Jefferson Lab
Thomas Jefferson National Accelerator Facility

Pion electromagnetic form factor at high Q^2 from lattice QCD

Bipasha Chakraborty

[With Raul Briceno, Robert Edwards, Adithia Kusno, Kostas Orginos, David Richards, Frank Winter]

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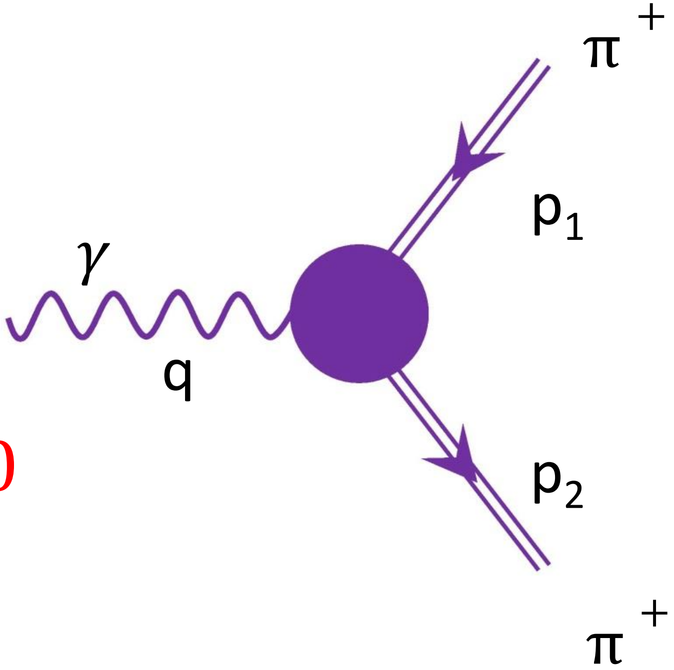
Definition

Simplest hadron

Space like "q":

