TMD factorization in SIDIS

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Outline

Normal steps in TMD factorization and issues

- Combining large and small transverse momentum
- Errors

A generalized formalism

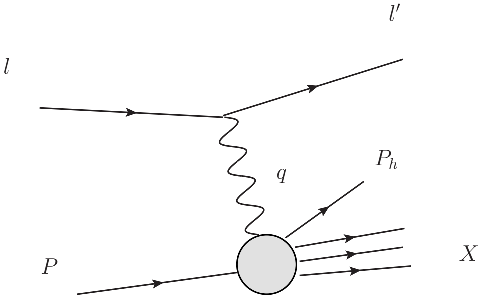
- Region of validity
- Matching perturbative and Nonperturbative transverse momentum

Further issues and possible solutions

- Issues with perturbative calculation
- Improved factorization?

Basic TMD factorization

SIDIS

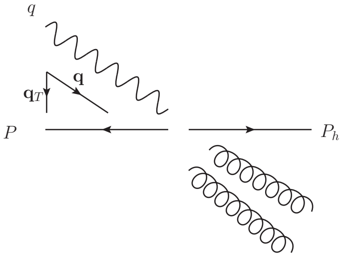


$$Q^2 = -q^2 = -(l' - l)^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

$$z = \frac{P \cdot P_h}{P \cdot q}$$

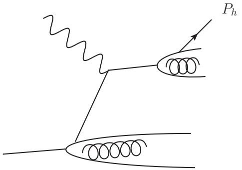
$$\mathbf{q}_T = -\mathbf{P}_{hT}/z$$

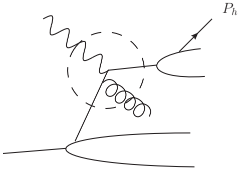


TMD physics at different q_T

$$q_T \sim m$$

$$q_T \gtrsim Q$$

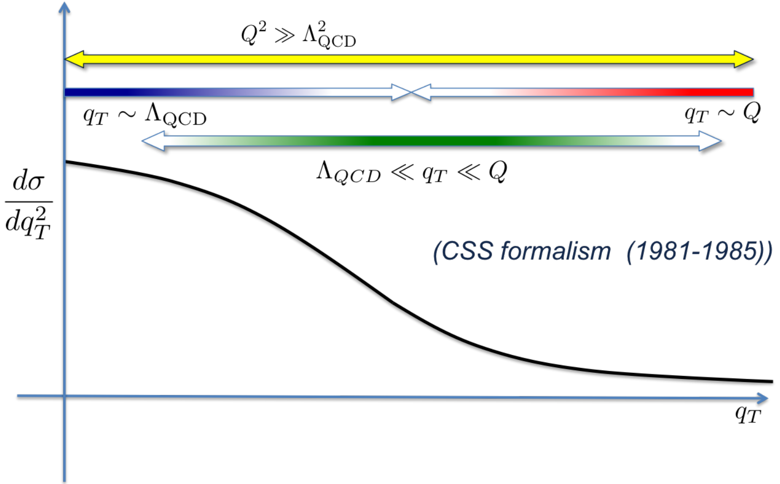




Small q_T approx.
TMD Factorization

Pert. approx.
Coll. Factorization

Unified: Transverse Momentum:



Structure of TMD cross sections

$$\Gamma = \mathbf{T}_{\text{TMD}}\Gamma + [\Gamma - \mathbf{T}_{\text{TMD}}\Gamma]$$
$$\approx \underbrace{\mathbf{T}_{\text{TMD}}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\text{coll}}\left[\Gamma - \mathbf{T}_{\text{TMD}}\Gamma\right]}_{\mathbf{Y}}$$

$\Gamma =$ Cross section

$\mathbf{T}_{TMD} =$ Small q_T approximant

$\mathbf{T}_{\text{coll}} =$ Large q_T approximant corrections

Accuracy in $m \lesssim q_T \lesssim Q$

Cross section

$$\begin{aligned}\Gamma =& \mathbf{T}_{\rm TMD}\Gamma + [\Gamma - \mathbf{T}_{\rm TMD}\Gamma] \\ \approx& \underbrace{\mathbf{T}_{\rm TMD}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\rm coll}\left[\Gamma - \mathbf{T}_{\rm TMD}\Gamma\right]}_{\mathbf{Y}}\end{aligned}$$

$$[\Gamma - \mathbf{T}_{\rm TMD}\Gamma] = O\left(\frac{q_T}{Q}\right)^a \Gamma$$

$$\begin{aligned}\mathbf{T}_{\rm coll}\left[\Gamma - \mathbf{T}_{\rm TMD}\Gamma\right] &= [\Gamma - \mathbf{T}_{\rm TMD}\Gamma]\left[1 + O\left(\frac{m}{q_T}\right)^b\right] \\ &= [\Gamma - \mathbf{T}_{\rm TMD}\Gamma] + O\left(\frac{q_T}{Q}\right)^a O\left(\frac{m}{q_T}\right)^b \Gamma \\ &= [\Gamma - \mathbf{T}_{\rm TMD}\Gamma] + \underbrace{O\left(\frac{m}{Q}\right)^{min(a,b)} \Gamma}_{\text{error for } \mathbf{W+Y}}\end{aligned}$$

