



# Nucleon Parton Structure from Dyson-Schwinger Equations

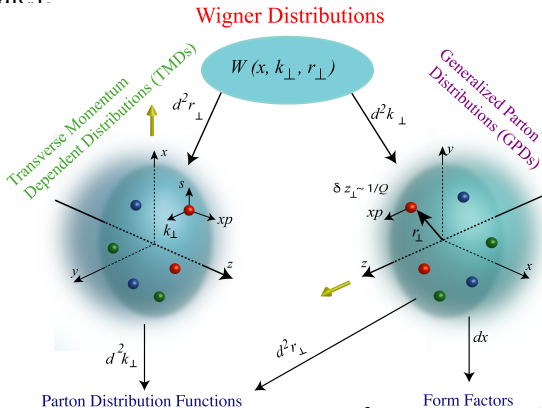


**Kyle Bednar** - Kent State University  
**Ian Cloët** - Argonne National Lab  
**Peter Tandy** - Kent State University

# Introduction



How do the fundamental degrees of freedom in QCD dynamically generate the mass, spin, motion, and spatial distribution of color charges inside hadrons with varying momentum resolution and energy scales?



# Dyson-Schwinger Equations



*What tools to use?*

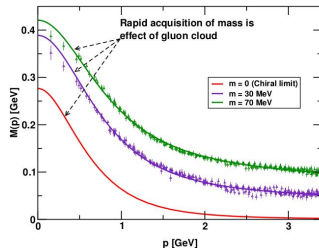
I.C. Cloët, C.D. Roberts, Explanation and prediction of observables using continuum strong QCD. Prog. Part. Nucl. Phys. 77, 169 (2014)

## Dyson-Schwinger Equations

- ▶ Nonperturbative, Poincaré covariant continuum QCD
- ▶ Coupled Integral Equations for QCD Schwinger Functions
- ▶ **Asymptotic Freedom**

→ *model-dependence restricted to infrared momenta,  $p < 1$  GeV*

- ▶ Quark mass function is expression of **DCSB**



**Truncations are necessary; Positive feedback between DSEs and Lattice-QCD**

Quark propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \circ \text{---} \text{---}^{-1}$$

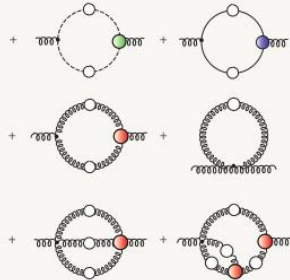
Ghost propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \circ \text{---} \text{---}^{-1}$$

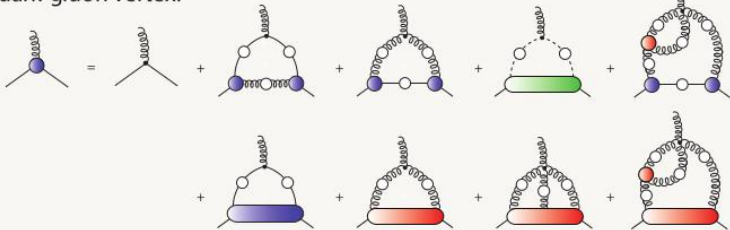
Ghost-gluon vertex:

Gluon propagator:

$$\text{---} \circ \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \circ \text{---} \text{---}^{-1}$$



Quark-gluon vertex:

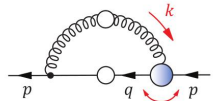


# Elements of PDFs with DSEs



## Gap Equation

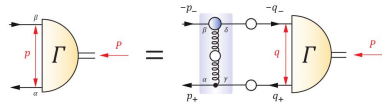
RL truncation; violation of reflection positivity (confinement); momentum-dependent mass function (DCSB);



Si-xue Qin et al. Phys. Rev. C 84, 042202(R)

## Bethe-Salpeter Equation

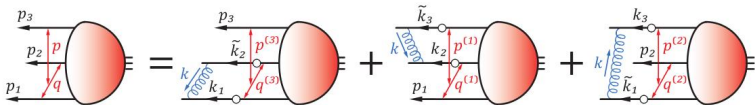
non-pointlike, dynamical diquark correlations;  $\Gamma_{0^+} = \Gamma_\pi C^\dagger$



Maris, Tandy, PRC 60 (1999)

## Faddeev Equation

Covariant  $qqq$  equation  $\rightarrow$  RL, remove irreducible 3-quark interactions;



