Nucleon matrix elements from Moments of Correlation Functions and the Proton Charge Radius

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**Speaker*

Proton EM form factors

- Nucleon Pauli and Dirac Form Factors described in terms of matrix element of vector current

$$\langle N \mid V_\mu \mid N \rangle(\vec{q}) = \bar{u}(\vec{p}_f)\left[F_q(q^2)\gamma_\mu + \sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2m_N}\right]u(\vec{p}_i)$$

- Alternatively, Sach's form factors determined in experiment

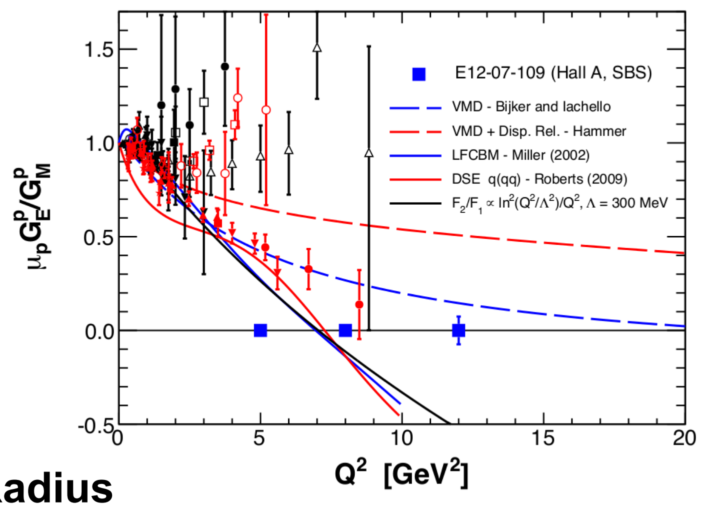
$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

Charge radius is slope at $Q^2 = 0$

$$\left.\frac{\partial G_E(Q^2)}{\partial Q^2}\right|_{Q^2=0} = -\frac{1}{6}\langle r^2 \rangle = \left.\frac{\partial F_1(Q^2)}{\partial Q^2}\right|_{Q^2=0} - \frac{F_2(0)}{4M^2}$$

