Energy dependence of azimuthal asymmetries

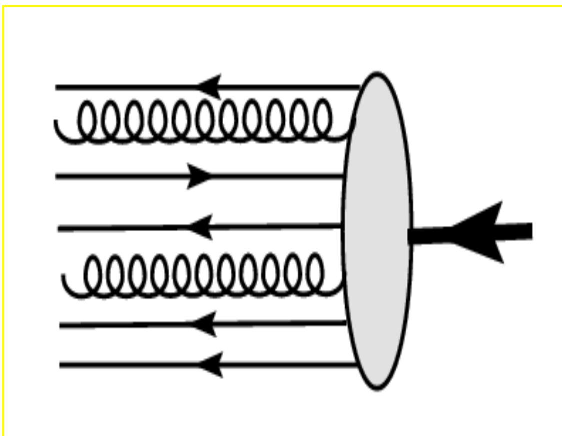
H. Avakian (Jlab)

Deep Processes Working Group Meeting 2016, Nov 3, 2016

- EVA framework
- Structure Function $F_{UU}^{\cos\phi}$ in SIDIS
- Sensitivity to kinematical cuts
- Summary & Conclusions

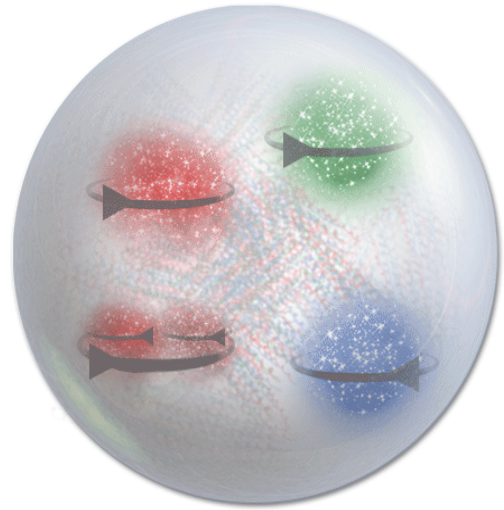
Partonic distributions from LC to real life

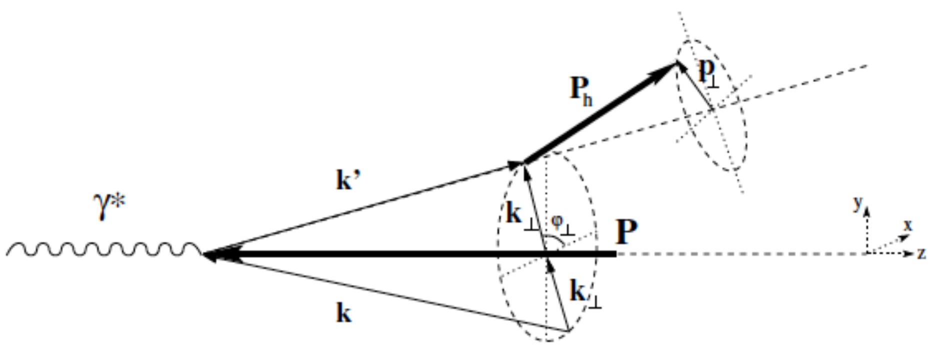
In the frame with fast moving hadron, with "frozen" interactions study longitudinal momentum distributions



q(x) - Probability to find a quark with a fraction x of proton momentum P, extended to q($x.k_T$)

real life





quark momentum in the γ*p CM frame (on shell quarks)

$$k = \left(xP_0 + \frac{k_\perp^2}{4xP_0}, \boldsymbol{k}_\perp, -xP_0 + \frac{k_\perp^2}{4xP_0} \right)$$

x and k_T not independent!

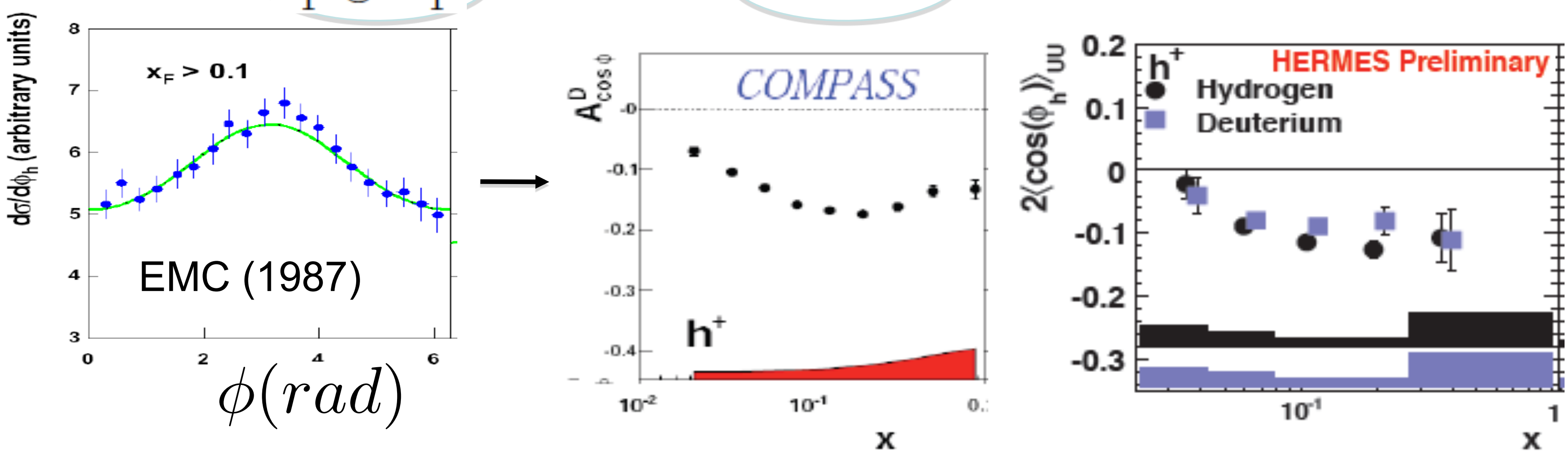
Azimuthal distributions in SIDIS

$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\, F_{UU}^{\cos\phi_h}$$

$$+ \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\, F_{LU}^{\sin\phi_h}\Big\},$$

$f_1 \otimes D_1$ h.t. h.t.

$h_1^\perp \otimes H_1^\perp$ h.t.



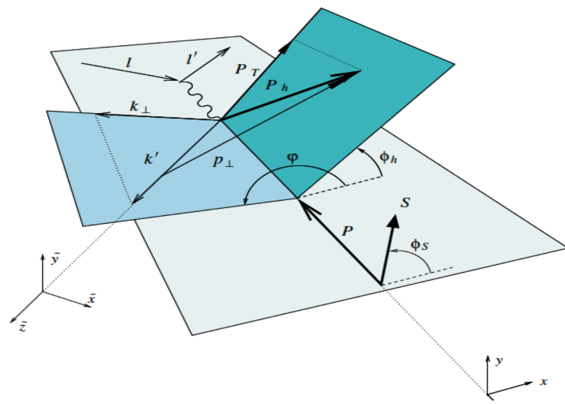
Understanding of cosϕ modulations observed by EMC, COMPASS and HERMES is crucial for interpretation of cos2ϕ and multiplicities

SIDIS cross section

$$\frac{d^5\sigma}{dx\,dQ^2\,dz\,d\phi_h\,dP_{h\perp}^2} \propto F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$\varepsilon = \frac{1-y-\gamma^2y^2/4}{1-y+y^2/2+\gamma^2y^2/4},$$

