Tracking in BONuS12

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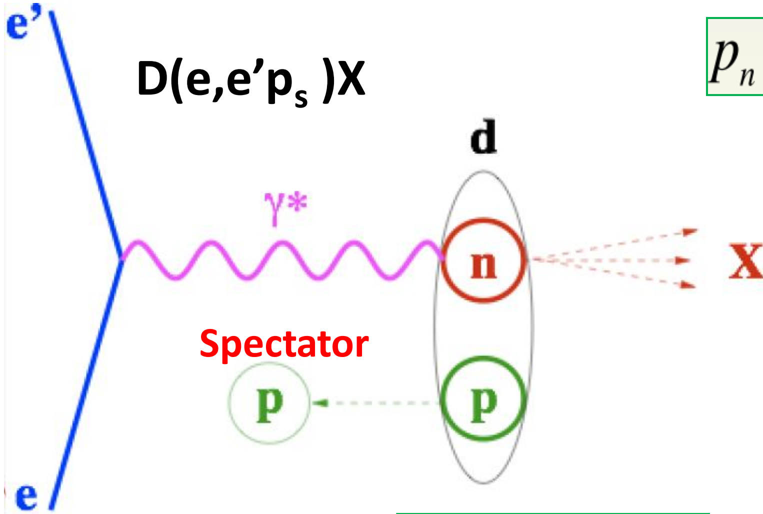
BoNuS12 Experiment

- BONuS12 (Barely Off-shell Nucleon Structure) experiment (E12-06-113 PAC36)
- Measurement of neutron SF: Q^2 1 to 14 GeV^2/c^2 and x ≈ 0.1 to 0.8.
  - Large x - Large Nuclear Effects
- "Spectator tagging" technique.
  - Detection of low momentum recoil proton (down to 70 MeV/c) in coincidence with scattered electrons.
  - Tagged spectator proton ensures the electron scattered from the neutron
  - Reduces model dependence
    - In the nuclear impulse approximation, the virtual photon interacts with the neutron on a short enough time scale such that the proton continues on unperturbed w/ momentum $\mathbf{p}_s = -\mathbf{p}_n$
- RTPC detector for detecting Recoil protons.

Spectator Tagging Technique



$$p_n = \left(M_D - E_S, -\vec{p}_S\right) \quad \alpha_n = 2 - \alpha_S \quad M^{*2} = p_n^{\mu} p_{n\mu}$$

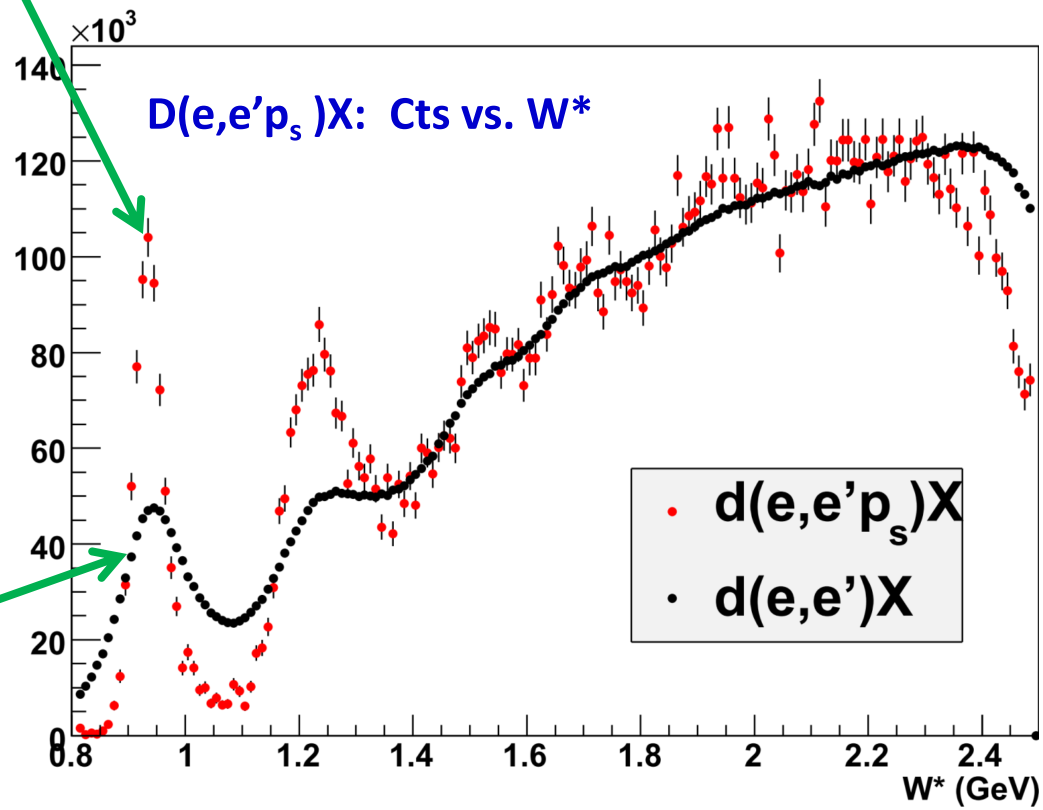
$$x = \frac{Q^2}{2 p_n{}^{\mu} q_{\mu}} \approx \frac{Q^2}{2M\nu(2-\alpha_S)}$$

$$W^{*2} = (p_n + q)^2 = M^{*2} + 2\left((M_D - E_s)\nu - \vec{p}_n \cdot \vec{q}\right) - Q^2$$
$$\approx M^{*2} + 2M\nu(2-\alpha_s) - Q^2$$

$$p_S = \left(E_S, \vec{p}_S\right) \quad \alpha_S = \frac{E_S - \vec{p}_S \cdot \hat{q}}{M_D/2}$$

Low momentum and large/backward angles minimizes:

- Final State Interactions
- Off-Shell Effects
- Target Fragmentation



$$W^2 = M^2 + 2M\nu - Q^2$$

The Recoil Detector - RTPC

BoNuS-6 Radial Time Projection Chamber (RTPC)

Sensitive to protons with momenta of 67-250 Mev/c

3 layers of GEM

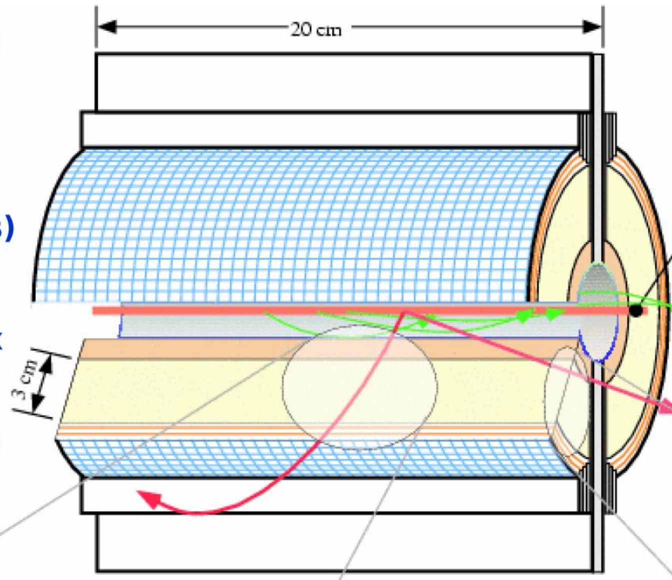
3200 pads (channels)

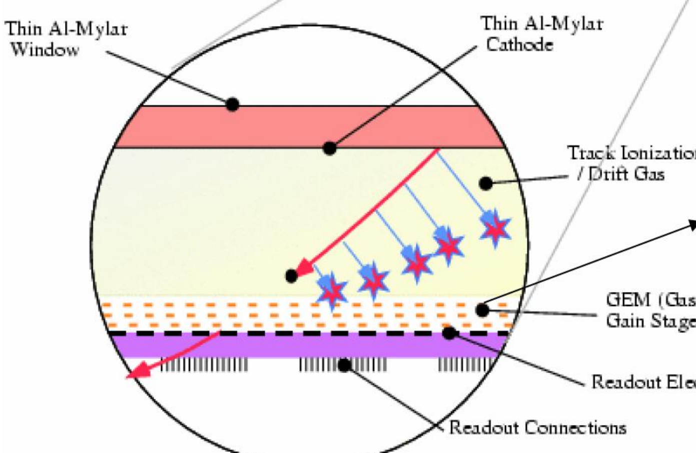
5 Tesla B field

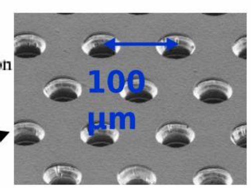
Particles ID by dE/dx

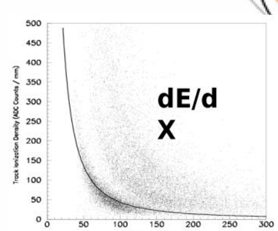
3-D tracking:
time of drift -> r
pad position -> φ, z

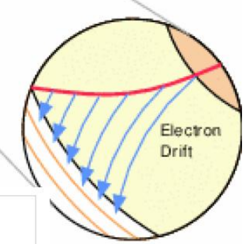
7Atm. D2 gas target, 20cm in length

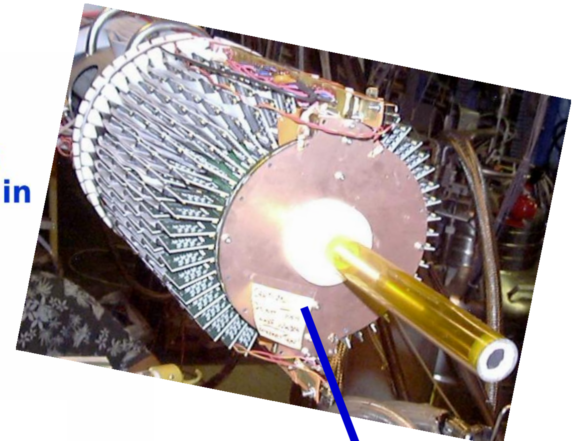




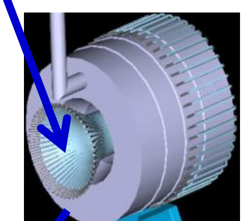


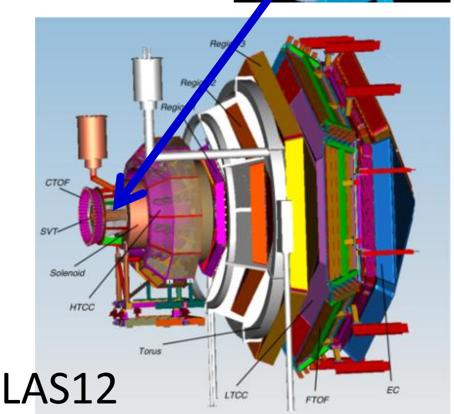






Central Detector





CLAS12

RTPC12 Design

Target: D2 gas, 293k, 7.0 ATM, 40 cm long

Target Wall: **28 um** kapton, 3 mm radius

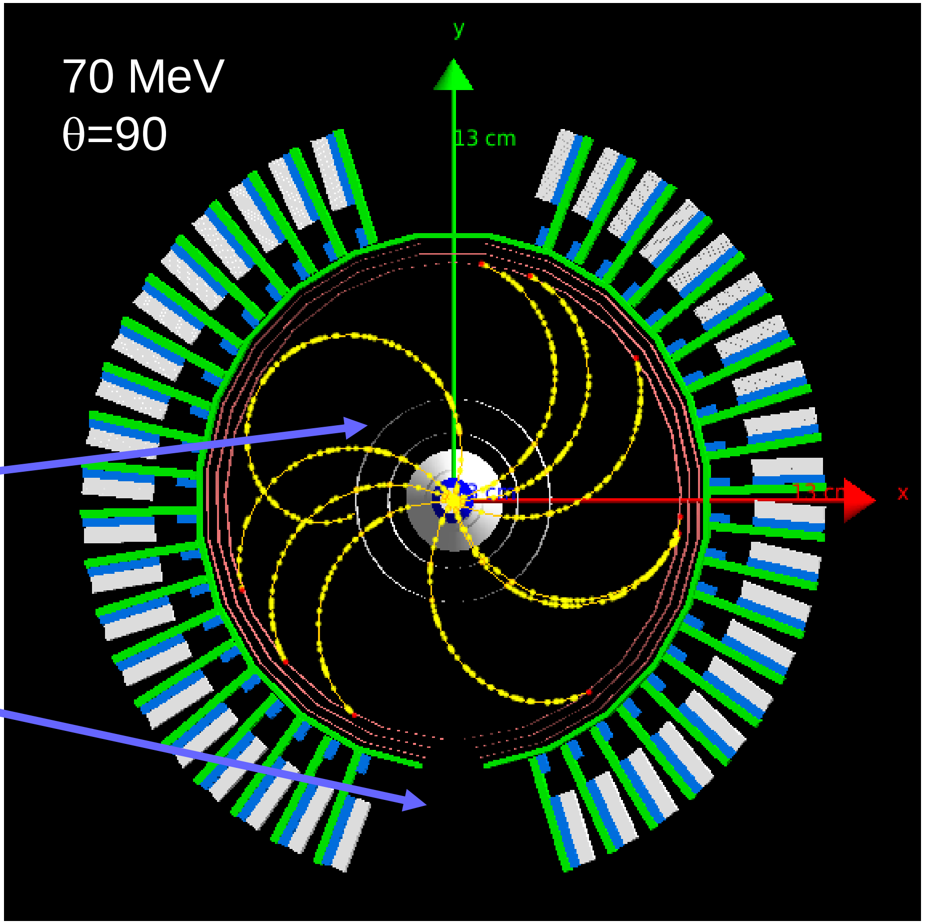
Drift Region: 3<R<7 cm

Drift Gas: 293k, 1 ATM, He/DME (90/10)