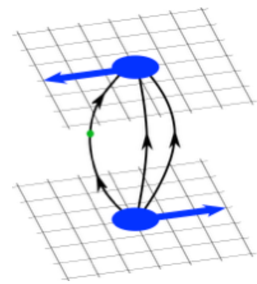
EM Form factors - II

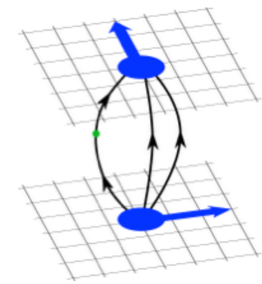
PRAD: E12-11-106



Approved expt E12-07-109

$Q^2 \lesssim 8.2\ \mathrm{GeV}^2$ $Q^2 \lesssim 4.1\ \mathrm{GeV}^2$





Nucleon Charge Radius

←

Direct calculation of charge radius through coordinate-space moments

UKQCD, Lellouch, Richards *et al.*, NPB444 (1995) 401

Bouchard, Chang, Orginos, Richards, Lattice 2016

→

Boosted interpolating operators

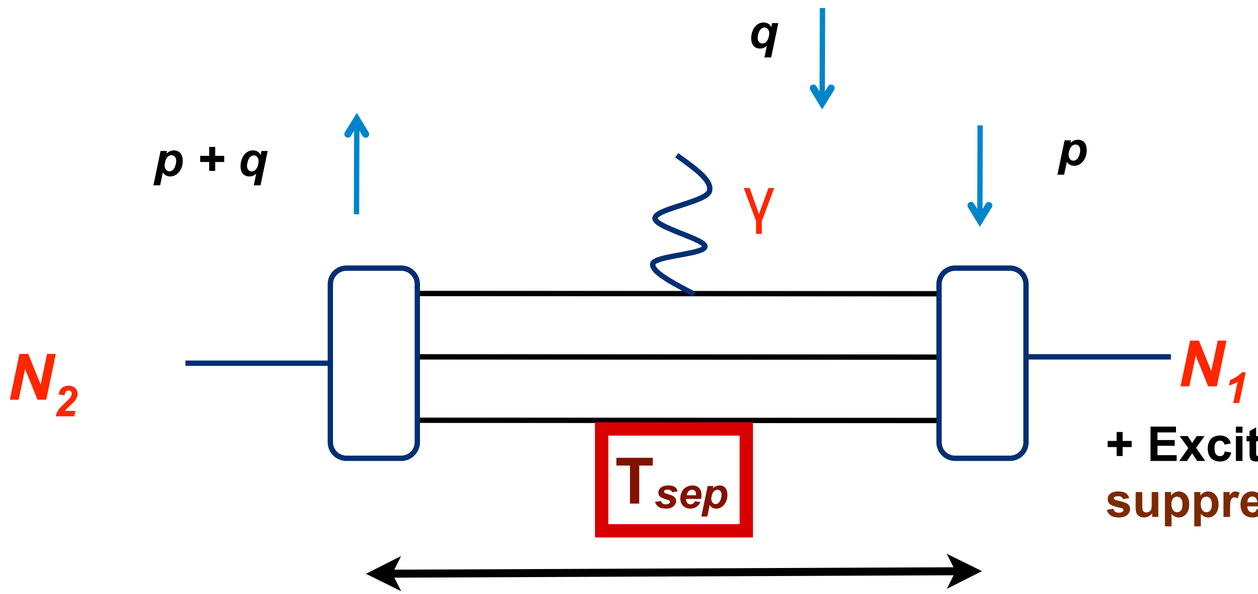
Bali *et al.*, Phys. Rev. D 93, 094515 (2016)

LHPC, Syritsyn, Gambhir, Orginos et al, Lattice 2016

Distillation + Operators for hadrons in flight

Dudek, Edwards, Thomas, Phys. Rev. D 85, 014507 (2012)

Form Factor in LQCD



+ Excited states - suppressed at large T

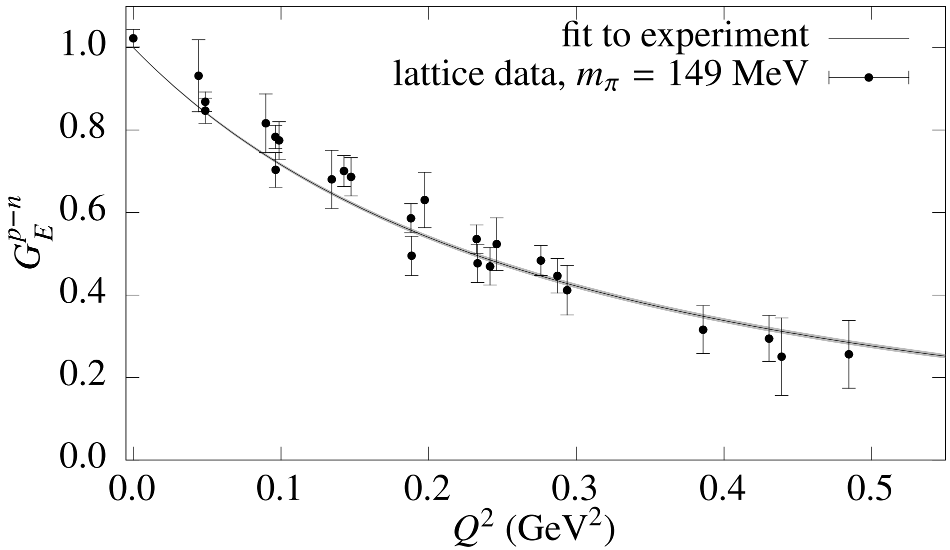
$$C_{3\mathrm{pt}}(t_{sep},t;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 \mid N(\vec{x},t_{\mathrm{sep}}) V_\mu(\vec{y},t) \bar{N}(\vec{0},0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

Resolution of unity – insert states

$$\longrightarrow \langle 0 \mid N \mid N, \vec{p}+\vec{q}\rangle\langle N,\vec{p}+\vec{q} \mid V_\mu \mid N\vec{p}\rangle\langle N,\vec{p} \mid \bar{N} \mid 0\rangle e^{-E(\vec{p}+\vec{q})(t_{\mathrm{sep}}-t)} e^{-E(\vec{p})t}$$

