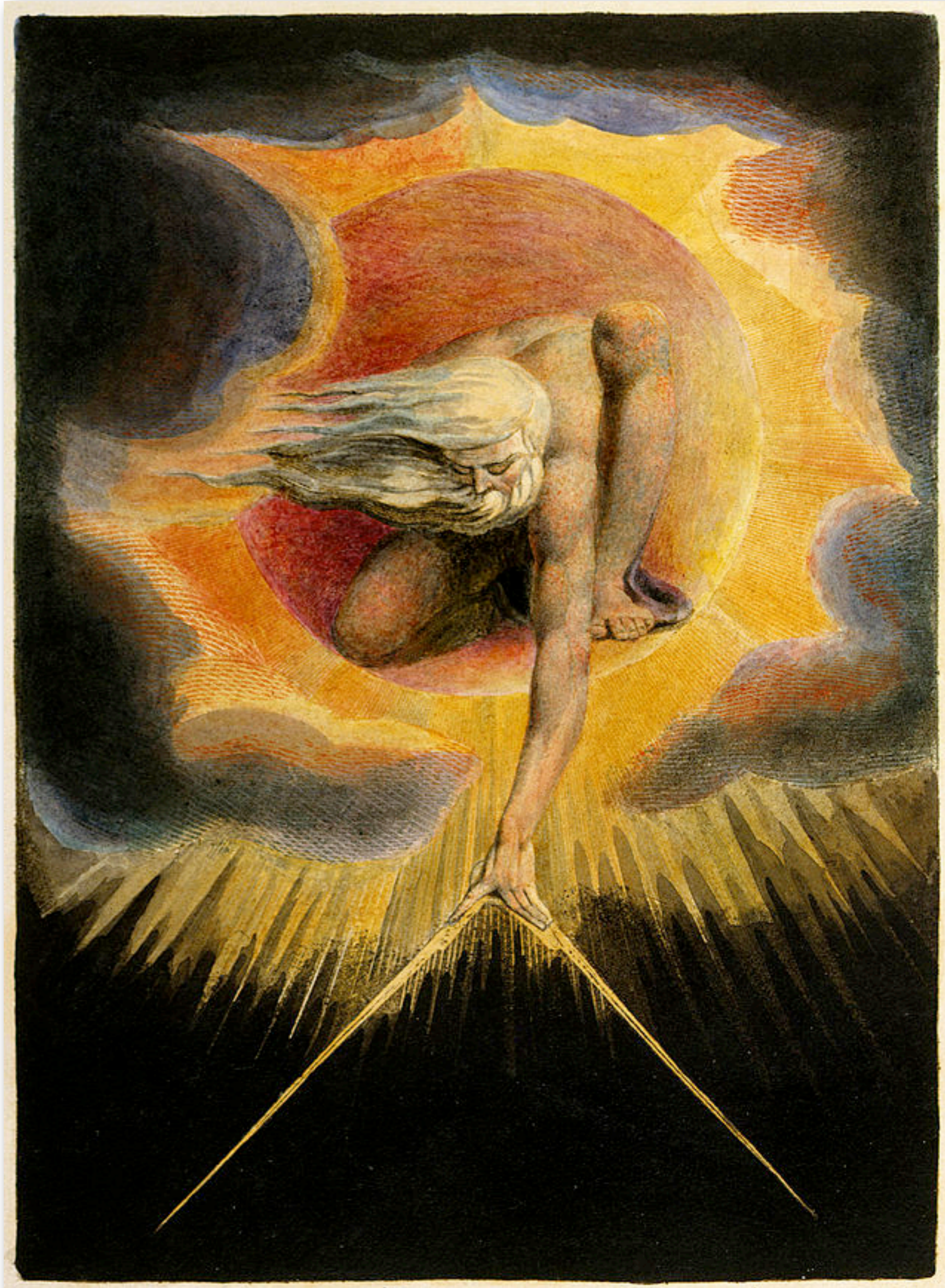
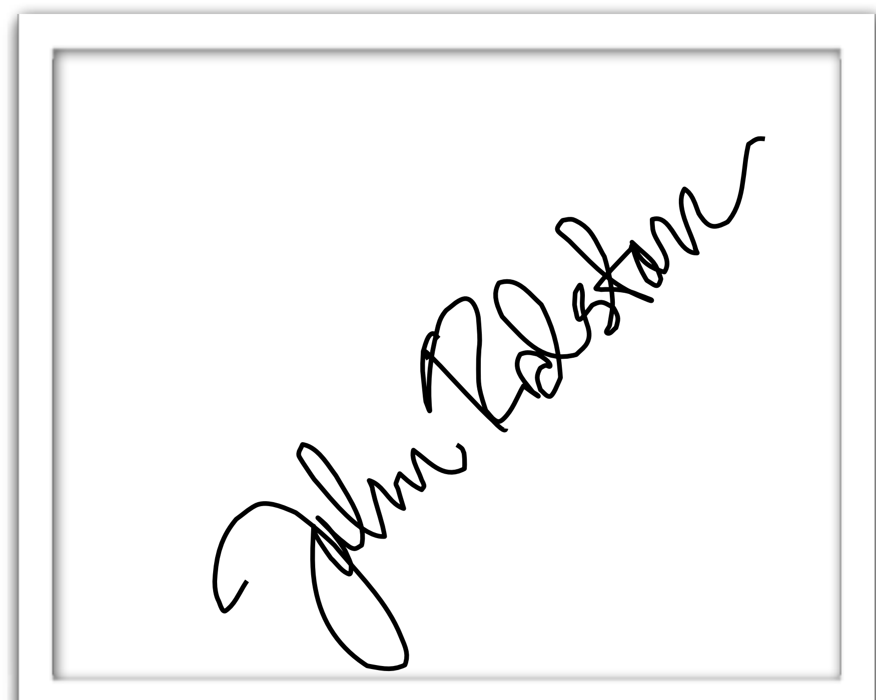


How Fundamental Are YOUR Constants?

The global approach to current anomalies



with John Martens

When did you last
make contact with your constants?

how did that make you feel?

Have you perhaps taken your
constants for granted?

do you feel any guilt ?

In your absence, who manages your
relationship with your constants?

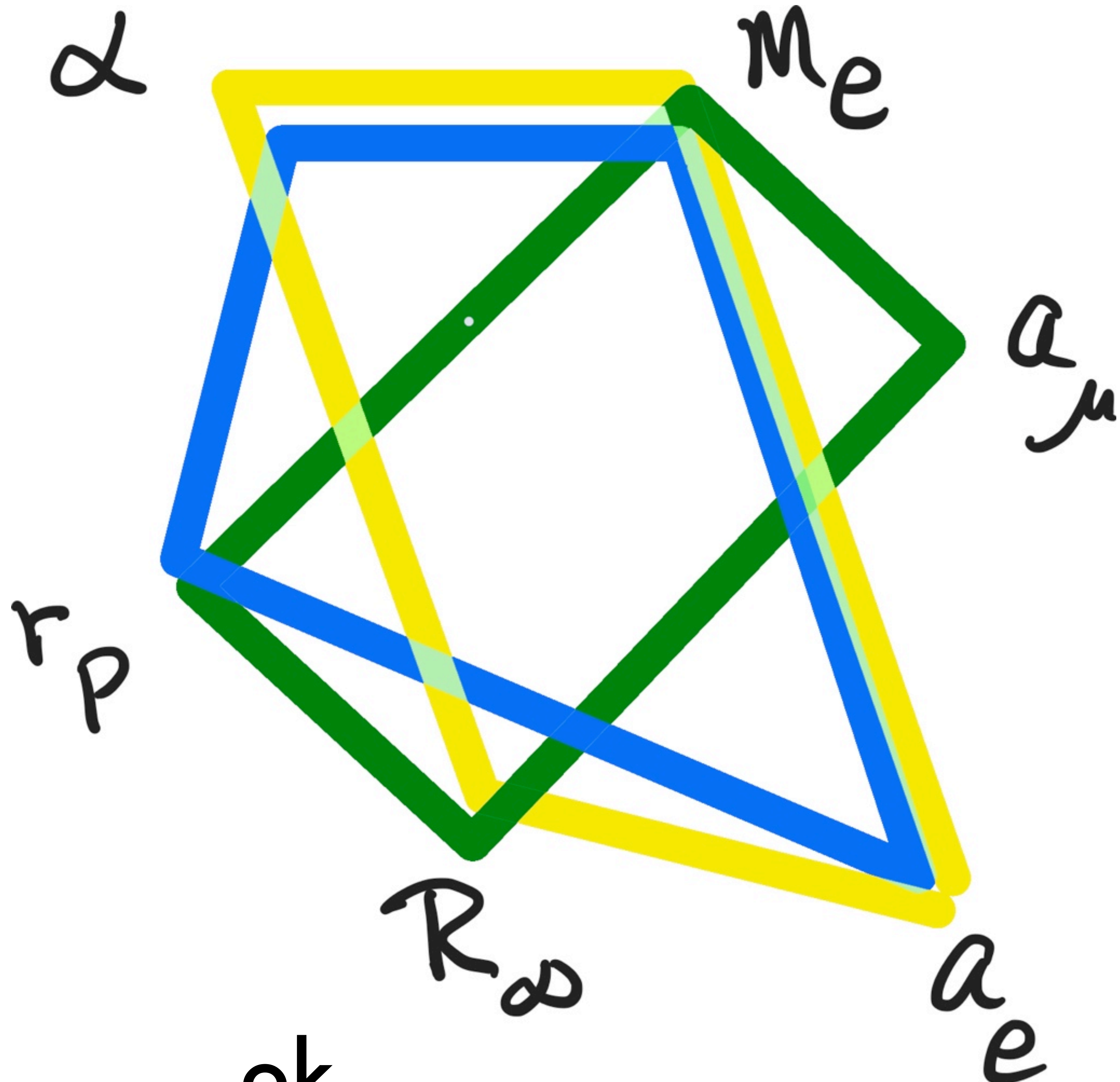
do you regret anything?

The biggest problem in physics as it now stands

the constants are not consistent

α	fine structure constant
$\mathrm{a}_e = (g_e - 2)/2$	electron anomalous moment
$\mathrm{a}_\mu = (g_\mu - 2)/2$	muon anomalous moment
m_e/h	electron Compton wavelength
$\mathrm{r}_p(\mu H)$	proton form factor, "size", muonic H
$\mathrm{r}_p(eH)$	proton form factor, "size", electronic H
R_∞	Rydberg constant

Three
Mysterious
Consistent
Solutions



each rejects
two items

why not think out of the box

We conducted the only existing global fit to all the data *using the entire body of precision Standard Model theory*

arXiv:1606.06209 [hep-ph]

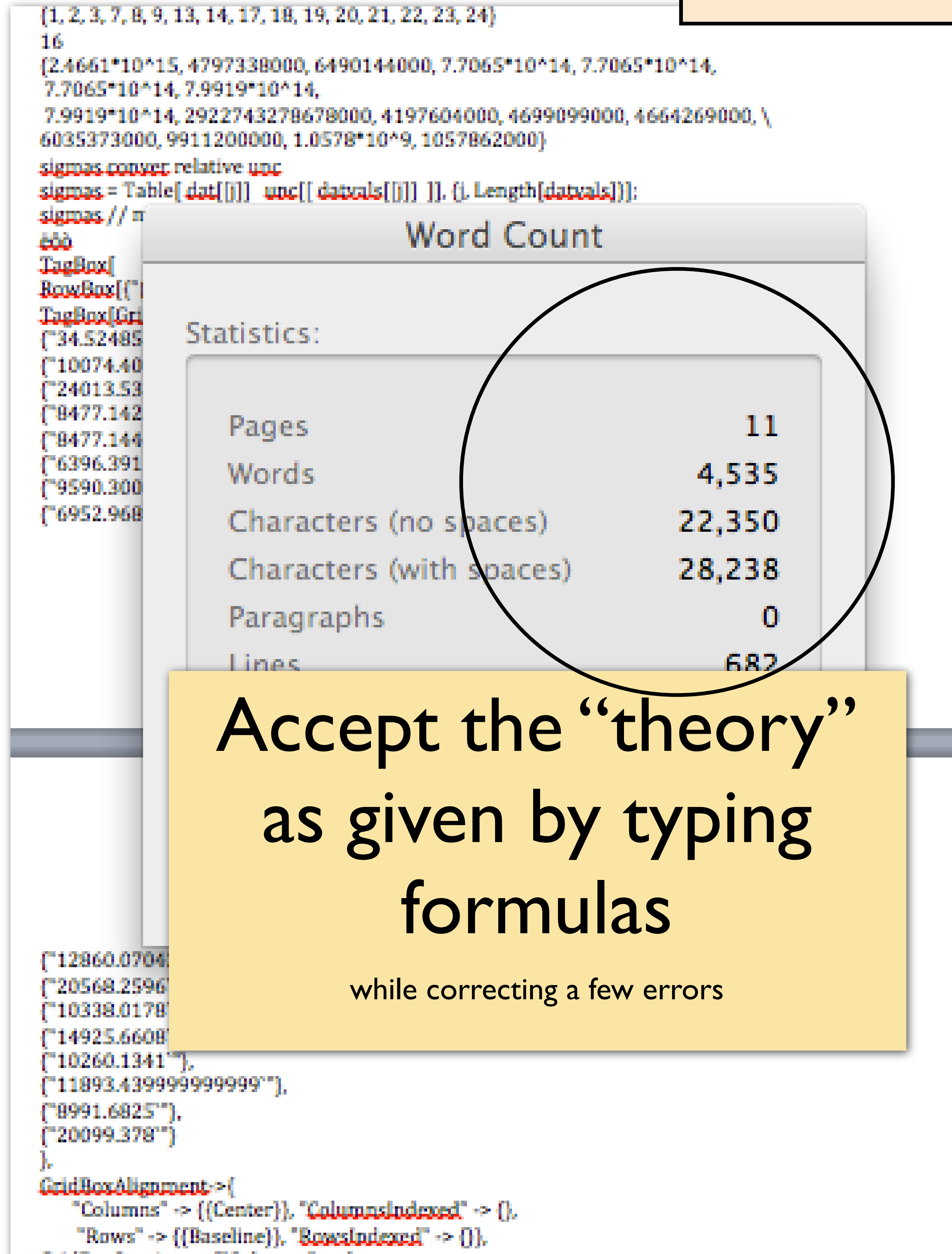
A GLOBAL RESOLUTION NEEDS A GLOBAL FIT

Review is over.
Our contribution
starts here

How do *inputs* affect *outputs*?

