The QCD coupling from CLAS data

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Outline

•Coupling constants are not constant at high energy. Why is that? (why are they running?) Effective couplings.

•For QCD, the perturbative definition of the coupling doesn't work at low energy. Can we extend the effective coupling approach to low energy?

•If so, can the CLAS data be used to get α_s at low energy?

•Now that we have some kind of coupling at low energy, is it useful? Does it work?

•What do we learn from all this?

Effective couplings

Force = coupling constant × charge$_1$ × charge$_2$ × f(r) ← $\frac{1}{r^2}$ (for linear theories with massless force carriers)

(2 static bodies)

magnitude of the force ~amount of matter

Faraday: $1/r^2$: weakening of the force flux as it spreads isotropically through space.

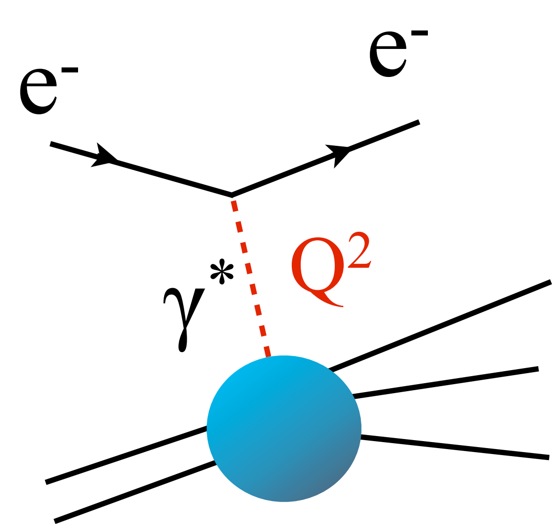
Effective couplings

Force = coupling constant × charge$_1$ × charge$_2$ × f(r) $\leftarrow \frac{1}{r^2}$

Faraday: $1/r^2$: weakening of the force flux as it spreads isotropically through space.

Nowadays: manifestation in the coordinate space of the propagator of the force carrier.

Ex: Electron scattering:

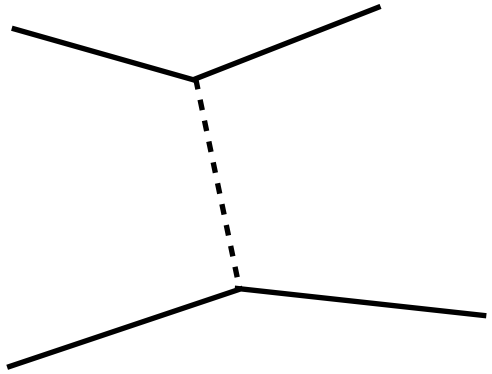


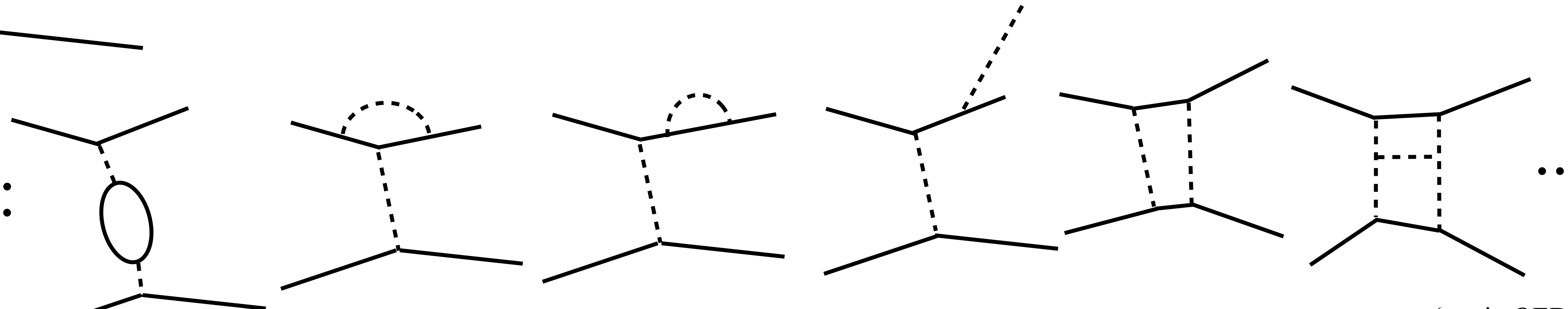
In momentum space, scattering amplitude ∝ propagator $1/Q^2$.

⇒ Potential in coordinate space ∝ FT(amplitude) ∝ 1/r.

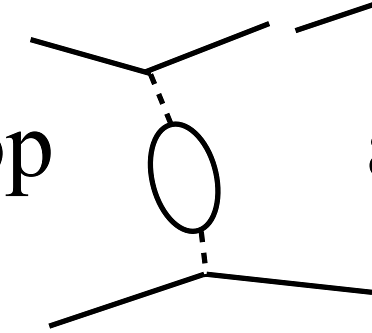
⇒ Force ∝ $1/r^2$.

Effective couplings

But  is a first order approximation.

Higher orders:  ...

a b c d e f (not in QED)

The loop  affects the propagator. (QED: Effect of other graphs cancel each others ("b+c=0") or do not affect definition of coupling (d). More complicated for QCD)

Force=coupling constant×charge$_1$×charge$_2$×f(r) $\cancel{\leftarrow}$ $\frac{1}{r^2}$

We keep f(r)=1/r^2 and fold the additional distance dependence in the coupling.
⇒ Effective coupling. Now depends on distance (i.e. energy) scale.

The strong coupling $\alpha_s(r)$

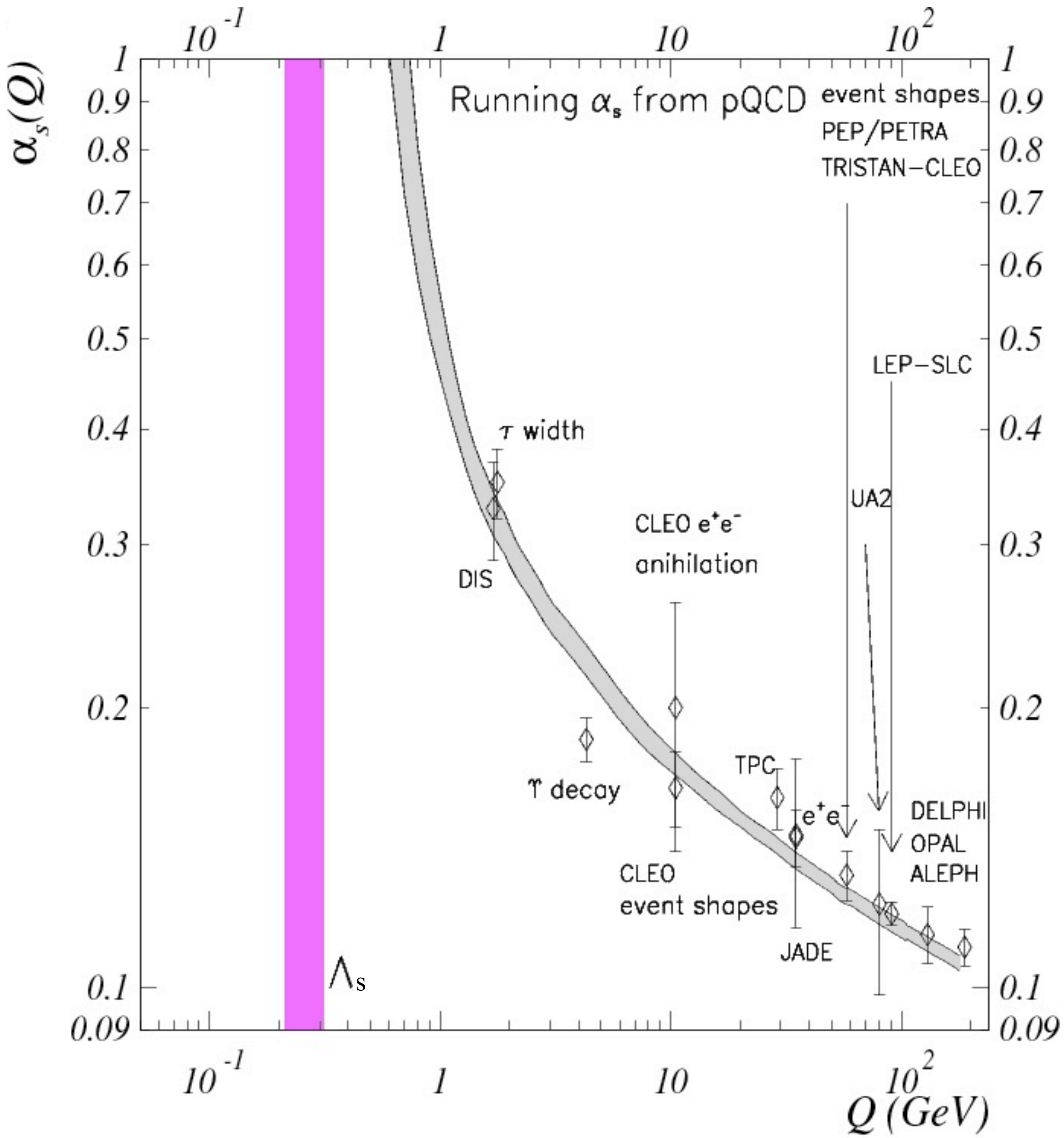
$\alpha_s(r)$ is well understood at short distances where it is small ($\alpha_s \sim 0.1$). (pQCD).

Very active research to understand it at long distances where it is large ($\alpha_s \sim 1$, non-perturbative domain).

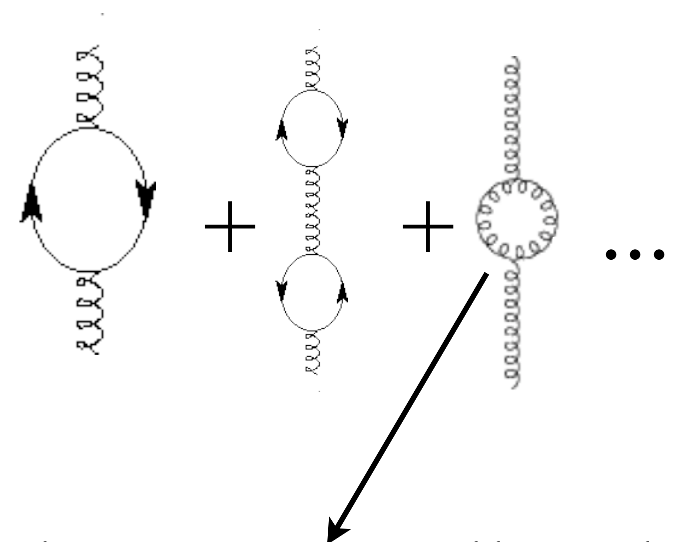
$\alpha_s(r)$ at large distance, work done in collaboration with:

- V. Burkert, J-P Chen and W. Korsch (experimental). PLB **650** 244 (2007), PLB **665** 349 (2008)
- S. J. Brodsky and G. de Teramond (phenomenology). PRD **81**,096010 (2010), PLB **750**, 528 (2015), PLB **757**, 275 (2016) arXiv:1604.04933
- Review on α_s with S. J. Brodsky and G. de Teramond. Prog. Part. Nuc. Phys. 90 1 (2016)

The strong coupling at short distances



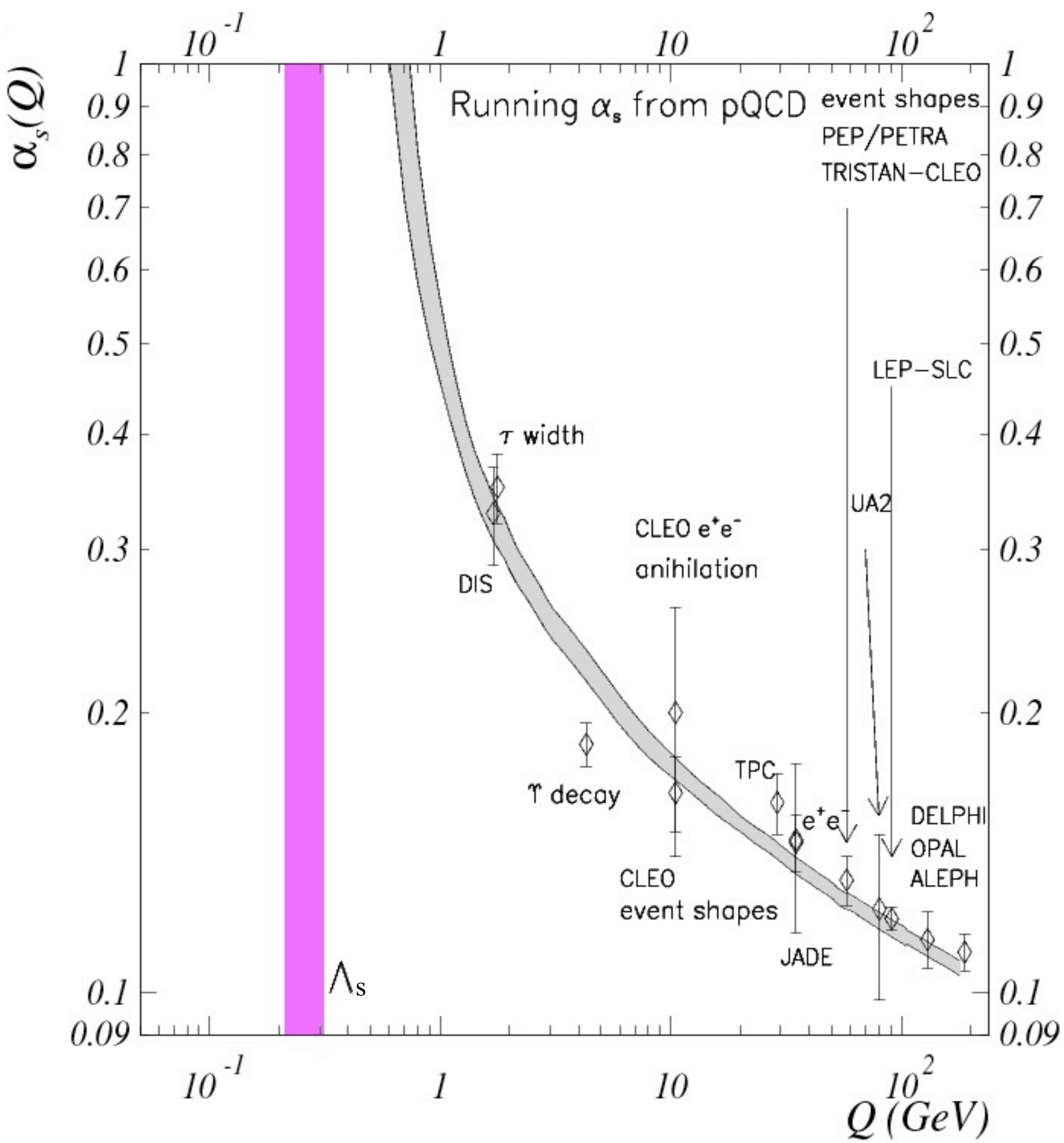
α_s is not constant due to loops in gluon propagator:



- α_s becomes small at short distances (large Q^2) $\Rightarrow$ Asymptotic freedom, pQCD.

