Resonance and QCD

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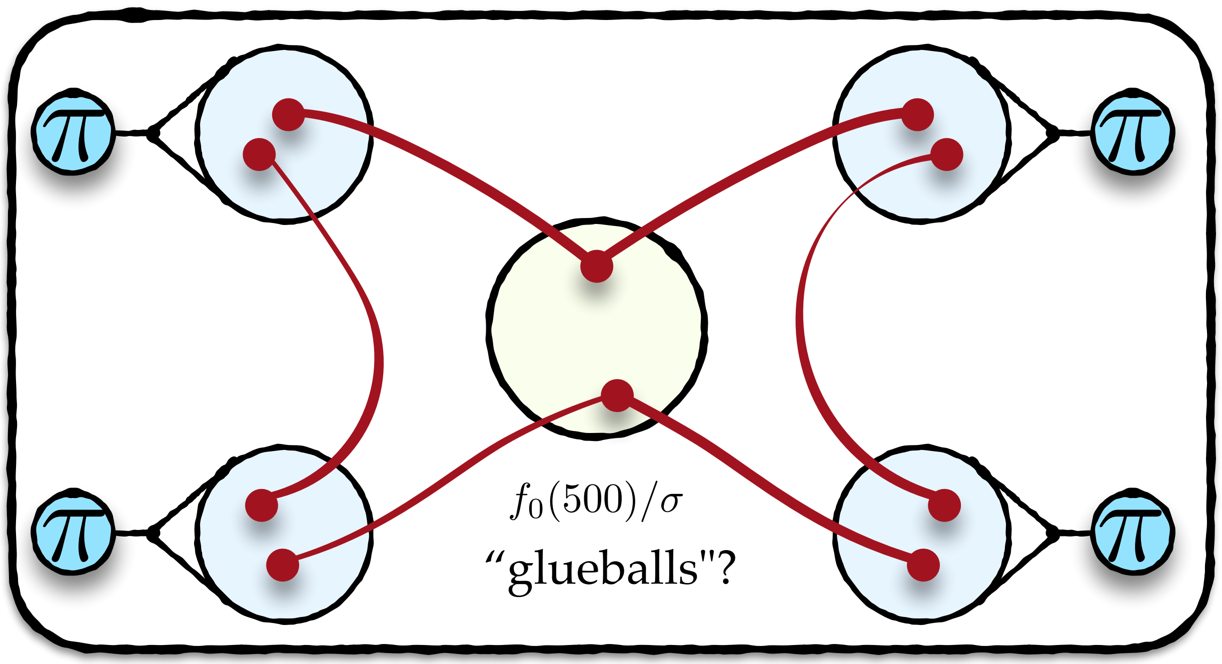
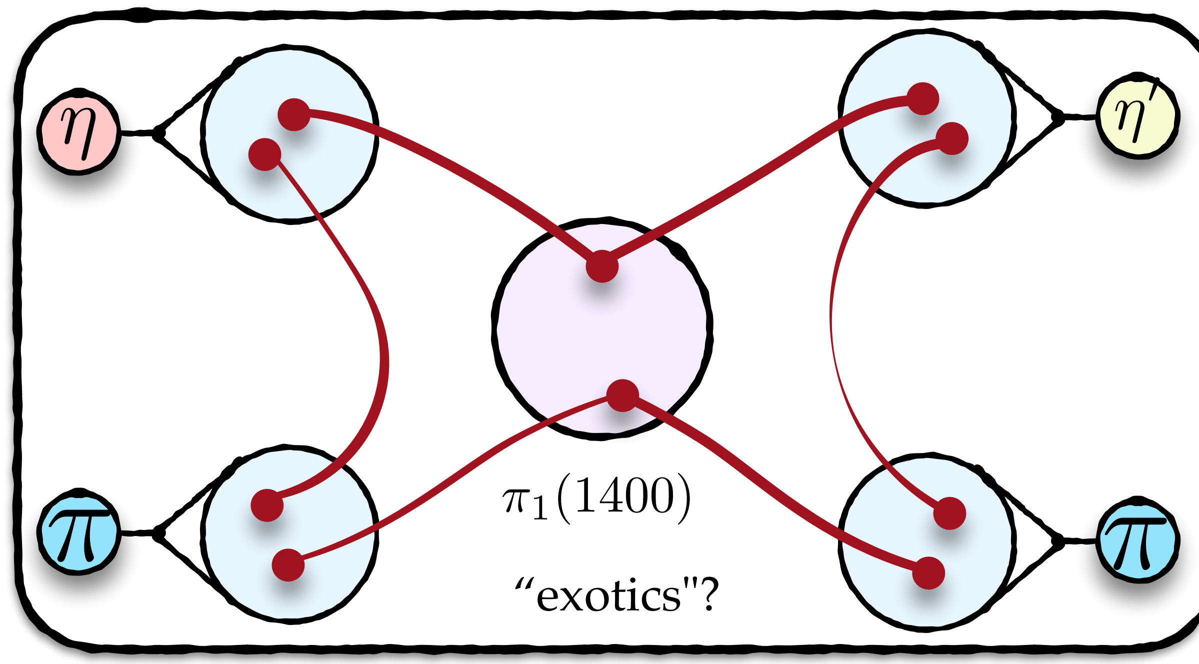
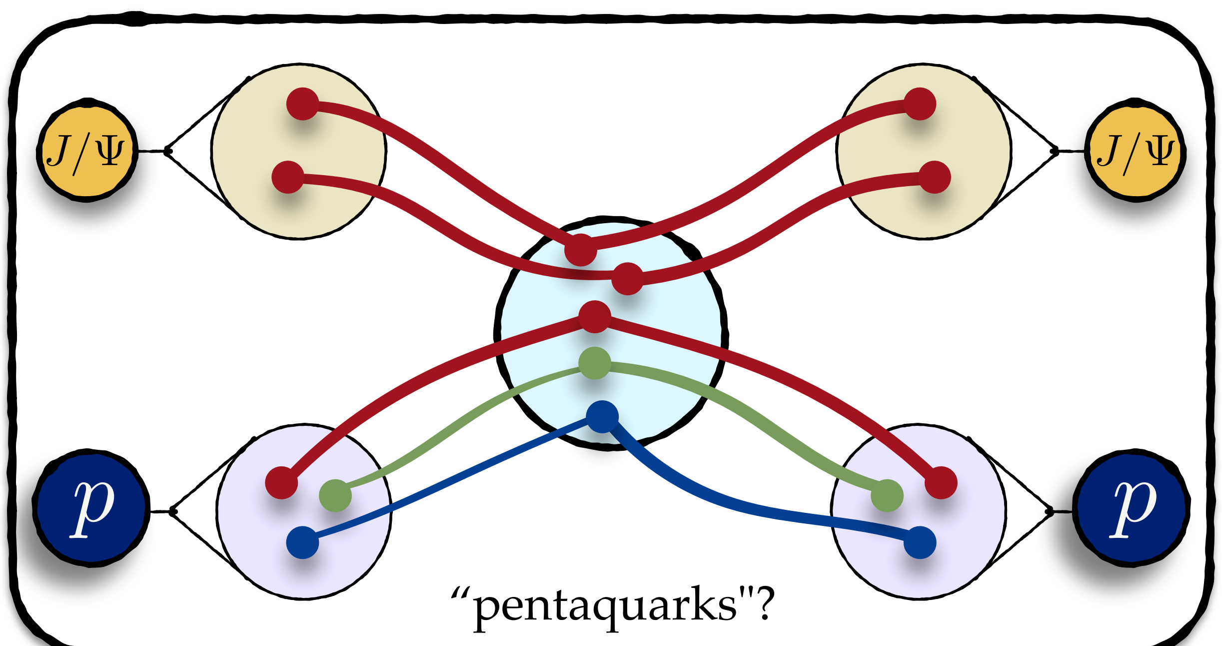
GHP, 2017

Resonances are the 99%

- Most hadrons are resonances
- Play a crucial role in a wide range of phenomenology

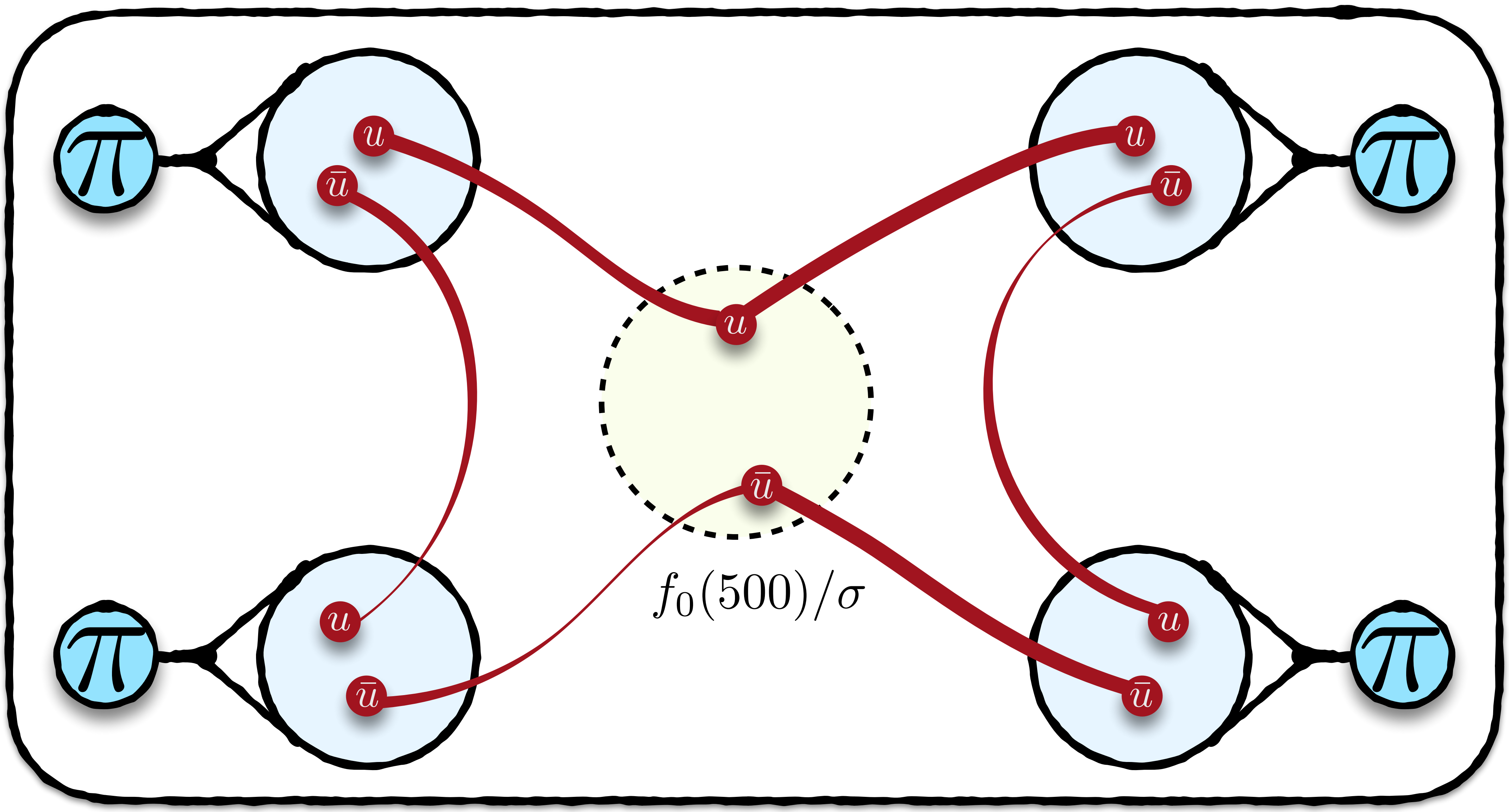




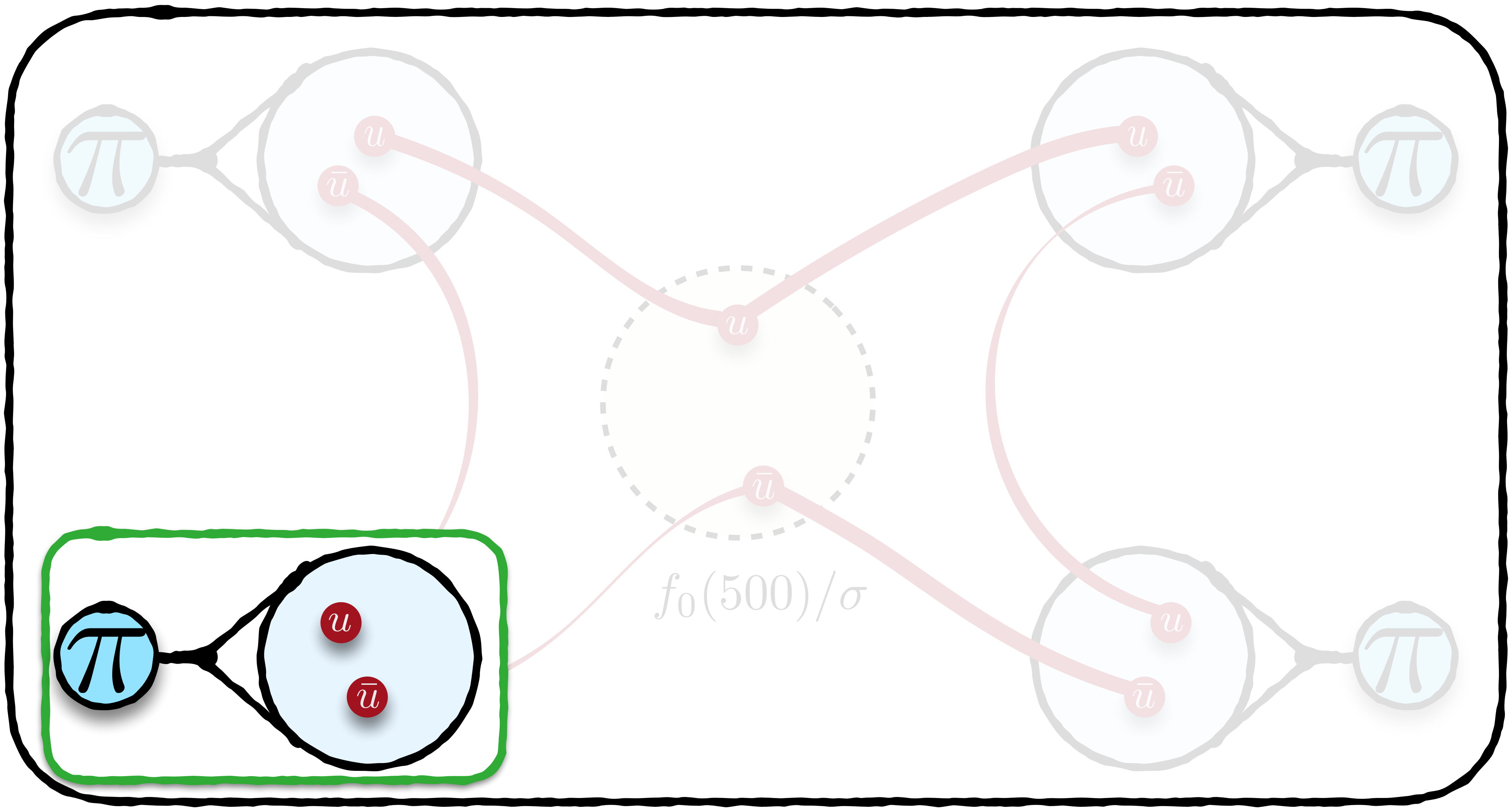


COMPASS GlueX LHCb BESIII CLAS12 …

Why lattice?

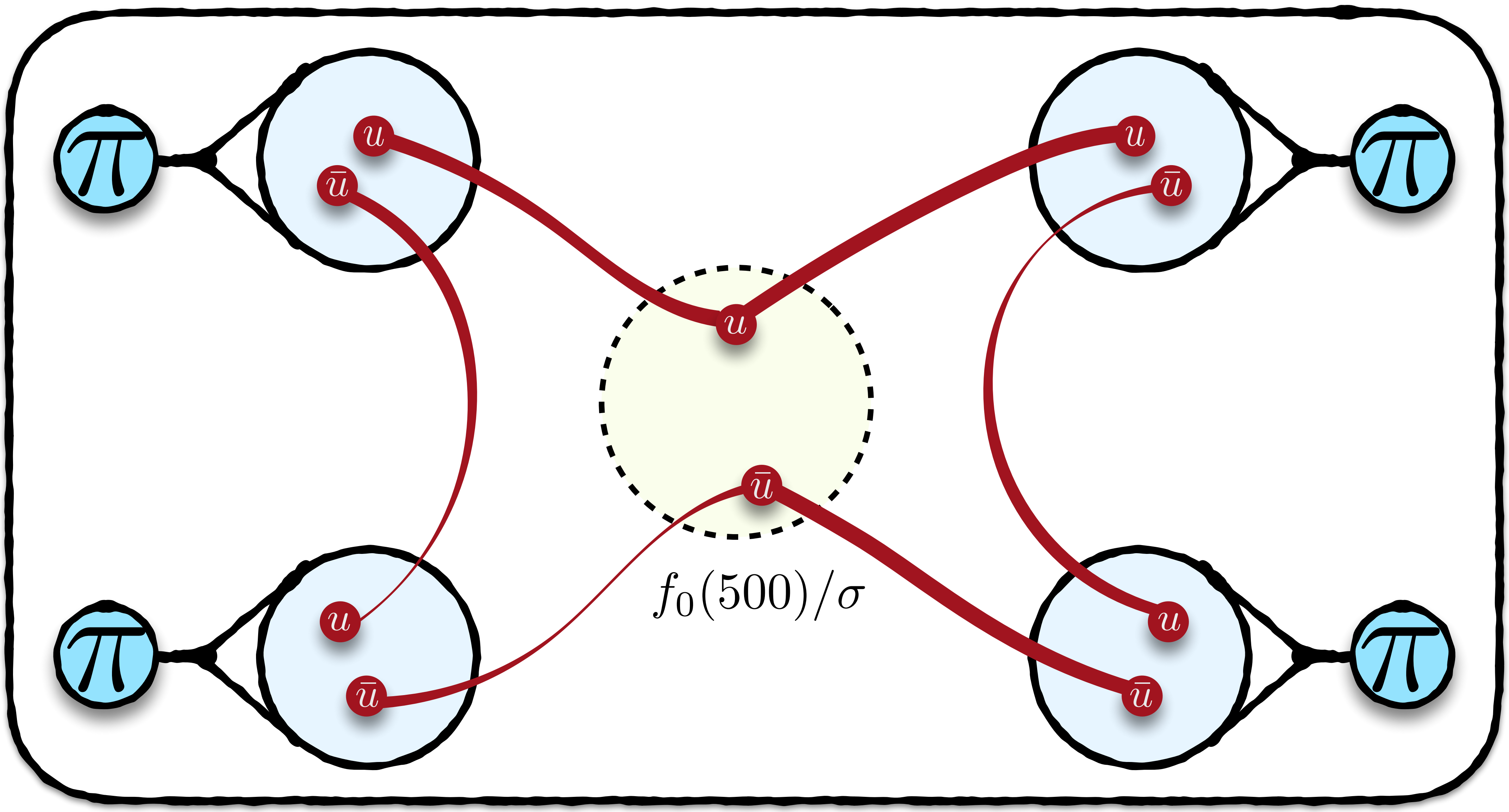


Why lattice?



- QCD-stable states are generated exactly

Why lattice?



- QCD-stable states are generated exactly
- N-body forces are incorporated exactly

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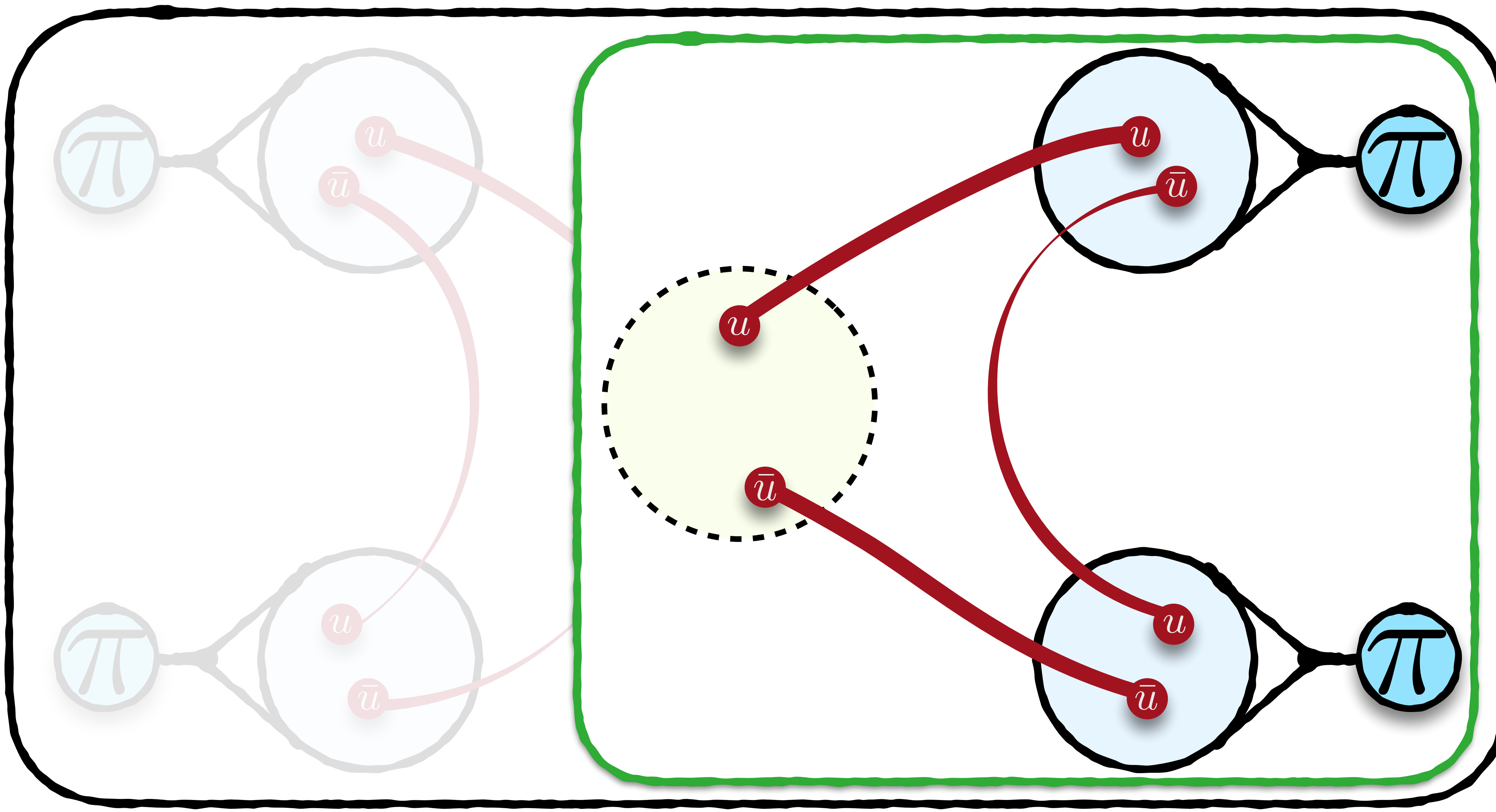
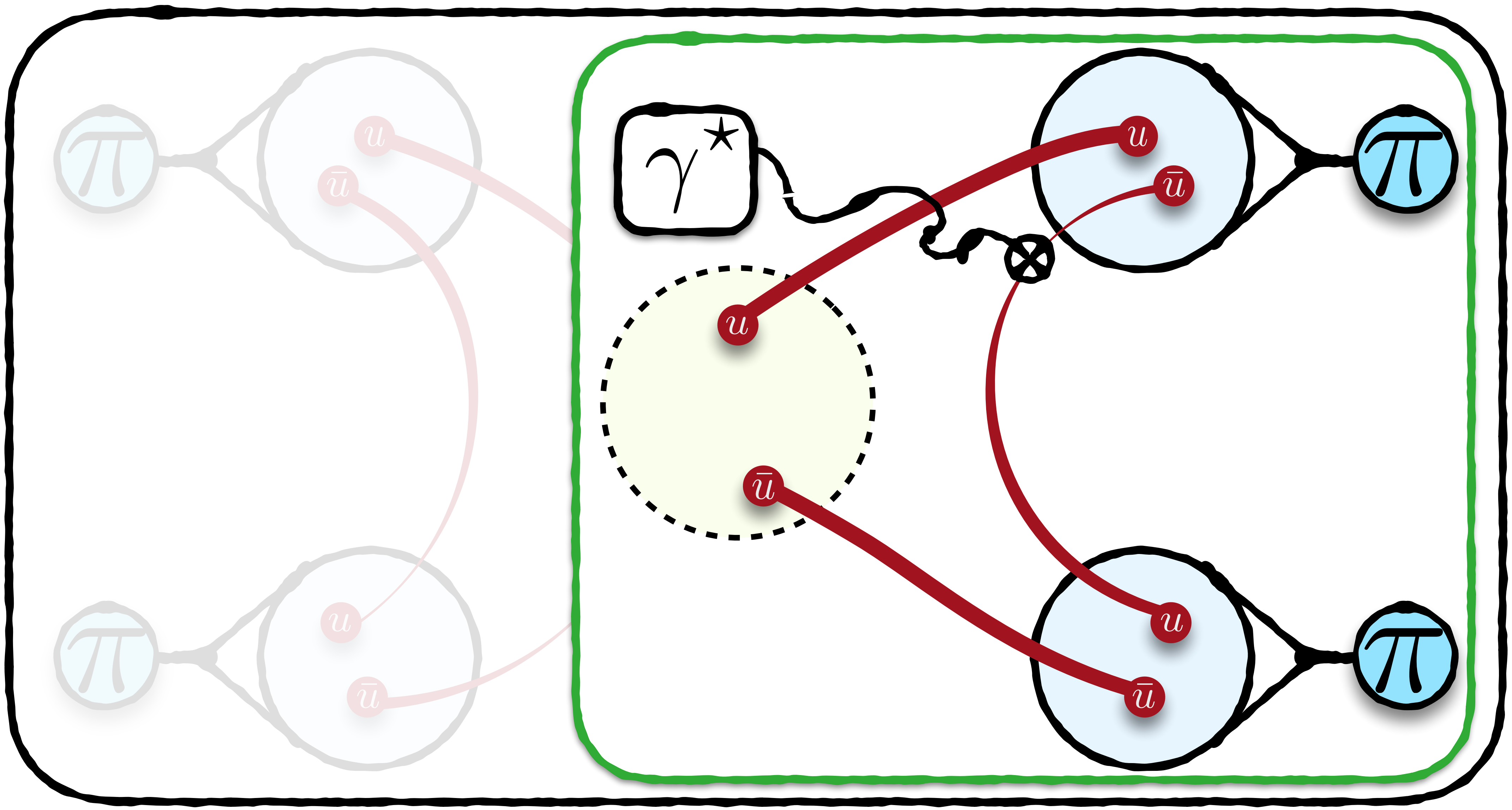


- QCD-stable states are generated exactly
- N-body forces are incorporated exactly
- Resonances are generated and decay

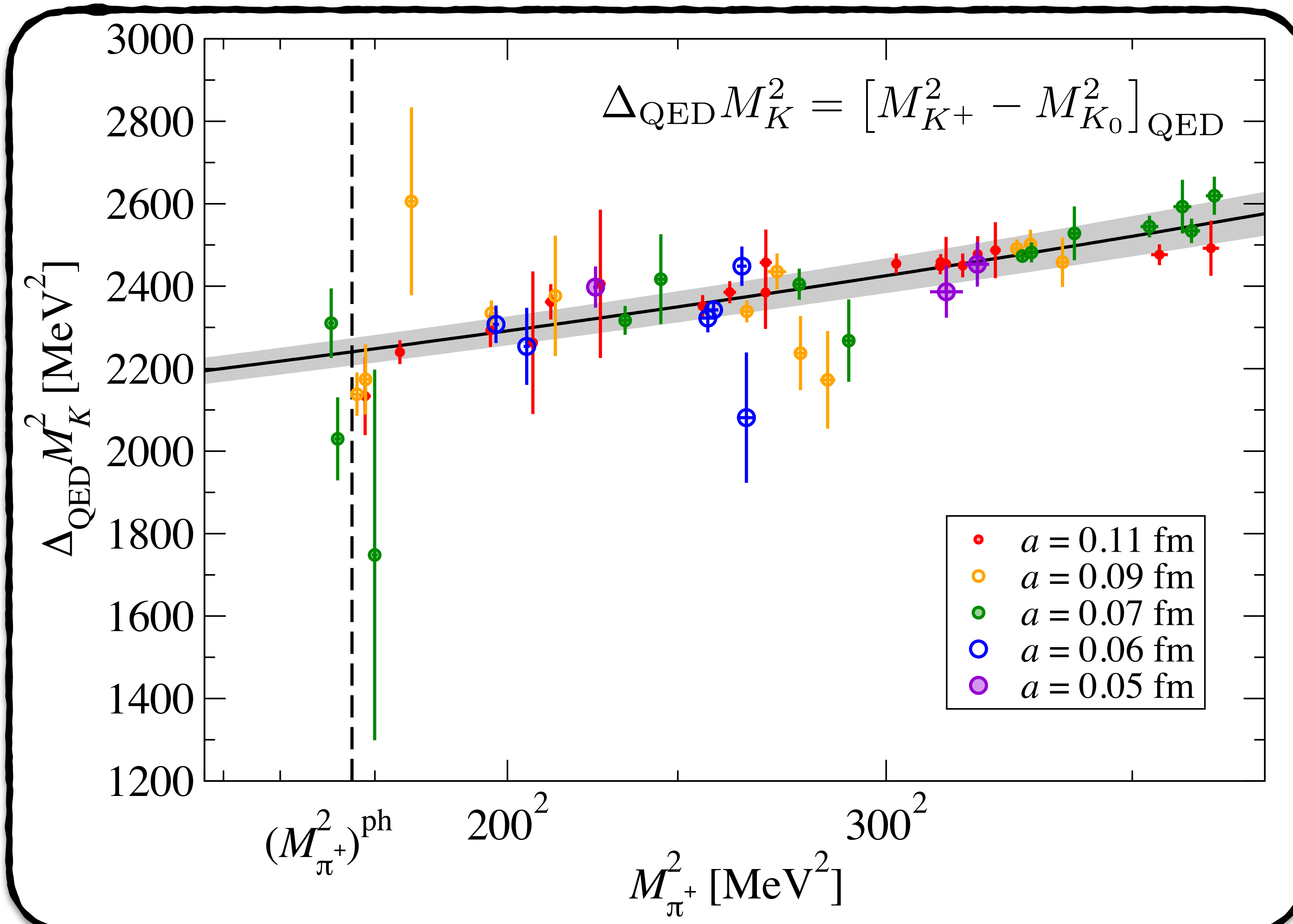
Why lattice?



