Twist Three Distribution Functions and the Role of Intrinsic Partonic Transverse Momentum in Orbital Angular Momentum in QCD

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People Involved

- Simonetta Liuti, University of Virginia
- Michael Engelhardt, New Mexico State University
- Gary Goldstein, Tufts University
- Aurore Courtoy, CINVESTAV Mexico
- Osvaldo Gonzalez, Old Dominion University and Jlab
- Brandon Kriesten, University of Virginia

Rajan, Courtoy, Engelhardt and Liuti PRD 94 (2016)
Coutroy et al, Phys.Lett. B 731(2014)

Outline

- Spin Crisis !
- Orbital Angular Momentum
  - GTMD definition
  - GPD definition Ji
- What's the connection ? Lorentz Invariance Relations
- Equations of Motion
- Model calculations
- Final state interactions
- Conclusions

Proton Spin Crisis

 $= g_1^P(x)$ Quark Spin Contribution

$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

Measured by EMC experiment in 1980s to be only 33% of total !!

Gluon Spin Contribution also small.

Proton Spin Crisis

 $= g_1^P(x)$ Quark Spin Contribution

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Measured by EMC experiment in 1980s to be only 33% of total !!

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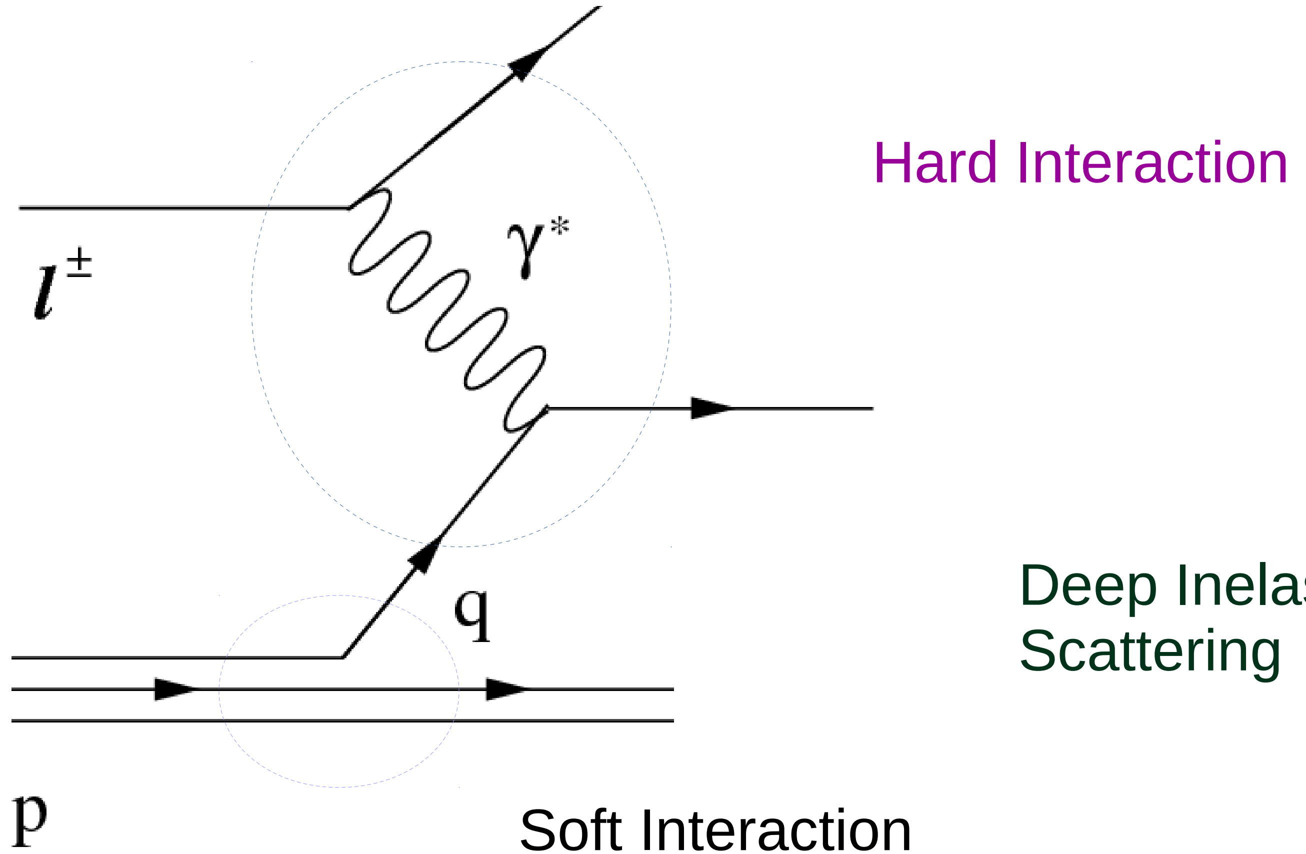
What are other sources ?
Partonic Orbital Angular Momentum