Electromagnetic Form Factors

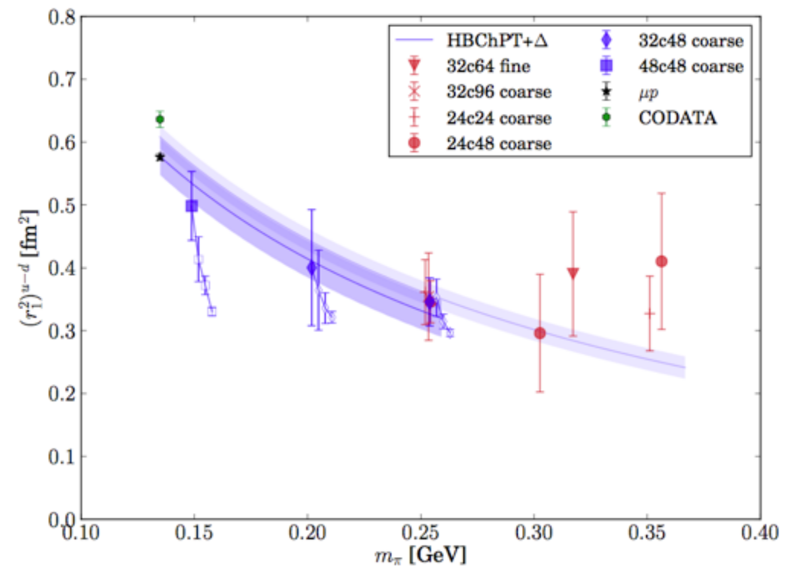
Wilson-clover lattices from BMW



Green et al (LHPC), Phys. Rev. D 90, 074507 (2014)

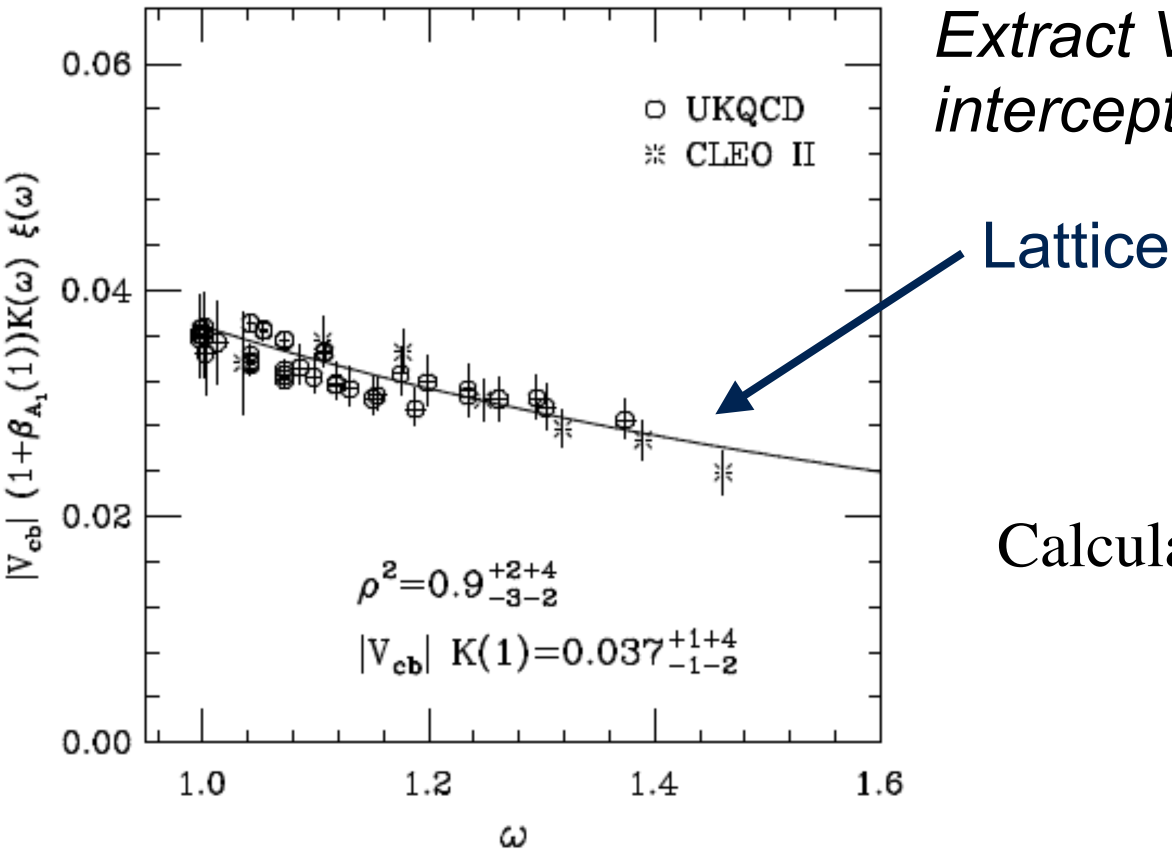
Hadron structure at nearly-physical quark masses

Why can't we get rid of those excited states!



