# QCD at the heart of the Standard Model 

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## Introduction and basic questions

- Standard model :) \&
- QCD \& Confinement? Chiral symmetry; left-right (a)symmetry
- Family structure
- Space time: $\mathrm{E}(1,3)$
- Mass ranges in standard model?

■ Naturalness? Supersymmetry?

- B-L? dark matter, gravity

■ But also, e.g. collinearity in QCD (jets)?

- Look for solution in vanishing dimensions

■ Stojkovic - 1406.2696
■ Calcagni - 0912.3142
■ PJM - 1601.0300

## Basic symmetries

- Translations:
$\left\{P^{\mu}\right\}:\left\{P^{+}, P^{-}, \ldots\right\}$ or $\{H, P\}$

$$
\left[P^{\mu}, P^{\nu}\right]=0
$$

■ Lorentz transformations

$$
\begin{array}{ll}
\left\{M^{\mu \nu}\right\}:\left\{M^{0 i}=K, M^{i j}=J\right\} & {[J, P]=P,[K, P]=P,} \\
M^{2}=g_{\mu \nu} P^{\mu} P^{\nu} & {[J, K]=K,[J, J]=J,[K, K]=J}
\end{array}
$$

- $P(1,3)=P(1,1) \bowtie S O(3)$
- Hilbert space
$\left\{\left(a^{\dagger}\right)^{n}|0\rangle, b^{\dagger}|0\rangle\right\}$

$$
\left[a, a^{\dagger}\right]=1, \quad\left\{b, b^{\dagger}\right\}=1
$$

■ Supercharges

$$
\begin{array}{ll}
|0\rangle=|0\rangle_{B} \otimes|0\rangle_{F} & Q=a^{\dagger} b-b^{\dagger} a \\
Q_{i k}^{\dagger}=b_{i} a_{k}^{\dagger} \text { and } Q_{i k}=b_{i}^{\dagger} a_{k} & P=a^{\dagger} a+\frac{1}{2}+b^{\dagger} b-\frac{1}{2} \\
a_{k}^{\dagger} \xrightarrow{Q_{i k}} b_{i}^{\dagger} \quad a_{k}^{\dagger} \stackrel{Q_{i k}^{\dagger}}{\rightleftarrows} b_{i}^{\dagger} & \left\{Q, Q^{\dagger}\right\}=2 P
\end{array}
$$

■ Extension to 'aligned' excitations (cf 3D harmonic oscillator)

$$
|0\rangle=|0,0,0\rangle_{B} \otimes|0,0,0\rangle_{F}
$$

■ Conformal embedding!

## Space-time and internal degrees of freedom

■ Harmonic oscillator levels $(\mathrm{SO}(3) \leftrightarrow \rightarrow$ internal symmetry \& more symmetry)

| level | degeneracy | $\left(n_{x}, n_{y}, n_{z}\right)$ | $\mathrm{SO}(3)(\ell)$ | $\mathrm{SU}(3)(\underline{n})$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | $(0,0,0)$ | 0 | $\underline{1}$ |
| 1 | 3 | $(1,0,0), \ldots$ | 1 | $\underline{3}$ |
| 2 | 6 | $(2,0,0),(1,1,0), \ldots$ | $0 \oplus 2$ | $\underline{6}$ |
| 3 | 10 | $(3,0,0),(2,1,0),(1,1,1), \ldots$ | $1 \oplus 3$ | $\underline{10}$ |
| 4 | 15 | $\ldots$ | $0 \oplus 2 \oplus 4$ | $\underline{15}$ |

■ Quark model: $\mathrm{SU}(6) \times \mathrm{O}(3)$

| N | configuration | $\mathrm{SU}(6) \times \mathrm{O}(3)$ multiplets |
| :--- | :---: | :--- |
| 0 | $(0 s)^{3}$ | $\left[56,0^{+}\right]$ |
| 1 | $(0 s)^{2}(1 p)$ | $\left(56,1^{-}\right)\left[70,1^{-}\right]$ |
| 2 | $(0 s)^{2}(2 s)$ | $\left(56,0^{+}\right)\left[70,0^{+}\right]$ |
|  | $(0 s)^{2}(2 d)$ | $\left(56,2^{+}\left[70,2^{+}\right]\right.$ |
|  | $(0 s)(1 p)^{2}$ | $\left[56,0^{+}\right]\left[56,2^{+}\right]\left(70,0^{+}\right)\left(70,1^{+}\right)\left(70,2^{+}\right)\left[20,1^{+}\right]$ |

