The QCD equation of state at high temperatures

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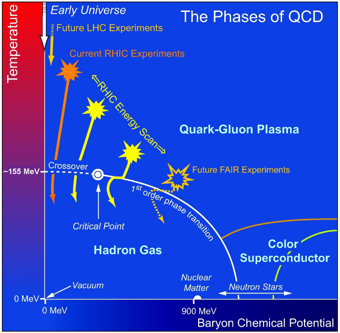
QCD phase diagram



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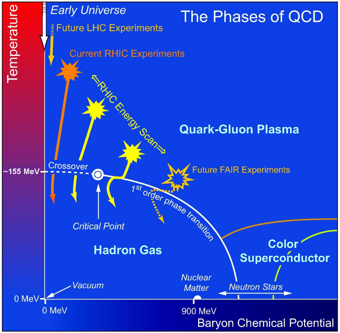

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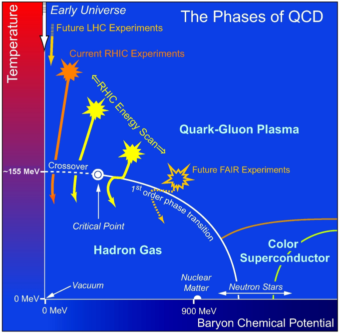

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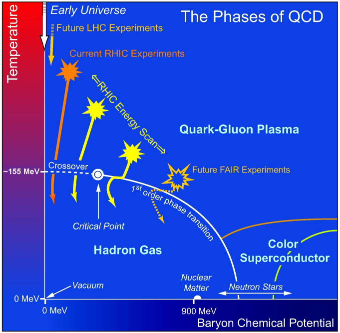

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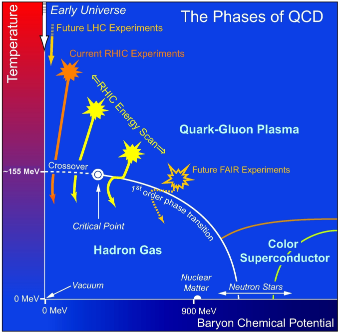

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- Experimental program: RHIC, LHC, FAIR, NICA
- High-temperature phase: deconfinement, restoration of chiral symmetry
- QCD equation of state at zero baryon density has been recently calculated up to $T = 400$ MeV

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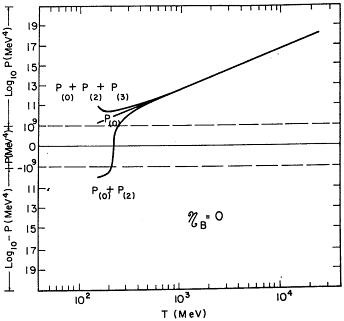
Earlier results on the equation of state



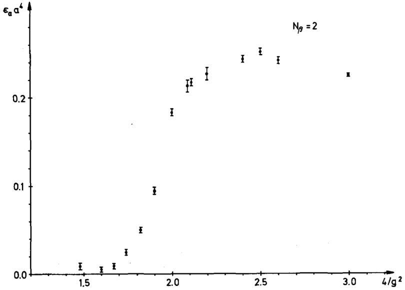


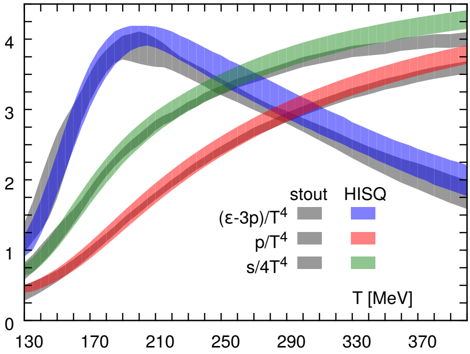
Fig. 3. Energy density of gluon matter versus $4/g^2$, at fixed lattice size $N_\beta = 2$, after about 500 iterations.