Sensor Wires are removed!
No wires here

φ coverage = 360 degrees,
NO φ acceptance loss here

Readout pad at R=8 cm
Pad size 2.79 (tran.) x 4 mm (z),
18000 pads in total

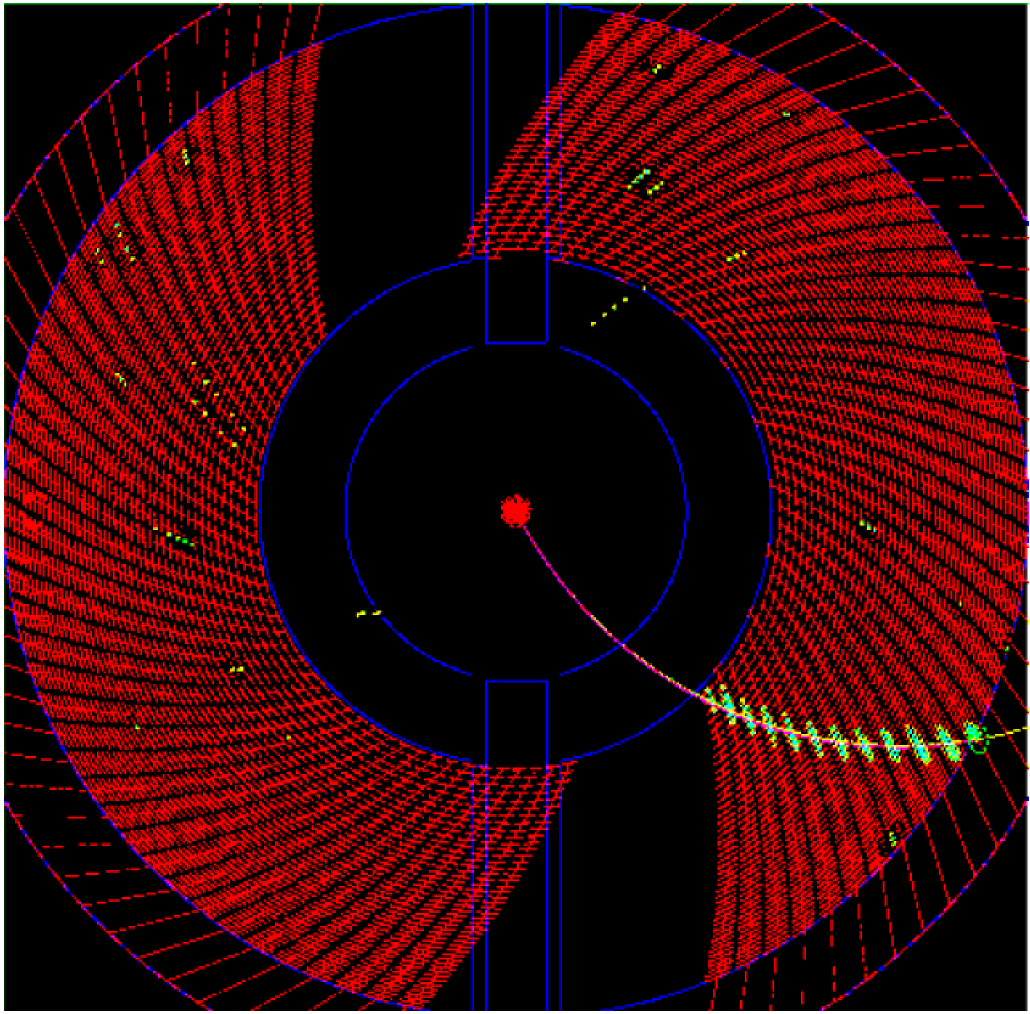


TIC window = 200ns

Use CLAS12 Solenoid with -5T field (pointing upstream)

The Drift Path of An Ionization Electron

- A MAGBOLTZ simulation of the crossed E and B fields in a drift gas mixture
  - determines the drift path and the drift velocity of the electrons.
- The red lines show the drift path of each ionization electron that would appear on a given channel.
- In green is the spatial reconstruction of where the ionization took place.
- Steps to reconstruction:
  - Close hits in space are linked together to form candidate tracks
  - The tracks are fit to helical trajectories.
- The resulting helices tell us the vertex position and the initial three momentum of the particle.

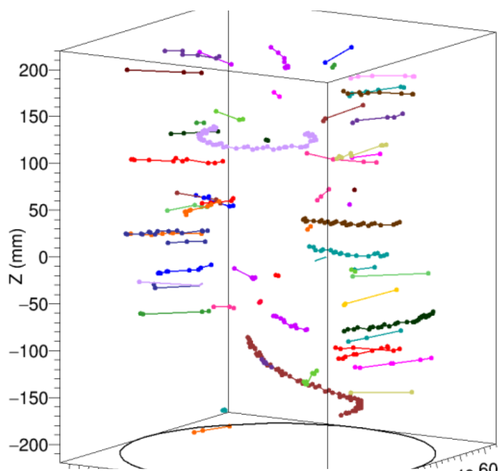


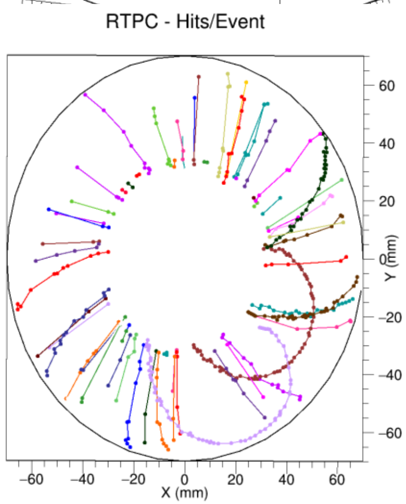
Track Finder

C. Ayerbe

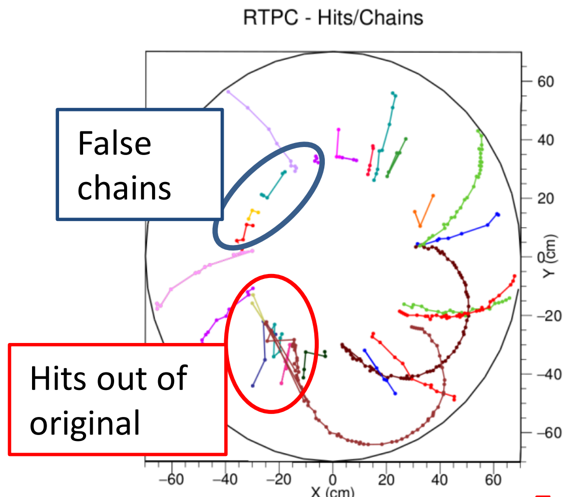
Naïve Track Following method, based on H. Fenker's code for BONuS6.

Simulated events (ELASTIC & QE)

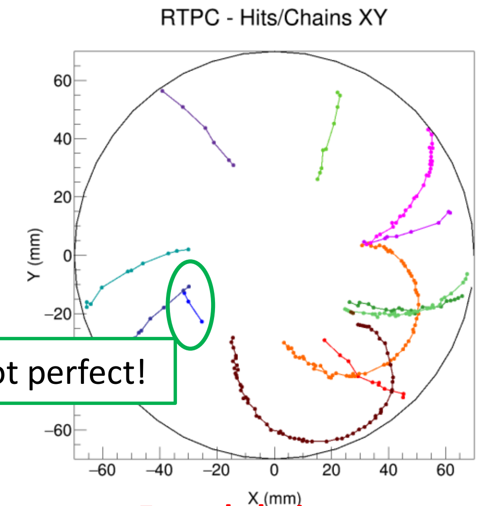




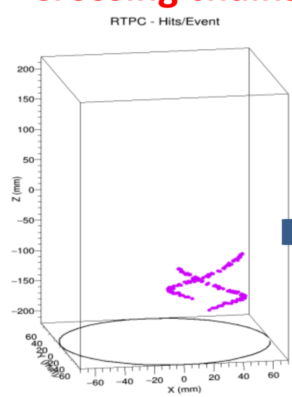
Found Events (simple space inspection)



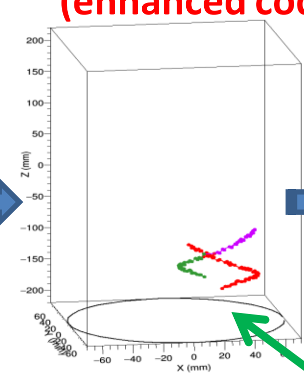
Found Events (angular space inspection)



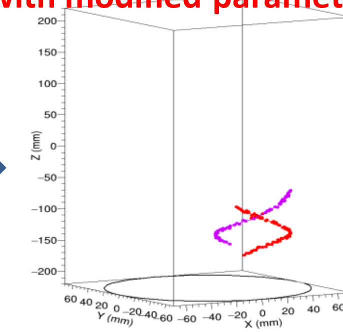
Crossing chains



Found chains (enhanced code)



Found chains (2nd pass of the code with modified parameters)



Now, we have 3 consecutive events

Helix Fitter

The Helix Equation

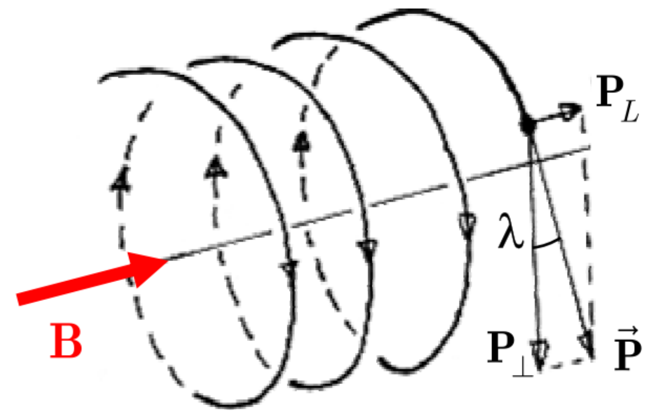
The helix is described in **parametric** form

$$x(s) = x_o + R\left[\cos\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_o\right]$$

$$y(s) = y_o + R\left[\sin\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_o\right]$$

$$z(s) = z_o + s\sin\lambda$$

$$\boxed{R(m) = \frac{p_\perp(GeV)}{0.3B(T)}}$$

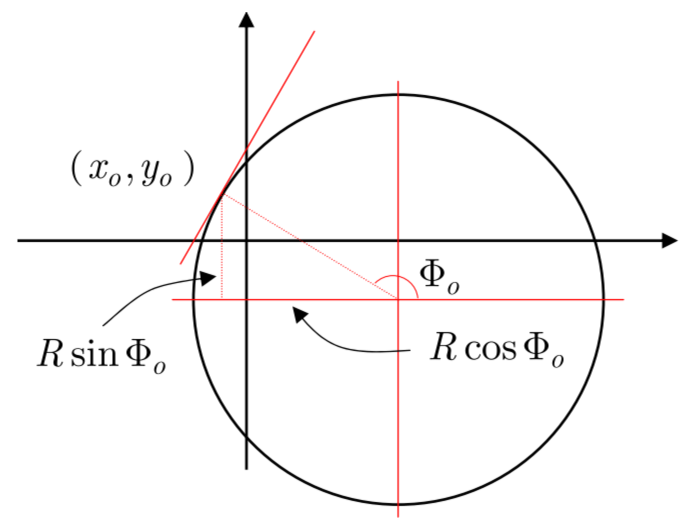


λ is the **dip angle**

$h = \pm 1$ is the **sense of rotation** on the helix

The **projection** on th x-y **plane** is a circle

$$(x - x_o + R\cos\Phi_o)^2 + (y - y_o + R\sin\Phi_o)^2 = R^2$$



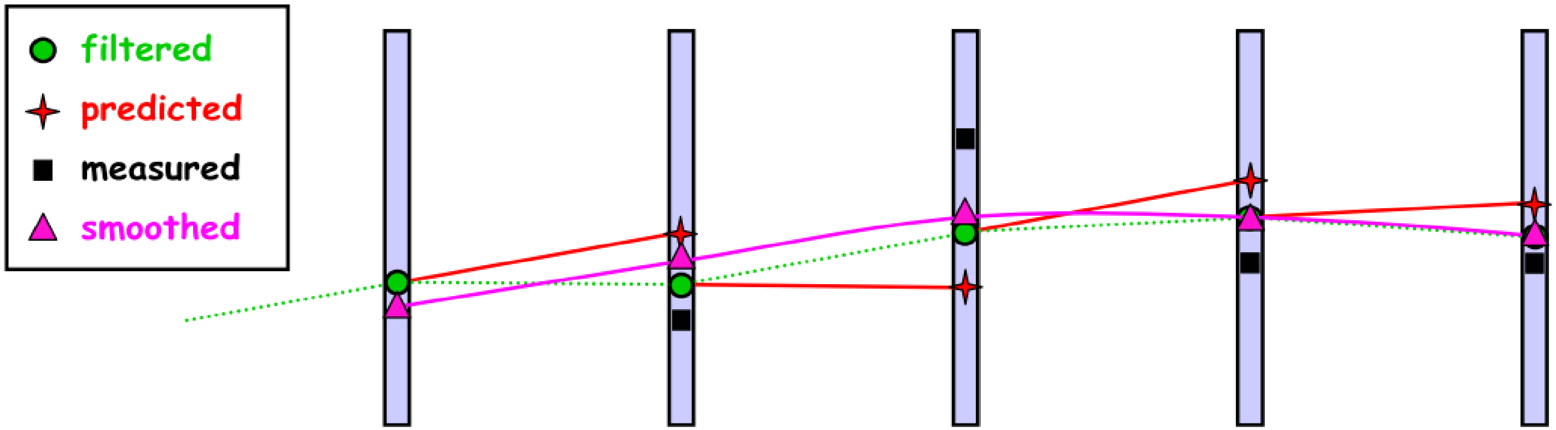
x_o and y_o the coordinates at $s = 0$

Φ_o is also related to the **slope** of the tangent to the circle at $s = 0$

Kalman Filter

Kalman filter is an algorithm that uses a series of measurements observed which contain noise (random variations e.g. multiple scattering) and other errors.

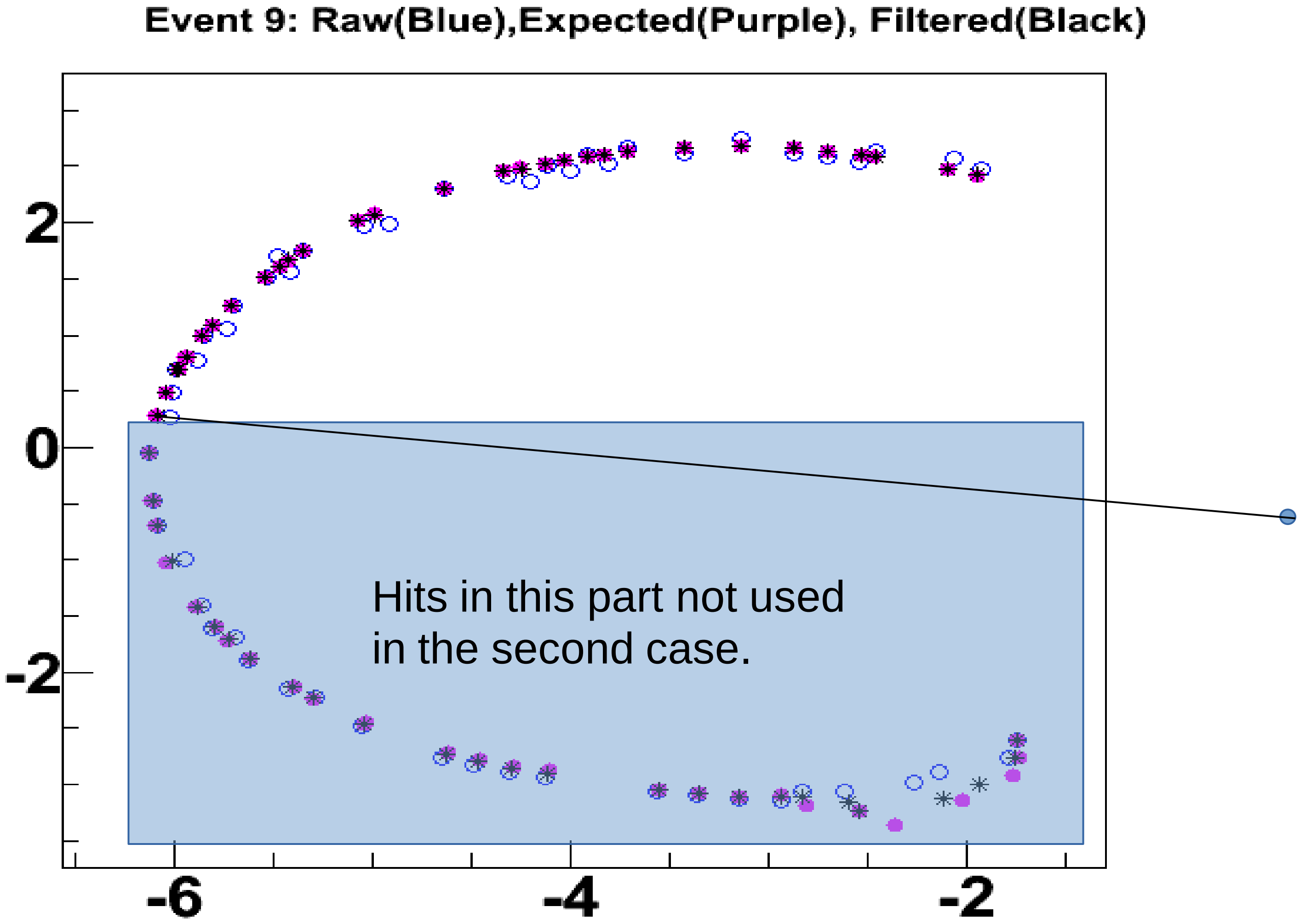
It produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone and it produces a statistically optimal estimate of the underlying system state.