QED

Theory: 75 years
28000 keystrokes
mathematica! In C++, estimate 260000
Breit, Dirac, Bethe...Yennie,
Sapirstein, Ericson,Brodsky...Eides,
Grotch, Shelyuto, Borie,
Karshenboim, Mohr, Kotochigova,
Pachucki,Yerokin et al, Jenstchura...



Accept the "theory" as given by typing formulas

while correcting a few errors

Validating 32k keystrokes of theory implementation

this data set: 16 eH transitions selected by CODATA for 20 years, 2010 includes 1S3S

2 free parameters →

QED only here

two versions of theory on two machines; round off errors controlled

John Martens

σ_{expt} Hz	f_{expt} Hz	$f_{our\ calc}$ Hz
35	$2.46606141319 \times 10^{15}$	$2.46606141319 \times 10^{15}$
10074	4.797338×10^{9}	$4.79733066539 \times 10^{9}$
24014	6.490144×10^{9}	$6.49012898284 \times 10^{9}$
8477	$7.70649350012 \times 10^{14}$	$7.70649350016 \times 10^{14}$
8477	$7.7064950445 \times 10^{14}$	$7.70649504449 \times 10^{14}$
6396	$7.70649561584 \times 10^{14}$	$7.70649561578 \times 10^{14}$
9590	$7.99191710473 \times 10^{14}$	$7.99191710481 \times 10^{14}$
6953	$7.99191727404 \times 10^{14}$	$7.99191727409 \times 10^{14}$
12860	$2.92274327868 \times 10^{15}$	$2.92274327867 \times 10^{15}$
20568	4.197604×10^{9}	$4.19759919778 \times 10^{9}$
10338	4.699099×10^{9}	$4.6991043085 \times 10^{9}$
14926	4.664269×10^{9}	$4.66425337748 \times 10^{9}$
10260	6.035373×10^{9}	$6.03538320383 \times 10^{9}$
11893	9.9112×10^{9}	$9.91119855042 \times 10^{9}$
8992	1.057845×10^{9}	$1.05784298986 \times 10^{9}$
20099	1.057862×10^{9}	$1.05784298986 \times 10^{9}$

JM+JPR

no theory errors listed here

Let's use ALL the data

$$a_e = 0.00115965218073 \pm 2.8 \times 10^{-13}$$

$$a_\mu = 0.00116592091 \pm 6.3 \times 10^{-10} \otimes \text{"}4.6\,\sigma\text{"}$$

$$\mu H:\ \Delta E_{2S-2P} = 202.3706 \pm 0.0026\,\mathrm{meV} \otimes \text{"}7\sigma\text{"}$$

$$h/m_e = 7.2738950972 \times 10^{-4} \pm 2 \times 10^{-12}\,\mathrm{m}^2 s^{-1}$$

$eH:$ 7 transitions listed in Table

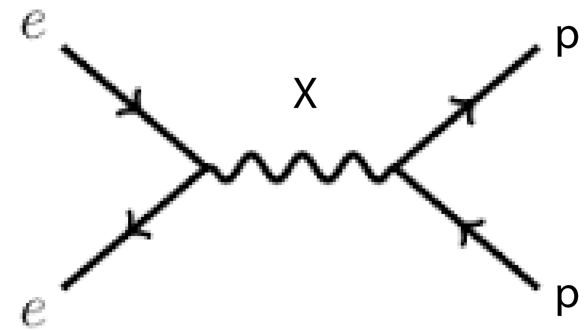
$eD:$ 7 transitions listed in Table

hep-ph 1606.06209

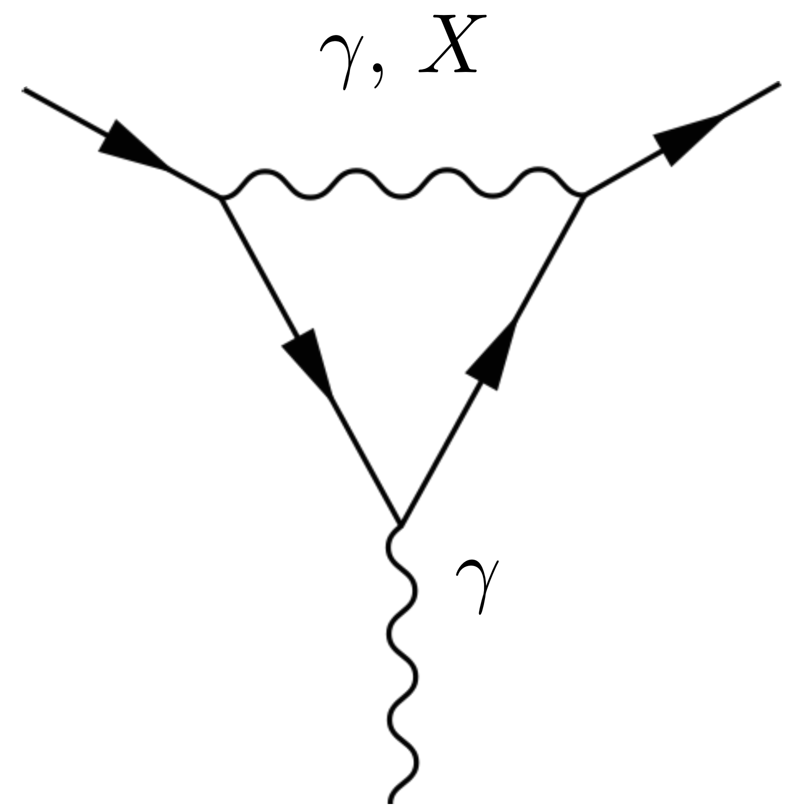
$eH =$ electronic hydrogen
$eD =$ electronic deuterium
$\mu H =$ muonic hydrogen

A global fit to everything, permitting an alternative

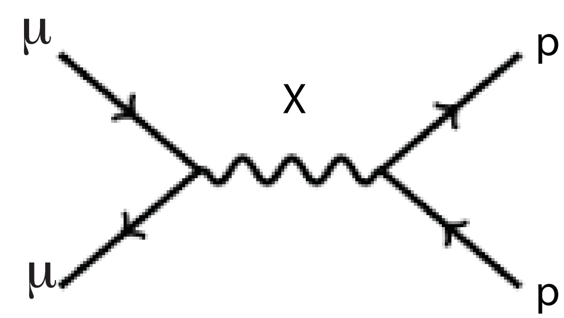
“no name theory”
of particle X



more general than dark photon

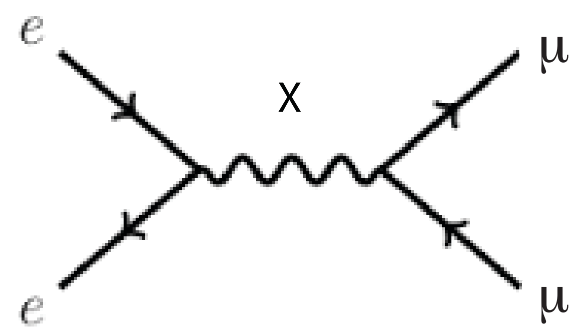


$$g_e^2 = g_\mu^2 = g_p^2 = g^2$$



$$\alpha_X = \xi m_X^2 = g^2/4\pi$$

minimal
“bottom-up”
data driven



No other assumptions

$$a_e^{theory} = 1.7147\times 10^{-12} + 0.159155\alpha - 0.0332818\alpha^2 + 0.0380966\alpha^3$$
$$- 0.0196046\alpha^4 + 0.0299202\alpha^5 + 0.027706\,\xi m_X^2 f(m_X/m_\ell)$$

integral f from Leveille 1978, Whisnant and Li 1985

$$\chi^2 = \sum_j \frac{(d_j - t_j(\theta_\ell))^2}{\sigma_j^2}$$

theory uncertainties later

$t_j = theory_j,\ d_j = data_j,\ \sigma_j = \text{experimental uncertainty}_j.$

Fitted parameters are $\theta_j = (\alpha,\ R_\infty,\ r_p,\ \theta_X)$.

New physics parameters are
$\theta_X = (\alpha_X,\ m_X)$ $(m_X \lesssim 50\ \text{MeV})$
or $\theta_X = \xi = \alpha_X / m_X^2$ $(m_X \gtrsim 50\ \text{MeV})$

$$\text{universal} \quad V(x) = \alpha_X \frac{e^{-m_X r}}{4\pi r}.$$

$$
\begin{aligned}
\chi^2 = & \frac{(a_e^{exp} - a_e^{theory}(\alpha,\, \theta_X))^2}{\sigma^2(a_e)} && \chi^2(a_e) \\
& + \frac{(a_\mu^{exp} - a_\mu^{theory}(\alpha,\, \theta_X))^2}{\sigma^2(a_\mu)} && \chi^2(a_\mu) \\
& + \sum_j^8 \frac{(\Delta f_{eH,j}^{exp} - \Delta f_{eH,j}^{theory}(\alpha,\, R_\infty,\, r_p,\, \theta_X))^2}{\sigma^2(\Delta f_{eH})} && \chi^2(eH) \\
& + \sum_j^8 \frac{(\Delta f_{eD,j}^{exp} - \Delta f_{eD,j}^{theory}(\alpha,\, R_\infty,\, r_p,\, \theta_X))^2}{\sigma^2(\Delta f_{eD})} && \chi^2(eD) \\
& + \frac{(\Delta f_{\mu H}^{exp} - \Delta f_{\mu H}^{theory}(r_p,\, \theta_X))^2}{\sigma^2(\Delta f_{\mu H})} && \chi^2(\mu H) \\
& + \frac{(4\pi c R_\infty/\alpha^2 - (m_e/h)^{exp})}{\sigma^2(m_e/h)} && \chi^2(\lambda_c)
\end{aligned}
$$

Results: A new region of local minimum χ^2

Omit	χ^2_{tot}	$\Delta\chi^2$	$(\delta R_\infty/R_\infty^\bullet)/10^{-12}$	$(\delta\alpha/\alpha^\bullet)/10^{-10}$	$r_p\,[fm]$	$\xi/10^{-11}$
none	14.3	13.5	610(430)	$-3.1(2.1)$	0.84113(27)	1.40(38)
λ_c	11.0	16.1	1290(910)	$-6.5(4.4)$	0.84117(27)	1.60(43)
μH	10.1	13.0	620(410)	$-3.1(2.1)$	0.88143(27)	1.39(38)
a_e	11.0	16.1	$-17(12)$	0.014(10)	0.84117(27)	1.60(43)
a_μ	11.7	0.3	60(42)	$-0.38(26)$	0.84074(27)	$-0.81(22)$
a_e,a_μ	6.9	4.6	$-8.3(5.9)$	0.058(40)	0.84650(27)	31.5(8.6)
μH, a_μ	6.9	0.6	$-8.3(5.9)$	0.058(40)	0.88453(27)	$-1.14(30)$
eH	7.4	13.1	610(430)	$-3.1(2.1)$	0.84112(27)	1.39(38)
eD	10.0	13.4	610(430)	$-3.1(2.1)$	0.84113(27)	1.40(38)
eH, eD	0.0	15.7	$-1310(920)$	$-6.5(4.4)$	0.84116(27)	1.57(43)

Table 1: The parameters for the best fit to all the data and for fits where observables are removed. Parentheses list the standard uncertainties. $\Delta\chi^2$ is the difference of χ^2 of the null model ($\alpha_X = 0$) with the best fit. Table made for the arbitrary value $m_X = 50$ MeV.

minimum region is not a global attractor, but it's where you find it

Reference values we don't use

CODATA2014 (C14)

$$\alpha_\bullet = 0.0072973525664$$ directly predicted by a_e

$$R_{\infty\bullet} = 10973731.5685080\,\mathrm{m}^{-1}$$ 99.9% correlated with r_p