Smallest non-zero Q^2 determined by spatial volume
⇒Calculate slope of form factor directly.

Isgur-Wise Function and CKM matrix

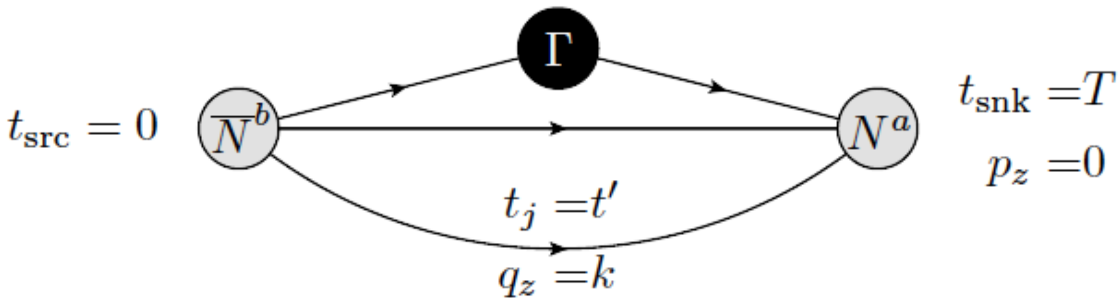


Extract V_{cb} if know intercept at zero recoil

Calculate slope at zero recoil..

UKQCD, L. Lellouch et al., Nucl. Phys. B444, 401 (1995), hep-lat/9410013

Moment Methods



- Introduce three-momentum projected three-point function

$$C^{3\rm pt}(t,t') = \sum_{\vec{x},\vec{x}'} \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle e^{-ikx'_z}$$

- Now take derivative w.r.t. k^2

$$C'_{3\rm pt}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'_z}{2k} \sin\left(kx'_z\right) \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle$$

whence

$$\lim_{k^2 \to 0} C'_{3\rm pt}(t,t') = \sum_{\vec{x},\vec{x}'} \frac{-x'^2_z}{2} \left\langle N^a_{t,\vec{x}} \Gamma_{t',\vec{x}'} \overline{N}^b_{0,\vec{0}} \right\rangle .$$

Odd moments vanish by symmetry

Moment Methods - II

- Analogous expressions for two-point functions:

$$C_{2\text{pt}}(t) = \sum_{\vec{x}} \left\langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \right\rangle e^{-ikx_z}$$

$$\Longrightarrow$$

$$C'_{2\text{pt}}(t) = \sum_{\vec{x}} \frac{-x_z}{2k} \sin(kx_z) \left\langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \right\rangle$$

$$\Longrightarrow$$

$$\lim_{k^2 \to 0} C'_{2\text{pt}}(t) = \sum_{\vec{x}} \frac{-x_z^2}{2} \left\langle N^b_{t,\vec{x}} \overline{N}^b_{0,\vec{0}} \right\rangle .$$

