

# Quark-Gluon Correlations in the Nucleon

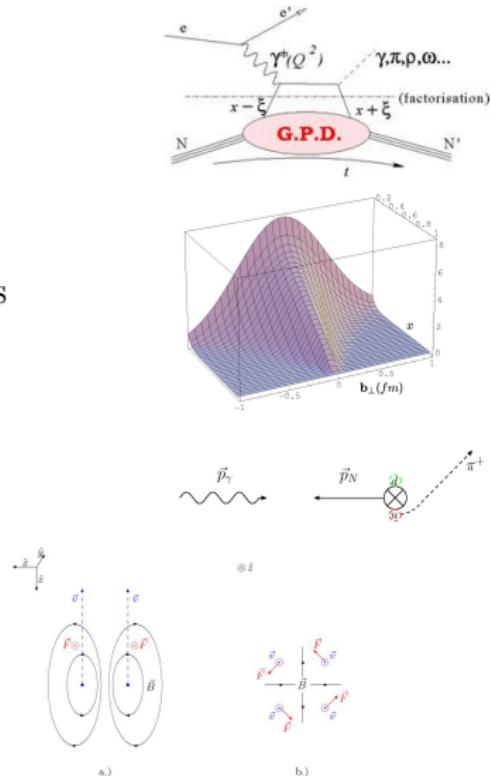
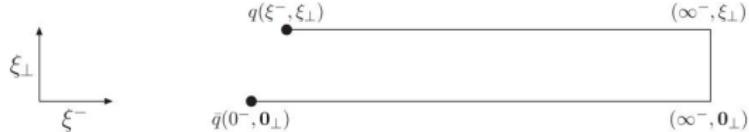
Matthias Burkardt

New Mexico State University

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# Outline

- GPDs  $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$  '3d imaging'
- $\perp$  polarization  $\Rightarrow \perp$  deformation
- $\hookrightarrow$   $\perp$  force from twist 3 correlations
- $\mathcal{L}_{JM}^q - L_{Ji}^q =$  change in OAM as quark leaves nucleon (due to torque from FSI)
- lattice calcs. of ISI/FSI &  $\mathcal{L}_{JM}^q$
- Summary



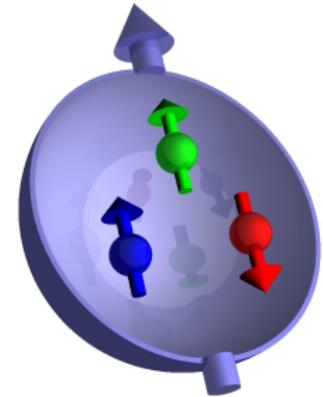
# Nucleon Spin Puzzle

## spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}$$

## Longitudinally polarized DIS:

- $\Delta\Sigma = \sum_q \Delta q \equiv \sum_q \int_0^1 dx [q_\uparrow(x) - q_\downarrow(x)] \approx 30\%$
- ↪ only small fraction of proton spin due to quark spins

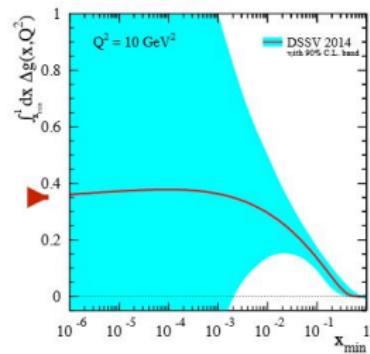


## Gluon spin $\Delta G$

could possibly account for remainder of nucleon spin, but still large uncertainties → EIC

## Quark Orbital Angular Momentum

- how can we measure  $\mathcal{L}_{q,g}$
- ↪ need correlation between **position & momentum**
- how exactly is  $\mathcal{L}_{q,g}$  defined



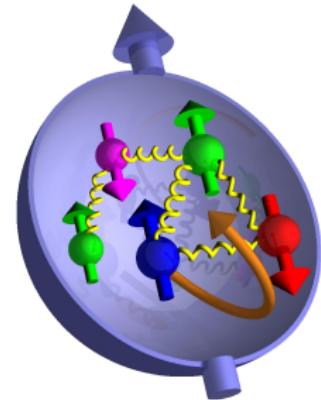
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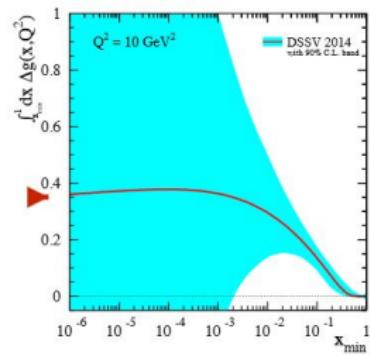


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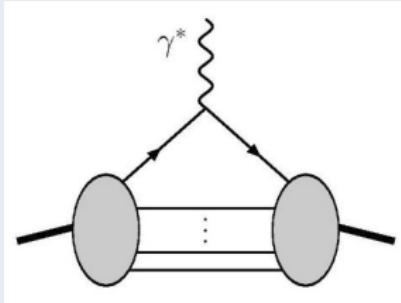
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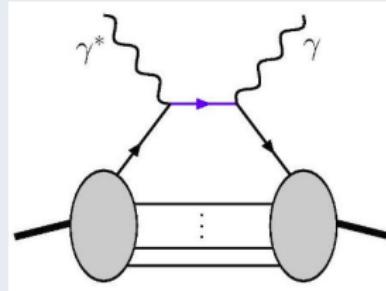