Hard and Soft Parts



Deep Inelastic Scattering

$$\int \frac{dz^-}{2\pi} e^{ik^+ z^-} \langle p, S \mid \bar{\psi}(-z/2)\gamma^+ \psi(z/2) \mid p, S\rangle_{z^+ = z_T = 0} = f_1(x)$$

$$a^{\pm} = \frac{a^0 \pm a^3}{\sqrt{2}}$$

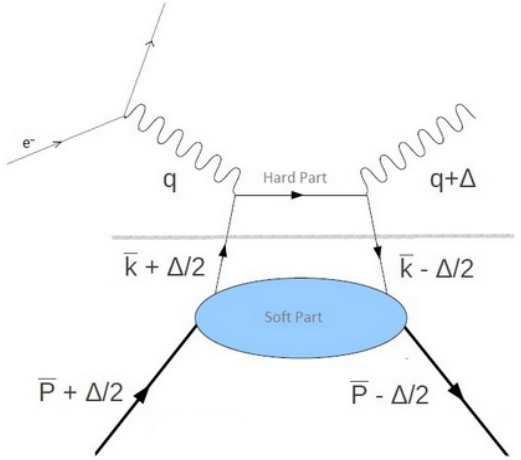
GPDs and GTMDs

- Generalized Parton Distributions : Off Forward PDFs

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0} = \bar{U}(P', \Lambda')(\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E(x,\xi,t))U(P,\Lambda)$$

$$\xi = \frac{\Delta^+}{P^+} \qquad t = \Delta^2 \qquad \Delta = P' - P$$

Xiangdong Ji, PRL 78.610,1997



Accessed in exclusive processes such as Deeply Virtual Compton Scattering (DVCS)

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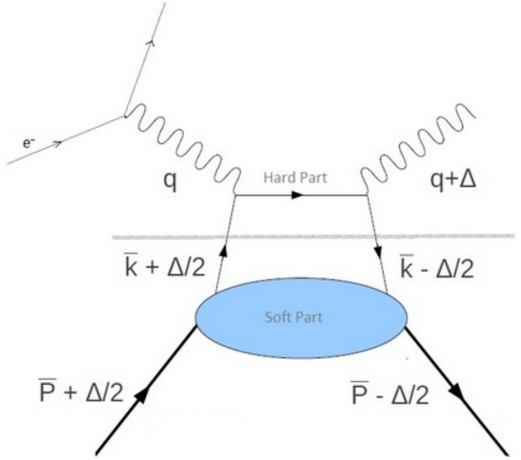


Accessed in exclusive processes such as Deeply Virtual Compton Scattering (DVCS)

- Generalised Transverse Momentum Distributions : Include intrinsic transverse momentum

$$W^{\gamma^+}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{u}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k^i_T}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta^i_T}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k^i_T\Delta^j_T}{M^2}F_{14}]u(p,\Lambda)$$

Orbital Angular Momentum

Functions of $x, k_T^2, k_T.\Delta_T, \xi, t$

Meissner Metz and Schlegel, JHEP 0908 (2009)

GPD based definition of Angular Momentum

$$J_q = \frac{1}{2}\int_{-1}^{1} dx x (H_q(x,0,0) + E_q(x,0,0))$$

Xiangdong Ji, PRL 78.610,1997

To access OAM, we take the difference between total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

Direct description of OAM

$$\int dx\, x G_2 = \int dx\, x (H + E) - \int dx \tilde{H}$$

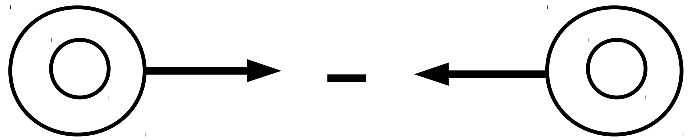
$$G_2 \equiv \tilde{E}_{2T} + H + E$$

Kiptily and Polyakov, Eur Phys J C 37 (2004)
Hatta and Yoshida, JHEP (1210), 2012

- The moment in x of the GPD G2 shown to be OAM
- Does not give us the distribution function for OAM

GTMDs that describe OAM

- How does F_{14} connect to OAM ?



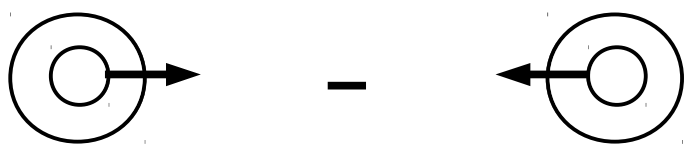
Unpolarized quark in a longitudinally polarized proton

$$\mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{ib.\Delta_T} \left[W^{\gamma^+}_{++} - W^{\gamma^+}_{--} \right]$$

$$L = \int dx \int d^2k_T \int d^2\mathbf{b} (\mathbf{b} \times \mathbf{k}_T) \mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce et al PRD84, (2011)

- Another GTMD relevant to OAM



G11 describes a longitudinally polarized quark in an unpolarized proton. Measures spin orbit correlation.