$$q^2 = (p_2 - p_1)^2 \leq 0$$

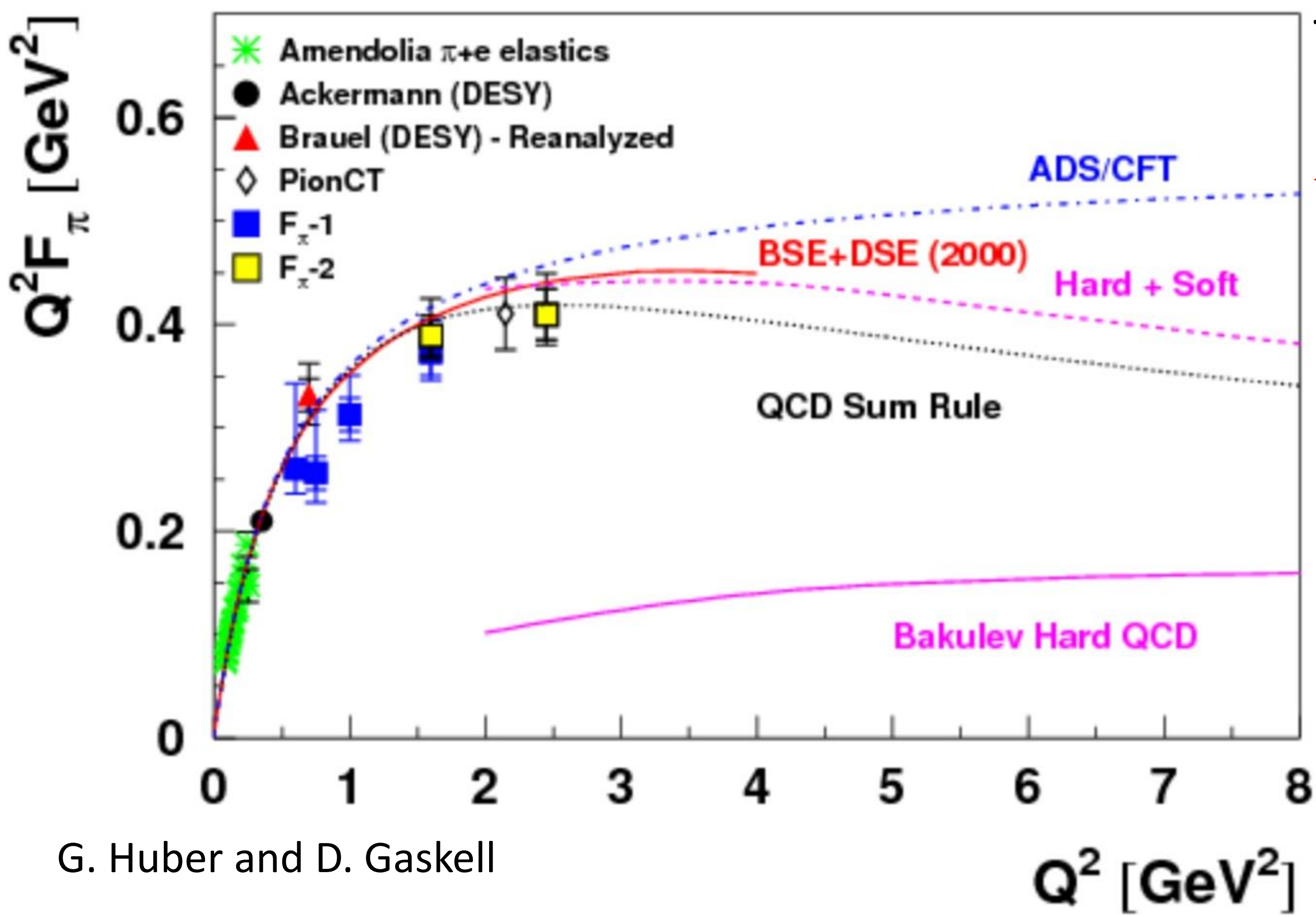
$$Q^2 = -q^2$$



$$\langle \pi^+(\vec{p}\,')\,|\,j^\mu\,|\,\pi^+(\vec{p})\rangle = (p+p')^\mu\, F_\pi(Q^2)$$

(in units of 'e')

Interplay between hard and soft scales



G. Huber and D. Gaskell

Hard tail ($Q^2 \rightarrow \infty$) from pQCD:

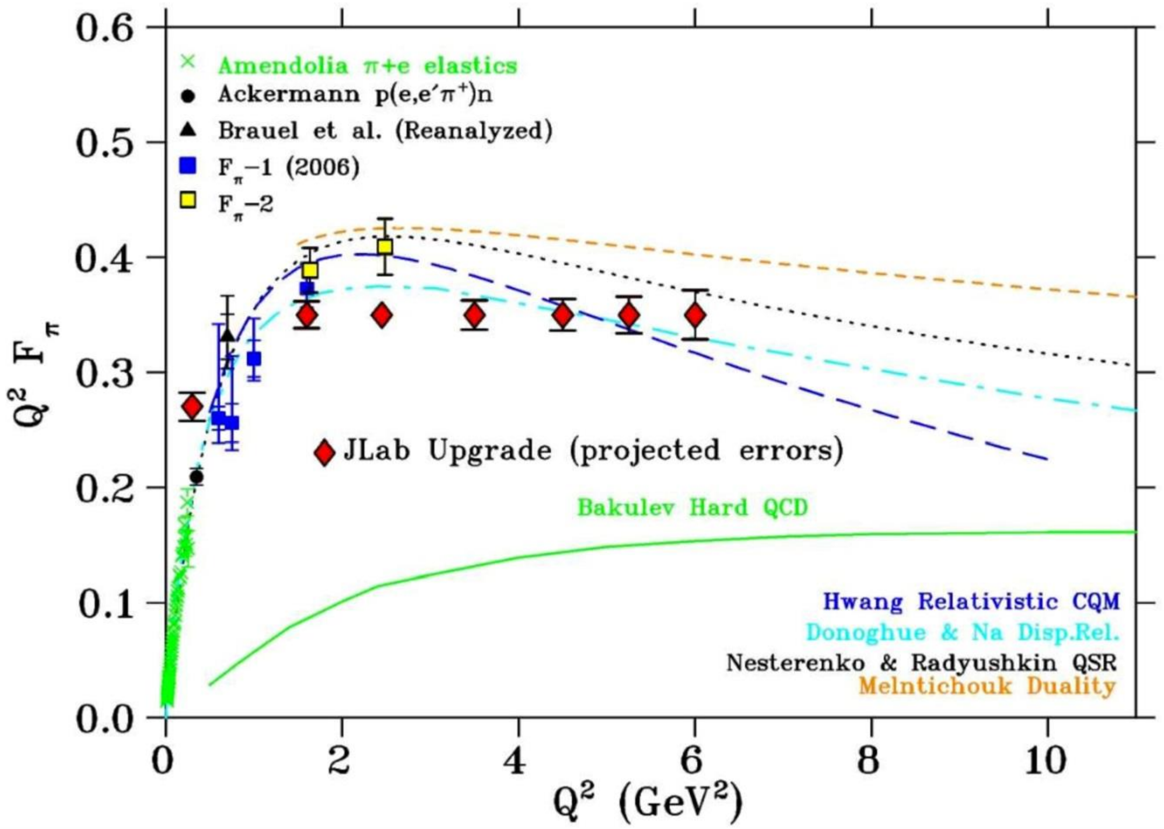
$$F_\pi(Q^2) \rightarrow \frac{16\pi\alpha_s(Q^2) f_\pi^{\,2}}{Q^2}$$