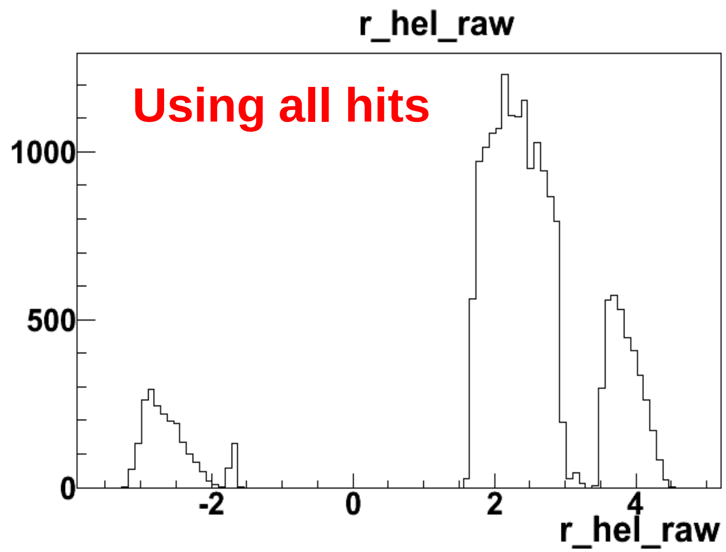
- To start the **Kalman Filter** we need an initial state (GHF output).
- The position on the next plane is **predicted**
- The **measurement** is considered
- **Prediction** and **measurement** are **merged (filtered)**
- then
    - New **prediction** .... **measurement**
    - **Filtering** ... **prediction** .... **measurement**
    - **Filtering** ... **prediction** .... **Measurement**

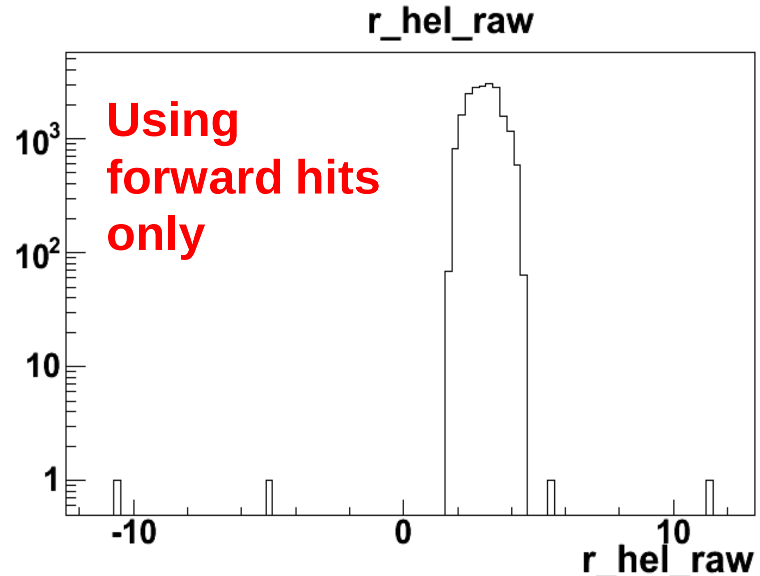
Two Choice for Helix Fitter on tracks that swim back

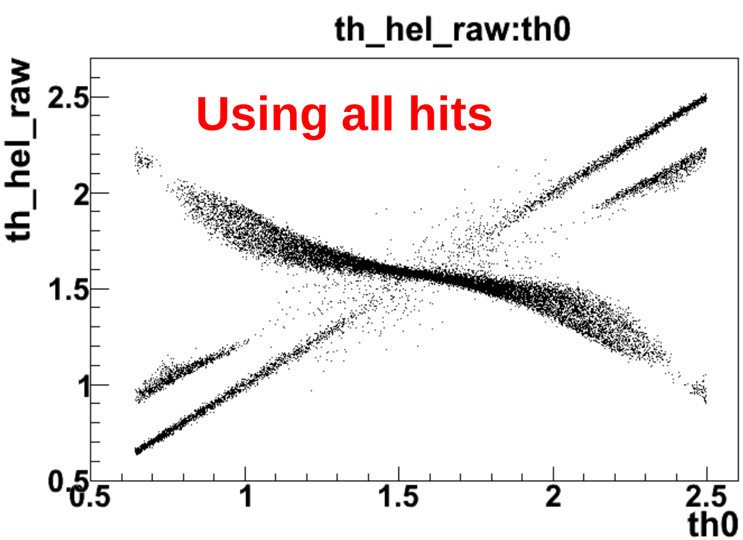
- Use all hits
- Use only hits from the forward part

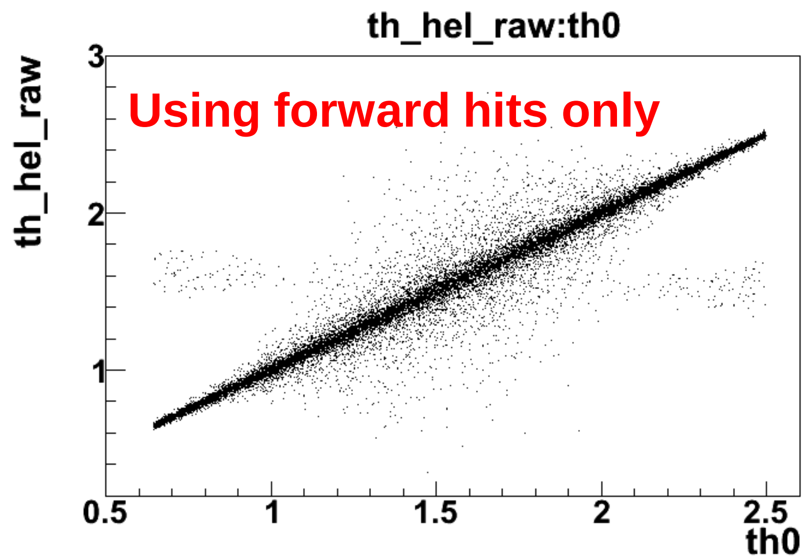


Fit to Only Forward Part of The Track in Global Helix Fitter





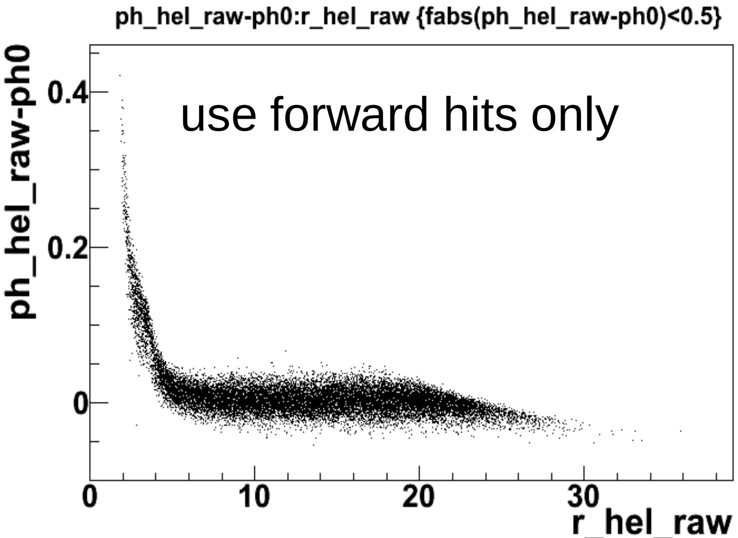


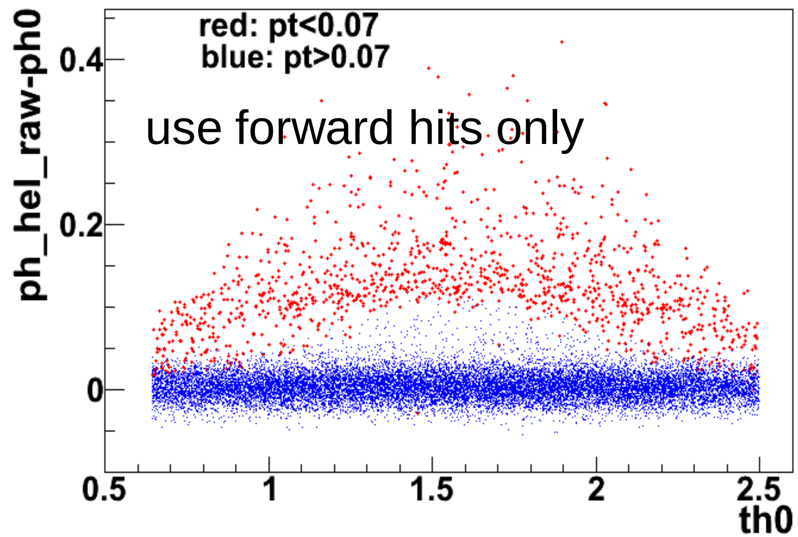


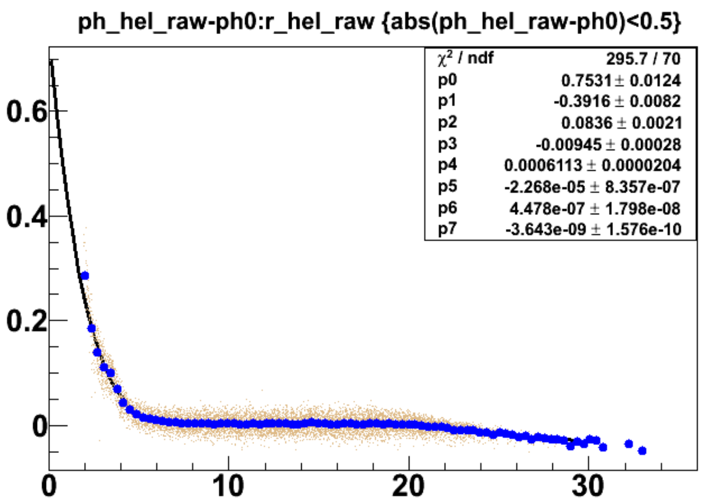
Energy loss is on. $0.05<P_t<0.07$, $-0.8<\cos\theta<0.8$

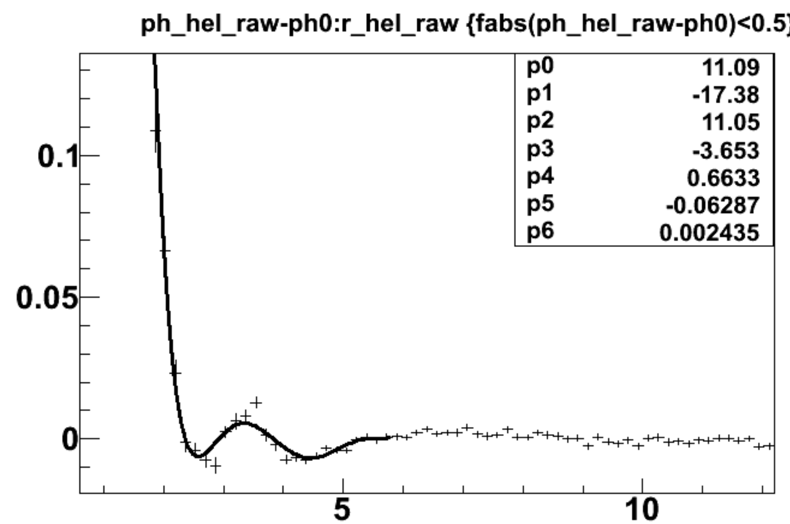
R and θ are much better, but ϕ still have problem, need to manually correct it back!

φ Correction (only useful for a given setup)









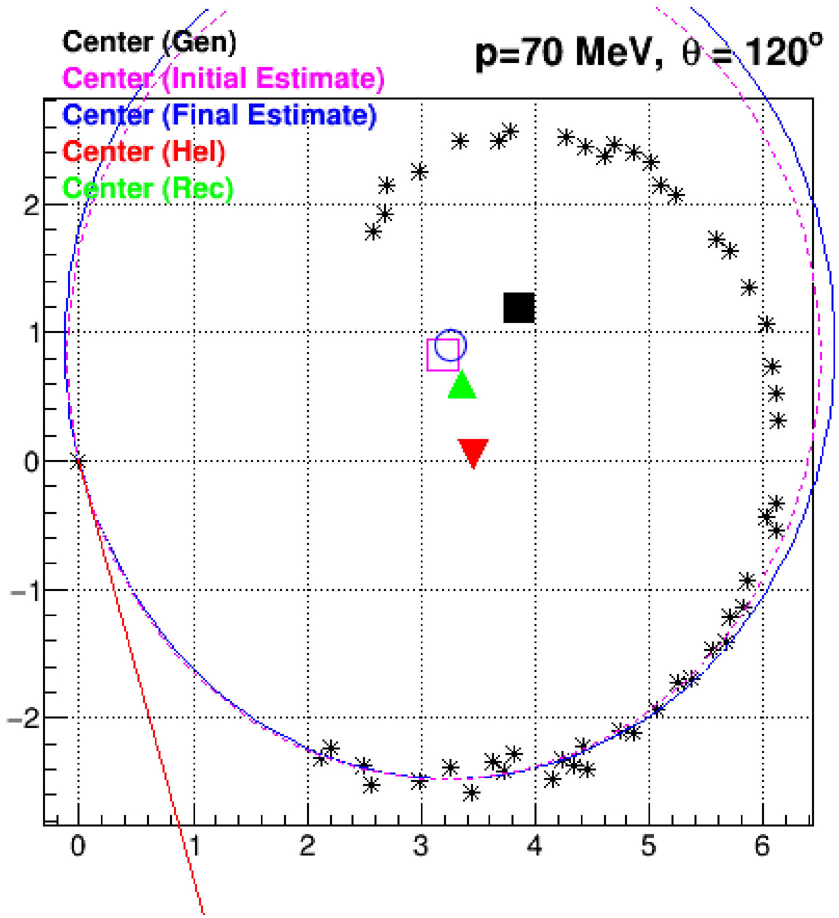
Energy loss is on. $0.05<P_t<0.4$, $-0.8<\cos\theta<0.8$

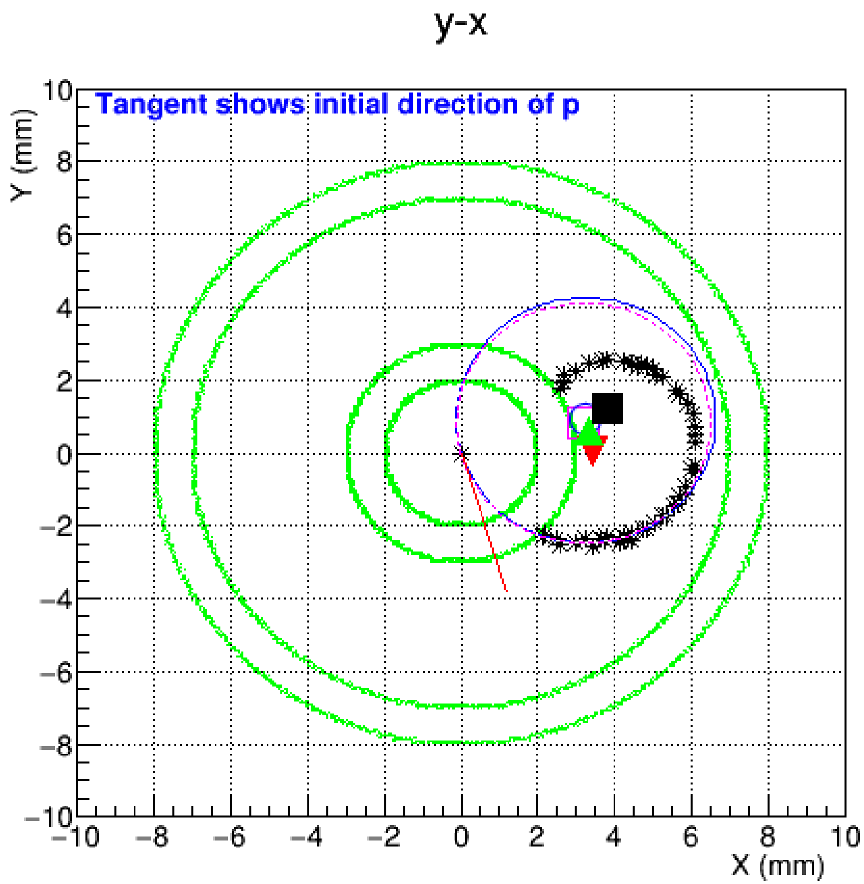
Apply 2-iteration-correction to φ.