The Two Definitions

- Weighted average of $b_T \times k_T$

$$L_z = -\int dx \int d^2 k_T \frac{k_T^2}{M^2} F_{14}$$

- Difference of total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x(H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

The Two Definitions

- Weighted average of $b_T \times k_T$

GTMD

$$L_z = -\int dx \int d^2 k_T \frac{k_T^2}{M^2} F_{14} \rightarrow F_{14}^{(1)}$$

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

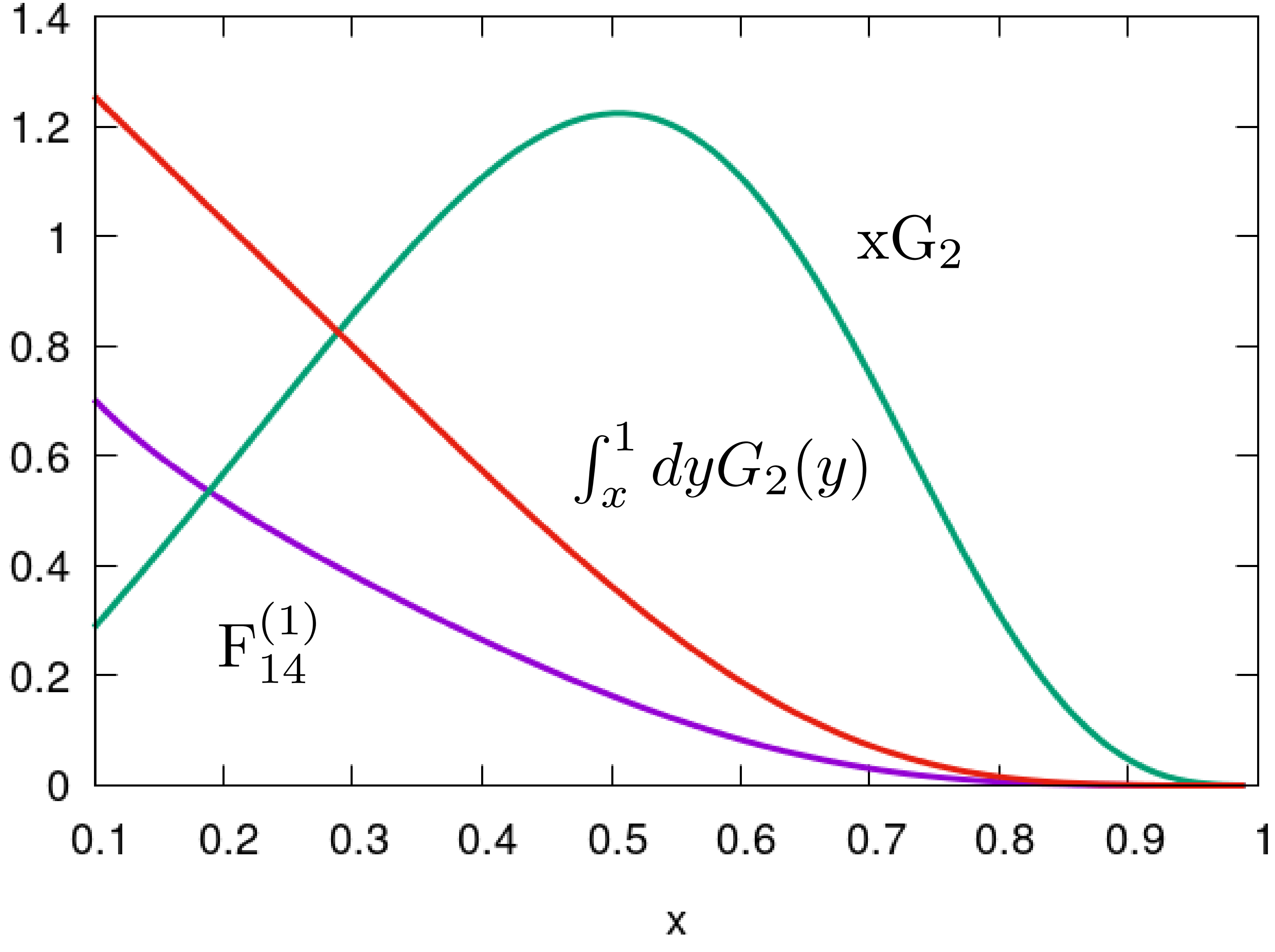
Is there a connection ?

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

- This is a form of Lorentz Invariant Relation (LIR)
- Unlike the previously known result, this is a distribution of OAM in x.
- Derived for a straight gauge link.

OAM Distributions as a function of x



$$xG_2 \neq F_{14}^{(1)}$$

Quark Quark Correlator Function

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+z^- - k_T.z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- Parametrization of the quark quark correlator at different levels
- LIRs occur because the number of GPCFs is less than the number of GTMDs.

$$\mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} = \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M}A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M}A_6^F + i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F)$$

Explicit kT coefficient

$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

$$F^{[\gamma^i]}_{\Lambda,\Lambda'} = \frac{1}{2(P^+)^2}\bar{U}\left[i\sigma^{+i}H_{2T} + \frac{\gamma^+\Delta_T^i}{2M}E_{2T} + \frac{P^+\Delta_T^i}{M^2}\tilde{H}_{2T} - \frac{P^+\gamma^i}{M}\tilde{E}_{2T}\right]U$$

Generalized Lorentz Invariance Relations

- The As are a function of the following scalar variables :

$$\sigma \equiv \frac{2k.P}{M^2}, \qquad \tau \equiv \frac{k^2}{M^2}, \qquad \sigma' \equiv \frac{k.\Delta}{\Delta^2} = \frac{k_T.\Delta_T}{\Delta_T^2}$$