G. P. Lepage, S.J.Brodsky, Phys. Lett. 87B(1979)359

Soft part ($Q^2 < 1$ GeV^2): vector meson dominance with $F_\pi(0) = 1$, data fits well

Need better understanding of the transition to the asymptotic region

JLAB 12 GeV upgrade



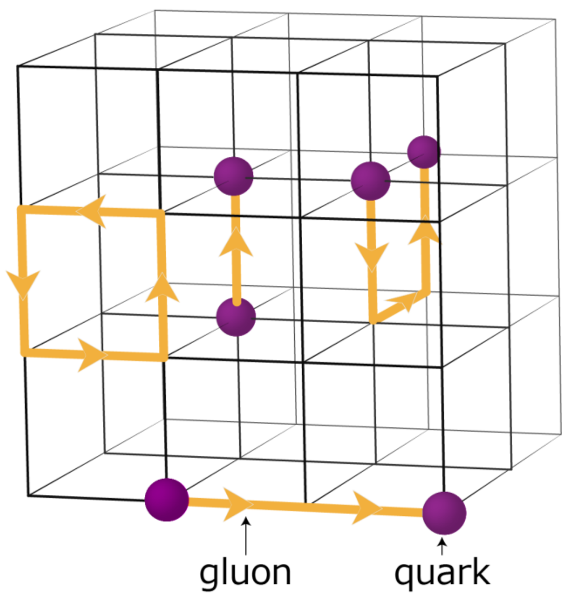
G. Huber and E. Gaskell

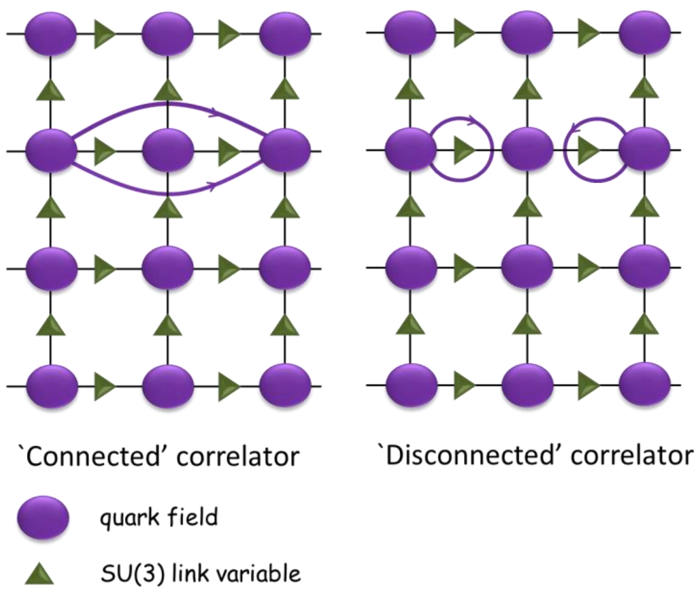
F_π measurements at $Q^2 \sim 6$ GeV^2: E12-06-101 at JLAB Hall C

Can we get some insight from first principles lattice QCD calculations to the question - where does the transition to pQCD happen?