The QED Rydberg is the most precise physical constant of a theory that does not fit all the data

$$\lambda_{C\bullet} = 2.4263102367 \times 10^{-12}\,\mathrm{m}$$

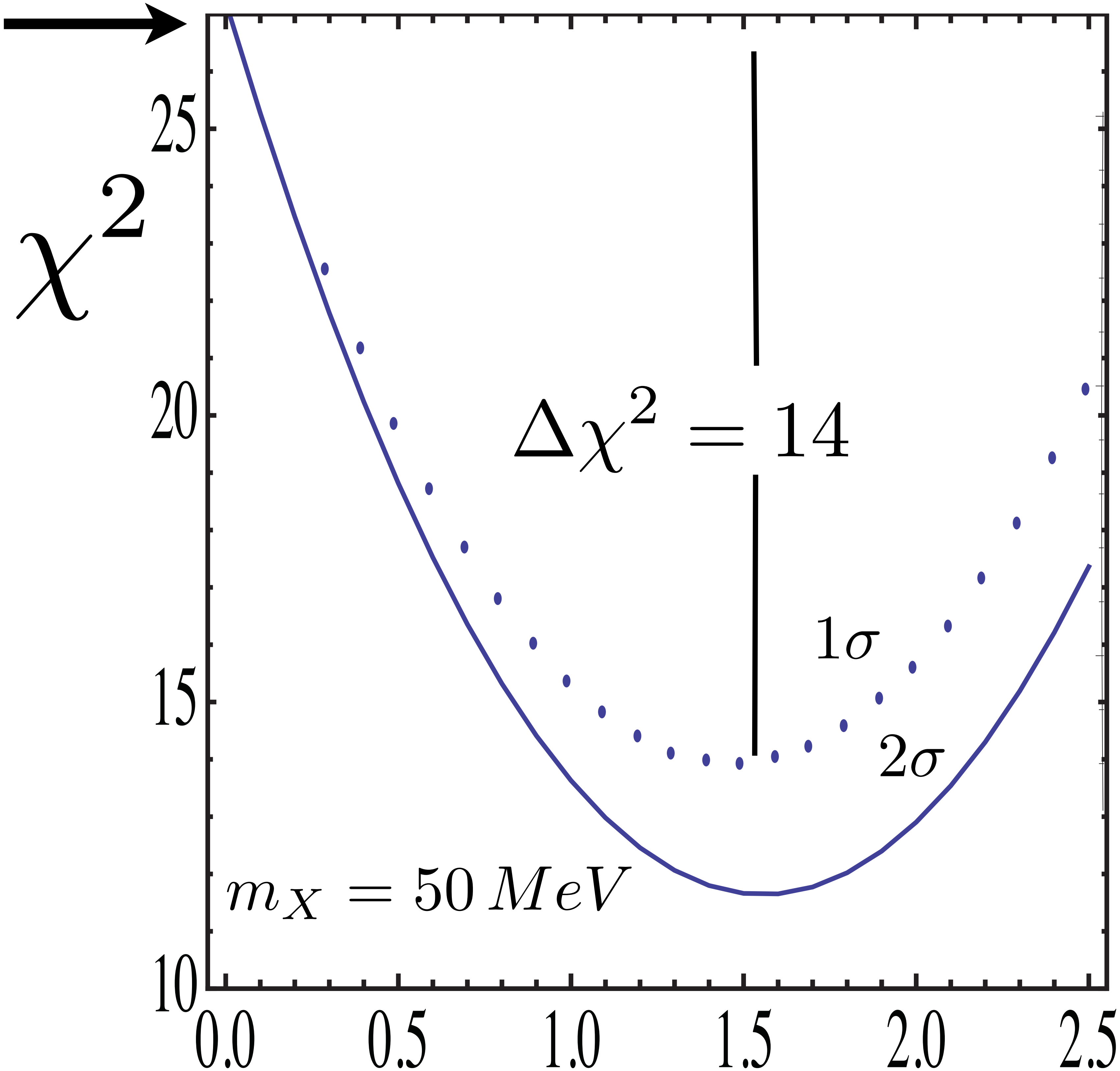
determines uncertainty of α

Values for comparison: we don't use 'em

The value and errors found for the Rydberg constant depend on your theory

$r_p = 0.841 \pm .001$ across the range

QED +SM →



Plots show
1 sigma-exp
2 sigma-exp

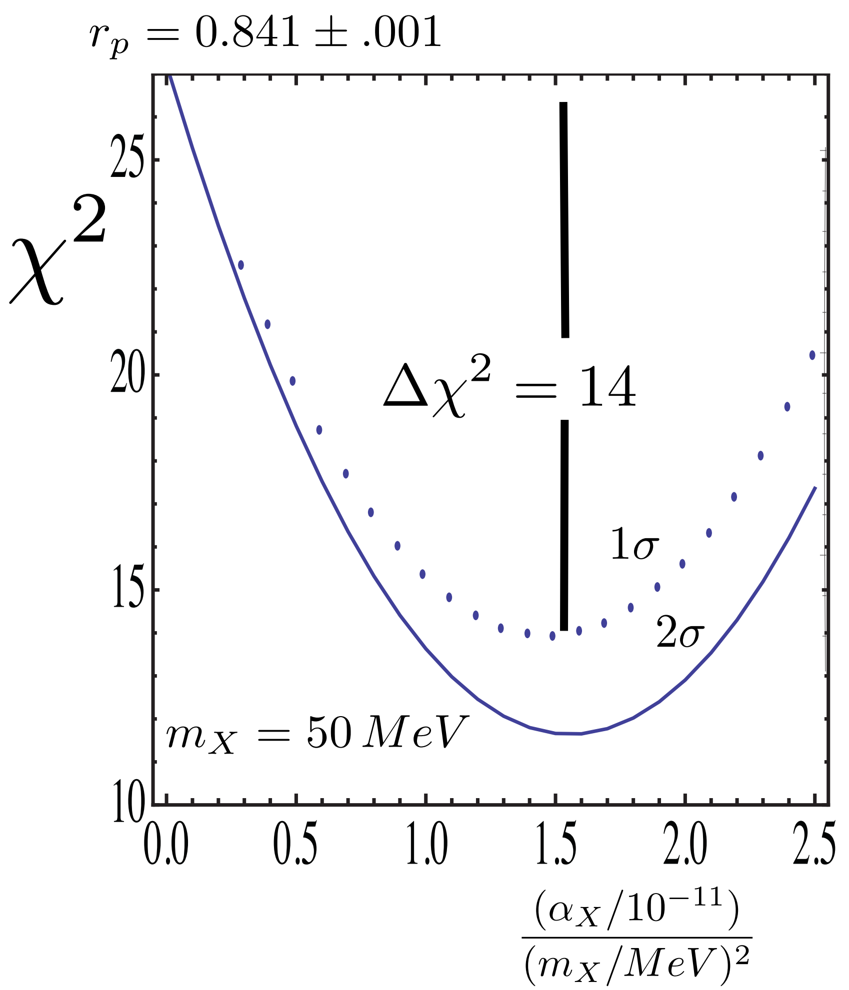
SM ruled out for
mX ≥ 20 MeV

ALL terms in χ^2
are a good fit

Arbitrary value $m_X = 50$ MeV chosen for figure

coupling axis $\dfrac{(\alpha_X/10^{-11})}{(m_X/MeV)^2}$

P-value 1.5×10^{-4}



$\Delta\chi^2 > 14$ for all $m_X > 50$ MeV does not mean the new model is the last word.
It is a comparison where the Standard Model is disfavored at nearly 4σ.

The first time a universal coupling reconciled a_e, a_μ, eH, eD, μH

BSM models always add 2 sigma to a_e… we did not

$$\frac{\alpha_X}{m_X^2}$$

arXiv:1606.06209 [hep-ph]

Omit	χ^2_{tot}	$\Delta\chi^2$	$(\delta R_\infty / R_\infty^\bullet)/10^{-12}$	$(\delta\alpha/\alpha^\bullet)/10^{-10}$	$r_p\ [fm]$	$\xi/10^{-11}$
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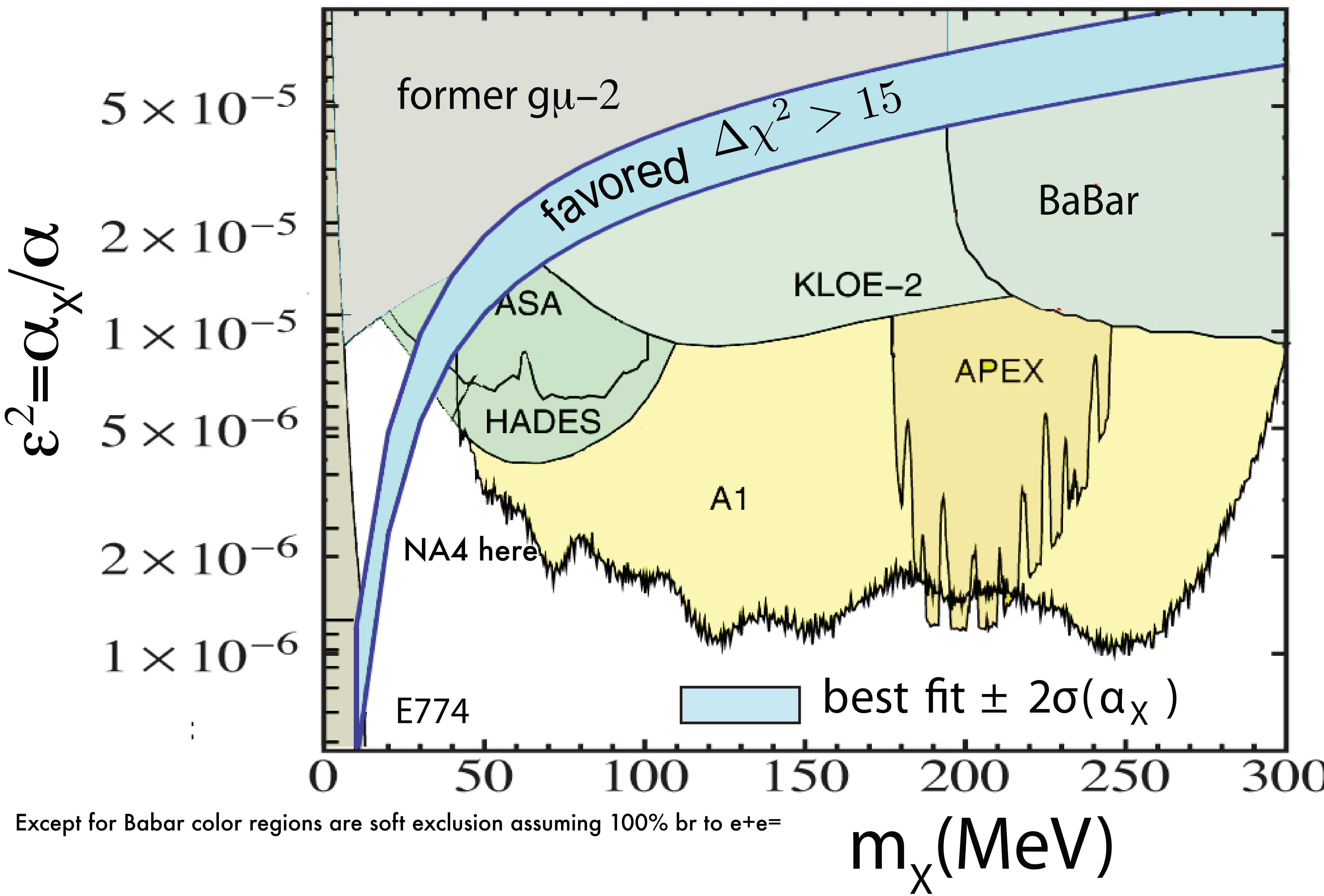
Table 1: The parameters for the best fit to all the data and for fits where observables are removed. Parentheses list the standard uncertainties. $\Delta\chi^2$ is the difference of χ^2 of the null model ($\alpha_X = 0$) with the best fit. Table made for the arbitrary value $m_X = 50$ MeV.

Results

No upper limit on m_χ is determined.



Except for Babar color regions are soft exclusion assuming 100% br to e+e=

VERY robust signal: fit eH and eD, μH, μD

Transition	f_{expt} Hz	$f_{our\ calc}$ Hz	σ_{exp} Hz
$\nu_H(2S_{1/2} - 8S_{1/2})$	$7.70649350012 \times 10^{14}$	$7.70649350006 \times 10^{14}$	8600
$\nu_H(2S_{1/2} - 8D_{3/2})$	$7.7064950445 \times 10^{14}$	$7.7064950444 \times 10^{14}$	8300
$\nu_H(2S_{1/2} - 8D_{5/2})$	$7.706495615842 \times 10^{14}$	$7.706495615680 \times 10^{14}$	6400
$\nu_H(2S_{1/2} - 12D_{3/2})$	$7.991917104727 \times 10^{14}$	$7.991917104715 \times 10^{14}$	9400
$\nu_H(2S_{1/2} - 12D_{5/2})$	$7.991917274037 \times 10^{14}$	$7.99191727409 \times 10^{14}$	7000
$\nu_H(2S_{1/2} - 2P_{3/2})$	9.9112×10^{9}	9.9112×10^{9}	12000
$\nu_H(2P_{1/2} - 2S_{1/2})$	1.057845×10^{9}	1.057846×10^{9}	9000
$\nu_H(2P_{1/2} - 2S_{1/2})$	1.057862×10^{9}	1.057846×10^{9}	20000
$\nu_D(2S_{1/2} - 8S_{1/2})$	$7.708590412457 \times 10^{14}$	$7.708590412336 \times 10^{14}$	6900
$\nu_D(2S_{1/2} - 8D_{3/2})$	$7.708591957018 \times 10^{14}$	$7.708591956914 \times 10^{14}$	6300
$\nu_D(2S_{1/2} - 8D_{5/2})$	$7.708592528495 \times 10^{14}$	$7.708592528361 \times 10^{14}$	5900
$\nu_D(2S_{1/2} - 12D_{3/2})$	$7.99409168038 \times 10^{14}$	$7.99409168032 \times 10^{14}$	8600
$\nu_D(2S_{1/2} - 12D_{5/2})$	$7.994091849668 \times 10^{14}$	$7.994091849642 \times 10^{14}$	6800
$\nu_D(2S_{1/2} - 2P_{3/2})$	9.91261×10^{9}	9.91280×10^{9}	300000
$\nu_D(2P_{1/2} - 2S_{1/2})$	1.05928×10^{9}	1.05923×10^{9}	60000
$\nu_D(2P_{1/2} - 2S_{1/2})$	1.05928×10^{9}	1.05923×10^{9}	60000

Table 1: The experimental values of electronic hydrogen (eH) and electronic deuterium (eD) transitions compared to our calculation using the best fit with $\alpha_X \neq 0$. The fit also reproduces the other transitions used in previous QED-EW fits as described in the text within a fraction of the experimental uncertainty.Table is made with an arbitrary value of $m_X = 50$ MeV.

chi-squared budget

Omit	$\chi^2(\lambda_c)$	$\chi^2(\mu H)$	$\chi^2(a_e)$	$\chi^2(a_\mu)$	$\chi^2(eH)$	$\chi^2(eD)$
none	1.6	0.00084	1.5	0.18	6.8	4.2
λ_c	–	0.00068	$4. \times 10^{-9}$	0.0030	6.8	4.2
μH	1.6	–	1.5	0.23	3.3	3.5
a_e	4.4×10^{-9}	0.00078	–	0.0030	6.8	4.2
a_μ	0.024	0.00087	0.023	–	7.4	4.3
a_e, a_μ	6.7×10^{-8}	2.0×10^{-12}	–	–	3.3	3.5
μH, a_μ	6.7×10^{-8}	–	9.5×10^{-13}	–	3.3	3.5
eH	1.5	5.6×10^{-6}	1.4	0.22	–	4.2
eD	1.6	0.00057	1.5	0.18	6.8	–
eH, eD	0.0	3.1×10^{-17}	9.7×10^{-14}	2.5×10^{-15}	–	–

Table 3: Contributions to χ^2 at a reference point $m_X = 50$ MeV. $\Delta\chi^2$ is the difference of χ^2 of the null model ($\alpha_X = 0$) with the best fit. Also shown are the contributions with different observables omitted. Fits are made with the arbitrary value $m_X = 50$ MeV. The columns of χ^2 and $\Delta\chi^2$ are the same as Table1, hence not repeated.