In WW approximation:



$$\hat{h} = \vec{P_T}/P_T$$

with HT functions

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\frac{\hat{h}\cdot\boldsymbol{p}_\perp}{zM_h}xh\,H_1^\perp - \frac{\hat{h}\cdot\boldsymbol{k}_\perp}{M}xzf^\perp D_1\right],$$

only LT functions

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\frac{\hat{h}\cdot\boldsymbol{p}_\perp}{zM_h}\frac{\boldsymbol{k}_\perp^2}{M^2}h_1^\perp H_1^\perp - \frac{\hat{h}\cdot\boldsymbol{k}_\perp}{M}zf_1D_1\right]$$

Main object to define

$$\boxed{\mathcal{C}[w,fD] = x\sum_a e_a^2\int_0^{k_{\perp max}} k_\perp dk_\perp\int_0^{2\pi} d\phi\, w(\boldsymbol{k}_\perp,\boldsymbol{p}_\perp(\boldsymbol{k}_\perp))f^a(x,\boldsymbol{k}_\perp^2)\,D^a(z,(\boldsymbol{P}_{h\perp}-z\boldsymbol{k}_\perp)^2)}$$

Possible solution: define grids for TMDs, then integrate numerically with interpolation

SIDIS cross-section

Expanding the contraction and integrating over ψ and the beam polarization, the cross-section for an unpolarized target can be written as

$$\frac{d^5\sigma}{dx\ dQ^2\ dz\ d\phi_h\ dP_{h\perp}^2} =$$

$$\underbrace{\frac{2\pi\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)(F_{UU,T}+\epsilon F_{UU,L})}_{A_0}\left\{1+\underbrace{\frac{\sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}}{(F_{UU,T}+\epsilon F_{UU,L})}}_{A_{UU}^{\cos\phi_h}}\cos\phi_h+\underbrace{\frac{\epsilon F_{UU}^{\cos 2\phi_h}}{(F_{UU,T}+\epsilon F_{UU,L})}}_{A_{UU}^{\cos 2\phi_h}}\cos 2\phi_h\right\}$$

According the the factorization theorem, structure functions can, in the Bjorken limit, be written as convolutions of TMDs and FFs $F = \sum \mathrm{TMD} \otimes \mathrm{FF}$

Bjorken Limit:

$Q^2 \to \infty$

$2P\cdot q \to \infty$

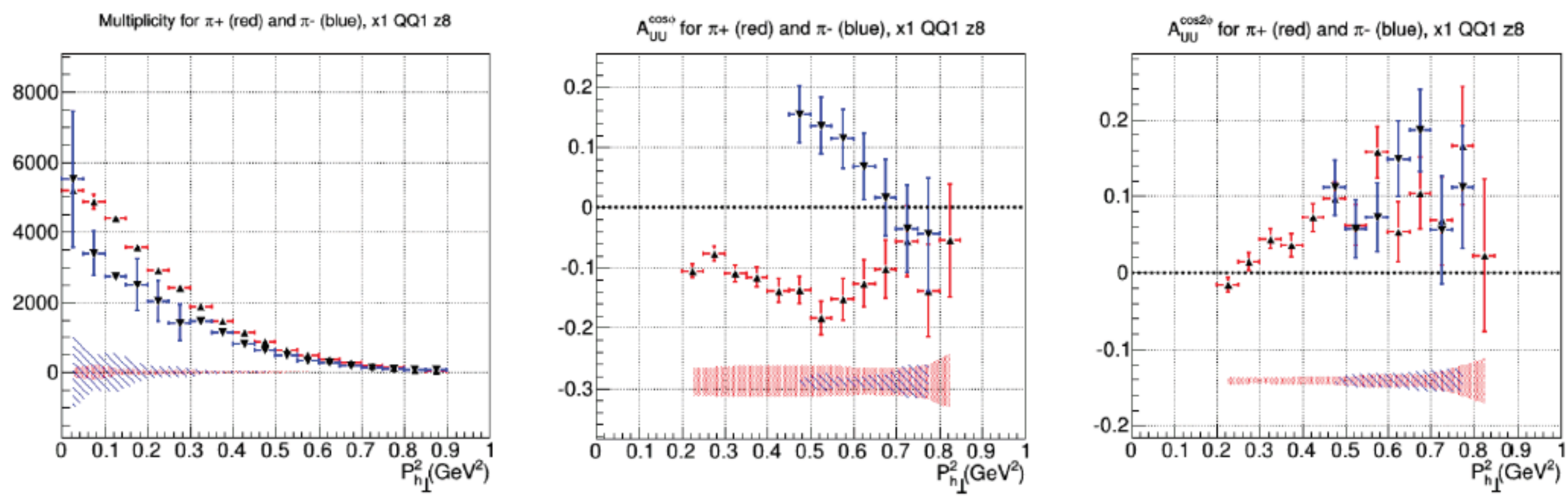
$P\cdot P_h \to \infty$

$$\text{fixed}\left\{\begin{array}{l} x = Q^2/2P\cdot q \\ z = P\cdot P_h/P\cdot q \end{array}\right.$$

Measuring SIDIS cross section

Fit with $a(1 + b\cos\phi_h + c\cos 2\phi_h)$

N. Harrison



in WW approximation

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{zM_h}\frac{\boldsymbol{k}_\perp^2}{M^2}h_1^\perp H_1^\perp - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M}z f_1 D_1\right],$$

Simetric behaviour indicates large BM contribution

SIDIS cross section: simple test

$$\frac{d^5\sigma}{dx\, dQ^2\, dz\, d\phi_h\, dP_{h\perp}^2} \propto F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h) F_{UU}^{\cos 2\phi_h}$$

$$\varepsilon = \frac{1-y-\gamma^2 y^2/4}{1-y+y^2/2+\gamma^2 y^2/4}$$

In WW approximation:

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\frac{\hat{h}\cdot p_\perp}{zM_h}\frac{k_\perp^2}{M^2}h_1^\perp H_1^\perp - \frac{\hat{h}\cdot k_\perp}{M} z f_1 D_1\right]$$

Boer-Mulders

Cahn

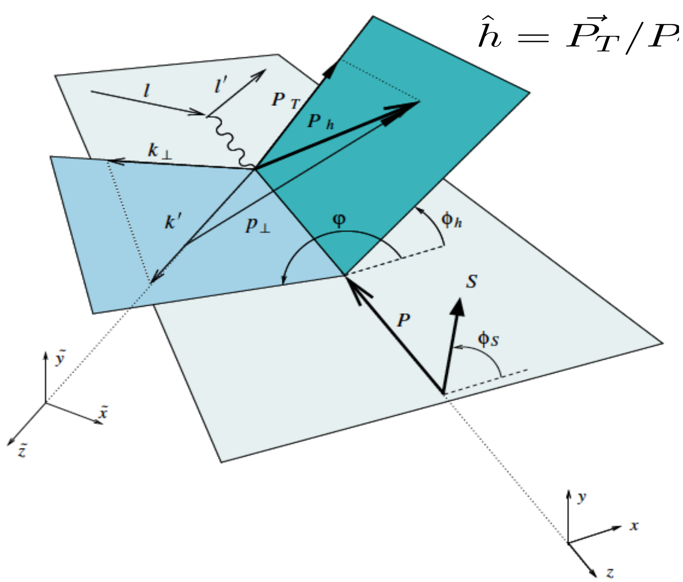
$$F_{UU} = \sum_q e_q^2\, x\, f_1^q(x)\, D_{h/q}(z_h)\frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle}}{\pi\langle P_{h\perp}^2\rangle},$$