Lattice recipe for meson correlators

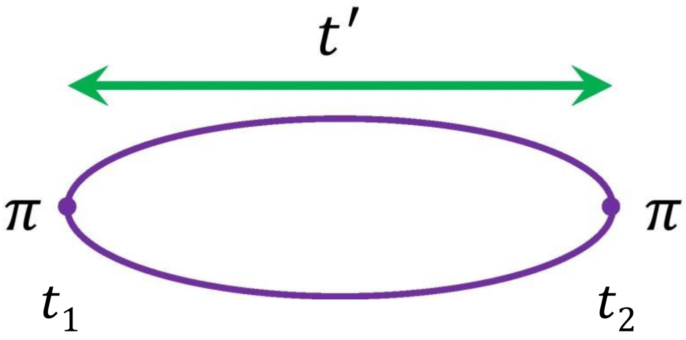
- Expectation values of observables : $\int DUD\psi D\bar{\psi} exp(-\int L_{QCD} d^4x)$
- 4-D space-time lattice
- Gauge configurations : gluons + sea quarks





- Discretise : $L_q \equiv \bar{\psi}(\gamma_\mu D^\mu + m)\psi \rightarrow \bar{\psi}(\gamma.\Delta + ma)\psi$
- Inversion of Dirac matrix : propagator
- 2-point, 3-point correlation functions : extract meson properties
- Corrections for lattice artifacts

Two-point correlator construction



$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

- Basis of operators

$$\mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \cdots \overleftrightarrow{D} \psi$$

- Optimized operator for state $|n>$

$$\Omega_{\mathfrak{n}}^\dagger = \sum_i w_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$$

in a variational sense by solving generalized eigenvalue problem-

$$C(t)\, v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t)\, C(t_0)\, v^{(\mathfrak{n})}$$

- Diagonalize the correlation matrix – eigenvalues

$$\lambda_n(t) = \exp[-En(t - t_0)]$$

Two-point correlator construction

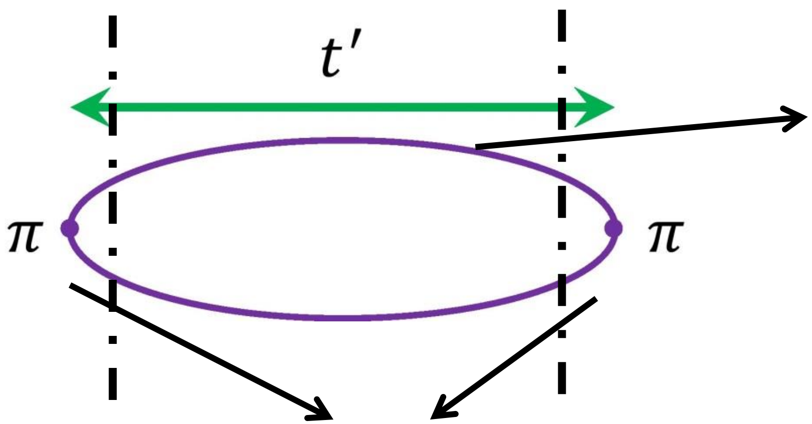
Correlator Construction: smearing of quark fields - 'distillation' with

$$\Box_{\vec{x}\vec{y}}(t) = \sum_{n=1}^{N_D} \xi_{\vec{x}}^{(n)}(t)\, \xi_{\vec{y}}^{(n)\dagger}(t)$$