- QCD-stable states are generated exactly
- N-body forces are incorporated exactly
- Resonances are generated and decay
- QED / weak sector can be treated perturbatively or non-perturbatively

Spectroscopy in LQCD

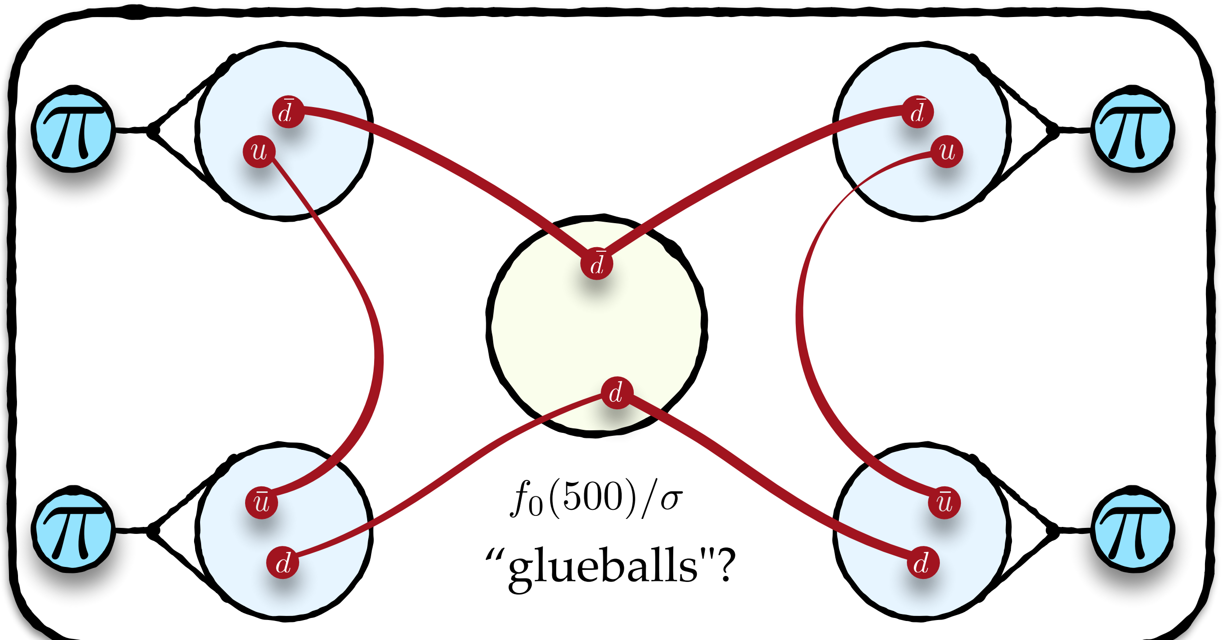
- *Vanilla* spectroscopy - QCD stable states [non-composite states]
  - Physical or lighter quark masses [down to m_π~120 MeV] ✓
  - Non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - Dynamical QED ✓

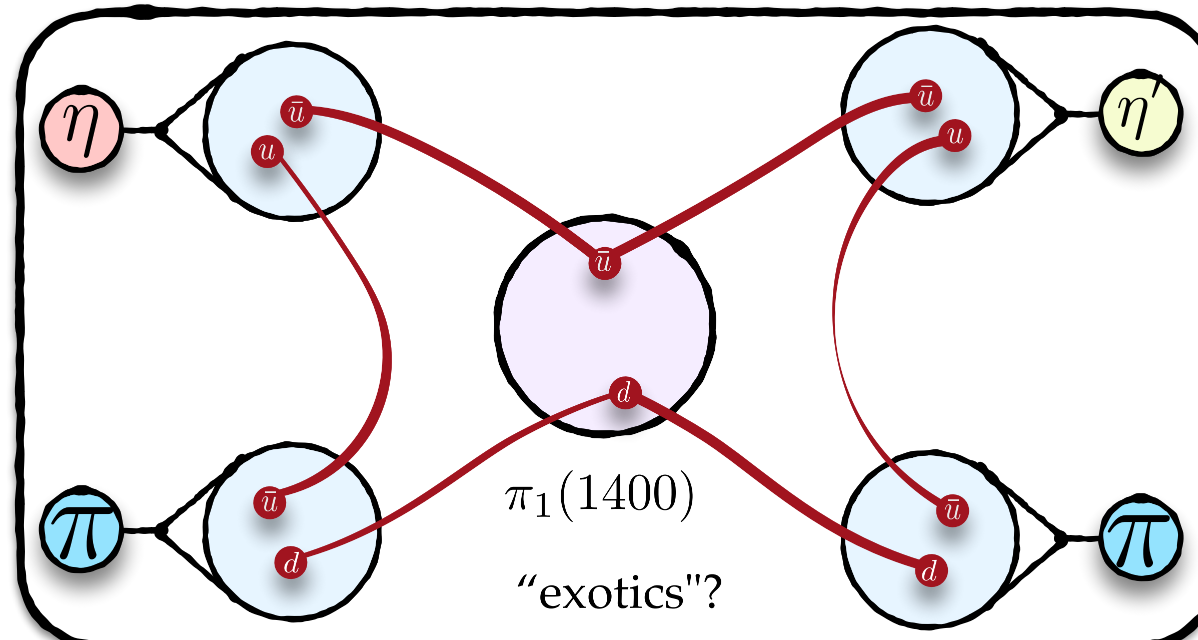


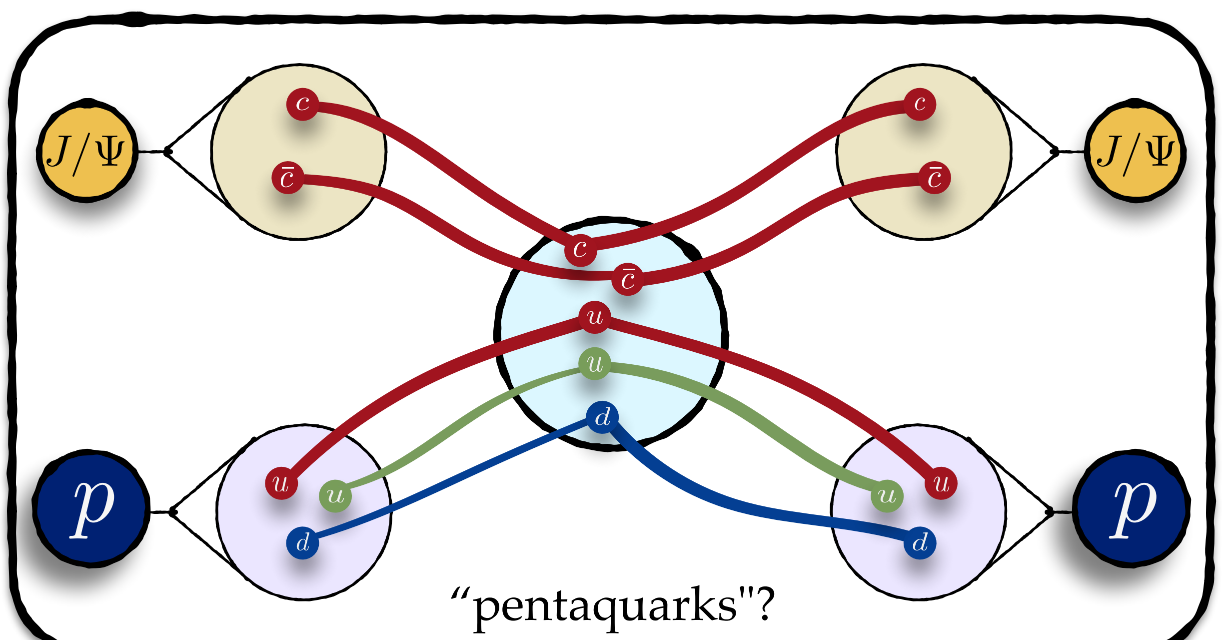
Fodor et al. [BMWc] (2016)

Spectroscopy in LQCD

- *Vanilla* spectroscopy - QCD stable states [non-composite states]
- the frontier of spectroscopy - hadronic resonances [composite states]

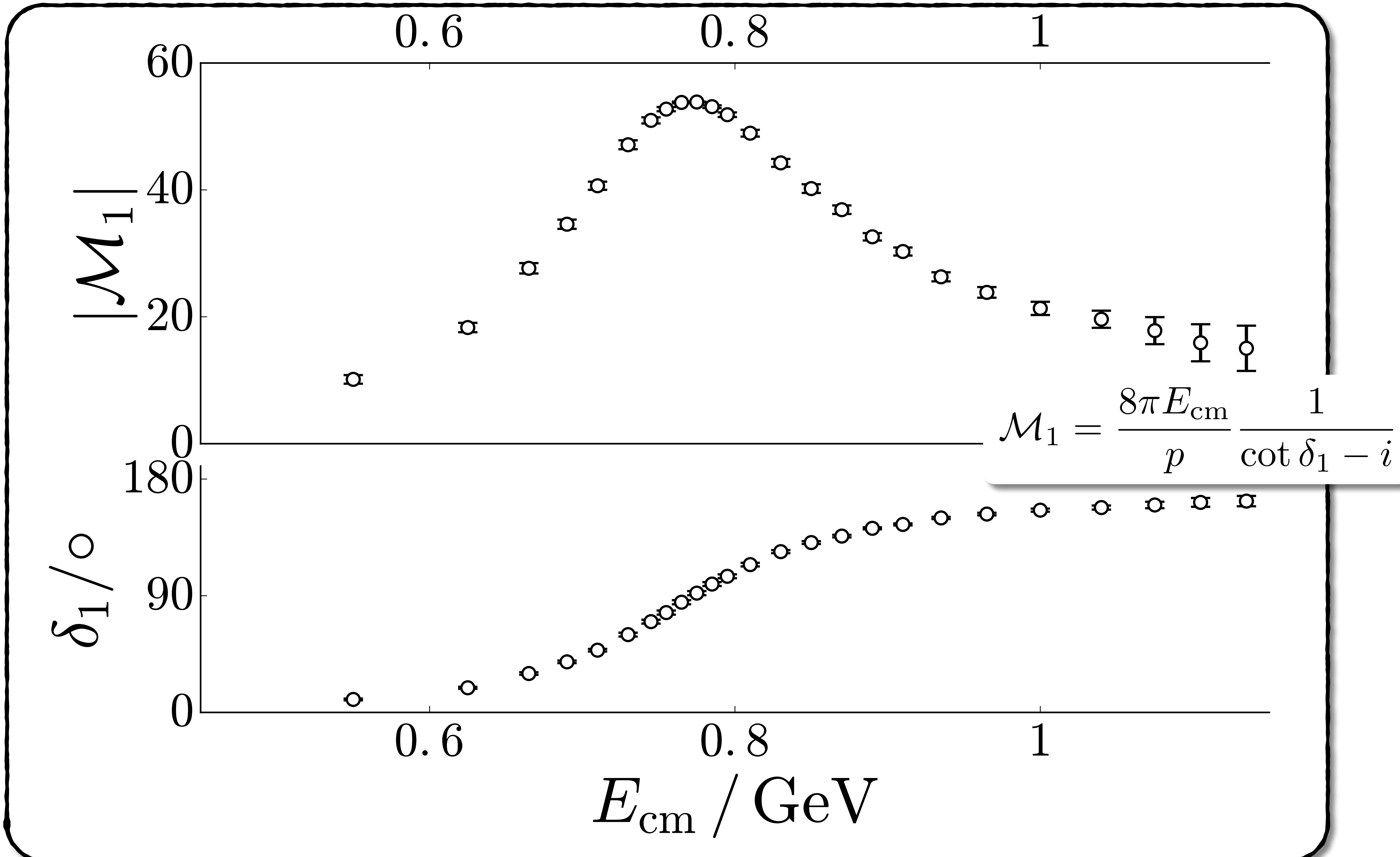






A pseudo-quantitative definition

(bump in cross sections/amplitude - e.g., ππ scattering in ϱ-channel)



Protopopescu et al. (1972)

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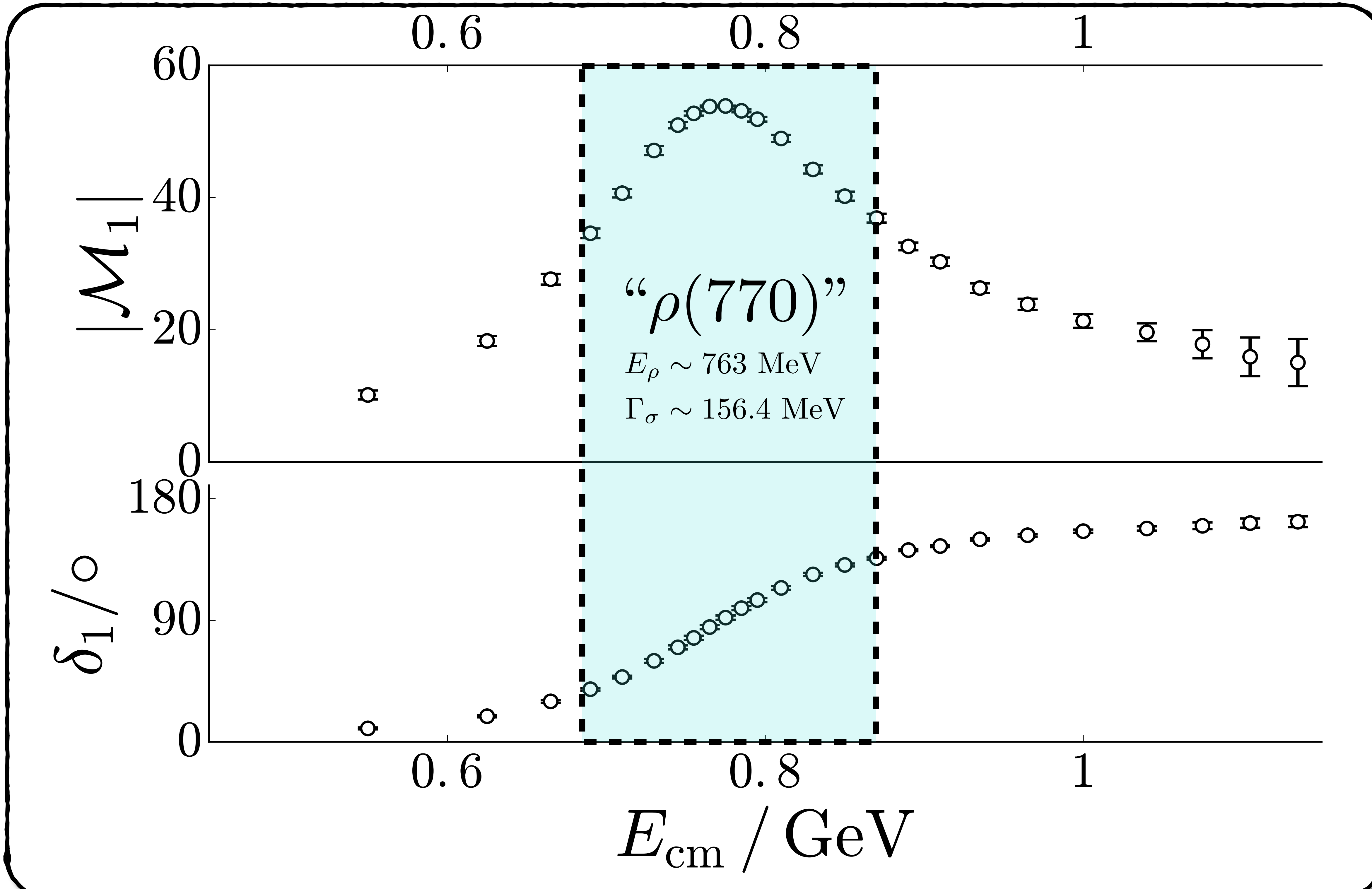
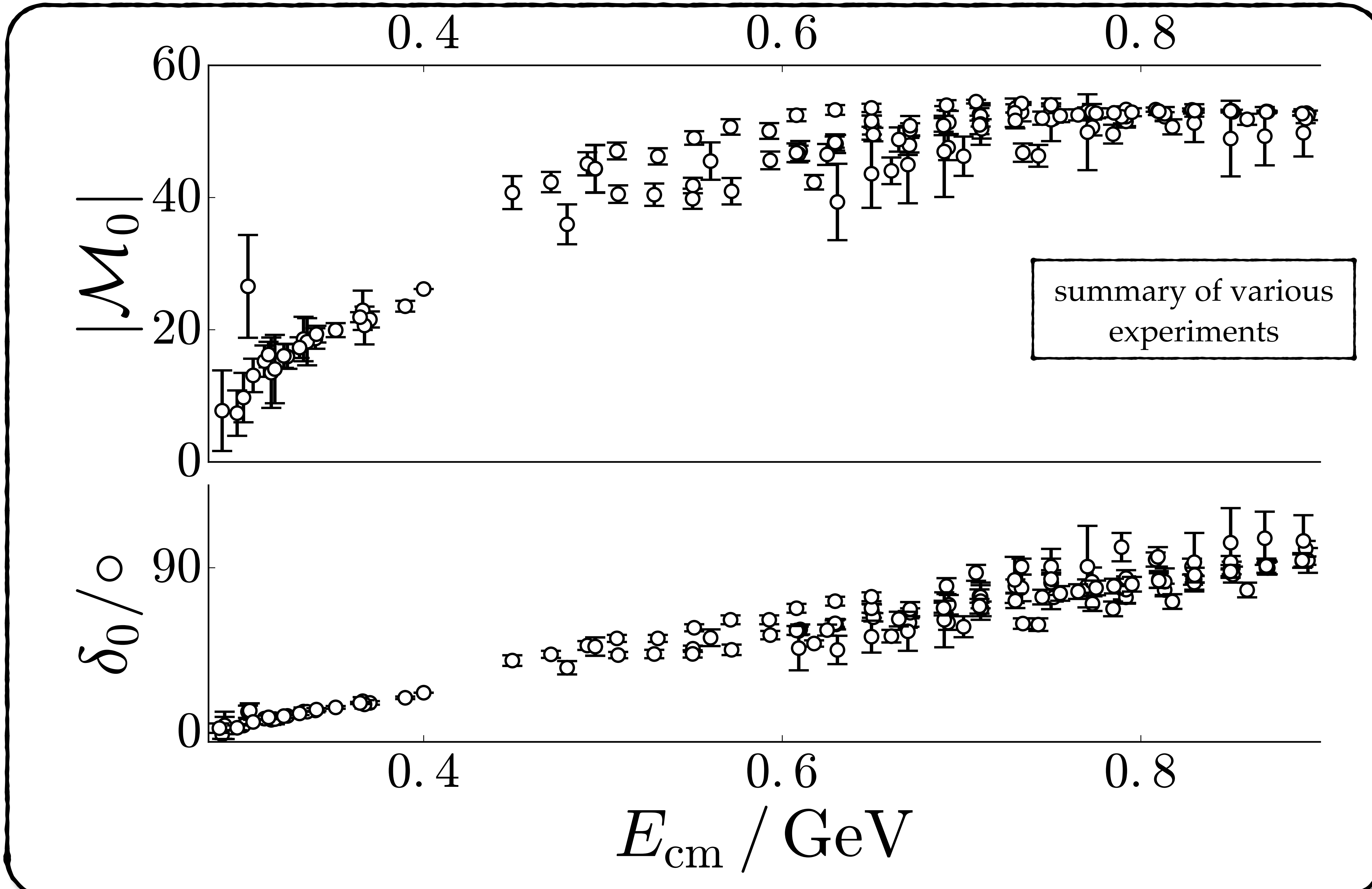


Protopopescu et al. (1972)

A counter example

(Isoscalar, scalar $\pi\pi$ scattering)



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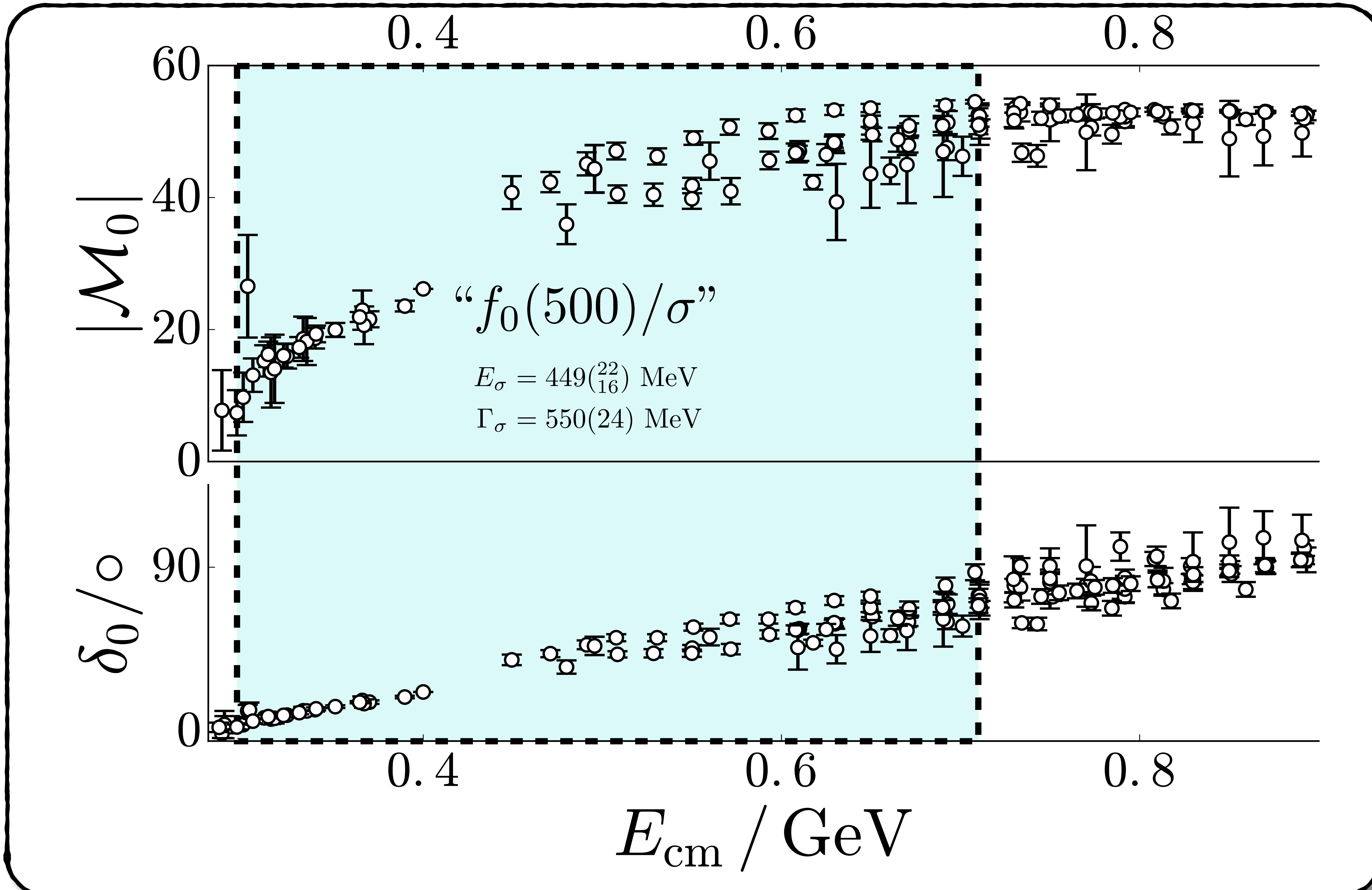
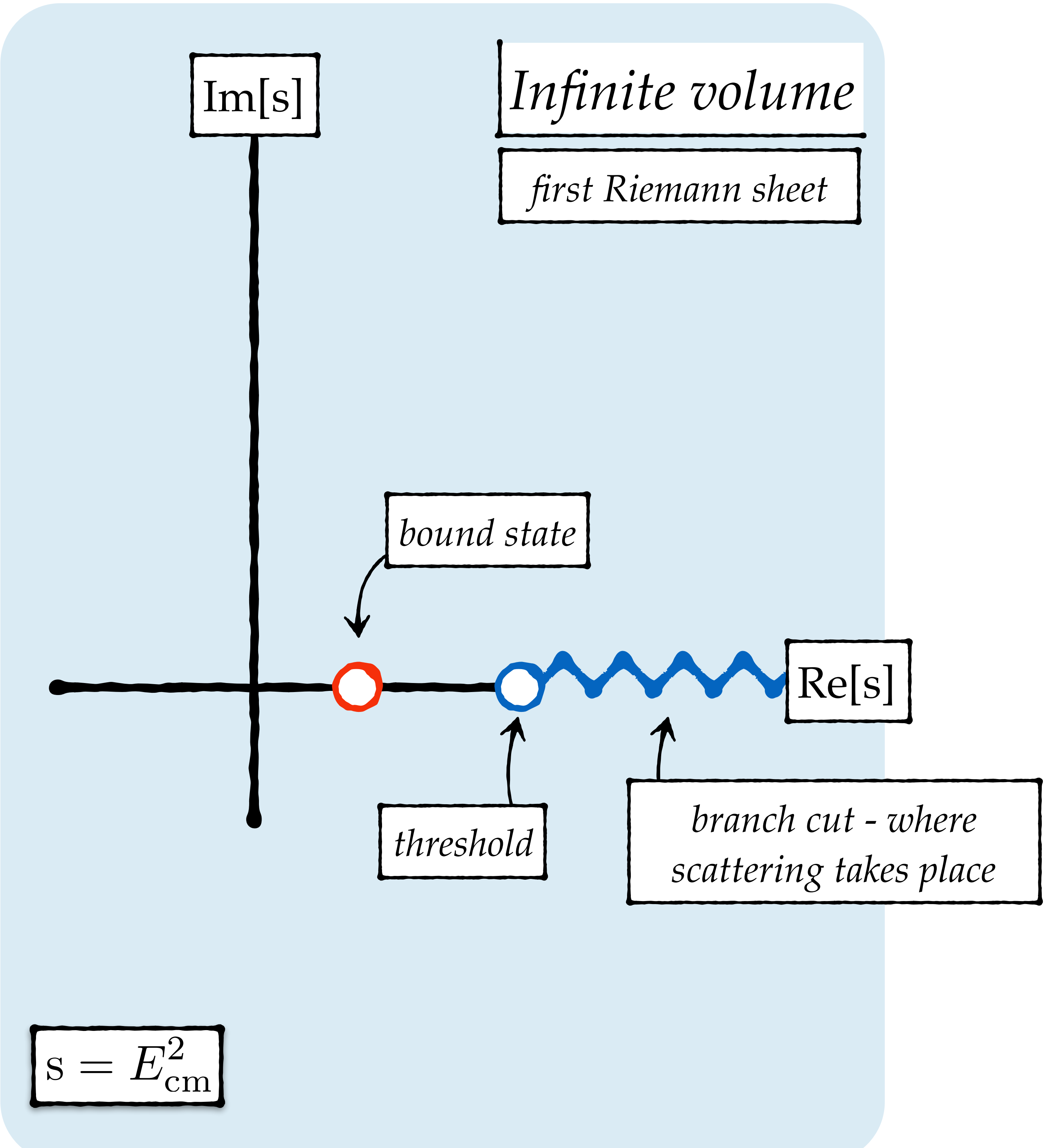
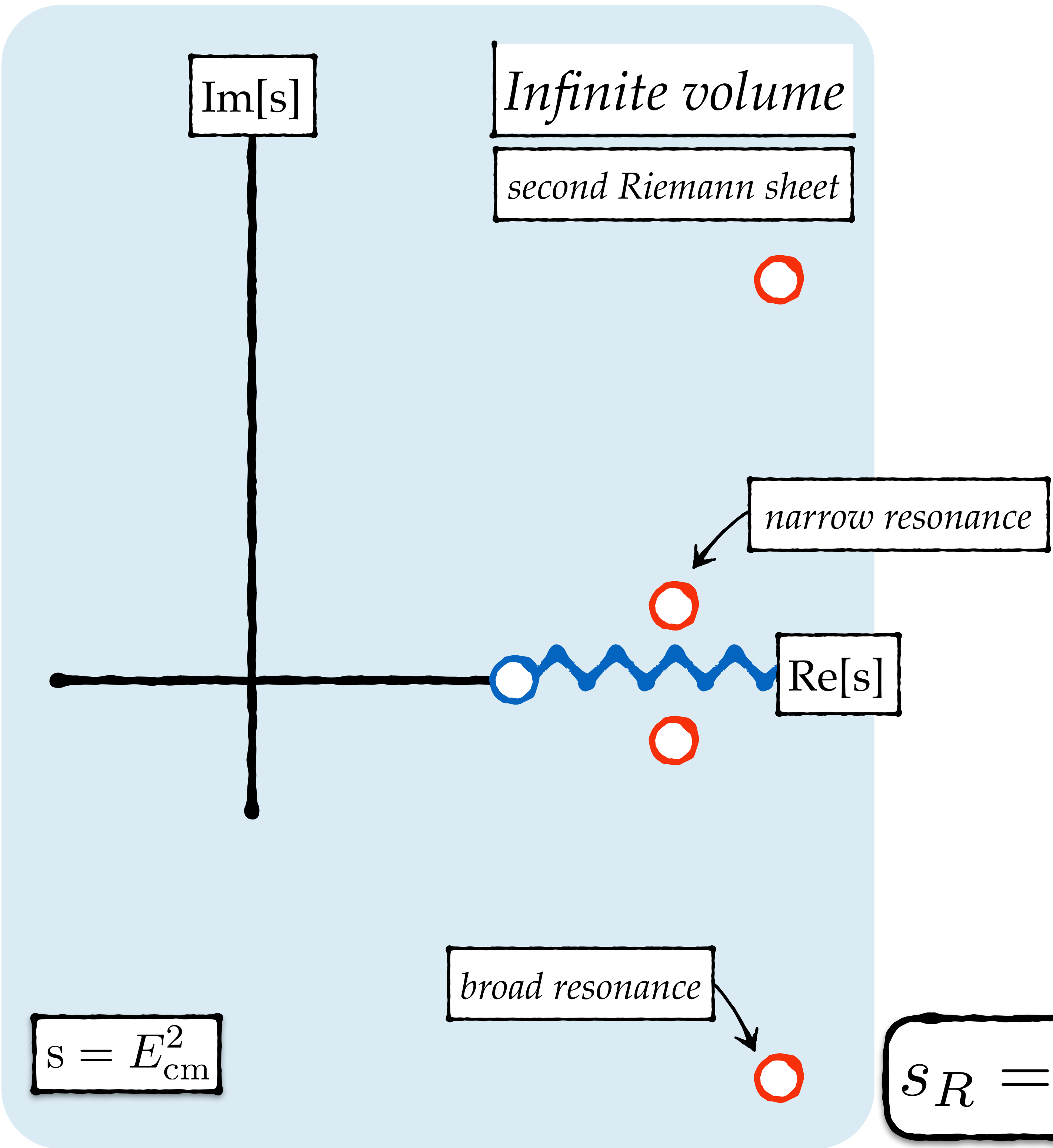


Spectroscopy recap



Spectroscopy recap

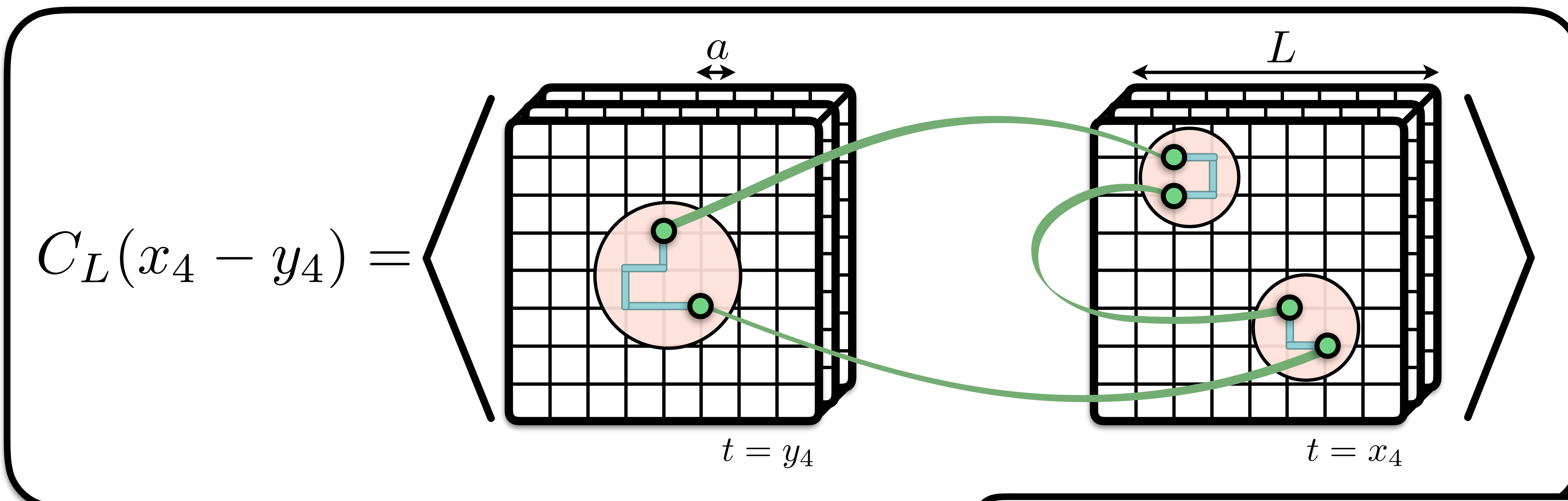


$$s_R = \left(E_R - \frac{i}{2}\Gamma_R\right)^2$$

Lattice QCD

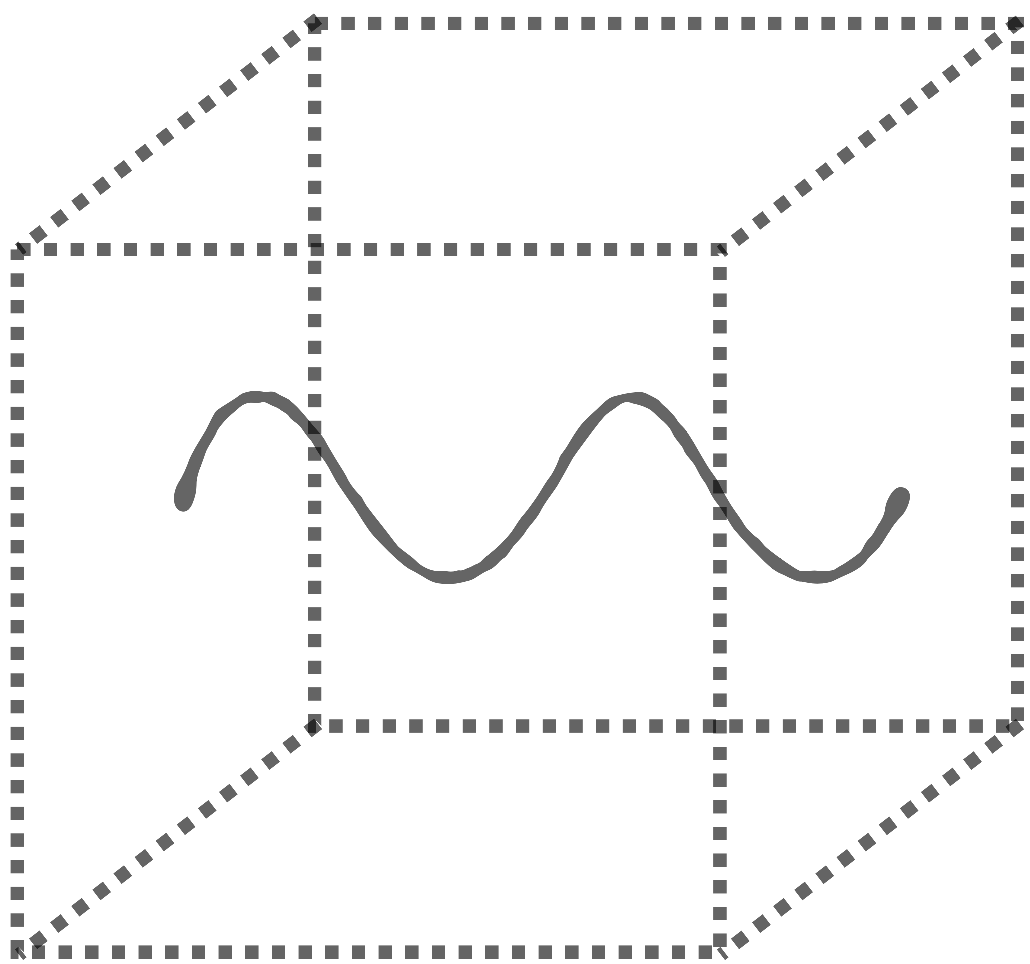
Correlation functions using:

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume
- quark masses: $m_q \to m_q^{\text{phys.}}$

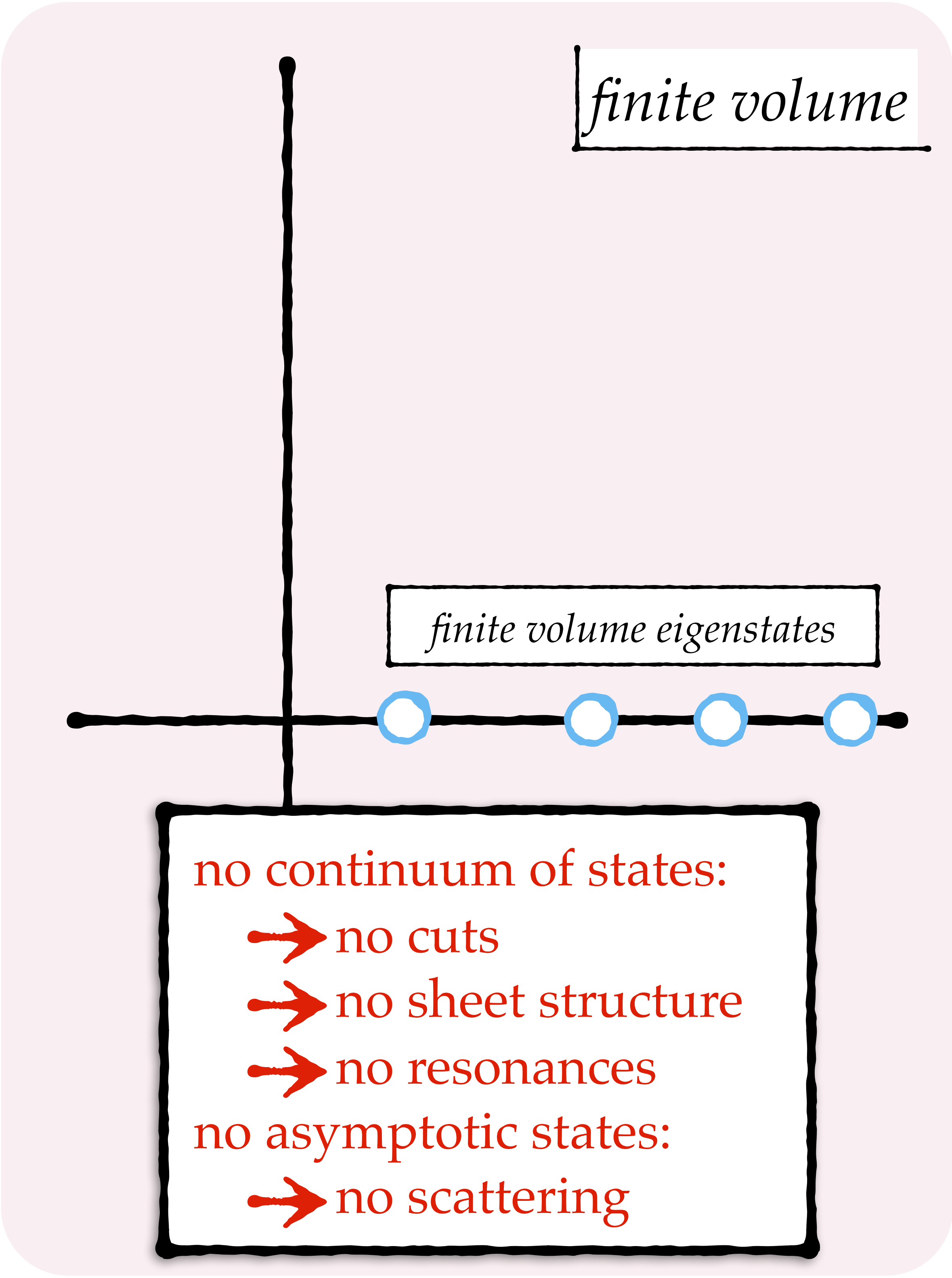


Have we *mangled* QCD too much?