## form factor



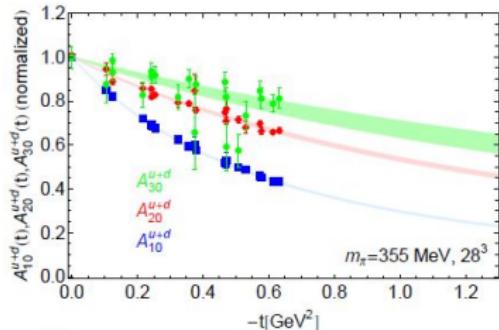
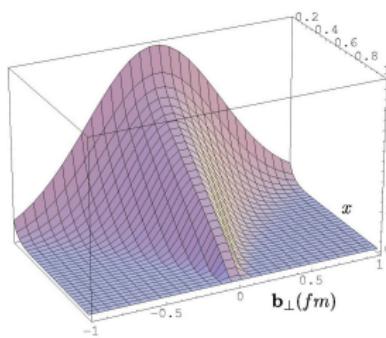
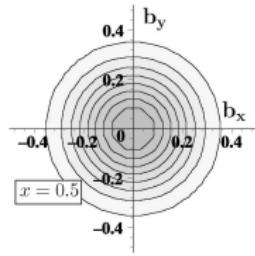
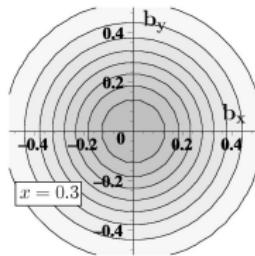
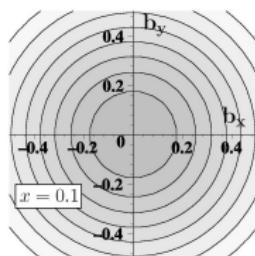
- electron hits nucleon & nucleon remains intact
- ↪ form factor  $F(q^2)$
- position information from Fourier trafo
- no sensitivity to quark momentum
- $F(q^2) = \int dx GPD(x, q^2)$
- ↪ GPDs provide momentum dissected form factors

## Compton scattering



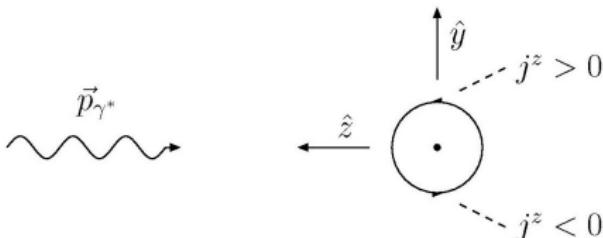
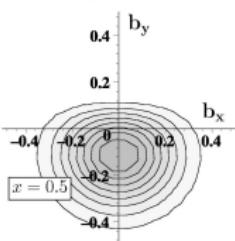
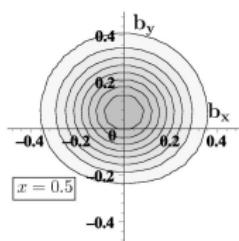
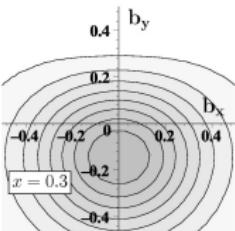
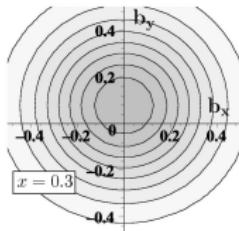
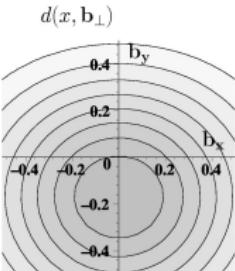
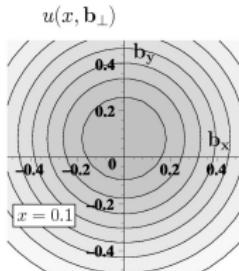
- electron hits nucleon, nucleon remains intact & photon gets emitted
- additional quark propagator
- ↪ additional information about momentum fraction  $x$  of active quark
- ↪ generalized parton distributions  $GPD(x, q^2)$
- info about both position and momentum of active quark

$q(x, \mathbf{b}_\perp)$  for unpol. p



## unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$
  - $F_1(-\Delta_\perp^2) = \int dx H(x, 0, -\Delta_\perp^2)$
  - $x$  = momentum fraction of the quark
  - $\mathbf{b}_\perp$  relative to  $\perp$  center of momentum
  - small  $x$ : large 'meson cloud'
  - larger  $x$ : compact 'valence core'
  - $x \rightarrow 1$ : active quark becomes center of momentum
- ↪  $\vec{b}_\perp \rightarrow 0$  (narrow distribution) for  $x \rightarrow 1$



proton polarized in  $+\hat{x}$  direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

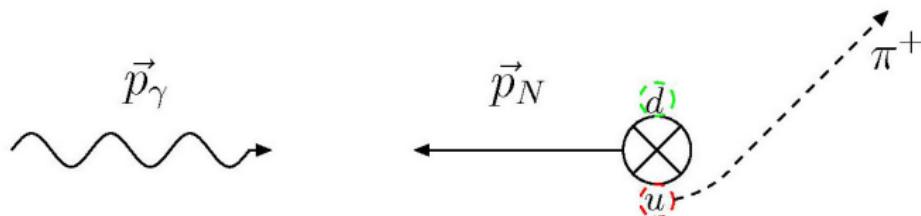
$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

- relevant density in DIS is  $j^+ \equiv j^0 + j^z$  and left-right asymmetry from  $j^z$
- av. shift model-independently related to **anomalous magnetic moments**:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q}{2M}$$

example:  $\gamma p \rightarrow \pi X$



- $u, d$  distributions in  $\perp$  polarized proton have left-right asymmetry in  $\perp$  position space (T-even!); sign “determined” by  $\kappa_u$  &  $\kappa_d$
- attractive final state interaction (FSI) deflects active quark towards the center of momentum
- ↗ FSI translates position space distortion (before the quark is knocked out) in  $+\hat{y}$ -direction into momentum asymmetry that favors  $-\hat{y}$  direction → **chromodynamic lensing**