Lowest coordinate-space moment ⇔ slope at zero momentum

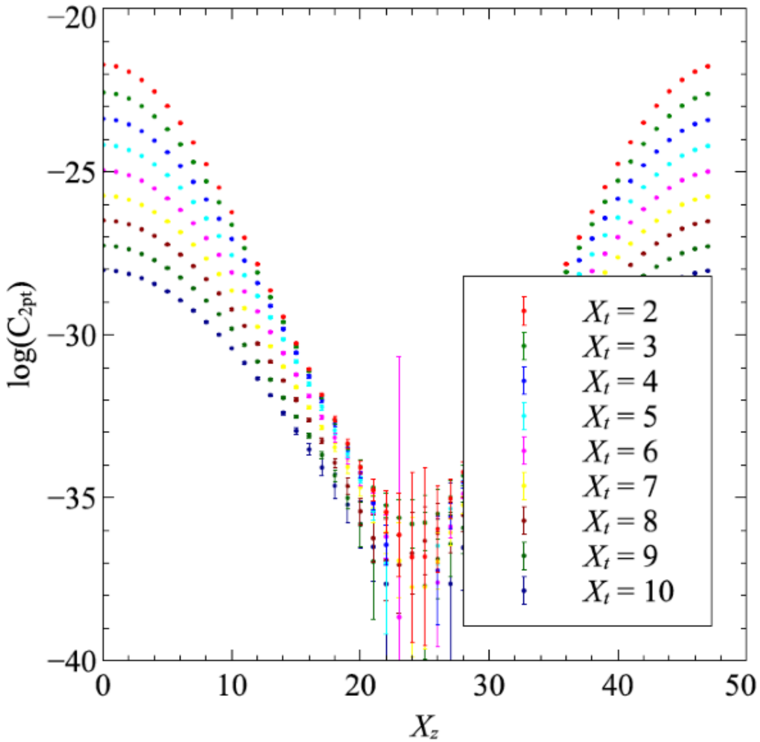
Lattice Details

- Two degenerate light-quark flavors, and strange quark set to its physical value

$$\begin{aligned} a &\simeq 0.12 \text{ fm} \\ m_\pi &\simeq 400 \text{ MeV} \\ \text{Lattice Size} &: 24^3 \times 64 \end{aligned}$$

- To gain control over finite-volume effects, replicate in z direction: $24 \times 24 \times 48 \times 64$