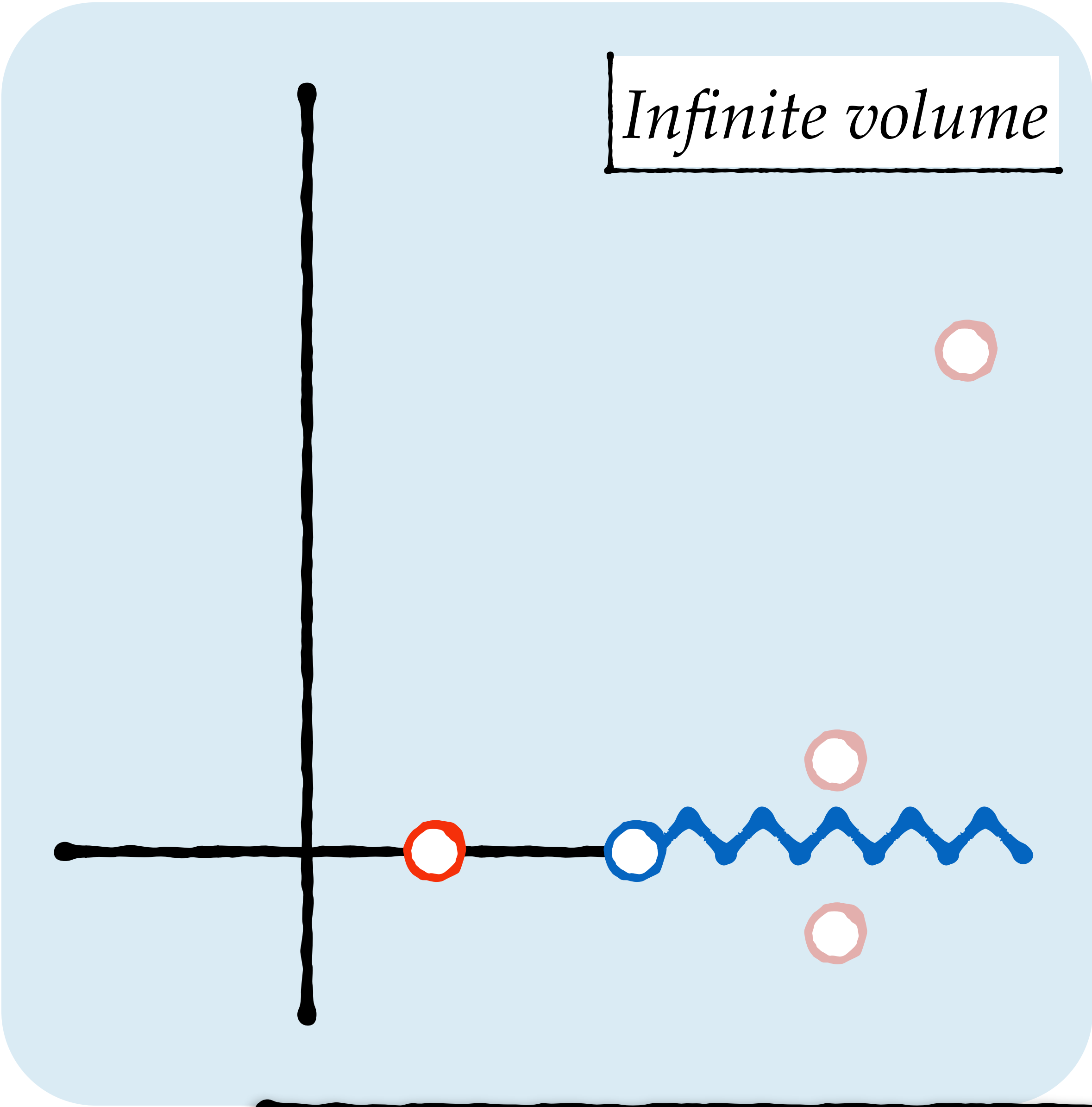
Finite volume spectrum

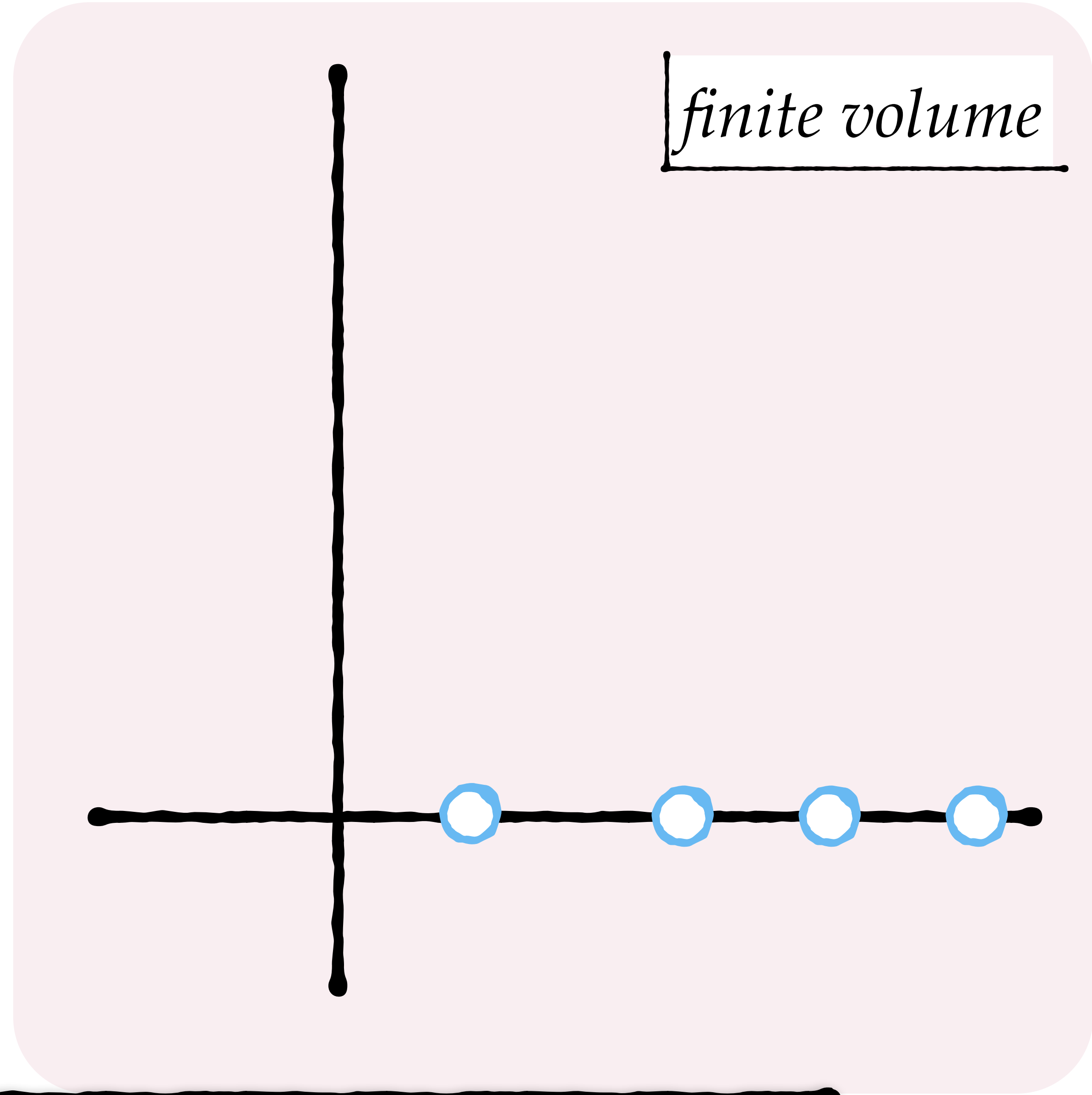


"only a discrete number of modes can exist in a finite volume"



Finite vs. infinite volume spectrum

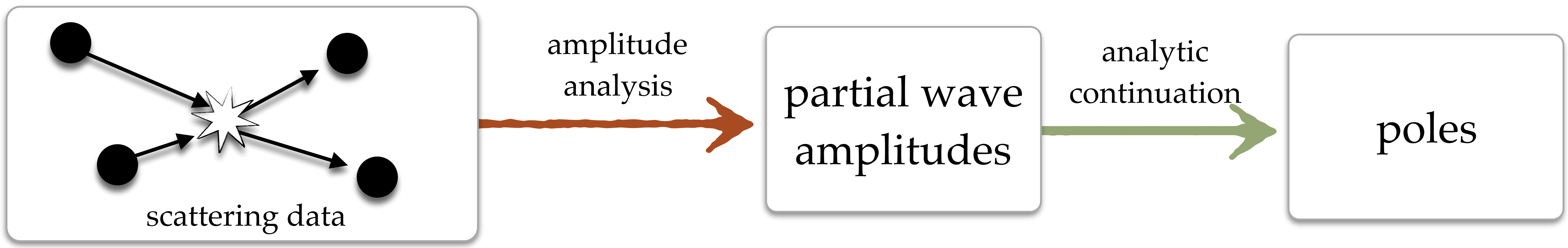


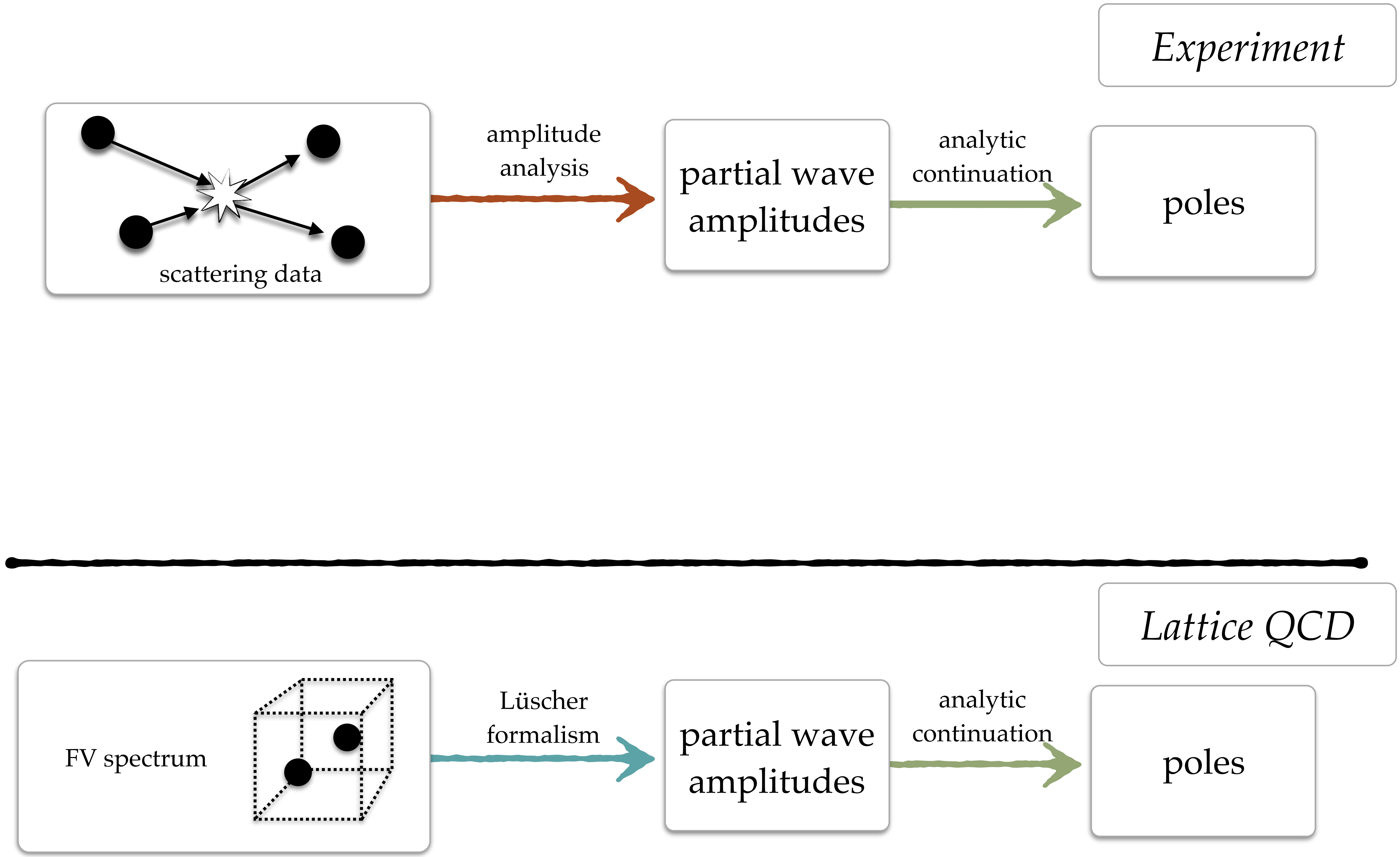


both pictures *are* QCD:

"Two analytic manifestations of QCD"

Experiment


scattering data
amplitude analysis
partial wave amplitudes
analytic continuation
poles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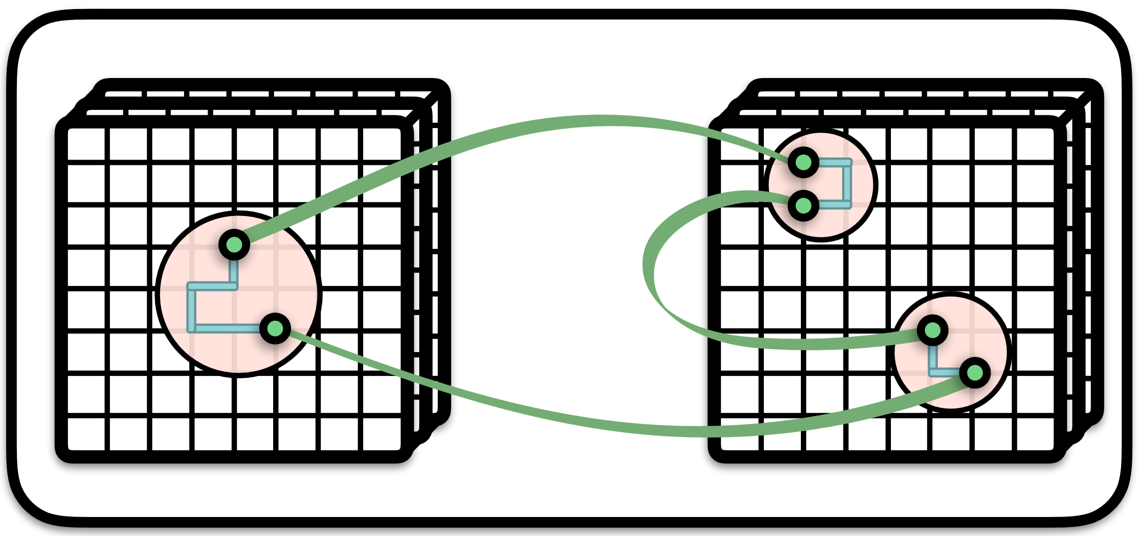
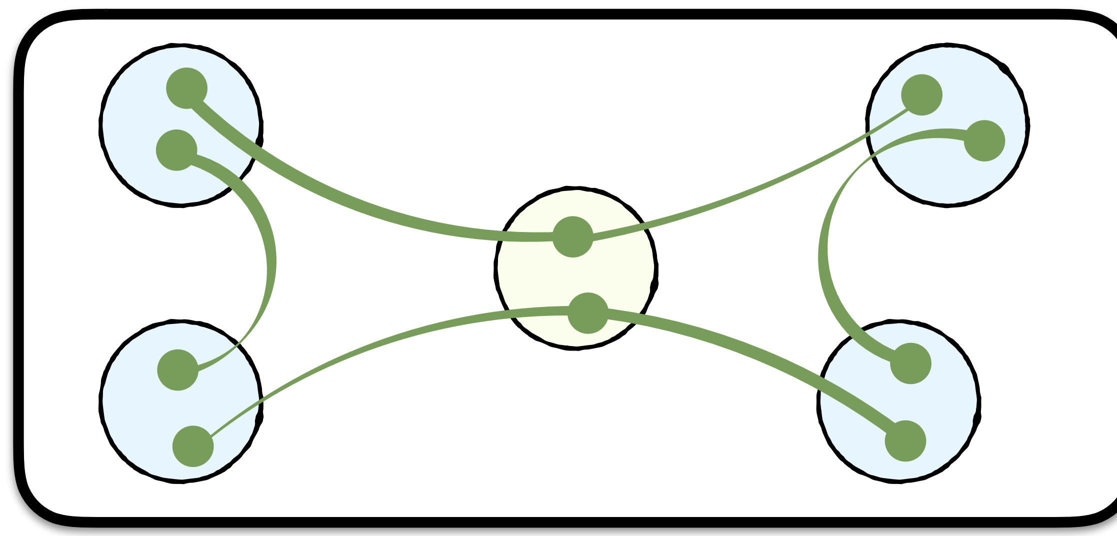
Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$



finite volume spectrum



scattering amplitude

E_L = finite volume spectrum

L = finite volume

F = known function

$\mathcal{M}$ = scattering amplitude

Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$

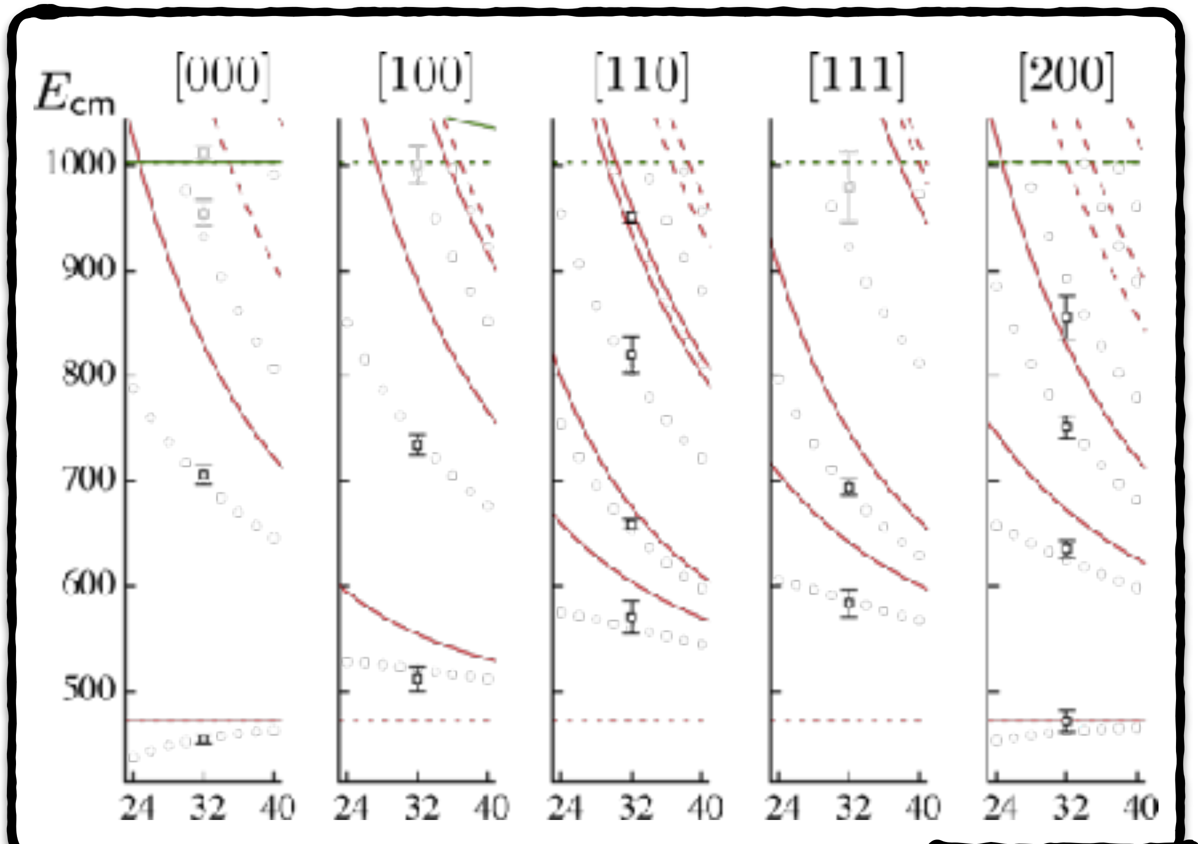
- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Bernard, Lage, Meißner & Rusetsky (2008) [Nπ systems]
- Gockeler, Horsley, et al. (2012) [Nπ systems]
- RB, Davoudi, Luu & Savage (2013) [generic spinning systems]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Bernarda, Lage, Meißner, and Rusetsky [inelastic scalar bosons in TBC]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) / RB & Hansen (2015) [moving inelastic spinning particles]

Extracting the spectrum

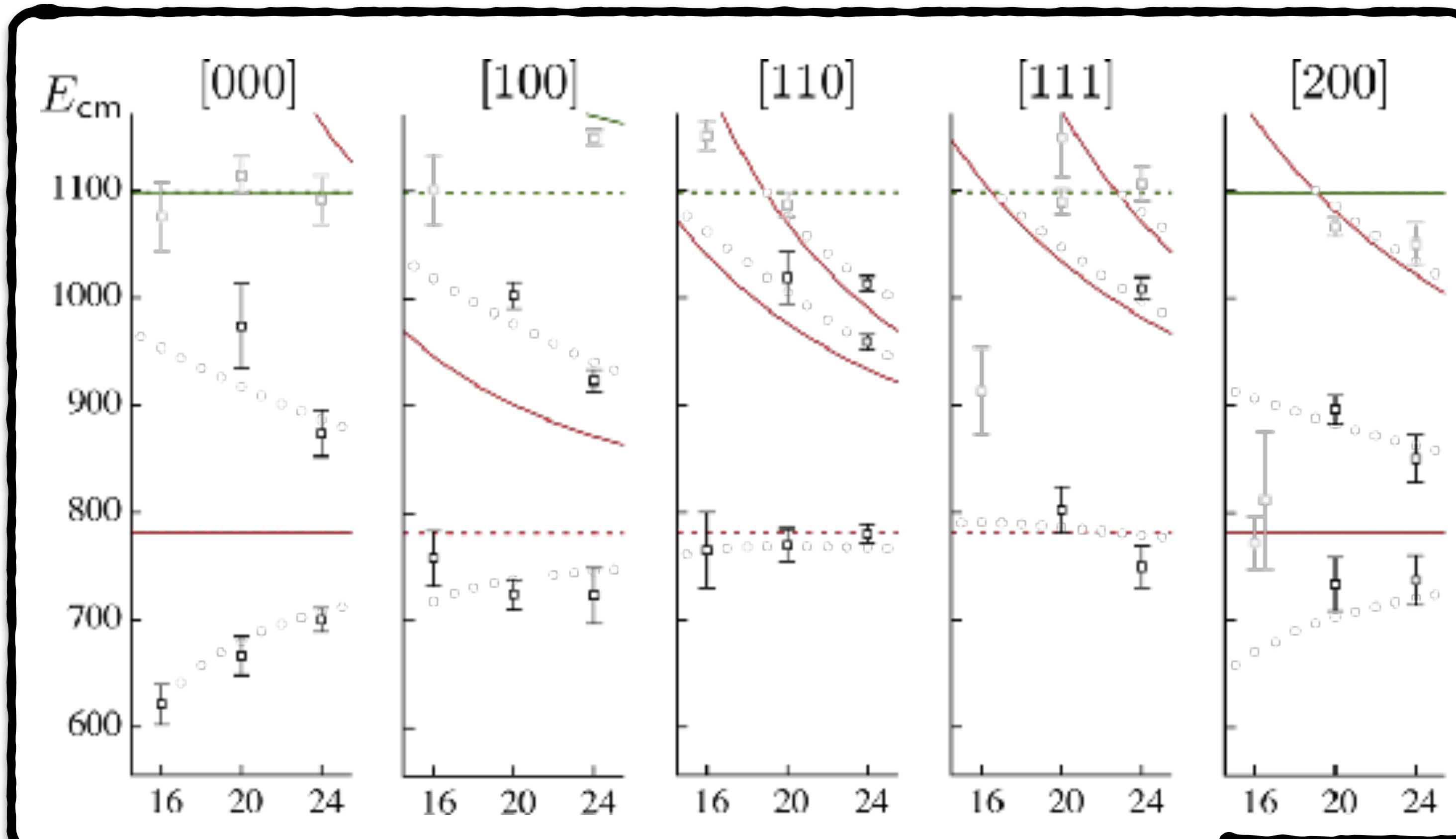
Two-point correlation functions:

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Evaluate **all** Wick contraction - [distillation - Peardon, *et al.* (Hadron Spectrum, 2009)]
- Use a large basis of operators with the same quantum numbers
- ‘Diagonalize’ correlation function *variationally*

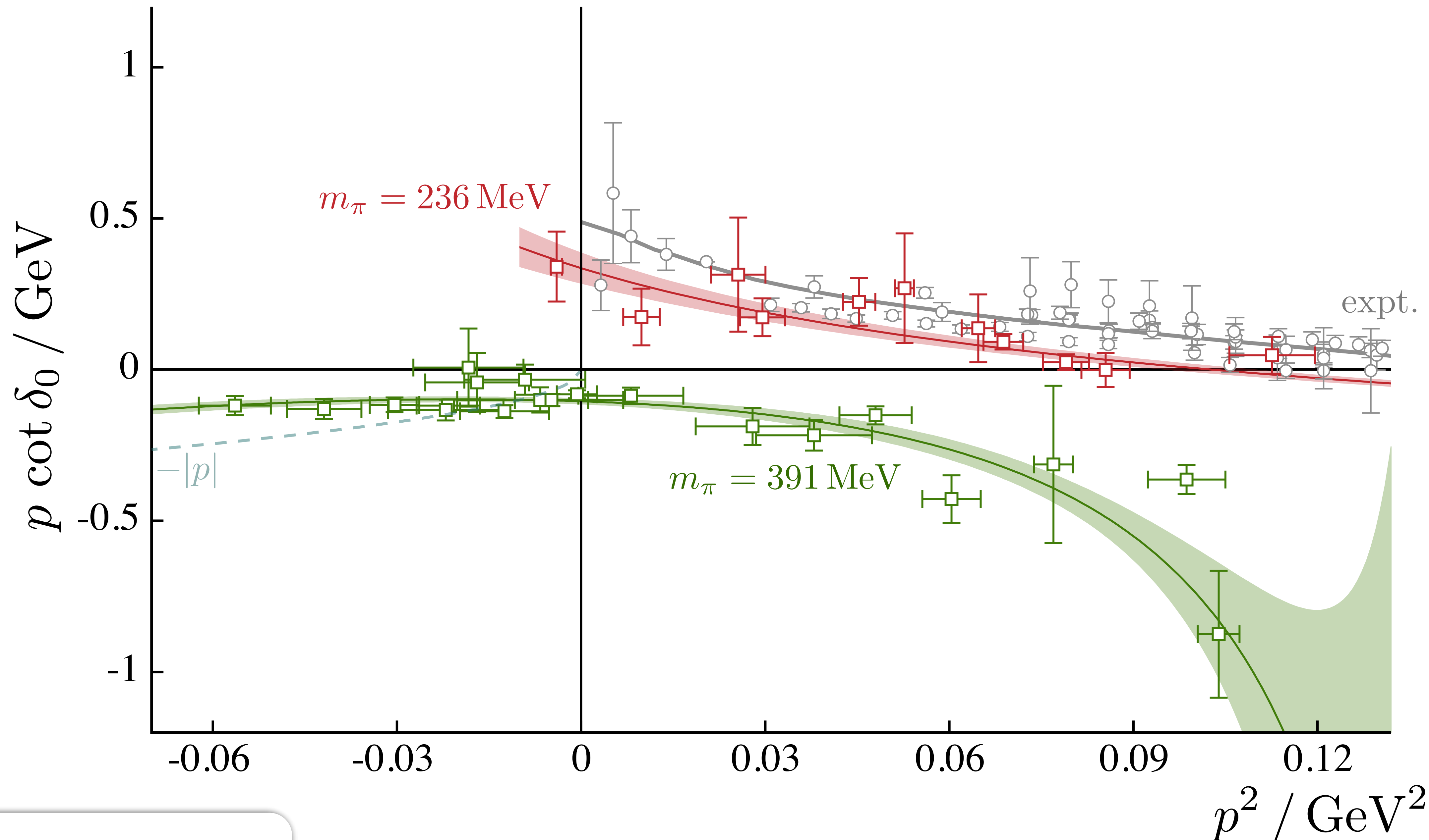


m_π=236 MeV



m_π=391 MeV

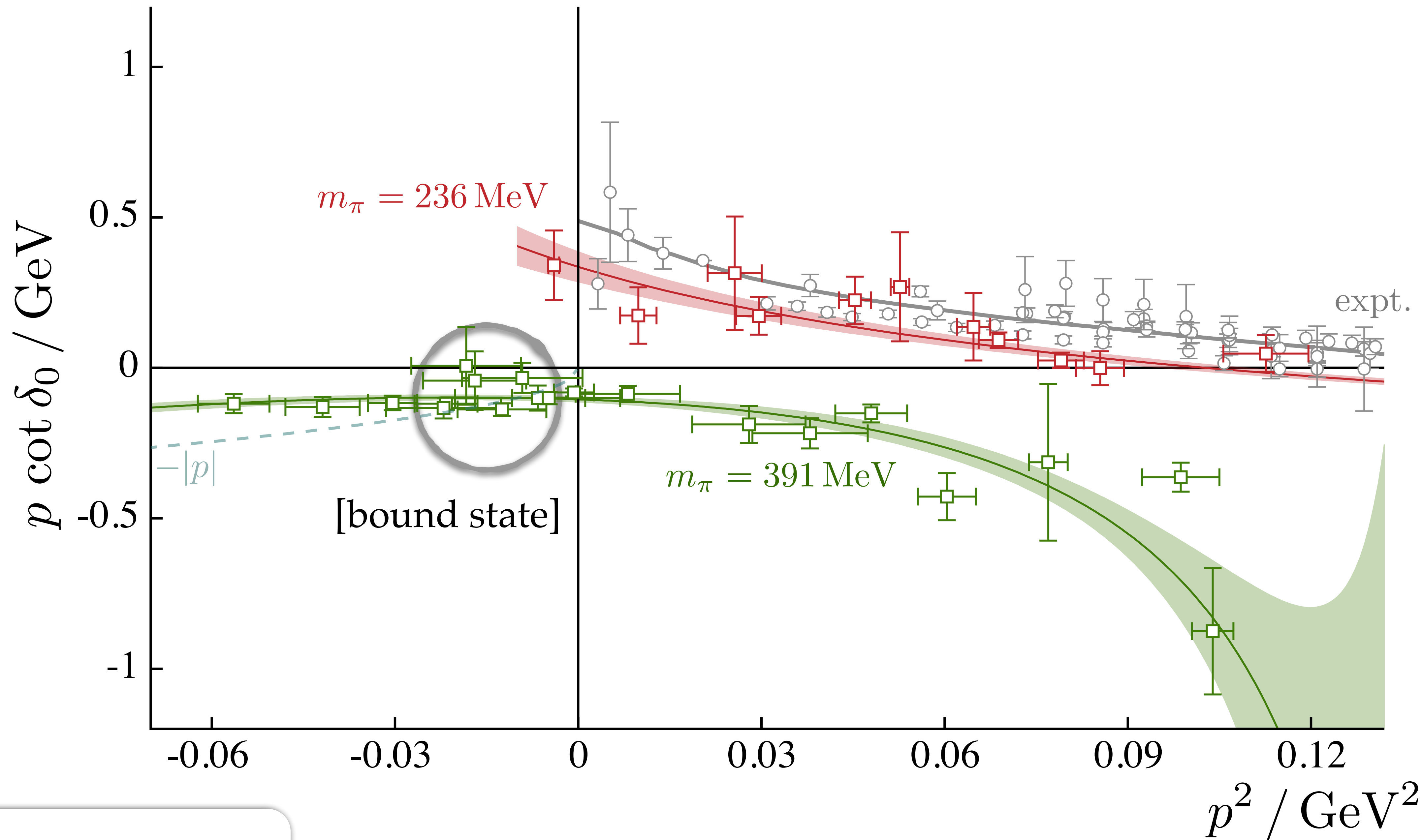
Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

RB, Dudek, Edwards & Wilson (2016)

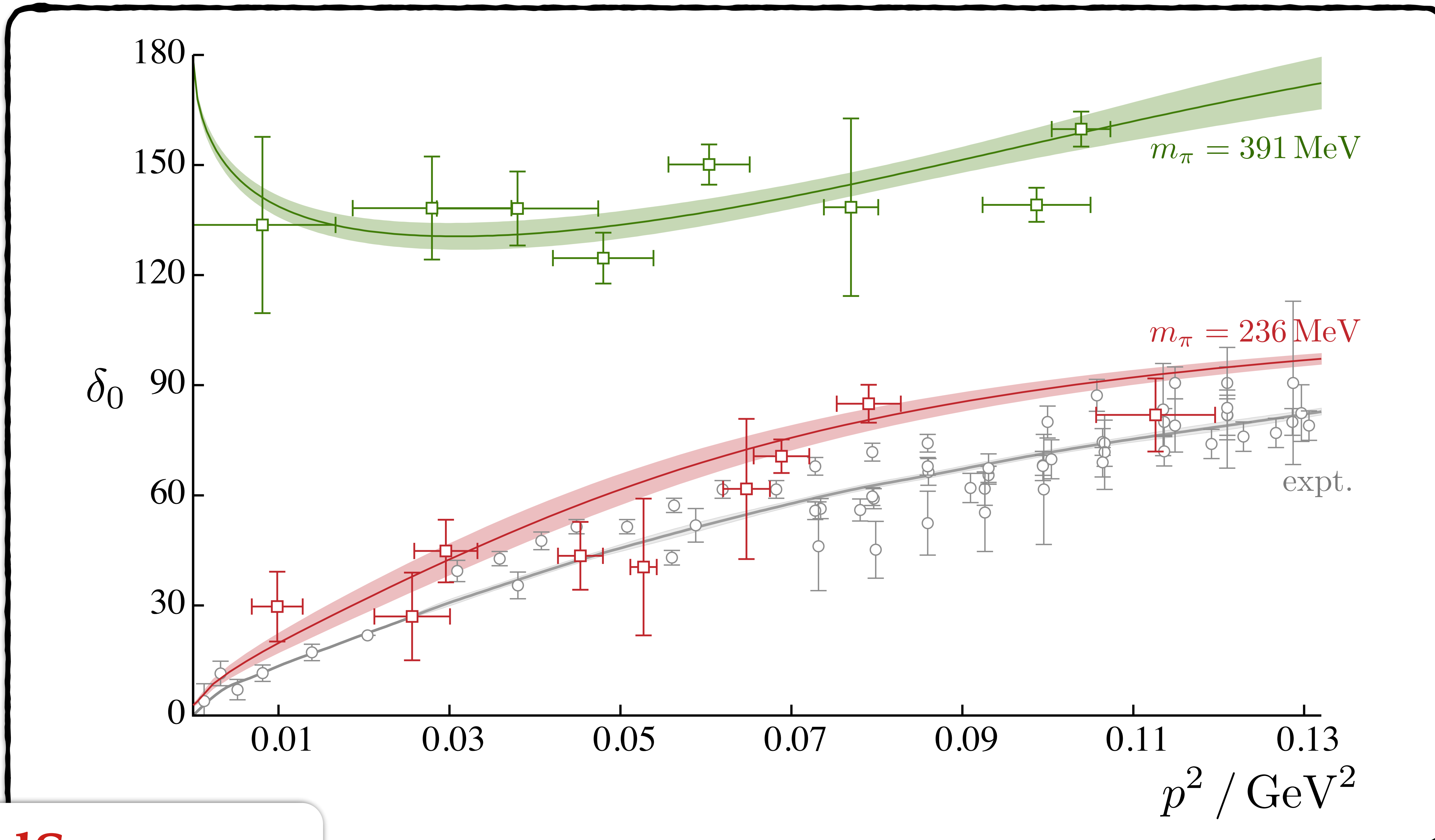
Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

$$\mathcal{M} \sim \frac{1}{p \cot \delta_0 - ip} \to \frac{1}{p \cot \delta_0 + |p|}$$