$\Rightarrow$

$\kappa_p, \kappa_n \longleftrightarrow$  sign of SSA!!!!!!! (MB,2004)

- confirmed by HERMES & COMPASS data

# Average $\perp$ Force on Quarks in DIS

$d_2 \leftrightarrow$  average  $\perp$  force on quark in DIS from  $\perp$  pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$

- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

→ 'clean' separation between  $g_2$  and  $\frac{1}{Q^2}$  corrections to  $g_1$

- $g_2 = g_2^{WW} + \bar{g}_2$  with  $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S | \bar{q}(0) \gamma^+ gF^{+y}(0) q(0) | P, S \rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

matrix element defining  $d_2 \leftrightarrow$  1<sup>st</sup> integration point in QS-integral

$d_2 \Rightarrow \perp$  force       $\leftrightarrow$       QS-integral  $\Rightarrow \perp$  impulse

sign of  $d_2$

- $\perp$  deformation of  $q(x, \mathbf{b}_\perp)$

→ sign of  $d_2^q$ : opposite Sivers

magnitude of  $d_2$

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$

- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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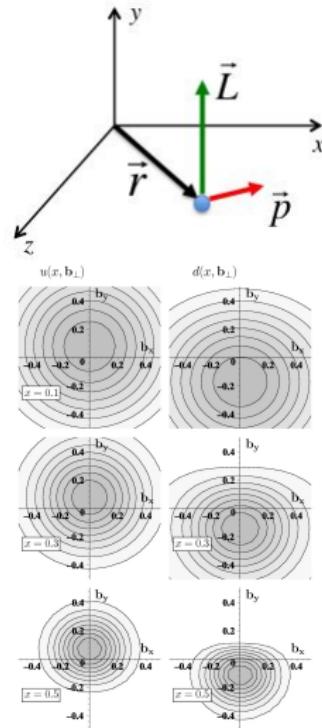
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consistent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)

- $L_x = yp_z - zp_y$
- if state invariant under rotations about  $\hat{x}$  axis then  $\langle yp_z \rangle = -\langle zp_y \rangle$
- $\hookrightarrow \langle L_x \rangle = 2\langle yp_z \rangle$
- GPDs provide simultaneous information about **longitudinal momentum** and **transverse position**
- $\hookrightarrow$  use quark GPDs to determine angular momentum carried by quarks

Ji sum rule (1996)

$$J_q^x = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$$



- parton interpretation in terms of 3D distributions (MB,2001,2005)

## QED with electrons

$$\begin{aligned}
 \vec{J}_\gamma &= \int d^3r \vec{r} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{r} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})] \\
 &= \int d^3r [E^j (\vec{r} \times \vec{\nabla}) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla}) \vec{A}] \\
 &= \int d^3r [E^j (\vec{r} \times \vec{\nabla}) A^j + (\vec{r} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}]
 \end{aligned}$$

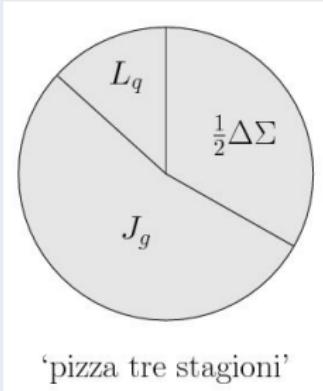
- replace 2<sup>nd</sup> term (eq. of motion  $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$ ), yielding

$$\vec{J}_\gamma = \int d^3r [\psi^\dagger \vec{r} \times e \vec{A} \psi + E^j (\vec{r} \times \vec{\nabla}) A^j + \vec{E} \times \vec{A}]$$

- $\psi^\dagger \vec{r} \times e \vec{A} \psi$  cancels similar term in electron OAM  $\psi^\dagger \vec{r} \times (\vec{p} - e \vec{A}) \psi$

↪ decomposing  $\vec{J}_\gamma$  into spin and orbital also shuffles angular momentum from photons to electrons!

## Ji decomposition



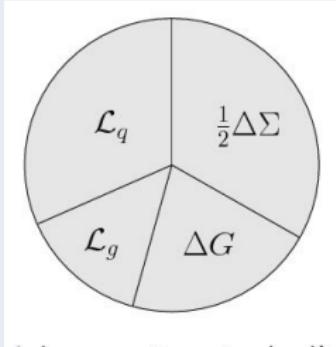
$$\frac{1}{2} = \sum_q \left( \frac{1}{2} \Delta q + L_q \right) + J_g$$

$$L_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i\vec{D})^3 q(\vec{x}) | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

- DVCS → GPDs →  $L^q$

## Jaffe-Manohar decomposition



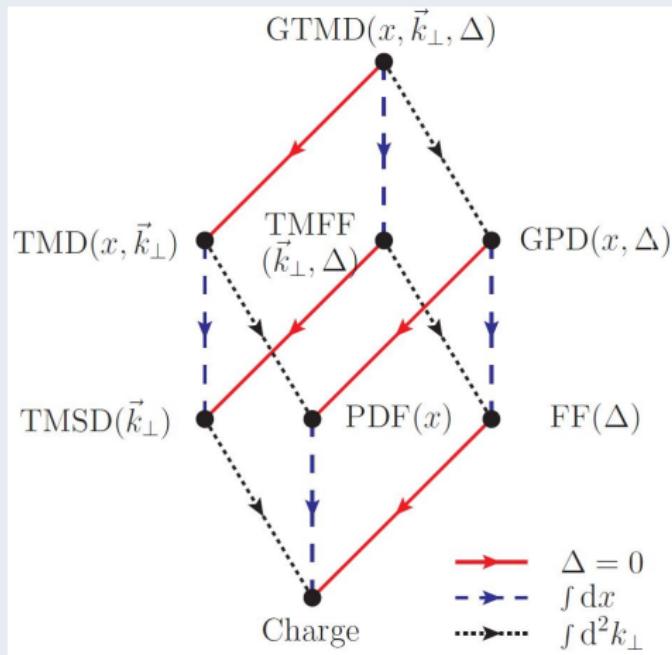
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$$L_q = \int d^3r \langle P, S | \bar{q}(\vec{r}) \gamma^+ (\vec{r} \times i\vec{\partial})^z q(\vec{r}) | P, S \rangle$$