Two-point correlator

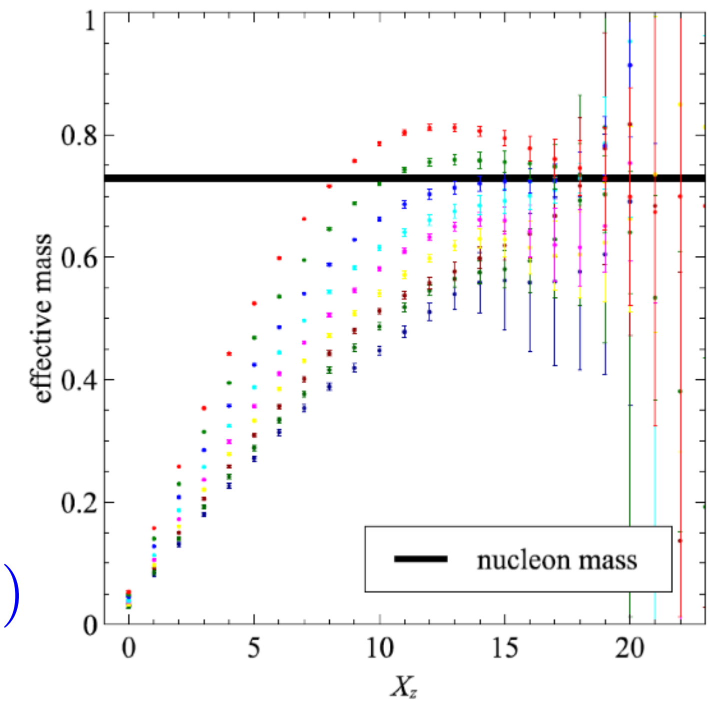


$$\ln\left[C_{2\mathrm{pt}}(t,x_z)\right]$$

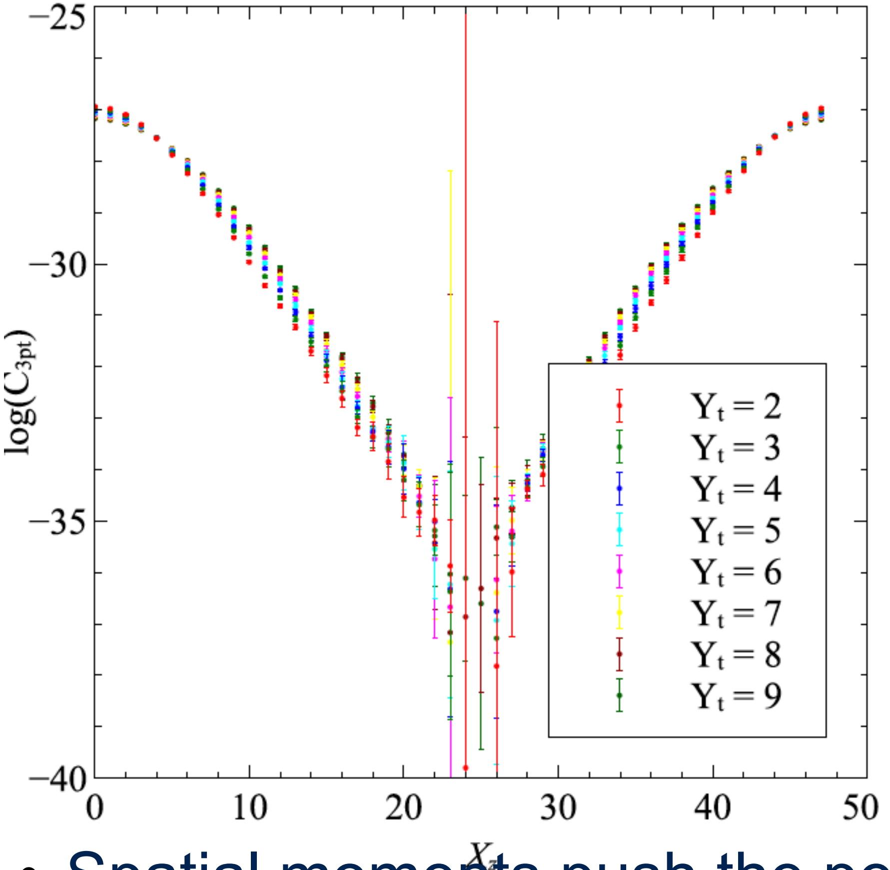
Any polynomial moment in x_z converges

"Effective mass"

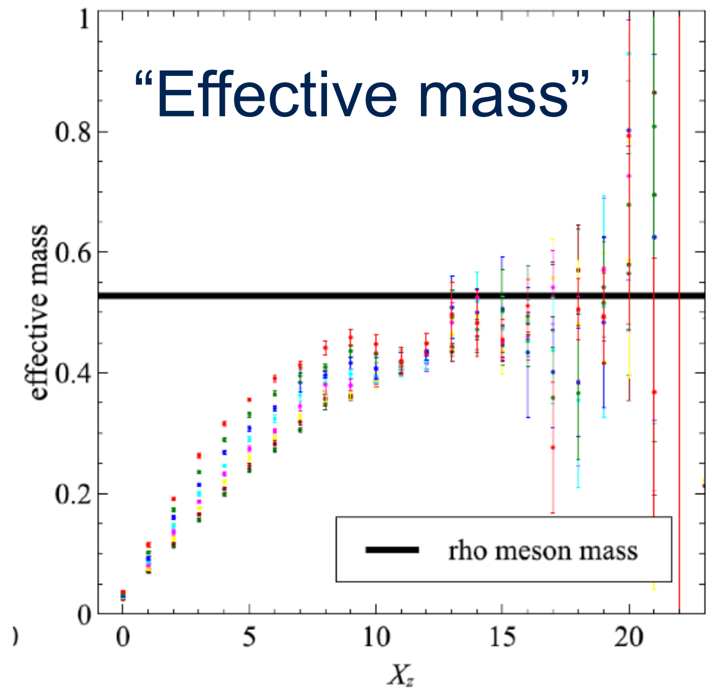
$$\ln C_{2\mathrm{pt}}(t,x_z)/C_{2\mathrm{pt}}(t,x_z+1)$$



Three-point correlator



$$\ln\left[C_{3\mathrm{pt}}(t', x'_z)\right]$$



- Spatial moments push the peak of the correlator away from origin
- Larger finite volume corrections compared to regular correlators

Fitting the data…

$$C^{3\mathrm{pt}}(t,t') = \sum_{n,m} \frac{Z_n^{\dagger a}(0)\Gamma_{nm}(k^2)Z_m^b(k^2)}{4M_n(0)E_m(k^2)} e^{-M_n(0)(t-t')} e^{-E_m(k^2)t'}$$

$$C_{2\mathrm{pt}}(t) = \sum_m \frac{Z_m^{b\dagger}(k^2)Z_m^b(k^2)}{2E_m(k^2)} e^{-E_m(k^2)t}$$

where

$$Z_n^{\dagger a}(0) \equiv \langle \Omega | N^a | n, p_i = (0,0,0)\rangle$$

$$Z_m^b(k^2) \equiv \langle m, p_i = (0,0,k) | \overline{N}^b | \Omega \rangle$$

$$\Gamma_{nm}(k^2) \equiv \langle n, p_i = (0,0,0) | \Gamma | m, p_i = (0,0,k)\rangle$$