$\alpha_s(Q^2)$ is well defined within pQCD.

The strong coupling at short distances

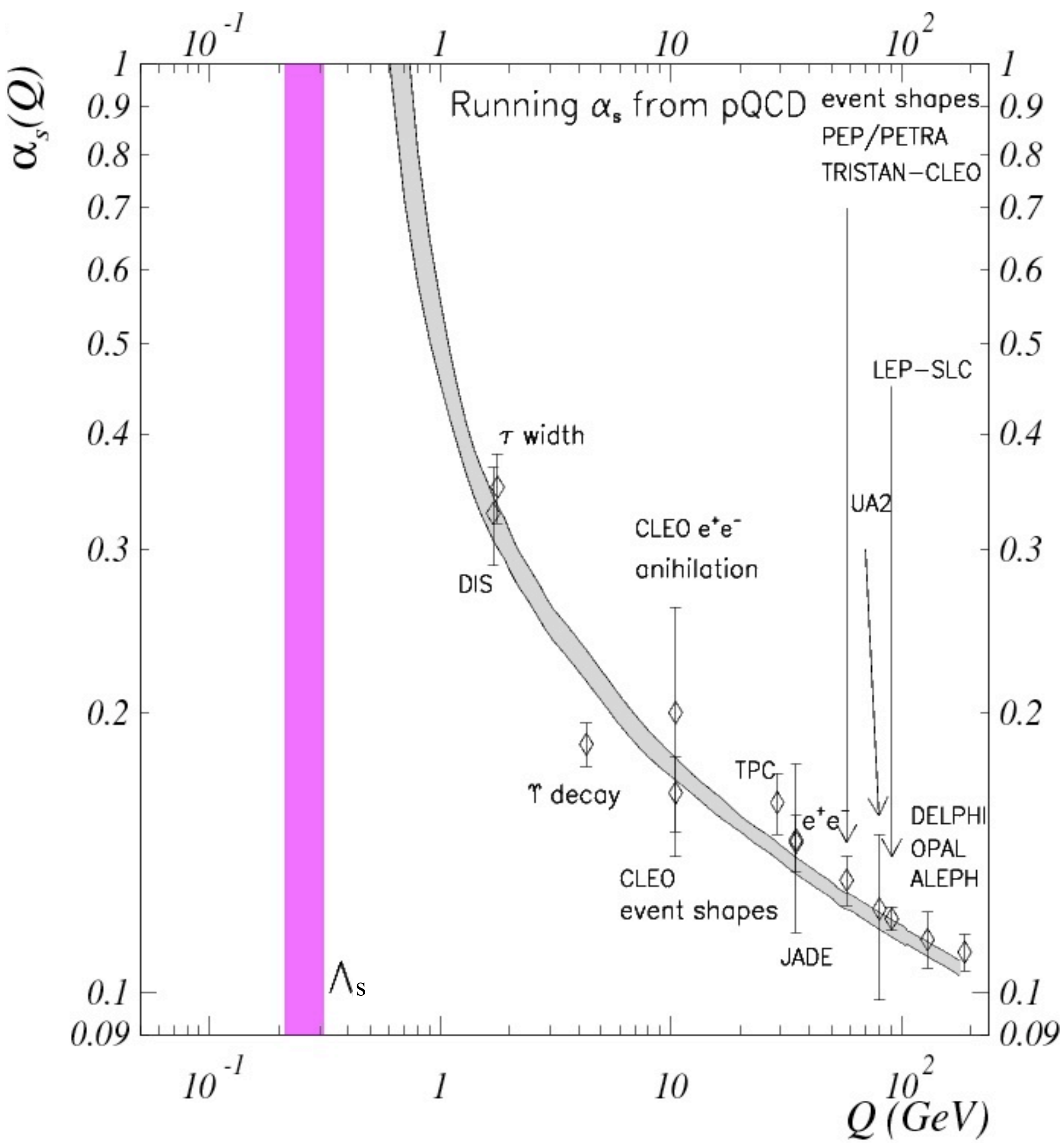


At low Q^2 ($\lesssim 1\text{GeV}^2$), pQCD cannot be used to define α_s: If pQCD is trusted,

$\alpha_s \rightarrow \infty$ for $Q \rightarrow \Lambda_s$.

Contradict the perturbative hypothesis

The strong coupling at short distances



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Contradict the perturbative hypothesis

Definition and computation of α_s at long distance?

$\alpha_s(r)$ at long distance (low Q^2)

Prescription: Define effective couplings from an observable's perturbative series truncated to first order in α_s.

G. Grunberg, PLB B95 70 (1980); PRD 29 2315 (1984); PRD 40 680(1989).

Proposed for pQCD. We tentatively extend it to non-perturbative QCD.

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Ex: Bjorken sum rule:

$$\int (g^p_1-g^n_1)dx \triangleq \Gamma_1^{p-n}=\frac{1}{6}g_A\left(1-\frac{\alpha_s}{\pi}-3.58\left(\frac{\alpha_s}{\pi}\right)^2-\ldots\right)+\frac{M^2}{9Q^2}\left[a_2(\alpha_s)+4d_2(\alpha_s)+4f_2(\alpha_s)\right]+\ldots$$

Spin structure functions.

Nucleon axial charge.

pQCD corrections. (Here in the $\overline{MS}$ scheme. 1^{st} order in α_s is scheme independent)

Higher twist corrections. Related to confinement forces.

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$$\Rightarrow \boxed{\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_{g1}}{\pi}\right)}$$

α_{g1} = "α_s in the g_1 scheme" i.e. α_s obtained using the Bjorken sum $\int g^{p-n}{}_1 dx$.

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Higher twist corrections. Related to confinement forces.

$$\Rightarrow \boxed{\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A\left(1 - \frac{\alpha_{g1}}{\pi}\right)}$$

This means that short distance pQCD effects and long distance confinement forces are now folded into the definition of α_s.

Analogy with the original coupling constant becoming an effective coupling when short distance quantum effects are folded into its definition.

$\alpha_s(r)$ at long distance (low Q^2)

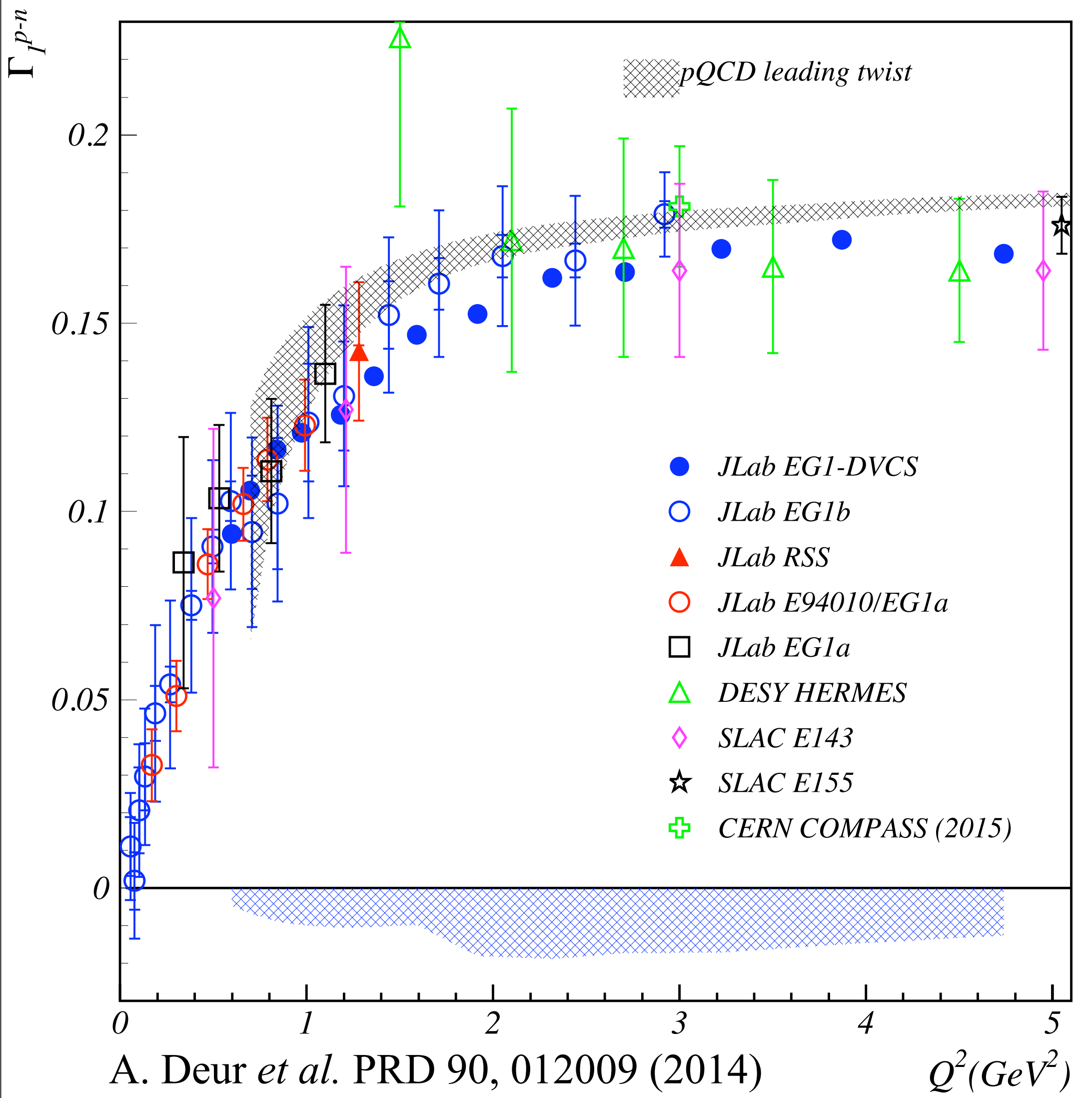
Advantages of extracting α_s from the Bjorken Sum Rule:

- Bjorken sum rule: simple perturbative series.
- Data (CLAS!) exist at low, intermediate, and high Q^2.
- Rigorous Sum Rules dictate the behavior of Γ_1^{p-n} in the unmeasured $Q^2 \to 0$ and $Q^2 \to \infty$ regions.

$\Rightarrow$ We can obtain α_{g1} at any Q^2.

α_{g1} from the Bjorken Sum data

Bjorken sum Γ_1^{p-n} measurement



A. Deur *et al.* PRD 90, 012009 (2014)

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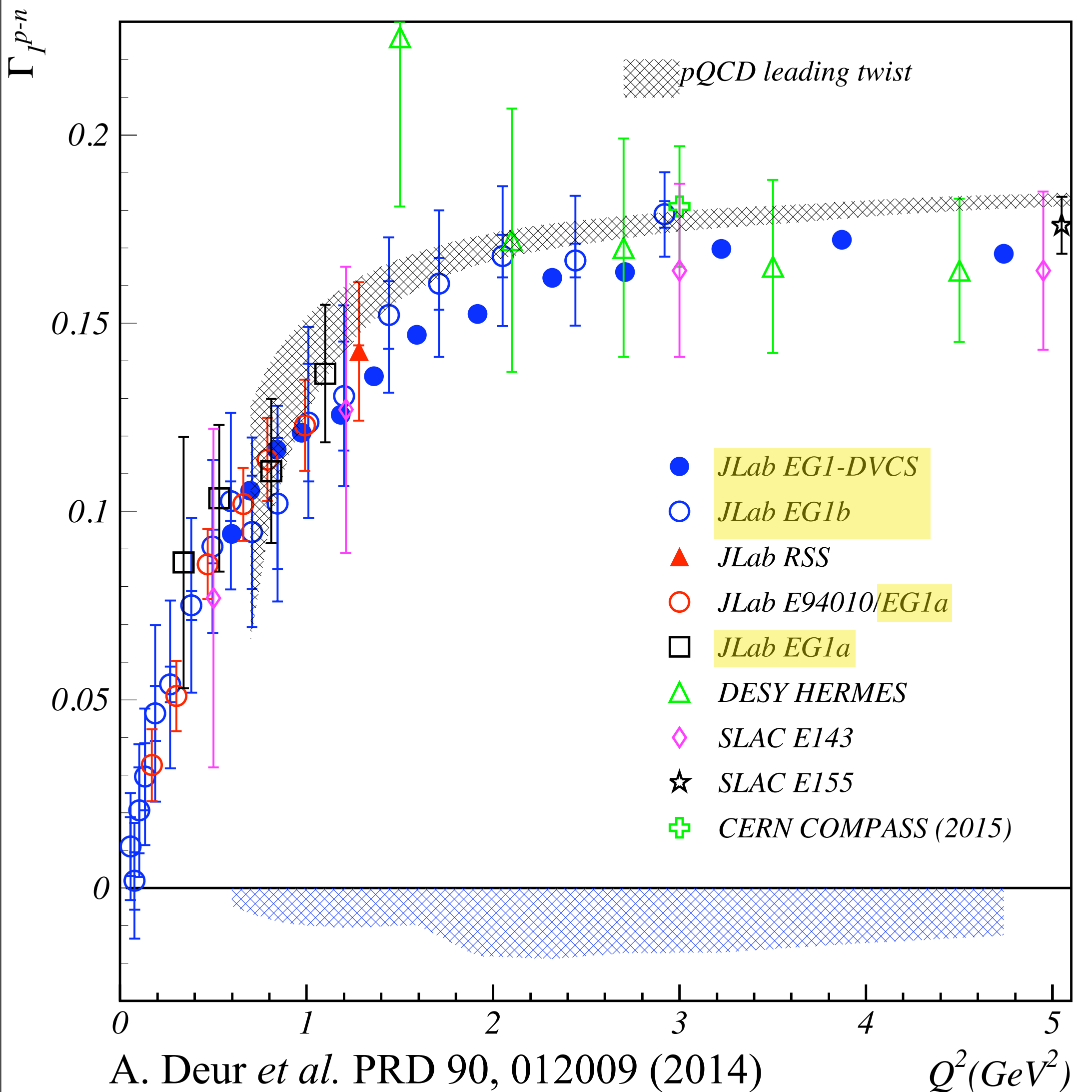
Bjorken sum $\Gamma_1^{p\text{-}n}$ measurement