■ Problematic at a fundamental level

# All Possible Symmetries of the S Matrix* 

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(Received 16 March 1967)
We prove a new theorem on the impossibility of combining space-time and internal symmetries in any but a trivial way. The theorem is an improvement on known results in that it is applicable to infinite-parameter groups, instead of just to Lie groups. This improvement is gained by using information about the $S$ matrix; previous investigations used only information about the single-particle spectrum. We define a symmetry group of the $S$ matrix as a group of unitary operators which turn one-particle states into one-particle states, transform many-particle states as if they were tensor products, and commute with the $S$ matrix. Let $G$ be a connected symmetry group of the $S$ matrix, and let the following five conditions hold: (1) $G$ contains a subgroup locally isomorphic to the Poincaré group. (2) For any $M>0$, there are only a finite number of one-particle states with mass less than $M$. (3) Elastic scattering amplitudes are analytic functions of $s$ and $t$, in some neighborhood of the physical region. (4) The $S$ matrix is nontrivial in the sense that any two oneparticle momentum eigenstates scatter (into something), except perhaps at isolated values of $s$. (5) The generators of $G$, written as integral operators in momentum space, have distributions for their kernels. Then, we show that $G$ is necessarily locally isomorphic to the direct product of an internal symmetry group and the Poincaré group.

## I. INTRODUCTION

UNTIL a few years ago, most physicists believed that the exact or approximate symmetry groups of the world were (locally) isomorphic to direct products of the Poincaré group and compact Lie groups. This world-view changed drastically with the publication of the first papers on $S U(6)^{1}$; these raised the dazzling possibility of a relativistic symmetry group which was not simply such a direct product. Unfortunately, all attempts to find such a group came to disastrous ends, and the situation was finallv settled hv the discnverv of
symmetry group of the $S$ matrix, which contains the Poincaré group and which puts a finite number of particles in a supermultiplet. Let the $S$ matrix be nontrivial and let elastic scattering amplitudes be analytic functions of $s$ and $t$ in some neighborhood of the physical region. Finally, let the generators of $G$ be representable as integral operators in momentum space, with kernels that are distributions. Then $G$ is locally isomorphic to the direct product of the Poincaré group and an internal symmetry group. (This is a loose statement of the theorem; a more precise one follows below.)

## Vacuum symmetries

- Symmetries of the vacuum

■ Poincaré symmetry $\mathrm{P}(1,3)$ with 3 aligned excitations (\# space dimensions)
$\square \underline{P(1,3)}=[\underline{P}(1,1), \underline{S O}(3)]$
■ Hilbert space transformations: $[\mathrm{P}(1,1), \underline{\mathrm{SU}(3)]}$ with $\underline{\mathrm{SU}(3)}=[\underline{\mathrm{SO}(3)}, \underline{\mathrm{SU}(2)} \times \underline{\mathrm{U}(1)}]$
■ How to evade Coleman-Mandula when moving $\mathrm{SO}(3)$ into $\mathrm{P}(1,3)$ ?
■ A guiding example could be supersymmetry (Haag-Luposzanski-Sohnius)

- Proposed way of proceeding
- Adding internal symmetries as central charges $(\mathrm{Y})$ to the algebra $\left\{Q_{i}, Q_{j}^{\dagger}\right\}=2 \delta_{i j} P+Y_{i j}$
- Internal symmetries that are moved into space-time symmetries need also to become visible in field space
- OD $\rightarrow$ 1D: use boost (fermionic + scalar and pseudoscalar bosonic fields)

■ 1D $\rightarrow$ 3D: use rotations (fermionic + scalar and vector bosonic fields)

## Procedure (illustrated in 0D)

■ Supercharges and Hamiltonian(s)
$[H, H]=[H, Q]=0,\left\{Q, Q^{\dagger}\right\}=2 H$

$$
\begin{aligned}
& {[a, H]=\{Q,[Q, a]\}=\sqrt{\omega}\{Q, b\}=\omega a} \\
& {[b, H]=[Q,\{Q, b\}]=\sqrt{\omega}[Q, a]=\omega b}
\end{aligned}
$$

- Look at the boson and fermion fields: $\varphi=\frac{1}{\sqrt{2 \omega}}\left(a+a^{\dagger}\right) \quad$ and $\quad \xi=\frac{1}{\sqrt{2}}\left(b+b^{\dagger}\right)$