Allow for multi-state contributions in the fit

Fitting - II

- Now look at the functional form of derivatives:

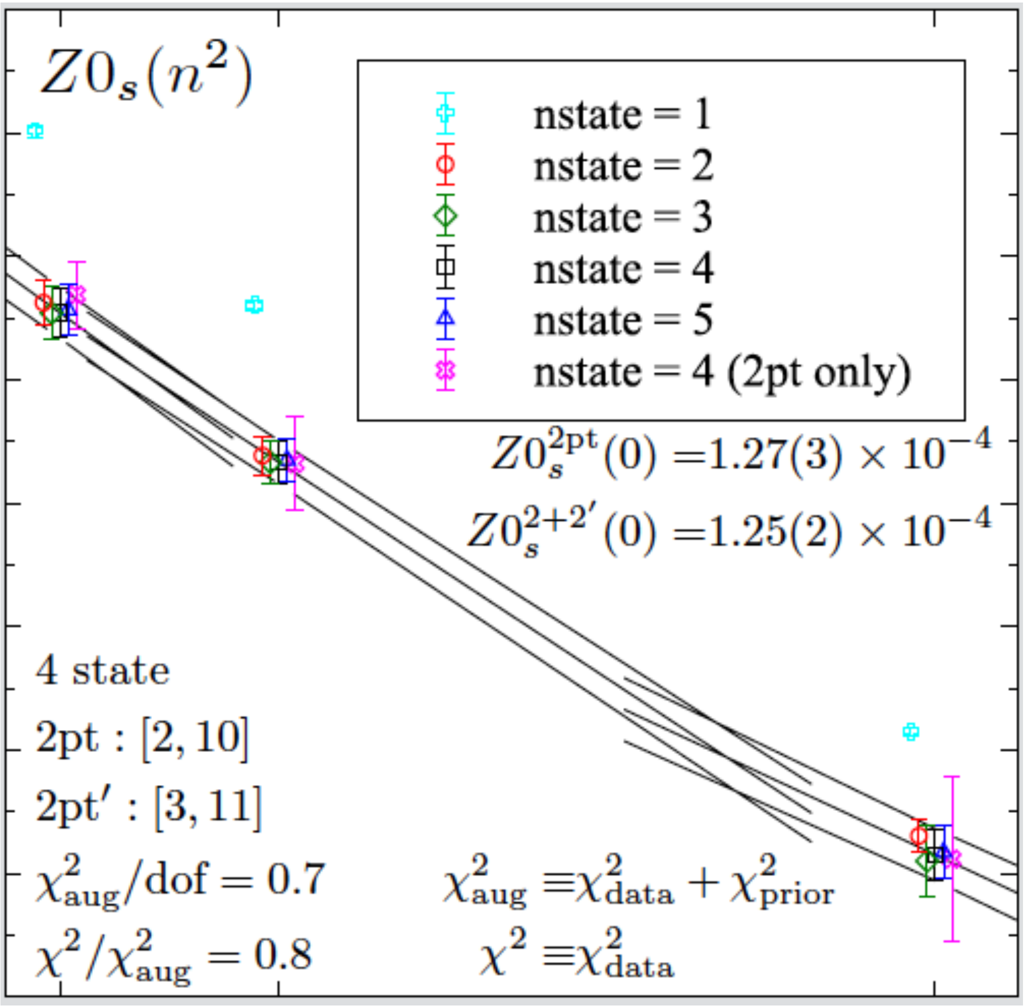
$$C'_{2\mathrm{pt}}(t) = \sum_m C_m^{2\mathrm{pt}}(t)\left(\frac{2Z_m^{b\prime}(k^2)}{Z_m^b(k^2)} - \frac{1}{2[E_m(k^2)]^2} - \frac{t}{2E_m(k^2)}\right)$$

$$C'_{3\mathrm{pt}}(t,t') = \sum_{n,m} C_{nm}^{3\mathrm{pt}}(t,t')\left\{\frac{\Gamma'_{nm}(k^2)}{\Gamma_{nm}(k^2)} + \frac{Z_m^{b\prime}(k^2)}{Z_m^b(k^2)} - \frac{1}{2[E_m(k^2)]^2} - \frac{t'}{2E_m(k^2)}\right\}$$