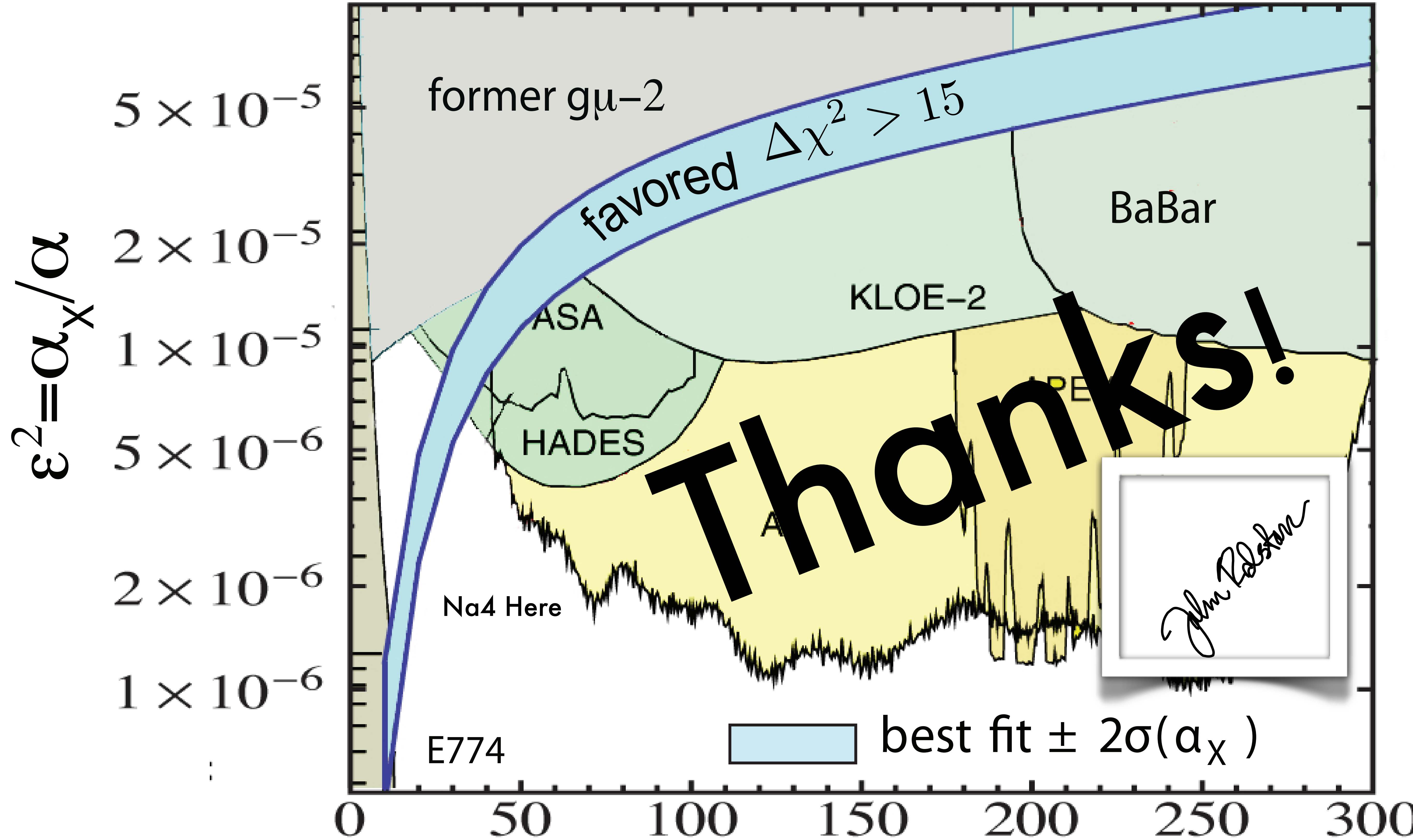
The Minimal Universal Solution
and tests

The minimal-universal solution finds the true proton charge radius $r_p \sim 0.84$ is very close to the one determined by muonic hydrogen experiments. There are no free parameters in a prediction of the muonic deuterium charge radius, whose experimental measurement is expected to be announced soon.

The universal nature of the interaction makes possible many tests that a muon-specific interaction could not confront. Spectroscopic tests include measuring more transitions in muonic hydrogen, detuerium and helium. Electronic hydrogen Rydberg states with $n >> 1$ will appear to indicate two different Rydberg constants. The model predicts effects that should be observable in positronium, muonium ($e^-\mu^+$ and $e^+\mu^-$) and true muonium ($\mu^+\mu^-$). Depending on m_X, the trend is that QED-EE theory will disagree with positronium while agreeing with true muonium, due to the relatively more significant effects of a light interaction on electrons. At the momentum transfer of existing experiments $\mu^\pm p$ and $e^\pm p$ scattering should both find the same apparent charge radius. The pole singularity of X is too small and too close to zero momentum transfer to be resolved with current methods, but might be observable in experiments dedicated to ultra-small momentum transfers. We are optimistic about the prospects for discovery.

Results

No upper limit on m_χ is determined.



Except for Babar color regions are soft exclusion assuming 100% br to e+e=

arXiv:1606.06209[hep-ph]

m_χ(MeV)

Appendix

We do NOT select special atomic data

we could have used 50 LINES

we fit the whole Kramida compilation

One μH is published. *Use it, don't discard it.*

Theory works for ALL eH or eD data as listed

We fit dozens of eH lines to fractions of uncertainty. *Including* μ*H, we get r_p \sim 0.84*

The1S2S is theoretically problematic. Its *theory* uncertainty is 500-1000 times its *experimental* uncertainty

(Karshenboim 2005 criticism)

Sophisticated efforts ("additive corrections") attempt to cover the 1S2S theory unreliability. Results depend directly on priors we don't want to defend *We just omit it.*

That leaves 8 eH and 7+(1 repeated) eD top quality transitions free from messing with 1S2S subtractions

Nevertheless including the 1S2S with existing method *does not change our fits significantly* as CODATA does it, every coupla' years

eH

determines *highly correlated*

$$\mathrm{R}_\infty \quad \mathrm{r}_p(eH)$$

subject to value of α

which is determined by $\mathrm{a}_e = (g_e - 2)/2$

which on its own tests nothing at all

with uncertainty found from m_e/h

dominated by *Rubidium recoil in classical physics...*

and selecting only data that verifies theory

Explore systematic theory uncertainty: chi-square with pull parameters

“additive theory corrections δ_j”

regulator, bayesian prior

$$\frac{(d_j - t_j(\theta_\ell))^2}{\sigma_j^2} \to \frac{(d_j - t_j(\theta_\ell) + \delta_j)^2}{\sigma_j^2} + \frac{\delta_j^2}{\sigma(\delta_j)^2}$$

Barlow

*makes it **easier** to fit data; not appropriate for our study*

most conservative method gives theory no help

when in doubt, leave it out

We repeated calculations including additive corrections and atomic experimental correlations, which made no significant difference

Necessary to deal with eH 1S2S, if one wants to defend some priors.
We tested that OK also. Simpler to omit.

The main reason to
care about ultra-precise
constants... is to find physical
discrepancies...which lead to the
exploration of alternatives

How theory is not tested

model of the elementary particles. On the experimental side the measurement of a_e by the Harvard group has reached the astonishing precision [1,2]:

$$a_e(\mathrm{HV}) = 1\,159\,652\,180.73(0.28) \times 10^{-12} \qquad [0.24\ \mathrm{ppb}]. \tag{1}$$

sentences are not about a "comparison"

The Eq. (13) shows clearly that the largest source of uncertainty is the fine-structure constant (12). To put it differently, it means that a non-QED α, even the best one available at present, is too crude to test QED to the extent achieved by the theory and measurement of a_e. Thus it makes more sense to test QED by an alternative approach, namely, compare $\alpha^{-1}(\mathrm{Rb10})$ with α^{-1} obtained from theory and measurement of a_e. This leads to

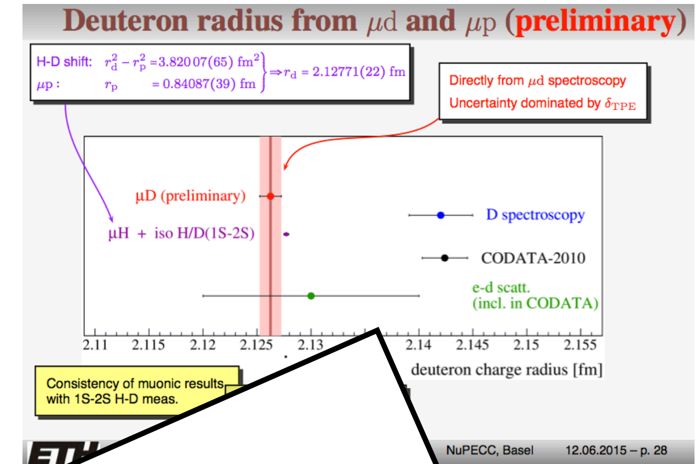
This →

$$\alpha^{-1}(a_e) = 137.035\,999\,1727(68)(46)(19)(331) \quad [0.25\ \mathrm{ppb}], \tag{15}$$

where the first, second, third, and fourth uncertainties come from the eighth-order and the tenth-order QED terms, the hadronic and electroweak terms, and the measurement of $a_e(\mathrm{HV})$ in (1), respectively. The uncertainty due to theory has been improved by a factor 4.5 compared with the previous one [22].

This $\alpha^{-1}(a_e)$ is a circular determination of α

ACTUALLY five (5) different muonic Lamb shift discrepancies in H and D



At least two (2) measured μH transitions

rium(

At least three (3) measured μD t

MA

preliminary)

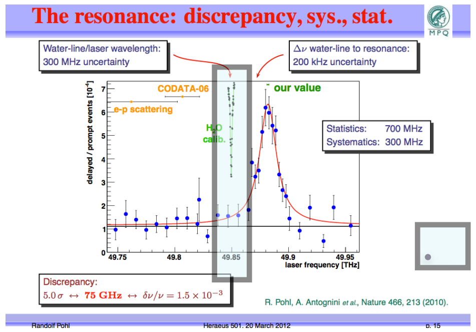
...plus many ordi

BANNED

$$eD = r^2_{eH} + r^2_{deut}$$

over-determined consistency

Analysis overview IV



μH: the most complete and reliable theory

Pachucki, Borie, Jentsura, Yerokin, Carlson, Miller...

$$\Delta E(\alpha,\,\xi,\,m_X,\,m_{red},\,r_p) = 206.0336\alpha^3/\alpha_\bullet^3 - 5.2275 r_p^2 \alpha^4/\alpha_\bullet^4$$
$$+\,0.0332 + 10^9 (m_X^4 \xi)/(2\alpha m_{red}(1 + m_X/(\alpha m_{red}))^4)$$

m_{red} = μp reduced mass

$$V(x) = \alpha_X \frac{e^{-m_X r}}{4\pi r}.$$

Interesting fact these days

The electron anomalous moment has come to define alpha, testing nothing, so both might be wrong

$$a_e^{theory} = 1.7147 \times 10^{-12} + 0.159155\alpha - 0.0332818\alpha^2 + 0.0380966\alpha^3 - 0.0196046\alpha^4 + 0.0299202\alpha^5$$

(OK, one 10 times weaker and one 100 weaker constraints do exist)

This

$\alpha^{-1}(a_e)$

is a circular determination of α →

The Eq. (13) shows clearly that the largest source of uncertainty is the fine-structure constant (12). To put it differently, it means that a non-QED α, even the best one available at present, is too crude to test QED to the extent achieved by the theory and measurement of a_e. Thus it makes more sense to test QED by an alternative approach, namely, compare α^{-1}(Rb10) with α^{-1} obtained from theory and measurement of a_e. This leads to

$$\alpha^{-1}(a_e) = 137.0359991727(68)(46)(19)(331) \quad [0.25\,\text{ppb}], \tag{15}$$

where the first, second, third, and fourth uncertainties come from the eighth-order and the tenth-order QED terms, the hadronic and electroweak terms, and the measurement of a_e(HV) in (1), respectively. The uncertainty due to theory has been improved by a factor 4.5 compared with the previous one [22].