$$F_{UU}^{\cos\phi} = -2\frac{P_{h\perp}}{Q}\sum_q e_q^2\, x_B\, f_{q/p}(x)\, D_{h/q}(z_h)\frac{z_h\langle k_\perp^2\rangle}{\langle P_{h\perp}^2\rangle}\frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle}}{\pi\langle P_{h\perp}^2\rangle}$$

$$+2e\frac{P_{h\perp}}{Q}\sum_q e_q^2\, x_B\frac{h_1^{\perp,q}(x)}{M_{BM}}\frac{H_1^{\perp,h/q}(z_h)}{M_C}\frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2\rangle_{BM}}}{\pi\langle P_{h\perp}^2\rangle_{BM}^4}$$

$$\times\frac{\langle k_\perp^2\rangle_{BM}^2\langle p_\perp^2\rangle_C^2}{\langle k_\perp^2\rangle\langle p_\perp^2\rangle}\left[z_h^2\langle k_\perp^2\rangle_{BM}\left(P_{h\perp}^2-\langle P_{h\perp}^2\rangle_{BM}\right)+\langle p_\perp^2\rangle_C\langle P_{h\perp}^2\rangle_{BM}\right]$$

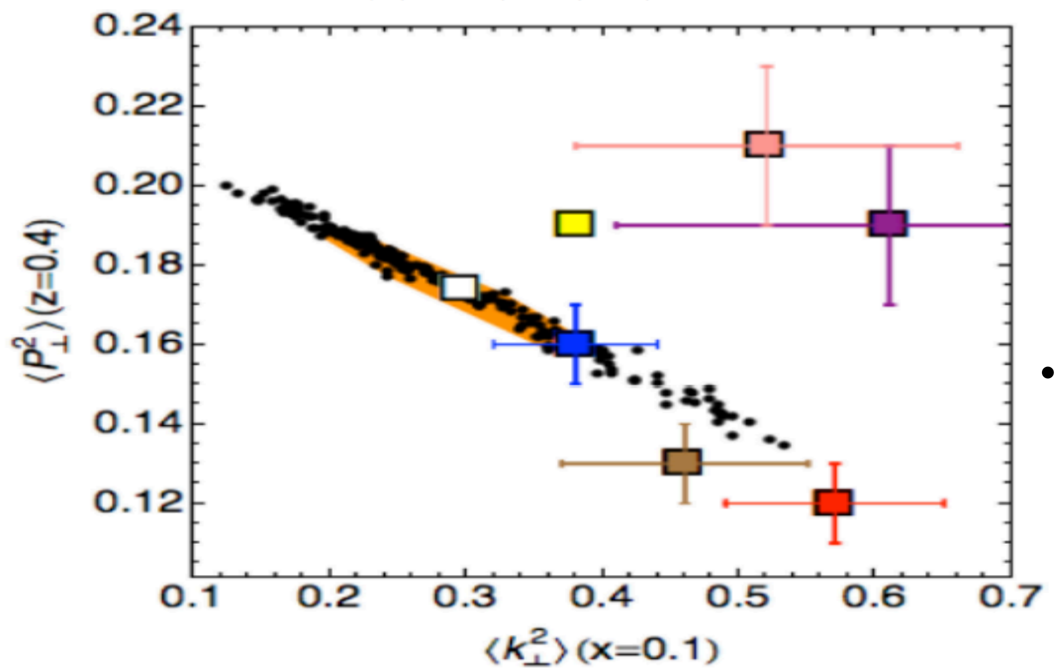
$$\hat{h} = \vec{P_T}/P$$

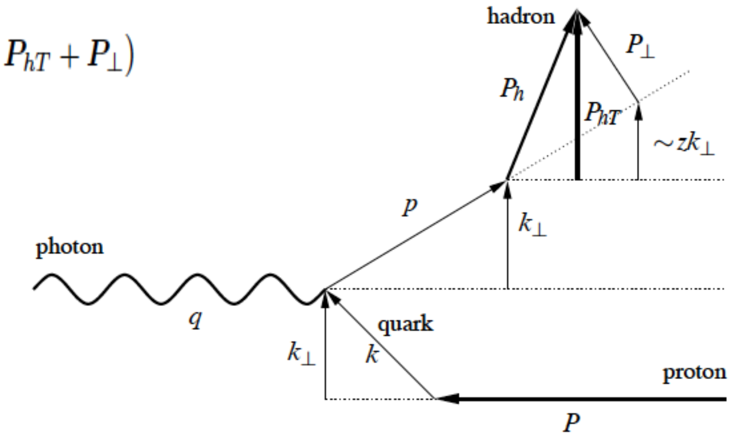


Extracting the average transverse momenta

Andrea Signori,[1,*] Alessandro Bacchetta,[2,3,†] Marco Radici,[3,‡] and Gunar Schnell[4,5,§]

$$F_{UU,T}(x,z,P_{hT}^2,Q^2) = \sum_a \mathcal{H}^a_{UU,T}(Q^2;\mu^2)\int dk_\perp\, dP_\perp\, f_1^a(x,k_\perp^2;\mu^2)\, D_1^{a\to h}(z,P_\perp^2;\mu^2)\,\delta(zk_\perp - P_{hT}+P_\perp)$$
$$+\, Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}(M/Q).$$





- Multiplicity alone may not be enough to separate $\langle k_T\rangle$ from average $\langle p_T\rangle$

$$m_N^h(x,z,P_{hT}^2) = \frac{\pi}{\sum_a e_a^2 f_1^a(x)}$$
$$\times \sum_a e_a^2\, f_1^a(x)\, D_1^{a\to h}(z)\, \frac{e^{-P_{hT}^2/\left(z^2\langle \boldsymbol{k}_{\perp,a}^2\rangle + \langle \boldsymbol{P}_{\perp,a\to h}^2\rangle\right)}}{\pi\left(z^2\langle \boldsymbol{k}_{\perp,a}^2\rangle + \langle \boldsymbol{P}_{\perp,a\to h}^2\rangle\right)}$$

$$\frac{\left(F_{UU}^{\cos\phi_h}\right)_{Cahn}}{F_{UU}} \propto \frac{\left\langle k_\perp^2\right\rangle}{\left\langle P_T^2\right\rangle}$$

- cos ϕ has much greater sensitivity to $\langle k_T\rangle$

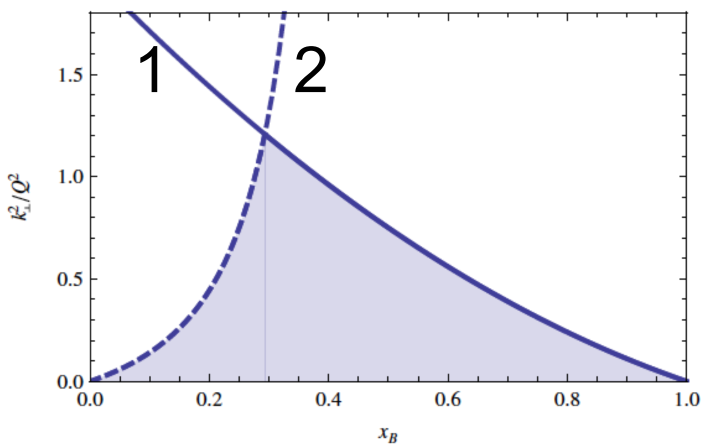
Corrections from "real life" limited phase space