Isoscalar $\pi\pi$ scattering



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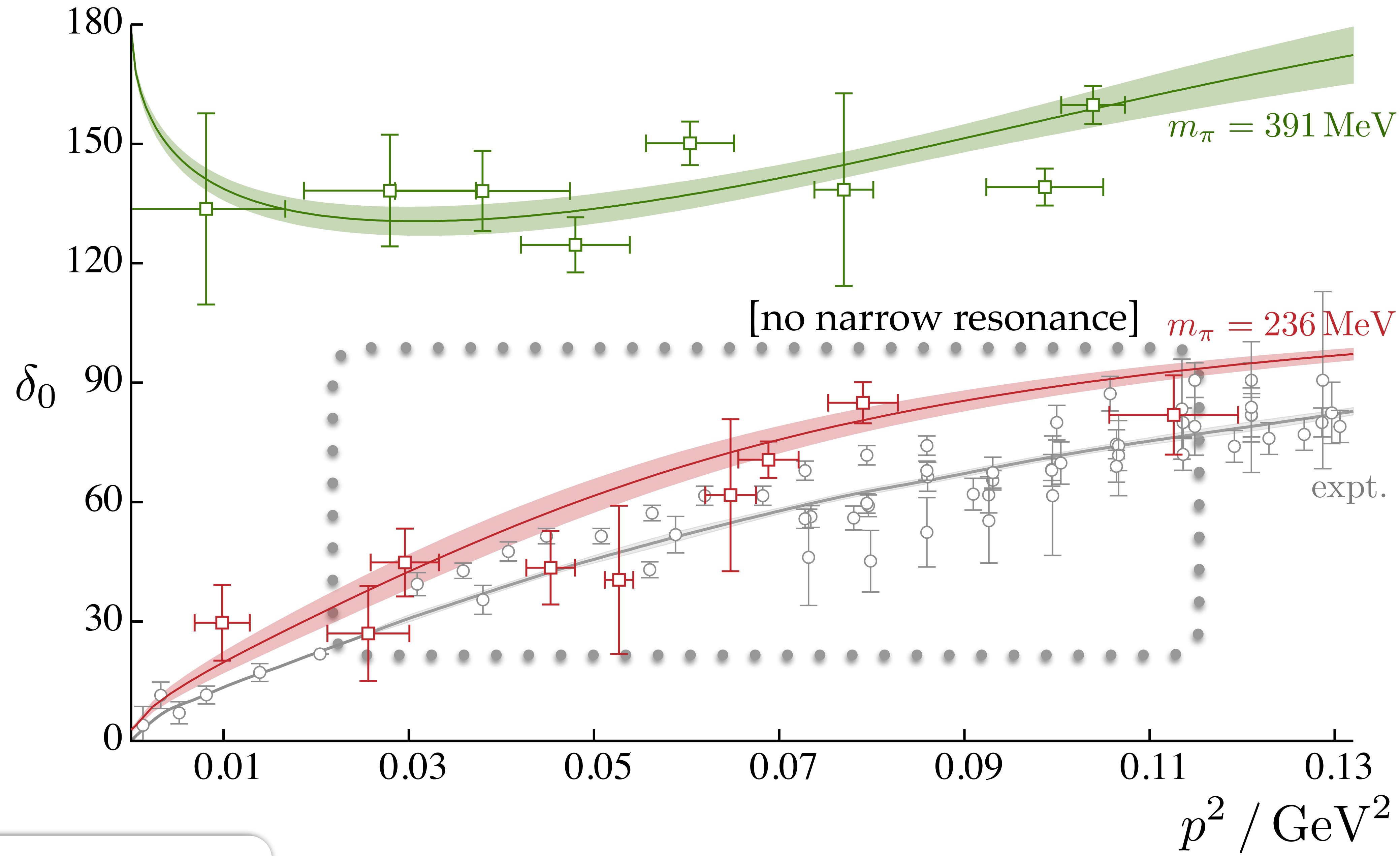

HadSpec Collaboration

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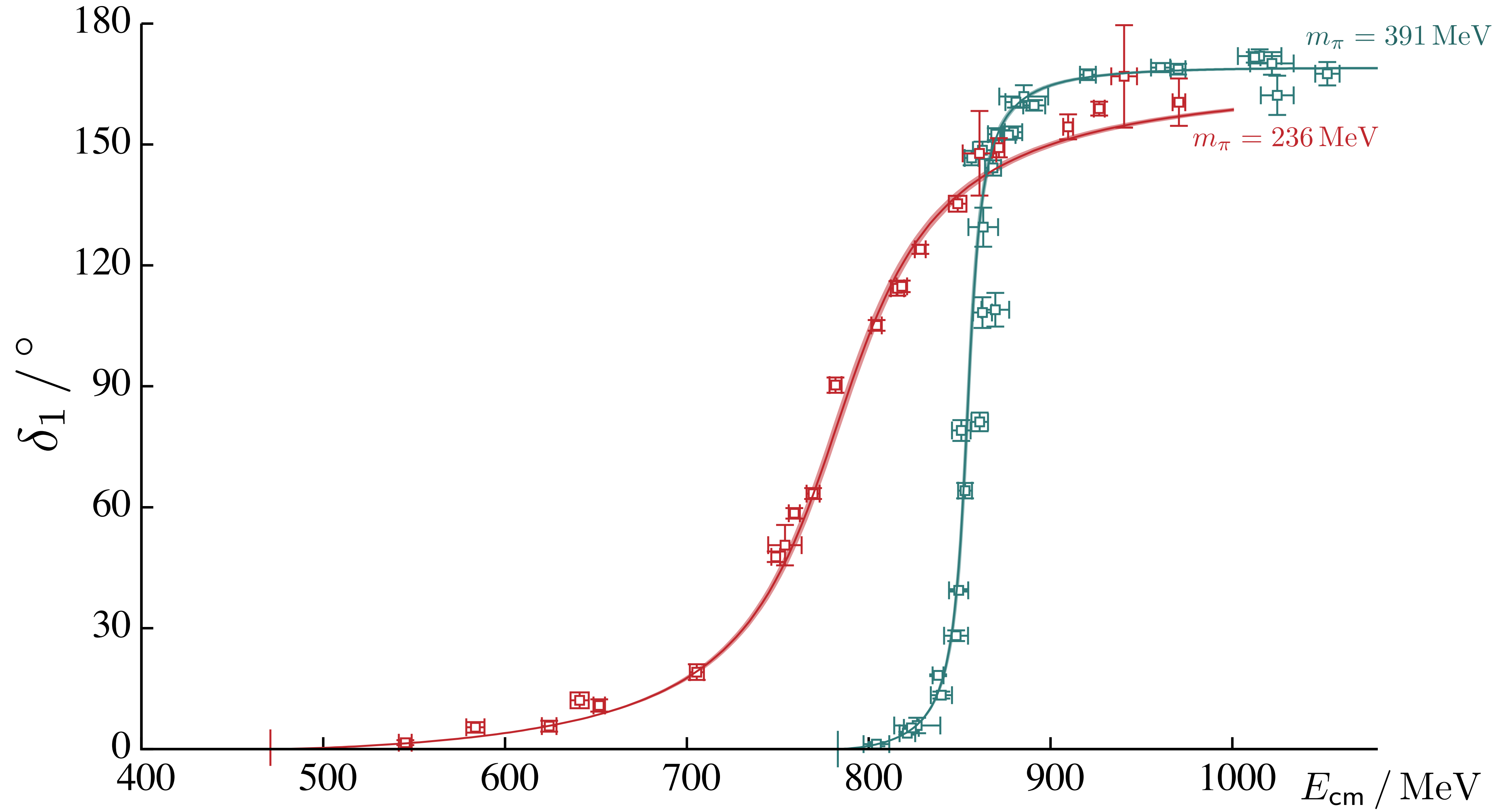
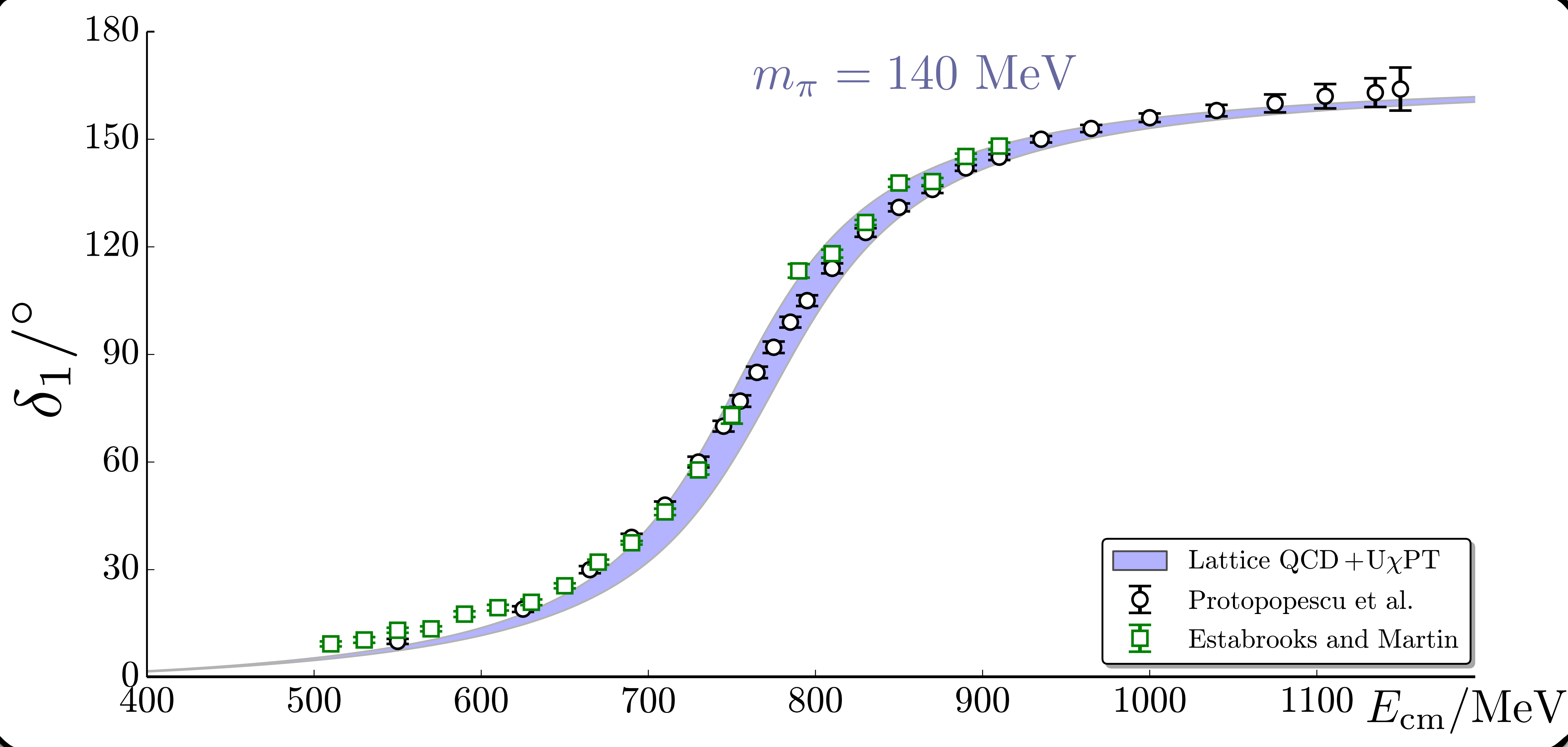
Isovector $\pi\pi$ scattering



HadSpec Collaboration

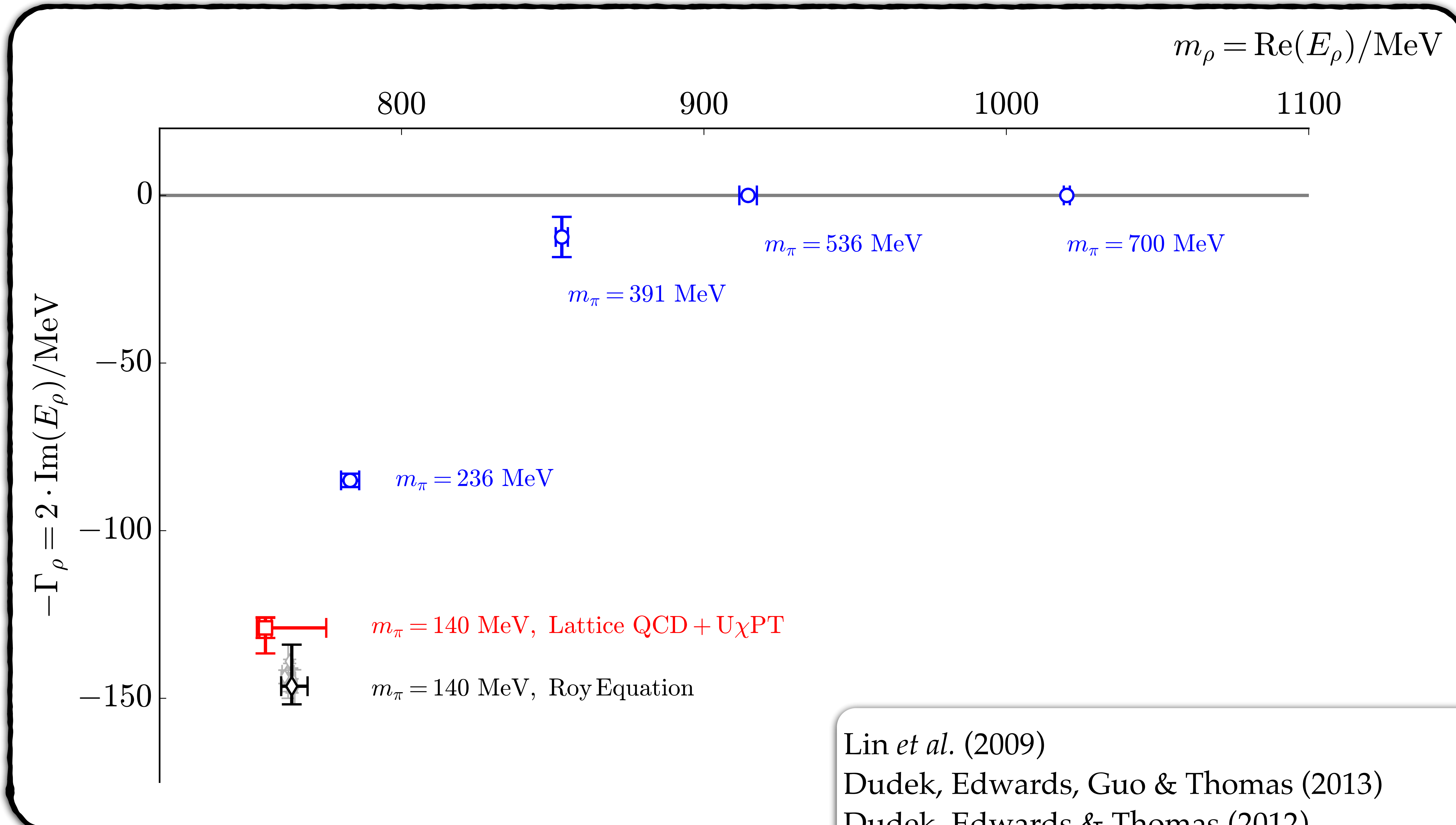
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Comparison with experiment



Bolton, RB, & Wilson (2016)

The ϱ vs m_π



Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
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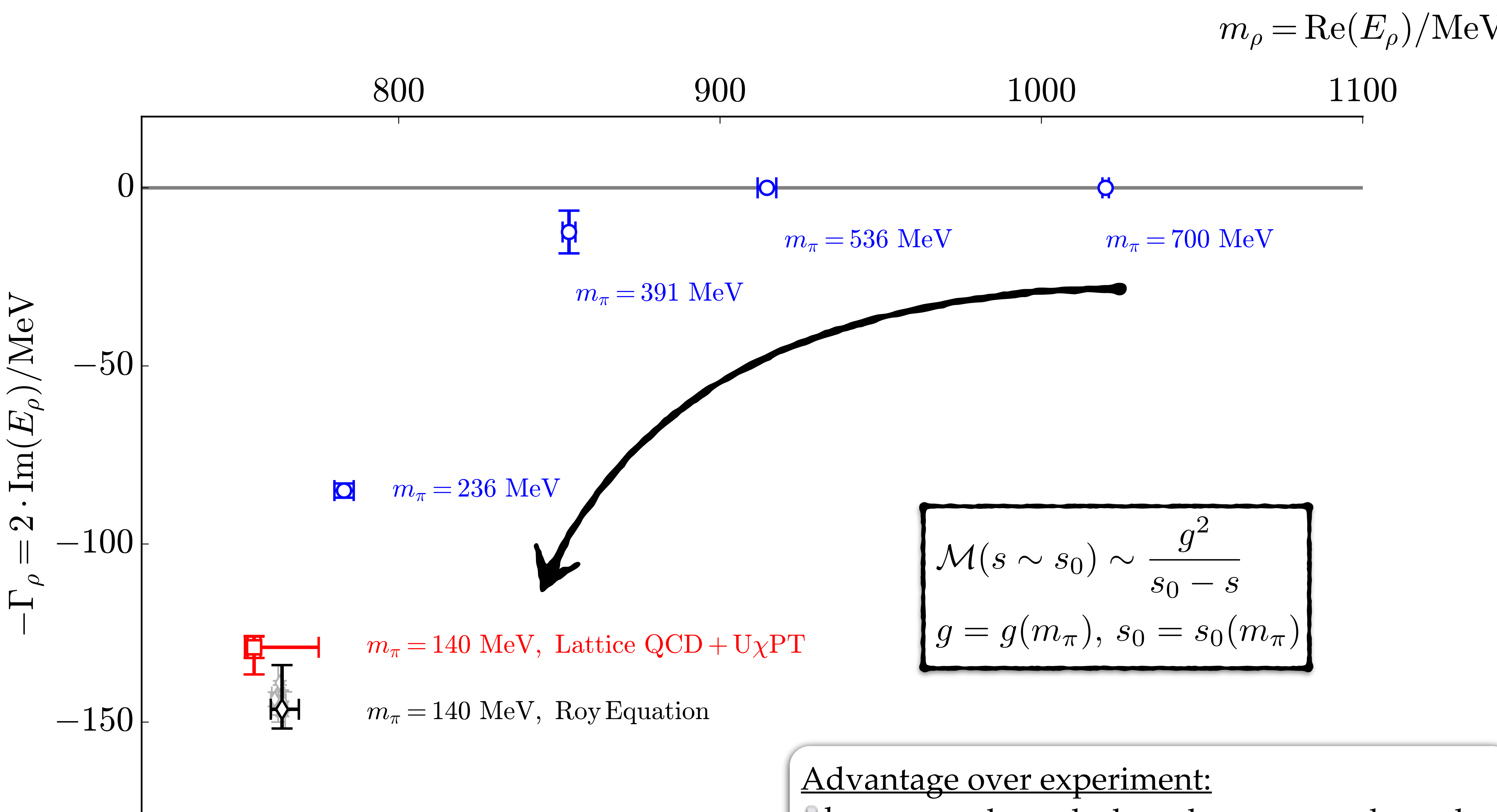


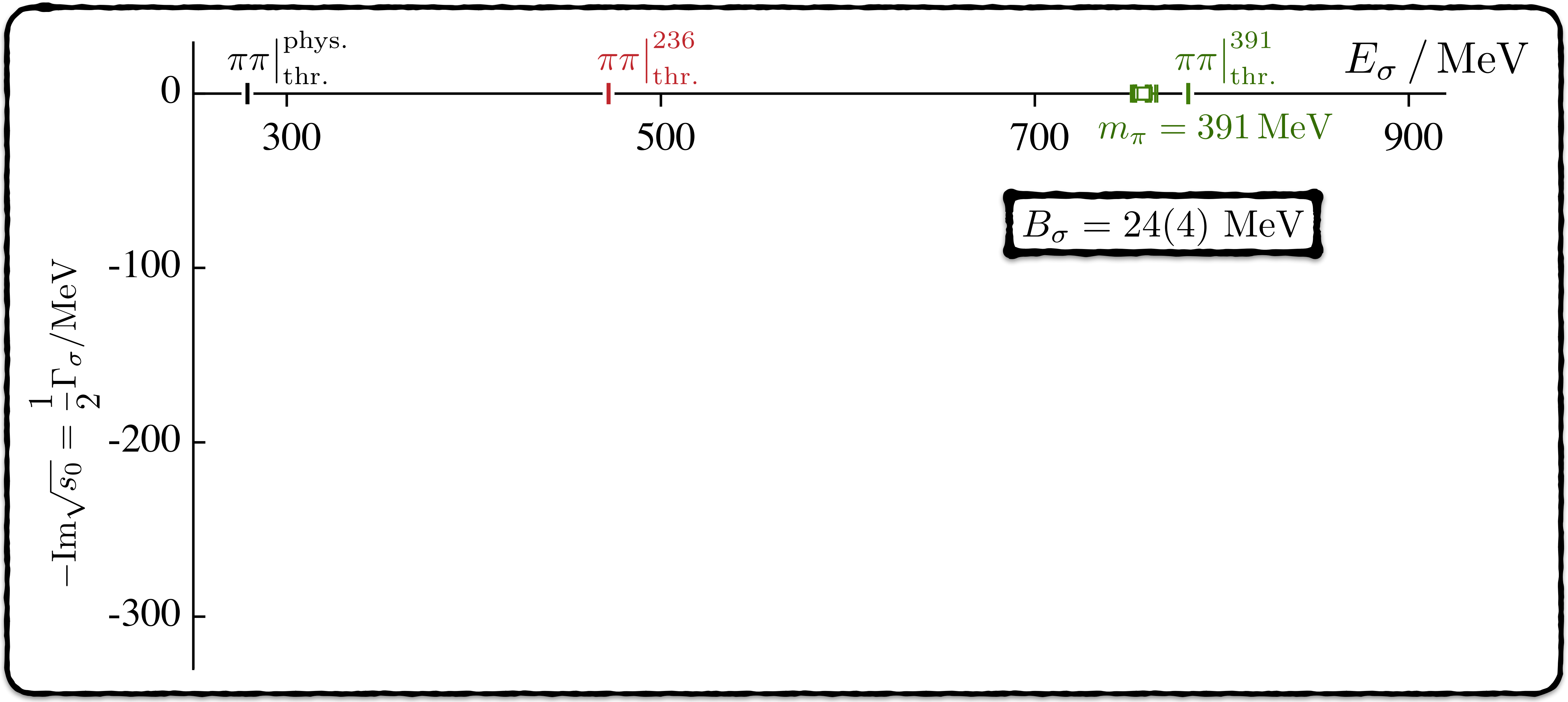
$$\mathcal{M}(s \sim s_0) \sim \frac{g^2}{s_0 - s}$$

$$g = g(m_\pi),\ s_0 = s_0(m_\pi)$$

Advantage over experiment:

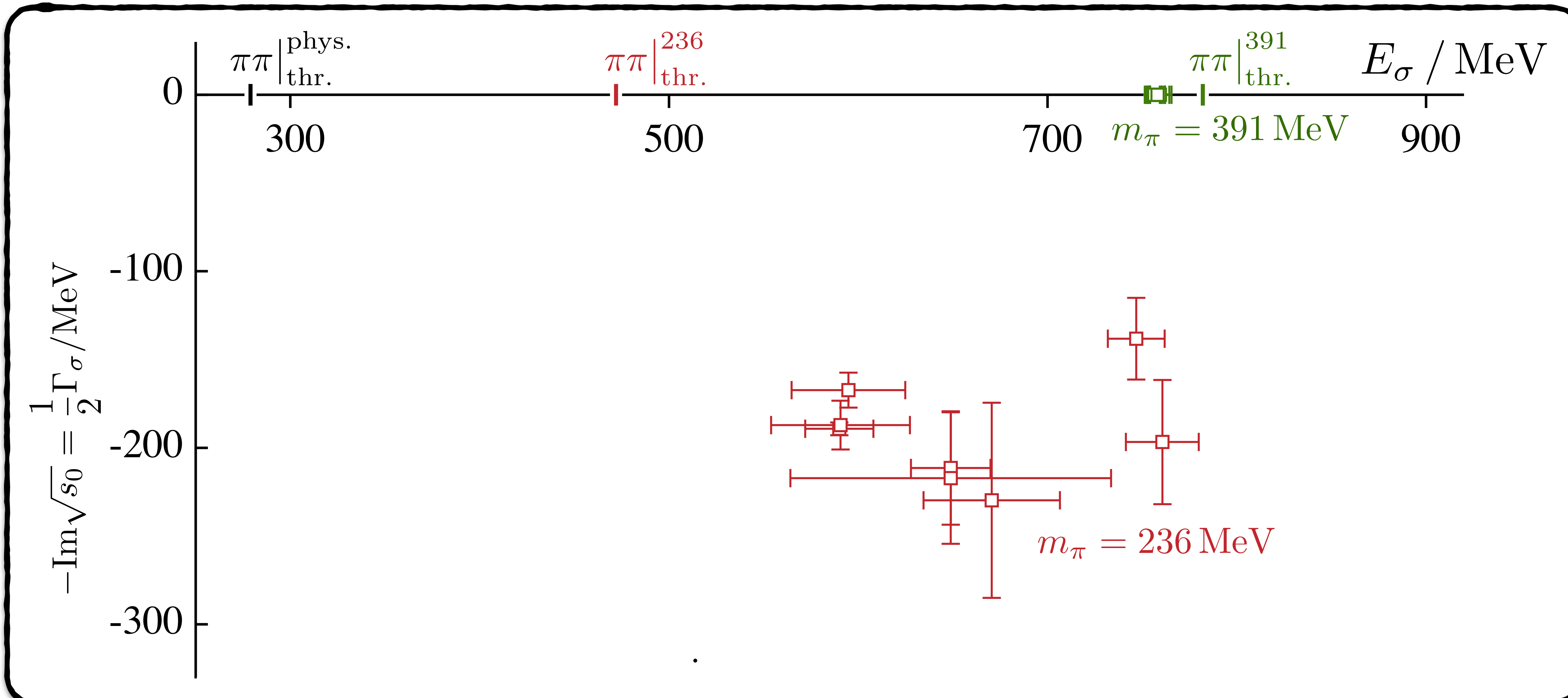
- heavy quarks make broad resonances bound
- **unambiguously track poles in complex plane**

The σ / $f_0(500)$ vs m_π



RB, Dudek, Edwards & Wilson (2016)

The σ / $f_0(500)$ vs m_π



RB, Dudek, Edwards & Wilson (2016)

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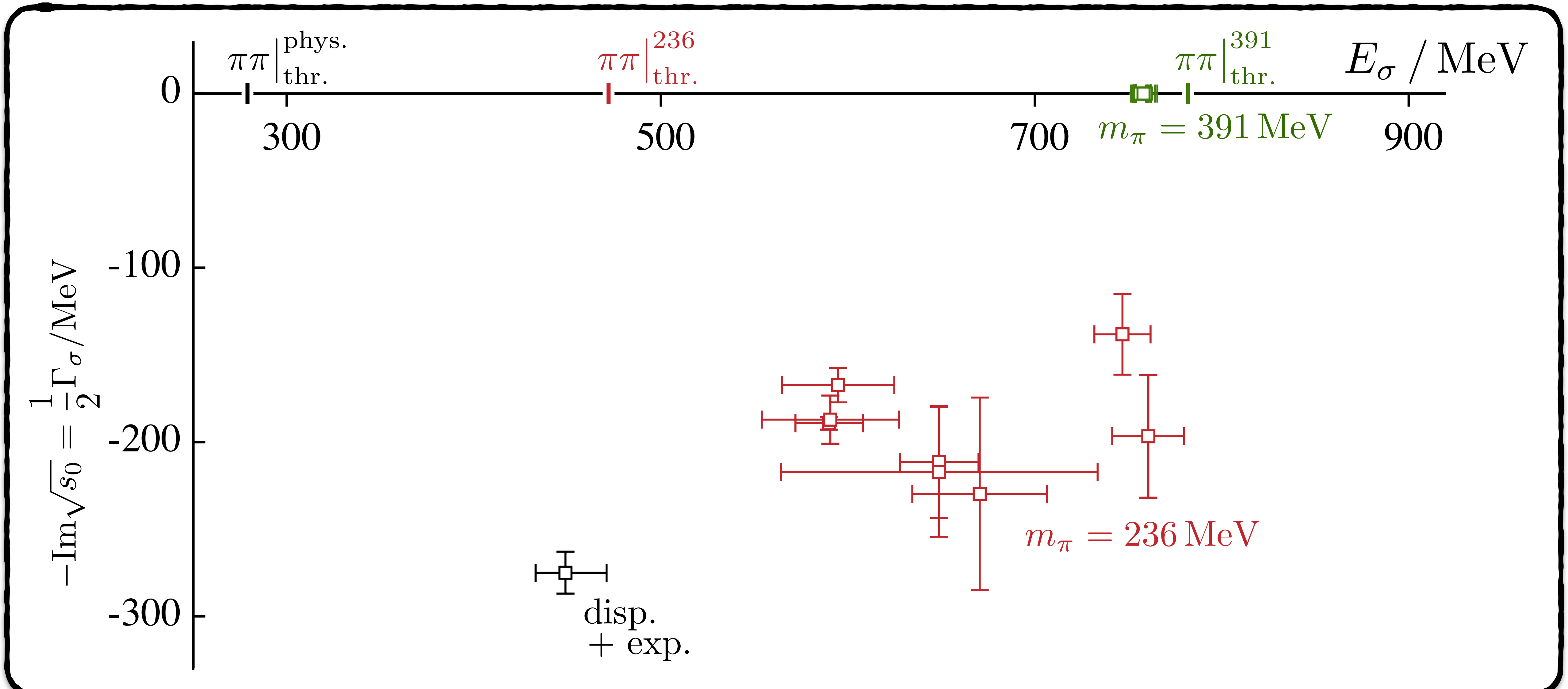


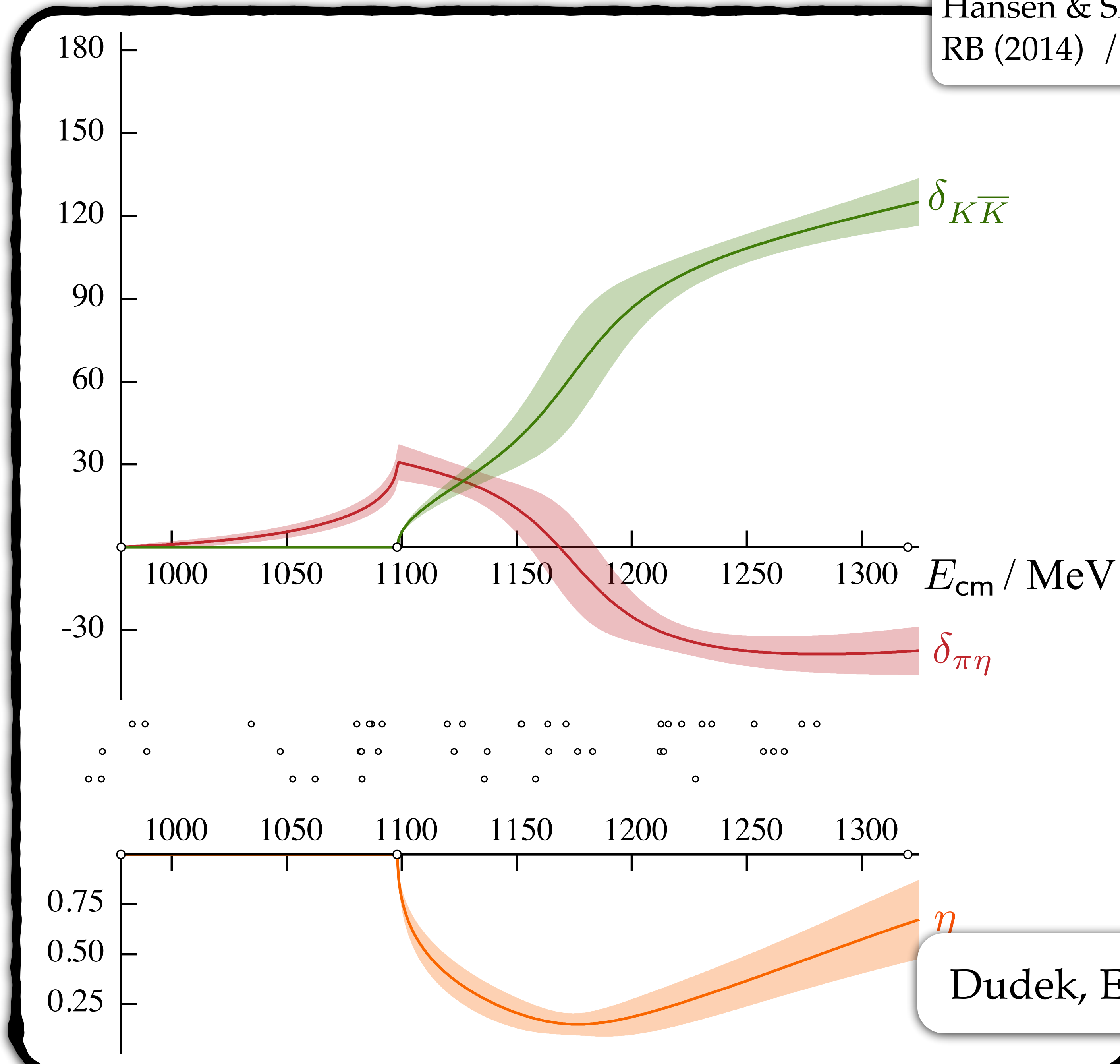
RB, Dudek, Edwards & Wilson (2016)

Going higher in energy

- *Coupled channels:*

$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

Hansen & Sharpe / RB & Davoudi (2012)
RB (2014) / RB & Hansen (2015)



m_π=391 MeV

~~RB~~

Dudek, Edwards & Wilson (2016)