REVIEWS OF MODERN PHYSICS, VOLUME 84, OCTOBER–DECEMBER 2012

CODATA recommended values of the fundamental physical constants: 2010*

Peter J. Mohr,† Barry N. Taylor,‡ and David B. Newell§

National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

Electron magnetic-moment anomaly, fine-structure constant, and QED. The most accurate value of the fine-structure constant α currently available from a single experiment has a relative standard uncertainty of 3.7×10^{-10}; it is obtained by equating the QED theoretical expression for the electron magnetic-moment anomaly a_e and the most accurate experimental value of a_e, obtained from measurements on a single electron in a Penning trap. This value of α is in excellent agreement with a competitive experimental value with an

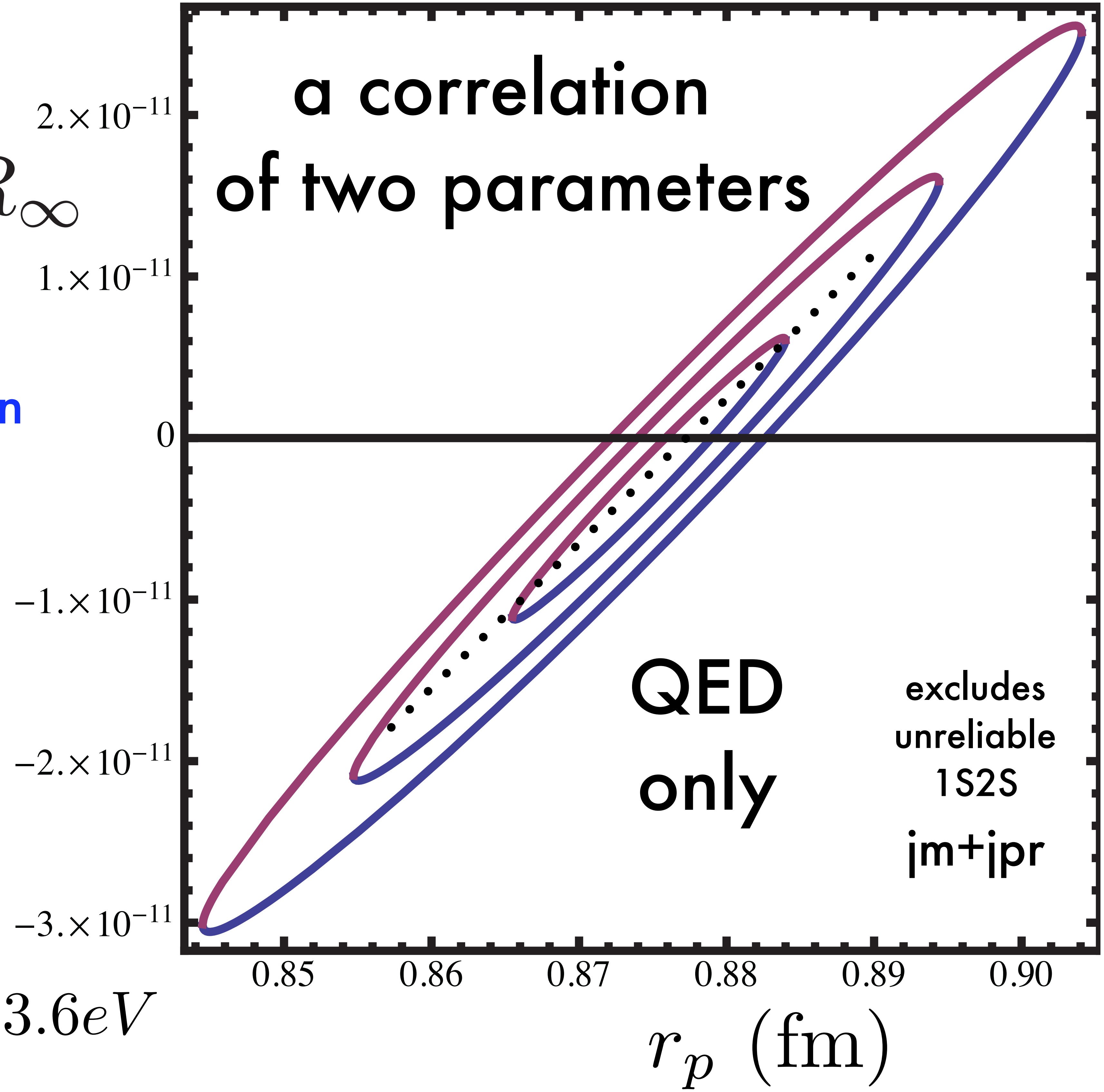
"Global fits" to α same as circular for 25 years

theory mistakes cause 4 sigma and 7 sigma revisions

e-Hydrogen spectra measures a correlation... a correlation...

$$r_p^2 = -\frac{1}{6}\frac{\partial G_E}{\partial q^2}\Big|_0$$

$$\delta R_\infty / R_\infty$$



"Rydberg uncertainty" omits needed information

σ_{expt} Hz	f_{expt} Hz	$f_{our\,calc}$ Hz
35	$2.46606141319 \times 10^{15}$	$2.46606141319 \times 10^{15}$
10074	4.797338×10^{9}	$4.79733066539 \times 10^{9}$
24014	6.490144×10^{9}	$6.49012898284 \times 10^{9}$
8477	$7.70649350012 \times 10^{14}$	$7.70649350016 \times 10^{14}$
8477	$7.7064950445 \times 10^{14}$	$7.70649504449 \times 10^{14}$
6396	$7.70649561584 \times 10^{14}$	$7.70649561578 \times 10^{14}$
9590	$7.99191710473 \times 10^{14}$	$7.99191710481 \times 10^{14}$
6953	$7.99191727404 \times 10^{14}$	$7.99191727409 \times 10^{14}$
12860	$2.92274327868 \times 10^{15}$	$2.92274327867 \times 10^{15}$
20568	4.197604×10^{9}	$4.19759919778 \times 10^{9}$
10338	4.699099×10^{9}	$4.6991043085 \times 10^{9}$
14926	4.664269×10^{9}	$4.66425337748 \times 10^{9}$
10260	6.035373×10^{9}	$6.03538320383 \times 10^{9}$
11893	9.9112×10^{9}	$9.91119855042 \times 10^{9}$
8992	1.057845×10^{9}	$1.05784298986 \times 10^{9}$
20099	1.057862×10^{9}	$1.05784298986 \times 10^{9}$

$$R_\infty = \text{Rydberg} \sim 13.6eV$$

We do NOT select special atomic data

we could have used 50 LINES

we fit the whole Kramida compilation

One μH is published. *Use it, don't discard it.*

Theory works for ALL eH or eD data as listed

We fit dozens of eH lines to fractions of uncertainty. *Including* μ*H, we get r_p \sim 0.84*

The1S2S is theoretically problematic. Its *theory* uncertainty is 500-1000 times its *experimental* uncertainty

(Karshenboim 2005 criticism)

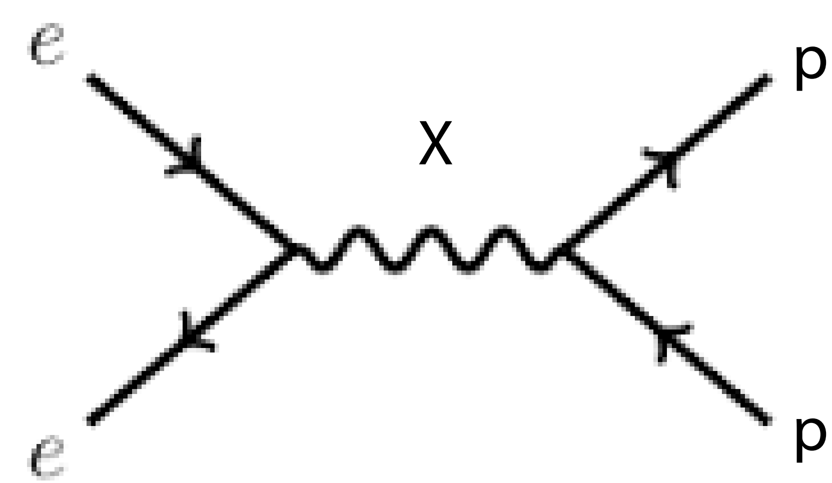
Sophisticated efforts ("additive corrections") attempt to cover the 1S2S theory unreliability. Results depend directly on priors we don't want to defend *We just omit it.*

That leaves 8 eH and 7+(1 repeated) eD top quality transitions free from messing with 1S2S subtractions

Nevertheless including the 1S2S with existing method *does not change our fits significantly*

Workshop question: does repulsion rule our vertex = scalar 1?

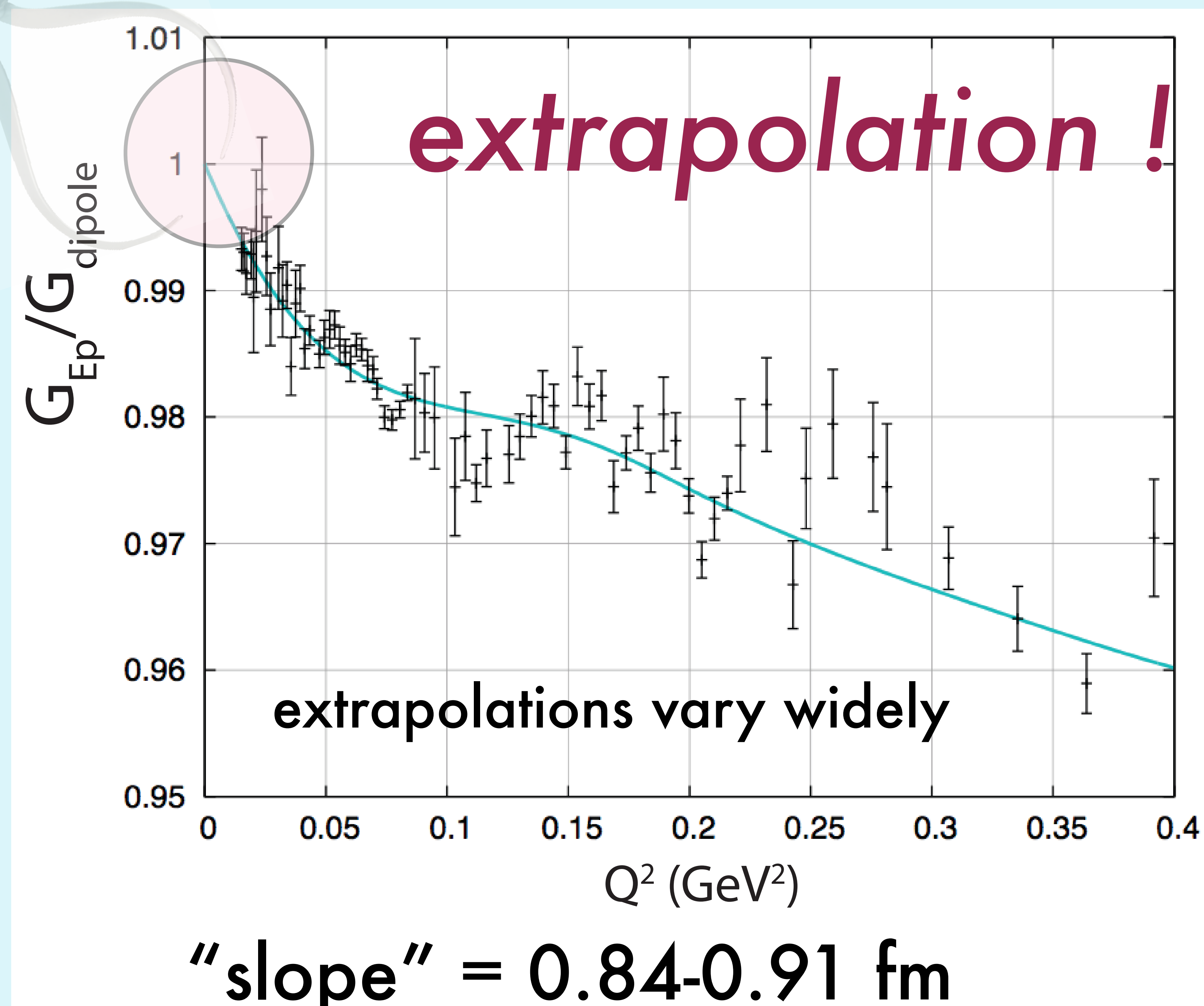
unitarity: sign of scalar propagator is minus the sign of vector propagator, (1,-1,-1,-1) metric



implies scalar attractive for identical particles or antiparticles, using g^2 >0

widely cited and widely repeated as totally general, which I think it ain't

electron scattering has not measured $r_p^2 = -\frac{1}{6}\frac{\partial G_E}{\partial q^2}\Big|_0$



especially see Higinbotham

fit from Vanderhagen & Walcher

is used for vertical focusing [8], the angular frequency difference, ω_a between the spin precession frequency and the cyclotron frequency, is given by

$$\vec{\omega}_a = \frac{e}{m\,c}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]. \qquad (1)$$

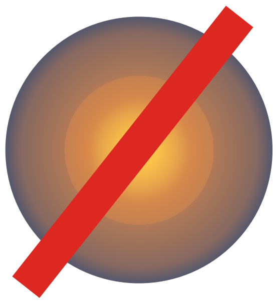
The dependence of ω_a on the electric field is eliminated by storing muons with the "magic" $\gamma = 29.3$ [9], which corresponds to a muon momentum $p = 3.09$ GeV/c. Hence measurement of ω_a and of B, in terms of the free proton NMR frequency ω_p and the ratio of muon to proton magnetic moments λ, determines a_μ. At the magic γ, the muon lifetime is approximately 64.4 μs and the $(g-2)$ precession period is 4.37 μs. With a field of 1.45 T in our storage ring [4], the central orbit radius is 7.11 m.

$$a_{\mu^-} = 11\,659\,214(8)(3)\times 10^{-10}\ \ (0.7\,\text{ppm}),$$

About 4-sigma discrepancy with QED/EW perturbation theory

From Bennet et al muon g-2

Why believe nuclear physics?