■ For free fields: $[Q, \varphi]=\xi$

$$
\begin{aligned}
& \{Q, \xi\}=F=[\varphi, H]=i \dot{\varphi} \\
& {[Q, F]=[\xi, H]=i \dot{\xi}}
\end{aligned}
$$

$$
F=M \varphi
$$

■ Adding additional degrees of freedom via auxiliary field(s)

$$
\begin{array}{lll}
{[Q, \varphi]=\xi} & \{Q, \xi\}=F=i D_{0} \varphi & i D_{0}=i \partial_{0}+g A_{0} \\
& {[Q, F]=i D_{0} \xi} &
\end{array}
$$

$\ldots$ and a nontrivial vacuum such that $\langle 0| \varphi|0\rangle=v_{0}$ and $\varphi=v_{0}(\phi-1)$

- For example in $0 \mathrm{D}: ~ i D_{0}=H=g A_{0}$ and $\phi(t)=U(t) \phi=\exp \left(-i g \int_{0}^{t} d \tau A_{0}\right) \phi$ (This is not conformal!)


## Symmetries in 1D

■ In OD a mass M breaks conformal invariance.

- For two fields in OD (right and left) mass provides the coupling, of $\left[P^{-}, \xi_{R}\right]=-i M \xi_{L}$

$$
H=\left[\begin{array}{cc}
P & M \\
M & -P
\end{array}\right]=\gamma_{0}(P P+M)
$$

$$
\begin{aligned}
& \gamma^{\mu}=\left[\begin{array}{cc}
0 & n_{-}^{\mu} \\
n_{+}^{\mu} & 0
\end{array}\right] \\
& \gamma_{5}=\left[\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right]
\end{aligned}
$$

■ Gateway to 1D: P as OD-Hamiltonian with H as central charge
■ Achieved via $[K, P]=i H$ or $\left[K, P^{ \pm}\right]= \pm i P^{ \pm}$

- $P(1,1)=\{H, P, K\}$ with $M^{2}$ as Casimir operator

■ Supersymmetric algebra with $\mathrm{Q}_{\mathrm{R} / \mathrm{L}}$ and $\left[K, Q_{R / L}\right]= \pm \frac{1}{2} i Q_{R / L}$
$\left[Q_{R / L}, \phi_{R / L}\right]=\xi_{R / L}$
$\left\{Q_{R / L}, Q_{R / L}^{\dagger}\right\}=2 P^{ \pm}$
$\left\{Q_{R / L}, \xi_{R / L}\right\}=F_{R / L}=i D_{\mp} \phi_{R / L}$
$\left\{Q_{R / L}, F_{R / L}\right\}=i D_{\mp} \xi_{R / L}$

■ Group fields as:
$\phi \sqrt{2}=e^{i \pi / 4} \phi_{R}+e^{-i \pi / 4} \phi_{L}=\phi_{S}+i \phi_{P}$

$$
\psi \sqrt{2}=\left[\begin{array}{c}
\xi_{R} \\
-i \xi_{L}
\end{array}\right]
$$

$\langle\phi\rangle=\left\langle\phi_{R}\right\rangle=\left\langle\phi_{L}\right\rangle=\frac{1}{\sqrt{2}} \quad\left\langle\phi_{S}\right\rangle=1 \quad\left\langle\phi_{P}\right\rangle=0$

## The supersymmetric 1D starting point

■ Wess-Zumino (1974) in d=2:
Interactions: M (couples right-left), $\mathrm{g}_{0}$ (Yukawa coupling), $\operatorname{dim}[\mathrm{M}]=\operatorname{dim}\left[\mathrm{g}_{0}\right]=1$

$$
\begin{aligned}
L & =\frac{1}{2} \partial_{-} \varphi_{R} \partial_{+} \varphi_{R}+\frac{1}{2} \partial_{+} \varphi_{L} \partial_{-} \varphi_{L}+\frac{i}{2} \xi_{R} \partial_{+} \xi_{R}+\frac{i}{2} \xi_{L} \partial_{-} \xi_{L}-V \\
& =\frac{1}{2} \partial^{\mu} \varphi_{S} \partial_{\mu} \varphi_{S}+\frac{1}{2} \partial^{\mu} \varphi_{P} \partial_{\mu} \varphi_{P}+\bar{\psi} i \gamma^{\mu} \partial_{\mu} \psi-V
\end{aligned}
$$

$$
V=\frac{1}{2}\left(M+g_{0} \varphi_{S}\right)^{2}\left(\varphi_{S}^{2}+\varphi_{P}^{2}\right)+\frac{1}{2} g_{0}^{2} \varphi_{P}^{2}\left(\varphi_{S}^{2}+\varphi_{P}^{2}\right)+\bar{\psi}\left(M+g_{0} \varphi_{S}+g_{0} \varphi_{P} \gamma_{0} \gamma_{5}\right) \psi
$$