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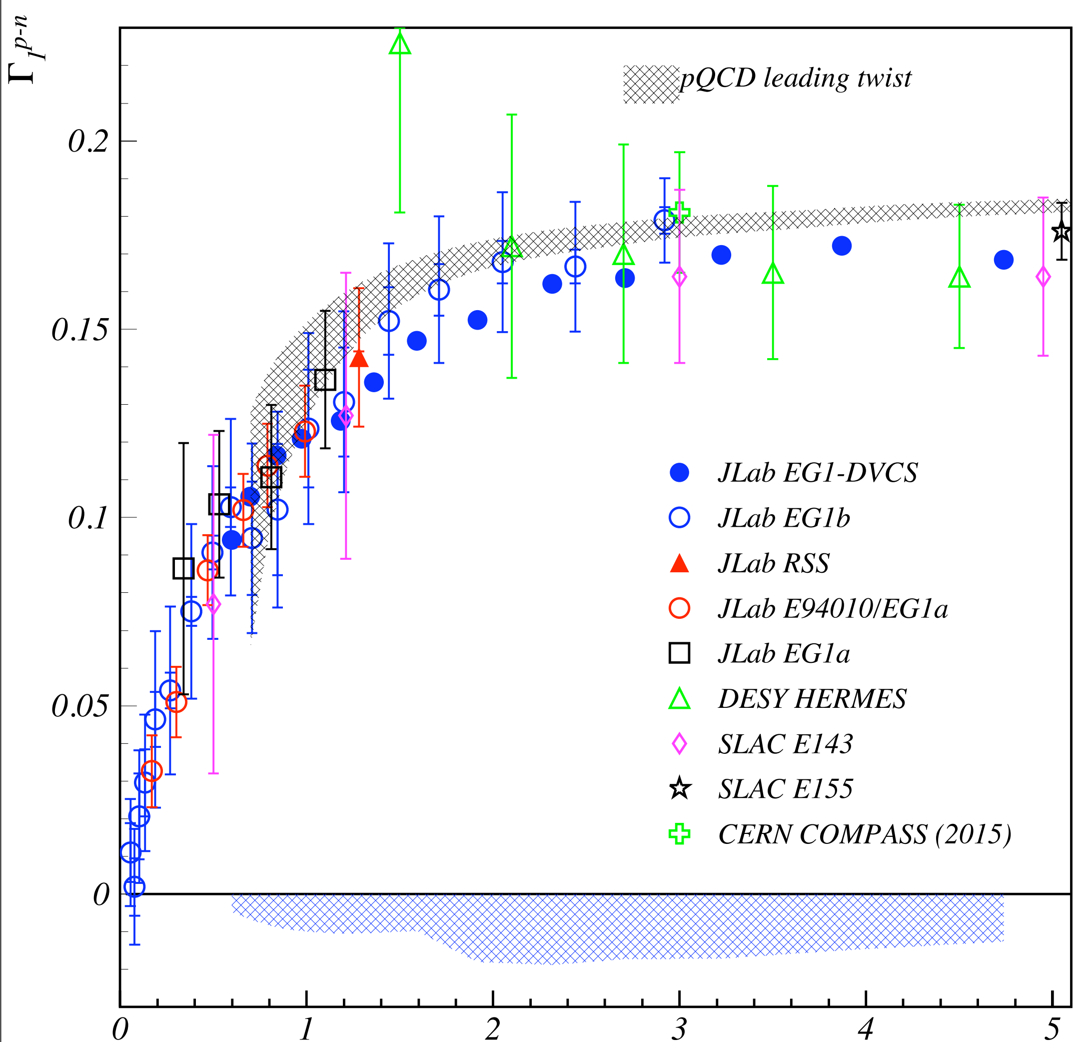
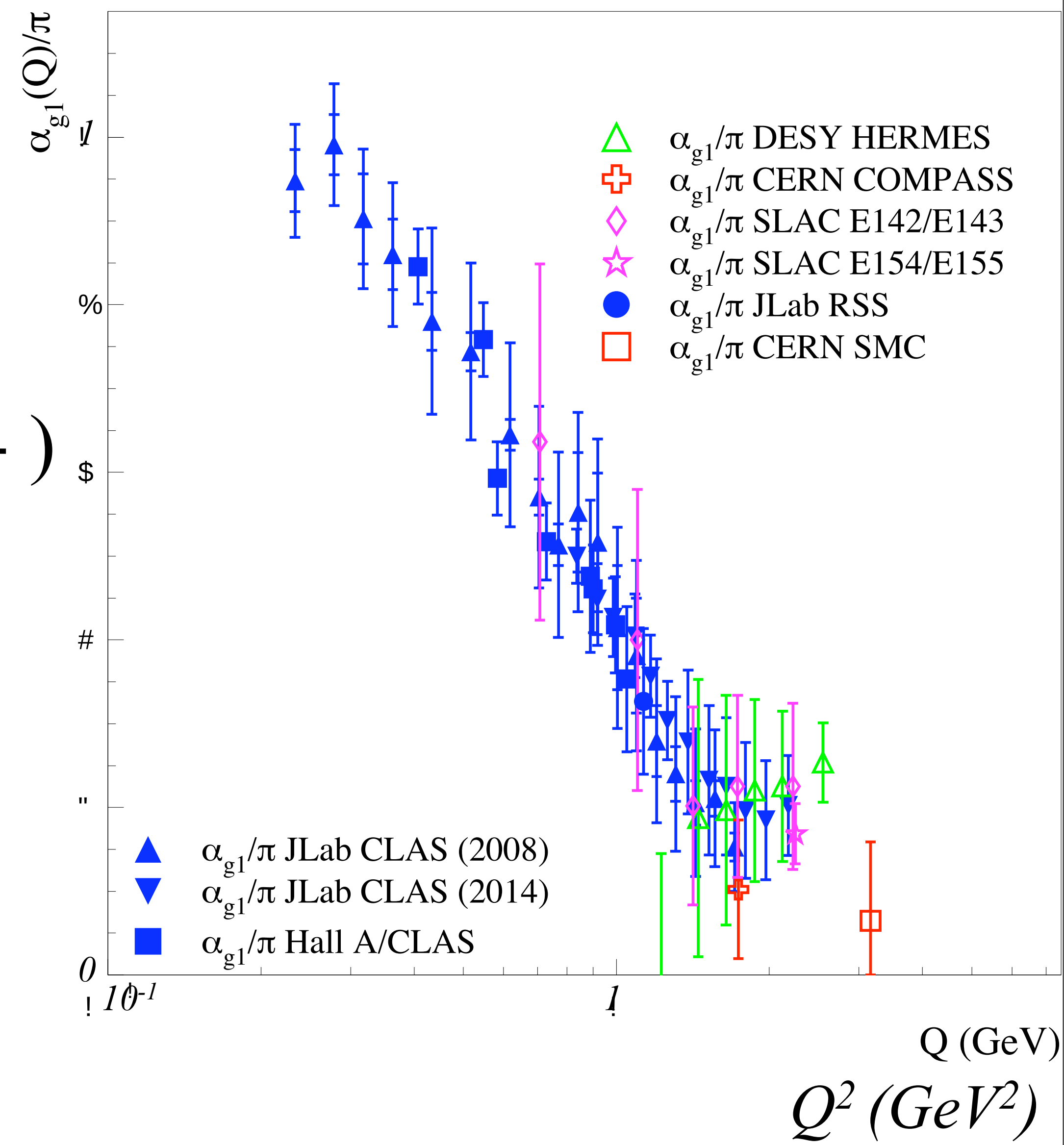


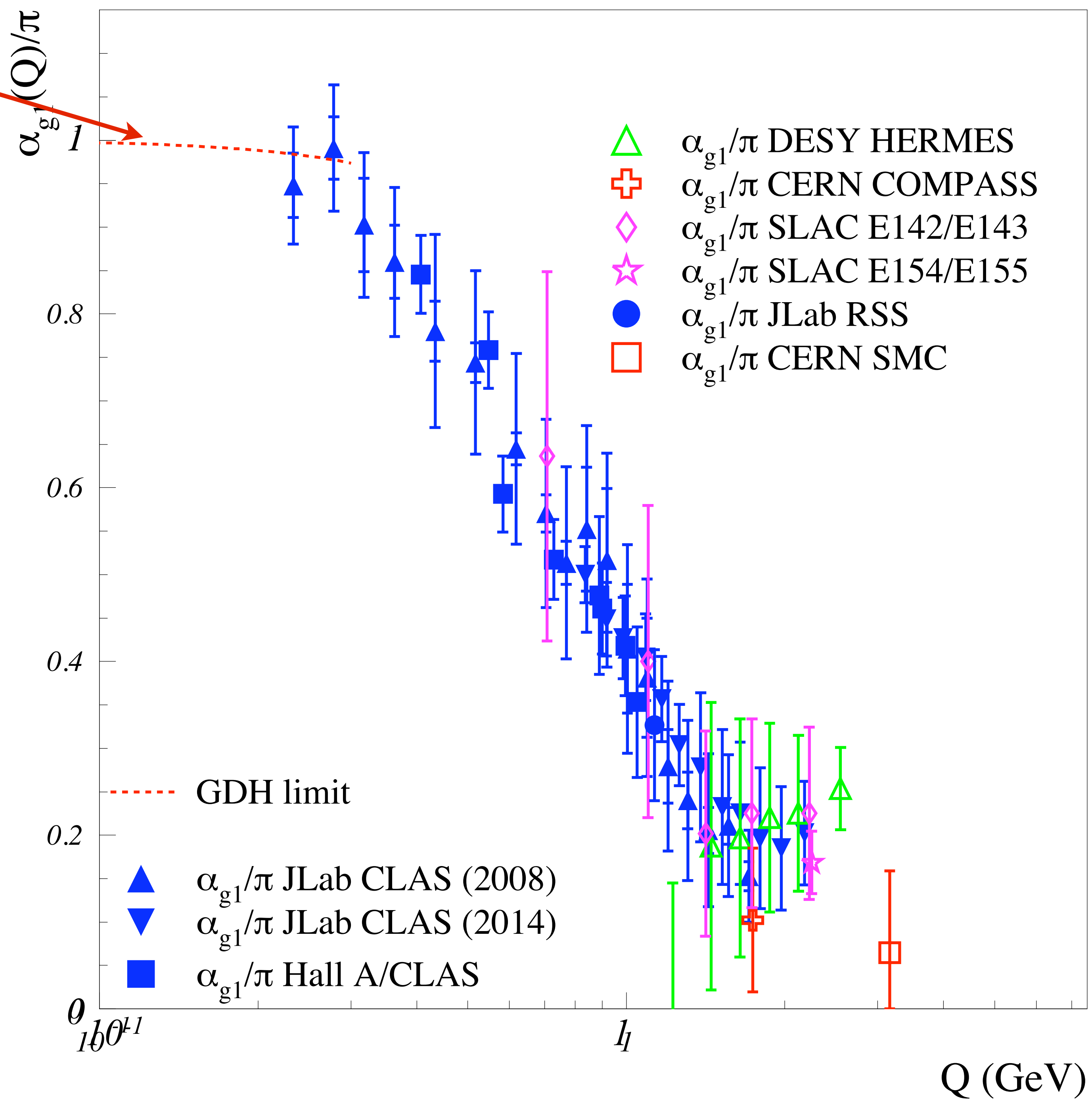
A. Deur *et al.* PRD 90, 012009 (2014)