For $\Delta^+ = 0$

$$\int dk^- A(k^2, k.P, k.\Delta \ldots) \quad \to \quad \frac{M^2}{2P^+} \int d\sigma A$$

$$\to \quad \frac{M^2}{2P^+} \int d\sigma' d\sigma d\tau \delta \left(\frac{k_T^2}{M^2} - x\sigma + \tau + \frac{x^2 P^2}{M^2} \right) \delta \left(\sigma' - \frac{k_T.\Delta_T}{\Delta_T^2} \right) A(\sigma, \tau, \sigma')$$

Generalized Lorentz Invariance Relations

$$F_{14}^{(1)} = \int d\sigma d\sigma' d\tau \frac{M^3}{2} J \left[A_8^F + x A_9^F\right] \qquad J = \sqrt{x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}}$$

$$\tilde{E}_{2T} = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \left[\left(x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}\right) A_9^F - \sigma' A_5^F - A_6^F\right]$$

$$H + E = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \sigma' A_5^F + A_6^F + \left(\frac{\sigma}{2} - \frac{x P^2}{M^2}\right)\left(A_8^F + x A_9^F\right)$$

$$-\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y)\right)$$

Distribution of OAM in x !

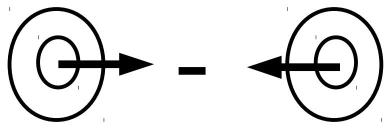
k_T^2 moment of a twist two function

Twist three function

Generalized Lorentz Invariance Relations

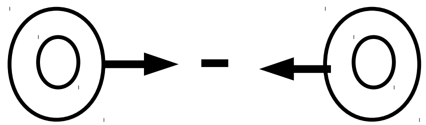
Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$



Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$

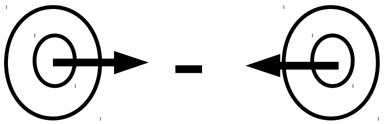


The GTMDs are complex in general.

$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

Generalized Lorentz Invariance Relations



Axial Vector

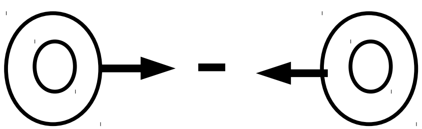
$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}'_{2T} + E'_{2T}\right) - \tilde{H}$$

$$\frac{dG_{12}^{e(1)}}{dx} = H'_{2T} - \frac{\Delta_T^2}{4M^2}E'_{2T} - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

Vector

$$\frac{dF_{12}^{o(1)}}{dx} = 0$$

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



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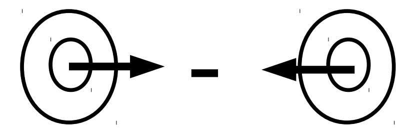

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$$\frac{dg_{1T}^{(1)}}{dx} = g_T + g_1$$

$\mathrm{f}_{1T}^{\perp}$

Vector

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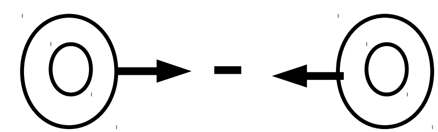
$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



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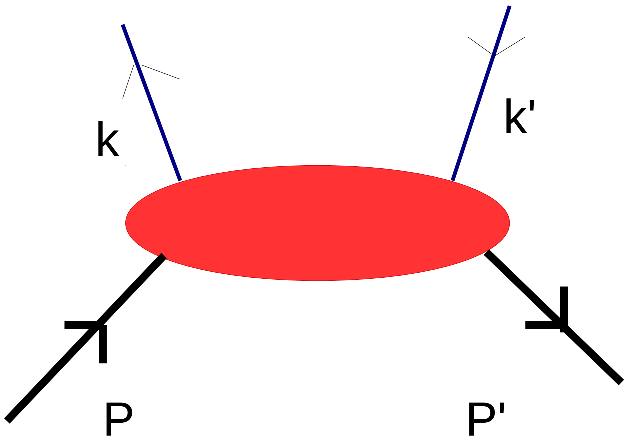
$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

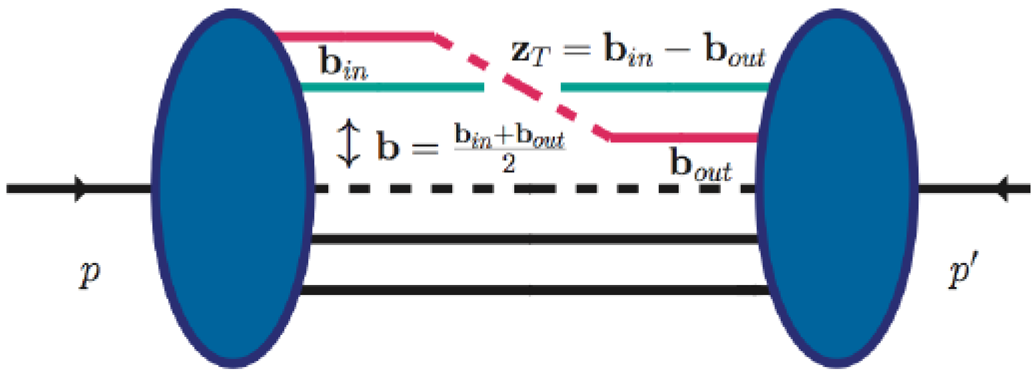
Intrinsic Momentum vs Momentum Transfer Δ