Levenberg-Marquardt Circle Fitter

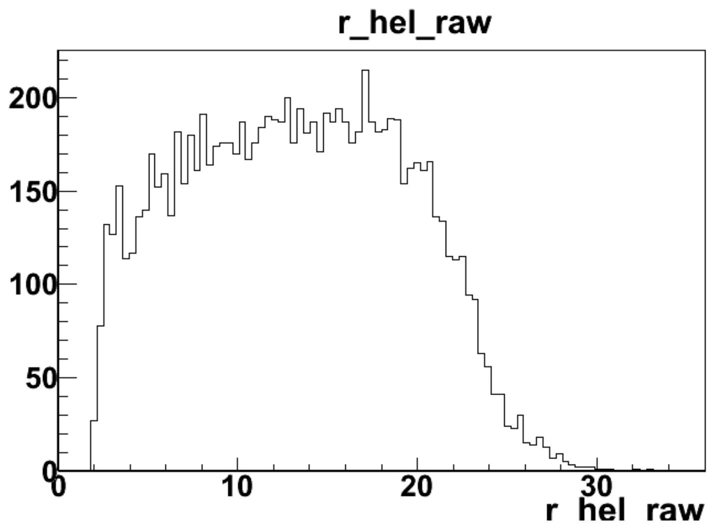
Initial guess: Average of the circumcircles for all non-aligned triplets of points.

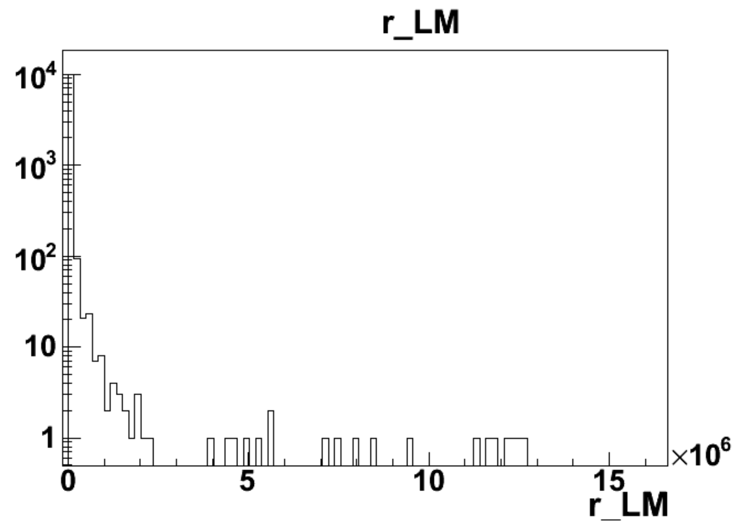
Iterative improvement: Using a least squares estimator based on the euclidean distance between the points and the circle

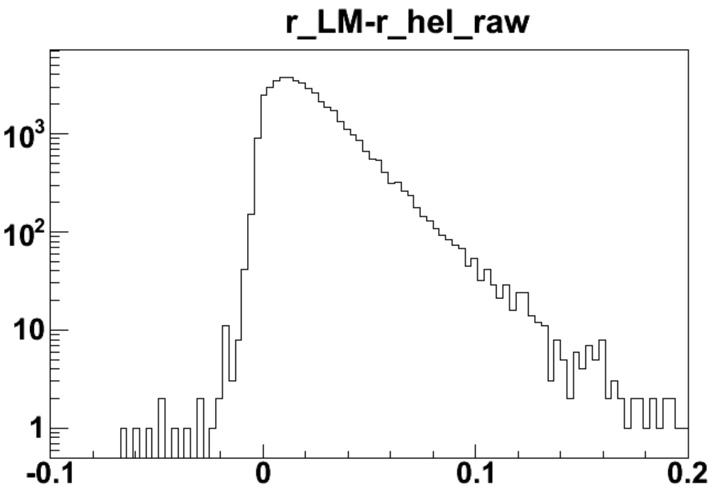


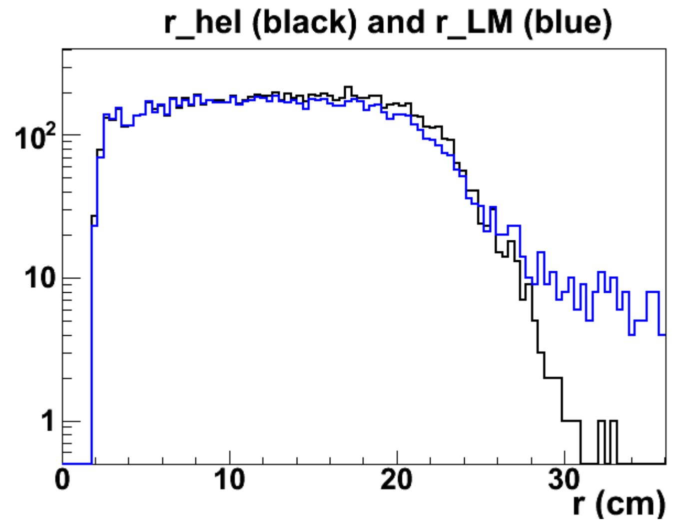


Levenberg-Marquardt Circle Fitter



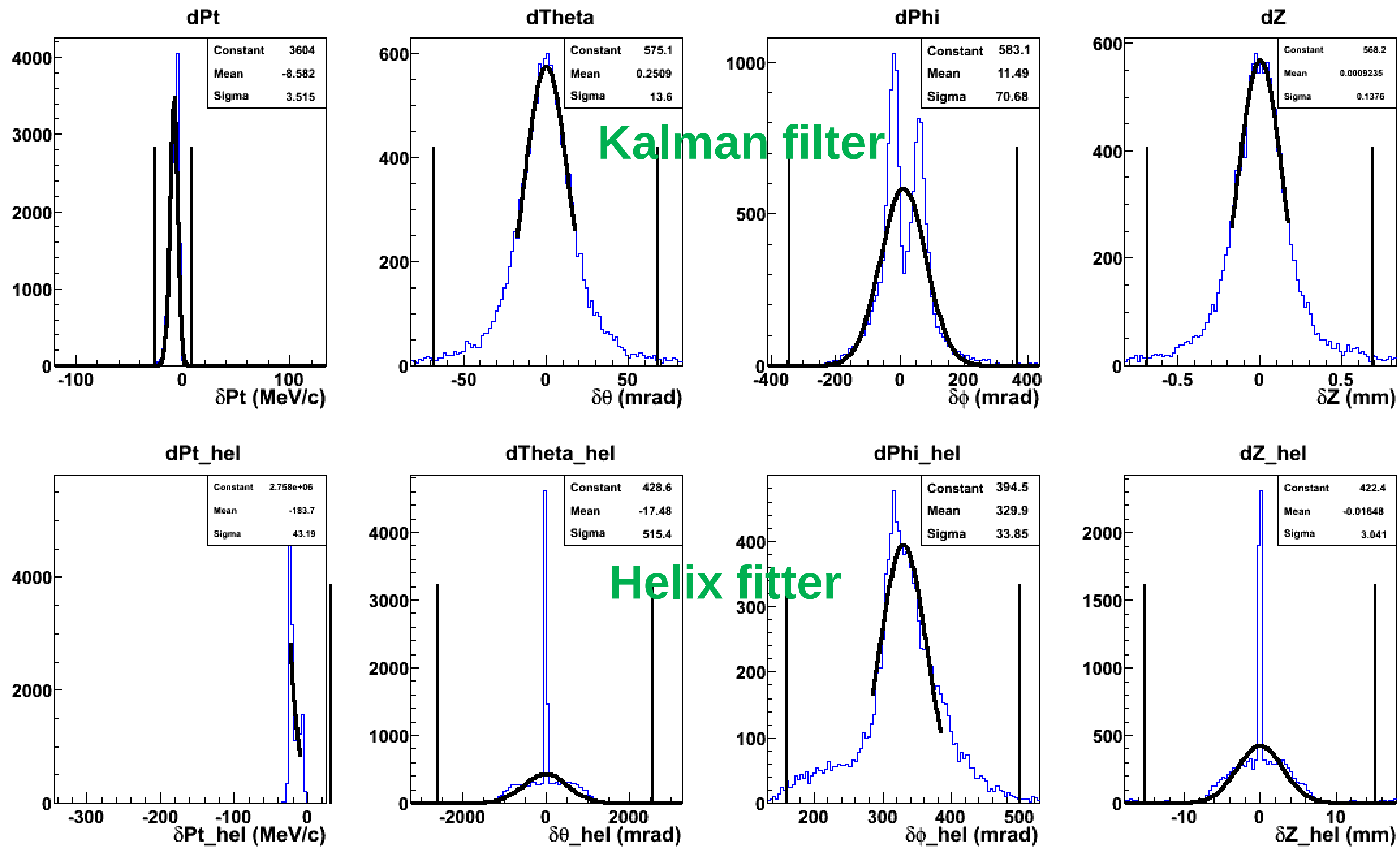






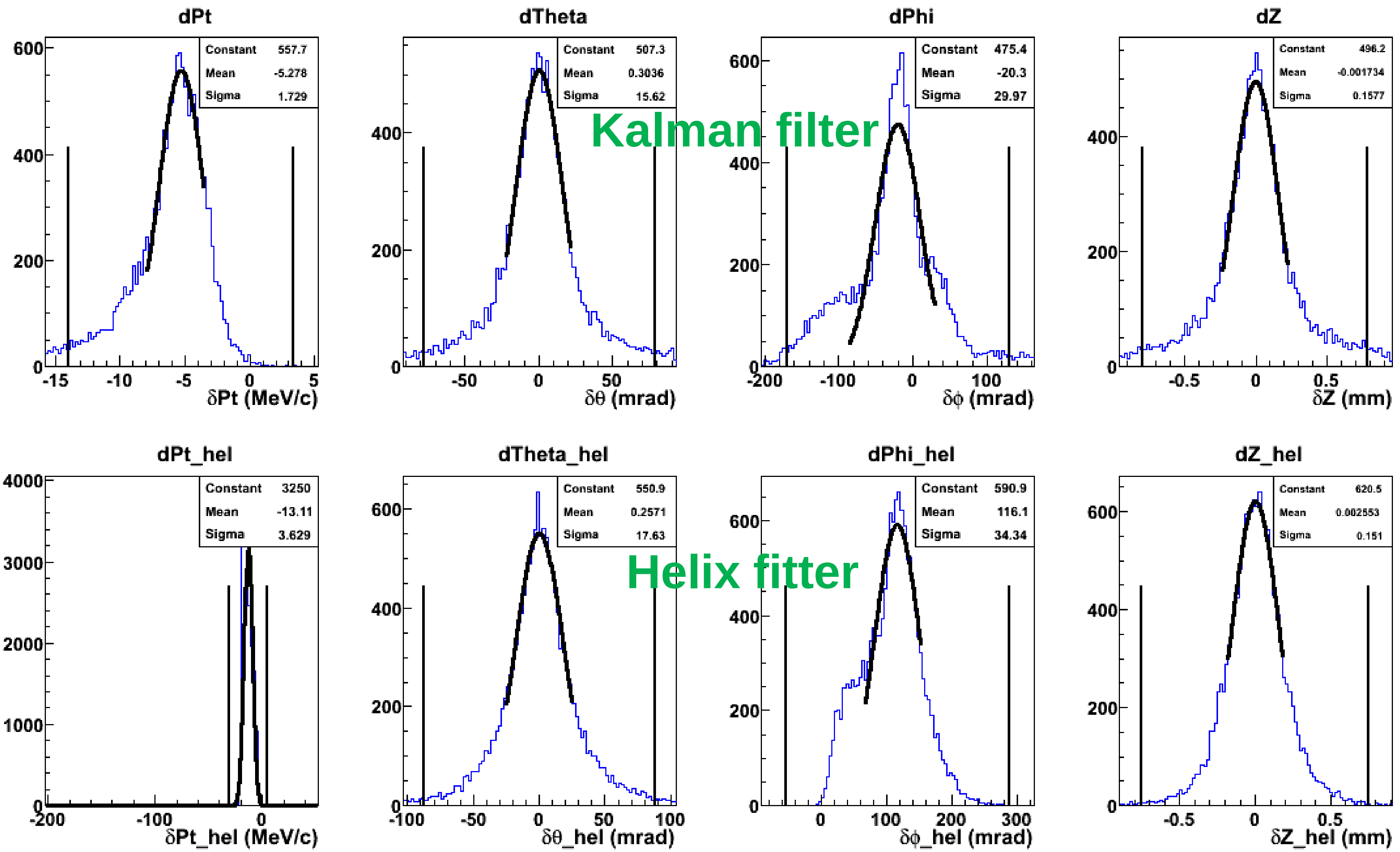
$0.05<P_t<0.40$, $-0.8<\cos\theta<0.8$

Kalman Filter Results



$0.05<P_t<0.07$, $-0.8<\cos\theta<0.8$, Using all hits (global helix fitter)

Kalman Filter Results



$0.05<P_t<0.07$, $-0.8<\cos\theta<0.8$, global helix fit forward hits only, KF uses all hits

What Has Been Found?