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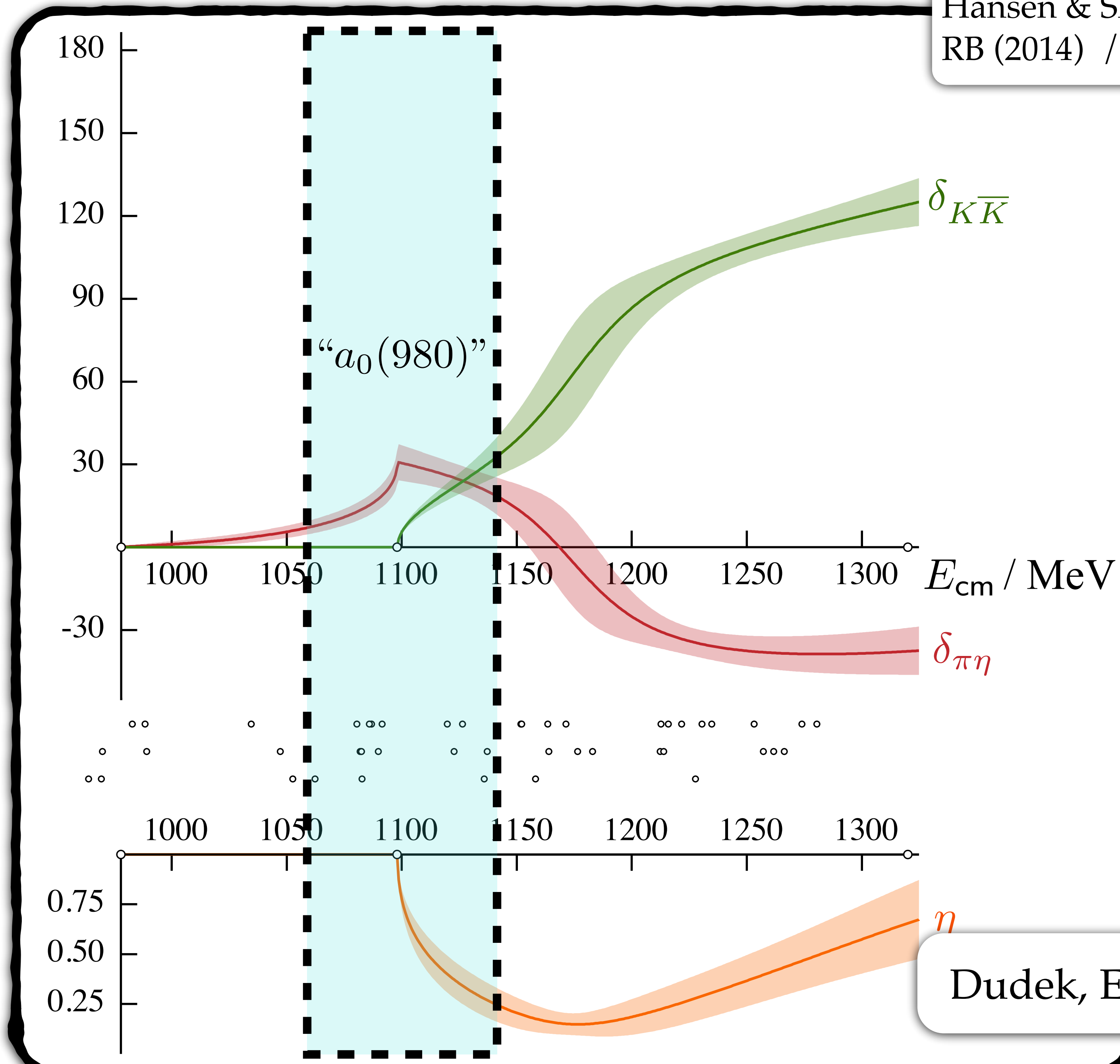


m$_\pi$=391 MeV

~~RB~~

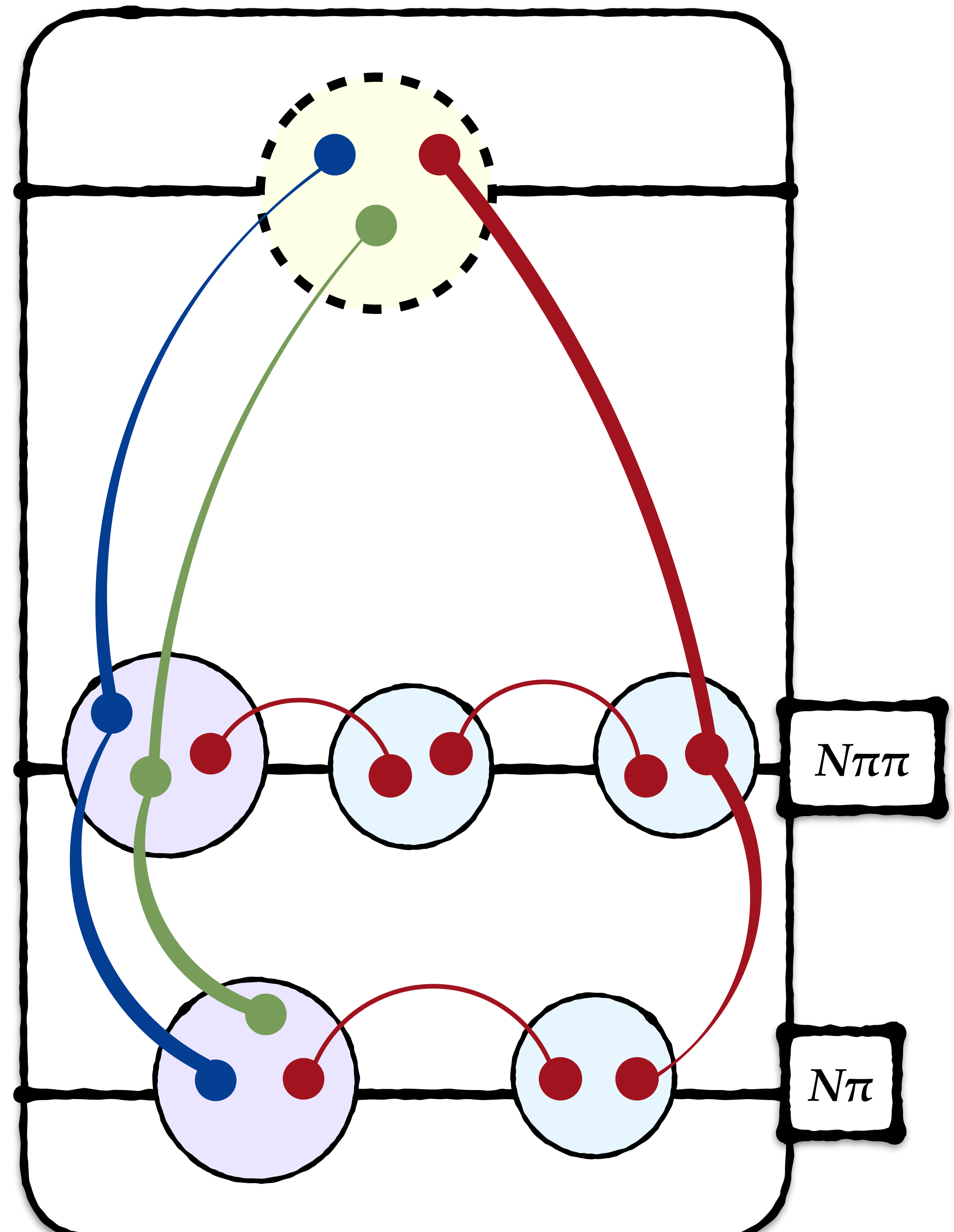
Dudek, Edwards & Wilson (2016)

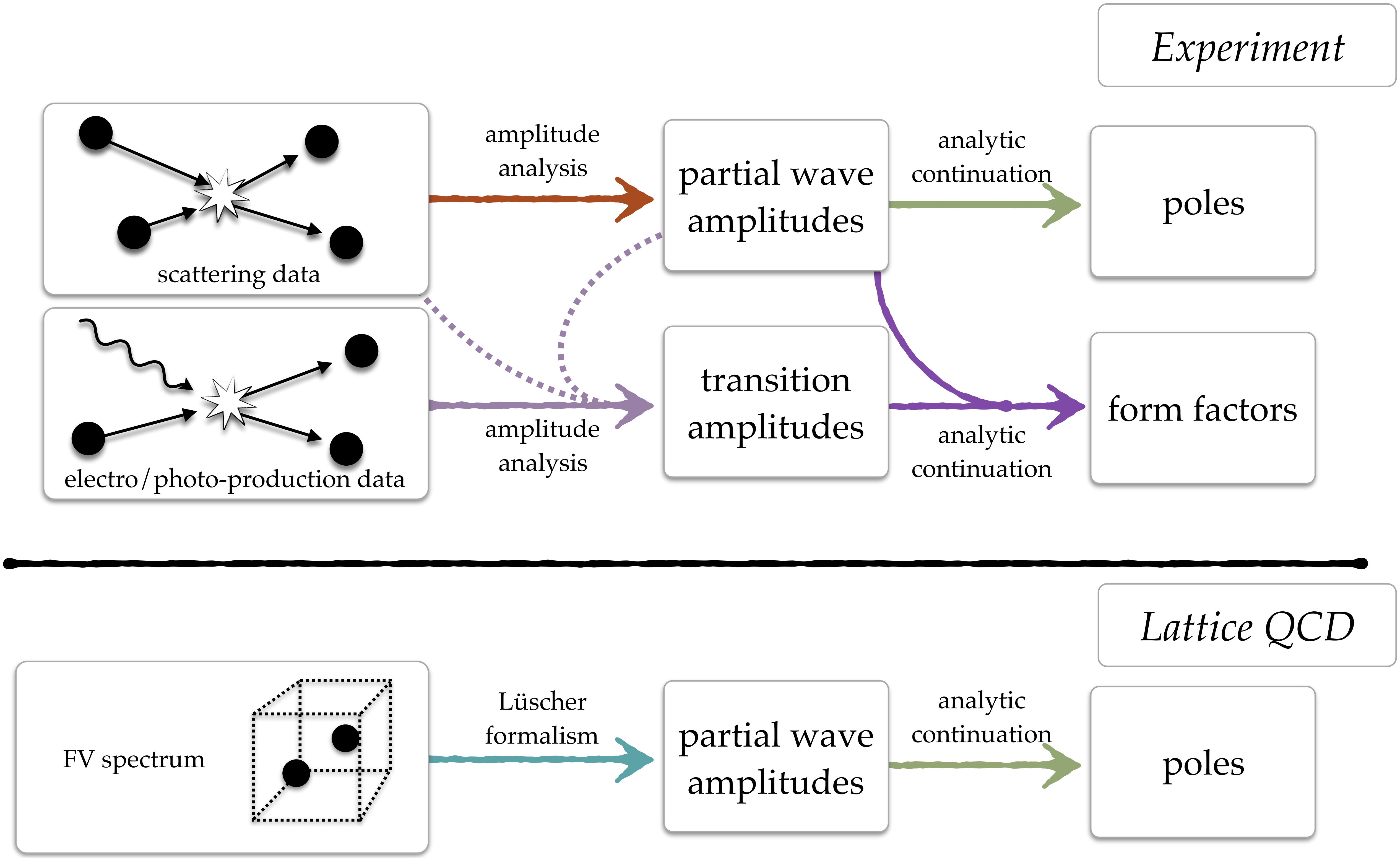
Going higher in energy

- *Coupled channels*
- *Beyond two particles:*

$$\det\left[1+\begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix}\begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right]=0$$

Hansen & Sharpe (2014)
RB, Hansen & Sharpe (2017)

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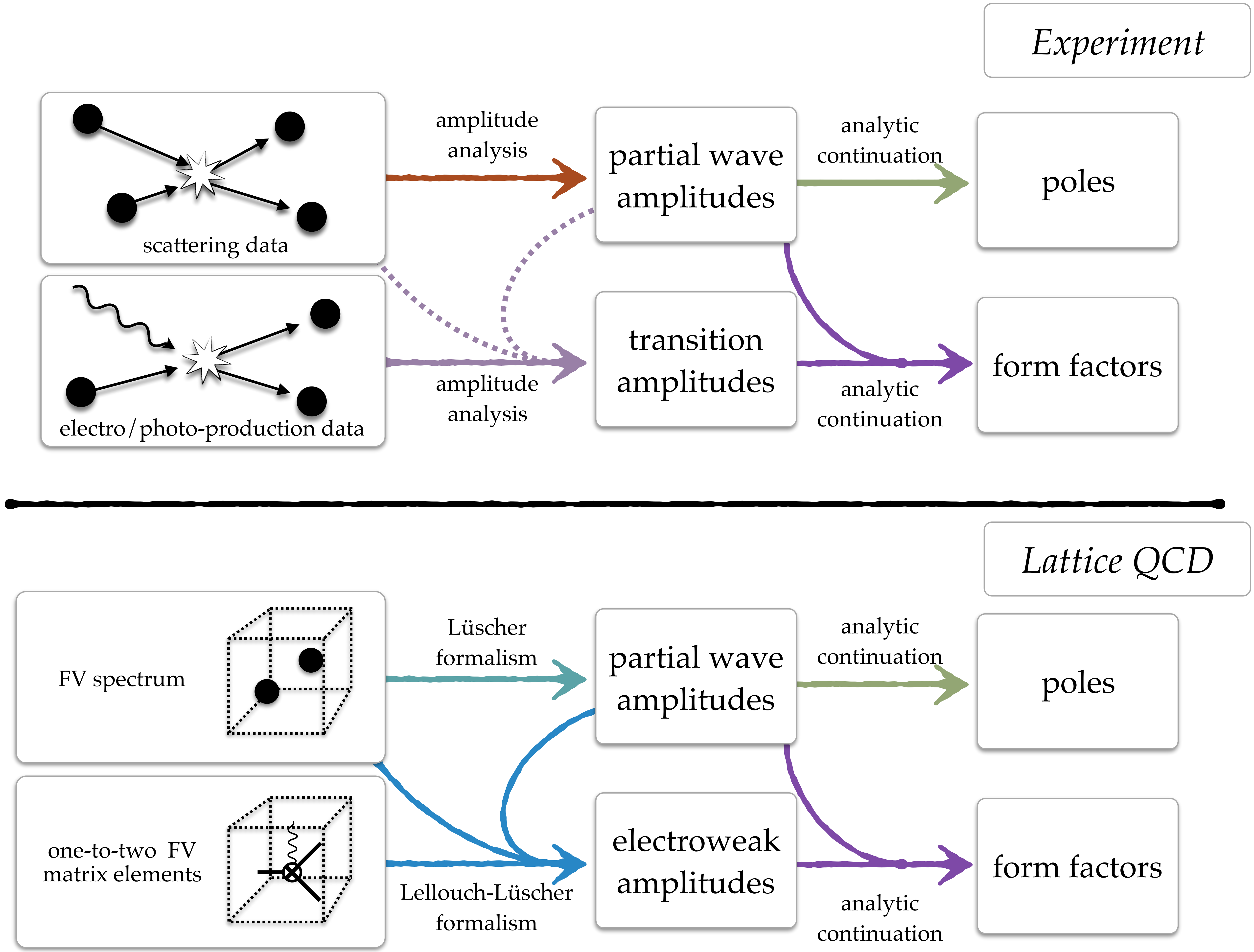
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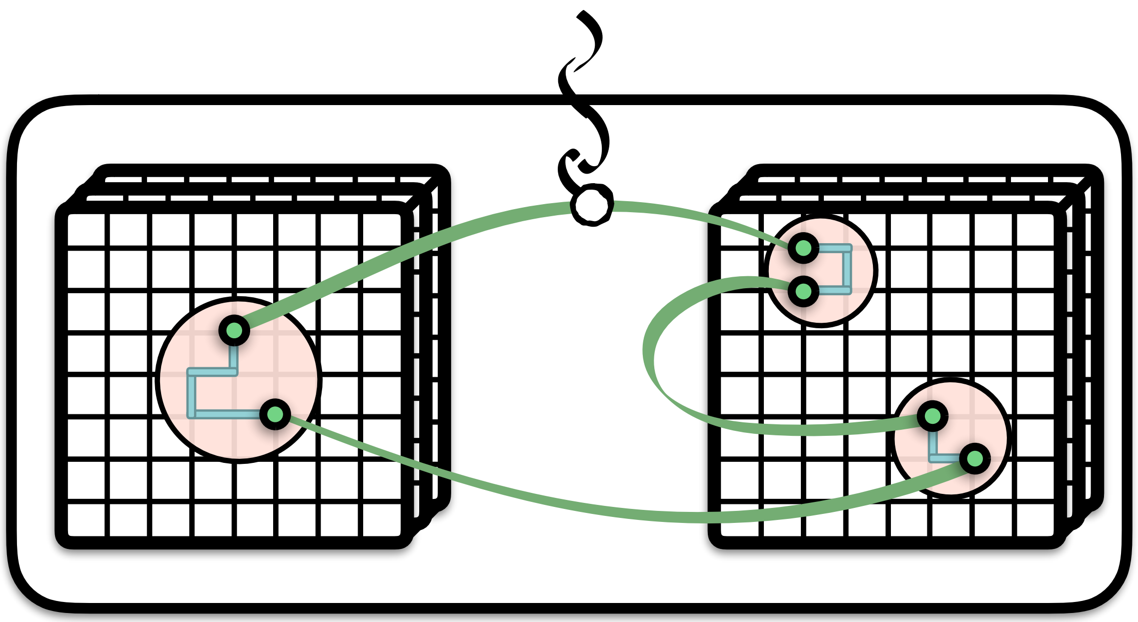
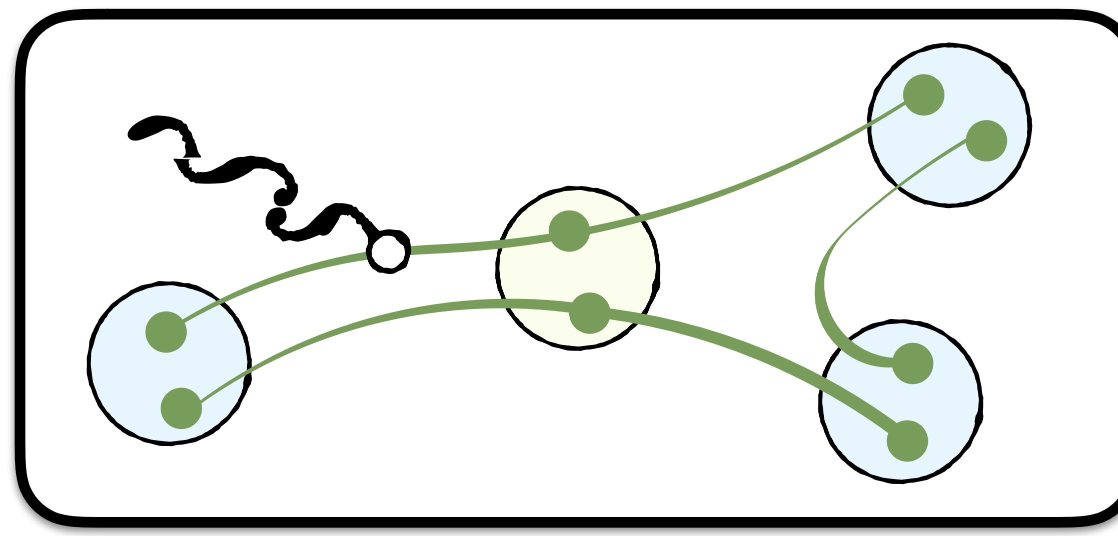
Electroweak properties

$$\left| \langle \mathbf{2} | \mathcal{J} | \mathbf{1} \rangle_L \right| = \sqrt{\mathcal{H}\ \mathcal{R}\ \mathcal{H}}$$



finite volume matrix element



electroweak amplitude

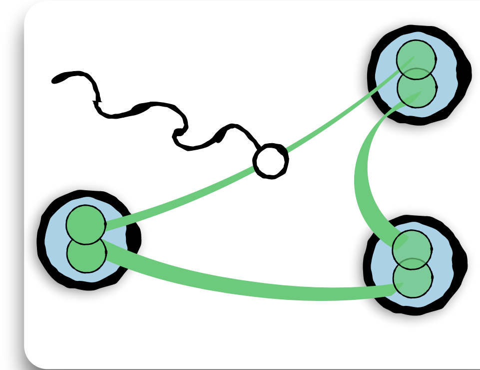
$\langle \mathbf{2} | \mathcal{J} | \mathbf{1} \rangle_L$ = finite matrix element

$\mathcal{R}$ = known function

$\mathcal{H}$ = electroweak amplitude

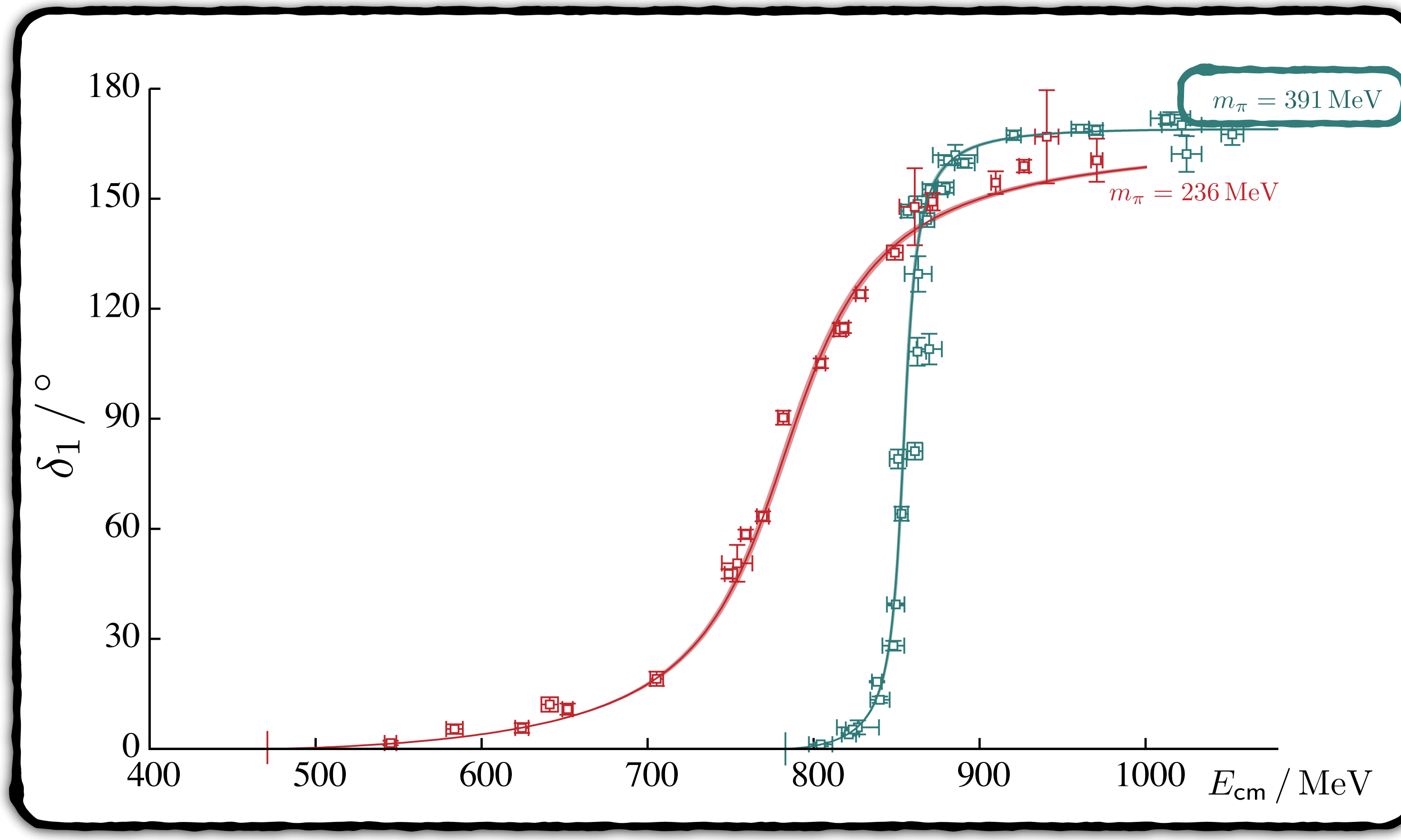
RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2014)

$\pi\gamma^*$-to-$\pi\pi$



Exploratory $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ calculation:

- $m_\pi = 391$ MeV

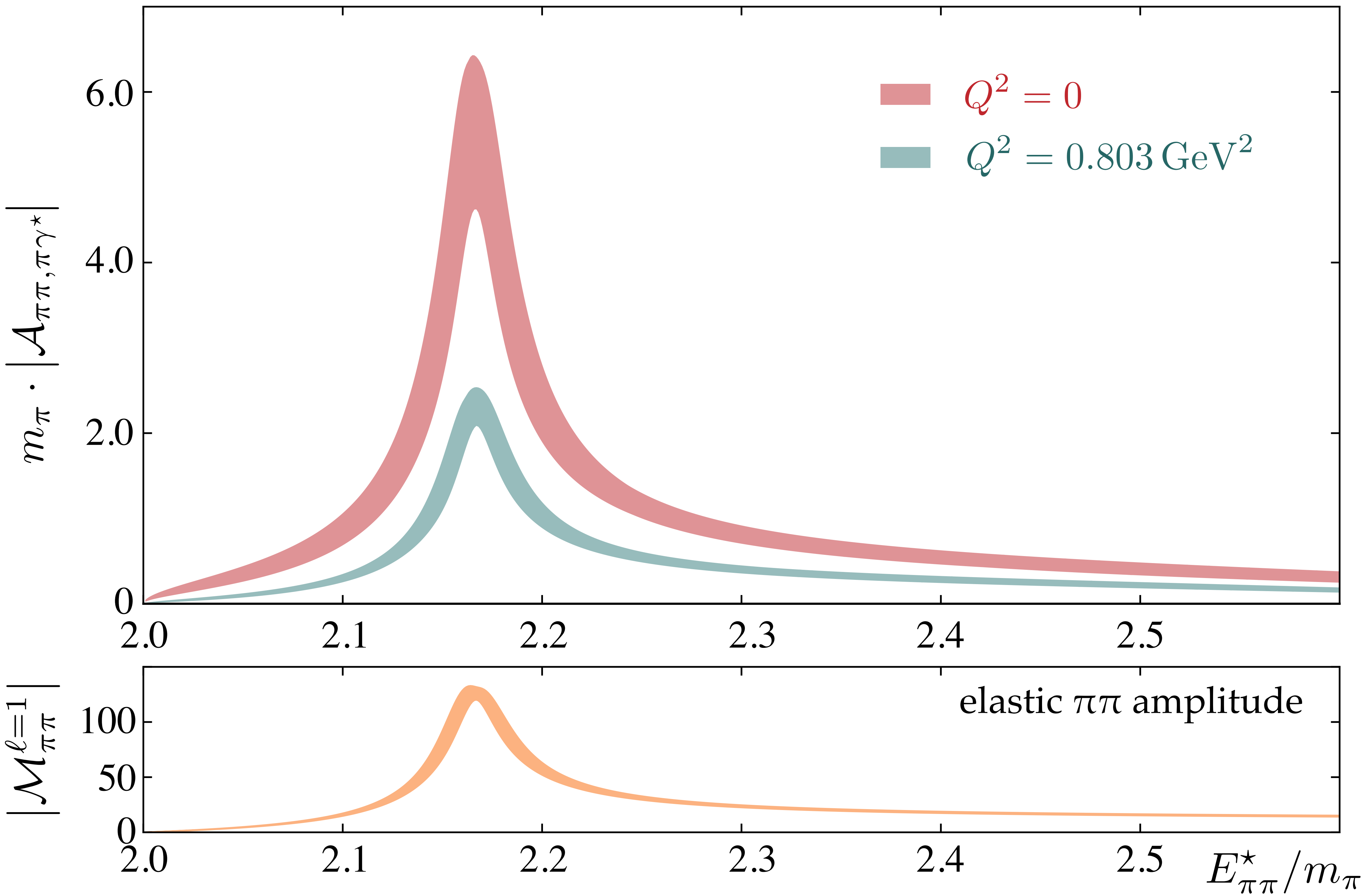


- Matrix element determined in **42** kinematic point: $(E_{\pi\pi}, Q^2)$
- Lorentz decomposition:

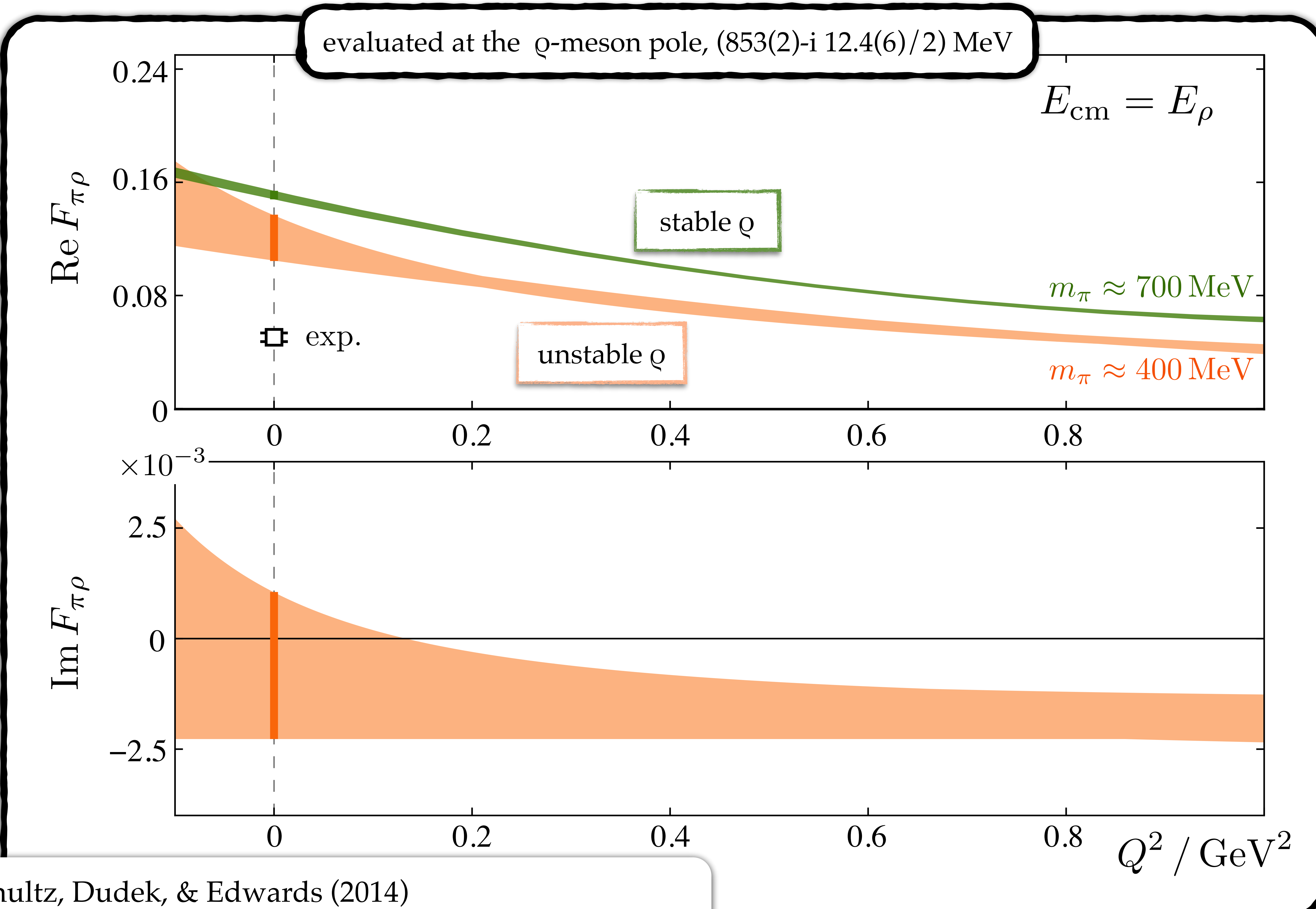
$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \epsilon^{\mu\nu\alpha\beta}\, P_{\pi,\nu}\, P_{\pi\pi,\alpha}\, \epsilon_\beta(\lambda_{\pi\pi}, \mathbf{P}_{\pi\pi})\, \frac{2}{m_\pi} \mathcal{A}_{\pi\pi,\pi\gamma^\star}$$