- light-cone gauge  $A^+ = 0$
- $\overleftrightarrow{p} \overleftarrow{p} \rightarrow \Delta G \rightarrow \mathcal{L} \equiv \sum_{i \in q, g} \mathcal{L}^i$
- manifestly gauge inv. def. exists

How large is difference  $\mathcal{L}_q - L_q$  in QCD and what does it represent?

## 5-D Wigner Functions (Lorcé, Pasquini)



$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i \vec{\Delta}_\perp \cdot \vec{b}_\perp} GTMD(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

## 5-D Wigner Functions (Lorcé, Pasquini)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ q(\xi) | PS \rangle.$$

- TMDs:  $f(x, \mathbf{k}_\perp) = \int d^2 \mathbf{b}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
  - GPDs:  $q(x, \mathbf{b}_\perp) = \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
  - $L_z = \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x)$
  - need to include Wilson-line gauge link  $\mathcal{U}_{0\xi} \sim \exp \left( i \frac{g}{\hbar} \int_0^\xi \vec{A} \cdot d\vec{r} \right)$  to connect 0 and  $\xi$
- ‘light-cone staple’ crucial for SSAs in SIDIS & DY

straight line for  $\mathcal{U}_{0\xi}$ straigth Wilson line from 0 to  $\xi$  yields  
Ji-OAM:

$$L^q = \int d^3 x \langle P, S | q^\dagger(\vec{x}) \left( \vec{x} \times i \vec{D} \right)^z q(\vec{x}) | P, S \rangle$$

Light-Cone Staple for  $\mathcal{U}_{0\xi}$ 'light-cone staple' yields  $\mathcal{L}_{Jaffe-Manohar}$

$\mathcal{L}_{\square}/\mathcal{L}_{\square}$ 

$\mathcal{L}$  with light-cone staple at  
 $x^- = \pm\infty$

## PT (Hatta)

- PT  $\rightarrow \mathcal{L}_{\square} = \mathcal{L}_{\square}$

(different from SSAs due to factor  $\vec{x}$  in OAM)

## Bashinsky-Jaffe

- $A^+ = 0$  no complete gauge fixing
- $\hookrightarrow$  residual gauge inv.  $A^\mu \rightarrow A^\mu + \partial^\mu \phi(\vec{x}_\perp)$
- $\vec{x} \times i\vec{\partial} \rightarrow \mathcal{L}_{JB} \equiv \vec{x} \times [i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp)]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-}$

Bashinsky-Jaffe  $\leftrightarrow$  light-cone staple

- $A^+ = 0$
- $\hookrightarrow \mathcal{L}_{\square/\square} = \vec{x} \times [i\vec{\partial} - g\vec{\mathcal{A}}_\perp(\pm\infty, \vec{x}_\perp)]$
- $\mathcal{L}_{JB} = \vec{x} \times [i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp)]$
- $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-} = \frac{1}{2} (\vec{A}_\perp(\infty, \vec{x}_\perp) + \vec{A}_\perp(-\infty, \vec{x}_\perp))$
- $\hookrightarrow \mathcal{L}_{JB} = \frac{1}{2} (\mathcal{L}_{\square} + \mathcal{L}_{\square}) = \mathcal{L}_{\square} = \mathcal{L}_{\square}$

straight line ( $\rightarrow$ Ji)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \textcolor{red}{L}_q + J_g$$

$$\textcolor{red}{L}_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \left( \vec{x} \times i\vec{D} \right)^z q(\vec{x}) | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple ( $\rightarrow$  Jaffe-Manohar)

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$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$$

difference  $\mathcal{L}^q - L^q$ 

$$\mathcal{L}^q - L^q = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \left[ \vec{x} \times \int_{x^-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) \right]^z q(\vec{x}) | P, S \rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

straight line ( $\rightarrow \text{Ji}$ )

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$$i\mathcal{D}^j = i\partial^j - gA^j(x^-, \mathbf{x}_\perp) - g \int_{x^-}^\infty dr^- F^{+j}$$

difference  $\mathcal{L}^q - L^q$ 

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color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left( \vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0, 0, -1)$$

straight line ( $\rightarrow J_i$ )

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \textcolor{red}{L}_q + J_g$$

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- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple ( $\rightarrow$  Jaffe-Manohar)

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color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

Torque along the trajectory of  $q$ 

$$T^z = \left[ \vec{x} \times \left( \vec{E} - \hat{\vec{z}} \times \vec{B} \right) \right]^z$$