M. Boglione, S. Melis & A. Prokudin Phys. Rev. D 84, 034033 2011

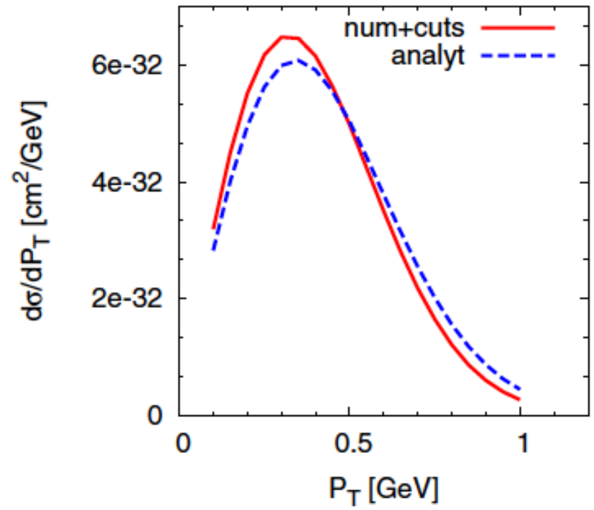
1. Energy of the parton less than the energy of the parent hadron
2. Parton moves forward with respect to the parent hadron direction

$$k_\perp^2 \le (2 - x_B)(1 - x_B)Q^2,$$

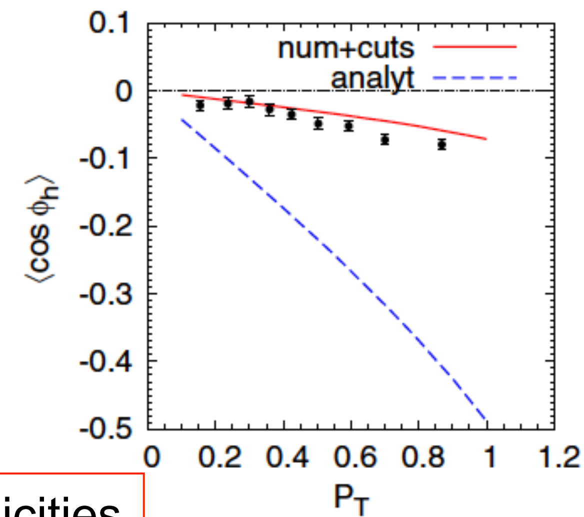
$$k_\perp^2 \le \frac{x_B(1 - x_B)}{(1 - 2x_B)^2} Q^2,$$



$$f_{q/p}(x) = \int_0^{2\pi} d\varphi \int_0^{k_\perp^{\max}} k_\perp dk_\perp f_{q/p}(x, k_\perp).$$



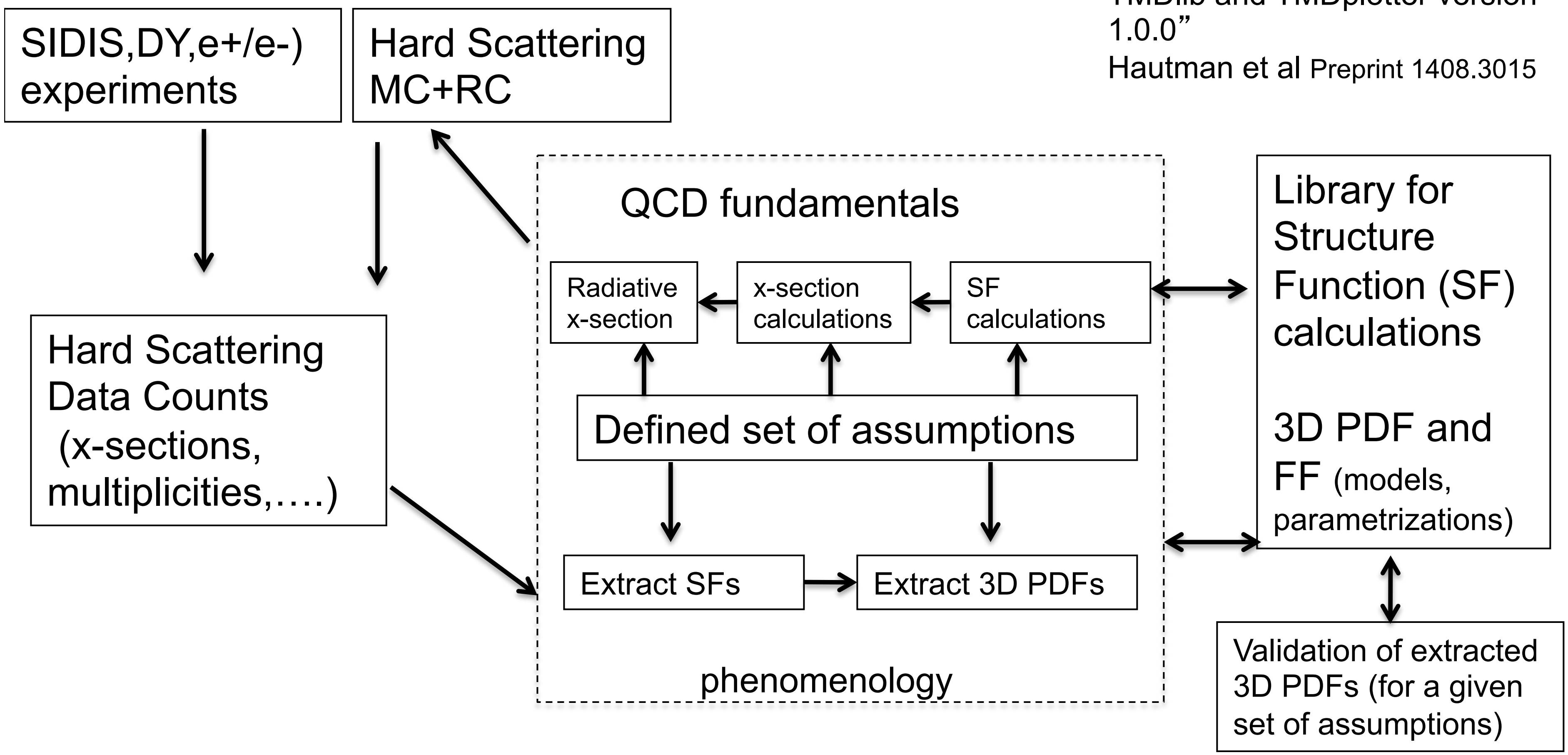
COMPASS Deuteron-π^+



<cos> more sensitive to kinematic limitations than multiplicities

Extraction and validation of 3D PDFs

TMDlib and TMDplotter version 1.0.0”
Hautman et al Preprint 1408.3015



Development of a reliable techniques for the extraction of 3D PDFs and fragmentation functions from the multidimensional experimental observables with controlled systematics requires close collaboration of experiment, theory and computing