→ Low lying hadron states

Meson creation operator :

$$\mathcal{O}^\dagger(\vec{p}) = \bar{\psi}_{\vec{x}} \Box_{\vec{x}\vec{y}}\, e^{-i\vec{p}\cdot\vec{y}}\, \mathbf{\Gamma}_{\vec{y}\vec{z}} \Box_{\vec{z}\vec{w}} \psi_{\vec{w}}$$



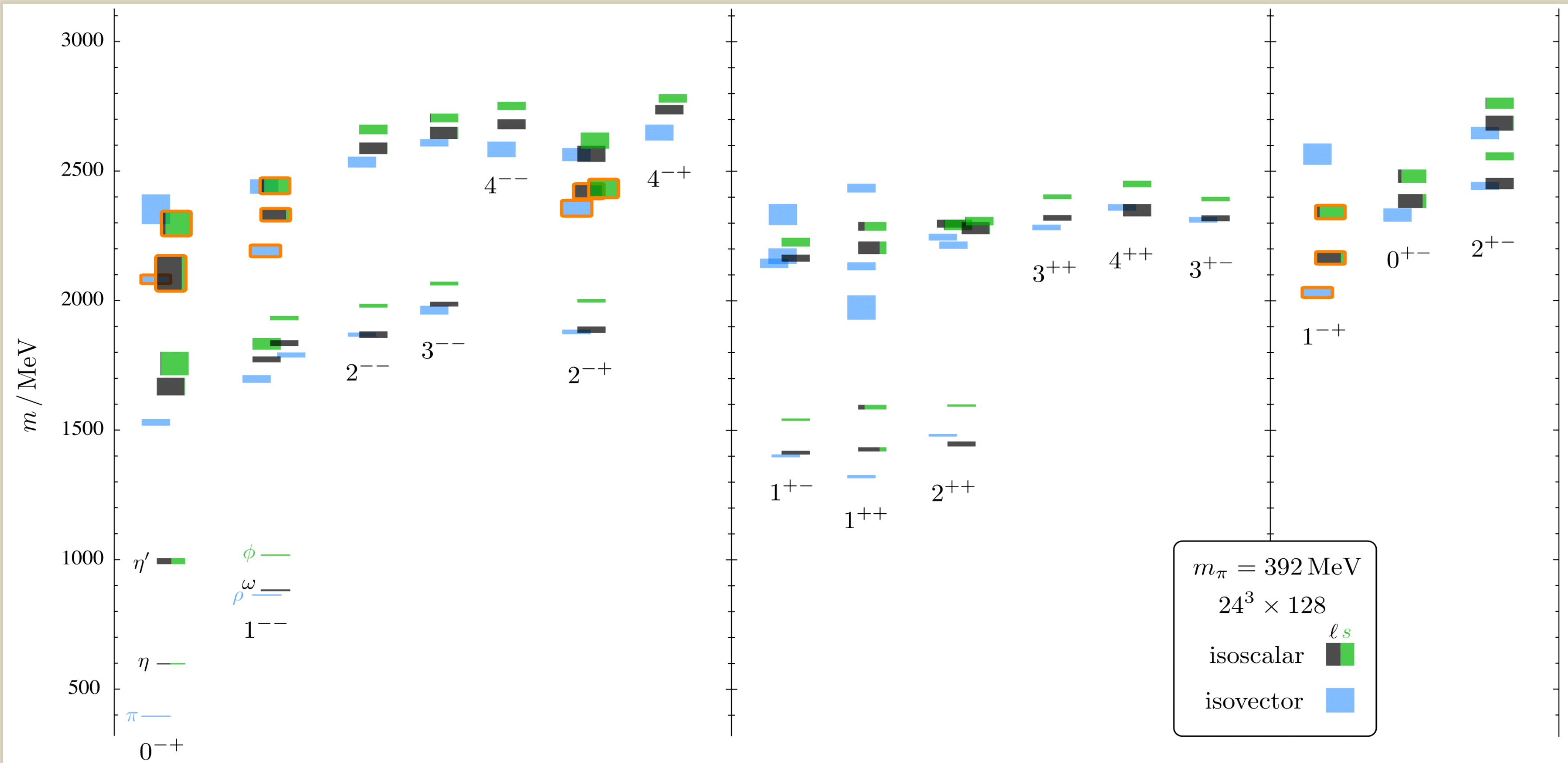
Parambulators by inverting the Dirac matrix