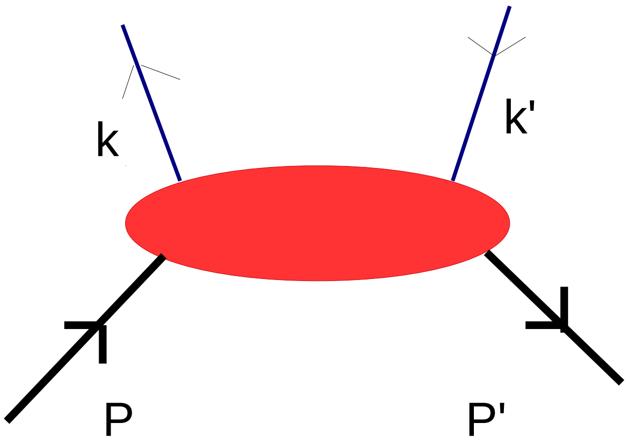


Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

Intrinsic Momentum vs Momentum Transfer Δ





$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

Equations of Motion Relations

Starting with the equation of motion and its conjugate we arrive at the following

$$-\frac{\Delta^+}{2} W_{\Lambda\Lambda'}^{[\gamma^i\gamma^5]} + ik^+\epsilon^{ij} W_{\Lambda\Lambda'}^{[\gamma^j]} = -\frac{\Delta^i}{2} W_{\Lambda\Lambda'}^{[\gamma^+\gamma^5]} + i\epsilon^{ij} k_T^j W_{\Lambda\Lambda'}^{[\gamma^+]} - \mathcal{M}^i$$

$$-k^+ W_{\Lambda\Lambda'}^{[\gamma^i\gamma^5]} + \frac{i\Delta^+}{2}\epsilon^{ij} W_{\Lambda\Lambda'}^{[\gamma^j]} + k^i W_{\Lambda\Lambda'}^{[\gamma^+\gamma^5]} = i\epsilon^{ij}\frac{\Delta^j}{2} W_{\Lambda\Lambda'}^{[\gamma^+]} - m W_{\Lambda\Lambda'}^{[i\sigma^{i+}\gamma^5]} - i\overline{\mathcal{M}}^i_{\Lambda\Lambda'}$$

Equations of Motion Relations

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$$-\frac{\Delta^+}{2}W^{[\gamma^i\gamma^5]}_{\Lambda\Lambda'} + ik^+\epsilon^{ij}W^{[\gamma^j]}_{\Lambda\Lambda'} = -\frac{\Delta^i}{2}W^{[\gamma^+\gamma^5]}_{\Lambda\Lambda'} + i\epsilon^{ij}k^j_T W^{[\gamma^+]}_{\Lambda\Lambda'} - \mathcal{M}^i$$

$$-k^+W^{[\gamma^i\gamma^5]}_{\Lambda\Lambda'} + \frac{i\Delta^+}{2}\epsilon^{ij}W^{[\gamma^j]}_{\Lambda\Lambda'} + k^iW^{[\gamma^+\gamma^5]}_{\Lambda\Lambda'} = i\epsilon^{ij}\frac{\Delta^j}{2}W^{[\gamma^+]}_{\Lambda\Lambda'} - mW^{[i\sigma^{i+}\gamma^5]}_{\Lambda\Lambda'} - i\overline{\mathcal{M}}^i_{\Lambda\Lambda'}$$

- Each W is a correlator that can be parameterized using GTMDs / GPDs.

$$W^{\Gamma}_{\Lambda\Lambda'} = \int \frac{dz^-\, d^2\mathbf{z}_T}{(2\pi)^3} e^{ixP^+z^- - i\bar{\mathbf{k}}_T\cdot\mathbf{z}_T} \langle p', \Lambda' \mid \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle\Big|_{z^+=0}$$

Equations of Motion Relations

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$$-\frac{\Delta^+}{2} W_{\Lambda\Lambda'}^{[\gamma^i\gamma^5]} + ik^+\epsilon^{ij} W_{\Lambda\Lambda'}^{[\gamma^j]} = -\frac{\Delta^i}{2} W_{\Lambda\Lambda'}^{[\gamma^+\gamma^5]} + i\epsilon^{ij} k_T^j W_{\Lambda\Lambda'}^{[\gamma^+]} - \mathcal{M}^i$$

$$-k^+ W_{\Lambda\Lambda'}^{[\gamma^i\gamma^5]} + \frac{i\Delta^+}{2}\epsilon^{ij} W_{\Lambda\Lambda'}^{[\gamma^j]} + k^i W_{\Lambda\Lambda'}^{[\gamma^+\gamma^5]} = i\epsilon^{ij}\frac{\Delta^j}{2} W_{\Lambda\Lambda'}^{[\gamma^+]} - mW_{\Lambda\Lambda'}^{[i\sigma^{i+}\gamma^5]} - i\overline{\mathcal{M}}_{\Lambda\Lambda'}^i$$

- Each W is a correlator that can be parameterized using GTMDs / GPDs.

$$W_{\Lambda\Lambda'}^{\Gamma} = \int \frac{dz^-\, d^2\mathbf{z}_T}{(2\pi)^3} e^{ixP^+z^- - i\bar{\mathbf{k}}_T\cdot\mathbf{z}_T} \langle p', \Lambda' \mid \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\psi\left(\frac{z}{2}\right) \mid p, \Lambda\rangle\Big|_{z^+=0}$$

- Different distributions enter for different incoming / outgoing proton helicity states.