■ Constraint: $\left(\varphi_{R} \sqrt{2}+v_{0}\right)\left(\varphi_{L} \sqrt{2}+v_{0}\right)-v_{0}^{2}=\left(\varphi_{S}+v_{0}\right)^{2}-\varphi_{P}^{2}-v_{0}^{2}=0$

$$
v_{0}=M / 2 g_{0}
$$

corresponds to a boost invariance in field space

- This enables $0 \mathrm{D} \rightarrow 1 \mathrm{D}$ extension starting from $\phi_{R / L}(\tau, 0)=\frac{1}{\sqrt{2}} \chi(\tau, 0)$

$$
\phi_{R / L}(t, z)=e^{ \pm \eta} \phi_{R / L}(\tau, 0) \text { and } i \partial_{\sigma} \phi_{R / L}(t, z)=e^{ \pm \eta} i D_{\sigma} \phi_{R / L}(\tau, 0)
$$

with $i D_{\sigma}=i \partial_{\sigma} \pm i \partial_{\sigma} \eta$ being a pure gauge in field space

## The resulting 1D supersymmetric lagrangian

■ Fields: identification $\left.M+g A^{\sigma}=\left(M+g_{0} \varphi_{S}\right) n_{0}^{\sigma}+\varphi_{P} n_{3}^{\sigma}\right)$

$$
\begin{aligned}
&\left.=\left(\left(\frac{M}{\sqrt{2}}+g_{0} \varphi_{R}\right) n_{+}^{\sigma}+\left(\frac{M}{\sqrt{2}}+\varphi_{L}\right) n_{-}^{\sigma}\right)\right) / \sqrt{2} \\
& \phi(t, z)\left.=\mathcal{P} \exp \left(-i g \int_{0}^{x} d s^{\sigma} A_{\sigma}(s)\right) \phi(\tau, 0)\right] \\
& \psi(t, z)\left.=\mathcal{P} \exp \left(-i g \int_{0}^{x} d s^{\sigma} A_{\sigma}(s) \gamma_{5}\right) \psi(\tau, 0)\right]
\end{aligned} \quad \phi=\chi e^{i \theta} \quad \begin{array}{ll} 
& g A_{\sigma}=-\partial_{\sigma} \theta
\end{array}
$$

$$
g F_{\tau \sigma}=\delta W[C] / \delta \sigma^{\tau \sigma}
$$

- Lagrangian:

$$
\begin{aligned}
\mathcal{L}= & -\frac{g^{2}}{4 g_{0}^{2}} F^{\mu \nu} F_{\mu \nu}+v_{0}^{2} D^{\mu} \phi^{*} D_{\mu} \phi-\frac{1}{8} M v_{0}^{2}\left(\phi^{2}+\phi^{* 2}-1\right)^{2}-\frac{1}{2} \lambda M v_{0}\left(\phi^{2}+\phi^{* 2}-1\right) \\
& +\frac{1}{2} \bar{\psi}\left(i \overleftrightarrow{\not D}-M\left(1+\frac{\phi^{*}}{\sqrt{2}}+\frac{\phi}{\sqrt{2}}\right)\right) \psi
\end{aligned}
$$

- Expanded around minimum ( $\phi \sqrt{2}=\chi=1+\varphi_{H} / v_{0}$ ) this gives $\mathrm{M}_{\mathrm{A}}=\mathrm{M}_{\mathrm{H}}=\mathrm{M}$ (in 1D massive A-field has one d.o.f., massless field just instantaneous potential)
- Also allowed is $\phi=\exp \left(i \phi_{\pi} / f_{\pi}\right)$

■ Lagrange multiplier $\lambda$ might be connected to space-time curvature.

## Multiple bosonic and fermionic excitations

- Many advantages of 1 D such as convergence properties, $\mathrm{d}[\mathrm{M}]=\mathrm{d}\left[\mathrm{g}_{0}\right]=1$ (e.g. take $v_{0}=M / 2 g_{0}=1$, thus $\left.g_{0}=M / 2\right), d[\phi]=0, d[\psi]=1 / 2$, naturalness, one-loop mass correction $\Sigma=M / 16 \pi, \ldots$
- Extend to three real fields with $\mathrm{SO}(3)$ field symmetry $(\mathrm{N}=3)$ and permutation symmetry, including complex phases embedded in an $\mathrm{SU}(3)$ symmetry (cf HO ).
■ Symmetry $G \supset P(1,1) \bowtie A(4) \bowtie S U(3)$