Please! The proton is
not a little ball
of classical charge

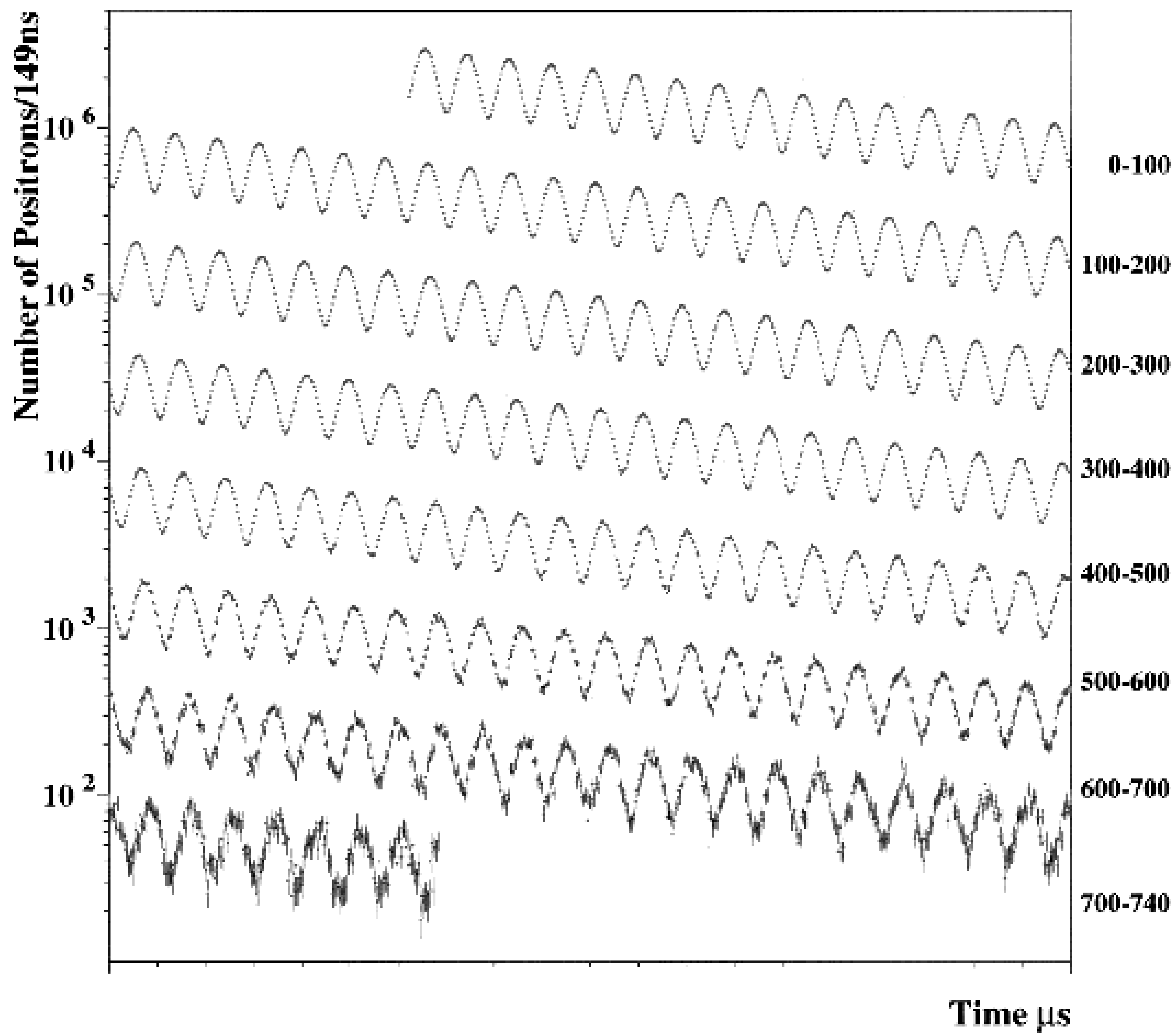
$$r_d^2 = r_p^2 + r_{deut}^2$$

You don't need this
when r_d is measured twice

μD error bars will beat eD

At least in our consistent picture,
r_{deut} is over-determined

During the muon storage runs the trolley was withdrawn into a special garage, (also under vacuum) and the field was monitored by 350 fixed NMR probes deployed above and below the vacuum chamber. The average field calculated from these fixed probes tracked with the average meaured by the trolley to within ±0.2 ppm.



Francis Farley

Very expensive classical physics experiment

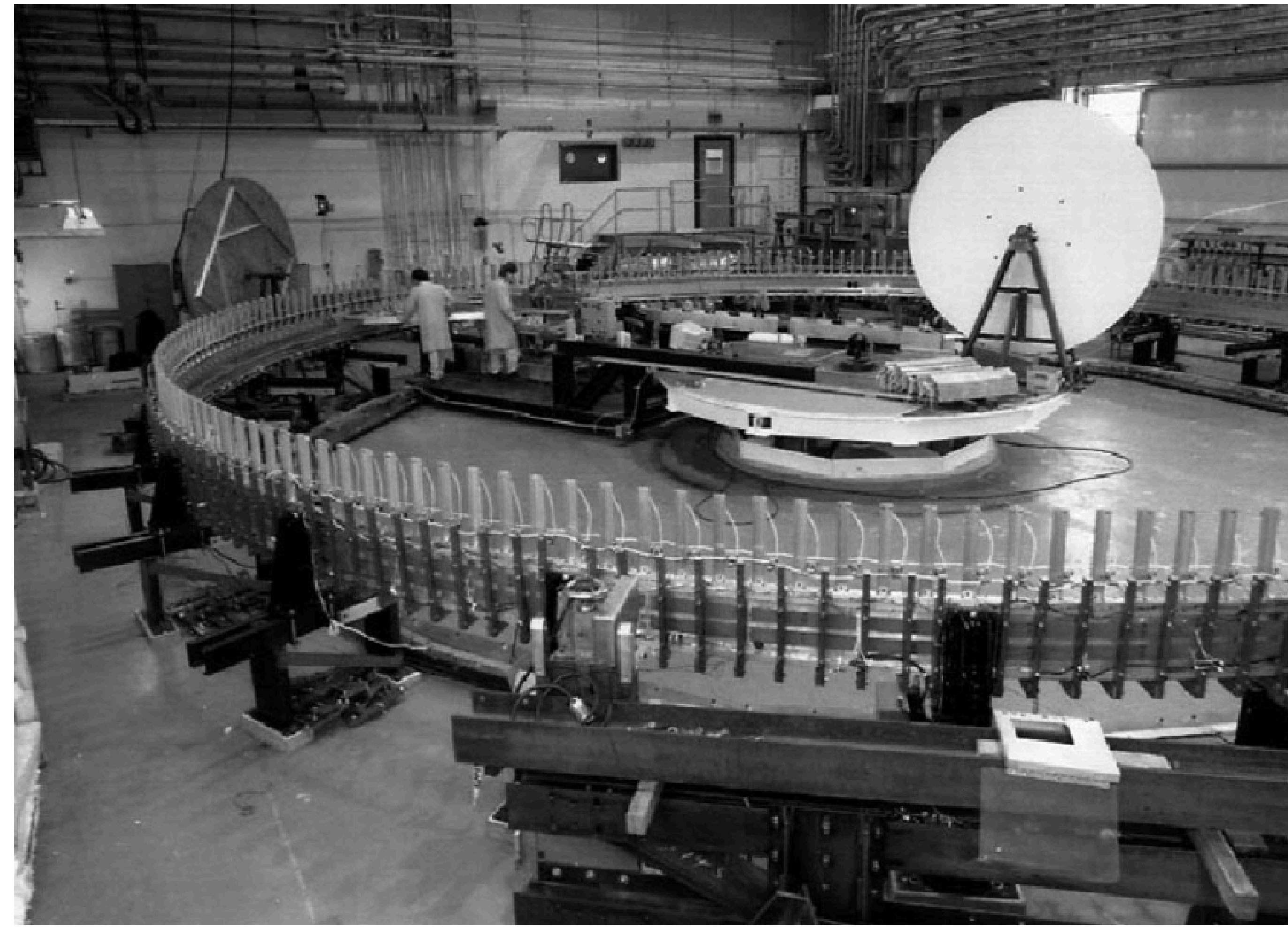


Fig. 7. The coil winding fixture, shown on the turntable. An inner coil is being wound.

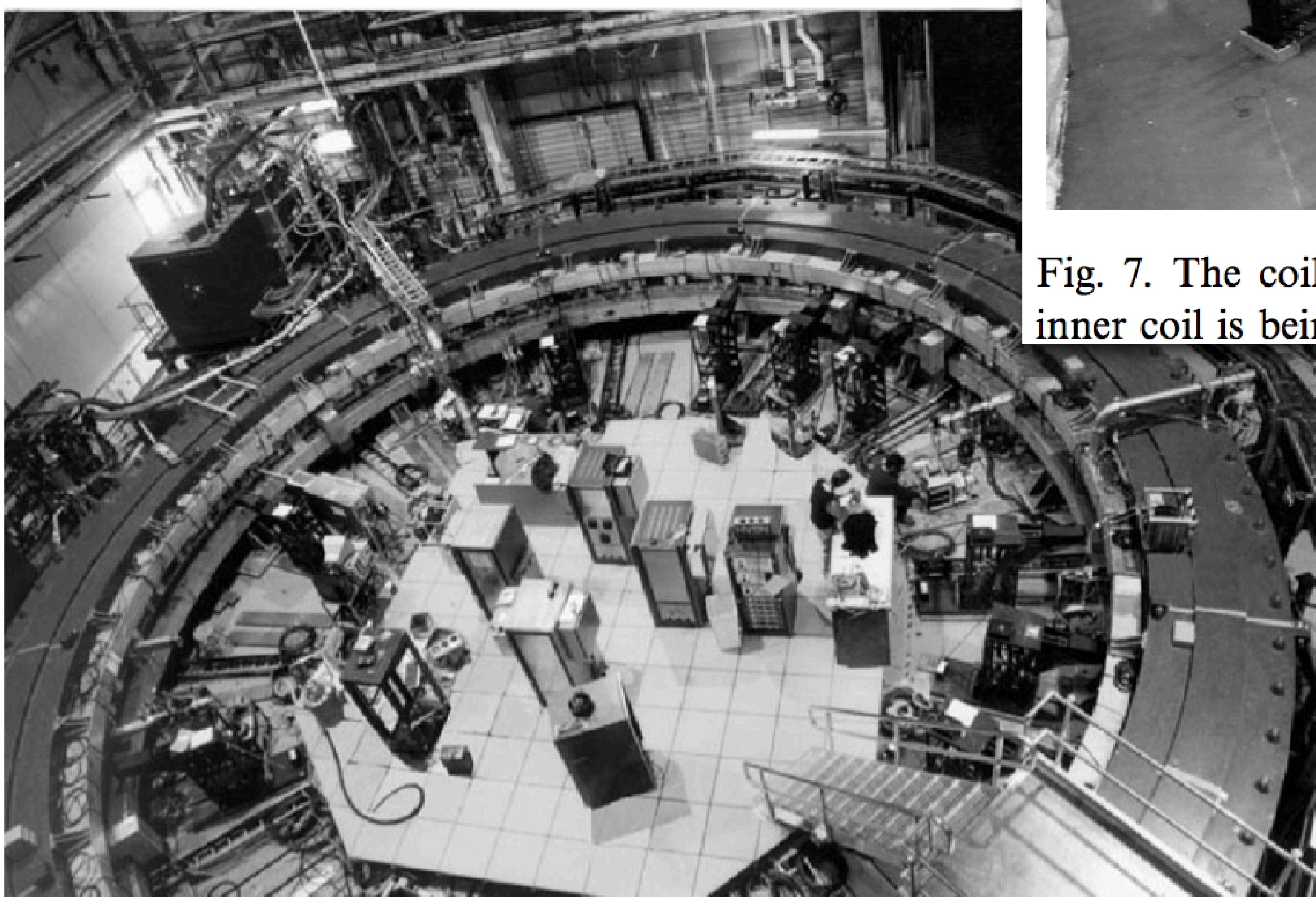


Fig. 1. Photograph of the muon g-2 storage ring magnet at

Gerry Bunce

Francis Farley (FRS) signed my superconductor

In 2010 comes the muonic lamb shift anomaly

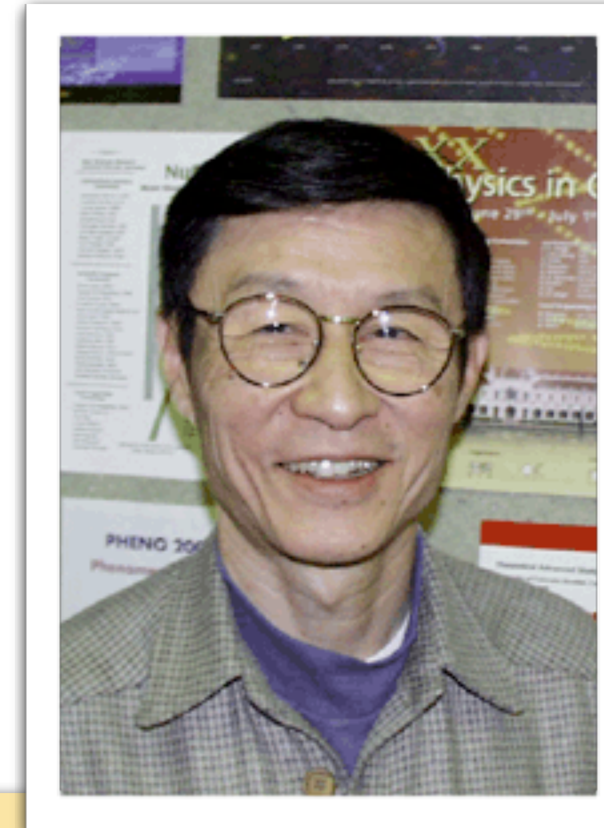
K. Pachucki, 1995, unknowingly used *Tung's Uncertainty Principle* to suggest muonic hydrogen *measurement* of *proton charge radius*.