Change in OAM

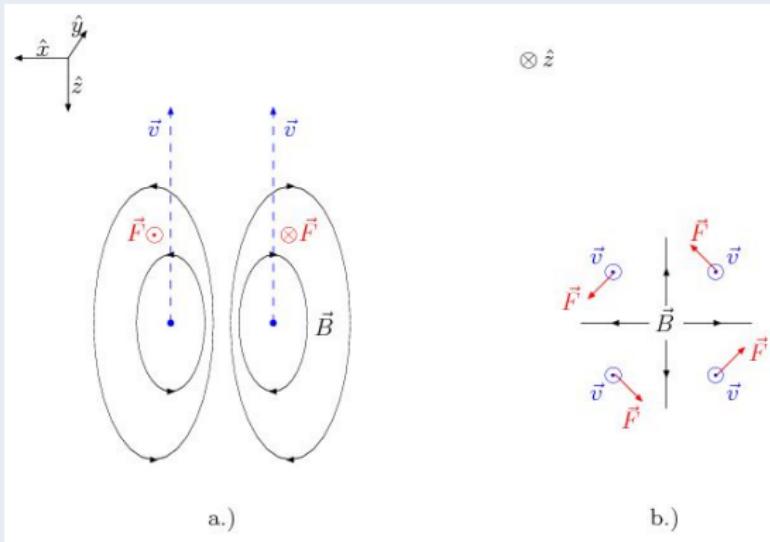
$$\Delta L^z = \int_{x^-}^\infty dr^- \left[ \vec{x} \times \left( \vec{E} - \hat{\vec{z}} \times \vec{B} \right) \right]^z$$

difference  $\mathcal{L}^q - L^q$

$\mathcal{L}_{JM}^q - L_{Ji}^q = \Delta L_{FSI}^q = \text{change in OAM as quark leaves nucleon}$

$$\mathcal{L}_{JM}^q - L_{Ji}^q = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_\perp)]^z q(\vec{x}) | P, S \rangle$$

$e^+$  moving through magnetic dipole field of  $e^-$



$$\mathcal{L}_{JM} - L_{Ji} = \langle \bar{q} \gamma^+ (\vec{r} \times \vec{A})^z q \rangle$$

in scalar diquark model

(Ji et al., 2016)

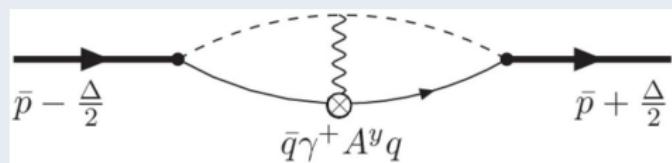
- for  $e^-$ :  $\mathcal{L}_{JM} - L_{Ji} = 0$  to  $\mathcal{O}(\alpha)$
- $\mathcal{L}_{JM} - L_{Ji} \stackrel{?}{=} 0$  in general?
- how significant is  $\mathcal{L}_{JM} - L_{Ji}$ ?

- pert. evaluation of  $\langle \bar{q} \gamma^+ (\vec{r} \times \vec{A})^z q \rangle$
- $\hookrightarrow \mathcal{L}_{JM} - L_{Ji} = \mathcal{O}(\alpha)$
- same order as Sivers
- $\hookrightarrow \mathcal{L}_{JM} - L_{Ji}$  as significant as SSAs

why scalar diquark model?

- Lorentz invariant
- 1<sup>st</sup> to illustrate: FSI  $\rightarrow$  SSAs  
(Brodsky,Hwang,Schmidt 2002)
- $\hookrightarrow$  Sivers  $\neq 0$

calculation



- nonforward matrix elem. of  $\bar{q} \gamma^+ A^y q$
- $\frac{d}{d\Delta^x} \Big|_{\Delta=0}$
- $\hookrightarrow \langle k_\perp^q \rangle = \frac{3m_q + M}{12} \pi \langle \bar{q} \gamma^+ (\vec{r} \times \vec{A})^z q \rangle$

difference  $\mathcal{L}^q - L^q$

$$\mathcal{L}_{JM}^q - L_{Ji}^q = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \int_{x^-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp)]^z q(\vec{x}) | P, S \rangle$$

- change in OAM as quark leaves nucleon due to torque from FSI on active quark

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 014014)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

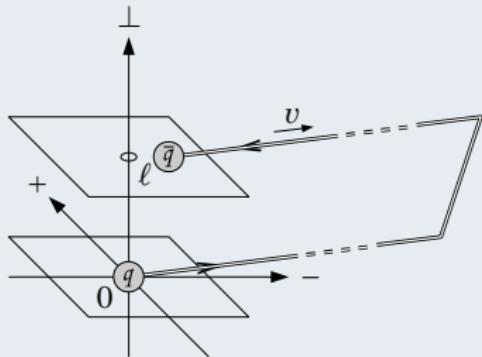
### Single-Spin Asymmetries (Qiu-Sterman)

- $\perp$  single-spin asymmetry in semi-inclusive DIS governed by 'Qiu-Sterman integral'

$$\langle k_\perp \rangle \sim \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x^-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x}) | P, S \rangle$$

- semi-classical interpretation:  $F^{+\perp}(r^-, \mathbf{x}_\perp)$  color Lorentz Force acting on active quark on its way out
- integral yields  $\perp$  impulse due to FSI