+

Operator construction with momentum projection

Meson Spectrum

Tools well established for spectroscopy

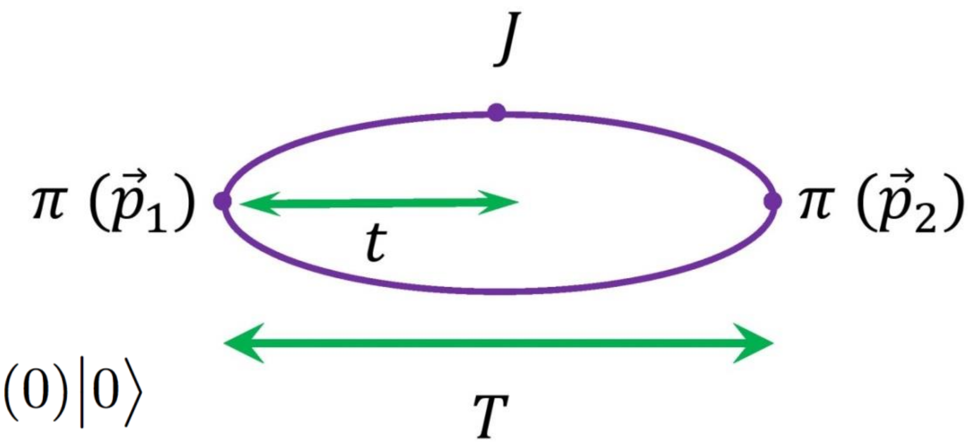


Hadron Spectrum Collaboration

Jozef J. Dudek *et. al*. Phys.Rev. D88 (2013)

Form factor calculation

Need three-point correlator



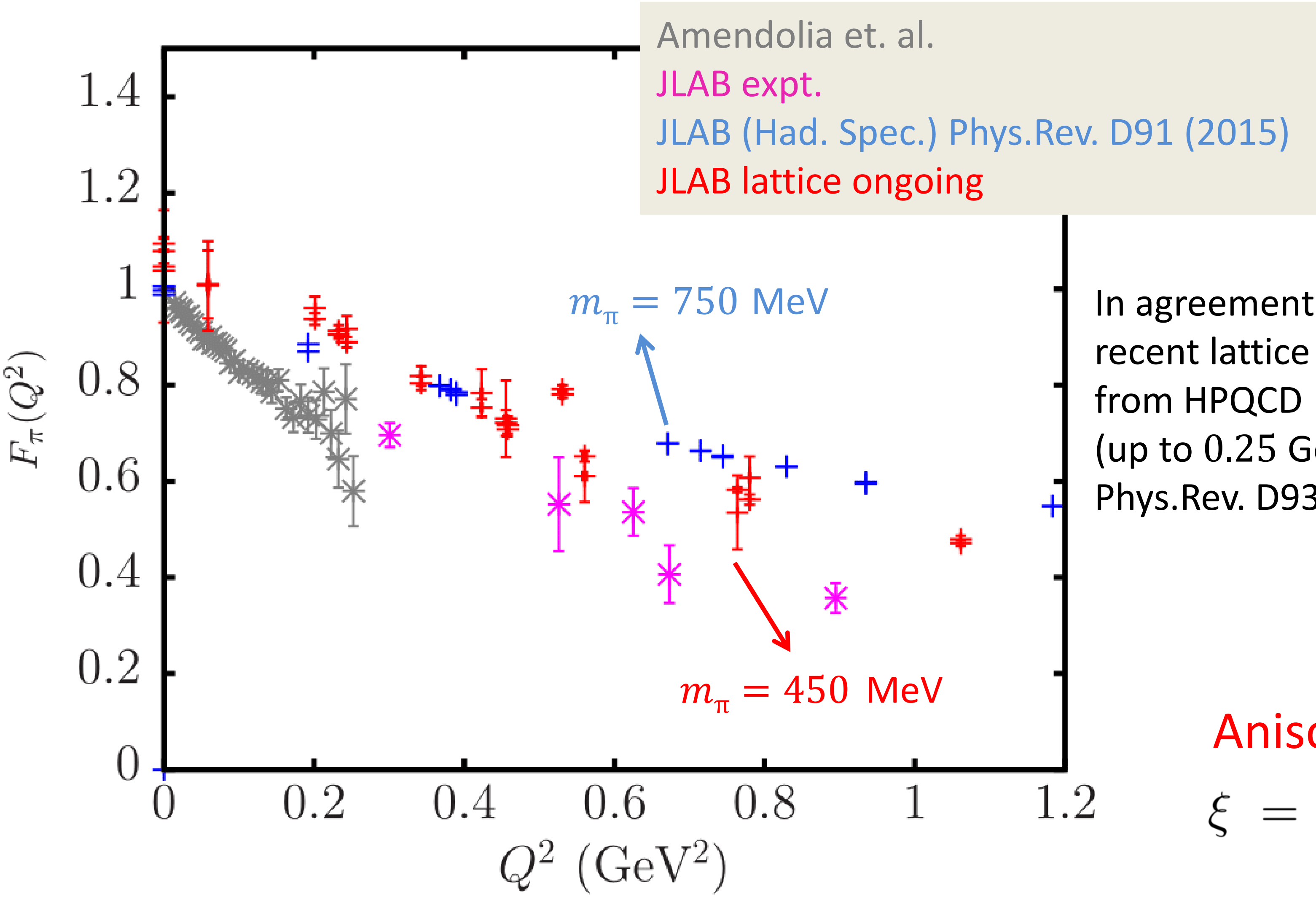
$$C_{\mathrm{f}\mu\mathrm{i}}(\Delta t, t) = \langle 0 | \mathcal{O}_{\mathrm{f}}(\Delta t)\, j_\mu(t)\, \mathcal{O}_{\mathrm{i}}^\dagger(0) | 0 \rangle$$