■ A(4) governs embedding: $n_{\sigma}^{0} \rightarrow n_{\mu}^{0}$ and $\hat{n}_{\sigma}^{3} \rightarrow \hat{n}_{\mu}^{i}$

- Ground state: $\langle\phi\rangle=\left\langle\phi_{R}\right\rangle=\left\langle\phi_{L}\right\rangle=\frac{1}{\sqrt{2}} \quad\left\langle\phi_{S}\right\rangle=1 \quad\left\langle\phi_{P}\right\rangle=0$ $|0\rangle=\left|0_{R}, 0_{L}\right\rangle=\left|(0,0,0)_{R},(0,0,0)_{L}\right\rangle$
■ Symmetry of ground state: $\mathrm{SO}(3)$, but also P and $\mathrm{T}, \mathrm{Z}(3)$


## Bosonic and fermionic excitations

- Bosonic fluctuations:
- Real SO(3) symmetric fluctuations identify space-time

■ Includes also $P$ and $T$ symmetry

- $\mathrm{P}(1,1) \times \mathrm{SO}(3)=>\mathrm{P}(1,3)$ : $\mathrm{d}=4$ Poincaré symmetry (3D)
- SO(3) part $\left(\{\vec{L}\} \subset\left\{F_{a}\right\}\right)$ used to lift fields into 3D: $\vec{r}=R(\vec{\theta}) r$

$$
\begin{aligned}
& \phi(\vec{r})=R(\hat{r}) \phi(r) \\
& \partial_{r} \phi(\vec{r})=R(\hat{r}) \partial_{r} \phi(r) \\
& \underbrace{-i \vec{r} \times \vec{\partial}}_{\vec{\ell}} \phi(\vec{r})=R(\hat{r}) \vec{L} \phi(r)
\end{aligned}
$$

- ... or
pure gauge that can also be used to rotate away some 1D fields:

$$
g \sqrt{2 / 3} \vec{A}_{\sigma}=-\partial_{\sigma} \vec{\theta} \Longleftrightarrow g_{0} \vec{\varphi}_{P} n_{\sigma}^{3}
$$

$$
\begin{aligned}
& R^{-1}(\hat{r}) \vec{\ell} R(\hat{r})=\vec{L} \\
& R^{-1}(\hat{r}) i \vec{\partial} R(\hat{r})=\hat{r} i \partial_{r}+\frac{1}{r^{2}}(\hat{r} \times \vec{L})
\end{aligned}
$$

■ ... other gauge fields also lifted to 3D

$$
R^{-1}(\hat{r}) \underline{\vec{A}}(\vec{r}) R(\hat{r})=\underline{A}(r)
$$

$$
\begin{aligned}
& \phi \rightarrow e^{i \underline{\theta}} \phi \text { with } \underline{\theta}=\theta^{a} F_{a} \\
& D_{\mu} \phi=\partial_{\mu} \phi+g \underline{A} \phi \text { with } \underline{A}=A_{\mu}^{a} F_{a} \\
& A_{\mu} \rightarrow e^{-i \underline{\theta}} A_{\mu} e^{i \underline{\theta}}+e^{-i \underline{\underline{\theta}}} i \partial_{\mu} e^{i \underline{\theta}}
\end{aligned}
$$

## Bosonic and fermionic excitations: gauge bosons

■ The additional gauge fields to account for 'complex' phases by realizing that the algebra $\underline{\mathrm{SU}(3)}=[\mathrm{SO}(3), \underline{\mathrm{SU}(2) \times U(1)]}$

- Incorporating SO(3) in 3D field space still requires accounting for the $\mathrm{Z}(3)$ symmetry in field space, coupling the embeddings to family structure. This fixes for each embedding the $\mathrm{SU}(2) \times \mathrm{U}(1)$ generators $\mathrm{T}_{1}, \mathrm{~T}_{2}, \mathrm{~T}_{3}$ (isospin plane) and Y . It also fixes the charge operator $\mathrm{Q}=\mathrm{T}_{3}+\mathrm{Y} / 2$.
■ Bosons become SU(3) rotated versions of scalar field (cf standard Higgs)
- Real rotations make fields 3D but limit them to isospin plane
- For 'imaginary' rotations we use the $\mathrm{SU}(2) \times \mathrm{U}(1)$ part

$$
\begin{aligned}
& \phi_{R}=\frac{1}{\sqrt{2}} \exp \left(+\frac{i}{2} \sum_{a=1,2,3,8} \theta^{a} \lambda_{a}\right)\left[\begin{array}{c}
1+\varphi_{H} \\
0 \\
0
\end{array}\right] \\
& \phi_{L}=\frac{1}{\sqrt{2}} \exp \left(-\frac{i}{2} \sum_{a=1,2,3,8} \theta^{a} \lambda_{a}\right)\left[\begin{array}{c}
0 \\
1+\varphi_{H} \\
0
\end{array}\right]
\end{aligned}
$$