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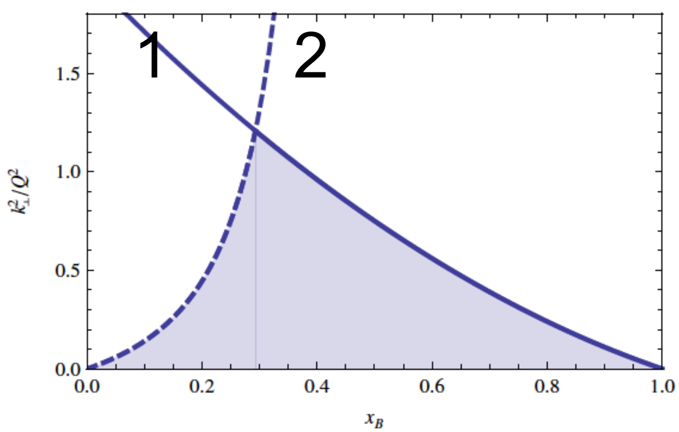


$$f_{q/p}(x) = \int_0^{2\pi} d\varphi \int_0^{k_\perp^{\max}} k_\perp dk_\perp f_{q/p}(x, k_\perp).$$

multiplicities are also sensitive to kinematic limitations

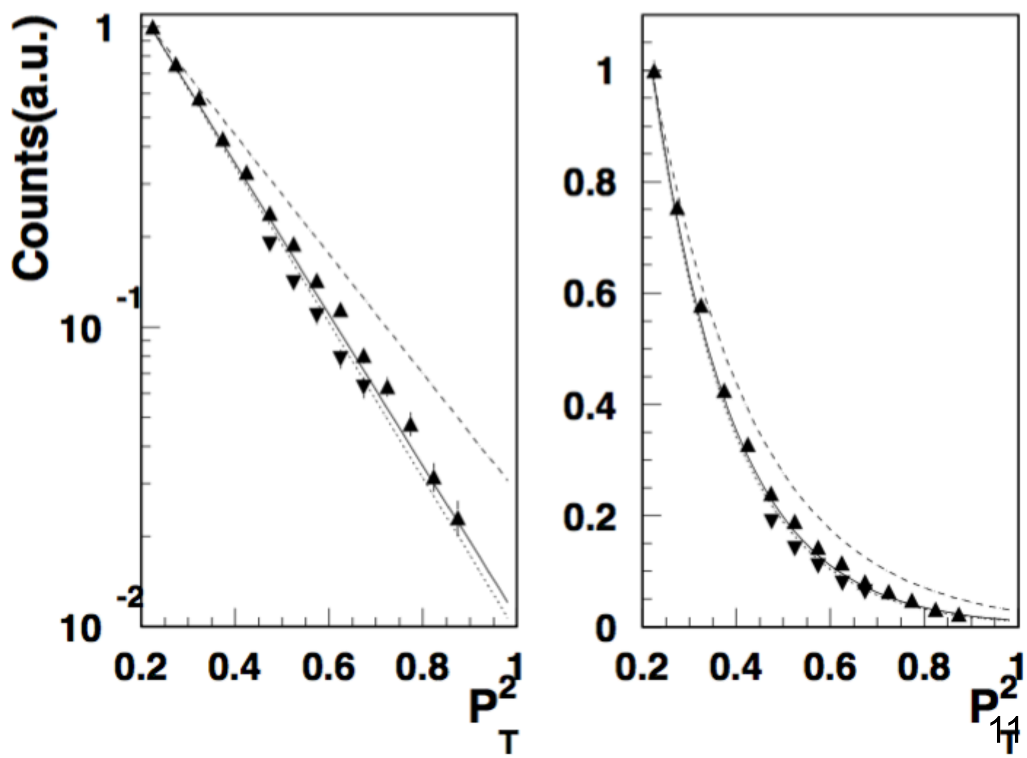
Aghasyan et al

$$\frac{k_\perp^2}{Q^2} \leq \frac{T_1(x, M, Q^2)}{T_2(x, M, Q^2)}$$

$$T_1 = Q^8x + 5M^2Q^6x^2 - Q^8x^2 + 8M^4Q^4x^3 - 4M^2Q^6x^3 + 4M^6Q^2x^4 - 4M^4Q^4x^4$$

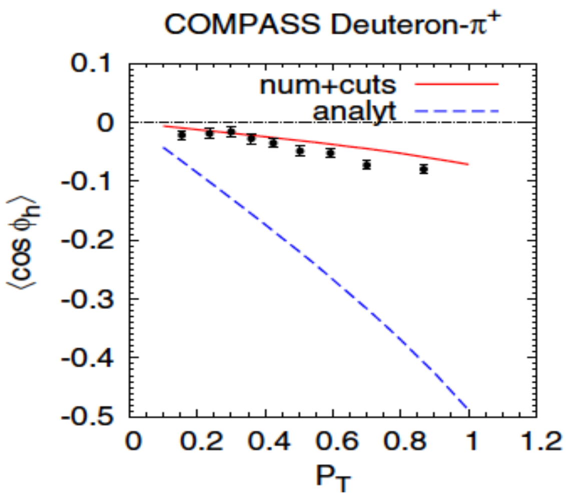
and

$$T_2 = Q^8 + 2M^2Q^6x - 4Q^8x + M^4Q^4x^2 - 12M^2Q^6x^2 + 4Q^8x^2 - 16M^4Q^4x^3 + 16M^2Q^6x^3 - 8M^6Q^2x^4 + 32M^4Q^4x^4 + 32M^6Q^2x^5 + 16M^8x^6$$

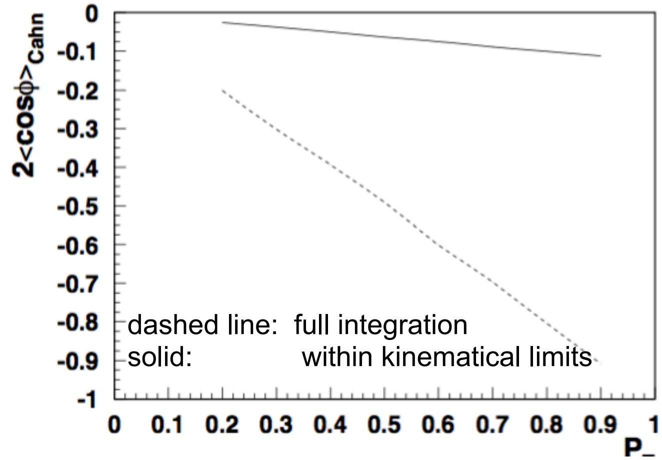


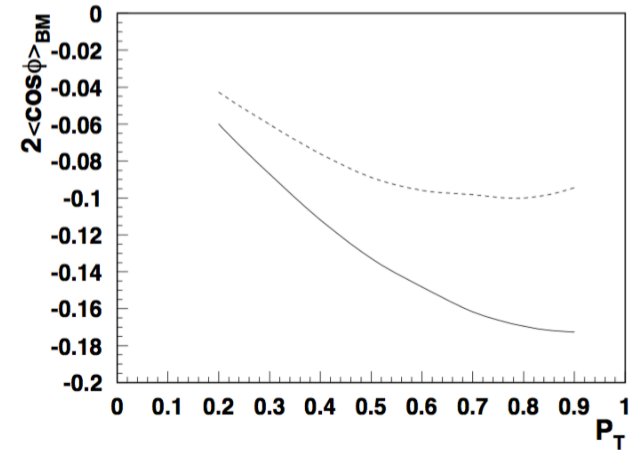
k_T-max: Effect on BM vs Cahn

M. Boglione, S. Melis & A. Prokudin
Phys. Rev. D 84, 034033 2011

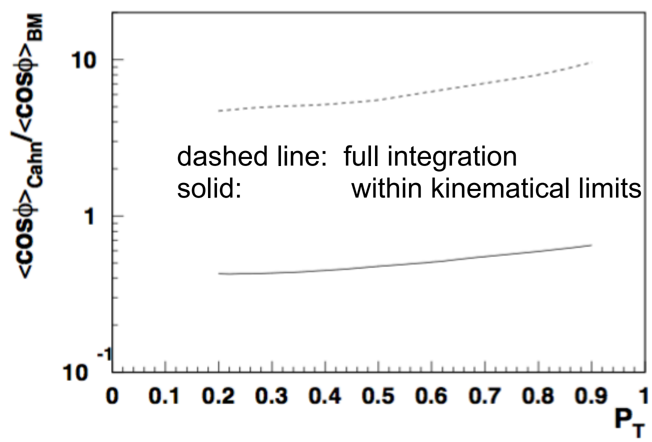


EVA tests: Cahn vs BM





$$C[w, fD] = x \sum_a e_a^2 \int_0^{k_{\perp max}} k_\perp dk_\perp \int_0^{2\pi} d\phi\, w(\mathbf{k}_\perp, \mathbf{p}_\perp(\mathbf{k}_\perp)) f^a(x, \mathbf{k}_\perp^2)\, D^a(z, (\mathbf{P}_{h\perp} - z\mathbf{k}_\perp)^2)$$