- First perturbative EoS calculation[2] (left)
- First lattice pure gauge $SU(2)$ EoS calculation[3] (right)

[2] Kapusta (1979)

[3] Engels et al. (1981)

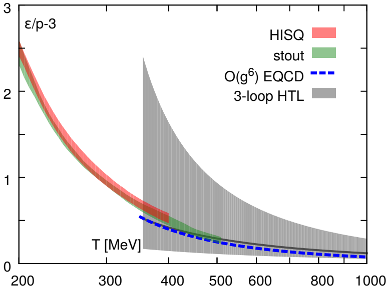
Recent results up to $T = 400$ MeV



- Comparison of the continuum results with HISQ[4] and stout[5] for the trace anomaly, pressure and entropy density
- About 2σ deviations in the integrated quantities at the highest temperature

[4]Bazavov et al. [HotQCD] (2014)
[5]Borsanyi et al. [WB] (2014)

Approach to the perturbative limit



- The ratio of the trace anomaly and the pressure

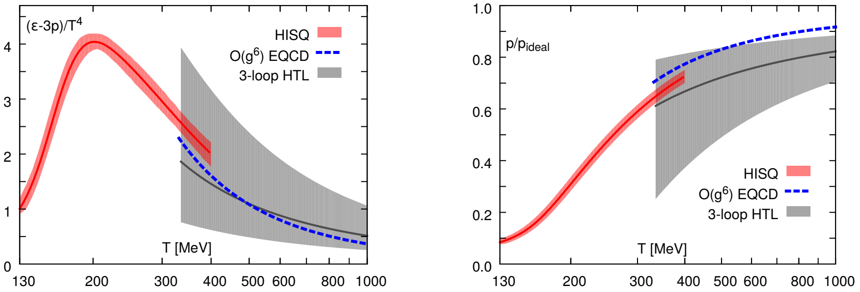
$$\frac{\Theta^{\mu\mu}}{p} = \frac{\epsilon}{p} - 3$$

compared with perturbative calculations in the Hard Thermal Loop (HTL)[6] and Electrostatic QCD (EQCD)[7] schemes

[6] Haque et al. (2014)

[7] Laine and Schroder (2006)

Approach to the perturbative limit



- The trace anomaly (left) and pressure (right) compared with HTL and EQCD calculations
- The black line is the HTL calculation with the renormalization scale $\mu = 2\pi T$

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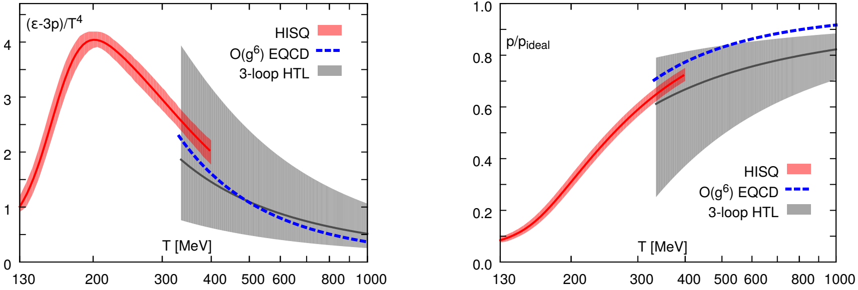

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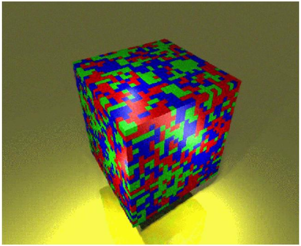
- Need to extend the lattice equation of state to higher temperature - THIS TALK

Lattice QCD



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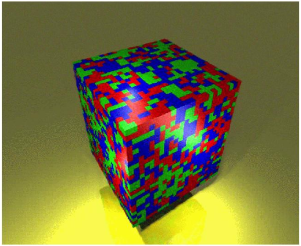

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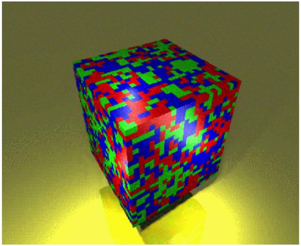

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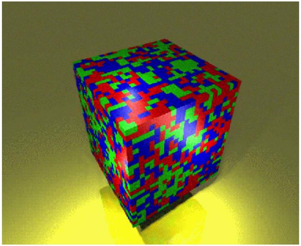

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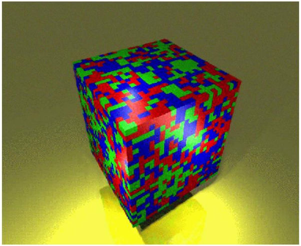