■ Covariant derivatives involve electroweak gauge bosons that eat three of the boson modes (as in standard model).

$$
i D_{\mu} \phi=i \partial_{\mu} \phi+\frac{g}{2}\left(\sum_{i=1}^{3} W_{\mu}^{i} \lambda_{i}+B_{\mu} \lambda_{8}\right) \phi
$$

■ D.o.f $=2 \times 6=1+3 \times 3+2 \times 1$

## Bosonic and fermionic excitations: electroweak symmetry breaking

- Incorporating the electroweak symmetry:

$$
P(1,1) \otimes S U(3) \supset \underbrace{P(1,1) \times S O(3)}_{P(1,3)} \bowtie Z(3) \bowtie \underbrace{\left[S U(2)_{I} \otimes U(1)_{Y}\right]}_{U(1)_{Q}}
$$

■ Symmetry breaking:

$$
\begin{aligned}
i D_{\mu} \phi & =i \partial_{\mu} \phi+\frac{g}{2}\left(\sum_{i=1}^{3} W_{\mu}^{i} \lambda_{i}+B_{\mu} \lambda_{8}\right) \phi \\
& =i \partial_{\mu} \phi+\frac{g}{\sqrt{2}}\left(W_{\mu}^{+} I_{-}+W_{\mu}^{-} I_{+}\right) \phi+\left(g W_{\mu}^{0} I_{3}+\frac{g}{2 \sqrt{3}} B_{\mu} Y\right) \phi
\end{aligned}
$$

■ $\mathrm{SU}(3)$ embedding gives embarrasingly good 'zeroth order' results:

- gives $M_{H}{ }^{2}=M^{2} / 2, e=g / 2=(3 / 2)^{1 / 2} g_{0} / 2 M=(3 / 32)^{1 / 2}$

Note: $128 \pi / 3=134$

- gives weak mixing angle $\sin ^{2} \theta_{\mathrm{w}}=1 / 4$ (Weinberg 1972)

■ gives $M_{w}{ }^{2}=3 M^{2} / 16, M_{z}^{2}=M^{2} / 4$

## Bosonic and fermionic excitations: lepton families

- 3D embedding fermions is straightforward:

- Families linked to three singlets of $Z(3)$ :

$$
\begin{aligned}
& Q_{\mathrm{ew}}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{array}\right] \quad Q_{\mathrm{mass}}=\left[\begin{array}{ccc}
0 & 0 & -i \\
0 & 0 & 0 \\
i & 0 & 0
\end{array}\right] \\
& Q_{\text {mass }}=U_{Q}^{\dagger} Q_{\mathrm{ew}} U_{Q} \quad U_{Q}^{\dagger}=\left[\begin{array}{ccc}
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & 1 & 0 \\
\frac{i}{\sqrt{2}} & 0 & \frac{-i}{\sqrt{2}}
\end{array}\right]
\end{aligned}
$$

$$
\begin{gathered}
\langle\widehat{\phi}\rangle_{\mathrm{fam}}=\left[\begin{array}{c}
\frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{3}}
\end{array}\right] \Leftrightarrow\langle\widehat{\phi}\rangle_{\mathrm{ew}}=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right] \\
\langle\widehat{\phi}\rangle_{\mathrm{fam}}=W\langle\widehat{\phi}\rangle_{\mathrm{ew}} \\
W=\frac{1}{\sqrt{3}}\left[\begin{array}{ccc}
1 & 1 & 1 \\
\omega^{2} & 1 & \omega \\
\omega & 1 & \omega^{2}
\end{array}\right]
\end{gathered}
$$

$$
Q_{\mathrm{fam}}=W Q_{\mathrm{ew}} W^{\dagger}=W U_{Q} Q_{\mathrm{mass}} U_{Q}^{\dagger} W^{\dagger} \quad U_{\mathrm{HPS}}=W U_{Q}=\left[\begin{array}{ccc}
\sqrt{2 / 3} & \sqrt{1 / 3} & 0 \\
-\sqrt{1 / 6} & \sqrt{1 / 3} & -\sqrt{1 / 2} \\
-\sqrt{1 / 6} & \sqrt{1 / 3} & \sqrt{1 / 2}
\end{array}\right]
$$