Interplay between $\mathbf{W}$ and $\mathbf{Y}$

Cross section

$$\Gamma = \mathbf{T}_{\text{TMD}}\Gamma + [\Gamma - \mathbf{T}_{\text{TMD}}\Gamma]$$
$$\approx \underbrace{\mathbf{T}_{\text{TMD}}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\text{coll}}[\Gamma - \mathbf{T}_{\text{TMD}}\Gamma]}_{\mathbf{Y}}$$

Region of $\mathbf{q_T \ll Q}$

- TMD approx. dominates $\rightarrow$ $\Gamma \approx \mathbf{T}_{\text{TMD}}\Gamma$
- Y term small

Region of $\mathbf{q_T \gtrsim Q}$

- Collinear approx. dominates $\rightarrow$ $\Gamma \approx \mathbf{T}_{\text{coll}}\Gamma$
- Consistency demands $\rightarrow \mathbf{T}_{\text{TMD}}\Gamma - \mathbf{T}_{\text{coll}}\mathbf{T}_{\text{TMD}}\Gamma \sim$ Small

Issues

Region of validity

- $\mathbf{W}$: $\mathrm{q_T} \ll \mathrm{Q}$.
- $\mathbf{T}_{\text{coll}}\Gamma$: $q_T \gg m$
- $\mathbf{W}+\mathbf{Y}$: $m \lesssim q_T \lesssim Q$
- What about $q_T \lesssim m$ and $q_T \gtrsim Q$?

Matching perturbative and Nonperturbative transverse momentum

- Large Q: broad region $m \ll q_T \ll Q$
- Small Q: transition rapid, need good parameterization to help with the transition.

Further issues

An improved W+Y formalism

J. Collins, L. Gamberg, A. Prokudin,
T. C. Rogers, N. Sato, and B. Wang,
arXiv:1605.0067

Region $q_T \gtrsim Q$: Modifications in $\mathbf{W}$

$$\mathbf{T}_{\mathrm{TMD}}\Gamma = \Gamma\left[1 + O\left(\frac{q_T}{Q}\right)^a + O\left(\frac{q_T}{Q}\right)^{a'} + \ldots\right]$$

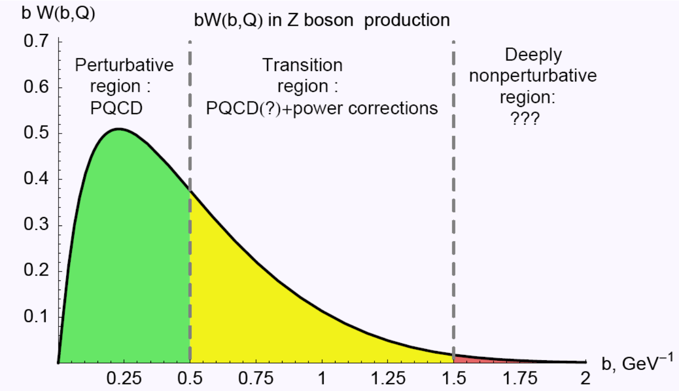
$\mathbf{W}$ term loses accuracy. Terms with $a' > a$ no loner negligible. Need to switch to the collinear term.

Transition can be difficult, especially at small Q.

Region $q_T \gtrsim Q$: Modifications in $\mathbf{W}$

Modification 1 in $\mathbf{W}$: regularization of small b_T

$$W(q_T, Q) = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\tilde{W}(b_T, Q)$$



Region $q_T \gtrsim Q$: Modifications in $\mathbf{W}$

Modification 1 in $\mathbf{W}$: regularization of small b_T region $b_T \lesssim 1/Q$.

$$W(q_T, Q) = \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T} \tilde{W}(b_T, Q)$$

$$\Rightarrow \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T} \tilde{W}(b_c(b_T), Q)$$