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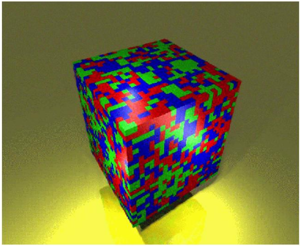

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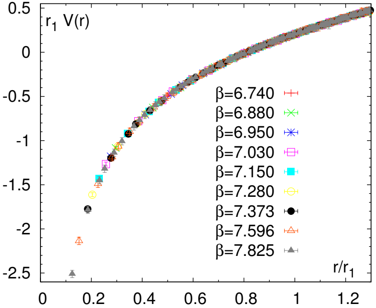
- Fix N_τ, dial the lattice spacing to cover a temperature range
- The continuum limit is reached as $1/N_\tau \to 0$

Static quark potential and setting the scale



- Fit the static quark potential to the form:

$$V(r) = C + \frac{B}{r} + \sigma r$$

- Define an interpolating quantity, r_1:

$$r^2 \frac{dV}{dr}\bigg|_{r_1} = 1$$

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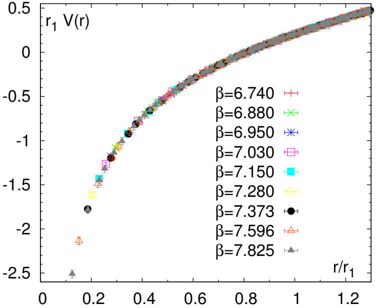

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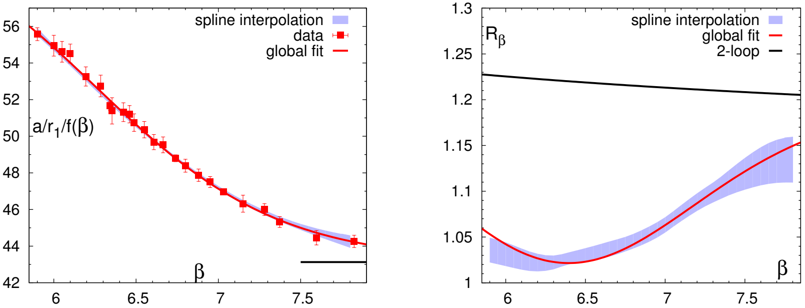
- The physical value $r_1 = 0.3106(14)(8)(4)$ fm
- Measuring r_1/a allows one to define the lattice spacing
- Other choices of scale setting are, of course, possible, *e.g.* f_K, m_Ω, w_0, etc.

Setting the scale



$$\frac{a}{r_1} = \frac{c_0 f(\beta) + c_2 (10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)}\,, \quad f(\beta) = \left(\frac{10 b_0}{\beta}\right)^{-b_1/(2b_0^2)} \exp(-\beta/(20 b_0))$$

$$R_\beta = -a\frac{d\beta}{da} = \frac{r_1}{a}\left(\frac{d(r_1/a)}{d\beta}\right)^{-1}\,, \quad R_\beta^{2-\mathrm{loop}} = 20 b_0 + 200 b_1/\beta$$

Trace anomaly

- The partition function

$$Z = \int DUD\bar{\psi}D\psi \exp\{-S\}, \quad S = S_g + S_f$$

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$$\Theta^{\mu\mu} \equiv \varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a} \quad \Rightarrow \quad \frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}$$

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- Requires subtraction of UV divergences (subtract divergent vacuum contribution evaluated at the same values of the gauge coupling):

$$\begin{aligned} \frac{\varepsilon - 3p}{T^4} &= R_\beta[\langle S_G\rangle_0 - \langle S_G\rangle_T] \\ &- R_\beta R_m[2m_l(\langle\bar{l}l\rangle_0 - \langle\bar{l}l\rangle_T) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)] \end{aligned}$$

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m}\frac{dm}{d\beta}, \quad \beta = \frac{10}{g^2}$$

HISQ data sets

- We use the Highly Improved Staggered Quarks[8] action for two degenerate light quarks and physical-mass strange quark and the tree-level Symanzik-improved gauge action
- Previous data set:

$$
\begin{aligned}
m_l &= m_s/20 \\
N_\tau &= 6, 8, 10, 12 \\
\beta &= 5.9, \ldots, 7.825
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[8] Follana et al. [HPQCD] (2007)