spatially extended sources

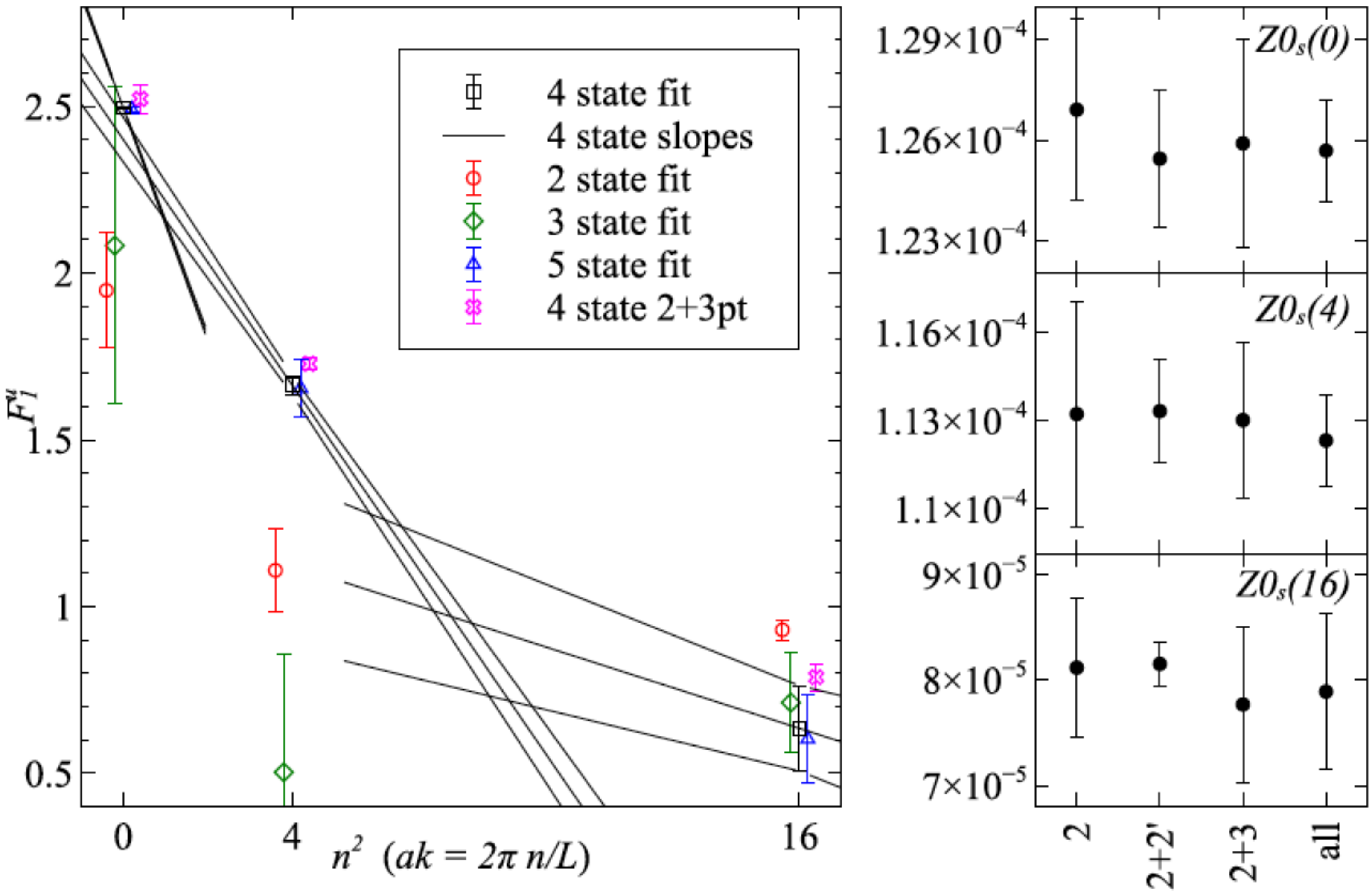
Second distance scale

Fitting - III



In practice we use multi-exponential, Bayesian fits

F_1 Form Factor



Conclusions

- Moment methods allow direct calculations of slopes of form factors at momenta allowed on lattice
- Lowest (even) moment gives the slope at $Q^2 = 0$.
- Larger finite-volume effects than regular correlators (perhaps expected - no free lunch).
- Illustrated here for u-quark contribution to EM form factor; d-quark and sea-quark contributions in progress….