■ Lepton masses? Just note: $\mathrm{M} / 8 \pi^{2}=2 \mathrm{GeV}$ (factor from $\mathrm{SO}(3)$ group measure)

## Bosonic and fermionic excitations: strong sector

■ Bosonic/fermionic modes in $\mathrm{E}(1,1): i D_{\mu} \phi^{i}=i \partial_{\mu} \phi^{i}+g \sum_{a \in \underline{G}} A_{\mu}^{a}\left(F_{a}\right)_{j}^{i} \phi^{j}$
■ Scalar fields turn into gauge fields, leaving d=2 QCD with a massless scalar field (cf Kaplan 2013)

$$
\mathcal{L}=\frac{1}{2} D^{\mu} \varphi_{H} D_{\mu} \varphi_{H}-\frac{1}{4} F^{\mu \nu} F_{\mu \nu}+\bar{\psi}\left(i \not D-M-g_{0} \varphi_{H}\right) \psi
$$

- 8 gluon fields
- Dynamics in Wilson loop $W[C]=\exp \left(-i g \oint_{C} d s^{\mu} A_{\mu}(s)\right)$
- A-fields dynamical if d>2

■ Used in TMD physics (ongoing work with a.o. Cotogno, van Daal, ...)
■ New basis for:
■ CFT approaches (Brodsky, de Téramond, Dosch, Lorcé)

- Color-kinematic duality

■ Soft Collinear Effective Theory

## Bosonic and fermionic excitations: valence quarks

- Asymptotic structure?

Manifestation of 'color' vacuum:


## Electroweak quantum numbers of colored excitations

- Asymptotic structure? Manifestation of 'color' vacuum:

$$
\begin{aligned}
|0\rangle & =\left|(0,0,0)_{R},(0,0,0)_{L}\right\rangle_{\text {leptons }} \\
& =\left|\left(0_{R}, 0_{R}, 0_{L}\right),\left(0_{L}, 0_{L}, 0_{R}\right)\right\rangle_{\text {quarks }}
\end{aligned}
$$

- Constraints for colored excitations (I-U-V spin)



## Fermionic excitations in the standard model

■ Quarks live in $\mathrm{E}(1,1)$, coming in 3 families and 3 colors. Confinement!
■ Go back to leptons:

- In d = $1: \xi^{0}, \xi^{+}$, or $\xi^{-}$charge/momentum eigenstates
$\square$ In $d=2$ : $\left(\xi^{0} \xi^{0}\right),\left(\xi^{+} \xi^{+}\right)$and $\left(\xi^{-} \xi^{-}\right)$charge/helicity eigenstates
$\square$ In $\mathrm{d}=3: \xi_{L}{ }^{0}\left(\xi_{L}{ }^{0} \xi_{L}{ }^{0}\right)$ is acceptable $\mathrm{SU}(3)$ root [ $\mathrm{I}_{3}$ quantum numbers] $\xi_{L}{ }^{0}\left(\xi_{L}{ }^{+} \xi_{L}{ }^{+}\right)$and $\xi_{L}{ }^{0}\left(\xi_{L}{ }^{-} \xi_{L}{ }^{-}\right)$are not acceptable!
- $\mathrm{E}(1,3)$ : gluons dynamical and 'electroweak properties' of quarks (= QCD)

Freeze color, e.g. $\mathrm{R}=$ red (in triplet 3 ), $\mathrm{L}=$ anti-red (in anti-triplet $3^{*}$ )
$\square$ For $\xi_{L}{ }^{0}$ only $\xi_{L}{ }^{0}\left(\xi_{R}+\xi_{R}{ }^{+}\right)$and $\xi_{L}{ }^{0}\left(\xi_{L}-\xi_{R}{ }^{-}\right)$are acceptable giving the quarks $u_{L}$ (red) and $\mathrm{u}_{\mathrm{L}}{ }^{*}$ (anti-red), the latter being an iso-singlet

- $\xi_{L}{ }^{0}\left(\xi_{R}{ }^{+} \xi_{R}{ }^{+}\right)$and $\xi_{L}{ }^{-}\left(\xi_{R}{ }^{0} \xi_{R}{ }^{0}\right)$ form a red iso-doublet $u_{L}$ and $d_{L}$
- Construction of quarks and leptons resembles rishon model without problem of compositeness (Harari \& Seiberg 1982)
■ In zeroth order one family takes all mass: top-quark, $\mathrm{t} \sim \xi^{0}\left(\xi^{+} \xi^{+}\right)$, CKM trivial, quark masses starting with mass proportional to $\Sigma=\mathrm{M} / 16 \pi=3.5 \mathrm{GeV}$ (b-mass?)