$\pi\pi/\rho$ polarization — *$\pi\pi/\rho$ helicity* — *Lorentz scalar*

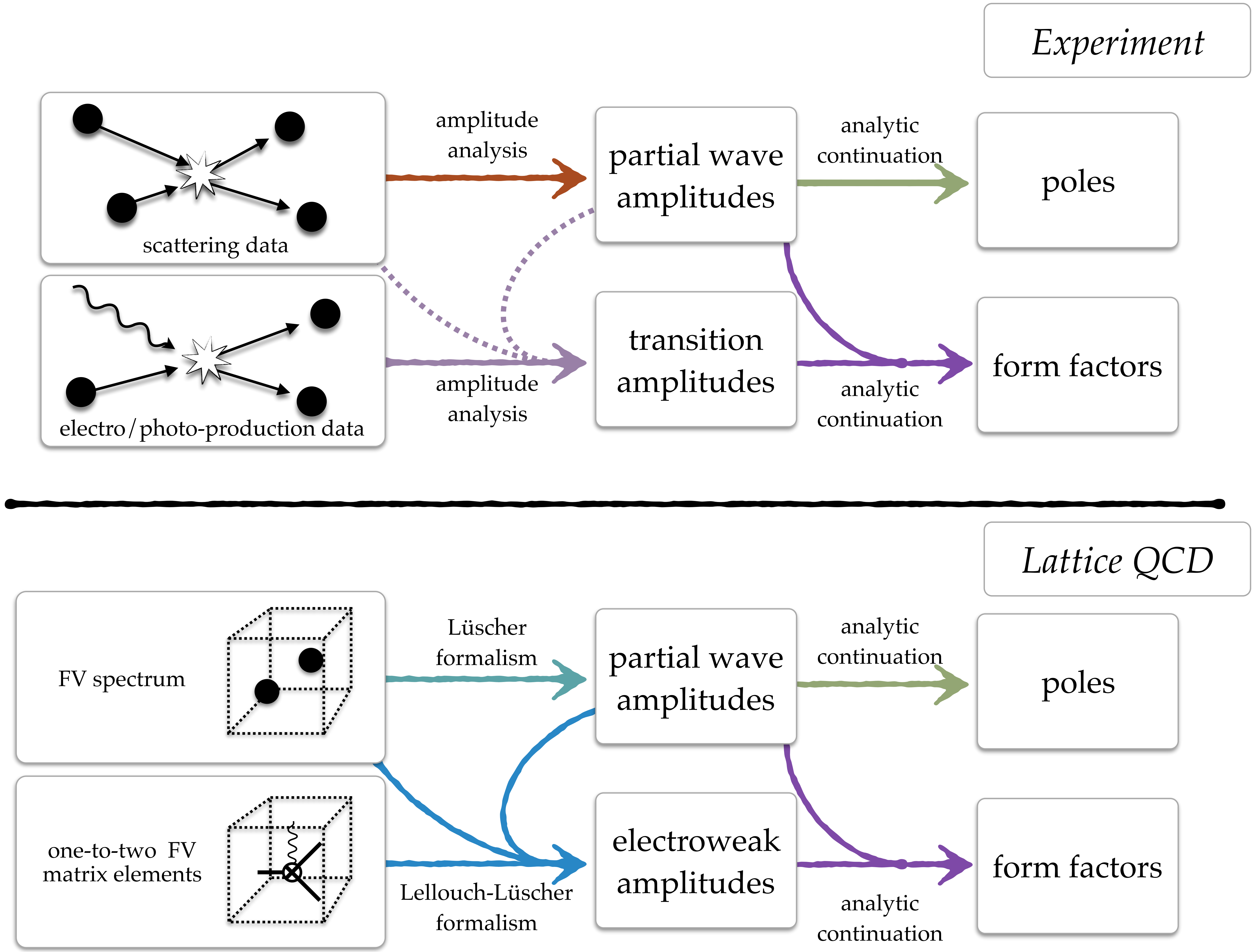
$\pi\gamma^*$-to-$\pi\pi$ amplitude



Form factor at ϱ pole



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)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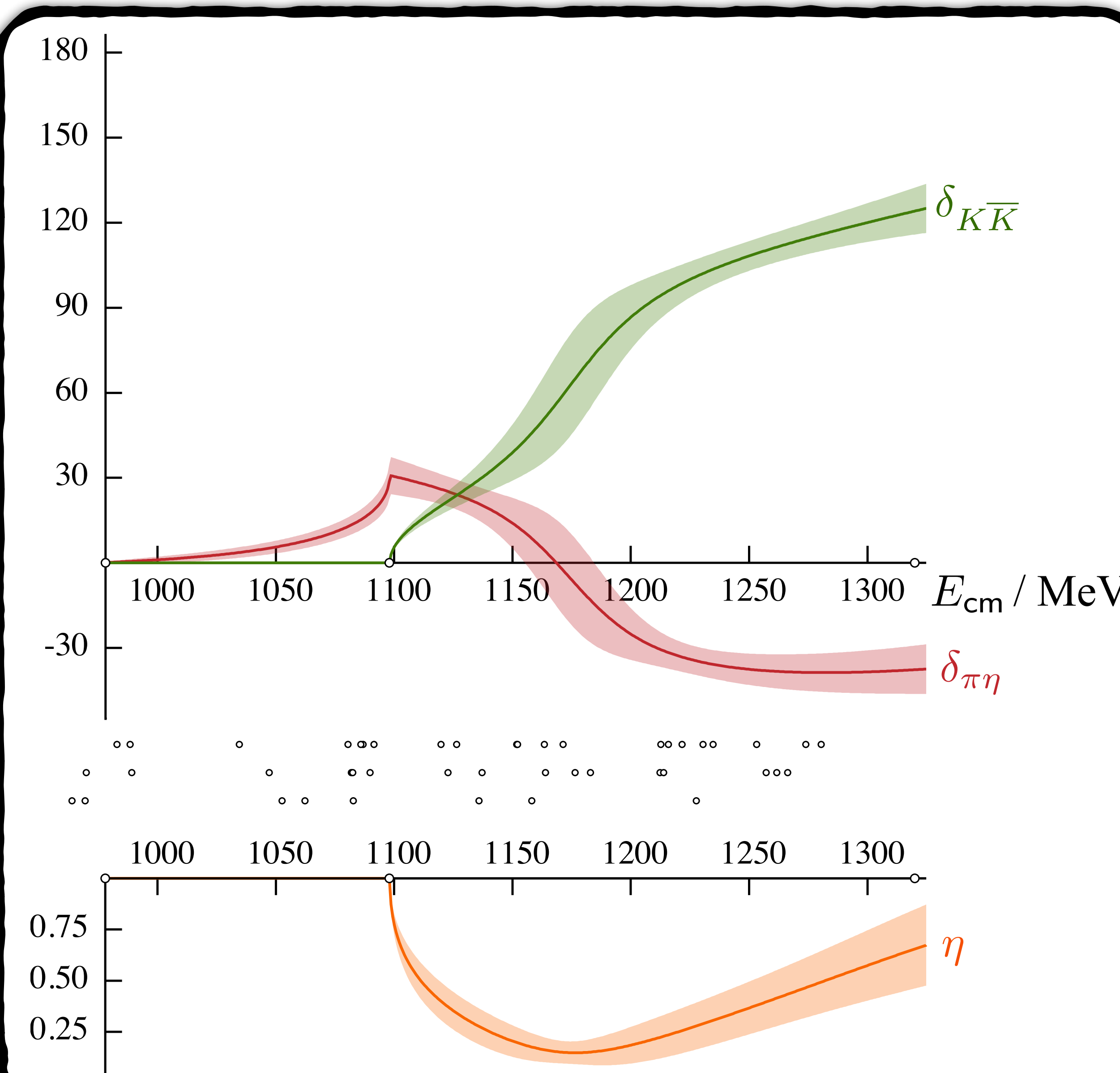
Summary / outlook

- *Coupled channels*

formalism understood:

Hansen & Sharpe / RB & Davoudi (2012)
RB (2014) / RB & Hansen (2015)

few implementations to date by HadSpec



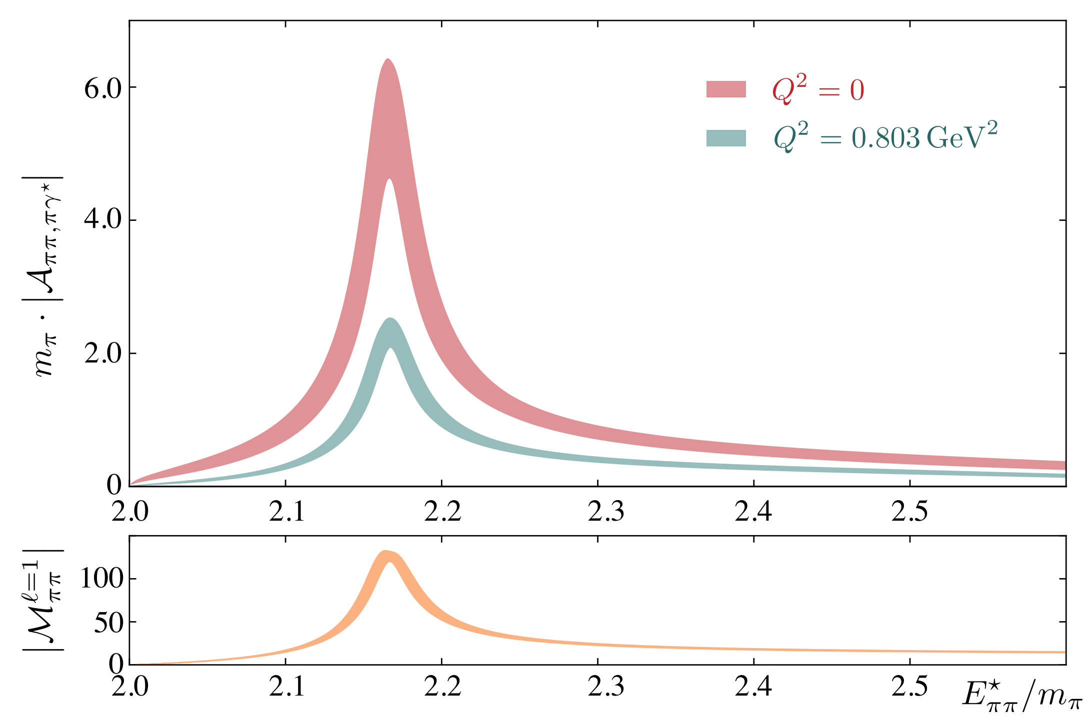
~~RB~~

Dudek, Edwards & Wilson (2016)

Summary / outlook

- *Coupled channels*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*

first implementation: $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ



formalism understood:

RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2015)

RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015, 2016)
RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015, 2016)

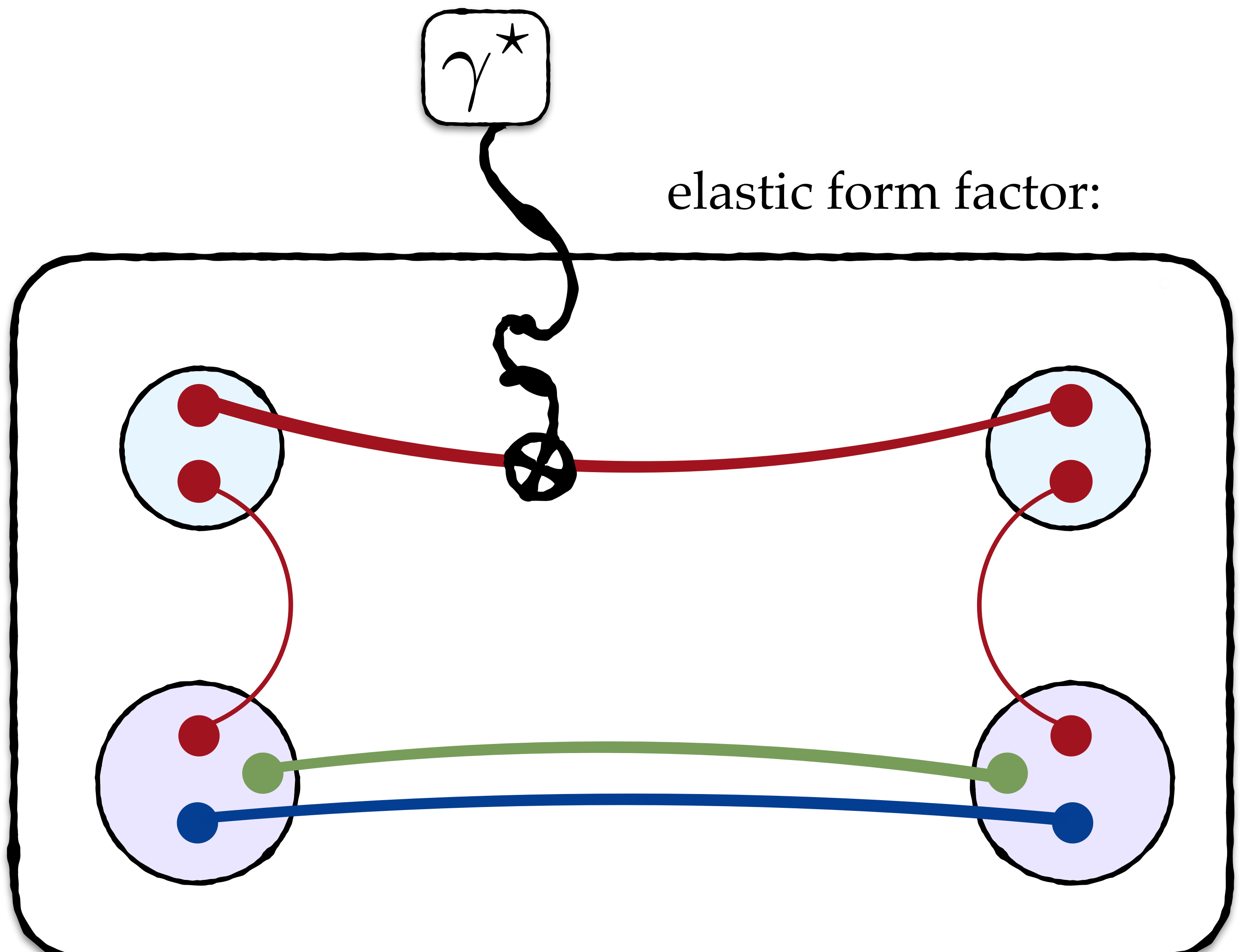
Summary / outlook

- *Coupled channels*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*

formalism understood:

RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2015)

elastic form factor:



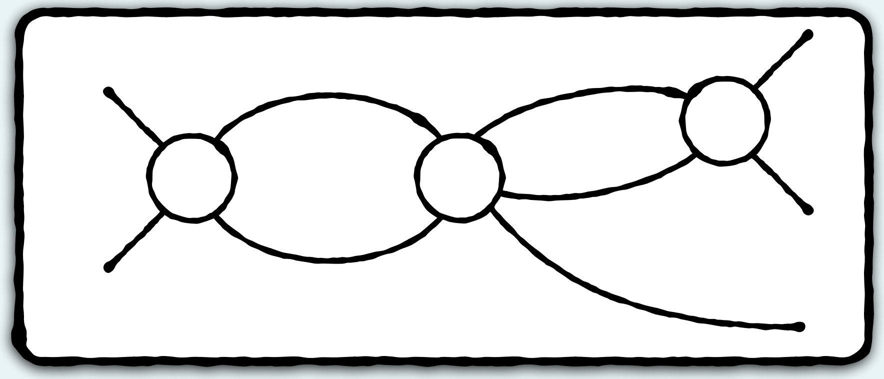
Summary / outlook

- *Coupled channels*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*

formalism under construction:

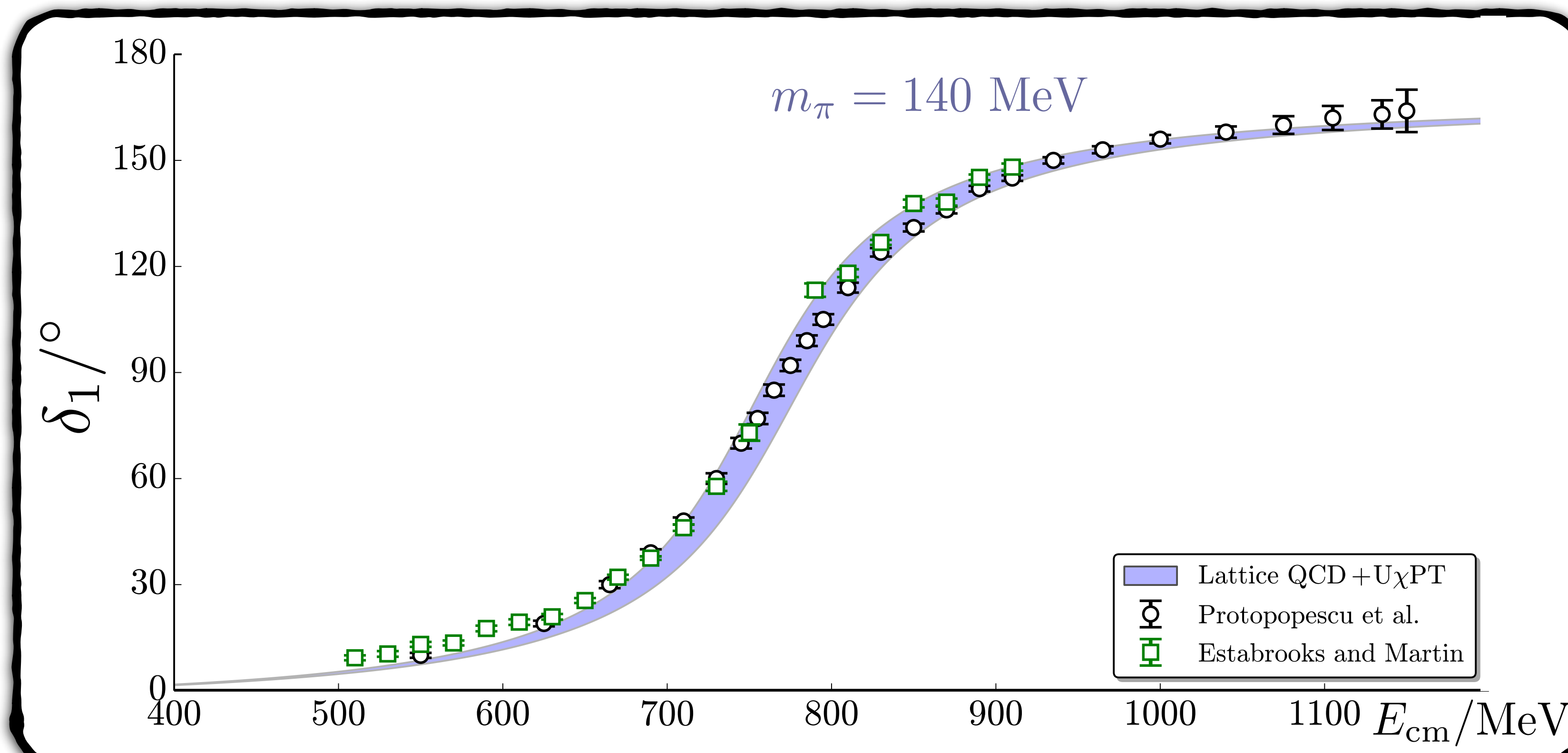
$$\det\left[1+\begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix}\begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right]=0$$

Hansen & Sharpe (2014)
RB, Hansen & Sharpe (2016)



Summary / outlook

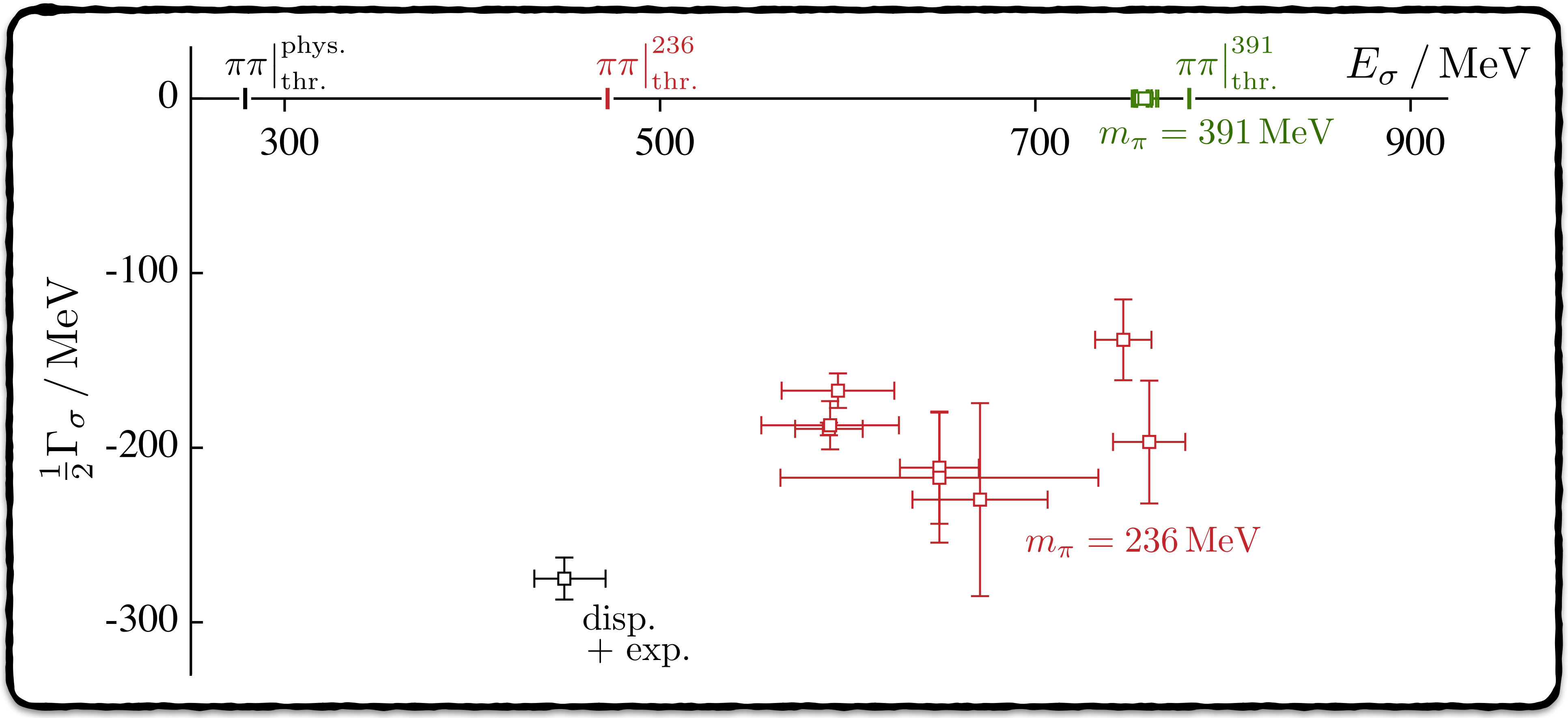
- *Coupled channels*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*



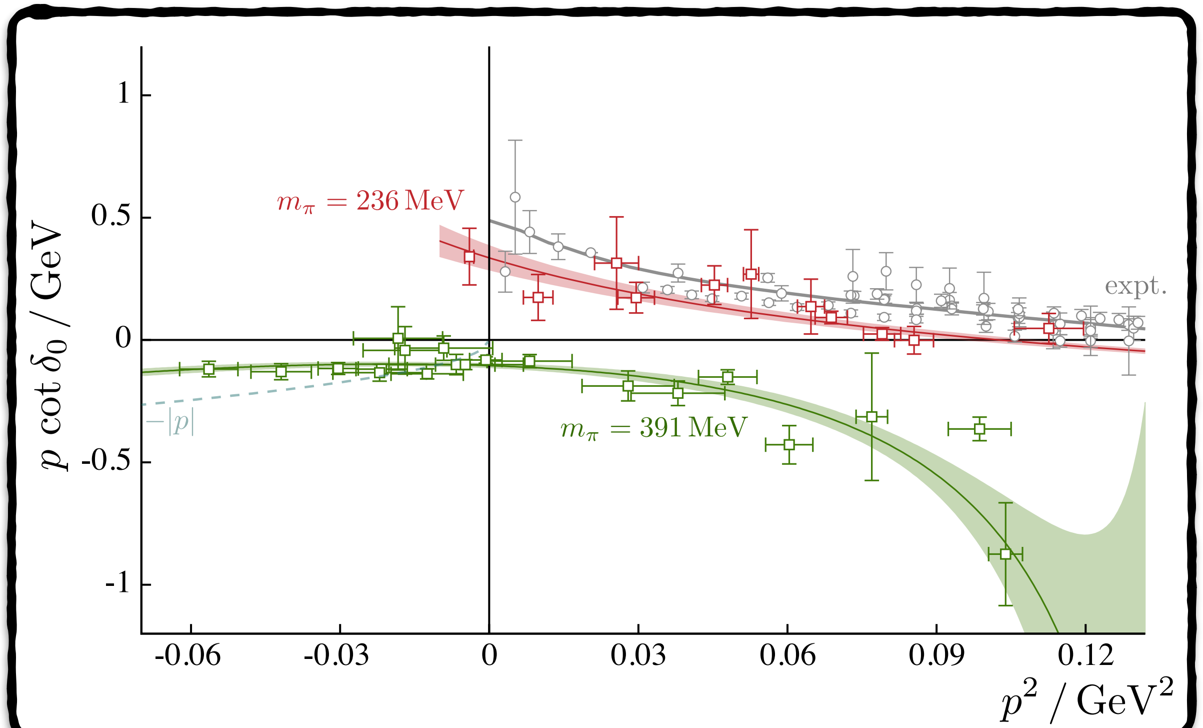
Bolton, RB, & Wilson (2016)

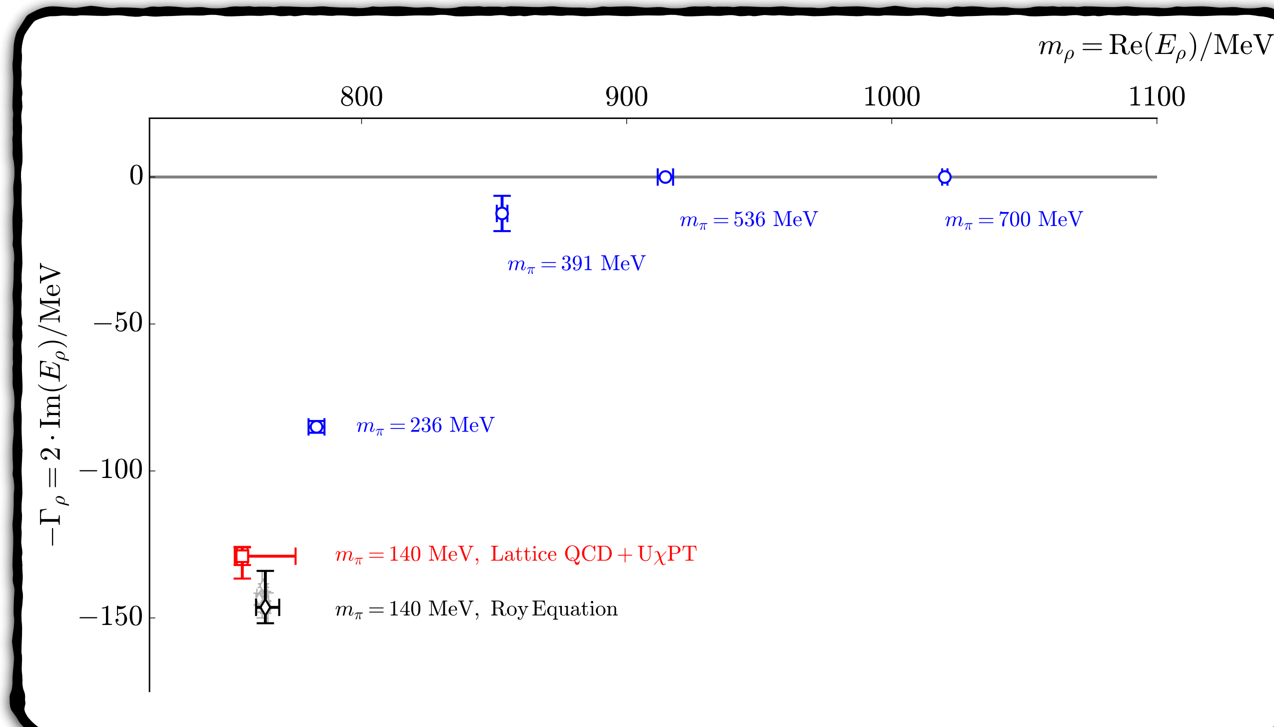
Summary / outlook

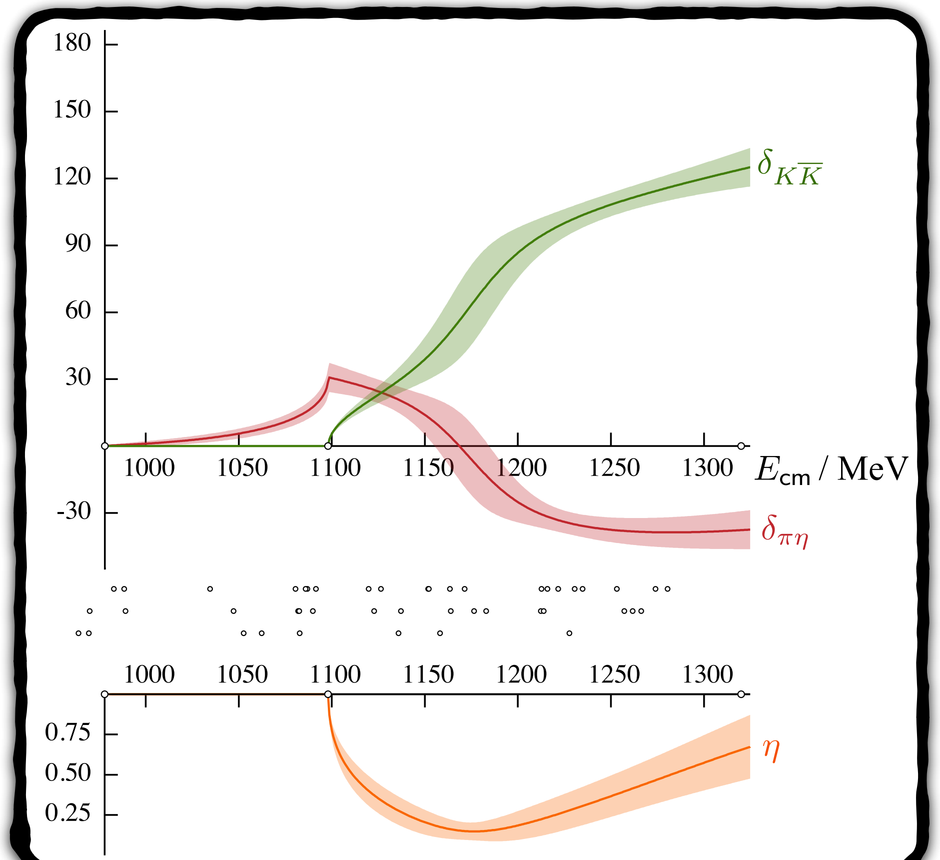
- *Coupled channels*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*
- *pole tracking*

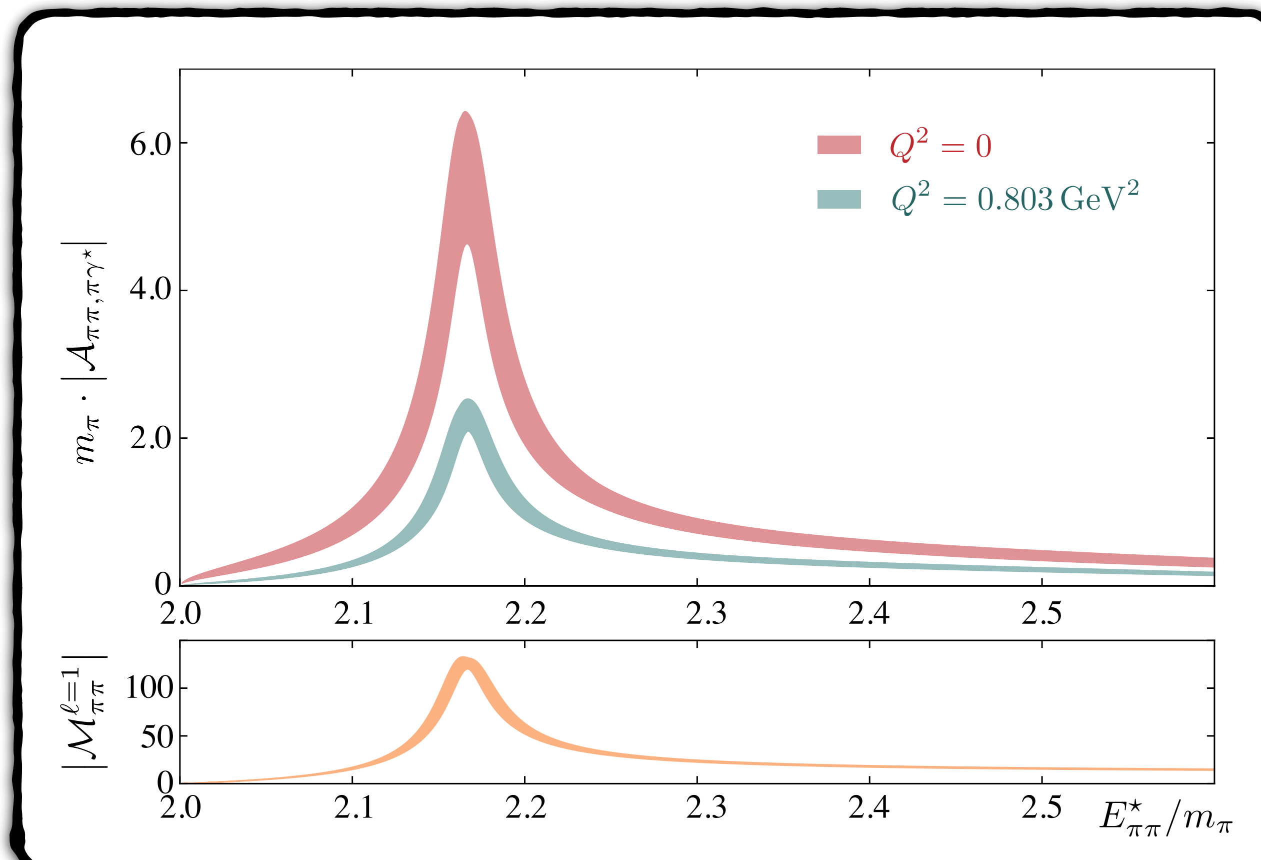


The big picture!









Collaborators & references

formalism

Hansen

Walker-Loud

Sharpe

numerical

Wilson

Shultz

Thomas

Dudek

Edwards

HadSpec Collaboration

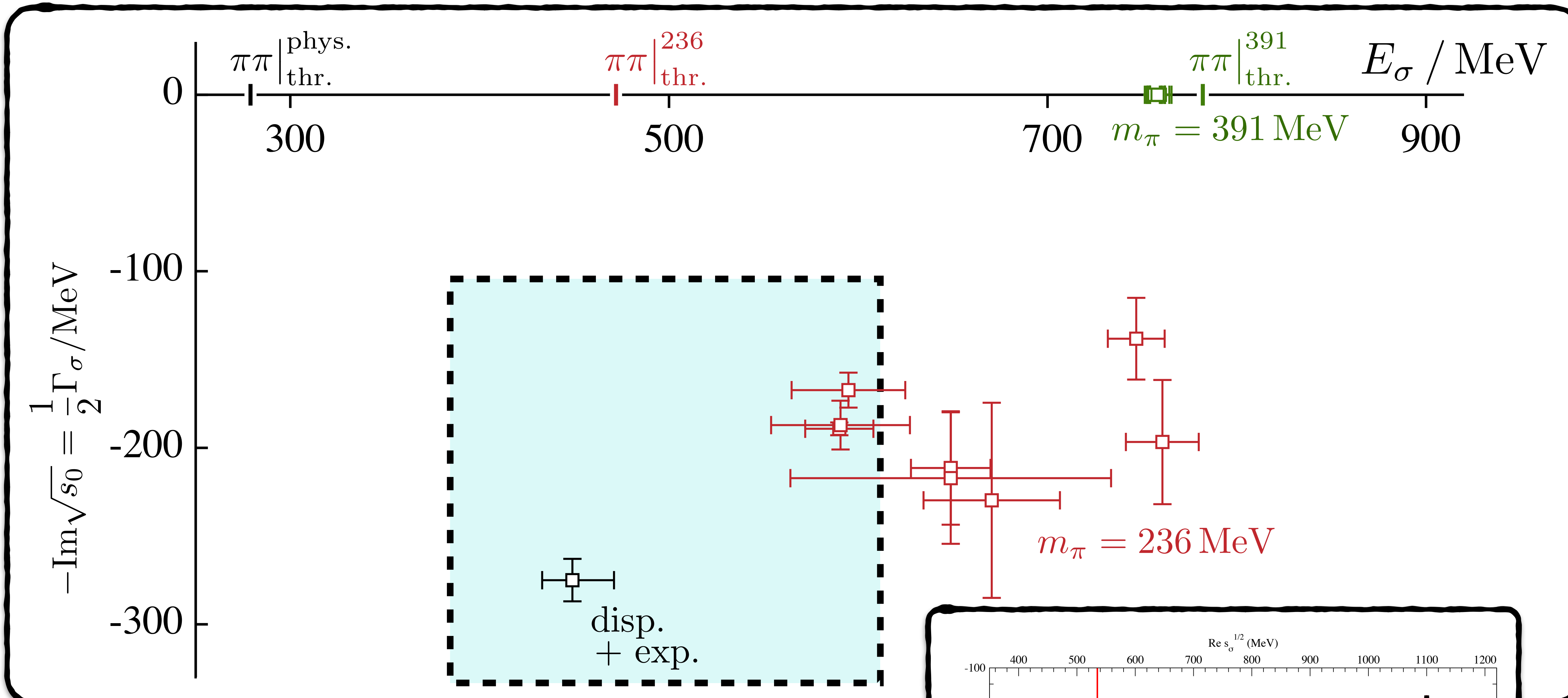
Bolton

RB, Hansen, Sharpe - arXiv:1701.07465 [hep-lat] (2017)
RB, Hansen - Phys.Rev. D94 (2016) no.1, 013008 .
RB, Hansen - Phys.Rev. D92 (2015) no.7, 074509.
RB, Hansen, Walker-Loud - Phys.Rev. D91 (2015) no.3, 034501.
RB - Phys.Rev. D89 (2014) no.7, 074507.