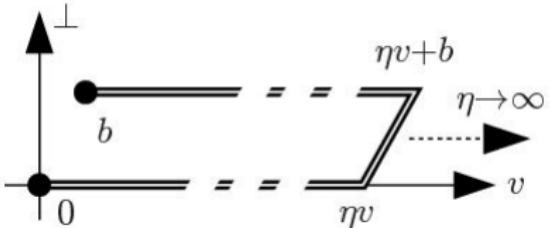
## challenge



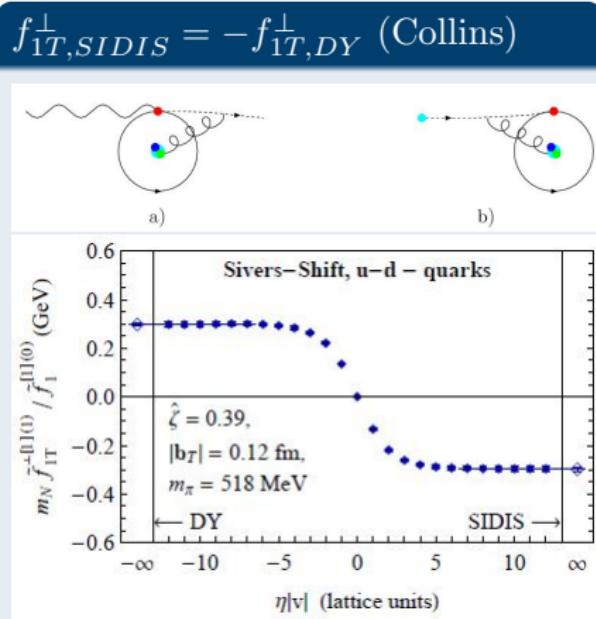
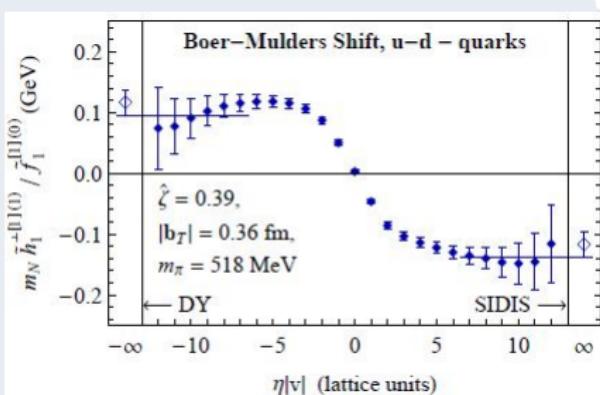
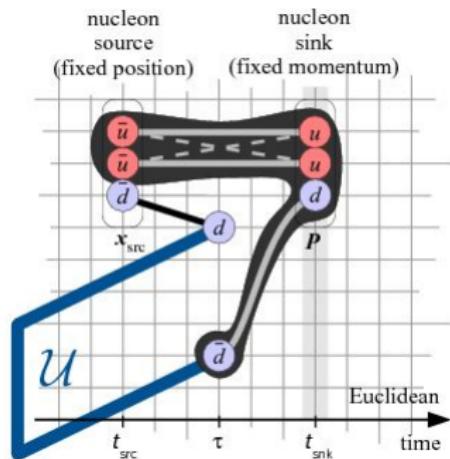
- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

## TMDs in lattice QCD

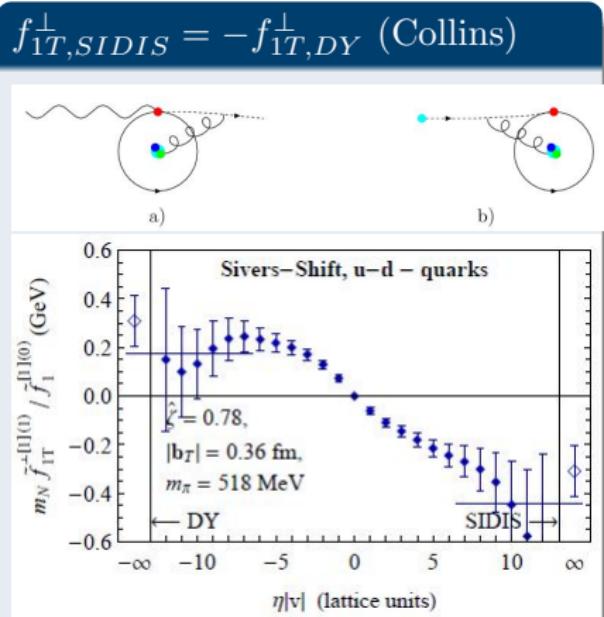
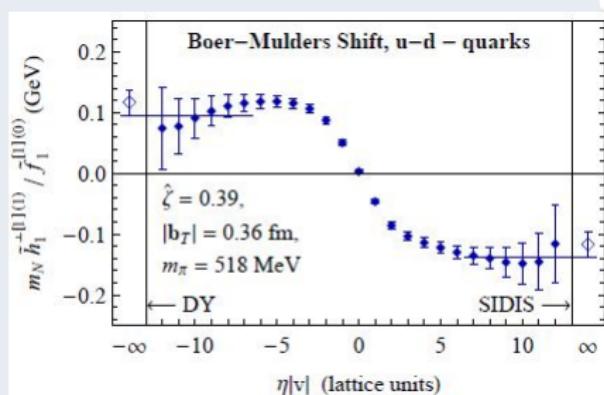
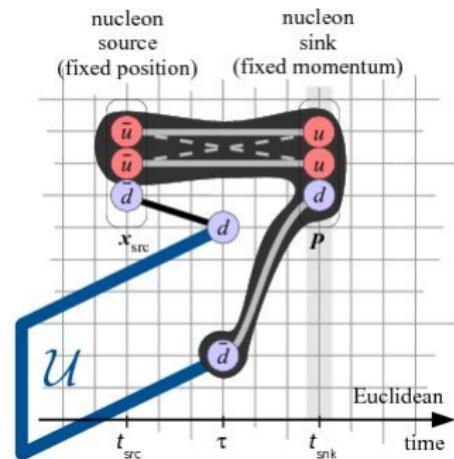
B. Musch, P. Hägler, M. Engelhardt



- calculate space-like staple-shaped Wilson line pointing in  $\hat{z}$  direction; length  $L \rightarrow \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios
- extrapolate/evolve to  $P_z \rightarrow \infty$



$f_{1T}(x, \mathbf{k}_\perp)$  is  $\mathbf{k}_\perp$ -odd term in quark-spin averaged momentum distribution in  $\perp$  polarized target



$f_{1T}^\perp(x, \mathbf{k}_\perp)$  is  $\mathbf{k}_\perp$ -odd term in quark-spin averaged momentum distribution in  $\perp$  polarized target

difference  $\mathcal{L}^q - L^q$

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$$\mathcal{L}_{JM}^q - L_{Ji}^q = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \int_{x^-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp)]^z q(\vec{x}) | P, S \rangle$$