Equations of Motion Relations

How do we obtain these ?

$$
\begin{aligned}
(i\not{D} - m)\psi(z_{out}) &= (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

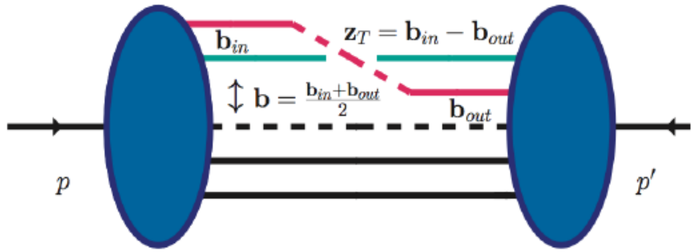
$$
\begin{aligned}
i\sigma^{i+}\gamma_5(i\not{D}-m)\psi(z_{out}) &= i\sigma^{i+}\gamma_5(i\not{\partial}+g\not{A}-m)\psi(z_{out})=0,\\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}}+m)i\sigma^{i+}\gamma_5 &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}}-g\not{A}+m)i\sigma^{i+}\gamma_5=0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

$$i\sigma^{i+}\gamma_5(i\not{D} - m)\psi(z_{out}) = i\sigma^{i+}\gamma_5(i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$
$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m)i\sigma^{i+}\gamma_5 = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m)i\sigma^{i+}\gamma_5 = 0$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2b_T e^{-ib\cdot\Delta} \int dz^- d^2z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{\not{D}} + m)i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5(i\overrightarrow{\not{D}} - m) \right] \psi | p, \Lambda \rangle = 0$$

EoM relations for Orbital Angular Momentum

$$x\tilde{E}_{2T} = -\tilde{H} + 2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T (\mathcal{M}^i_{++} - \mathcal{M}^i_{--})$$

$$x\left(E'_{2T} + 2\tilde{H}'_{2T}\right) = -H + \frac{m}{M}(E_T + 2\tilde{H}_T) - 2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} G_{11} - \mathcal{M}_{G_{11}}$$

$\rightarrow G_{11}^{(1)}$

$$\mathcal{M}_{G_{11}} = \frac{2i\epsilon^{im}\Delta^m}{\Delta_T^2}(\overline{\mathcal{M}}^i_{++} + \overline{\mathcal{M}}^i_{--})$$

Valid in the forward as well as off forward limit.

Wandzura Wilczek Relations

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

Twist three vector GPD

Twist two

Axial vector GPD contributes to a vector GPD

Genuine Tw 3

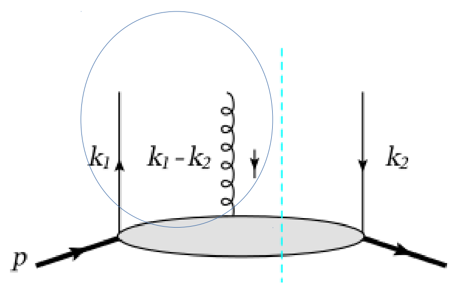
Wandzura Wilczek Relations

$$2\tilde{H}'_{2T} + E'_{2T} = -\int_x^1 \frac{dy}{y}\tilde{H} + \left[\frac{H}{x} - \int_x^1 \frac{dy}{y^2} H\right]$$

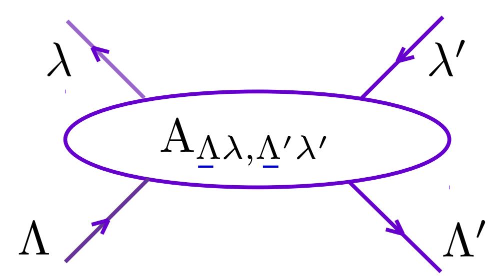
$$+ \quad \frac{m}{M}\left[\frac{1}{x}(2\tilde{H}_T + E_T) - \int_x^1 \frac{dy}{y^2}(2\tilde{H}_T + E_T)\right] + \frac{\mathcal{M}_{G_{11}}}{x} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{G_{11}}$$