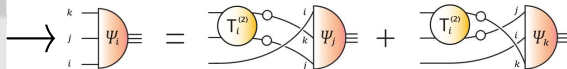
# Diquark Correlations



## Faddeev Components

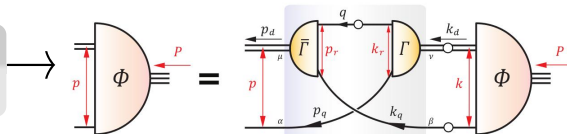
Dyson's Equation

$$T^{(2)} = (1 + T^{(2)})K^{(2)}$$



## Diquark Ansatz

$$T = \Gamma D \bar{\Gamma} + \Gamma^\mu D^{\mu\nu} \bar{\Gamma}^\nu$$



I. C. Cloët, G. Eichmann, B. El-Bennich, et al., Few Body Syst. 46, 1 (2009)

**Realistic qq interactions show nonpointlike color-antitriplet diquark correlations**

- 1 Scalar diquarks only (for now)
- 2  $M_D = 791$  MeV

**Scalar diquark correlations are a direct consequence of dynamical chiral symmetry breaking in QCD**

# Nucleon PDFs from DSEs

## Nakanishi Representations

Quark Prop:  $S(q) = \sum \left[ \frac{Z_i}{i\gamma \cdot q + m_i} + \frac{Z_i^*}{i\gamma \cdot q + m_i^*} \right]$

Vertices:  $\int_{-1}^1 d\alpha \frac{\rho(\alpha) C_0 \Lambda^{2n}}{(k^2 + \frac{2}{3} \alpha k \cdot P + \Lambda^2)^n}$

- ▶ Interpolates numerical solutions
- ▶ Semi-analytic evaluation
- ▶ Many moments  $\int_k (k^+ / P^+)^m$

### Bjorken limit

$Q^2 \rightarrow \infty, 2p \cdot q \rightarrow \infty, x = \text{fixed.}$

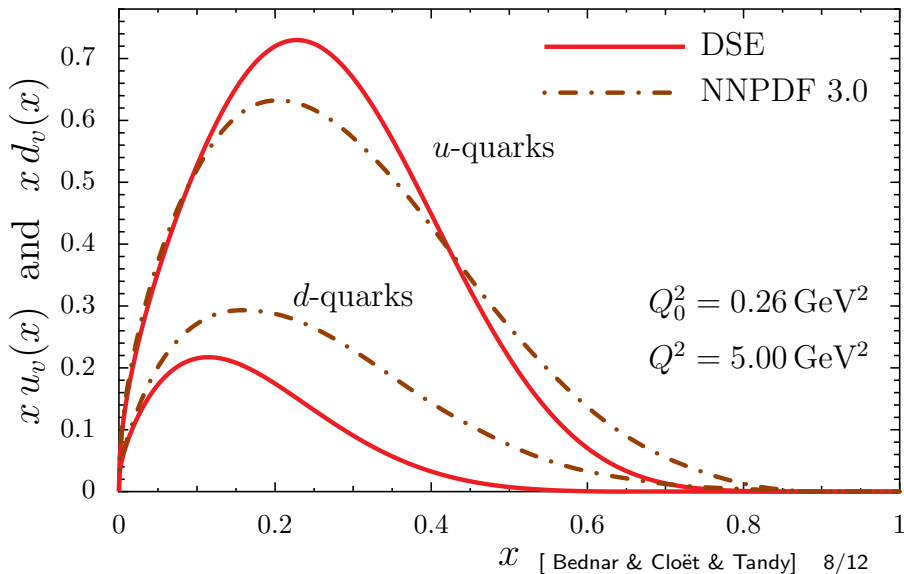
### Parton Distribution Function

$$q(x) = \int \frac{d\lambda}{4\pi} e^{-ixP \cdot n\lambda} \langle P | \bar{\psi}(\lambda n) \gamma \cdot n \psi(0) | P \rangle_c$$