## Orbital Angular Momentum

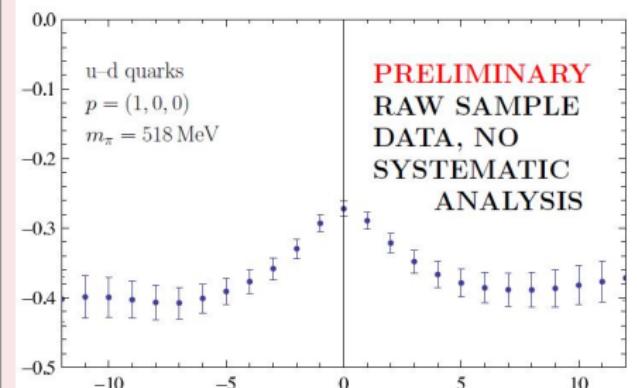
- same operator as for TMDs, only nonforward matrix elements:
  - momentum transfer provides position space information ( $\rightarrow \mathbf{r}_\perp \times \mathbf{k}_\perp$ )
  - staple with long side in  $\hat{z}$  direction
  - (large) nucleon momentum in  $\hat{z}$  direction
  - small momentum transfer in  $\hat{y}$  direction

→ generalized TMD  $F_{14}$  (Metz et al.)

- quark OAM

## lattice QCD (M.Engelhardt)

- $L_{staple}$  vs. staple length
- $L_{Ji}^q$  for length = 0
- $\mathcal{L}_{JM}^q$  for length  $\rightarrow \infty$



- shown  $L_{staple}^u - L_{staple}^d$
- similar result for each  $\Delta L_{FSI}^q$

difference  $\mathcal{L}^q - L^q$

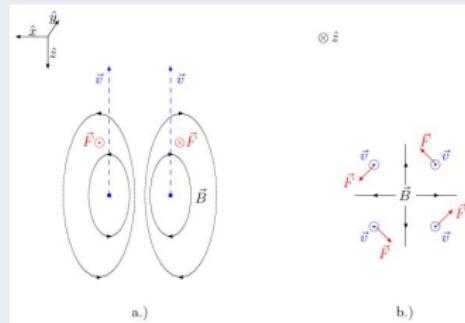
$\mathcal{L}_{JM}^q - L_{Ji}^q = \Delta L_{FSI}^q =$  change in OAM as quark leaves nucleon

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local torque

$$\int d^2x_\perp \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times F^{+\perp}(x^-, \mathbf{x}_\perp)]^z q(\vec{x}) | P, S \rangle = 0$$

- formal argument: PT
- intuitive: front vs. back cancellation in ensemble average

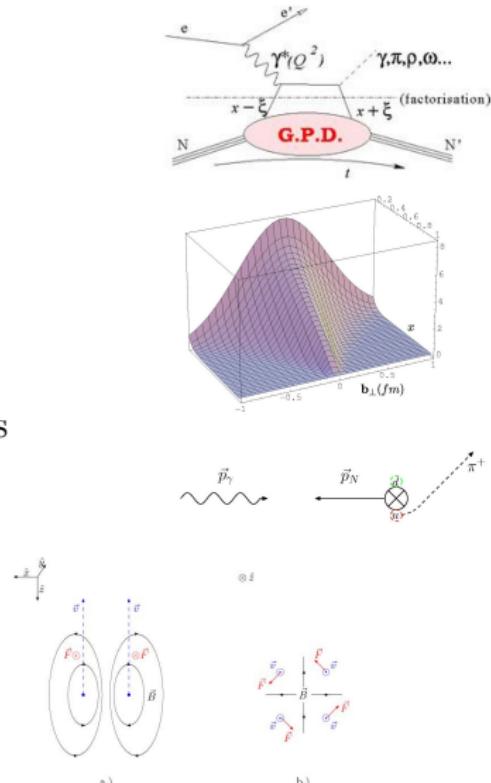
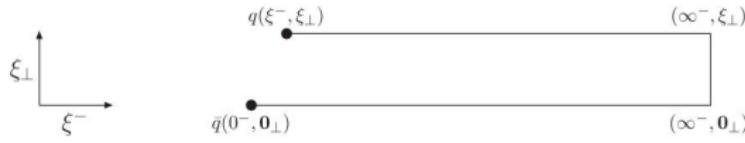


- lowest nontrivial moment

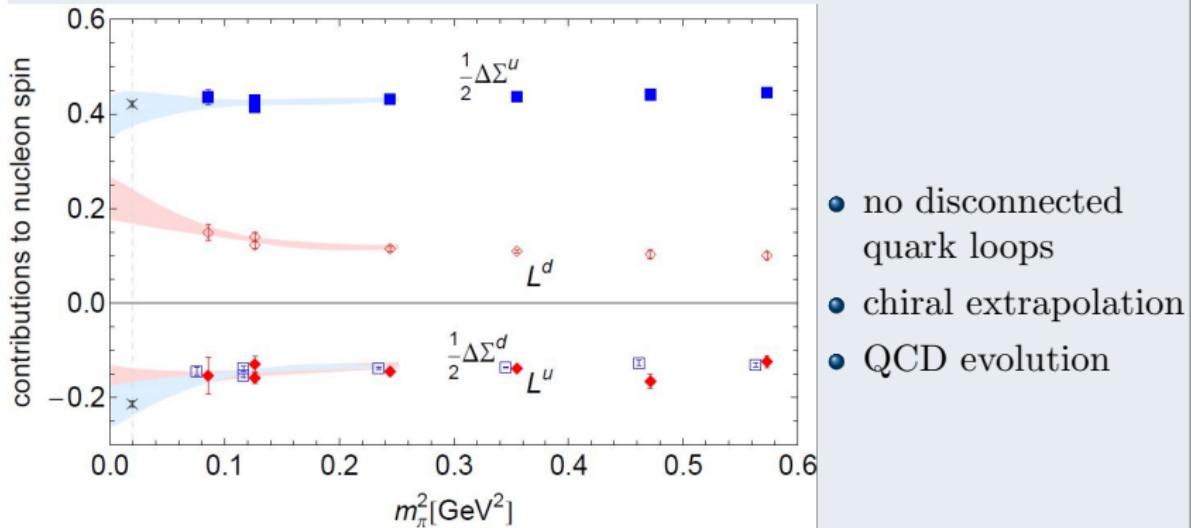
$$\int d^2x_\perp \langle P, S | \bar{q}(\vec{x}) \gamma^+ [\vec{x} \times \partial_- F^{+\perp}(x^-, \mathbf{x}_\perp)]^z q(\vec{x}) | P, S \rangle = 0$$