$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q} C\left[\frac{\hat{h}\cdot p_\perp}{zM_h}\frac{k_\perp^2}{M^2} h_1^\perp H_1^\perp - \frac{\hat{h}\cdot k_\perp}{M} z f_1 D_1\right]$$

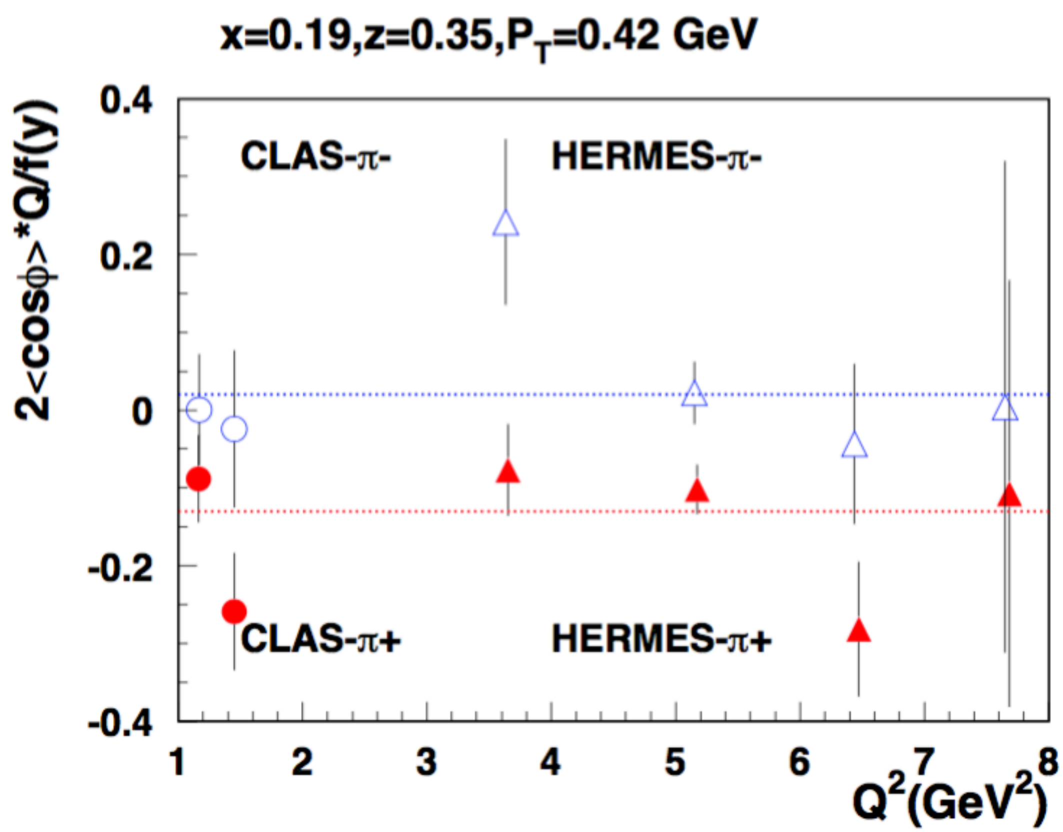
BM contribution seem to be less sensitive to phase space limitations
Need cross check.

Comparing with HERMES

$$F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\cos\phi_h\, F_{UU}^{\cos\phi_h}$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_\perp}{zM_h}\frac{\boldsymbol{k}_\perp^2}{M^2}h_1^\perp H_1^\perp - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M}z f_1 D_1\right],$$

CLAS data consistent with HERMES (27.5 GeV)
π- above π+!



SUMMARY

Kinematic limitations due to finite beam energies may change significantly all spin-azimuthal asymmetries (smaller the beam energy, higher affected x-values)!

- Define correct kinematical space for current fragmentation hadrons
- Use MC to test the program for x-section calculations
- Use CLAS data to choose reasonable constants for MC
- Analyze <cos> and extract Cahn and BM contributions from MC and data.
- Define the data input (x-sections, normalized counts in ϕ-bins)

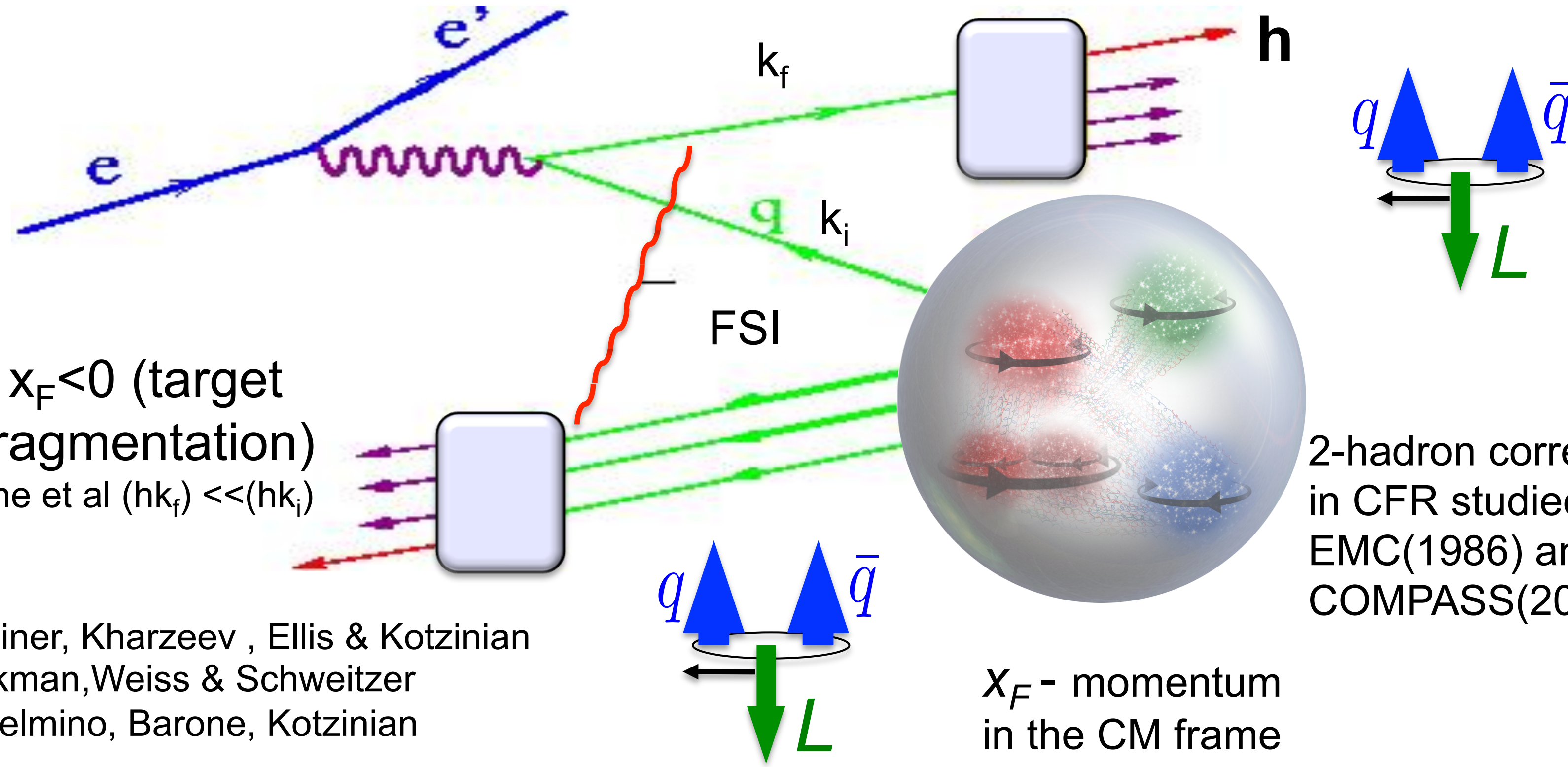
- A self consistent procedure for extraction of TMDs with validation should be used to test the sensitivity of different observables to k_T structure of nucleon.