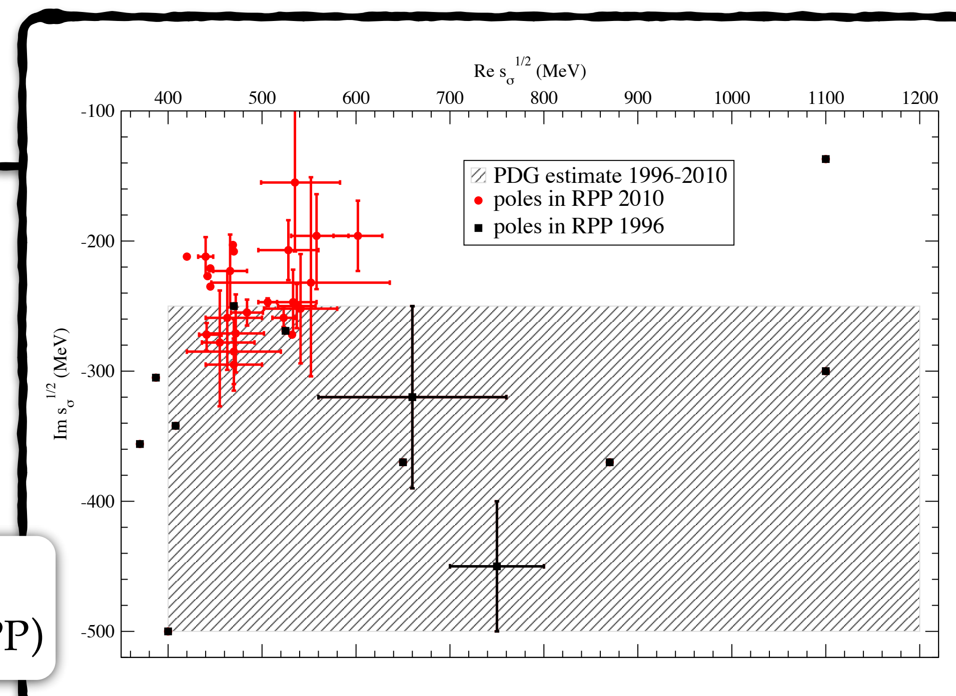
RB, Dudek, Edwards, Wilson - arXiv:1607.05900 [hep-ph].
Moir, Peardon, Ryan, Thomas, Wilson - arXiv:1607.07093 [hep-lat].
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev. D93 (2016) 114508.
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev.Lett. 115 (2015) 242001
Wilson, RB, Dudek, Edwards, Thomas - Phys.Rev. D92 (2015) no.9, 094502

Back-up slides

The σ / $f_0(500)$ vs m_π

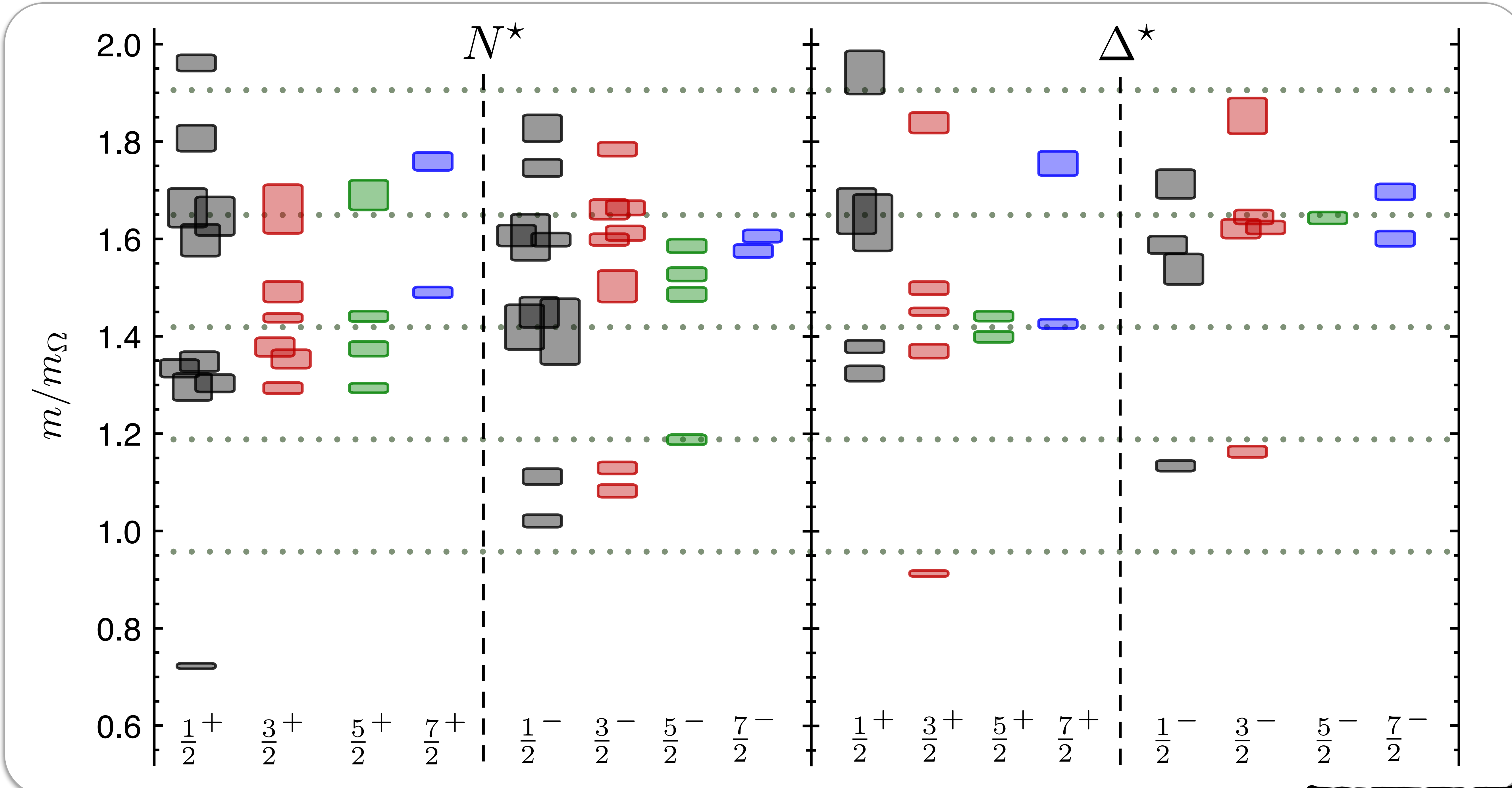


RB, Dudek, Edwards & Wilson (2016)



J. R. Peláez (2015)
Review of Particle Physics (RPP)

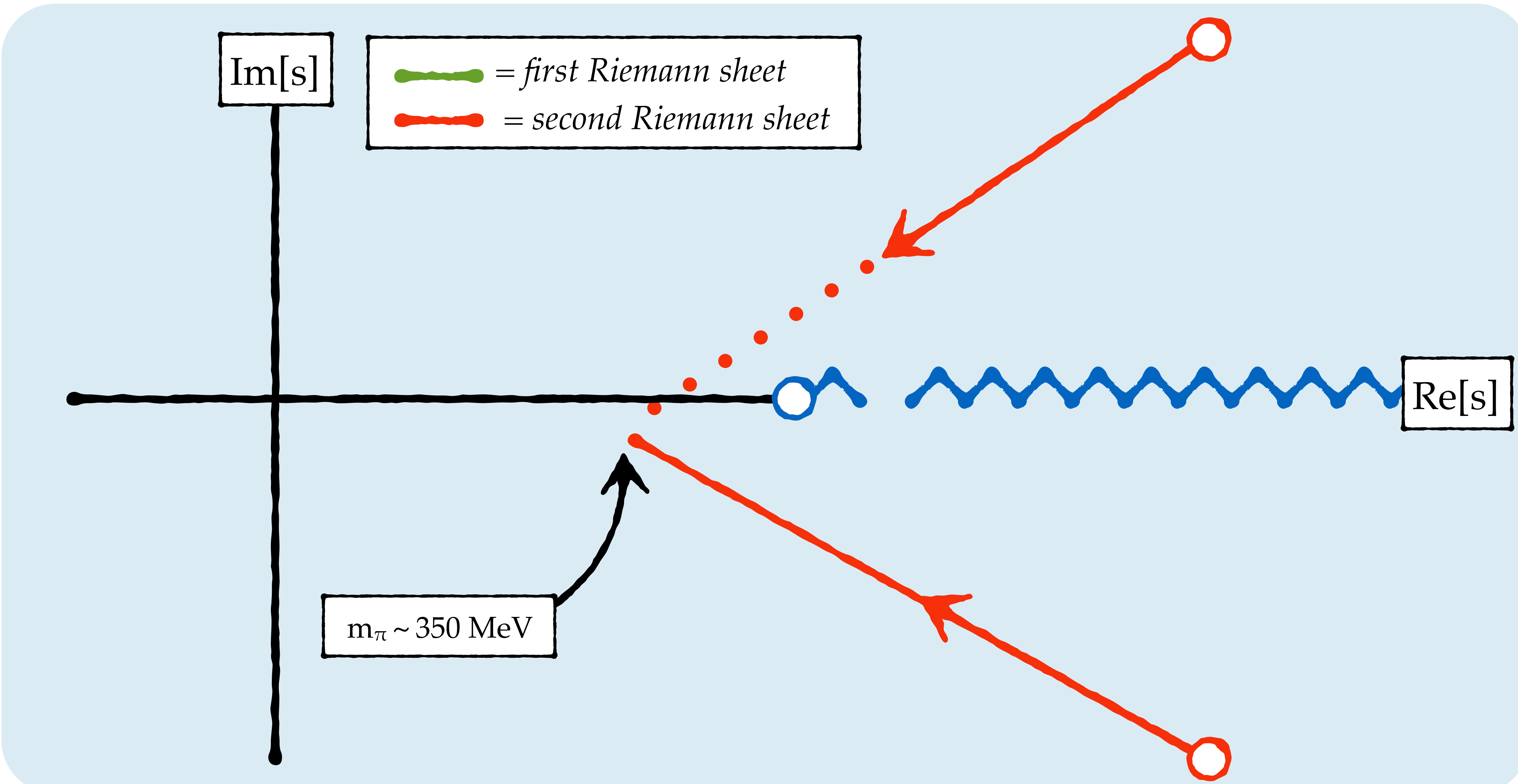
Going higher in energy



m_π=391 MeV

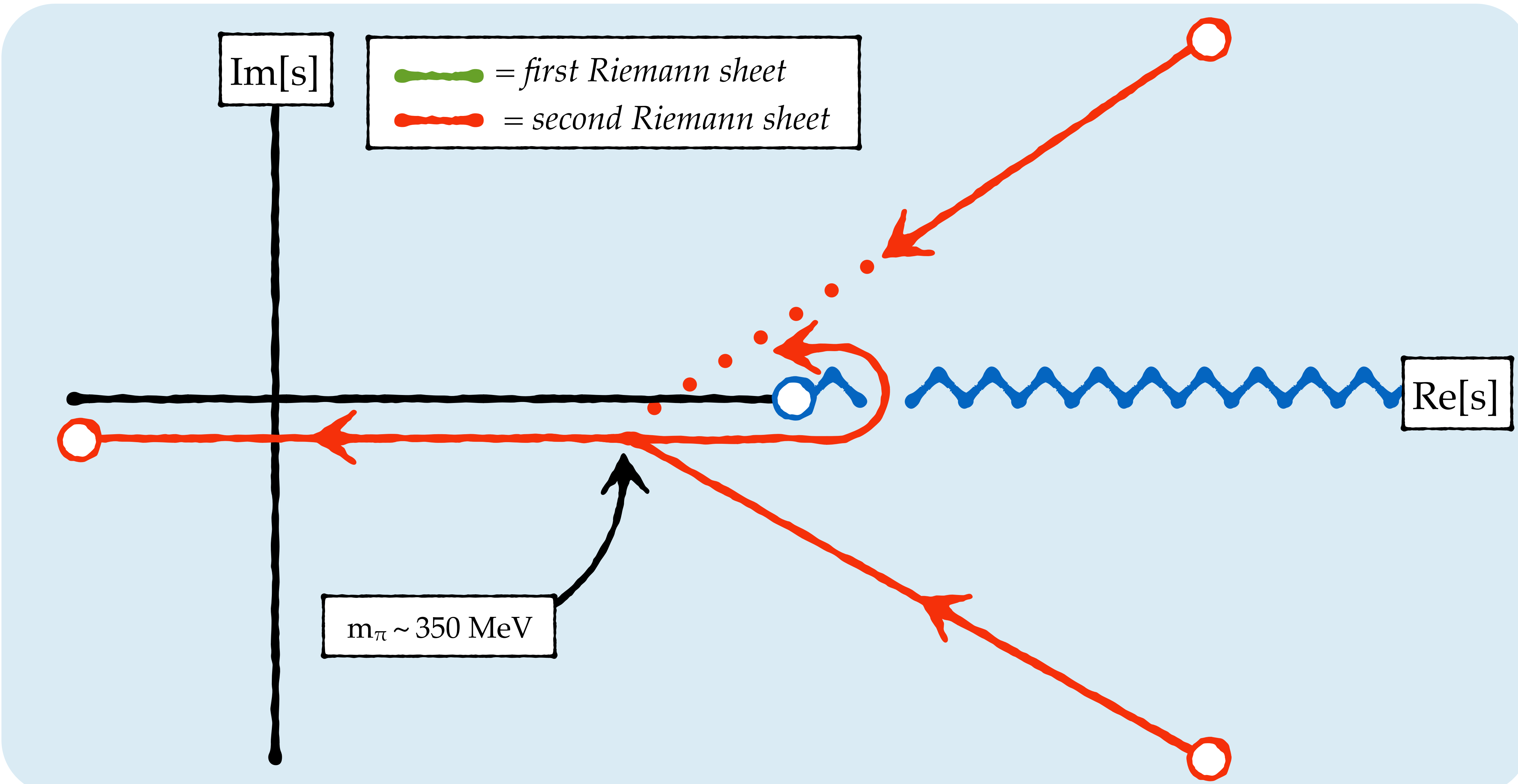
Edwards, Dudek, Richards, Wallace [Hadspec Collab.] (2011)

Qualitative understanding



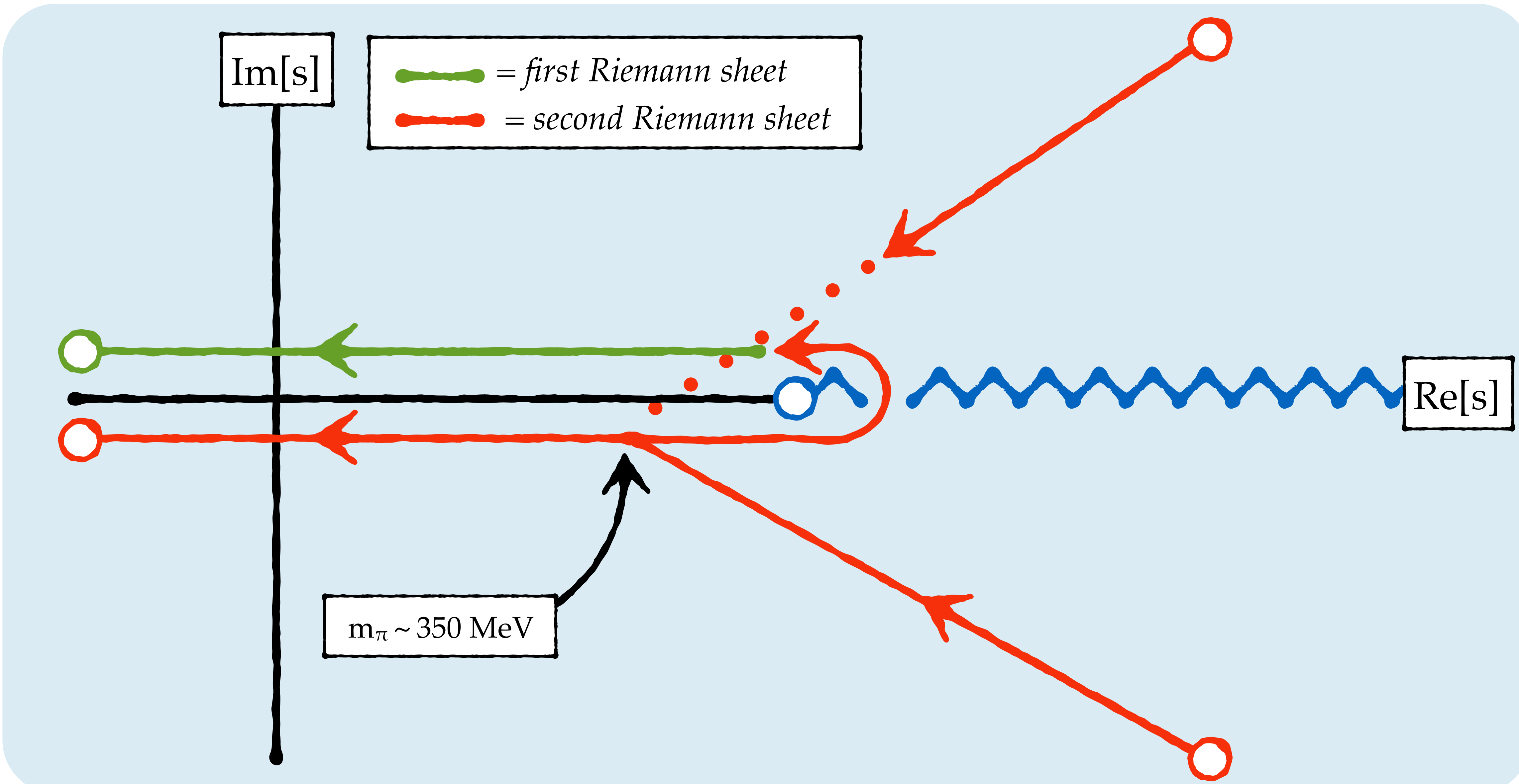
Hanhart, Peláez, and Ríos (2008)

Qualitative understanding



Hanhart, Peláez, and Ríos (2008)

Qualitative understanding

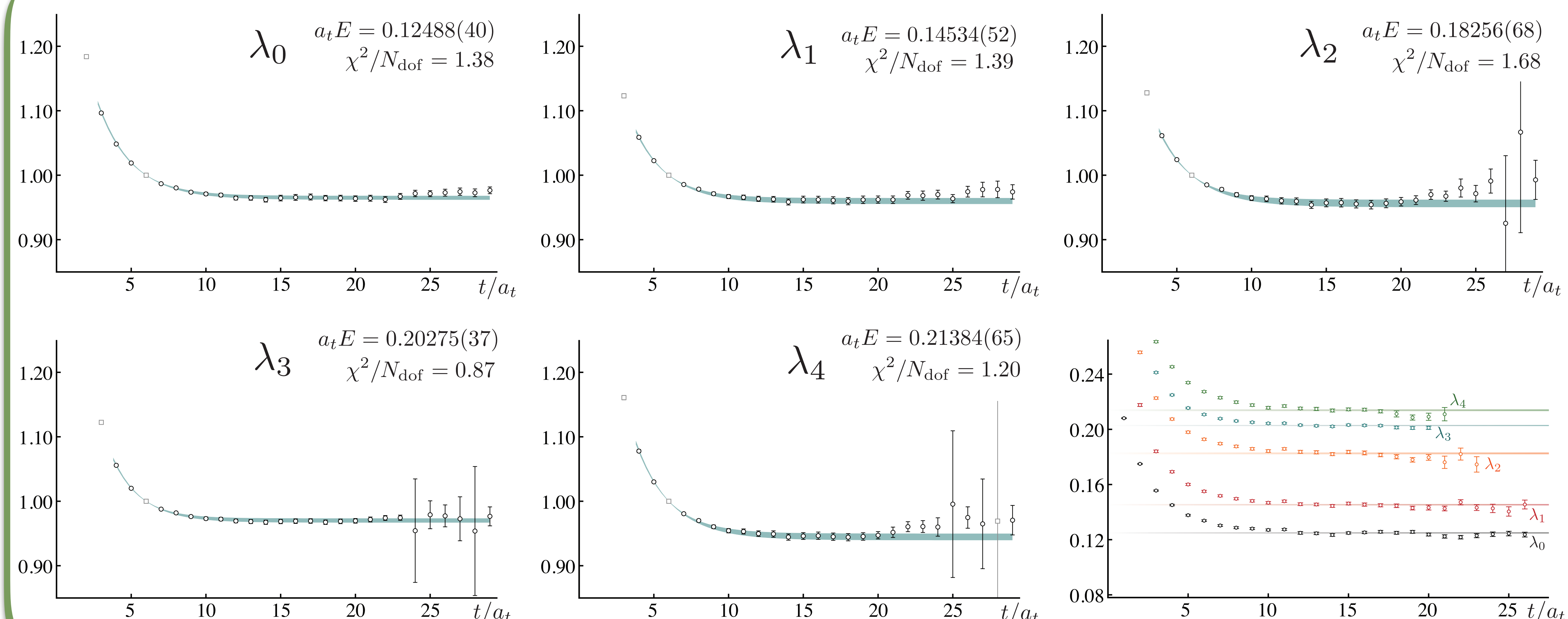


Hanhart, Peláez, and Ríos (2008)

Determining spectrum

$$C(t)v_n(t) = \lambda_n(t)C(t_0)v_n(t),$$
$$\lambda_n(t) \sim e^{-E_n(t-t_0)}$$

$[000]\ T_1^-$



Parametrization

$$t(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\,\Gamma(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)},$$

$$\Gamma(s) = \frac{g_R^2}{6\pi} \frac{k^3}{s}$$

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s)\,,$$

$$\operatorname{Im} I_{ij}(s) = -\delta_{ij}\,\rho_i(s)$$

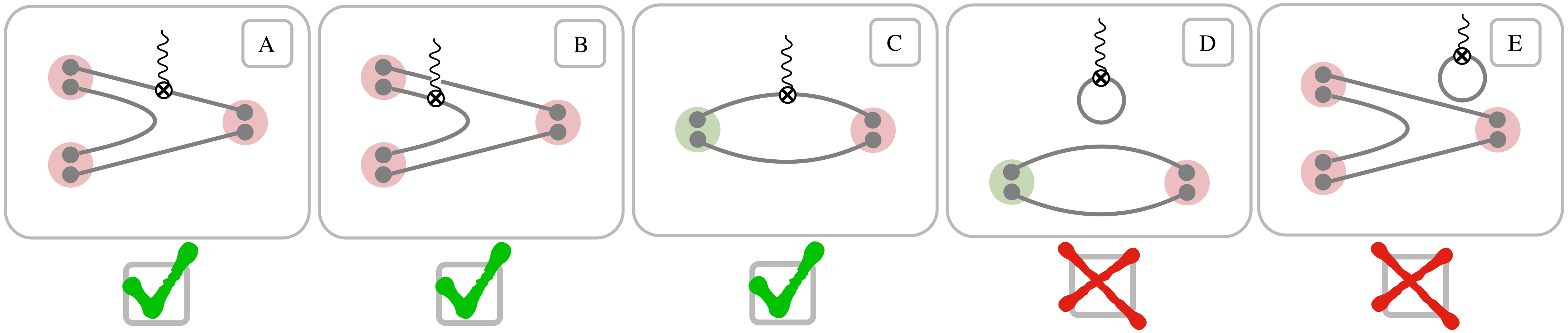
$$K_{ij}(s) = \frac{g_i\,g_j}{m^2 - s} + \sum_{n=0}^{N} \gamma_{ij}^{(n)} \left(\frac{s}{s_0}\right)^n,$$

$$K_{ij}^{-1} = \sum_{m=0}^{M} c_{ij}^{(m)} s^m,$$

$\pi\gamma^*$-to-$\pi\pi$ amplitude

Correlation functions

Contractions:



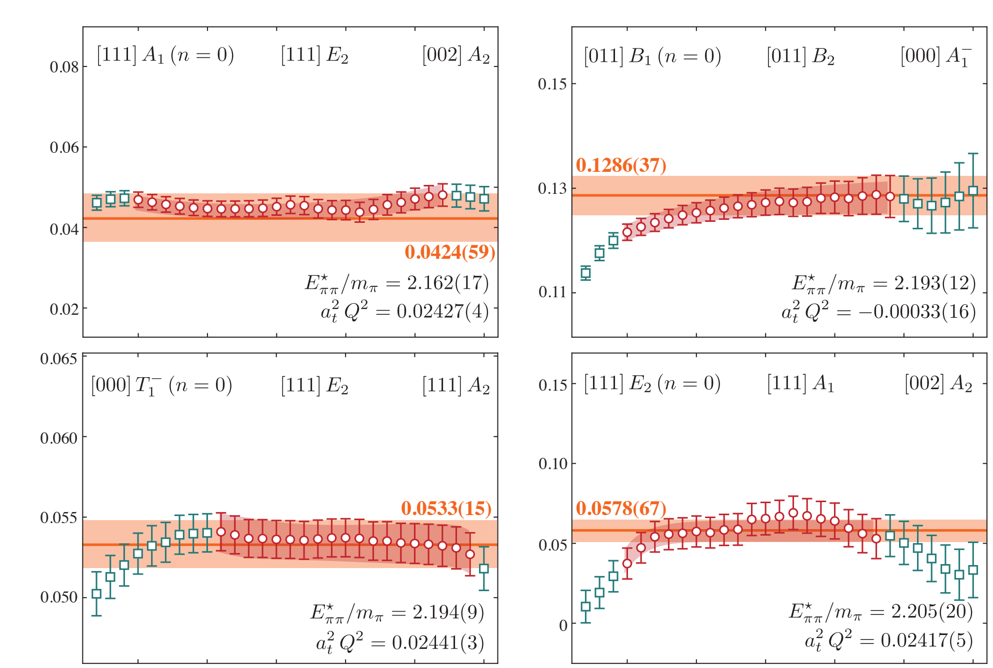
Operators and matrix elements:

$$C^{(3)}_{\pi\pi_n,\mu,\pi}(\mathbf{P}_\pi,\mathbf{P}_{\pi\pi};\Delta t,t) = \langle 0|\Omega_\pi(\Delta t,\mathbf{P}_\pi)\,\widetilde{\mathcal{J}}_\mu(t,\mathbf{P}_\pi-\mathbf{P}_{\pi\pi})\,\Omega^\dagger_{\pi\pi}(0,\mathbf{P}_{\pi\pi})|0\rangle$$
$$= e^{-(E_{\pi\pi}-E_\pi)t}\,e^{-E_\pi\Delta t}\,\langle\pi;L|\widetilde{\mathcal{J}}_\mu|\pi\pi;L\rangle + \dots$$

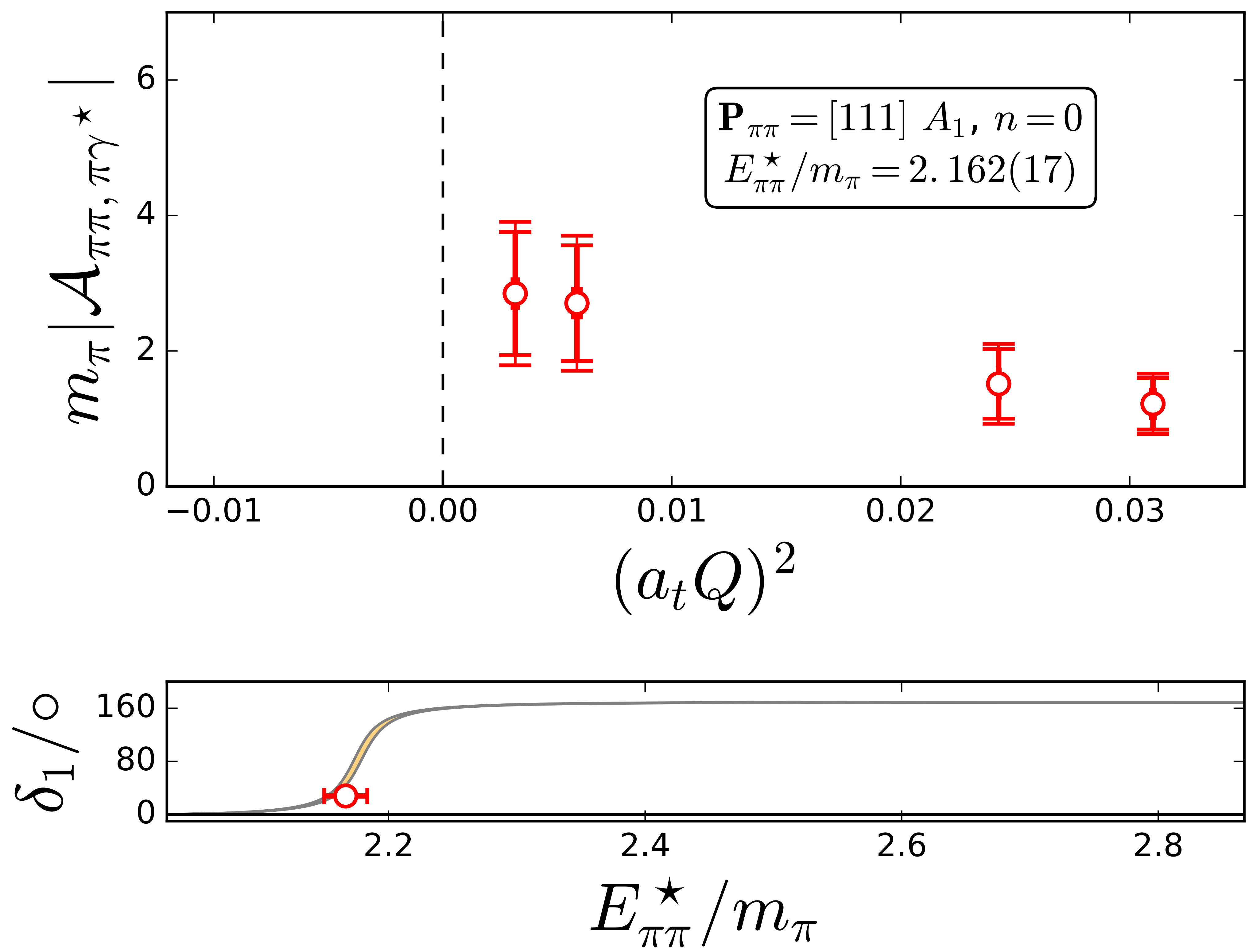
Ω_π =optimized 'π' operator,
linear combo. of $\sim$ 10 ops.

$\Omega_{\pi\pi}$ =optimized '$\pi\pi$' operator,
linear combo. of $\sim$ 20-30 ops.