$\hookrightarrow$  off-forward matrix element of  $\bar{q}\gamma_5\gamma_\perp D_{-\perp}^3 q$  (twist-3,  $x^3$  moment)

- GPDs  $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$  '3d imaging'
- $\perp$  polarization  $\Rightarrow \perp$  deformation
- $\hookrightarrow$  sign of qg-correlations
- $\hookrightarrow$  sign and magnitude of  $d_2$
- simultaneous info about  $\perp$  position & long. momentum
- $\hookrightarrow$  Ji sum rule for  $J_q$
- $\mathcal{L}_{JM}^q - L_{Ji}^q =$  change in OAM as quark leaves nucleon (due to torque from FSI)
- ISI/FSI and  $\mathcal{L}_{JM}^q$  vs.  $L_{Ji}^q$  in lattice QCD



lattice: (lattice hadron physics collaboration - LHPC)



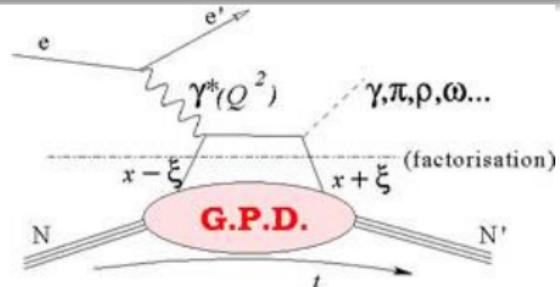
$$J^q = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$$

$$L^q = J^q - \frac{1}{2} \Delta\Sigma^q$$

# $\mathcal{A}_{DVCS} \rightsquigarrow GPDs$

interesting GPD physics:

- $J_q = \int_0^1 dx x [H(x, \xi, 0) + E(x, \xi, 0)]$   
requires  $GPDs(x, \xi, 0)$  for (common)  
fixed  $\xi$  for all  $x$
- $\perp$  imaging requires  $GPDs(x, \xi = 0, t)$



- $\xi$  longitudinal momentum transfer on the target  $\xi = \frac{p^{+'} - p^+}{p^{+'} + p^+}$
- $x$  (average) momentum fraction of the active quark  $x = \frac{k^{+'} + p^+}{p^{+'} + p^+}$

$$\Im \mathcal{A}_{DVCS}(\xi, t) \rightarrow GPD(\xi, \xi, t)$$

- only sensitive to ‘diagonal’  $x = \xi$
- limited  $\xi$  range

$$\Re \mathcal{A}_{DVCS}(\xi, t) \rightarrow \int_{-1}^1 dx \frac{GPD(x, \xi, t)}{x - \xi}$$

- limited  $\xi$  range
- most sensitive to  $x \approx \xi$
- some sensitivity to  $x \neq \xi$ , but

## Polynomiality/Dispersion Relations (GPV/AT DI)

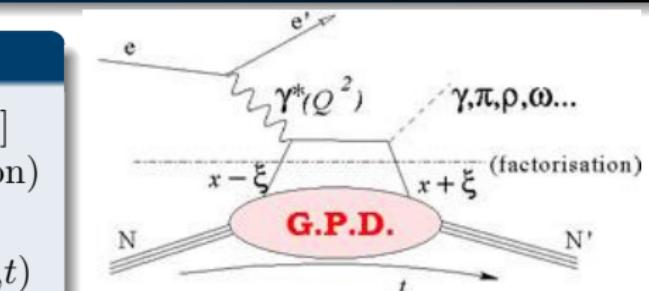
$$\Re \mathcal{A}(\xi, t, Q^2) = \int_{-1}^1 dx \frac{H(x, \xi, t, Q^2)}{x - \xi} = \int_{-1}^1 dx \frac{\textcolor{red}{H(x, x, t, Q^2)}}{x - \xi} + \Delta(t, Q^2)$$

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$$\Im \mathcal{A}_{DVCS}(\xi, t) \longrightarrow GPD(\xi, \xi, t)$$



$$\Re \mathcal{A}_{DVCS}(\xi, t) \longrightarrow \int_{-1}^1 dx \frac{GPD(x, \xi, t)}{x - \xi}$$

## Polynomiality/Dispersion Relations (GPV/AT DI)

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- Can 'condense' all information contained in  $\mathcal{A}_{DVCS}$  (fixed  $Q^2$ ) into  $GPD(x, x, t, Q^2)$  &  $\Delta(t, Q^2)$
- if two models both satisfy polynomiality and are equal for  $x = \xi$  (but not for  $x \neq \xi$ ) and have same  $\Delta(t, Q^2)$  then DVCS at fixed  $Q^2$  cannot distinguish between the two models

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need Evolution!

$$\mu^2 \frac{d}{d\mu^2} H^{q(-)}(x, \xi, t) = \int_{-1}^1 dx' \frac{1}{|\xi|} V_{\text{NS}}\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) H^{q(-)}(x', \xi, t)$$

- $Q^2$  evolution changes  $x$  distribution in a known way for fixed  $\xi$   
→ measure  $Q^2$  dependence to disentangle  $x$  vs.  $\xi$  dependence