Objective was to "improve the determination of the Rydberg constant"

12 digit precision was good, but not good enough...

Tung's Uncertainty Principle:

$$\Delta\mathcal{E}\Delta t \gtrsim 1$$



$\Delta\mathcal{E}$ = difficulty of experiment

Δt = difficulty of theory

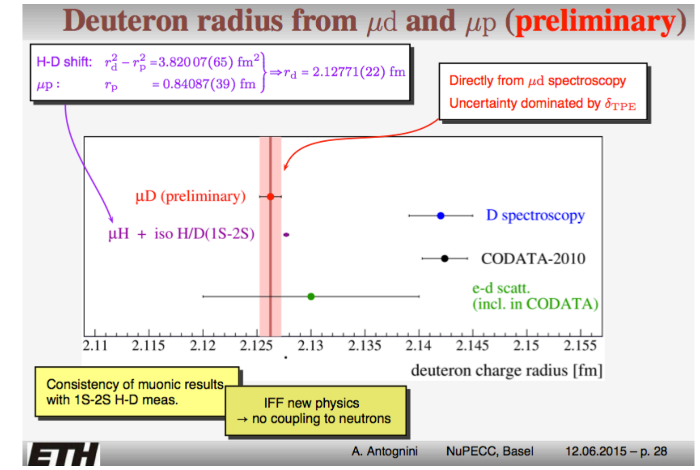
$$\Delta t(electronic\, hydrogen) >> 1$$ harder theory, easier experiment

$$\Delta\mathcal{E}(muonic\, hydrogen) >> 1$$ easier theory, harder experiment

Wu-Ki says: *to make theory easier, choose harder experiment*

Lamb shift in muonic atoms is easy theory !

ACTUALLY five (5) different muonic Lamb shift discrepancies in H and D



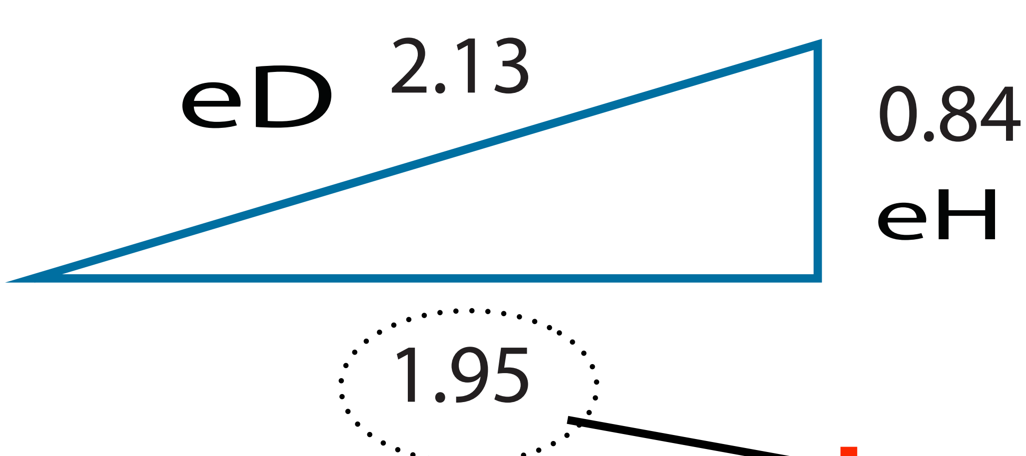
At least two (2) measured μH transitions

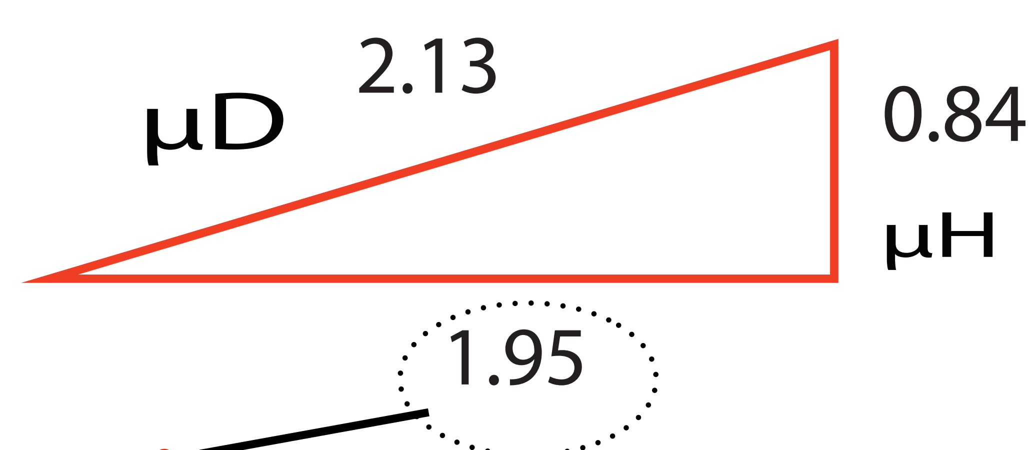
D=deuterium
(CREMA preliminary)

At least three (3) measured μD transitions

...plus many ordinary eH, eD Lamb shifts...

classic relation $$r_{eD}^2 = r_{eH}^2 + r_{deut}^2$$





over-determined consistency

About 200 papers explore ideas

*PRA **81**, 060501 (2010) JETP Lett. **92**, 8 (2010) PRL **105**, 242001 (2010) PRD **82**, 125020 (2010) PLB **693**, 555 (2010) Nucl.Phys.News **21**, 14 (2011) Can. J. Phys. **89**, 109 (2011) PRC **83**, 012201(R) (2011) PRD **82**, 113005 (2010) PLB **697**, 26 (2011) PLB **696**, 343 (2011) EPJD **61**, 7 (2011) PRL **106**, 153001 (2011) PRA **83**, 012507 (2011) PRD **83**, 101702(R) (2011) Ann. Phys. **326**, 500 (2011) Ann. Phys. **326**, 516 (2011) Few-Body Syst. **50**, 367 (2011) PRD **83**, 035020 (2011) PRA **83**, 042509 (2011) PRL **106**, 193007 (2011) PRL **107**, 011803 (2011) PRA **84**, 012506 (2011) PRA **84**, 012505 (2011) PRA **84**, 020101(R) (2011) PRA **84**, 020102(R) (2011)Karshenboim et al.: "Nonrelativistic contributions of order $\alpha 5 m \mu c 2$ to the Lamb shift in muonic ..." (1005.4879) Karshenboim et al.: "Contribution of light-by-light scattering to energy levels of light muonic atoms" (1005.4880) Bernauer et al: "High-precision determination of the electric and magnetic form factors of the proton" (1007.5076) Jaeckel, Roy: "Spectroscopy as a test of Coulomb's law" (1008.3536) De Rujula: "QED is not endangerec by the proton's size" (1008.3861) Vanderhaeghen, Walcher: "Long range structure of the nucleon" (1008.4225) Jentschura: "From first principles of QED to an application: hyperfine structure of P states of muonic hydrogen" Cloet, Miller: "Third Zemach moment of the proton" (1008.4345) Hill, Paz: "Model-independent extraction of the proton charge radius from electron scattering" (1008.4619) De Rujula: "QED confronts the proton's radius" (1010.3421) Distler et al: "The RMS radius of the proton and Zemach moments" (1011.1861) Jentschura: "Proton radius, Darwin-Foldy tern and radiative corrections" (1012.4029) Barger, Chiang, Keung, Marfatia: "Proton size anomaly" (1011.3519) Yerokhin: "Nuclea size corrections to the Lamb shift of one-electron atoms" (1011.4272) Tucker-Smith, Yavin: "Muonic hydrogen and MeV forces" (1011.4922)Jentschura: "Lamb shift in muonic hydrogen I: Verification and update of theoretical predictions" (1011.527 Jentschura: "Lamb shift in muonic hydrogen II: Analysis of the discrepancy of theory and experiment" (1011.5453) Sick: "Troubles with the proton rms radius" Brax, Burrage: "Atomic precision tests and light scalar couplings" (1010.5108) Carlson et al.: "Proton-structure corrections to hyperfine splitting in muonic hydrogen" (1101.3239) Pachucki: "Nuclear structur corrections in muonic deuterium" (1102.3296) Batell, McKeen, Pospelov: "New parity-violating muonic forces and the proton charge radius" (1102.3296) Carroll et al.: "Nonperturbative relativistic calculation of the muonic hydrogen spectrum" (1104.297 Jentschura: "Relativistic reduced-mass and recoil corrections to vacuum polarization in muonic hydrogen, ..." (1107.1737) Miller, Thomas, Carroll, Rafelski: "Toward a resolution of the proton size puzzle" (1101.4073) Carlson, Vaderhaeghen: "Higher-order proton structure corrections to the Lamb shift in muonic hydrogen" (1101.5965)....*

One astonishing QED prediction now explained

Jentschura, Kotochigova, LeBigot, Mohr, Taylor

PRL **95,** 163003 (2005) PHYSICAL REVIEW LETTERS week ending 14 OCTOBER 2005

TABLE I. Transition frequencies in hydrogen ν_H and in deuterium ν_D used in the 2002 CODATA least-squares adjustment of the values of the fundamental constants and the calculated values. Hyperfine effects are not included in these values.

Experiment	Frequency interval(s)	Reported value ν/kHz	Calculated value ν/kHz
Niering *et al.* [1]	$\nu_H(1S_{1/2} - 2S_{1/2})$	2 466 061 413 187.103(46)	2 466 061 413 187.103(46)
Weitz *et al.* [2]	$\nu_H(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	4 797 338(10)	4 797 331.8(2.0)
	$\nu_H(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	6 490 144(24)	6 490 129.9(1.7)
	$\nu_D(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_D(1S_{1/2} - 2S_{1/2})$	4 801 693(20)	4 801 710.2(2.0)
	$\nu_D(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_D(1S_{1/2} - 2S_{1/2})$	6 494 841(41)	6 494 831.5(1.7)

$$\sigma_{theory} << \sigma_{expt}$$

1S2S exact agreement experiment v calculated

"However, one thing can be stated with certainty: the exact agreement of those two ultra-precise 1S2S measurements with the QED calculations cannot be considered as a confirmation of the QED theory, because it is the result of the fitting of the fundamental constants based on these (and other) transitions." A. Kramida, Atomic Data and Nuclear Data Tables, 96, 586 (2010)

`` the values of the constants... are correlated, particularly those for R_{∞} and r_{p}... The uncertainty of the calculated value for the $1s-2s$ frequency in hydrogen is increased by a factor of about 500 if such correlations are neglected."