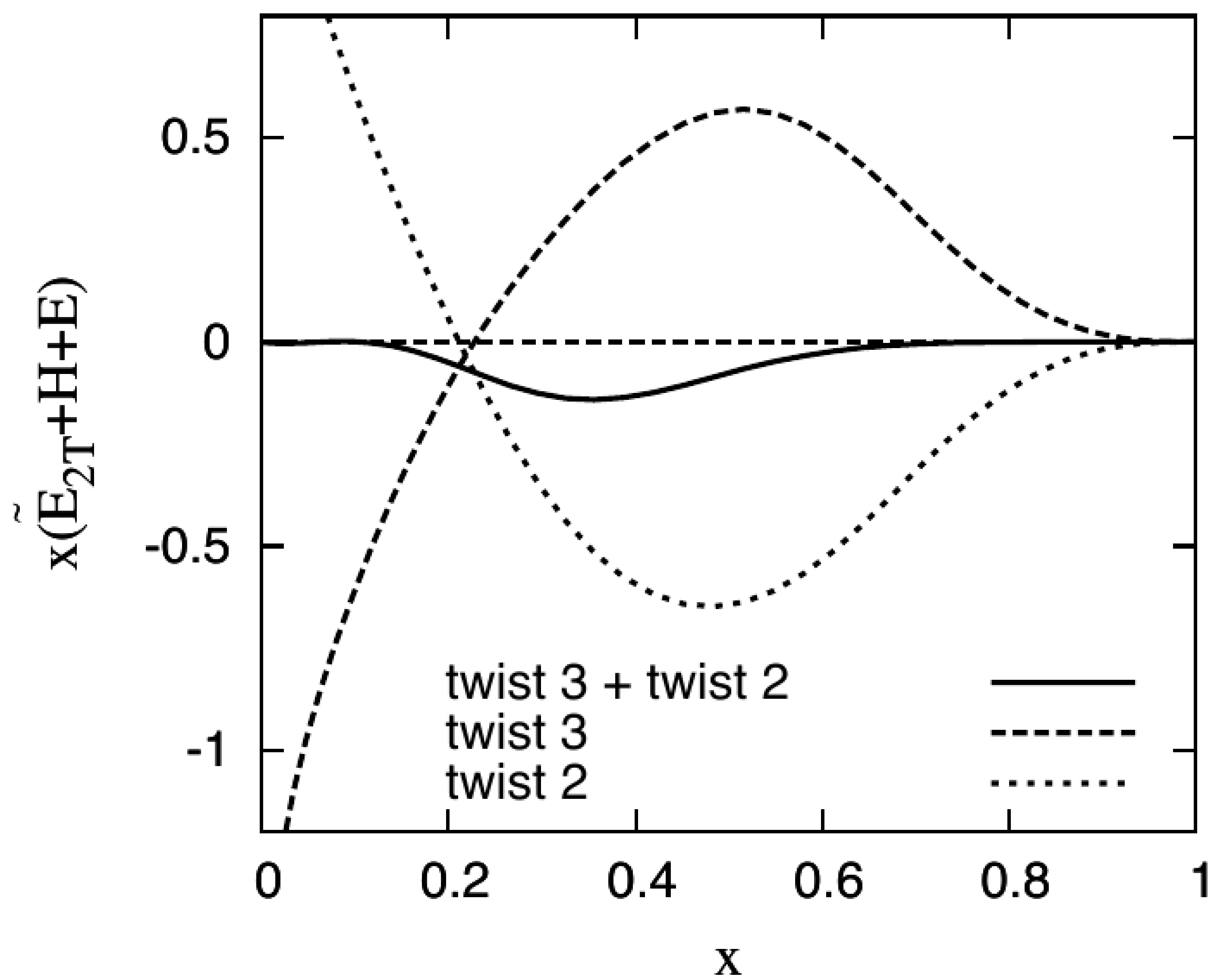
Model Calculations

- Diquark model at twist three
- Use projection into good and bad components to form helicity amplitudes
- Bad component is a composite quark gluon structure



$$\bar{\psi}\gamma^1\psi = \chi_R^*\phi_R - \chi_L^*\phi_L - \phi_L^*\chi_L + \phi_R^*\chi_R$$



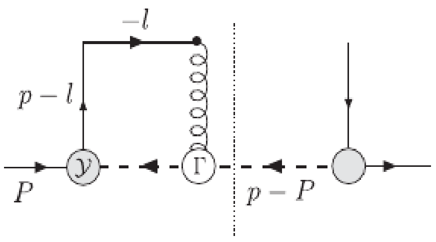


Rajan, Courtoy, Engelhardt and Liuti, PRD94, 2016

Including Final State Interactions

Work by Brandon Kriesten

- Ji → Straight Gauge link
- Jaffe Manohar → Staple Link



- The difference is the torque

$$L_q^{JM} - L_q^{Ji} = \int \frac{d^2 z_T dz^-}{(2\pi)^3} \langle P',\Lambda' | \bar{\psi}(z)\gamma^+(-g)\int_{z^-}^{\infty} dy^- U\left[z_1 G^{+1}(y^-) - z_2 G^{+2}(y^-)\right] U\psi(z) | P,\Lambda\rangle \Big|_{z^+=0}$$

Burkardt (2013)

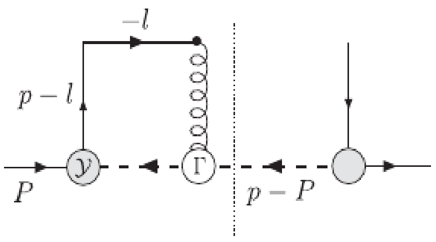
Including Final State Interactions

Talk by Simonetta Liuti

Work by Brandon Kriesten

- Ji → Straight Gauge link

- Jaffe Manohar → Staple Link



- The difference is the torque

$$\mathcal{L}_q^{JM} - L_q^{Ji} = \int \frac{d^2 z_T dz^-}{(2\pi)^3} \langle P',\Lambda' | \bar{\psi}(z)\gamma^+(-g)\int_{z^-}^{\infty} dy^- U \left[z_1 G^{+1}(y^-) - z_2 G^{+2}(y^-) \right] U \psi(z) | P,\Lambda \rangle \Big|_{z^+=0}$$