- For non-curve-back tracks($R>3.5$cm), global helix fitter and KF both work. KF is a little better but not obvious.
- For curve-back tracks ($R<3.5$ cm):
  - ➢ Neither global helix fitter nor LM circle fitter works for the whole track if the track swims back. KF manages to work but not performs well in R and ϕ.
  - ➢ Using only the forward hits will give better R and ϕ for all 3 fitters.
  - ➢ Using the whole track in KF will give better θ and z, but ruins R and ϕ.
  - ➢ Both GHF & KF do not give reliable ϕ reconstruction.
- We should use only forward hits to fit R and ϕ, use all hits to fit θ and Z in KF.
- LM circle fitter shows no advantage to helix fitter. It loses efficiency for large R tracks ($R>15$cm).

Initial Parameters for KF

Helix state vector:

$$\begin{cases} d_\rho & : \ \text{the distance between the helix and the pivot in the } x\text{-}y \text{ plane} \\ \phi_0 & : \ \text{the azimuthal angle of the pivot with respect to the center of the helix} \\ \kappa & : \ \equiv Q/P_t : \text{the charge in units of that of proton / the transverse momentum} \\ d_z & : \ \text{the distance between the helix and the pivot in the } z \text{ direction} \\ \tan\lambda & : \ \text{the dip angle, i.e., the angle of the helix to the } x\text{-}y \text{ plane,} \end{cases}$$

tanλ = Pz/Pt = 1/tanθ , a/k = r

$$\sigma_{MS} = \frac{0.0141}{P(\text{GeV})\beta}\sqrt{X_L}\left(1 + \frac{1}{9}\log_{10} X_L\right)$$

$$Q_m = \sigma_{MS}^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1+\tan^2\lambda & 0 & 0 & 0 \\ 0 & 0 & (\kappa\tan\lambda)^2 & 0 & \kappa\tan\lambda(1+\tan^2\lambda) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \kappa\tan\lambda(1+\tan^2\lambda) & 0 & (1+\tan^2\lambda)^2 \end{pmatrix}$$

Initial covariance matrix uses 0.05 for all diagonal elements. All others are 0.
Use only the forward hits in a track in both global helix fitter and KF

Use parameters (k, tanλ, ϕ) at last site inferred from global helix fitter as inputs to KF

Dependence on Initial Values to KF?

Kalman Filter prefers the parameters at last site as input, especially for P_t and φ_0.

φ reconstruction is not reliable at all if the track loses too much energy inside the drift region.

P_t and φ reconstruction has strong correlation on initial R and φ_0, weak correlation on θ.

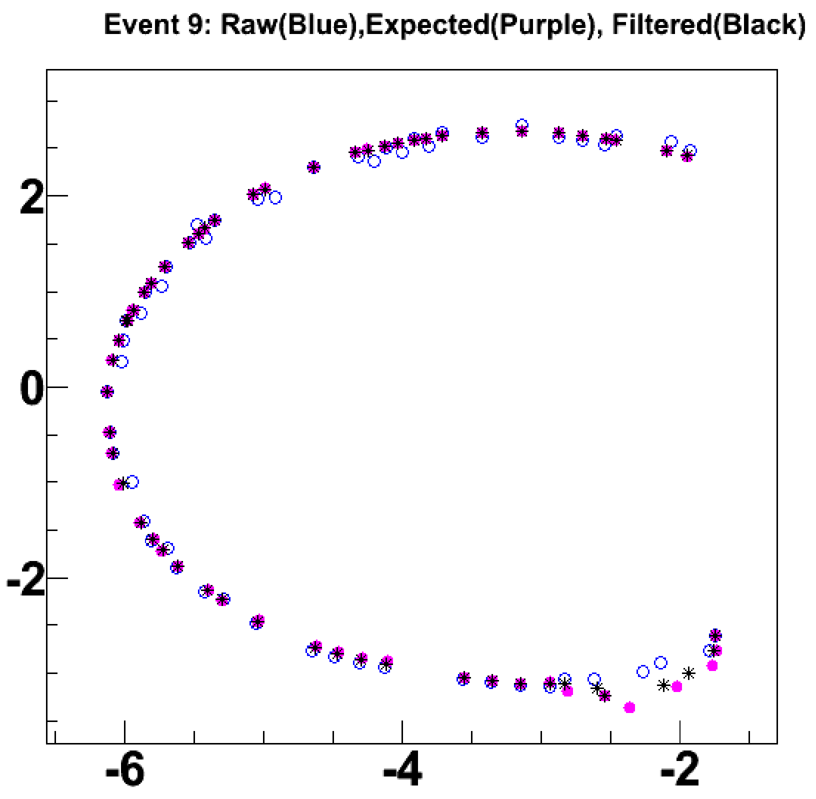
θ reconstruction has strong correlation on initial θ, weak correlation on R or φ.

Apply second-Iteration in Kalman Filter

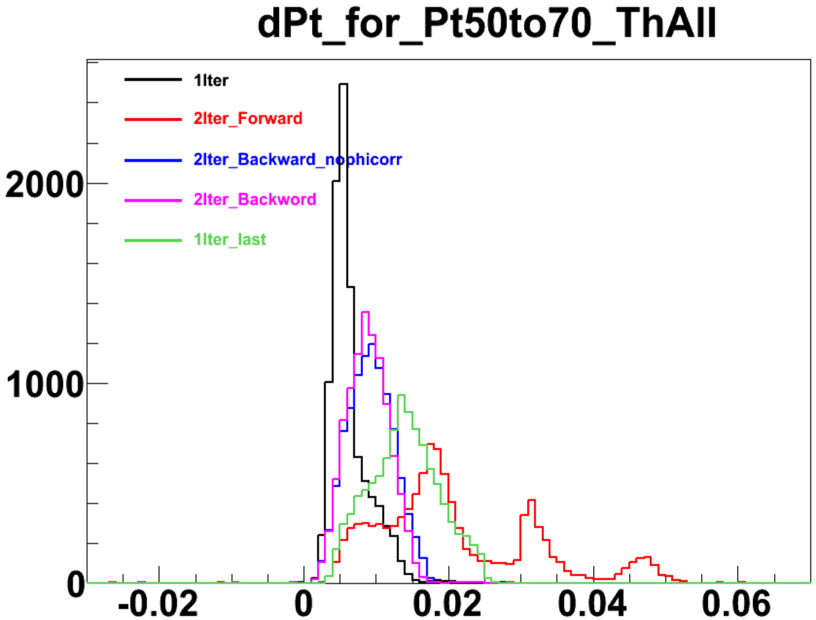
First Iteration: use parameters (k, tanλ,ϕ_0) at last site inferred from helix fitter as inputs. Two options: start from first (forward) or last site (backward+smooth back).

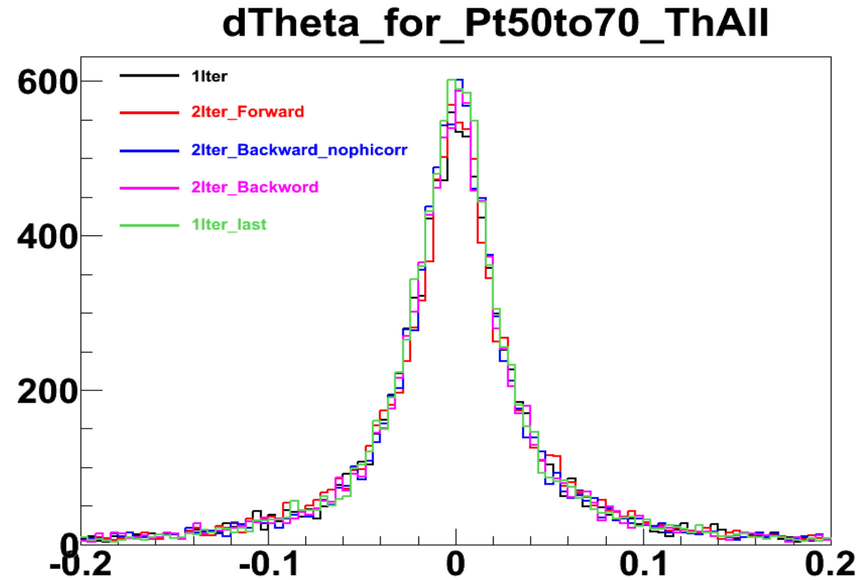
Second Iteration: use parameters (k, tanλ,ϕ_0) at last site inferred from first iteration as inputs, also use the outcome covariance matrix

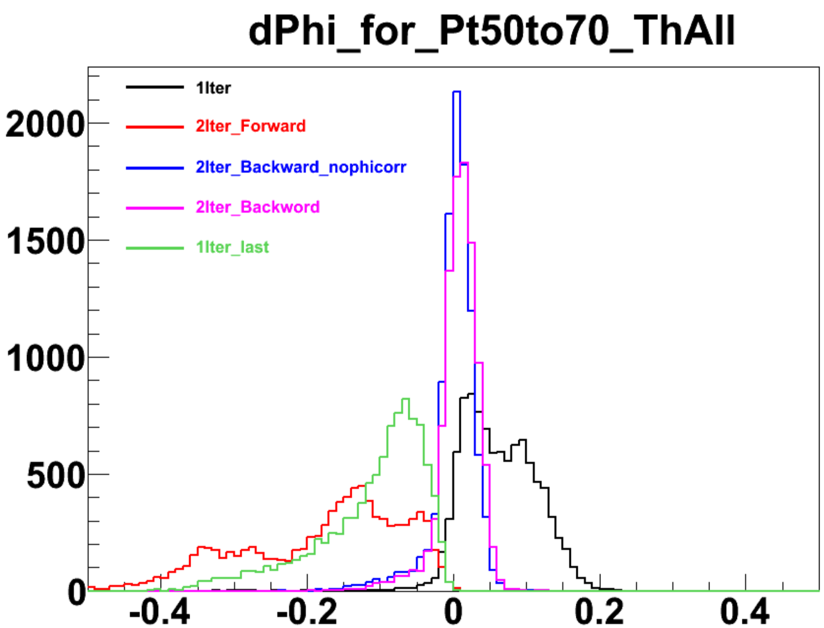
Option 1: iteration-1 goes forward,
Option 2: iteration-2 goes backward



Compare Iter2-KF (50< P_t<70)





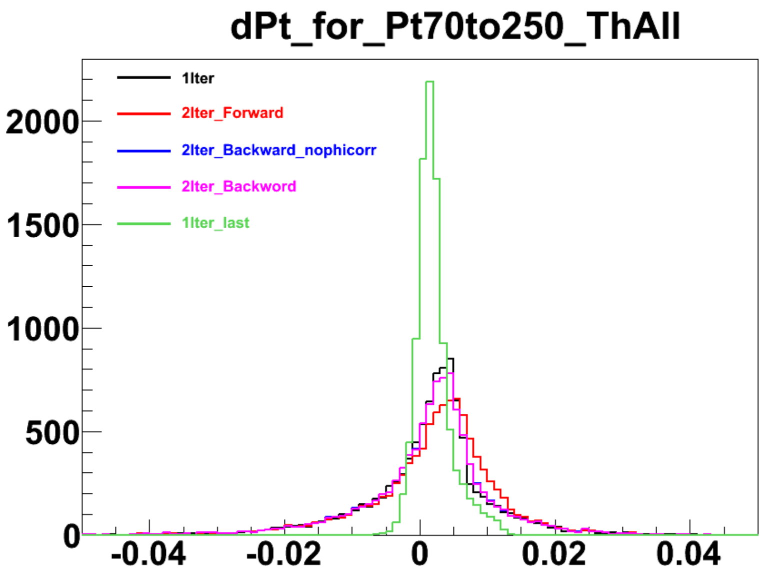


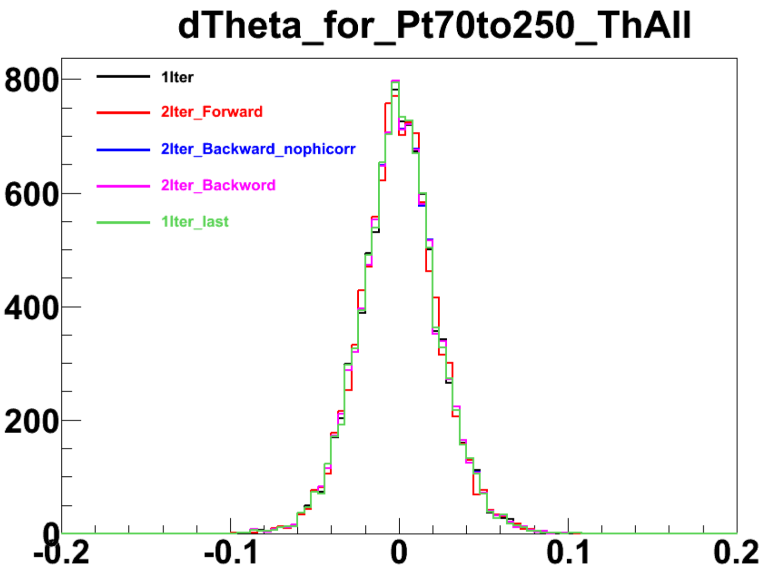
50< P_t<70, -0.8<Cosθ<0.8

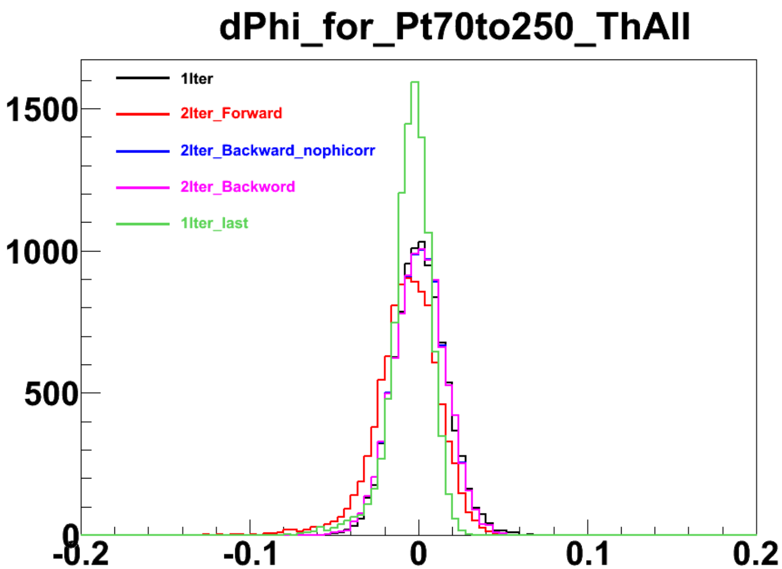
Both 2Iter-Backward and 2Iter-Backward_nophicorr work! With φ correction P_t is better.

P_t is better with only one iteration!