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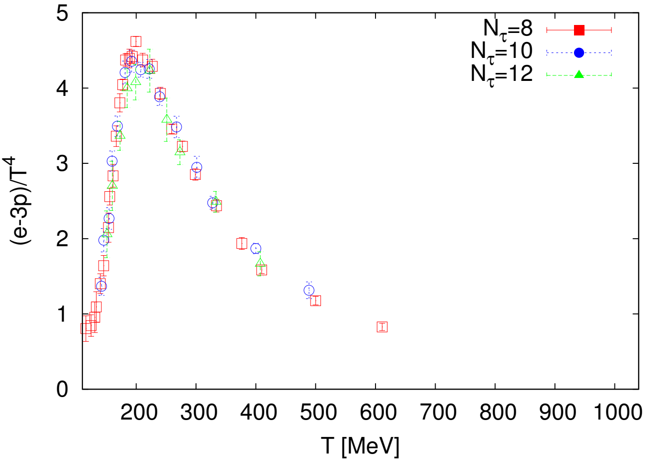
- New data set:

$$\begin{aligned} m_l &= m_s/5 \\ N_\tau &= 8, 10, 12 \\ \beta &= 8, 8.2, 8.4 \end{aligned}$$

[8] Follana et al. [HPQCD] (2007)

Results: trace anomaly



- The trace anomaly with HISQ $m_l = m_s/20$

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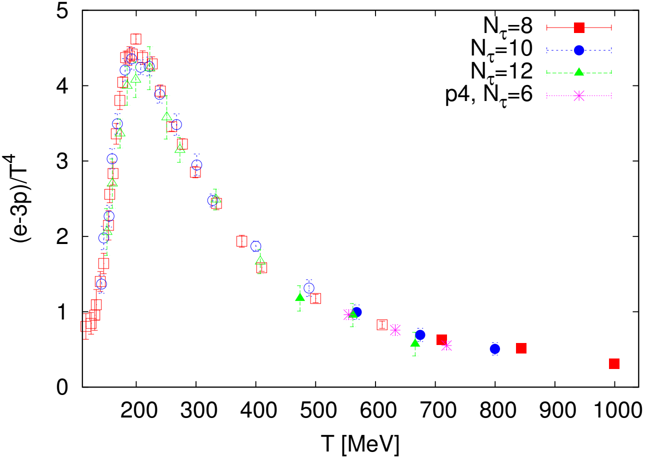
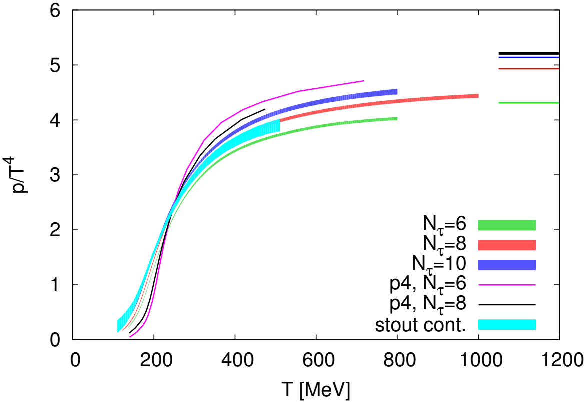


- The trace anomaly with HISQ $m_l = m_s/20$ and $m_l = m_s/5$ at $T > 400 MeV$

Results: pressure



- Pressure with HISQ at $N_\tau = 6$, 8 and 10. Continuum stout result and p4 at $N_\tau = 6$ and 8 are shown for comparison.
- The cutoff effects with HISQ are consistent with those of free theory.

Conclusion

- Previous result by the HotQCD collaboration for the 2+1 QCD equation of state at zero baryon chemical potential is being extended to higher temperatures
- At temperatures above 400 MeV we use ensembles with $m_l = m_s/5$
- Quark mass (in)dependence at high temperatures needs to be quantified
- More statistics is required for $N_\tau = 12$ ensembles to do the continuum extrapolation
- The continuum limit at high temperature may be somewhat above the stout result (as earlier results also indicate)