1)^j$

a_1 trajectory: $P=(-1)^{j+1}$ $C=(-1)^{j+1}$

rho trajectory: $P=(-1)^j$ $C=(-1)^j$

There are no "quark model exotics" with $P=(-1)^j$ and $C=(-1)^{j+1}$

π, a_1 and ρ spectra

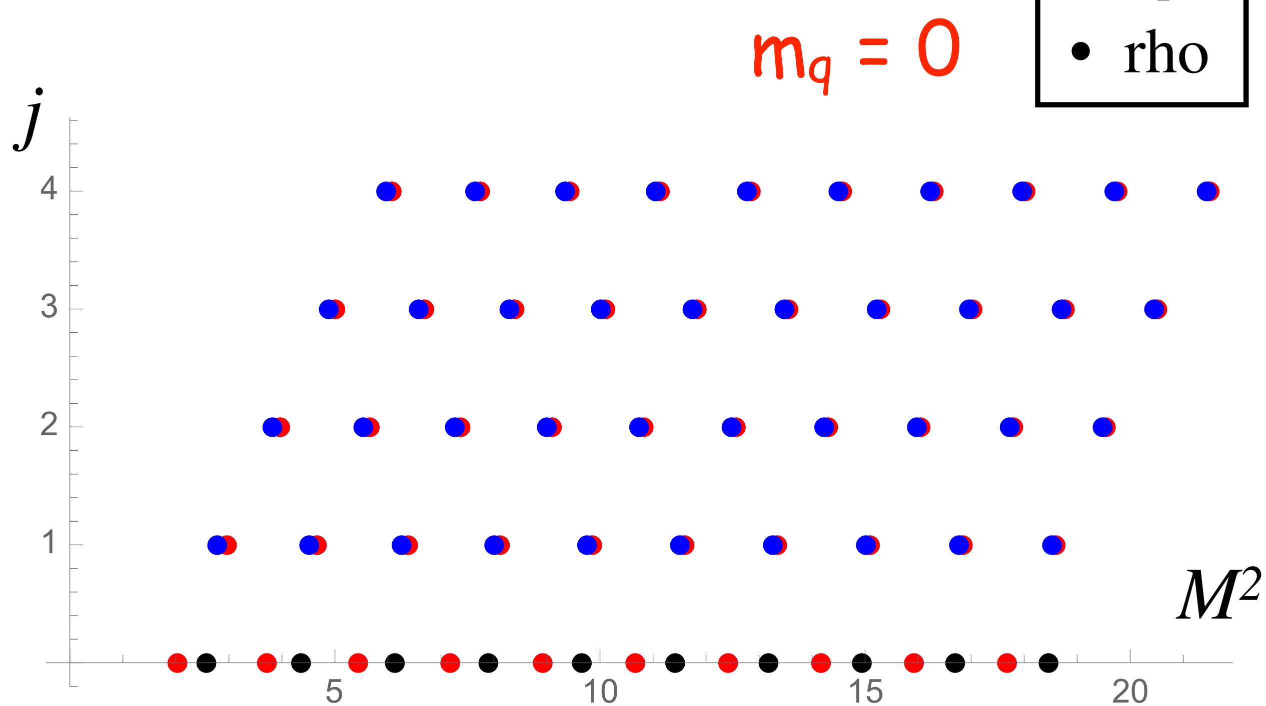
The π, a_1 and ϱ trajectories are nearly linear

There are parallel daughter trajectories

Mass from dynamics!

Spectrum similar to dual models

Chiral symmetry is unbroken.



There are also $M = 0$ states.

The massless 0^{++} (σ) state has vacuum quantum numbers.

Its mixing with the chirally symmetric vacuum would cause chiral symmetry breaking.

Promising prospects

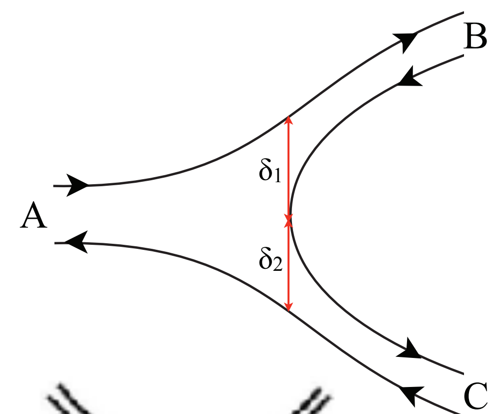
The approach is guided by:

- Phenomenological observations
- QCD framework: $\hbar$ expansion

Open issues, not yet sufficiently studied:

- Boost covariance for relativistic dynamics
- Phenomenology, e.g., DIS (done in D=1+1)
- Chiral symmetry breaking

- String breaking (determined by $q\bar{q}$ states)
- Hadron loops, unitarity



- Quark-hadron duality (seen in D=1+1)
- Hadron scattering amplitudes