Compare Iter2-KF ($70 < P_t < 250$)







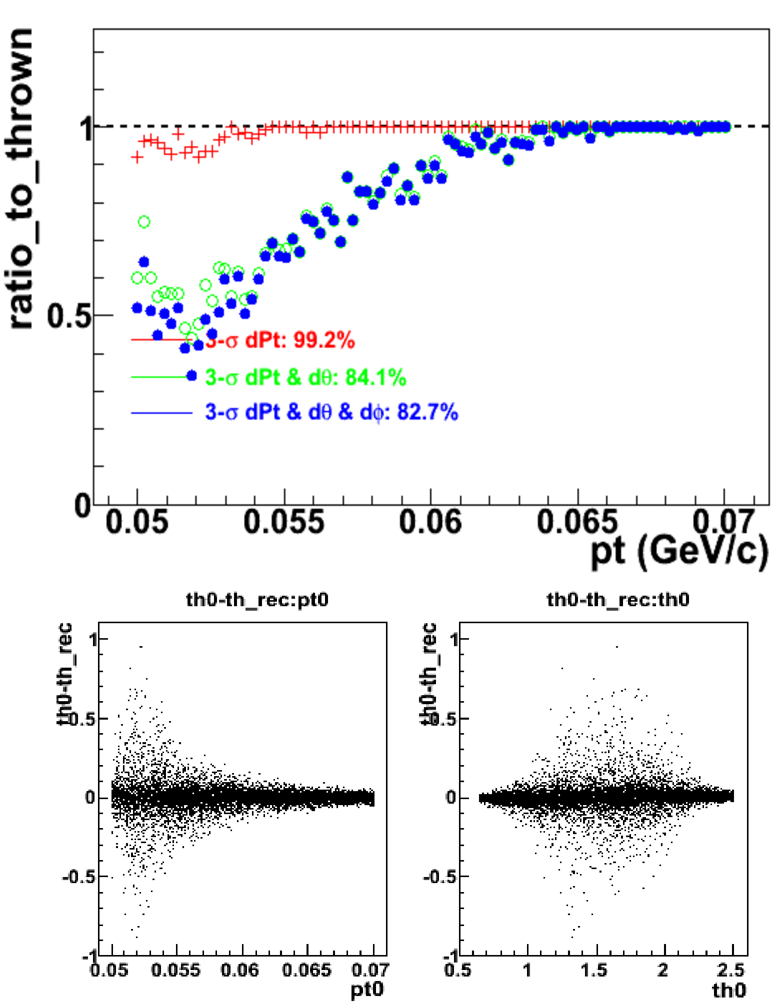
$70 < P_t < 250$, $-0.8 < \cos\theta < 0.8$

P_t and φ are better with only one iteration!

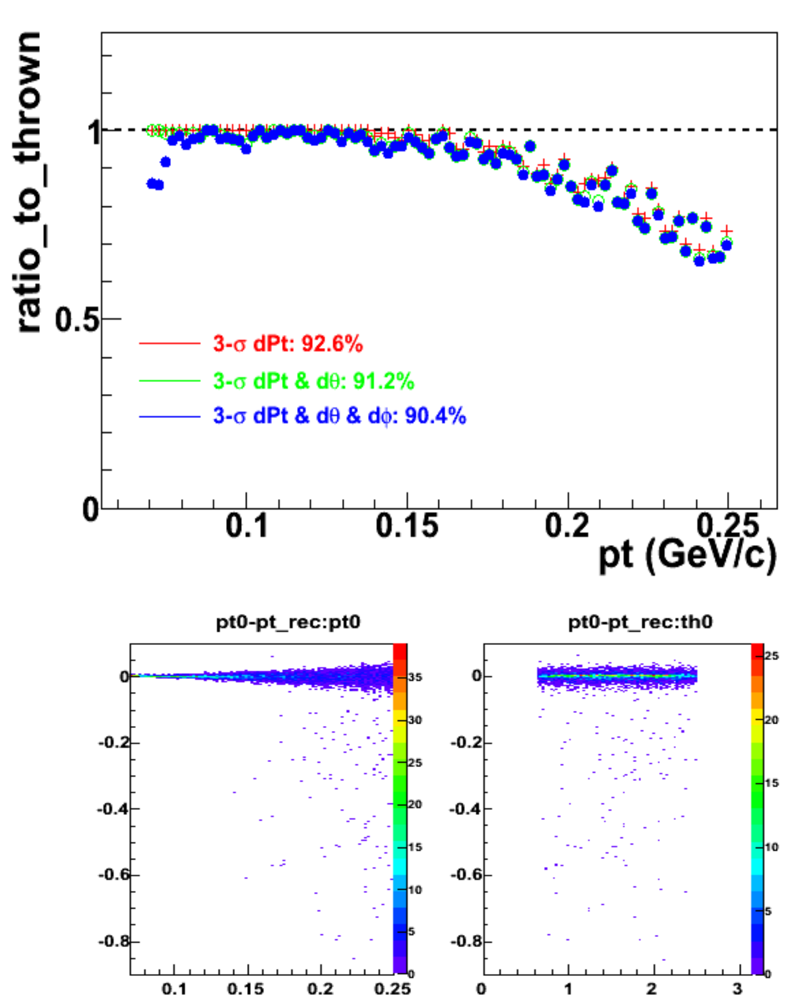
Iter2-KF show tiny advantage in θ!

NO need to apply second iteration for these tracks!

Kalman Filter Performance



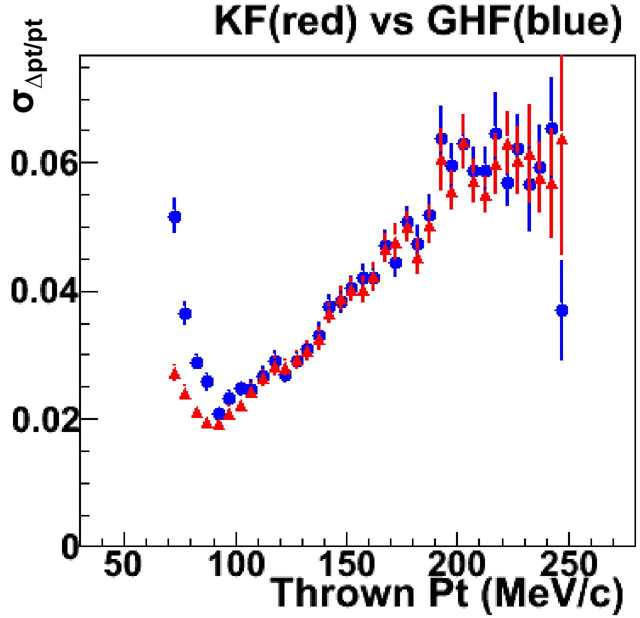
50<P_t<70, -0.8<Cosθ<0.8

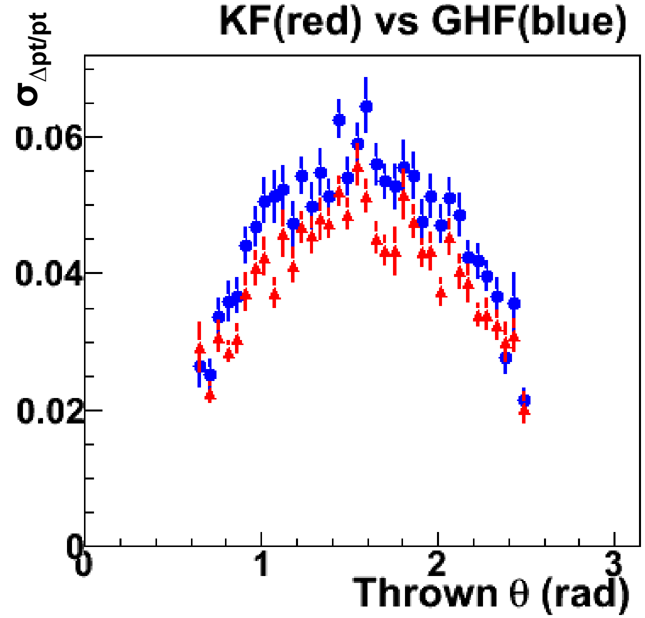


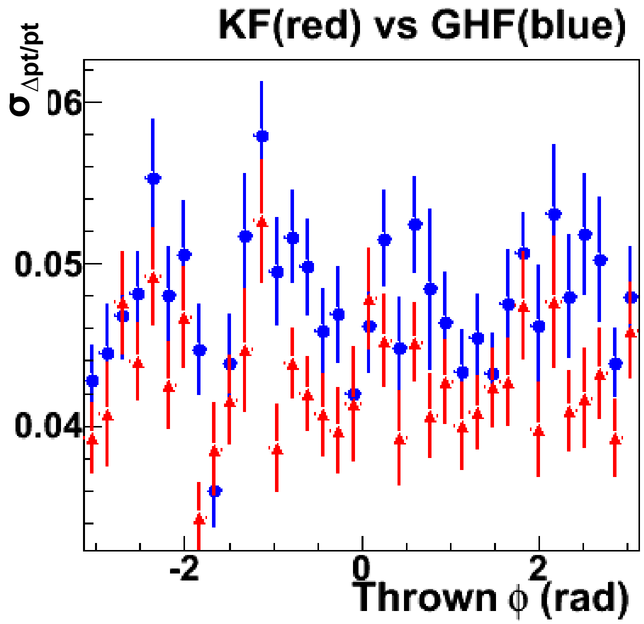
70<P_t<250, -0.8<Cos θ<0.8

50<Pt<70: 82.7% valid reconstruction, 15% lost due to θ reconstruction at small Pt region (Pt<0.63)
70<Pt<250: 90.4% valid reconstruction, 7.4% lost due to Pt reconstruction in large Pt region (Pt>0.17)

Kalman Filter Resolution: P_t (MeV/c)

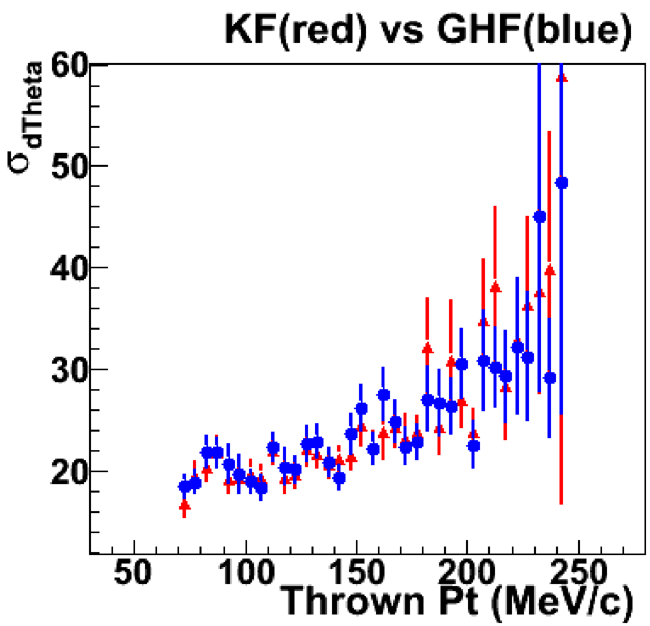


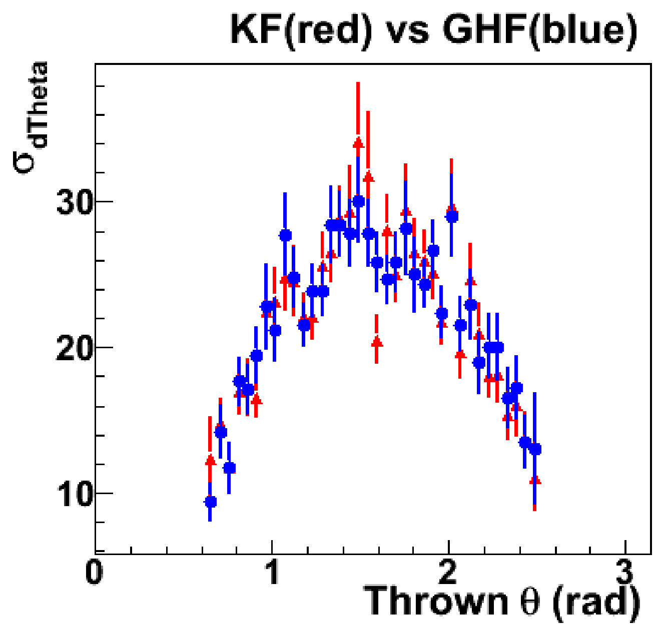


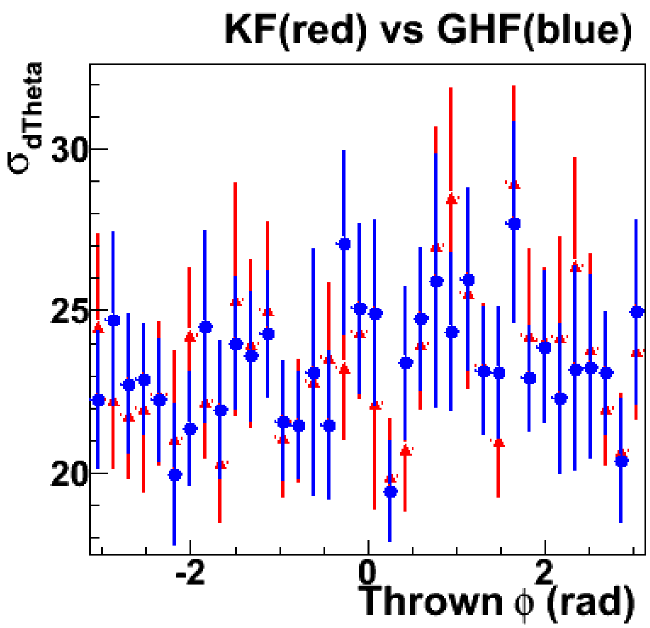


$70<P_t<250$, $-0.8<\mathrm{Cos}\theta<0.8$, Use fitted parameters at last site from helix fitter as input

Kalman Filter Resolution: **θ (mrad)**

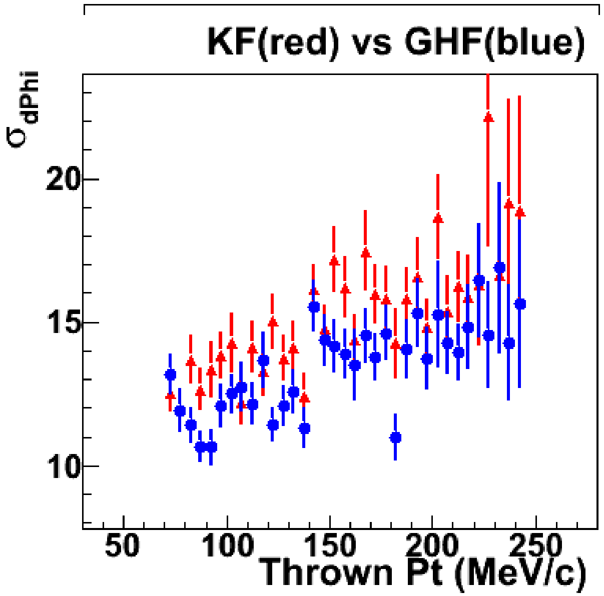


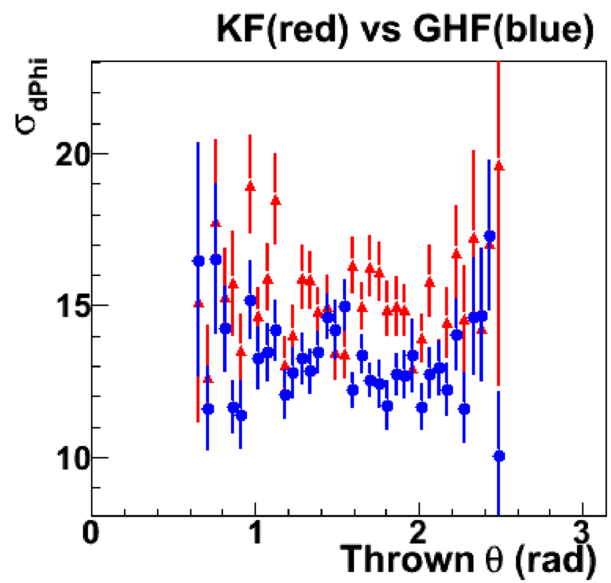


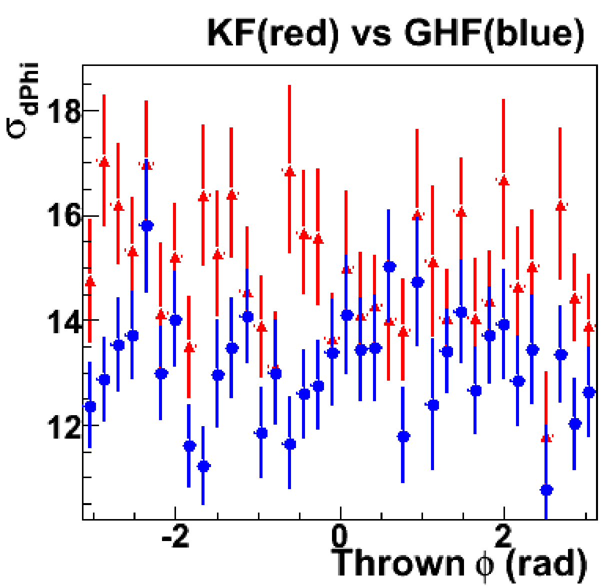


$70<P_t<250$, $-0.8<\text{Cos}\theta<0.8$, Use fitted parameters at last site from helix fitter as input

Kalman Filter Resolution: φ (mrad)







$70<P_t<250$, $-0.8<\cos\theta<0.8$, Use fitted parameters at last site from helix fitter as input

Conclusion

- ❖Fitting only forward part of the track if a track swims back works!
- ❖Levenberg-Marquardt circle fitter show no improvement to global helix fitter
- ❖ KF is sensitive to initial values. To first order, reconstructed P_t and ϕ are sensitive to initial R and ϕ, while reconstructed θ is sensitive to initial θ. If there are offsets in these initial values, these offsets are still seen in the final results.
- ❖ For non-curve-back tracks, global helix fitter or Iter1-KF works fine. Iter1-KF is a little bit better.
- ❖ For curve-back tracks, Iter1-KF will not reconstruct ϕ well. Iter2-KF (with the first iteration going backward then smooth back to the last site) will fit ϕ well.