$$Z_V < \pi^+(p_2) | J^\mu{}_\pi(0) | \pi^+(p_1) > = e(p_1 + p_2)^\mu F_\pi(q^2)$$

Clover discretised fermion action

Z_V calculated using $F_\pi(q^2 = 0) = 1$

Pion electromagnetic form factor: up to $Q^2 = 1\ \text{GeV}^2$



In agreement with recent lattice result from HPQCD (up to 0.25 GeV^2) Phys.Rev. D93 (2016)

Anisotropy

$$\xi = 3.44$$

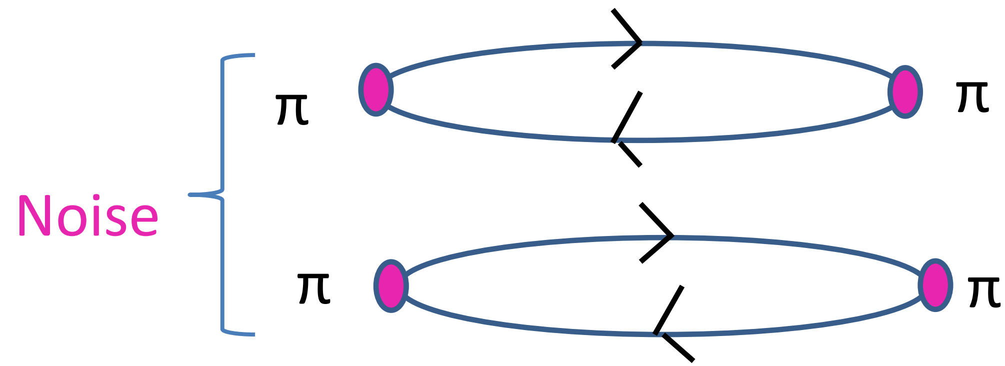
$$(L/a_s)^3 \times (T/a_t) = 20^3 \times 128$$

$$a_s = 0.12\ \text{fm},\ \frac{a_s}{a_t} = 3.44$$

Towards higher Q^2

More difficult on lattice for higher momenta

Signal-to-noise ratio:



2-point correlators :

$$\exp[-(E_\pi(p) - 2m_\pi)t]$$

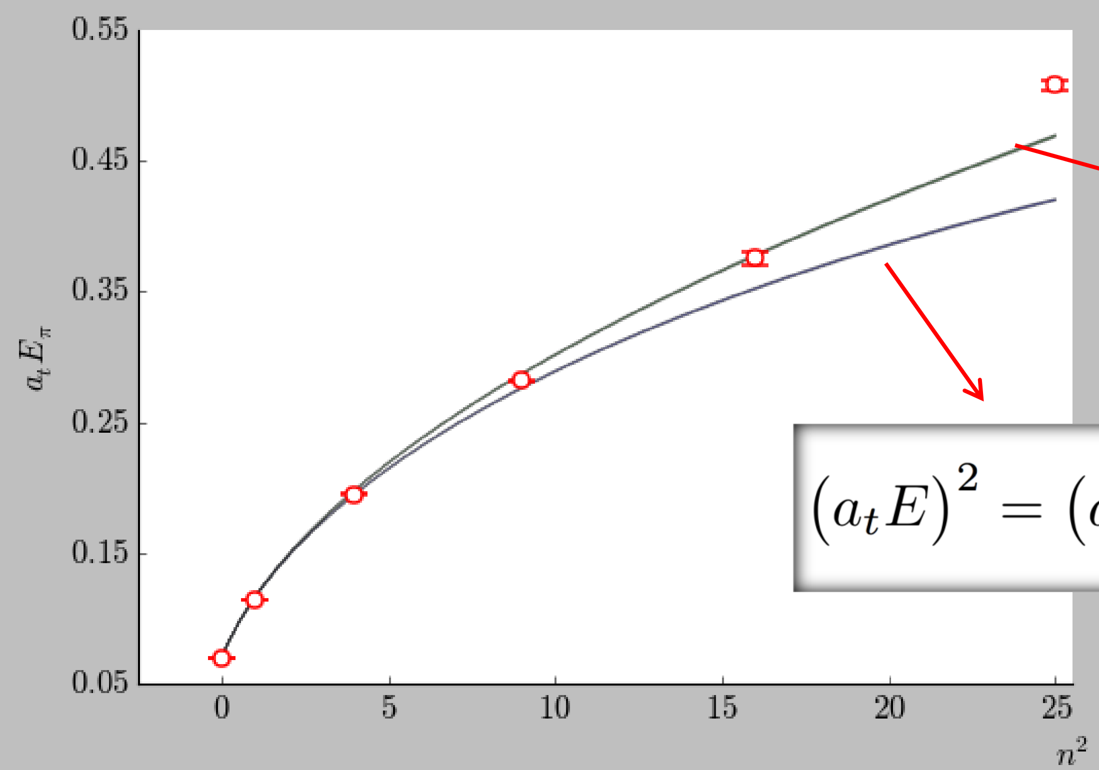
3-point correlators :

$$\exp[-(E_\pi(pi) + E_\pi(pf) - 2m_\pi)t/2]$$

in the middle of the plateau

Minimize energies
for a given Q^2
to get better signal

Towards higher Q^2



$\vec{n}_{\vec{p}} = [0,0,0],\ [0,0,1],\ [0,1,1],\ [1,1,1],\ [0,0,2]$ ...

Dispersion relation:

$$E^2 = m^2 + p^2$$

$$(a_t E)^2 = (a_t m)^2 + \left(\frac{2\pi}{\xi(L/a_s)}\right)^2 |\vec{n}_{\vec{p}}|^2$$

Achieve maximum Q^2 by using Breit frame : $\overrightarrow{P_f} = -\overrightarrow{P_i}$

Outlook

Immediate goals:

- Pion form factor at $Q^2 \geq 6$ GeV^2
- Extend to more ensembles with lighter pion masses , multiple volumes, multiple lattice spacings
- Take care of lattice artifacts

Long term goals:

- Hadron structure program – distribution amplitude, PFDs, Quasi PDFs
- Extend to nulceons & more – charges, moments, TMDs, GPDs ….