$$\Gamma_1^{p-n} = \frac{1}{6} g_A \left(1 - \frac{\alpha_{g1}}{\pi}\right)$$



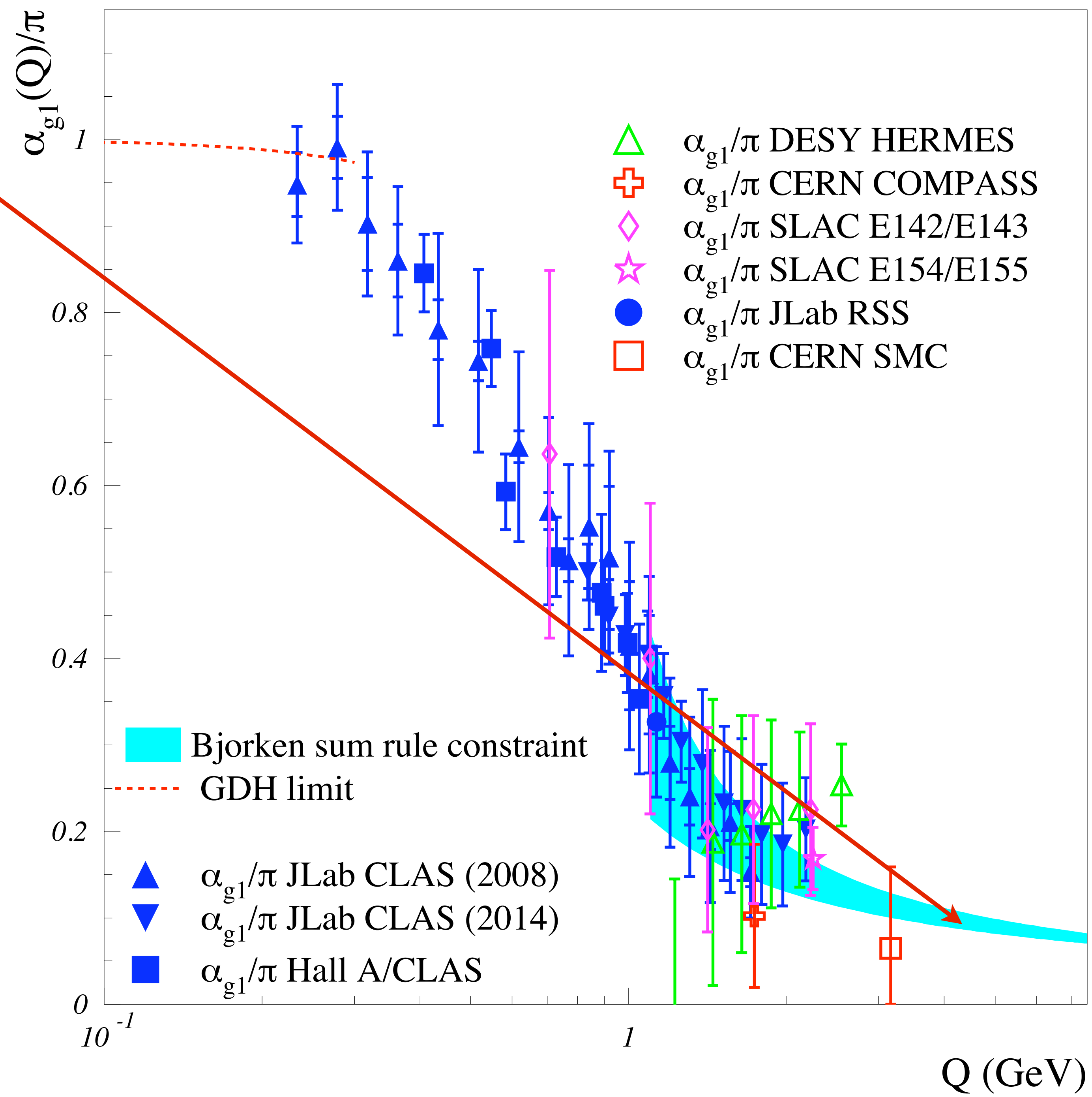
Low Q^2 limit

At $Q^2 = 0$, constraint from Gerasimov-Drell-Hearn (GDH) sum rule:



Large Q^2 limit

At large Q^2, constraint from Bjorken sum rule:



Deur, Burkert, Chen, Korsch. PLB 650 244 (2007), PLB 665 349 (2008)

$\Rightarrow$ We know α_{g1} at any Q^2.

Does it work?
•Comparison with theory
•Is α_{g1} bringing us useful information on QCD?

First, we will compare with AdS/QCD approach to QCD. Then check with Lattice, SDE,... approaches.

AdS/QCD:

•Analytical method to study non-perturbative QCD.

•Based on QCD Lagrangian expressed on the light front and reasonable approximations (neglect quark masses and short range quantum fluctuations).

•Provides a semi-classical approximation for QCD that is fully determined: Equations determined by QCD Lagrangian and by QCD's conformal symmetry).

•One universal parameter κ. (Minimal amount of parameter for a strong force description (if quark masses are neglected): pQCD has one: Λ_s.)

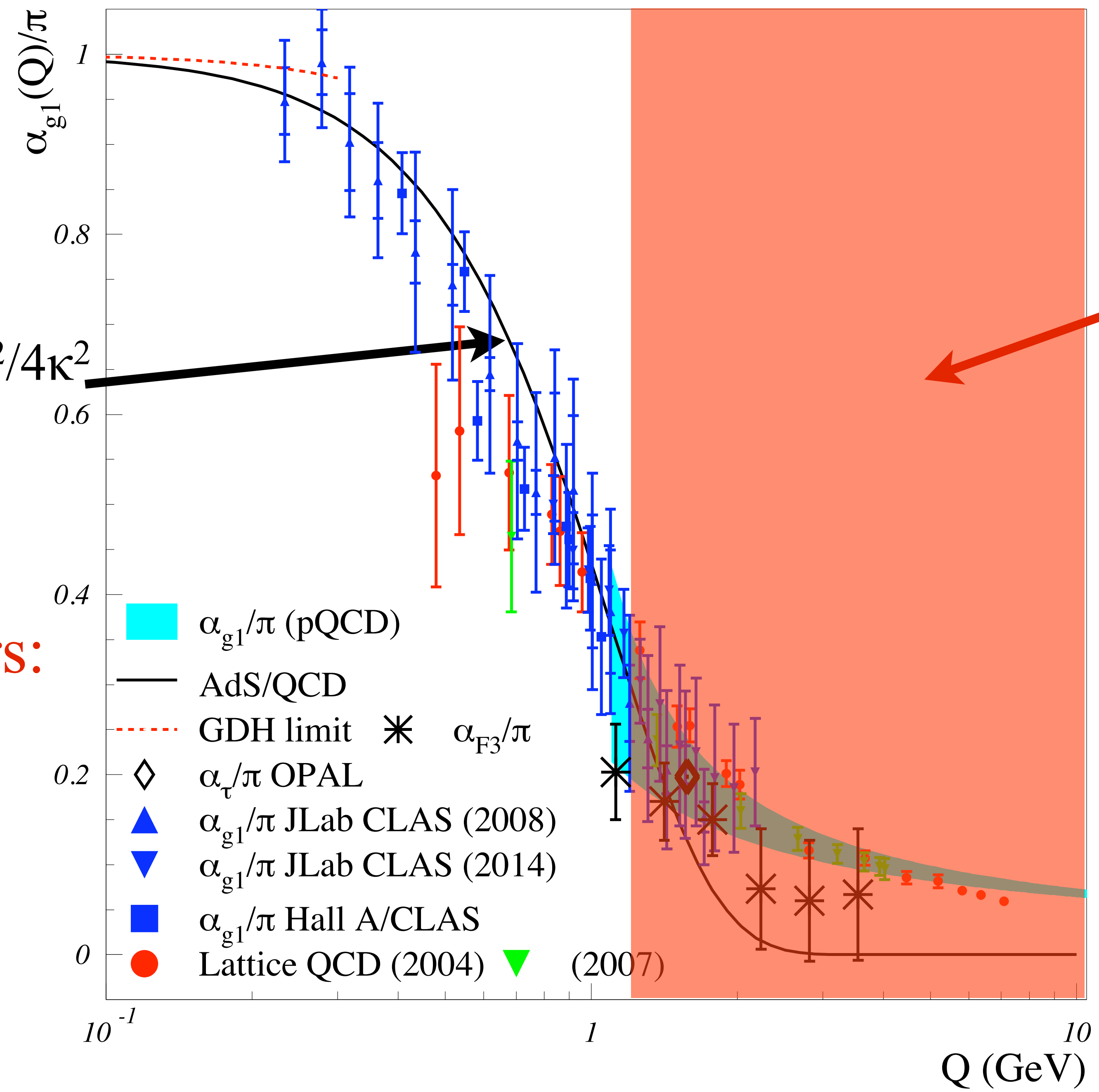
•Very successful in describing hadron form factors, hadron mass spectrum.

Review: Brodsky, de Teramond, Dosch, Erlich, Phys. Rep. 05 (2015) 001 [arXiv:1407.8131]

Does it work? Comparison with theory

$$\alpha_s^{AdS}(Q^2)/\,\pi = e^{-Q^2/4\kappa^2}$$

Brodsky, de Teramond, Deur.
PRD 81,096010 (2010),



AdS/QCD not valid

No free parameters:

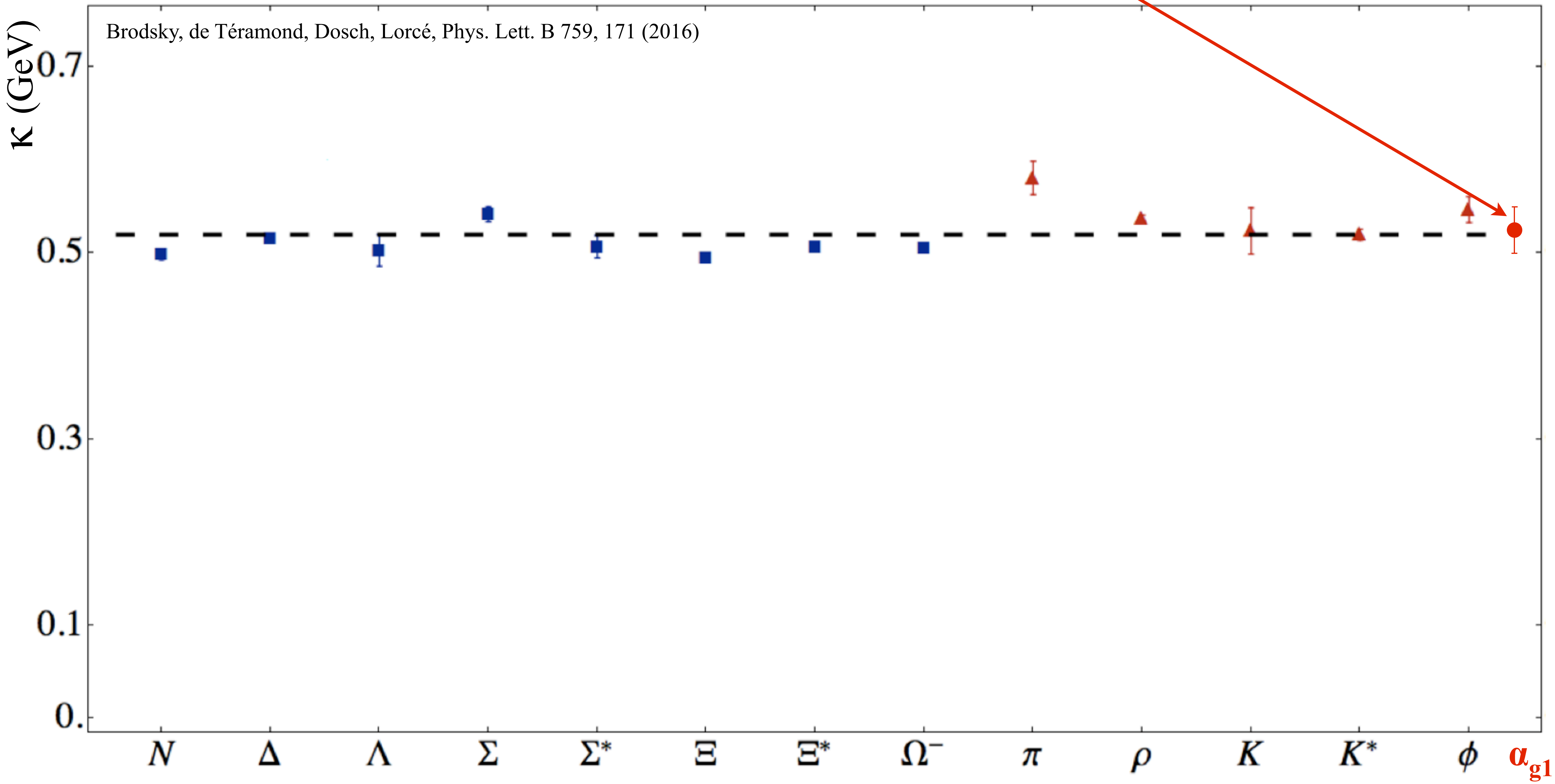
$\kappa = M_\rho/\sqrt{2}$

$\alpha_s(0) = \pi$ imposed either by sum rules or obtained from κ.

ADS/QCD prediction agrees very well with the α_s extracted from JLab's Bjorken sum data. No free parameters.

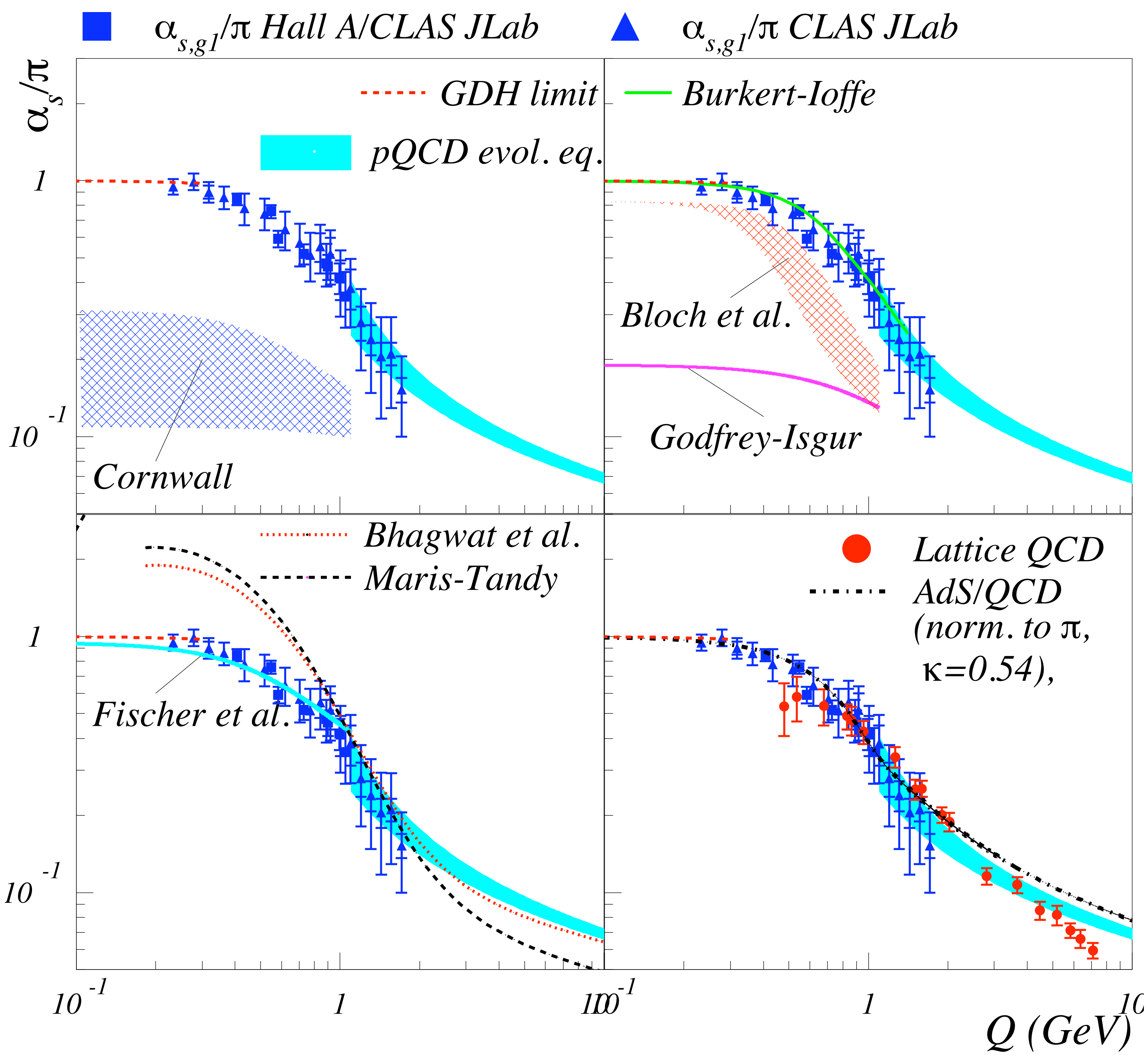
Does it work? Comparison with theory

One can also fit the $\alpha_{g1}(Q^2)$ data to get κ: $\kappa = 0.513 \pm 0.025$ GeV