Backup

Helix Function

Helix function:

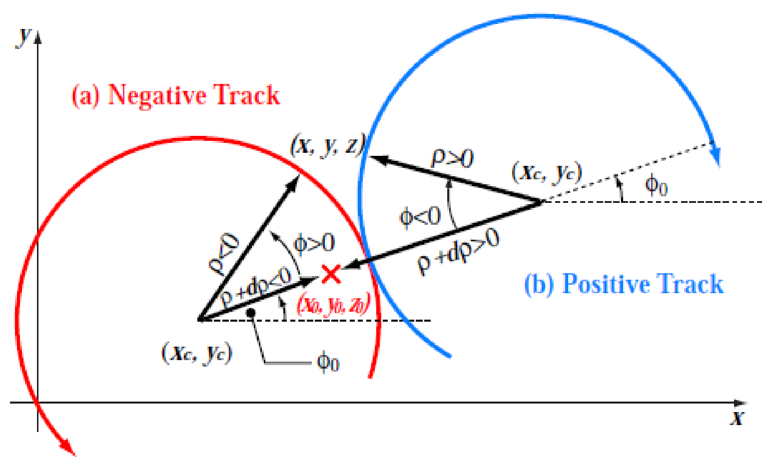
$$\left\{\begin{array}{lllllll} x & = & x_0 & + & d_\rho \cos\phi_0 & + & \frac{\alpha}{\kappa}(\cos\phi_0 - \cos(\phi_0+\phi)) \\ y & = & y_0 & + & d_\rho \sin\phi_0 & + & \frac{\alpha}{\kappa}(\sin\phi_0 - \sin(\phi_0+\phi)) \\ z & = & z_0 & + & d_z & - & \frac{\alpha}{\kappa}\tan\lambda\cdot\phi, \end{array}\right.$$

,where f is the angle deflection in phi in helix coordinate system.

Helix state vector:

$$\left\{\begin{array}{lll} d_\rho & : & \text{the distance between the helix and the pivot in the } x\text{-}y \text{ plane} \\ \phi_0 & : & \text{the azimuthal angle of the pivot with respect to the center of the helix} \\ \kappa & : & \equiv Q/P_t : \text{the charge in units of that of proton / the transverse momentum} \\ d_z & : & \text{the distance between the helix and the pivot in the } z \text{ direction} \\ \tan\lambda & : & \text{the dip angle, i.e., the angle of the helix to the } x\text{-}y \text{ plane,} \end{array}\right.$$

tanl = Pz/Pt = 1/tanq , a/k = r



Geometric interpretation of track parameters for (a) negatively and (b) positively charged tracks.

KalRTPC

- **Detector**: construct materials for target, target wall, helium gas, drift gas, aluminized mylar. Build them according to design.

- **Field**: CLAS12 solenoid field map included. Currently only Bz is used.
- Todo: Need to upgrade the structure to apply Bx and By as well.

- **MeasLayer**: build 35 measurement layers in the drift region, each layer is associated with a measurement uncertainty. In each event, the radius of
- the measurement layer will be shifted to match measurement point.
- Todo: Its Bz field should also be modified......

- **Track generator:** hard-coded, can do
  - 1) generate a perfect circle track without msc or eloss, smear with measurement layer uncertainties;
  - 2) generate a helix track with msc alone the trajectory, smear with measurement layer uncertainties;
  - 3) load track from geant4 output root tree, no smearing.

  - **Code can be download from github:**

    git clone https://github.com/jixie/KalmanFilter.git
    git clone https://github.com/jixie/KalRTPC.git

Energy Loss For Heavy Particles

Bethe- Bloch equation:
(From PDG boklet)

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

1. This equation only work in 0.1<bg<1000 with a few % accuracy

2. Radiation Effect starts at bg>1000

3. For 0.05<bg<0.1, the following effects must be included:
A) Shell correction C_e/Z
B) Barkas effect
C) High order correction

4. For bg<0.05, other appromimation is used.

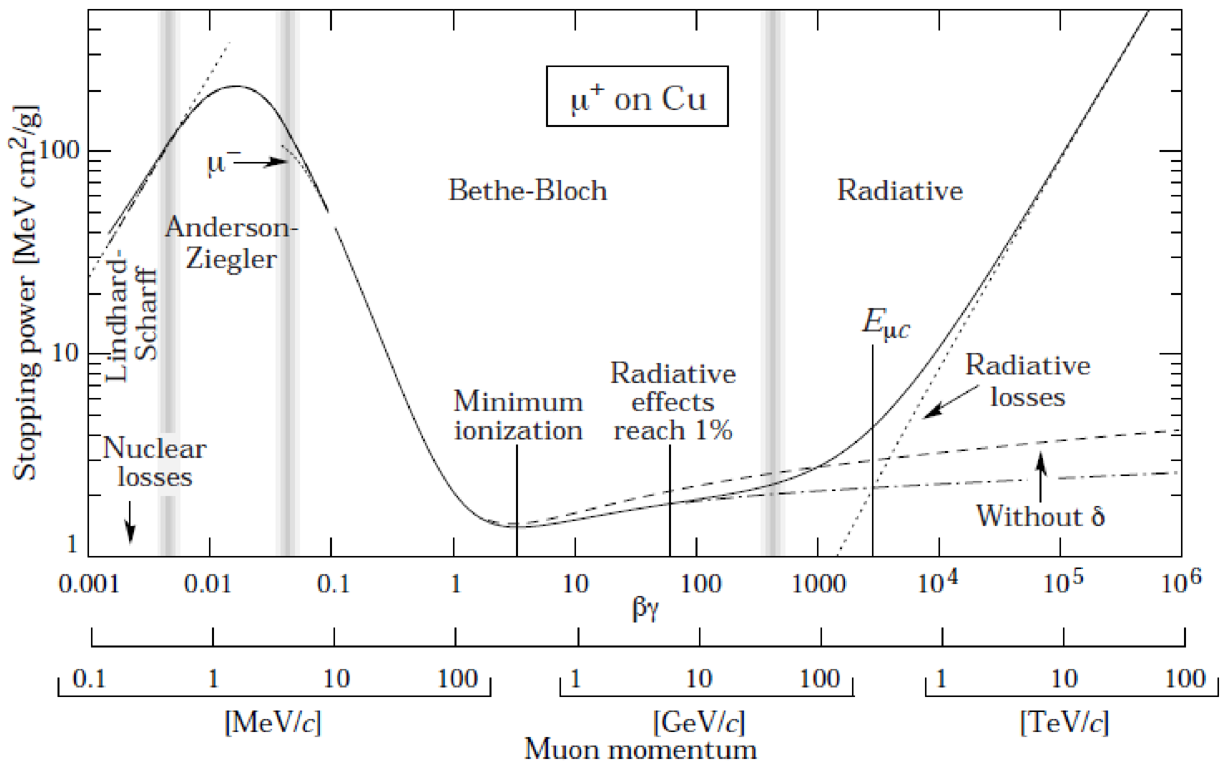
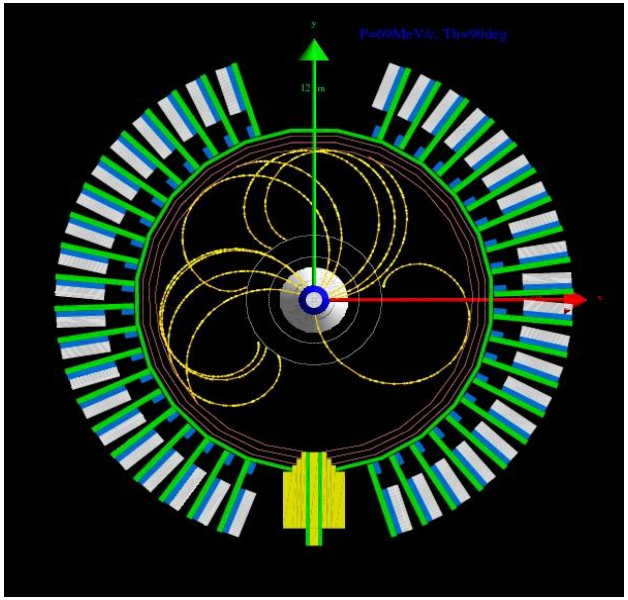
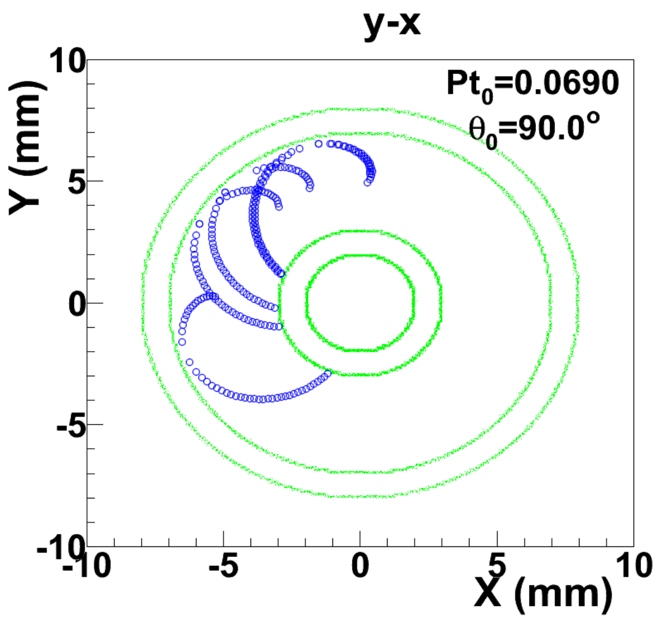


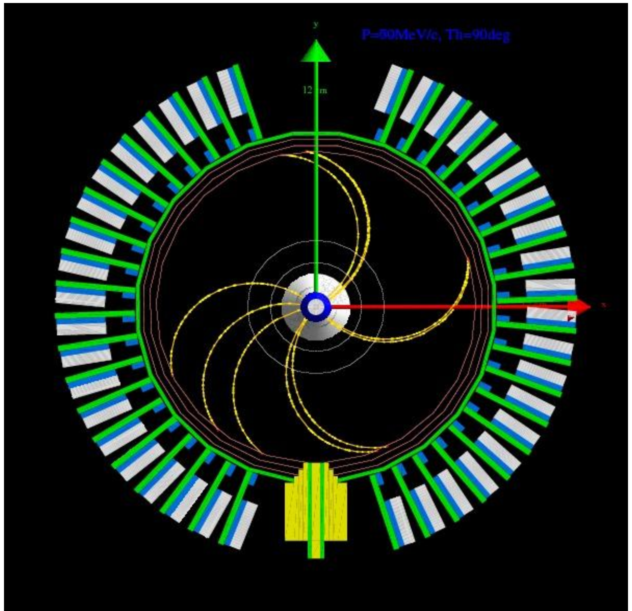
Fig. 27.1: Stopping power ($= \langle -dE/dx \rangle$) for positive muons in copper as a function of $\beta\gamma = p/Mc$ over nine orders of magnitude in momentum (12 orders of magnitude in kinetic energy). Solid curves indicate the total stopping power. Data below the break at $\beta\gamma \approx 0.1$ are taken from ICRU 49 [2], and data at higher energies are from Ref. 1. Vertical bands indicate boundaries between different approximations discussed in the text. The short dotted lines labeled "μ^-" illustrate the "Barkas effect," the dependence of stopping power on projectile charge at very low energies [3].