M. Oettel, M. Pichowsky and L. von Smekal, Eur. Phys. J. A 8, 251 (2000)

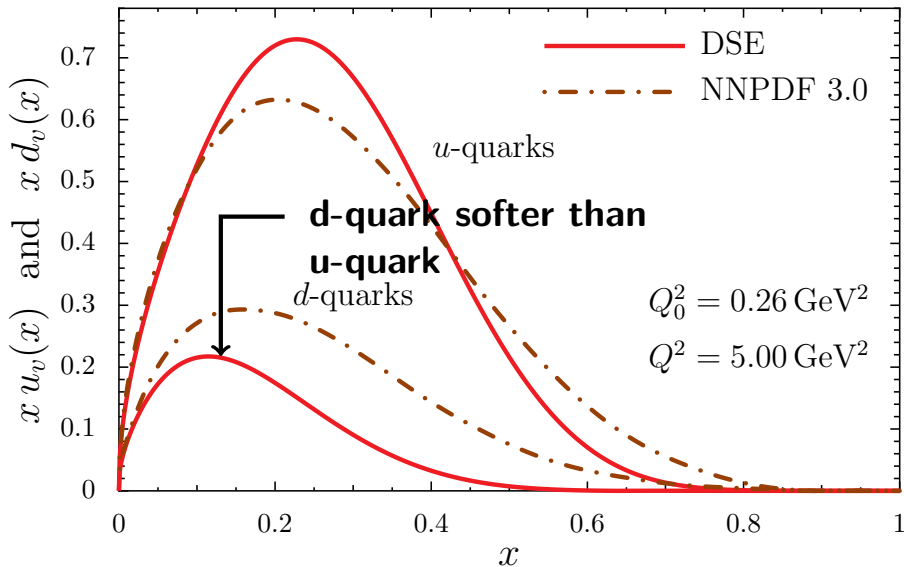
Exchange Terms + Seagull Terms

# Results

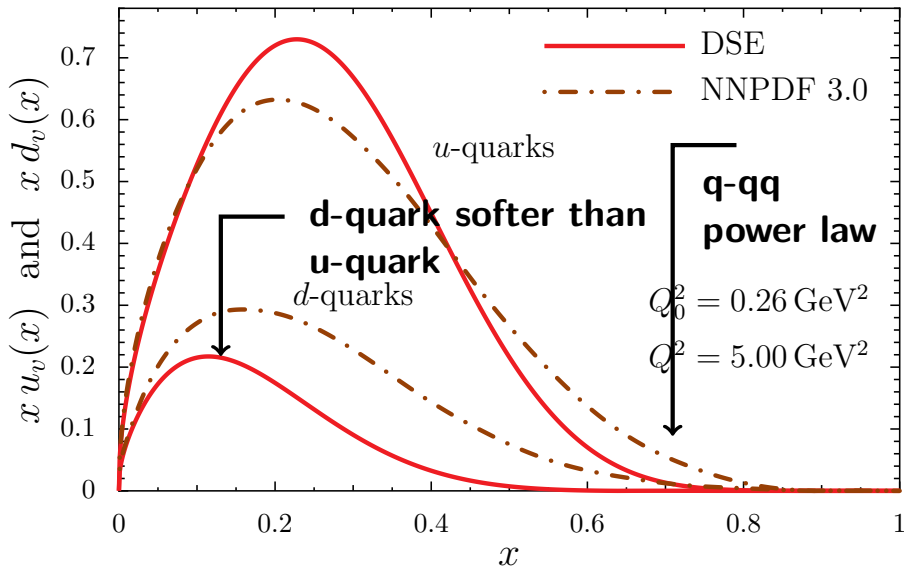




# Results and Large- $x$



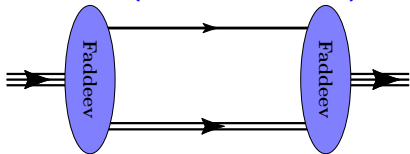
# Results and Large- $x$



# PDFs as $x \rightarrow 1$

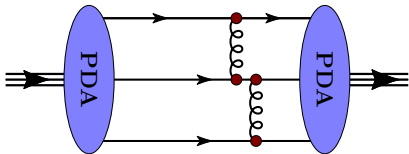


## DSE (Scalar Diquarks)



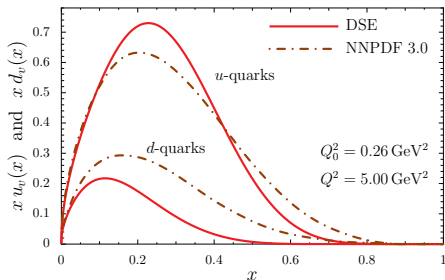
$$q(x) \stackrel{x \rightarrow 1}{\simeq} (1-x)^5$$

## Conformal QCD



$$q(x) \stackrel{x \rightarrow 1}{\simeq} (1-x)^3$$

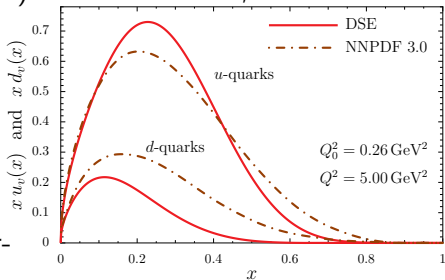
- ▶ DSE Quark + Scalar Diquark gives  $(1-x)^5$
- ▶ Conformal QCD gives  $(1-x)^3$
- ▶ **Resolution:**
  - ▶ Include AV diquarks?
  - ▶ Diquark approx. breaks down?



# Conclusions and Future

## Conclusions

- 1 DSEs produce moments ( $> 20$ ) of nucleon PDFs;  
Nakanishi forms for semi-analytic calculations
- 2 Non-pointlike diquark (quark-quark) correlations play an important role
- 3 Quark + Scalar-diquark approx. gives incorrect power-law behavior as  $x \rightarrow 1$



## Future

- 1 Include Axial-Vector Diquarks
- 2 Include Seagulls and Exchange terms
- 3 Move to TMDs!