Excellent agreement

Comparison with other predictions



Fisher *et al.*
Bloch *et al.*
Maris-Tandy
Bhagwat *et al.*
Cornwall
} Schwinger -Dyson

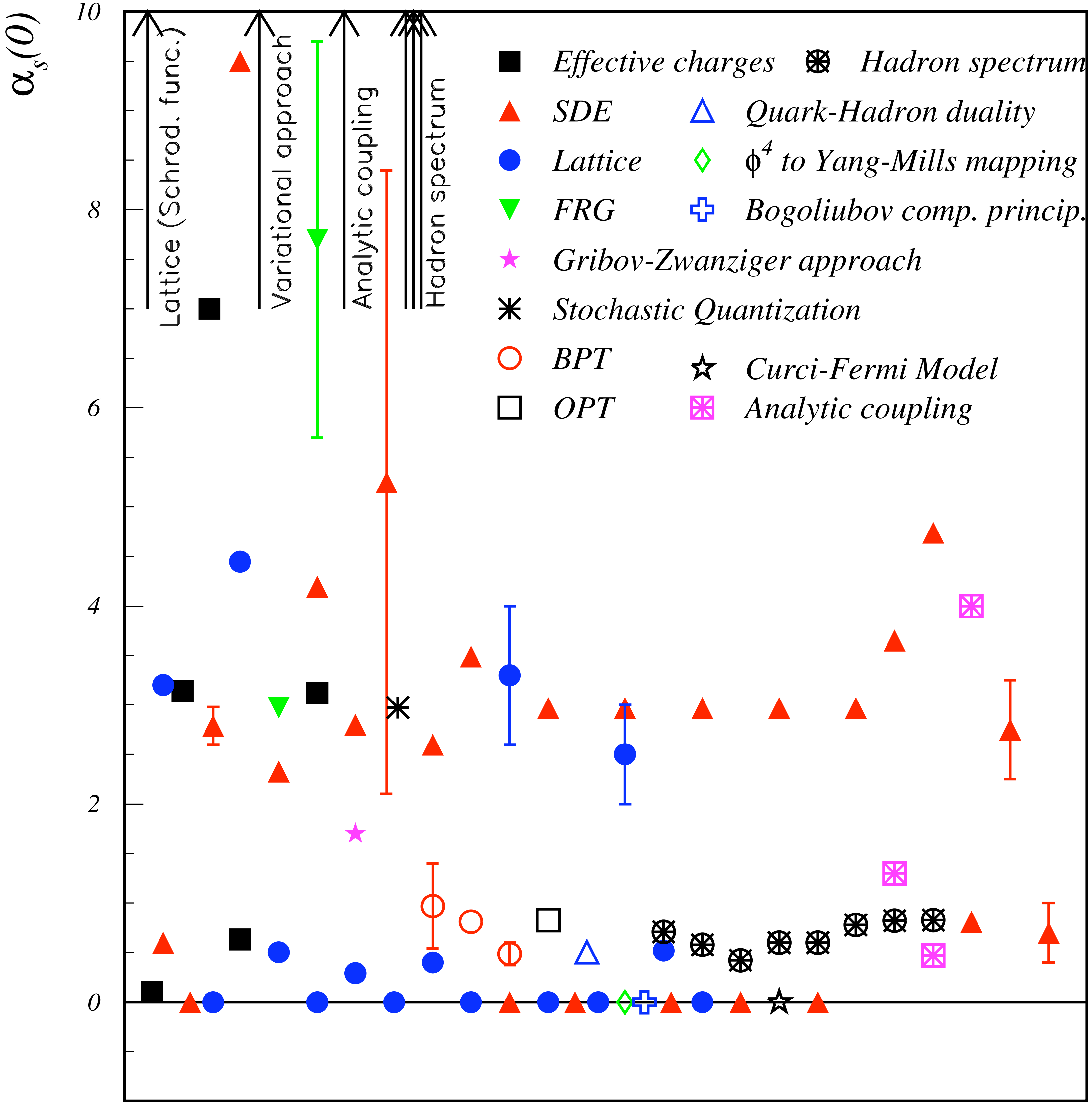
Godfrey-Isgur: Constituent Quark Model

Furui & Nakajima: Lattice QCD

Brodsky, de Teramond, AD: AdS/CFT

These calculations agree qualitatively (α_s freezes, transition occurs at similar Q^2 scale) but they disagree on $\alpha_s(0)$ value.

The many values of $\alpha_s(0)$ (from literature)



Deur, Brodsky, de Teramond. Prog. Part. Nuc. Phys. 90 1 (2016)

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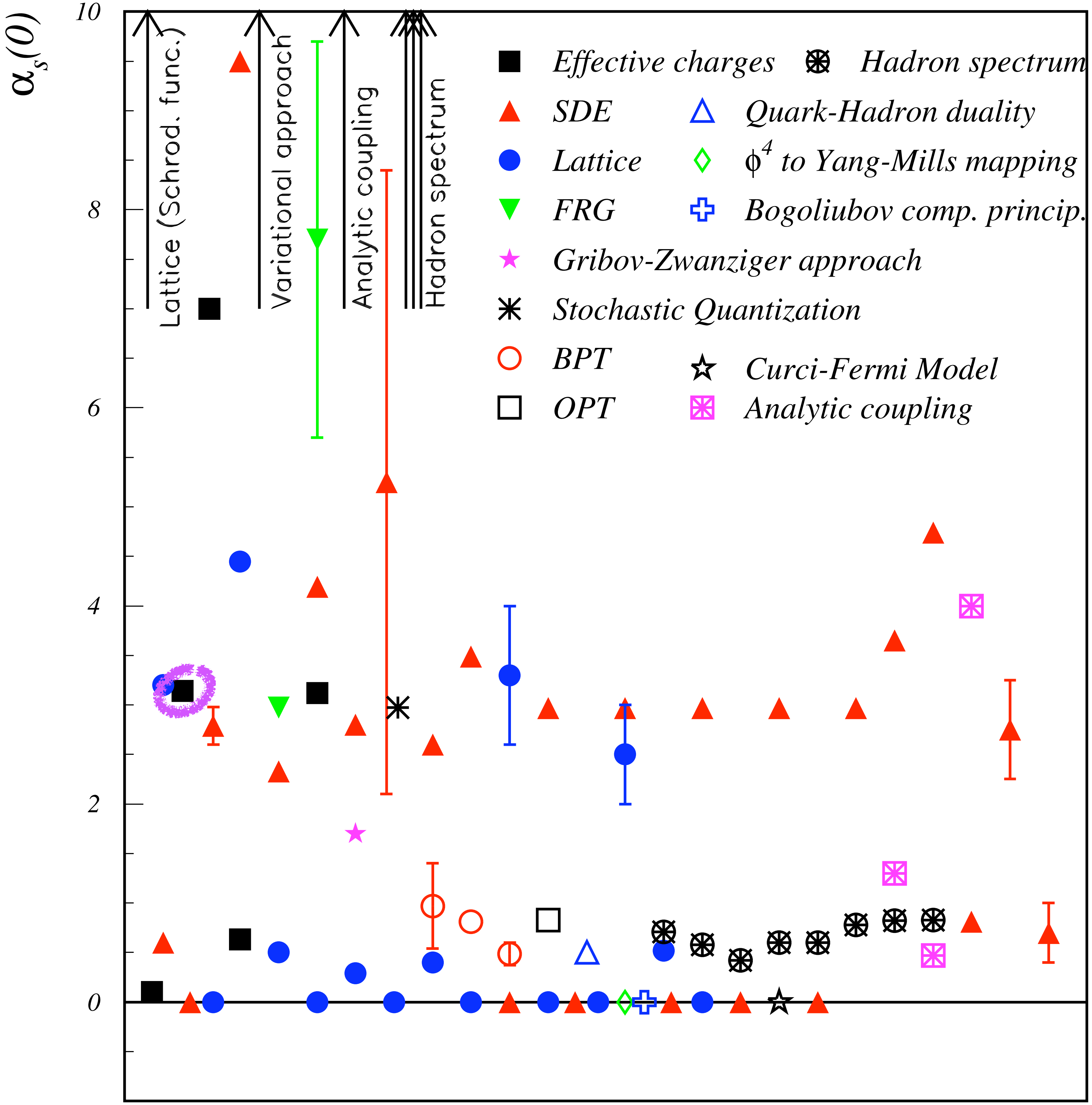
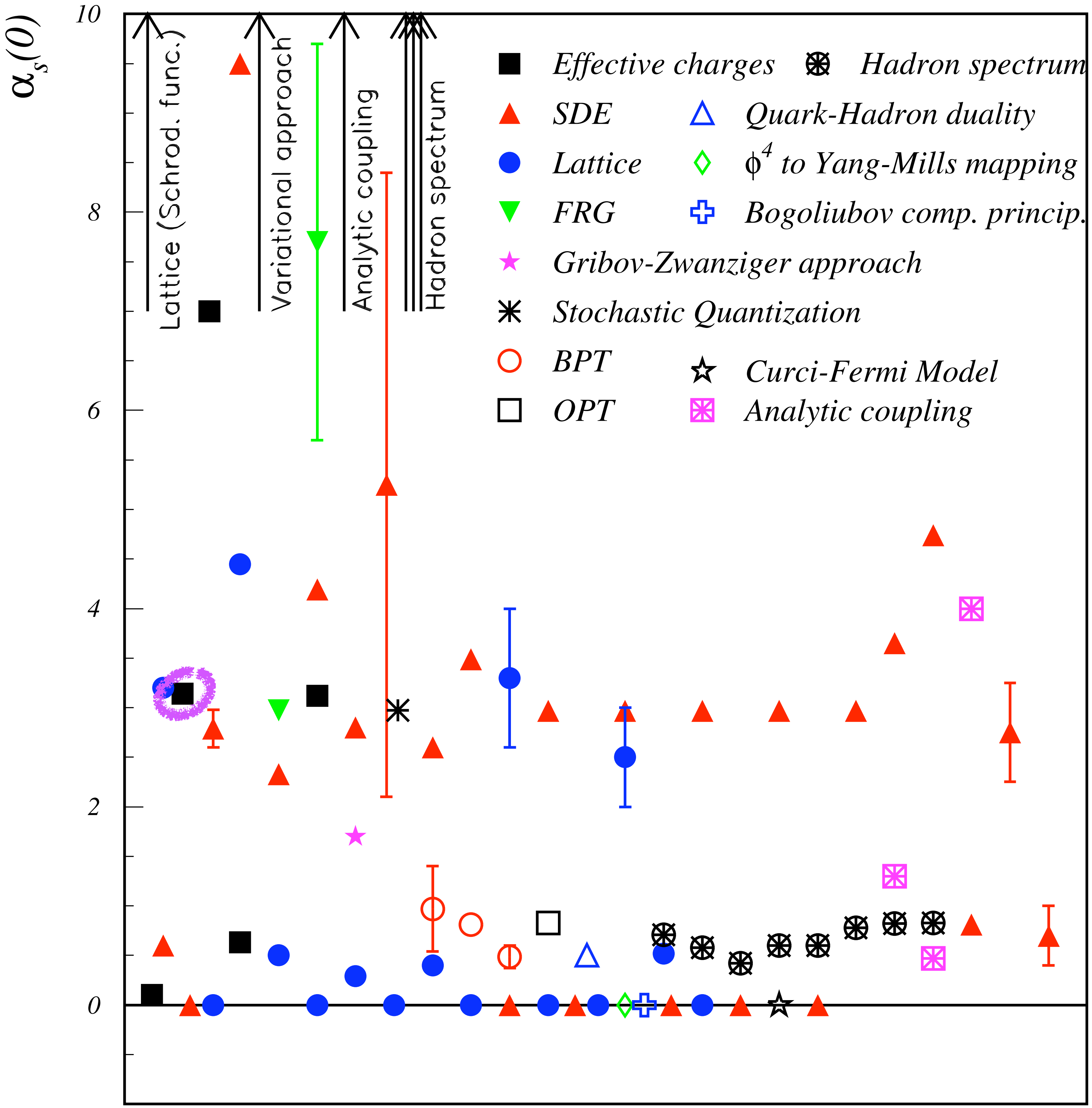
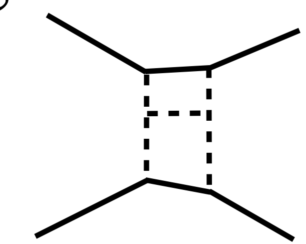


Deur, Brodsky, de Teramond. Prog. Part. Nuc. Phys. 90 1 (2016)



∞

Mostly calculations using V scheme. (problematic because of multi-gluon H-diagram divergences)



Calculations mostly using MOM scheme.

Calculations mostly using $\overline{\text{MS}}$ scheme.

(Separate and coexistent solution of SDE and Lattice. Unphysical?)