Compare Energy Loss to Geant4

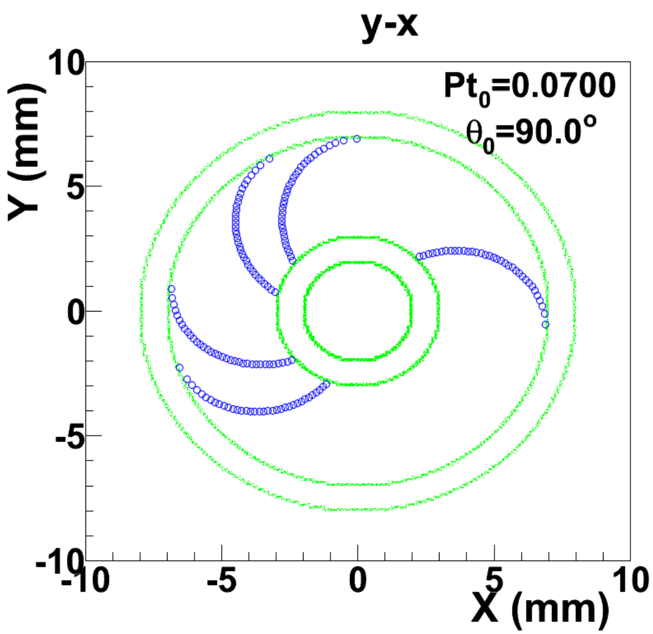


P = 69 MeV/c,
Theta = 90 deg





P = 70 MeV/c,
Theta = 90 deg



Apply second iteration in Kalman Filter

dZ(cm)	dTheta(rad)	dPhi(rad)	dPt(GeV/c)	FileName
0.002 +/- 0.163	0.000 +/- 0.020	0.001 +/- 0.014	0.0038 +/- 0.0058	global_helix_fit
0.003 +/- 0.160	0.000 +/- 0.020	0.001 +/- 0.015	0.0020 +/- 0.0069	70to250MeV_All_1Iter
0.003 +/- 0.155	0.000 +/- 0.020	-0.003 +/- 0.010	0.0015 +/- 0.0015	70to250MeV_All_1Iter_last ← Ideal
0.003 +/- 0.178	0.000 +/- 0.025	-0.005 +/- 0.017	0.0037 +/- 0.0057	70to250MeV_All_2Iter_ForwardNDiagC
0.002 +/- 0.189	0.000 +/- 0.020	-0.005 +/- 0.016	0.0037 +/- 0.0057	70to250MeV_All_2Iter_Forward
0.002 +/- 0.184	0.001 +/- 0.027	0.001 +/- 0.016	0.0026 +/- 0.0047	70to250MeV_All_2Iter_BackwardNDiagC
0.003 +/- 0.173	0.000 +/- 0.020	0.001 +/- 0.015	0.0026 +/- 0.0048	70to250MeV_All_2Iter_Backward
0.003 +/- 0.173	0.000 +/- 0.020	0.001 +/- 0.015	0.0026 +/- 0.0048	70to250MeV_All_2Iter_Backward_nophicorr

dZ(cm)	dTheta(rad)	dPhi(rad)	dPt(GeV/c)	FileName
-0.002 +/- 0.180	-0.001 +/- 0.023	-0.001 +/- 0.029	0.0167 +/- 0.0039	global_helix_fit
-0.002 +/- 0.203	-0.000 +/- 0.026	~~0.026 +/- 0.094~~	~~0.0053 +/- 0.0013~~	50to70MeV_All_1Iter
-0.002 +/- 0.177	-0.000 +/- 0.024	~~-0.066 +/- 0.053~~	0.0140 +/- 0.0046	50to70MeV_All_1Iter_last
-0.003 +/- 0.278	-0.000 +/- 0.030	~~-0.115 +/- 0.072~~	~~0.0160 +/- 0.0049~~	50to70MeV_All_2Iter_ForwardNDiagC
-0.003 +/- 0.295	-0.000 +/- 0.025	~~2.121 +/- 0.772~~	~~0.0167 +/- 0.0045~~	50to70MeV_All_2Iter_Forward
-0.003 +/- 0.216	-0.000 +/- 0.028	0.020 +/- 0.024	0.0085 +/- 0.0027	50to70MeV_All_2Iter_BackwardNDiagC
-0.003 +/- 0.199	-0.000 +/- 0.025	0.012 +/- 0.020	0.0087 +/- 0.0032	50to70MeV_All_2Iter_Backward
-0.002 +/- 0.200	-0.000 +/- 0.024	0.007 +/- 0.017	0.0091 +/- 0.0036	50to70MeV_All_2Iter_Backward_nophicorr

multi-peak distribution,
can not do a good fit

Only one iteration can not reconstruct phi well

Apply second iteration in KF:

1) If the first iteration goes backward, phi reconstruction is reliable.

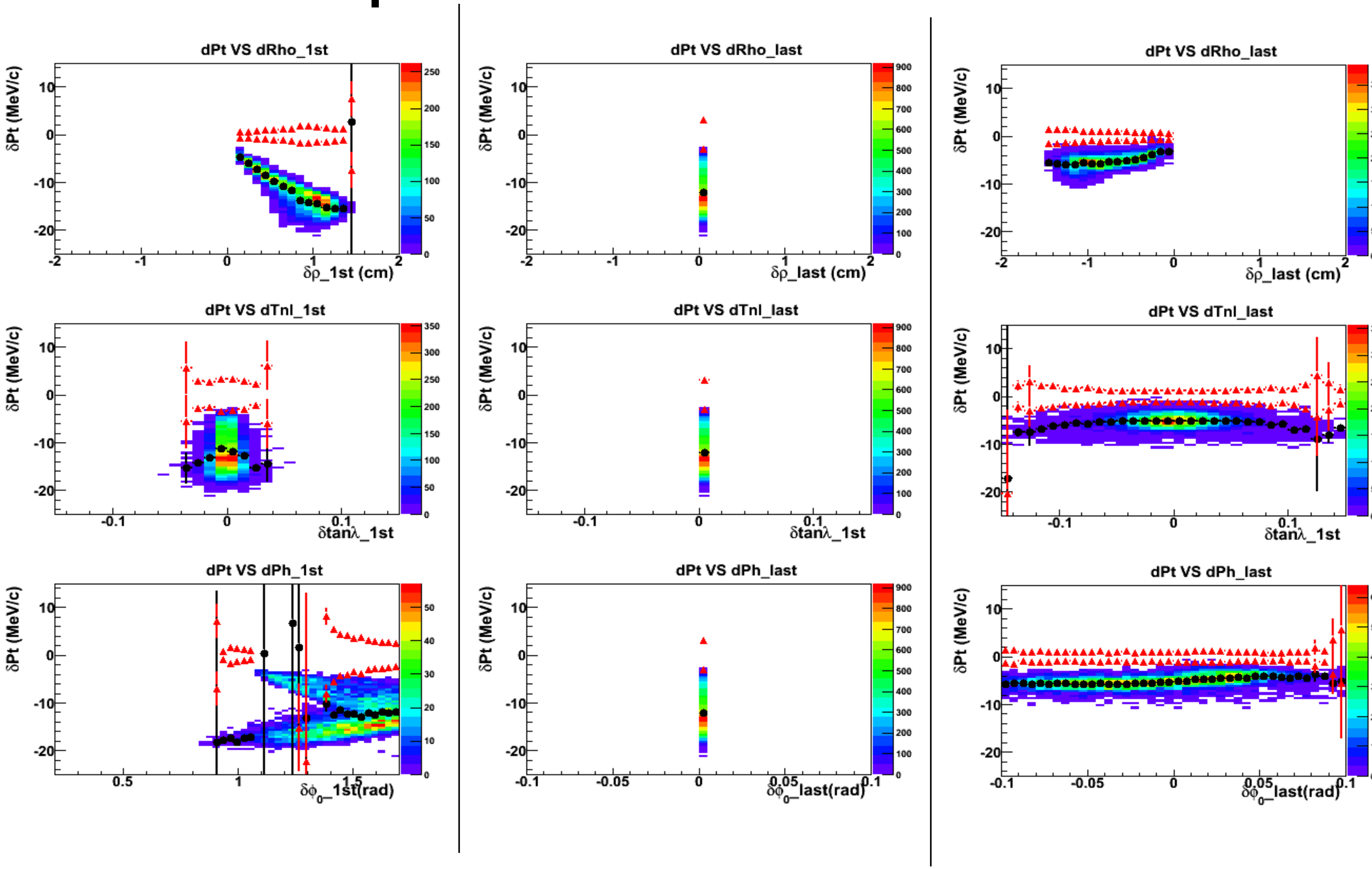
2) Use the whole covariance matrix is better than diagonal elements only.

3) For non-curve back tracks, Iter2-KF shows no advantage to Iter1-KF, and both KF are a little better than the global helix fitter.

4) For curve back tracks, Iter2-KF will improve Pt and Phi reconstruction.

5) In Iter2-KF, applying phi correction to global helix fit result will help Pt but injure Phi reconstruction

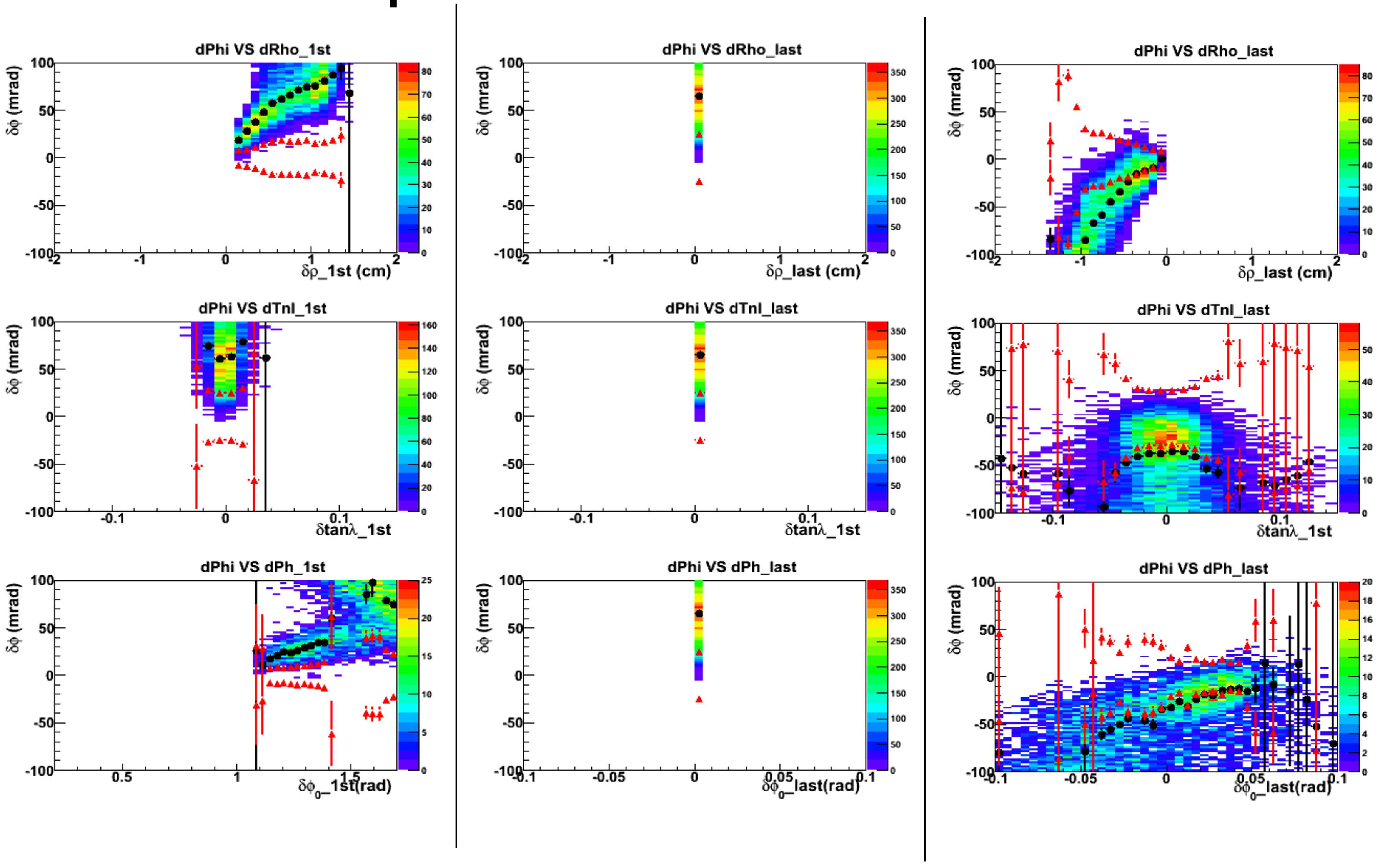
Pt Dependence: 50<Pt<70,-0.8<CosTh<0.8



Left, Middle: Use thrown parameters at last site as input to KF

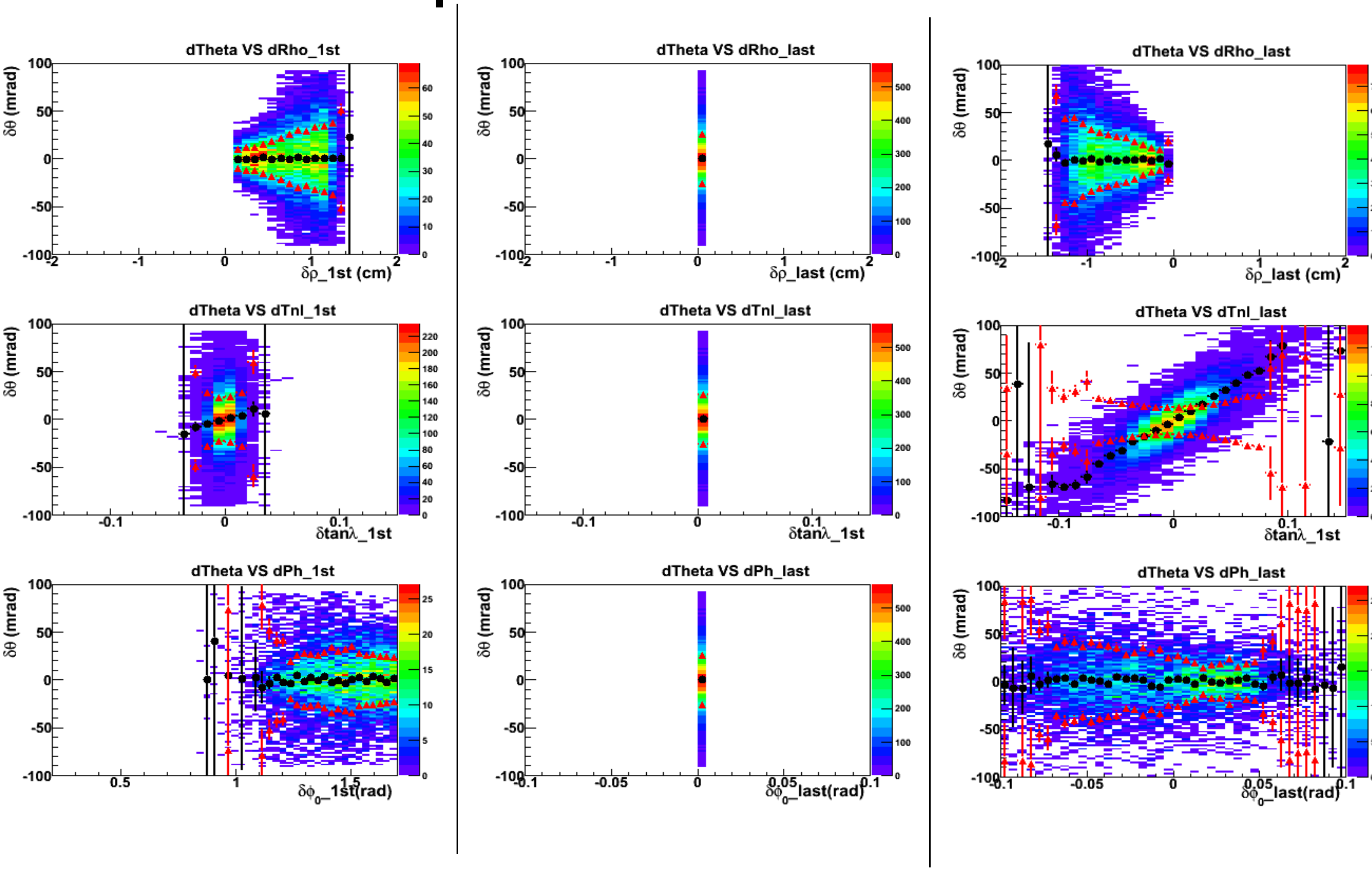
Right: Use fitted parameters at last site from helix fitter as input to KF

Phi Dependence: 50<Pt<70,-0.8<CosTh<0.8



Left, Middle: Use thrown parameters at last site as input to KF
Right: Use fitted parameters at last site from helix fitter as input to KF

Theta Dependence: 50<Pt<70,-0.8<CosTh<0.8



Left, Middle: Use thrown parameters at last site as input to KF
Right: Use fitted parameters at last site from helix fitter as input to KF