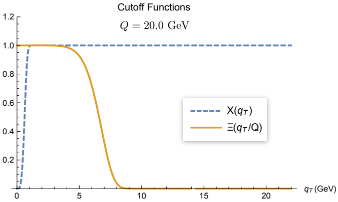
$$b_c(b_T) = \sqrt{b_T^2 + C_1^2/\mu_{max}^2}, \quad \frac{C_1}{\mu_{max}} \sim \frac{1}{Q}$$

Region $q_T \gtrsim Q$: Modifications in $\mathbf{W}$

Modification 2 in $\mathbf{W}$: q_T space cutoff.

$$W(q_T, Q) = \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T} \tilde{W}(b_T, Q)$$
$$\Rightarrow \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T} \tilde{W}(b_c(b_T), Q)$$

$$\Rightarrow W_{New}(q_T, Q) = \Xi(q_T/Q) \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i \boldsymbol{q}_T \cdot \boldsymbol{b}_T} \tilde{W}(b_c(b_T), Q)$$



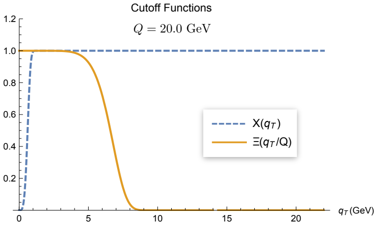
Region $q_T \lesssim m$: Modifications in $\mathbf{Y}$

$$\mathbf{T}_{\text{coll}} \left[\Gamma - \overline{\mathbf{T}_{\text{TMD}}}\Gamma\right] = \left[\Gamma - \overline{\mathbf{T}_{\text{TMD}}}\Gamma\right] \left[1 + O\left(\frac{m}{q_T}\right)^{b} + O\left(\frac{m}{q_T}\right)^{b'} + \ldots\right]$$

Terms with power $b' > b$, etc. dominate this region.

Modification in $\mathbf{Y}$: cutoff function $X(q_T/\lambda)$.

$$\mathbf{T}_{\text{coll}} \left[\Gamma - \overline{\mathbf{T}_{\text{TMD}}}\Gamma\right] \Rightarrow X(q_T/\lambda)\mathbf{T}_{\text{coll}} \left[\Gamma - \overline{\mathbf{T}_{\text{TMD}}}\Gamma\right]$$



Relation to q_T integrated cross section

Old formalism:

$$\int d^2\boldsymbol{q}_T W(q_T, Q) = \tilde{W}(b_T = 0, Q) = 0$$

Modified:

$$\int d^2\boldsymbol{q}_T W_{New}(q_T, Q) = \tilde{W}_{New}(b_c(b_T = 0) = O(1/Q), Q)$$

$$= H^{(0)} f(x, \mu_c) d(z, \mu_c) + O(\alpha_S(Q)),$$

$$\mu_c \sim O(Q)$$

Further issues

Accuracy in fixed-order calculation

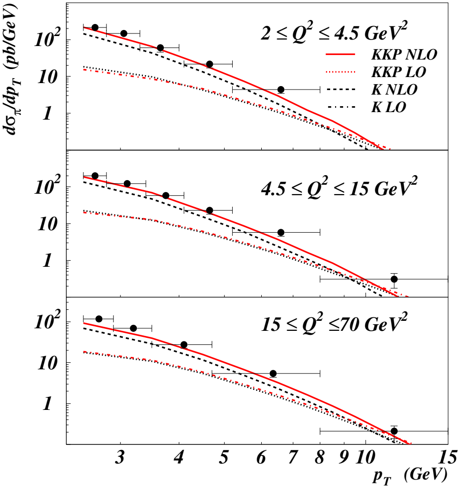
Cross section

$$\begin{aligned}\Gamma =&\overline{\mathbf{T}_{\mathrm{TMD}}}\Gamma + \mathbf{T}_{\mathrm{coll}}\left[\Gamma - \overline{\mathbf{T}_{\mathrm{TMD}}}\Gamma\right] + \mathrm{O}\left(\frac{\mathrm{m}}{\mathrm{Q}}\right)^{\mathrm{c}}\Gamma \\ =&\overline{\mathbf{T}_{\mathrm{TMD}}}\Gamma + (\mathbf{T}_{\mathrm{coll}})_{\mathrm{FO}}\left[\Gamma - \overline{\mathbf{T}_{\mathrm{TMD}}}\Gamma\right] \\ &+ O\left(\frac{m}{Q}\right)^{c}\Gamma + \text{higher order corrections}\end{aligned}$$

Fix-order calculation can be a bad approximation for small Q, which introduces more error than expected.