Deur, Brodsky, de Teramond. Prog. Part. Nuc. Phys. 90 1 (2016)

AdS/QCD results can be used to obtain $\alpha_s(0)$ in any scheme:

$\Rightarrow$ Quantify scheme-dependence of $\alpha_s(0)$ in the non-perturbative domain.

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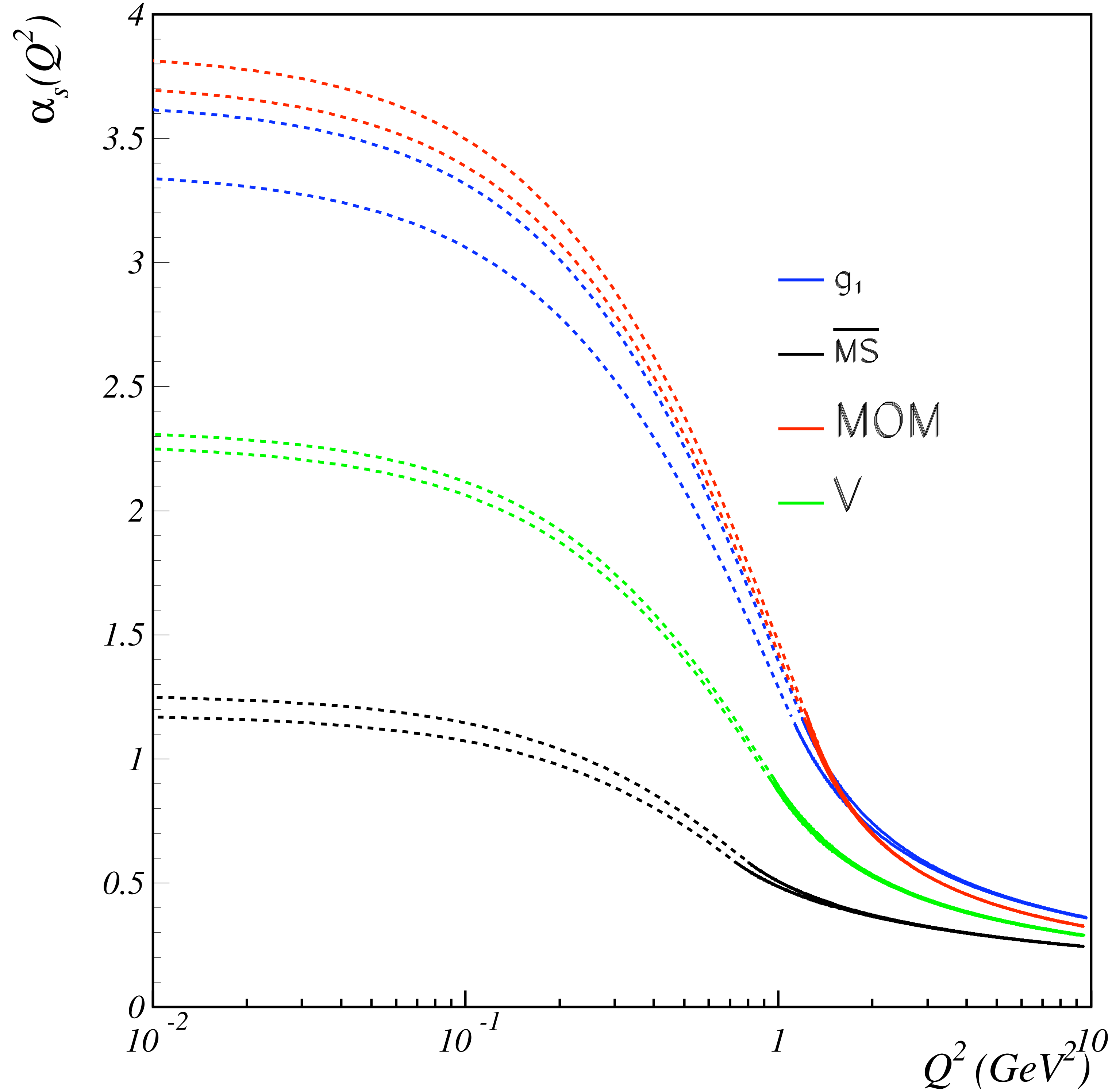
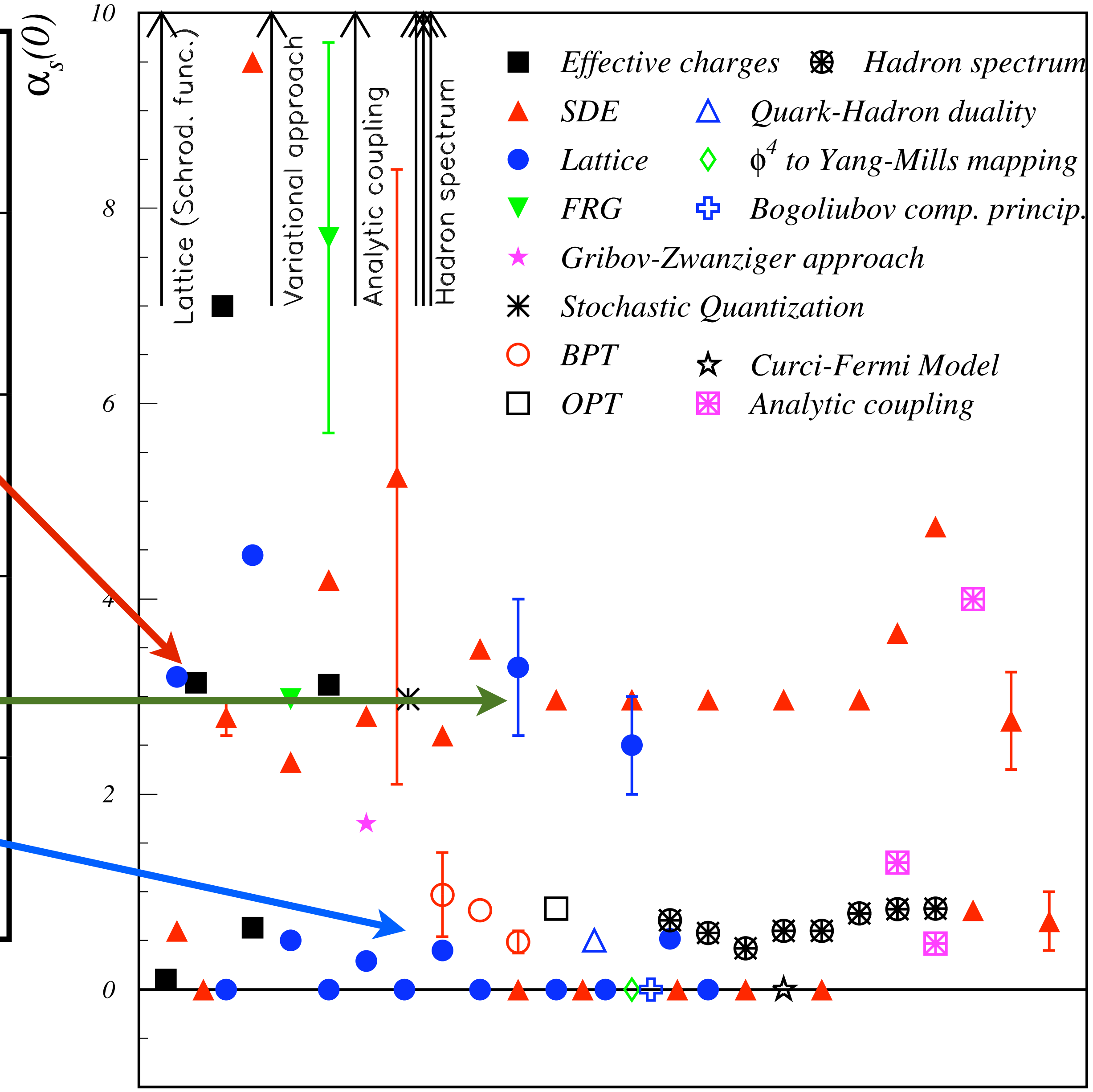


Deur, Brodsky, de Teramond PLB 757, 275 (2016)

Comparison with literature

$\alpha_s(0)$	Scheme	Expectation from Literature
3.51±0.62 (n_f=3 at Q_0)	g_1	π
2.30±0.35 (n_f=3 at Q_0)	V	-
2.84±0.60 (n_f=0 at Q=0)	MOM	2.97 (n_f=0)
0.80±0.10 (n_f=0 at Q=0)	$\overline{\text{MS}}$	~0.6 (n_f=0)



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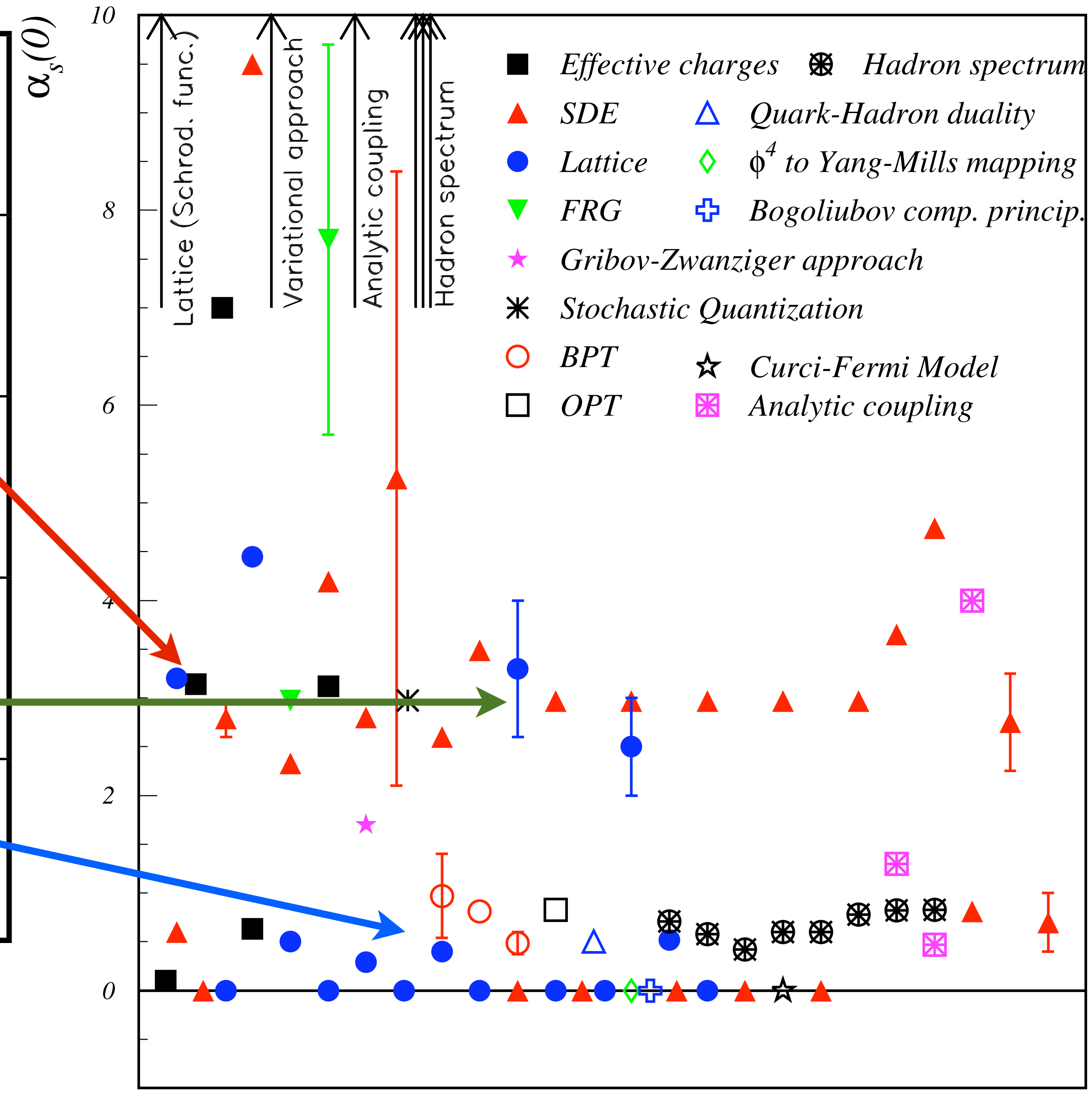
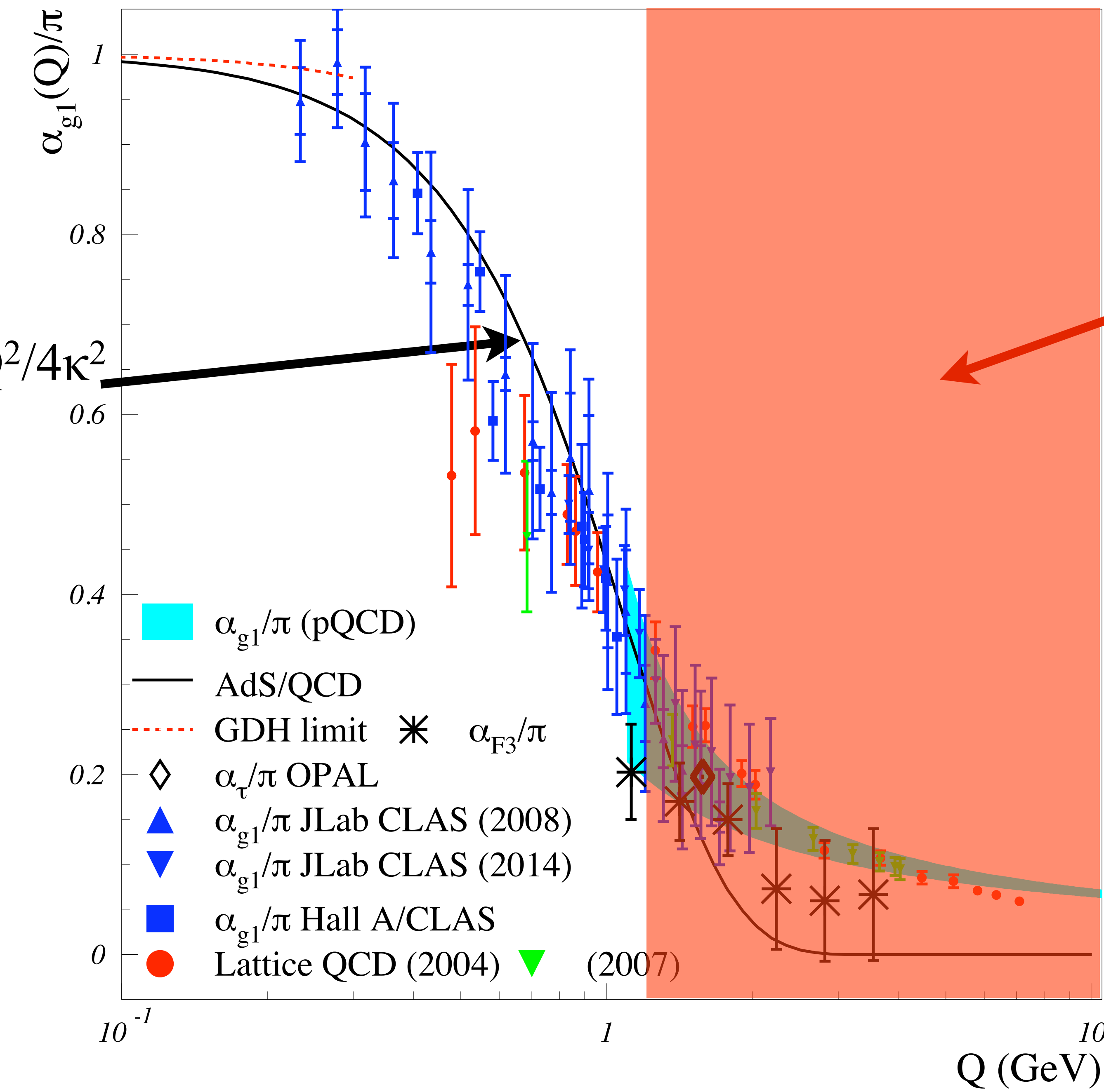
Deur, Brodsky, de Teramond
arXiv:1601.06568