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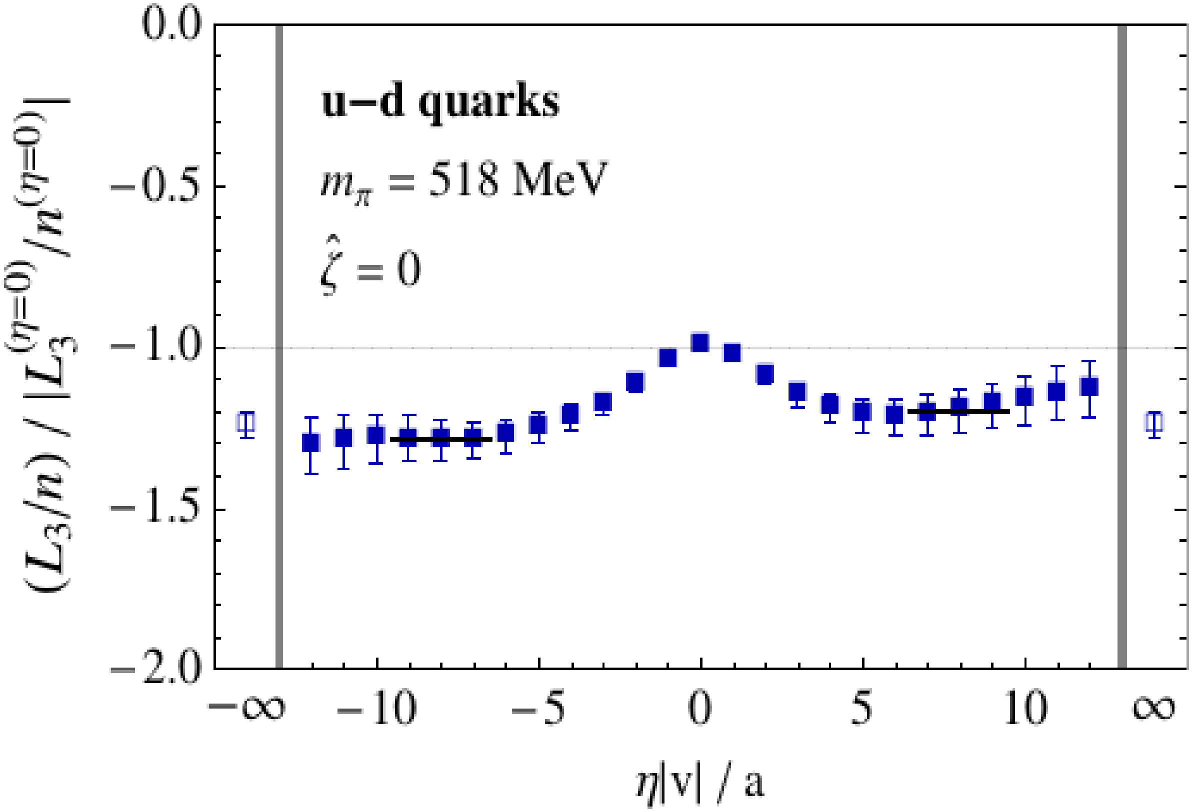

Michael Engelhardt

arxiv: 1701.01536

Including Final State Interactions

- The twist three GPDs are obtained by integrating over the gluon momentum for a quark quark gluon quark operator

$$g_T(x) = \frac{1}{2x} \int \mathrm{d}y \Big[\tilde{G}(x,y) + \tilde{G}(y,x) + G(x,y) - G(y,x)\Big]$$

Jaffe and Ji, 1992

- Final state interactions on the other hand connect to a gluonic pole of Qiu Sterman like term

$$T_{q,F}(x,x) = \frac{1}{M} \int d^2k_\perp k_\perp^2 f_{1T}^\perp(x,k_\perp^2)$$

Kang, Qiu and Zhang PRD 81, 2010

- Hence in some sense different limits of the same correlator

The Parametrization and the gauge link structure

$$W^{\gamma^+}_{\Lambda\Lambda'} = \int \frac{dz_- d^2 z_T}{2\pi} e^{ixP^+ z^- - k_T . z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\gamma^+ U(-z/2, z/2|n)\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

- Without the gauge link, the number of As matches that of the GPDs.
- If we include the gauge link, new As are introduced and the number then matches the number of GTMDs.
- Hence, LIRs need not exist anymore, the new terms also called 'LIR breaking' terms.

Conclusions

- We find relations between GPDs and GTMDs that highlight the role of quark gluon interactions.
- These relations are valid for both the forward and off forward case.
- This also provides a way to measure effects that were solely associated with GTMDs by measuring the associated GPD.
- Quark gluon quark interactions are at the heart of twist three effects.
- Alternate way of deriving the genuine twist twist three contributions and 'wandzura wilczek' terms. Allows us to write out precisely quark gluon contribution.