Support slides….

Hadron production in hard scattering

x_F>0 (current fragmentation)

X. Artru & Z. Belghobsi



x_F<0 (target fragmentation)

Boglione et al (hk_f) <<(hk_i)

2-hadron correlations in CFR studied in EMC(1986) and COMPASS(2015)

Karliner, Kharzeev , Ellis & Kotzinian
Strikman,Weiss & Schweitzer
Anselmino, Barone, Kotzinian

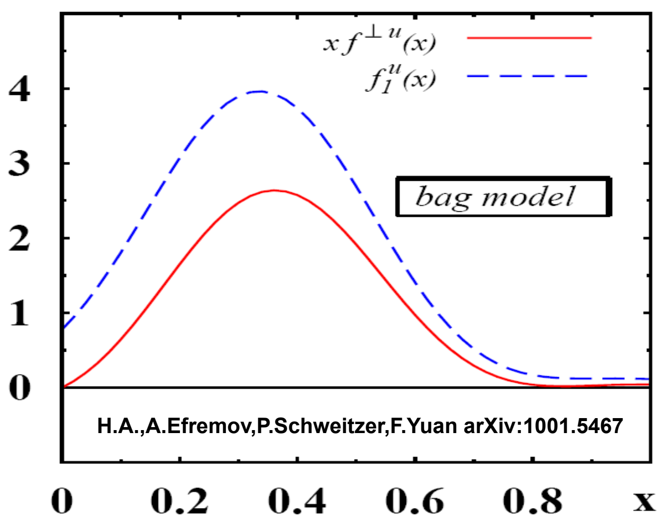
x_F - momentum in the CM frame

Correlations of the spin of the target or/and the momentum and the spin of quarks, combined with final state interactions define the azimuthal distributions of produced particles

Model predictions for cosϕ

$$F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(x f^{\perp} D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$



$$F_{UU}^{\cos\phi} \propto f^{\perp q} D_1^q$$

$$x f^{\perp q} = x\tilde{f}^{\perp q} + f_1^q$$

"interaction dependent"

Models agree on a large HT distributions

Extracting the average transverse momenta

V. Barone, M. Boglione, J. O. Gonzalez Hernandez, S. Melis

$$F_{UU}^{\cos\phi}|_{\mathrm{Cahn}} = -2\sum_q e_q^2 x \int \mathrm{d}^2\boldsymbol{k}_\perp \frac{(\boldsymbol{k}_\perp \cdot \boldsymbol{h})}{Q} f_q(x, k_\perp) D_q(z, p_\perp), \quad (9)$$

$$\frac{\left(F_{UU}^{\cos\phi_h}\right)_{Cahn}}{F_{UU}} \propto \frac{\langle k_\perp^2 \rangle}{\langle P_T^2 \rangle} \qquad \langle cos(\phi)\rangle \propto \frac{\left(F_{UU}^{\cos\phi_h}\right)_{Cahn}}{F_{UU}} + \frac{\left(F_{UU}^{\cos\phi_h}\right)_{BM}}{F_{UU}}$$

$$F_{UU}^{\cos\phi}|_{\mathrm{BM}} = \sum_q e_q^2 x \int \mathrm{d}^2\boldsymbol{k}_\perp \frac{k_\perp}{Q} \frac{P_T - z(\boldsymbol{k}_\perp \cdot \boldsymbol{h})}{p_\perp} \times \Delta f_{q^\uparrow/p}(x, k_\perp) \Delta D_{h/q^\uparrow}(z, p_\perp). \quad (10)$$

$$\Delta f_{q^\uparrow/p}(x, k_\perp) = \Delta f_{q^\uparrow/p}(x) \sqrt{2e} \frac{k_\perp}{M_{\mathrm{BM}}} \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_{\mathrm{BM}}}}{\pi\langle k_\perp^2\rangle}$$

$$F_{UU} = \sum_q e_q^2 x_B f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle},$$

cos ϕ has much greater sensitivity to <k$_T$>

$$F_{UU}^{\cos\phi}|_{\mathrm{Cahn}} = -2\frac{P_T}{Q} \sum_q e_q^2 x_B f_{q/p}(x_B) D_{h/q}(z_h) \frac{z_h\langle k_\perp^2\rangle}{\langle P_T^2\rangle} \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle},$$

$$F_{UU}^{\cos\phi}|_{\mathrm{BM}} = 2e\frac{P_T}{Q} \sum_q e_q^2 x_B \frac{\Delta f_{q^\uparrow/p}(x_B)}{M_{\mathrm{BM}}} \frac{\Delta D_{h/q^\uparrow}(z_h)}{M_C} \frac{e^{-P_T^2/\langle P_T^2\rangle_{\mathrm{BM}}}}{\pi\langle P_T^2\rangle_{\mathrm{BM}}^4} \times \frac{\langle k_\perp^2\rangle_{\mathrm{BM}}^2 \langle p_\perp^2\rangle_C^2}{\langle k_\perp^2\rangle\langle p_\perp^2\rangle} [z_h^2\langle k_\perp^2\rangle_{\mathrm{BM}}(P_T^2 - \langle P_T^2\rangle_{\mathrm{BM}}) + \langle p_\perp^2\rangle_C \langle P_T^2\rangle_{\mathrm{BM}}],$$

Example of a EBC table

N. Harrison
(e1f:CLAS@5.5)

5D tables (counts in bins of x, Q^2, z, PT^2, \phi_h):

column 1: x bin number (0-4)
column 2: Q^2 bin number (0-1)
column 3: z bin number (0-17)
column 4: PT^2 bin number (0-19)
column 5: phi bin number (0-35)
column 6: <x>
column 7: <Q^2> (GeV^2)
column 8: <z>
column 9: <PT^2> (GeV^2)
column 10: <phi> (degrees)

column 11: <y>
column 12: number of counts, corrected for acceptance and radiative effects
column 13: statistical error on the the number of counts
column 14: the radiative correction factor