Also compatible with $\alpha_s(0)\rightarrow\infty$ results: based on $V \propto r$: linear static quark-quark potential. AdS/QCD harmonic oscillator potential on Light-Front form equivalent to linear potential in usual frame (Instant-Front form).

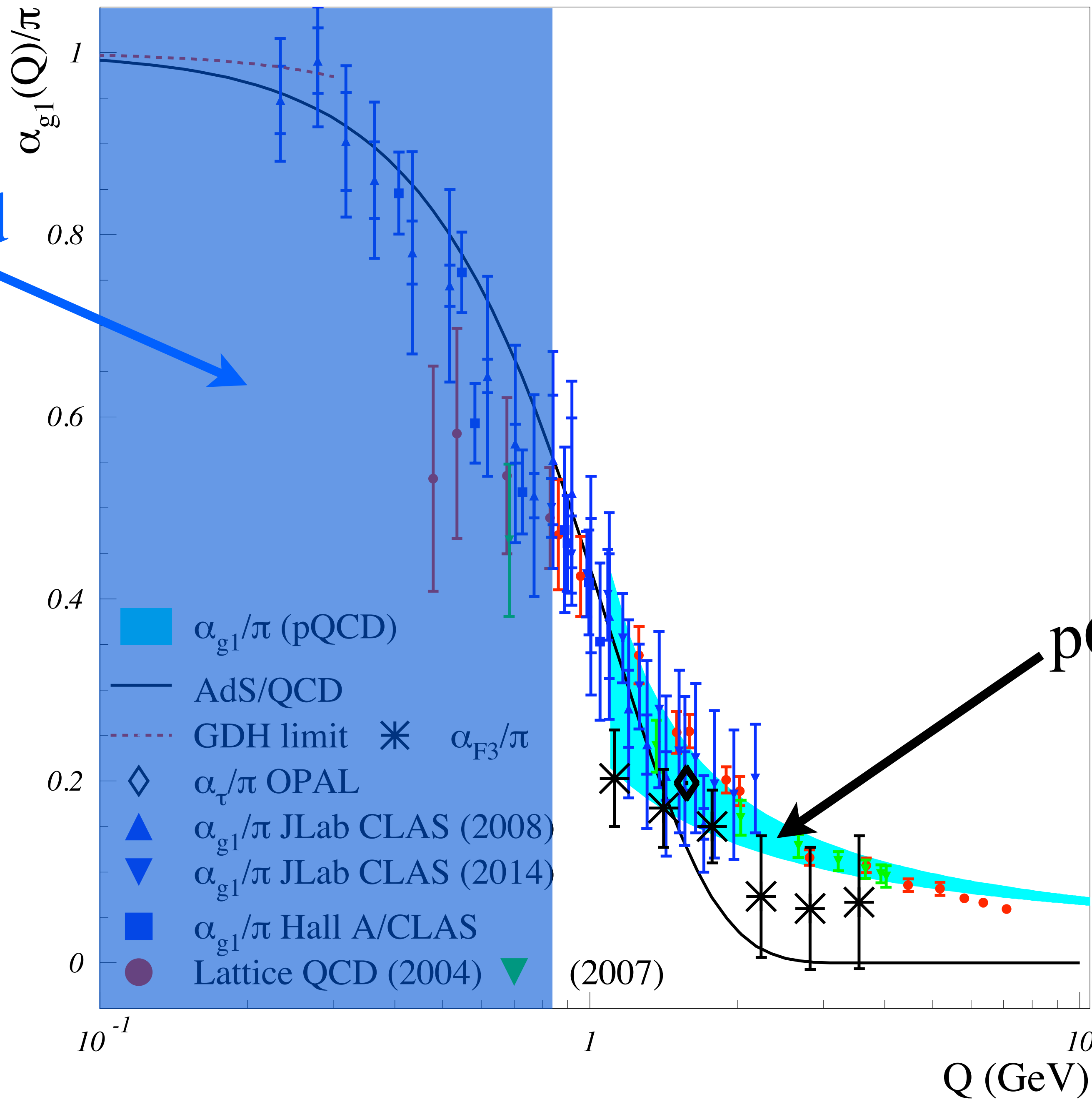
⇒ Discrepancy in non-perturbative α_s behavior seen in literature can be explained by scheme-dependence and ~mismatch in coordinate system used.

$$\alpha_s^{AdS}(Q^2)/\,\pi = e^{-Q^2/4\kappa^2}$$



AdS/QCD not valid

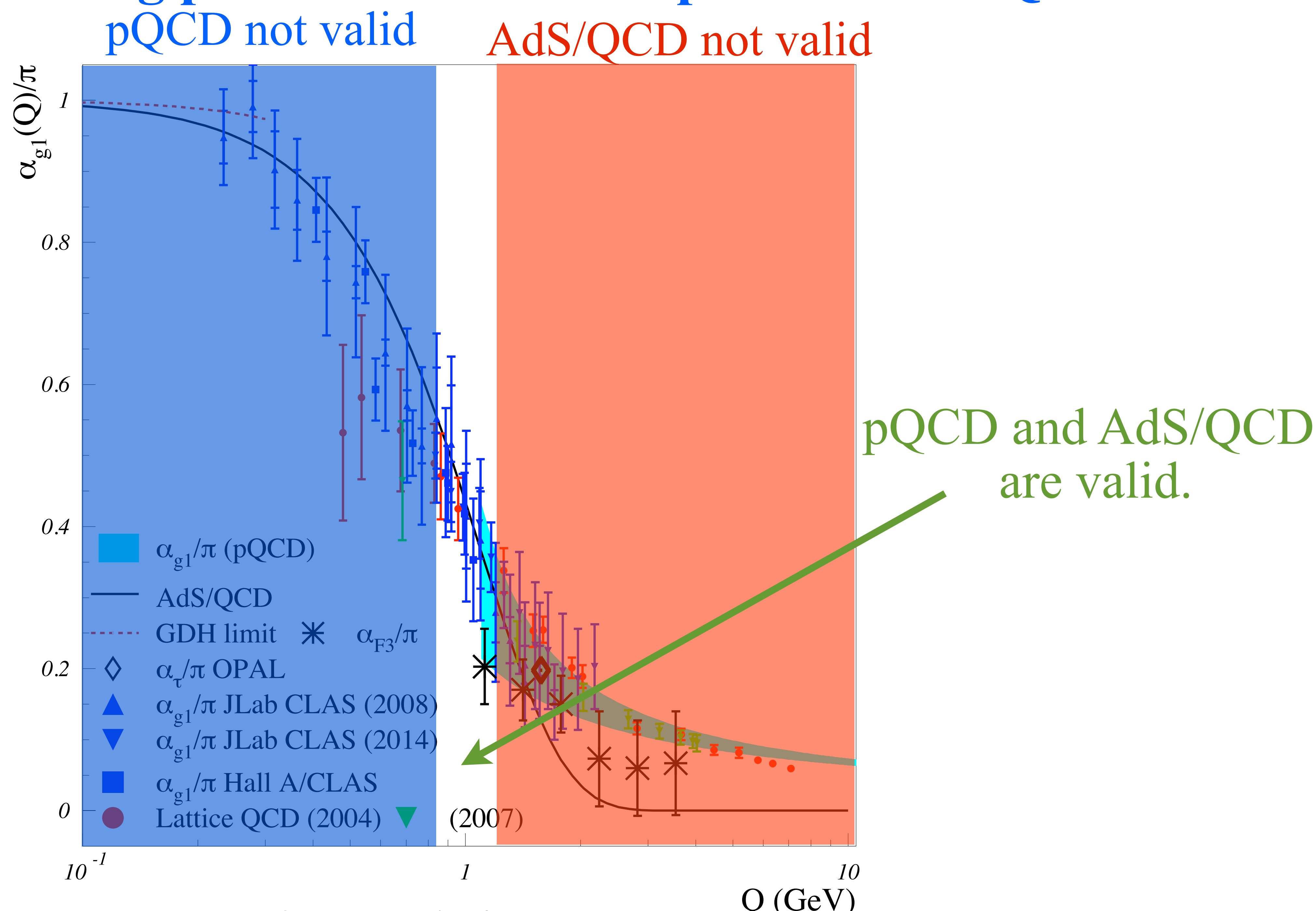
pQCD not valid



pQCD

Good agreement between JLab α_{g1} data and pQCD prediction.

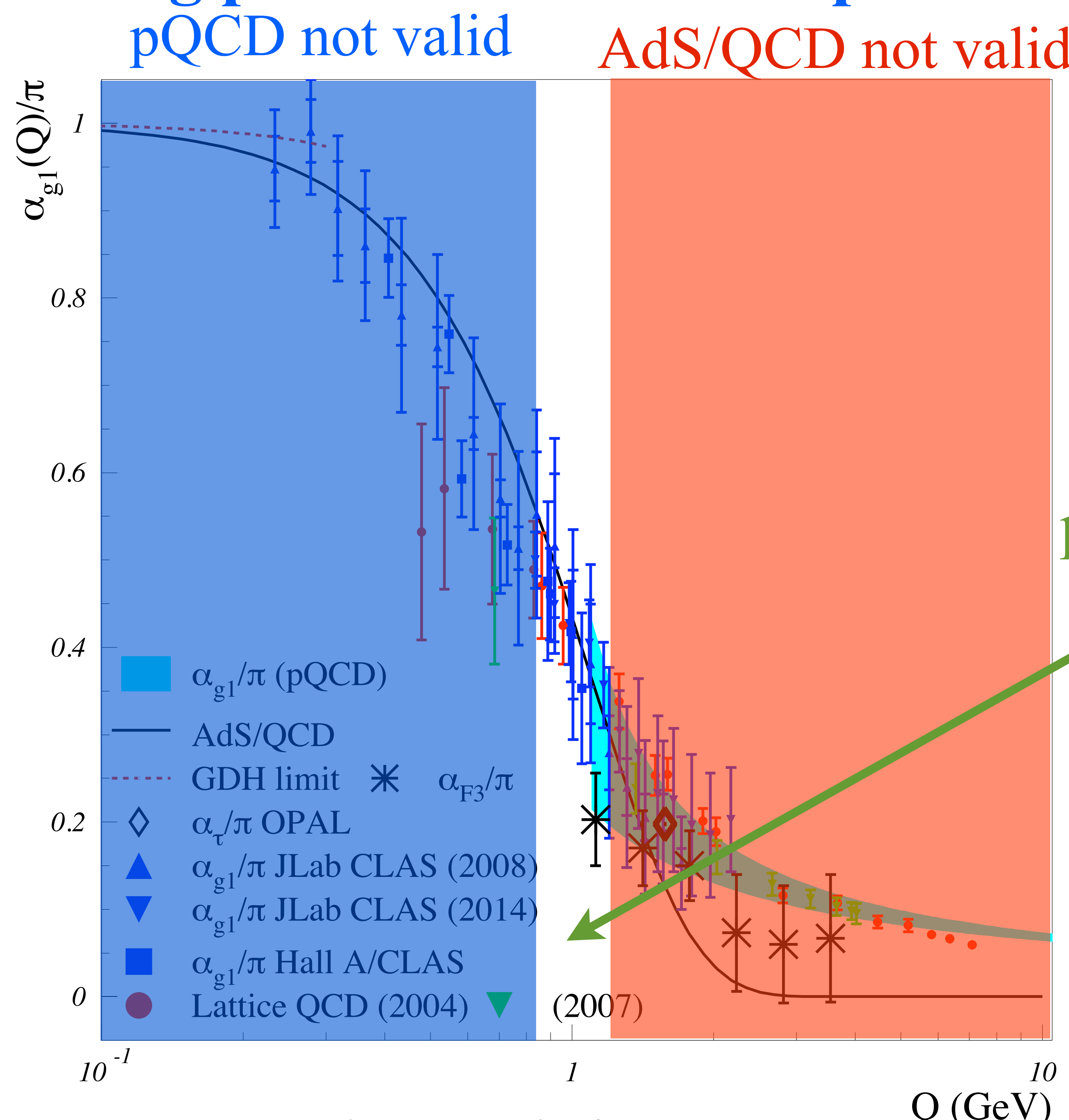
Connecting perturbative to non-perturbative QCD



Can match α_s from pQCD to α_s from AdS/QCD.