Okay. 500 x 46 Hz = 23000 Hz theory uncertainty

PRL 109, 111807 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

Tenth-Order QED Contribution to the Electron $g-2$ and an Improved Value of the Fine Structure Constant

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[2]*Nishina Center, RIKEN, Wako, Japan 351-0198*

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(Received 24 May 2012; published 13 September 2012)

This Letter presents the complete QED contribution to the electron $g-2$ up to the tenth order. With the help of the automatic code generator, we evaluate all 12 672 diagrams of the tenth-order diagrams and obtain $9.16(58)(\alpha/\pi)^5$. We also improve the eighth-order contribution obtaining $-1.9097(20)(\alpha/\pi)^4$, which includes the mass-dependent contributions. These results lead to $a_e(\text{theory}) = 1\,159\,652\,181.78(77) \times 10^{-12}$. The improved value of the fine-structure constant $\alpha^{-1} = 137.035\,999\,173\,(35)$ [0.25 ppb] is also derived from the theory and measurement of a_e.

alpha has been found circularly in the tables

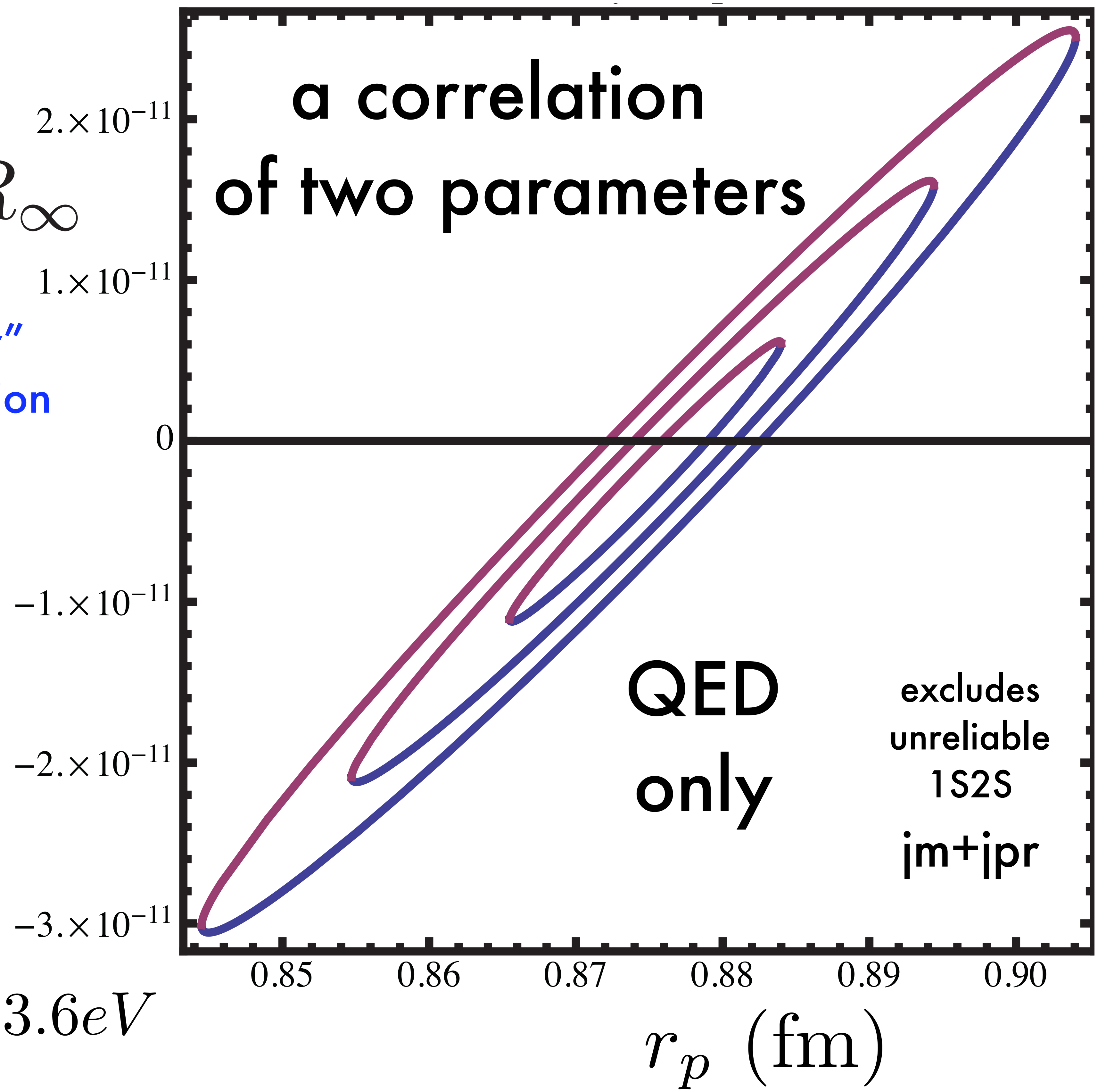
To compare the theoretical prediction with the measurement (1), we need the value of the fine-structure constant α determined by a method independent of $g-2$. The best α available at present is the one obtained from the measurement of h/m_{Rb} [35], combined with the very precisely known Rydberg constant and m_{Rb}/m_e [3] :

$$\alpha^{-1}(\text{Rb10}) = 137.035\,999\,049\,(90) \quad [0.66\,\text{ppb}]. \qquad (12)$$

e-Hydrogen spectra measures a correlation... a correlation...

$$r_p^2 = -\frac{1}{6}\frac{\partial G_E}{\partial q^2}\Big|_0$$

$$\delta R_\infty / R_\infty$$



"Rydberg uncertainty"
omits needed information

σ_{expt} Hz	f_{expt} Hz	$f_{our\,calc}$ Hz
35	$2.46606141319 \times 10^{15}$	$2.46606141319 \times 10^{15}$
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8992	1.057845×10^{9}	$1.05784298986 \times 10^{9}$
20099	1.057862×10^{9}	$1.05784298986 \times 10^{9}$

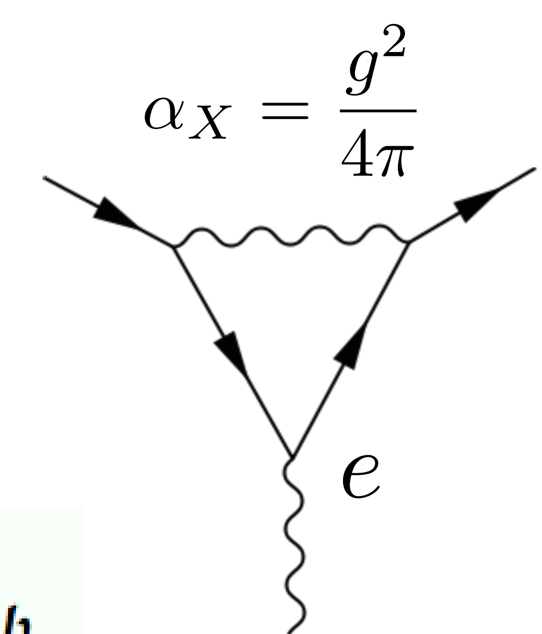
$$R_\infty = \text{Rydberg} \sim 13.6 eV$$

null model: the ENTIRE BODY of atomic QED and Standard Model calculations

test model: the null plus "universal coupling" of X to charge

$$V(x) = \alpha_X \frac{e^{-m_X r}}{4\pi r}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m_X^2}{2}A^2 + \bar{\psi}(i\partial\!\!\!/ - g_X A\!\!\!/)\psi$$

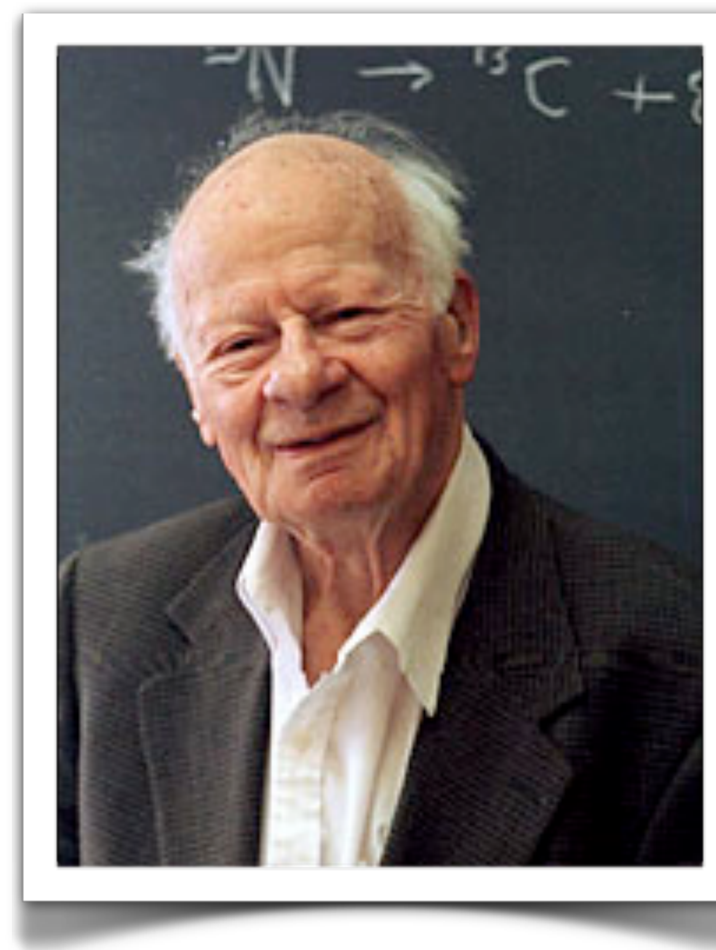


which hypothesis wins?

How to speak Atomic

$$E \neq h\nu$$

* natural units are frequency. It's what's measured
* planck's constant errors are unacceptably large
* ground state frequency $R_\infty c = 3 \times 10^{15}$ Hz
* proton size effect 1.5 Mhz
* To measure size to 0.1% needs 1 kHz theory errors



the term "Lamb shift" can mean the particular splitting of one transition observed by Willis Lamb in 1945, or it (more often) means everything beyond the bound state prediction of the Dirac equation as relativistic quantum mechanics...not quantum field theory

TABLE XVIII. Summary of principal input data for the determination of the 2010 recommended value of the Rydberg constant R_∞.

Item No.	Input datum	Value	Relative standard uncertainty[a] u_r	Identification	Sec.
$A1$	$\delta_H(1S_{1/2})$	0.0(2.5) kHz	$[7.5 \times 10^{-13}]$	Theory	IV.A.1.l
$A2$	$\delta_H(2S_{1/2})$	0.00(31) kHz	$[3.8 \times 10^{-13}]$	Theory	IV.A.1.l
$A3$	$\delta_H(3S_{1/2})$	0.000(91) kHz	$[2.5 \times 10^{-13}]$	Theory	IV.A.1.l
$A4$	$\delta_H(4S_{1/2})$	0.000(39) kHz	$[1.9 \times 10^{-13}]$	Theory	IV.A.1.l
$A5$	$\delta_H(6S_{1/2})$	0.000(15) kHz	$[1.6 \times 10^{-13}]$	Theory	IV.A.1.l
$A6$	$\delta_H(8S_{1/2})$	0.0000(63) kHz	$[1.2 \times 10^{-13}]$	Theory	IV.A.1.l
$A7$	$\delta_H(2P_{1/2})$	0.000(28) kHz	$[3.5 \times 10^{-14}]$	Theory	IV.A.1.l
$A8$	$\delta_H(4P_{1/2})$	0.0000(38) kHz	$[1.9 \times 10^{-14}]$	Theory	IV.A.1.l
$A9$	$\delta_H(2P_{3/2})$	0.000(28) kHz	$[3.5 \times 10^{-14}]$	Theory	IV.A.1.l
$A10$	$\delta_H(4P_{3/2})$	0.0000(38) kHz	$[1.9 \times 10^{-14}]$	Theory	IV.A.1.l
$A11$	$\delta_H(8D_{3/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A12$	$\delta_H(12D_{3/2})$	0.000 00(13) kHz	$[5.7 \times 10^{-15}]$	Theory	IV.A.1.l
$A13$	$\delta_H(4D_{5/2})$	0.0000(35) kHz	$[1.7 \times 10^{-14}]$	Theory	IV.A.1.l
$A14$	$\delta_H(6D_{5/2})$	0.0000(10) kHz	$[1.1 \times 10^{-14}]$	Theory	IV.A.1.l
$A15$	$\delta_H(8D_{5/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A16$	$\delta_H(12D_{5/2})$	0.000 00(13) kHz	$[5.7 \times 10^{-15}]$	Theory	IV.A.1.l
$A17$	$\delta_D(1S_{1/2})$	0.0(2.3) kHz	$[6.9 \times 10^{-13}]$	Theory	IV.A.1.l
$A18$	$\delta_D(2S_{1/2})$	0.00(29) kHz	$[3.5 \times 10^{-13}]$	Theory	IV.A.1.l
$A19$	$\delta_D(4S_{1/2})$	0.000(36) kHz	$[1.7 \times 10^{-13}]$	Theory	IV.A.1.l
$A20$	$\delta_D(8S_{1/2})$	0.0000(60) kHz	$[1.2 \times 10^{-13}]$	Theory	IV.A.1.l
$A21$	$\delta_D(8D_{3/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A22$	$\delta_D(12D_{3/2})$	0.000 00(13) kHz	$[5.6 \times 10^{-15}]$	Theory	IV.A.1.l
$A23$	$\delta_D(4D_{5/2})$	0.0000(35) kHz	$[1.7 \times 10^{-14}]$	Theory	IV.A.1.l
$A24$	$\delta_D(8D_{5/2})$	0.000 00(44) kHz	$[8.5 \times 10^{-15}]$	Theory	IV.A.1.l
$A25$	$\delta_D(12D_{5/2})$	0.000 00(13) kHz	$[5.7 \times 10^{-15}]$	Theory	IV.A.1.l
$A26$	$\nu_H(1S_{1/2} - 2S_{1/2})$	2 466 061 413 187.080(34) kHz	1.4×10^{-14}	MPQ-04	IV.A.2
$A27$	$\nu_H(1S_{1/2} - 3S_{1/2})$	2 922 743 278 678(13) kHz	4.4×10^{-12}	LKB-10	IV.A.2
$A28$	$\nu_H(2S_{1/2} - 8S_{1/2})$	770 649 350 012.0(8.6) kHz	1.1×10^{-11}	LK/SY-97	IV.A.2
$A29$	$\nu_H(2S_{1/2} - 8D_{3/2})$	770 649 504 450.0(8.3) kHz	1.1×10^{-11}	LK/SY-97	IV.A.2
$A30$	$\nu_H(2S_{1/2} - 8D_{5/2})$	770 649 561 584.2(6.4) kHz	8.3×10^{-12}	LK/SY-97	IV.A.2
$A31$	$\nu_H(2S_{1/2} - 12D_{3/2})$	799 191 710 472.7(9.4) kHz	1.2×10^{-11}	LK/SY-98	IV.A.2
$A32$	$\nu_H(2S_{1/2} - 12D_{5/2})$	799 191 727 403.7(7.0) kHz	8.7×10^{-12}	LK/SY-98	IV.A.2

here additive theory adjustments are called "principal data"