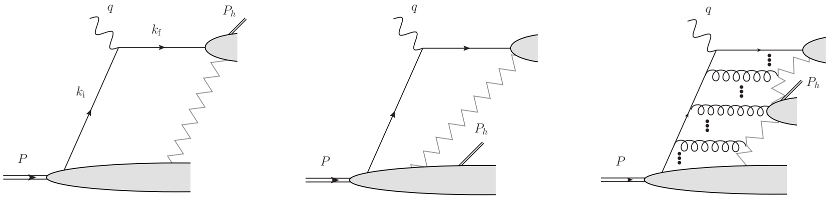
Accuracy in fixed-order calculation

Perturbative cross section in SIDIS. A. Daleo, *etal* (2004)



Current and target fragmentation

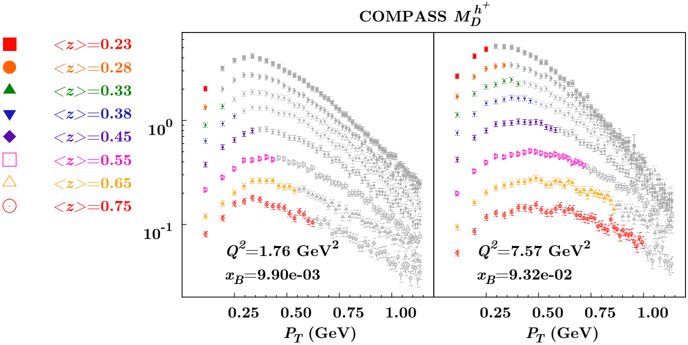
Theory designed for fragmentation of the struck quark (current fragmentation)



M. Boglion, J.Collins, L.Gamberg, J.O.Gonzalez, T.C.Rogers, N.Sato, arXiv:1611.10329

Are all the data in the current region?

- Need to isolate a current region to apply TMD factorization.



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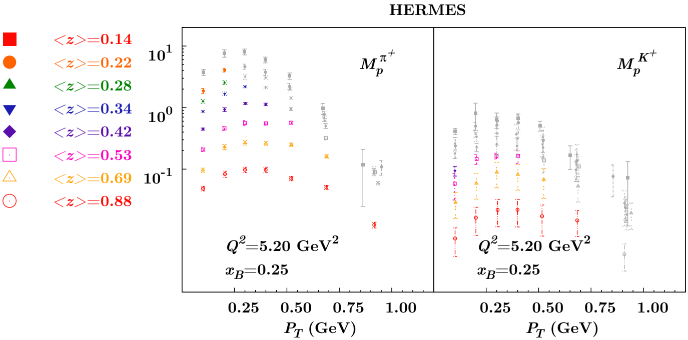
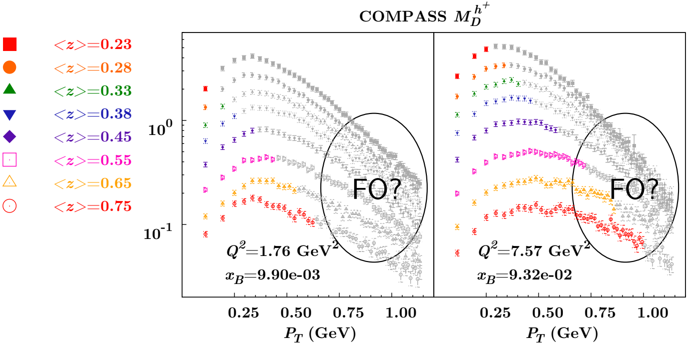


M. Boglion, J.Collins, L.Gamberg, J.O.Gonzalez, T.C.Rogers, N.Sato, arXiv:1611.10329

What to do with data not in the current region?

- No problem for large enough q_T — collinear factorization



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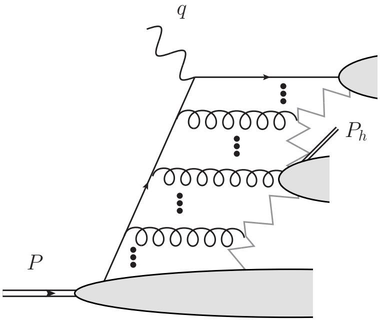
Current and target fragmentation

What to do with data not in the current region?

- Some intermediate q_T may be in neither current nor hard region.

- Need better factorization formalism to handle this region. Hints may come from hadronization models used by event generators. (see talk by Markus)



Summary

Improved TMD formalism

- $m \lesssim q_T \lesssim Q \Rightarrow$ all q_T
- Matching improved
- Consistent with Collinear cross section.

Outlook

- Higher order perturbative calculation
- Phenomenology: Isolate current region in data
- Theory for target/soft fragmentation?