$\Rightarrow$ can relate κ, i.e. **hadronic masses,** to fundamental QCD parameter Λ_s.

Connecting perturbative to non-perturbative QCD



pQCD and AdS/QCD are valid.

Can match α_s from pQCD to α_s from AdS/QCD.

$\Rightarrow$ can relate κ, i.e. **hadronic masses,** to fundamental QCD parameter Λ_s.

$$\boxed{\Lambda_{\overline{MS}} = 0.440 M_\rho \sim M_\rho e^{-(a+1)} a^{-1/2}}$$

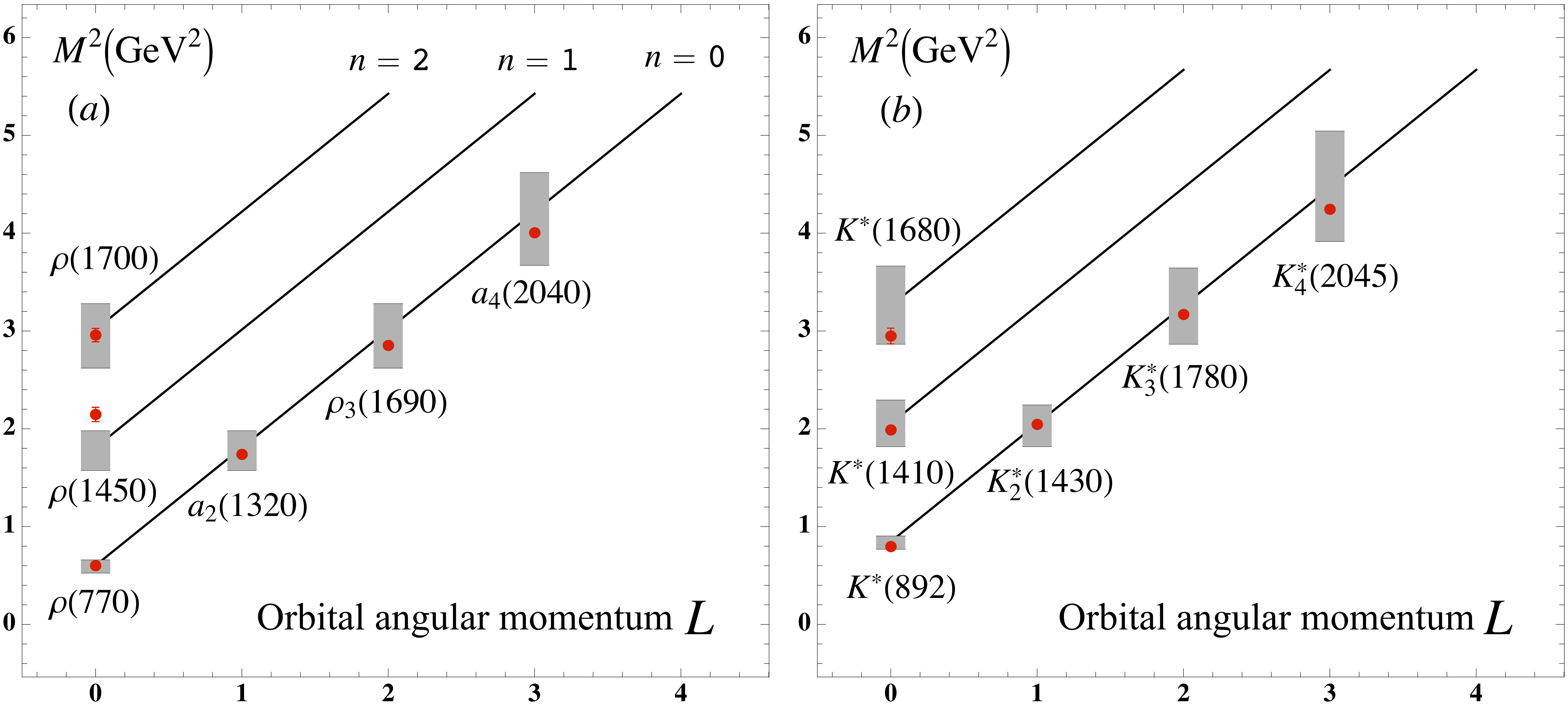
$a = 4(\sqrt{\ln(2)^2 - 1 + \beta_0/4} - \ln(2))/\beta_0$

Deur, Brodsky, de Teramond, Phys. Lett. B 750, 528 (2015)

At N^3LO

At LO (here, equations are solvable analytically).

Higher order predictions for meson spectrum



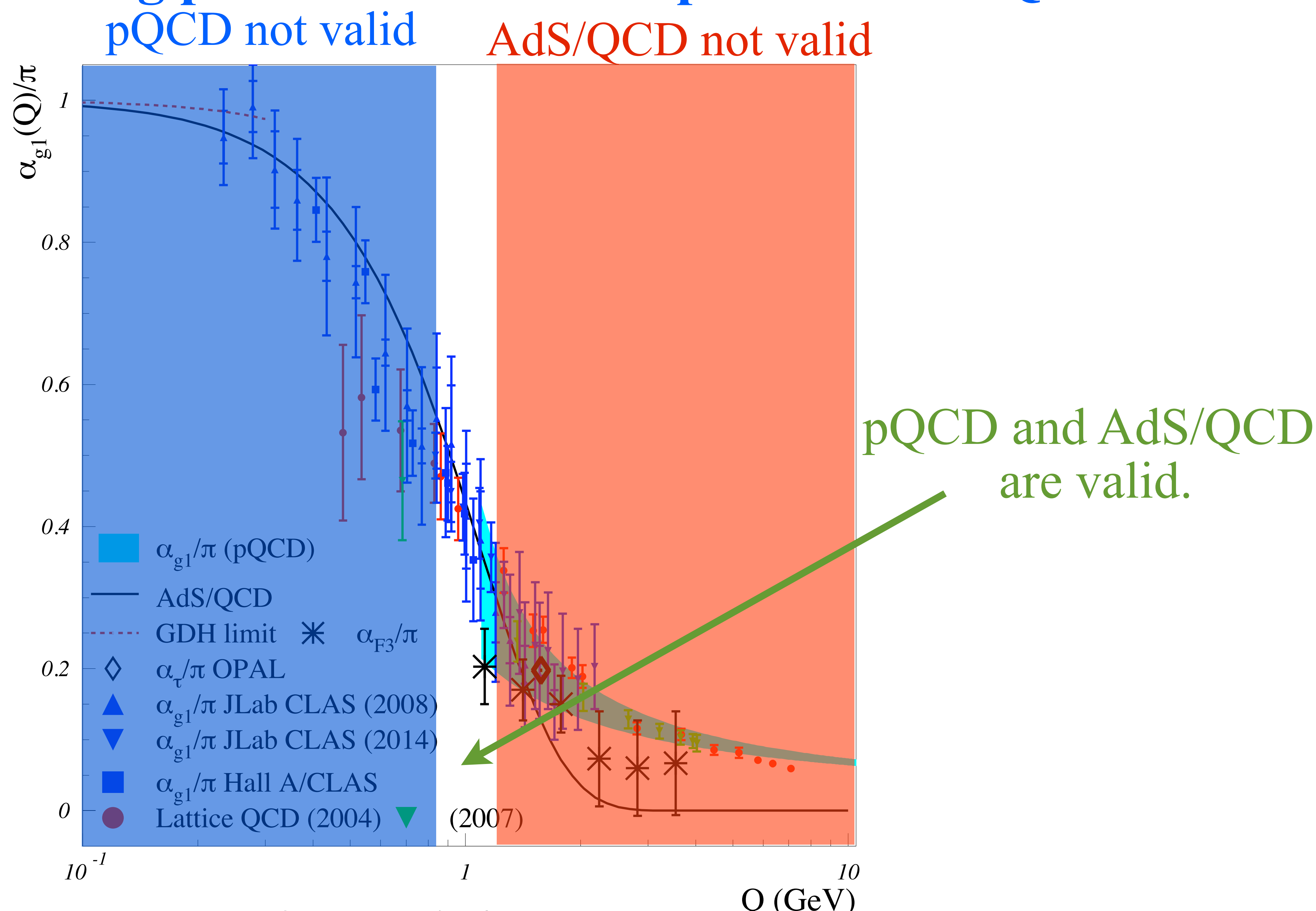
: AdS/QCD predictions with Λ_s from PDG as (only) input.

: Slopes predicted by AdS/QCD.

: Measurements.

Analytic determination of hadron spectrum from Λ_s

Connecting perturbative to non-perturbative QCD



Can match α_s from pQCD to α_s from AdS/QCD.

Matching equations also determines pQCD-Strong QCD transition scale

$$\alpha_{g_1}^{pQCD}(Q_0)=\alpha_{g_1}^{AdS/QCD}(Q_0)$$
$$\beta^{pQCD}(Q_0) = \beta^{AdS/QCD}(Q_0)$$

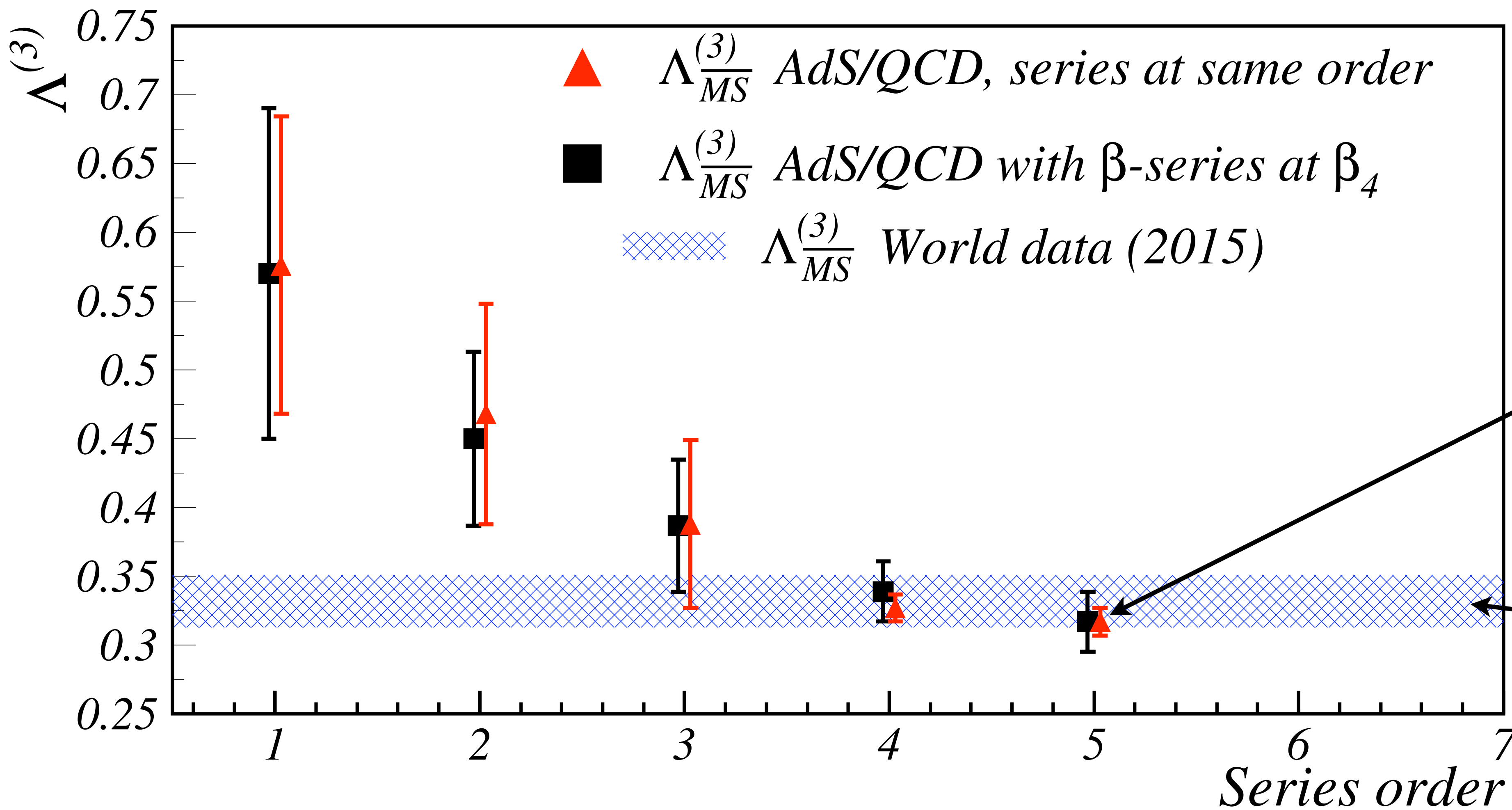
(β is the log derivative of α_s)

$Q_0 \sim 1.07$ GeV.

($\overline{MS}$ scheme)

Prediction of Λ_s from hadronic observable

Conversely, one can use κ and use the same matching procedure to predict QCD's fundamental parameter Λ_s.



Λ_s^{AdS}=0.339(19) GeV

Deur, Brodsky, de Téramond, arXiv:1608.04933

Λ_s^{PDG}=0.332(19) GeV

Particle Data Group, Chin. Phys. C, 38, 090001 (2014) and 2015 update

κ from hadron masses, n_f=3, use recent 5-loop α_s calculation.

Determination of Λ_s in excellent agreement with PDG world average and with similar uncertainty.

Summary

- α_s is the fundamental parameter of QCD.
- It is well understood at short distances but no so at long distances.
- Bjorken Sum Rule is advantageous to define an effective coupling α_{g1}.
- Data -essentially from CLAS- and sum rules allow to obtain α_{g1} at all Q^2.
- α_{g1} "freezes" at low Q^2
- Remarkable agreement with AdS/QCD prediction. No free parameters.
  - Analytic determination of hadron spectrum from Λ_s.
  - Conversely, high precision determination of Λ_s. (Other means: Lattice or experimental data).

  | Long thought goal of QCD.

  - Q_0 can be used as starting scale for QCD's DGLAP and ERBL evolution equations.
- Agreement with most other theoretical predictions once scheme-dependence accounted for. Solve long-standing confusion on what is the freezing value of α_s.

Hall A and CLAS data at the origin of these progresses.