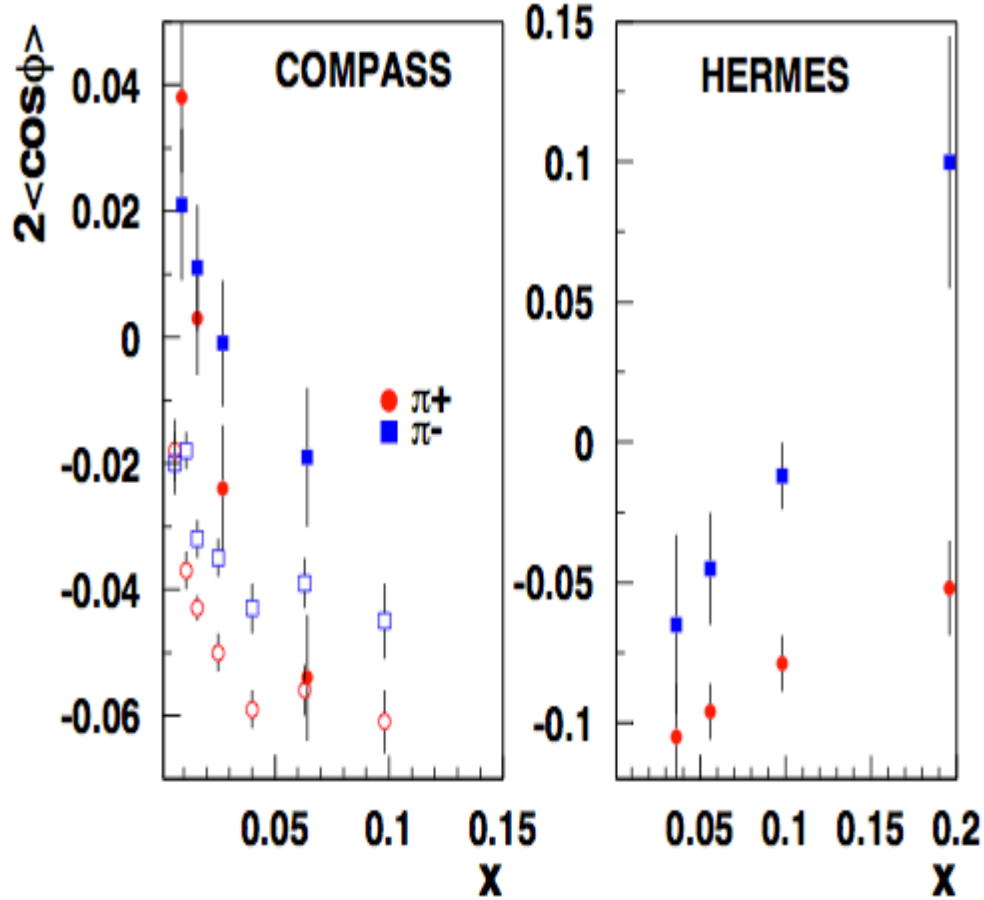
0	0	2	3	19	0.147459	1.16316	0.126884	0.171938	15	0.770322	20528	472.849	1.06035
0	0	2	3	20	0.147459	1.16316	0.126884	0.171938	25	0.770322	19958.1	619.905	1.06123
0	0	2	3	21	0.147459	1.16316	0.126884	0.171938	35	0.770322	20775.6	541.396	1.06257
0	0	2	3	22	0.147459	1.16316	0.126884	0.171938	45	0.770322	19948.5	434.023	1.06435
0	0	2	3	23	0.147459	1.16316	0.126884	0.171938	55	0.770322	21764.5	465.939	1.06671
0	0	2	3	24	0.147459	1.16316	0.126884	0.171938	65	0.770322	20436.3	445.162	1.06951
0	0	2	3	25	0.147459	1.16316	0.126884	0.171938	75	0.770322	20714.1	495.978	1.07289
0	0	2	3	26	0.147459	1.16316	0.126884	0.171938	85	0.770322	20714.4	634.193	1.07689
0	0	2	3	27	0.147459	1.16316	0.126884	0.171938	95	0.770322	21371.5	523.387	1.08116
0	0	2	3	28	0.147459	1.16316	0.126884	0.171938	105	0.770322	21770.1	460.747	1.08614
0	0	2	3	29	0.147459	1.16316	0.126884	0.171938	115	0.770322	21471.5	452.809	1.09134
0	0	2	3	30	0.147459	1.16316	0.126884	0.171938	125	0.770322	22028.4	467.693	1.09713
0	0	2	3	31	0.147459	1.16316	0.126884	0.171938	135	0.770322	24086.5	536.874	1.10245
0	0	2	3	32	0.147459	1.16316	0.126884	0.171938	145	0.770322	21488.1	616.541	1.10712
0	0	2	3	33	0.147459	1.16316	0.126884	0.171938	155	0.770322	23926.8	605.209	1.11166

$A_{UU}^{\cos\phi}$: From measurements to interpretation

N/q	U	L	T
U	$f^\perp$	$g^\perp$	h, e
L	$f_L^\perp$	$g_L^\perp$	h_L, e_L
T	$f_T, f_T^\perp$	$g_T, g_T^\perp$	$h_T, e_T, h_T^\perp, e_T^\perp$

q/h	U	L	
U	D_1		
L		G_{1L}	
T	$H_1^\perp$	$H_{1L}^\perp$	H

$$A_{UU}^{\cos\phi} \propto f^\perp D_1 + hH_1^\perp$$



π^0 SSA less sensitive polarized fragmentation effects (Collins function suppressed)

Azimuthal moments with unpolarized target

quark polarization

N/q	U	L	T
U	$\mathbf{f_1}$		$h_1^\perp$
L		$\mathbf{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\mathbf{h_1}\ h_{1T}^\perp$

N/q	U	L	T
U	$f^\perp$	$g^\perp$	$h, \mathbf{e}$
L	$f_L^\perp$	$g_L^\perp$	$\mathbf{h_L}, e_L$
T	$f_T, f_T^\perp$	$\mathbf{g_T}, g_T^\perp$	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{UU}^{\cos\phi} \propto \frac{M_h}{M}\mathbf{f_1}\frac{D^\perp}{z} - \frac{M}{M_h} x f^\perp D_1$$

q/h	U	L	T
U	$D^\perp$	$D_L^\perp$	$D_T, D_T^\perp$
L	$G^\perp$	$\mathbf{G_L^\perp}$	$\mathbf{G_T}, G_T^\perp$
T	$H, \mathbf{E}$	$\mathbf{H_L}, E_L$	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	$\mathbf{D_1}$		$D_{1T}^\perp$
L		$\mathbf{G_{1L}}$	$G_{1T}^\perp$
T	$H_1^\perp$	$H_{1L}^\perp$	$\mathbf{H_1}\ H_{1T}^\perp$

Azimuthal moments with unpolarized target

quark polarization

N/q	U	L	T
U	$\mathbf{f_1}$		$h_1^\perp$
L		$\mathbf{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\mathbf{h_1}$ $h_{1T}^\perp$

N/q	U	L	T
U	$f^\perp$	$g^\perp$	$h, \mathbf{e}$
L	$f_L^\perp$	$g_L^\perp$	$\mathbf{h_L}, e_L$
T	$f_T, f_T^\perp$	$\mathbf{g_T}, g_T^\perp$	$h_T, e_T, h_T^\perp, e_T^\perp$

$$A_{UU}^{\cos\phi} \sim -h_1^\perp \frac{H}{z} + xhH_1^\perp$$

q/h	U	L	T
U	$D^\perp$	$D_L^\perp$	$D_T, D_T^\perp$
L	$G^\perp$	$\mathbf{G_L^\perp}$	$\mathbf{G_T}, G_T^\perp$
T	$H, \mathbf{E}$	$\mathbf{H_L}, E_L$	$H_T, E_T, H_T^\perp, E_T^\perp$

q/h	U	L	T
U	$\mathbf{D_1}$		$D_{1T}^\perp$
L		$\mathbf{G_{1L}}$	$G_{1T}^\perp$
T	$H_1^\perp$	$H_{1L}^\perp$	$\mathbf{H_1}$ $H_{1T}^\perp$