## Particle content of standard model

| particle | space |  |  | isospin |  | $\begin{gathered} \text { hypercharge } \\ Y \end{gathered}$ | charge $Q$ | color $\underline{c}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $L$ | $T_{1}$ | $T_{2}$ | $I$ | $I_{3}$ |  |  |  |
| $\nu_{L}$ | $\xi_{L}^{0}$ | $\xi_{L}^{0}$ | $\xi_{L}^{0}$ | 1/2 | +1/2 | -1 | 0 | $\underline{1}$ |
| $e_{L}^{-}$ | $\xi_{L}^{-}$ | $\xi_{L}^{-}$ | $\xi_{L}^{-}$ | $1 / 2$ | $-1 / 2$ | -1 | -1 | 1 |
| $e_{L}^{+}$ | $\xi_{L}^{+}$ | $\xi_{L}^{+}$ | $\xi_{L}^{+}$ | 0 | 0 | +2 | +1 | 1 |
| $\nu_{R}$ | $\xi_{R}^{0}$ | $\xi_{R}^{0}$ | $\xi_{R}^{0}$ | 1/2 | $-1 / 2$ | +1 | 0 | $\underline{1}$ |
| $e_{R}^{+}$ | $\xi_{R}^{+}$ | $\xi_{R}^{+}$ | $\xi_{R}^{+}$ | $1 / 2$ | +1/2 | +1 | +1 | $\underline{1}$ |
| $e_{R}^{-}$ | $\xi_{R}^{-}$ | $\xi_{R}^{-}$ | $\xi_{R}^{-}$ | 0 | 0 | -2 | -1 | $\underline{1}$ |
| $u_{L}$ | $\xi_{L}^{0}$ | $\left(\xi_{R}^{+}\right.$ | $\left.\xi_{R}^{+}\right)$ | 1/2 | +1/2 | +1/3 | +2/3 | $\underline{3}$ |
| $d_{L}$ | $\xi_{L}^{-}$ | $\left(\xi_{R}^{0}\right.$ | $\left.\xi_{R}^{0}\right)$ | $1 / 2$ | $-1 / 2$ | $+1 / 3$ | $-1 / 3$ | $\underline{3}$ |
| $\bar{u}_{L}$ | $\xi_{L}^{0}$ | $\left(\xi_{L}^{-}\right.$ | $\left.\xi_{R}^{-}\right)$ | 0 | 0 | $-4 / 3$ | $-2 / 3$ | $\underline{3}^{*}$ |
| $\overline{\bar{d}}_{L}$ | $\xi_{L}^{+}$ | $\left(\xi_{L}^{0}\right.$ | $\left.\xi_{R}^{0}\right)$ | 0 | 0 | $+2 / 3$ | +1/3 | $\underline{3}^{*}$ |
| $\overline{\bar{u}}_{R}$ | $\xi_{R}^{0}$ |  |  | 1/2 | $-1 / 2$ | $-1 / 3$ | $-2 / 3$ | $\underline{3}^{*}$ |
| $\bar{d}_{R}$ | $\xi_{R}^{+}$ | $\left(\xi_{L}^{0}\right.$ | $\left.\xi_{L}^{0}\right)$ |  | +1/2 | $-1 / 3$ | +1/3 | $\underline{3}^{*}$ |
| $u_{R}$ | $\xi_{R}^{0}$ | $\left(\xi_{L}^{+}\right.$ | $\left.\xi_{R}^{+}\right)$ | 0 | 0 | $+4 / 3$ | $+2 / 3$ | $\underline{3}$ |
| $d_{R}$ | $\xi_{R}^{-}$ | $\left(\xi_{L}^{0}\right.$ | $\left.\xi_{R}^{0}\right)$ | 0 | 0 | $-2 / 3$ | $-1 / 3$ | $\underline{3}$ |

## Standard model particle content



## Conclusions: emergent symmetries in Standard Model

■ Basic supersymmetric starting point, solves hierarchy and naturalness problems

- Links \# space dimensions, \# colors, \# families
- Provides spectrum of bosons and fermions in standard model
- Allows for family mixing ( M and g can become complex symmetric)

■ Left-right symmetric starting point and custodial symmetry
■ Provides a new view for many phenomena in QCD (Confinement, Bloom-Gilman duality, importance of SCET for PDFs, FFs including TMDs, multitude of effective models for QCD)

- B-L symmetry

■ Proton involves all excitations in lowest family. Family-breaking effects when different families meet, e.g. proton radius puzzle

- However, there are still many open ends!


## The Portal