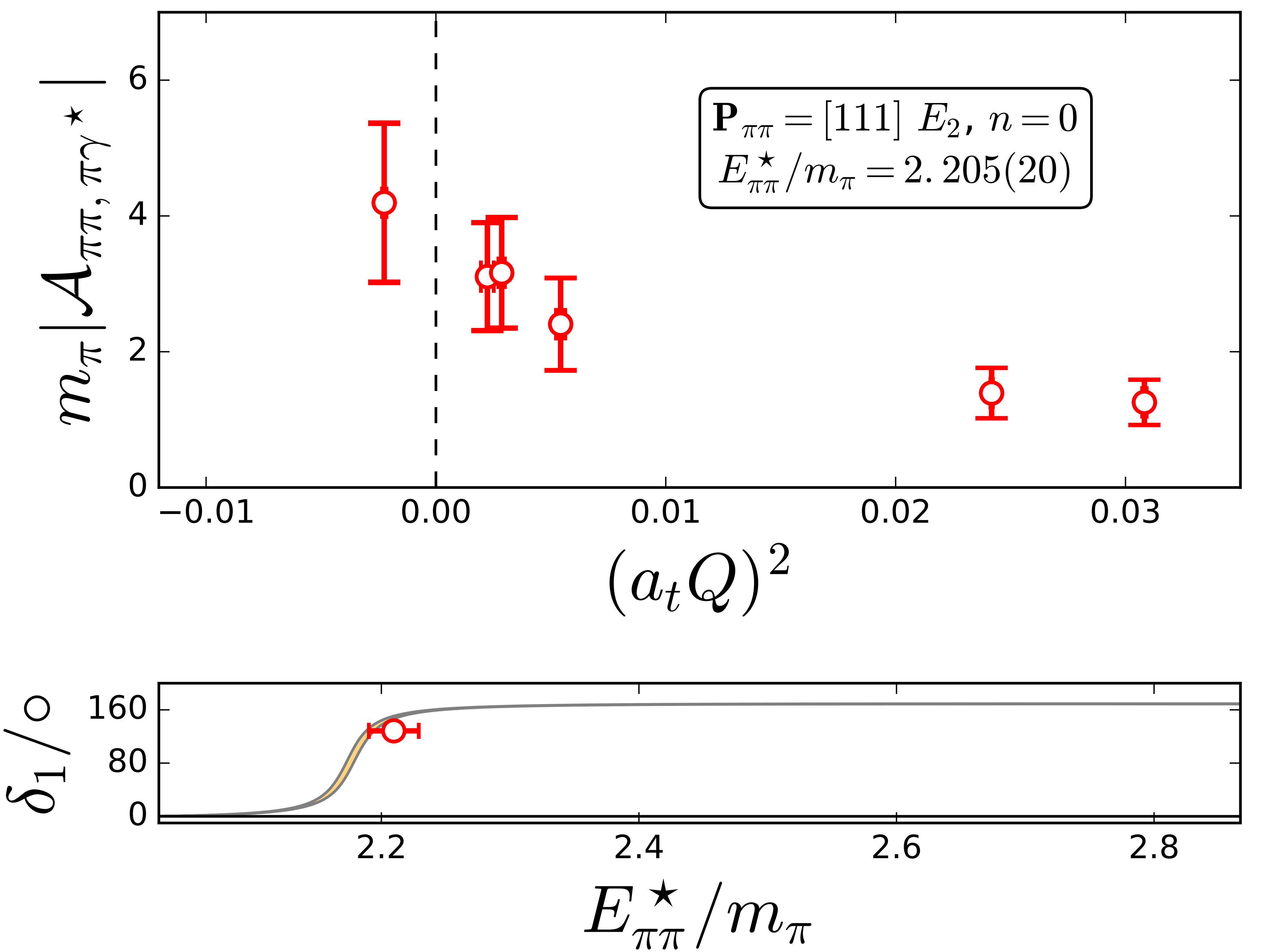
$\widetilde{\mathcal{J}}_\mu$ =electromagnetic current



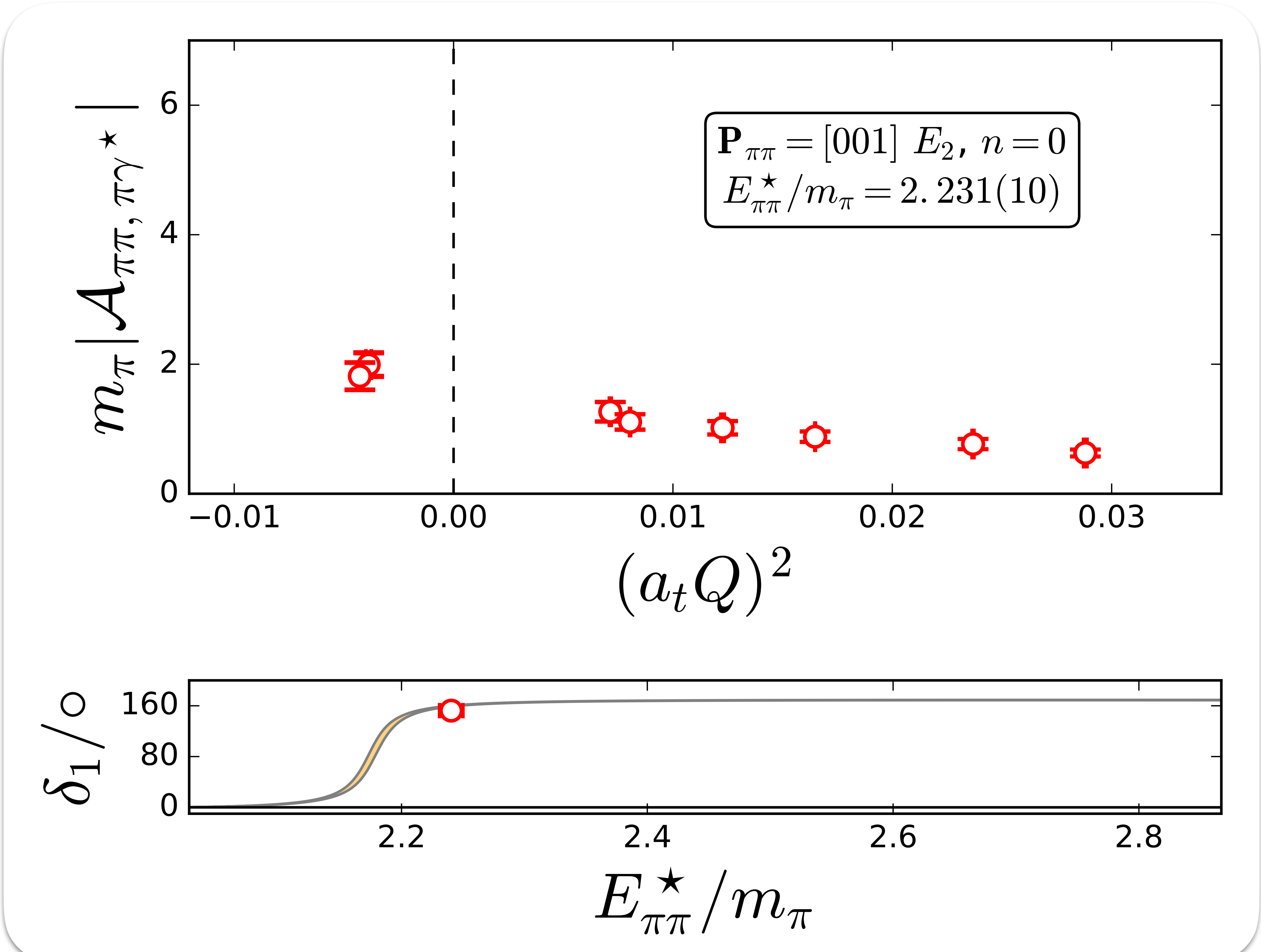
$\pi\gamma^*$-to-$\pi\pi$ amplitude



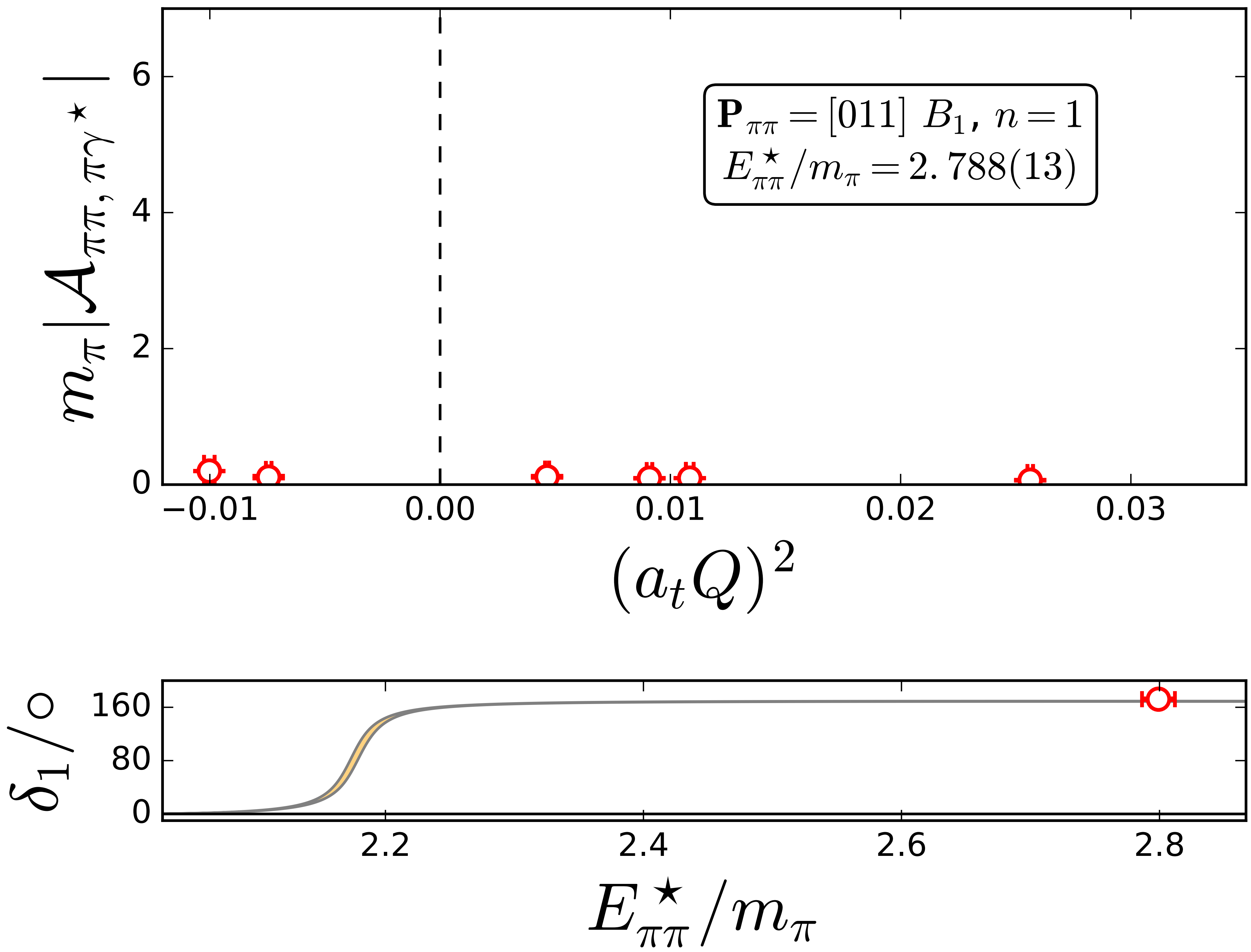
$\pi\gamma^{*}$-to-$\pi\pi$ amplitude



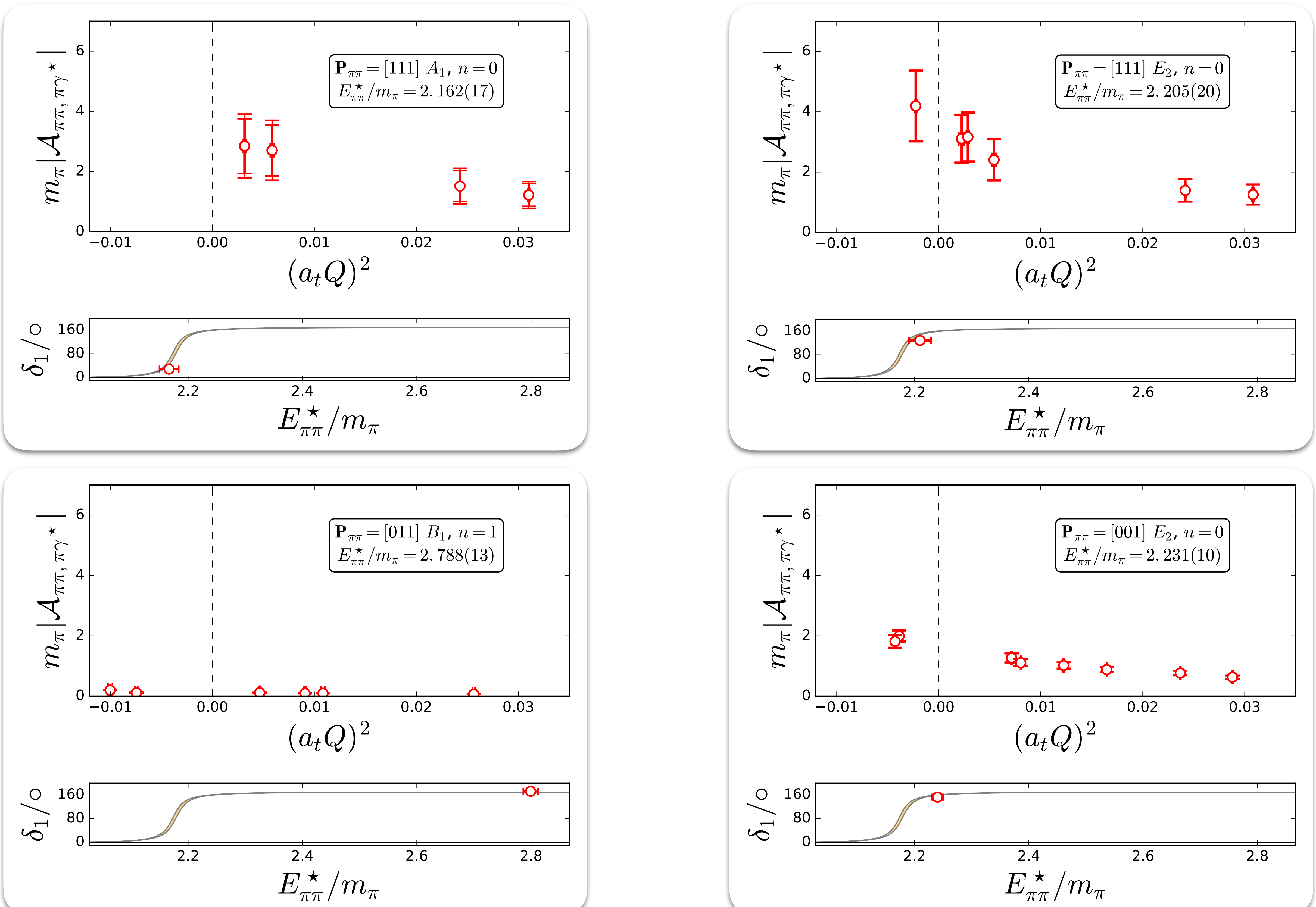
$\pi\gamma^*$-to-$\pi\pi$ amplitude



$\pi\gamma^*$-to-$\pi\pi$ amplitude

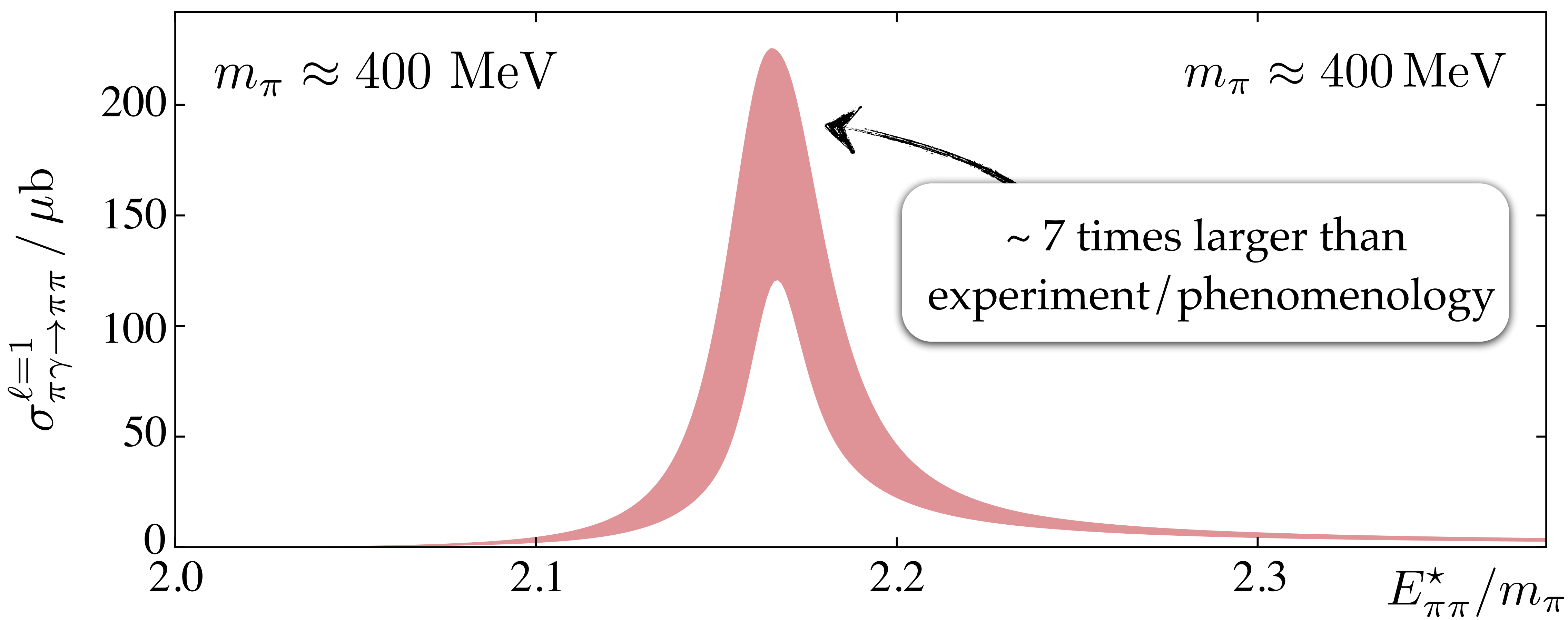


$\pi\gamma^*$-to-$\pi\pi$ amplitude

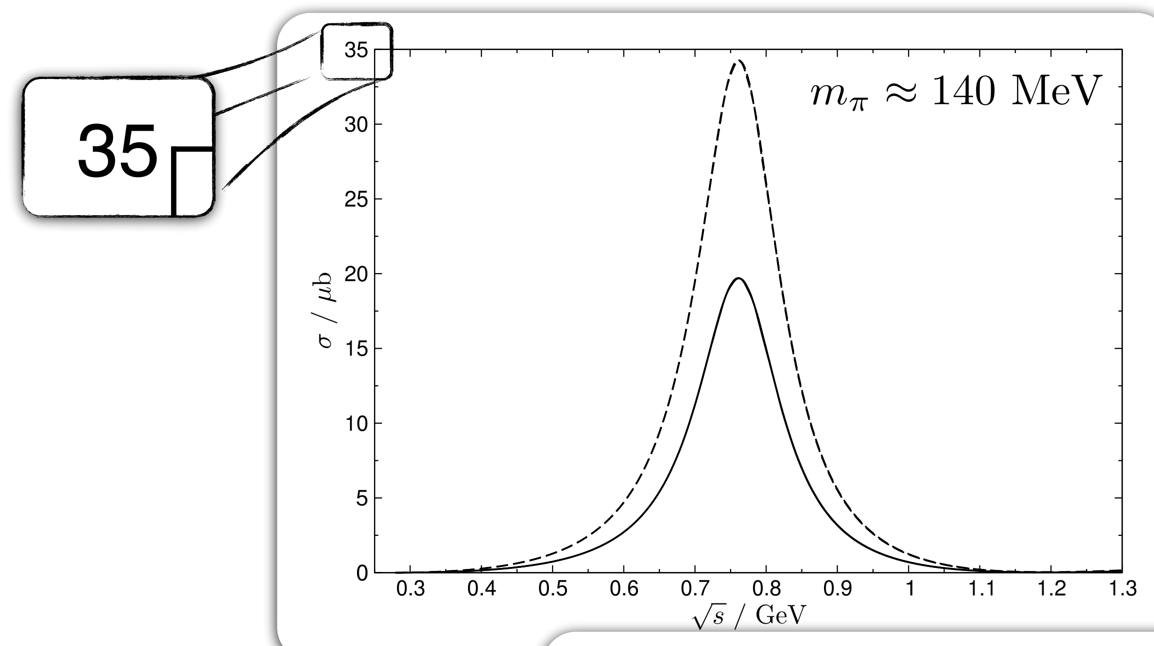


Comparison with phenomenology

$\pi\gamma$-to-$\pi\pi$ cross section



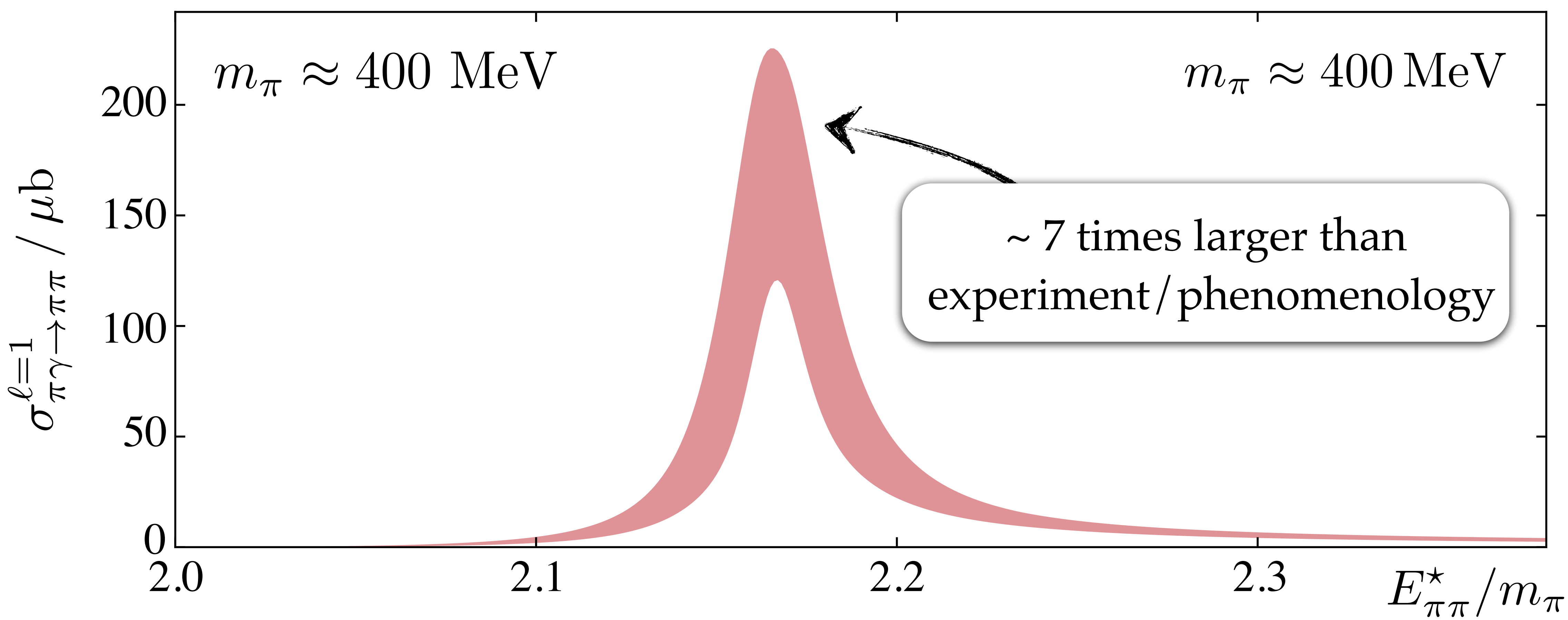
~ 7 times larger than experiment / phenomenology



non trivial quark-mass dependence!

Hoferichter, Kubis, & Sakkas (2012)

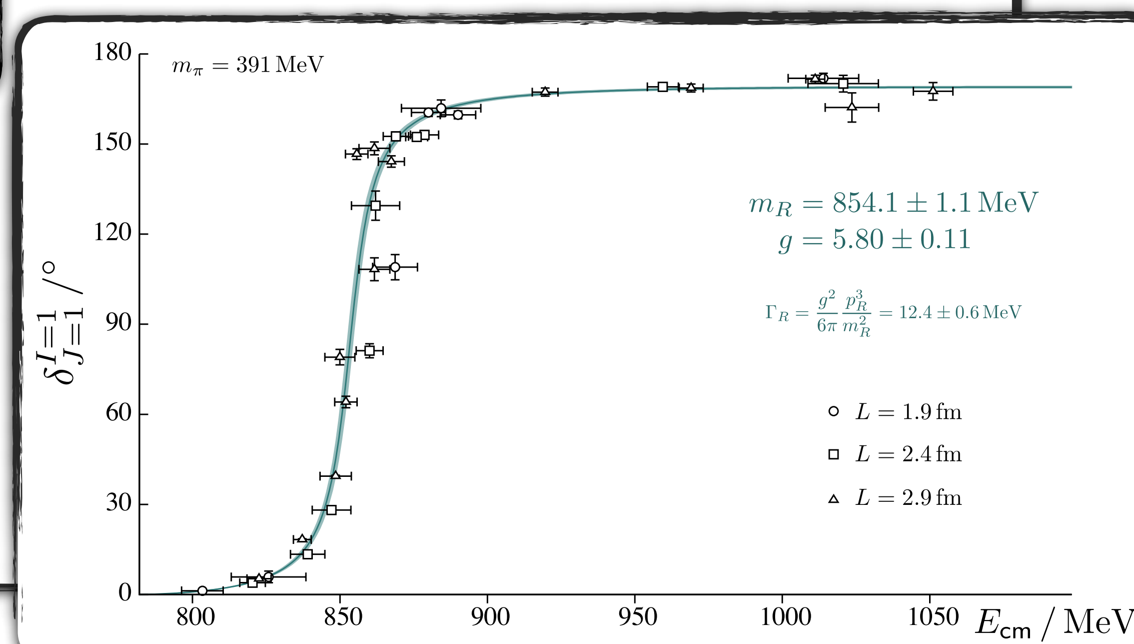
$\pi\gamma$-to-$\pi\pi$ cross section



$$\lim_{E^\star_{\pi\pi}\to m_\rho} \sigma(\pi^+\gamma \to \pi^+\pi^0) \propto \frac{q^\star_{\pi\gamma}\, F^2_{\pi\rho}(m_\rho, 0)}{m_\pi^2} \times \frac{1}{\Gamma_1(m_\rho)}$$

0.60 x (physical)

12 x (physical)



On determining correlation function using small basis of operators

Extracting the spectrum

Two-point correlation functions:

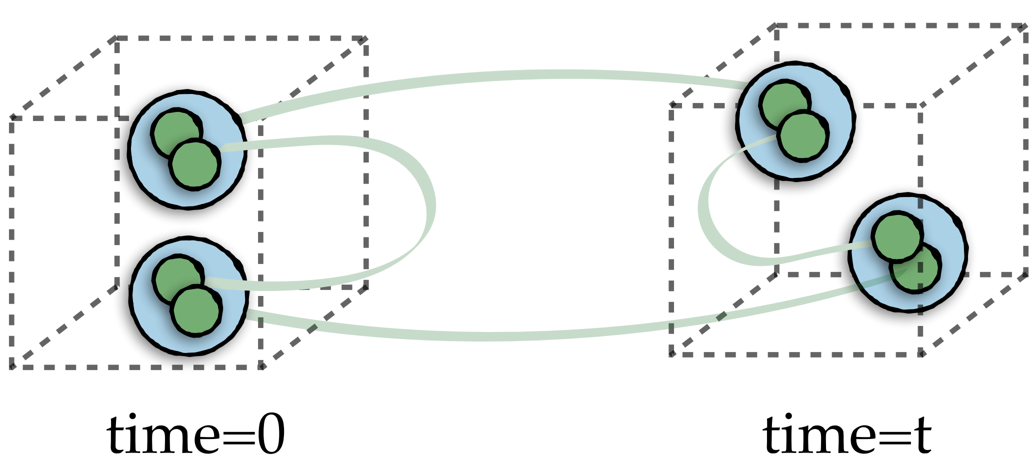
$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle$$

$$= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle$$

$$= \sum_n \langle 0|e^{t\hat{H}_{QCD}}\mathcal{O}_b(0,\mathbf{P})e^{-t\hat{H}_{QCD}}|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle$$

$$= \sum_n Z_{b,n}Z_{a,n}^\dagger e^{-E_n t}$$

spectrum

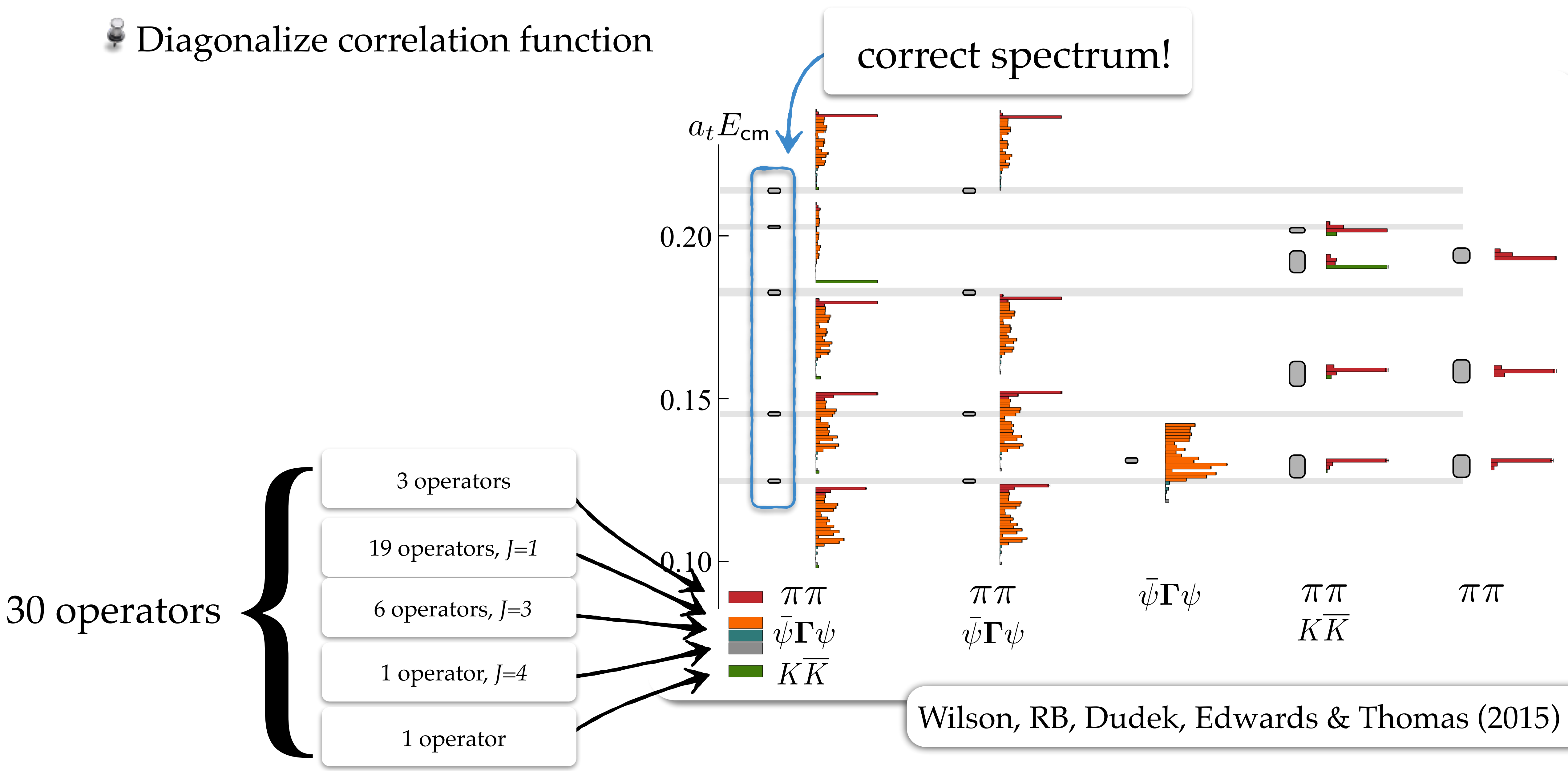


Extracting the spectrum

Two-point correlation functions:

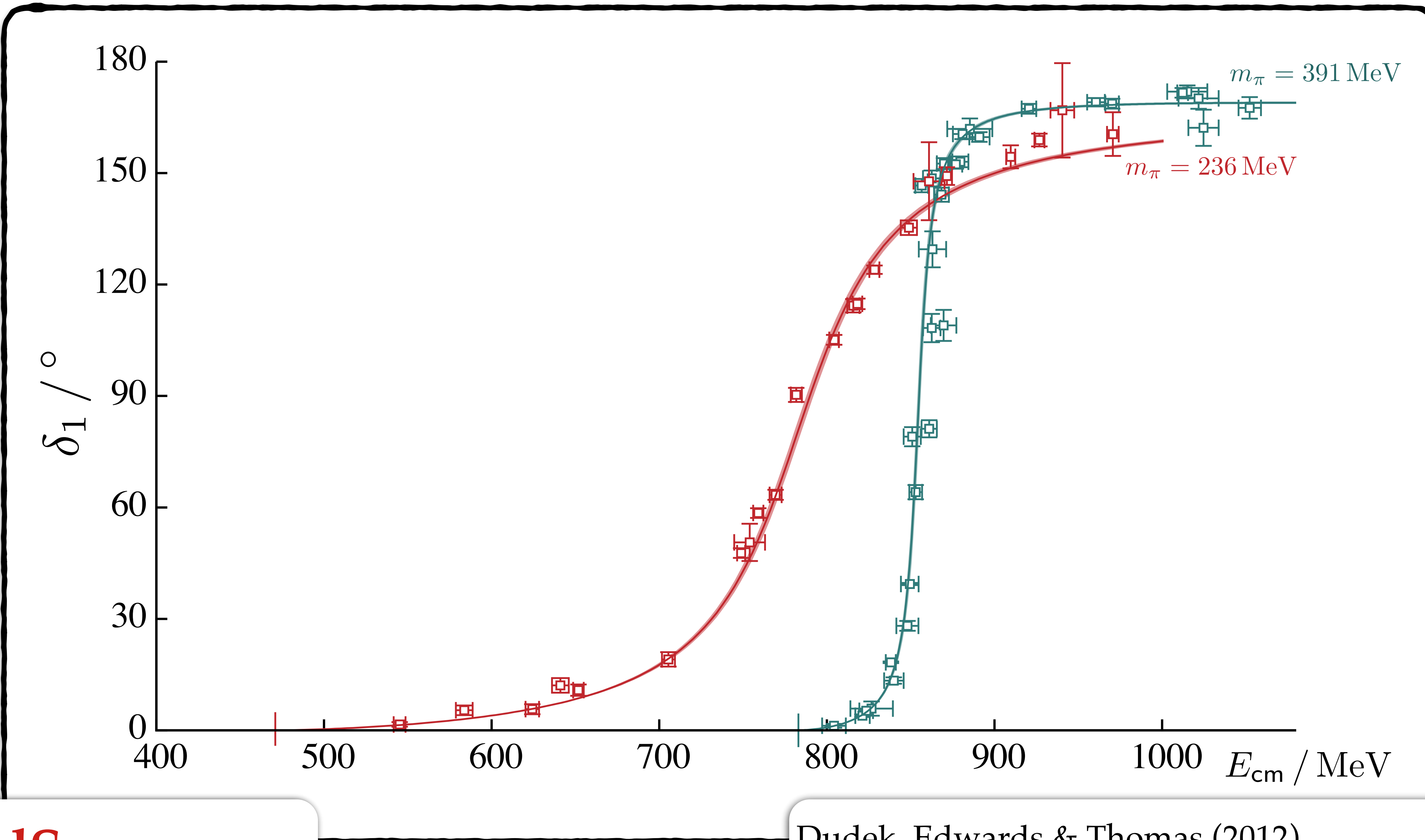
$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t, \mathbf{P})\mathcal{O}_a^\dagger(0, -\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Use a large basis of operators with the same quantum numbers
- Diagonalize correlation function



Wilson, RB, Dudek, Edwards & Thomas (2015)

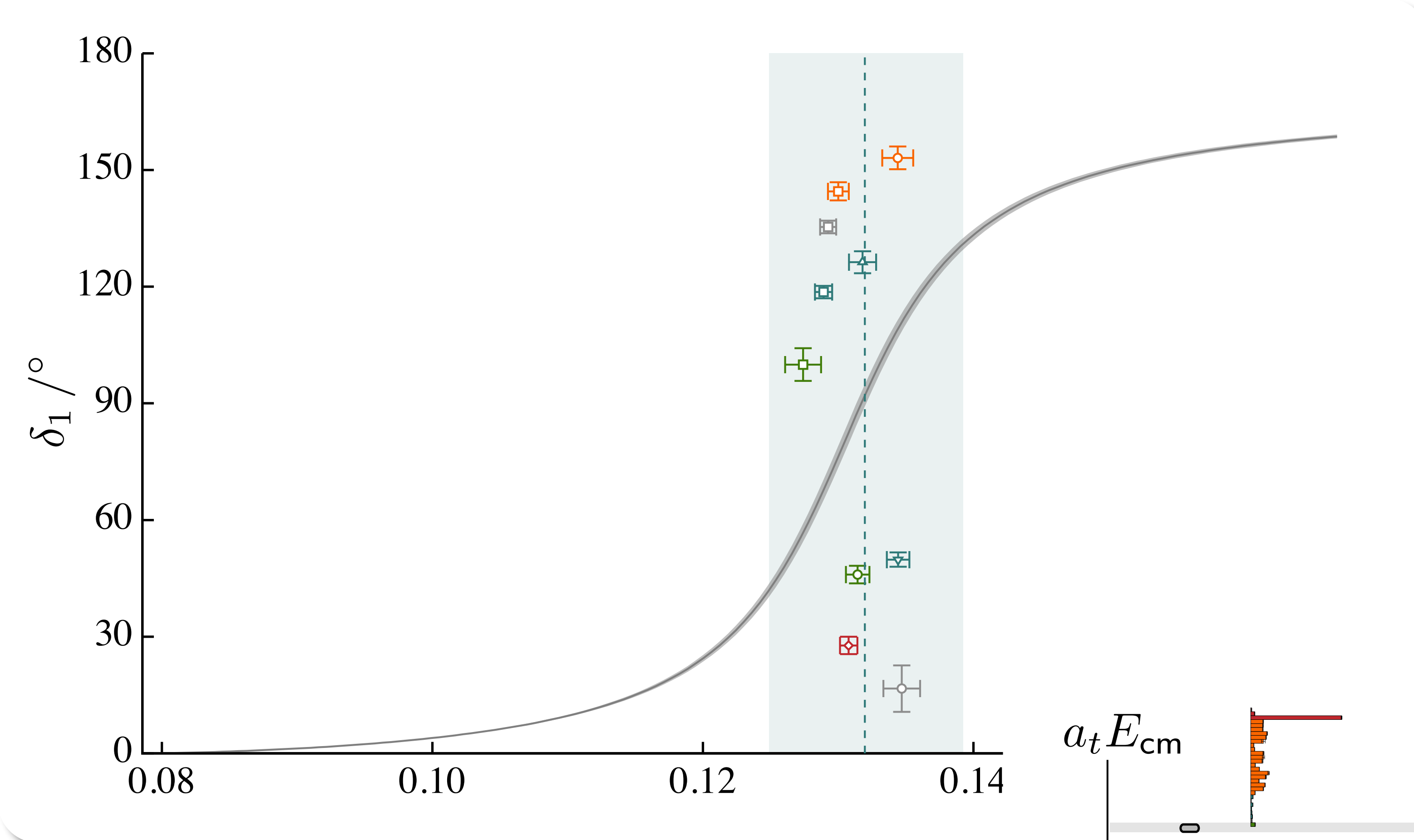
Isovector $\pi\pi$ scattering

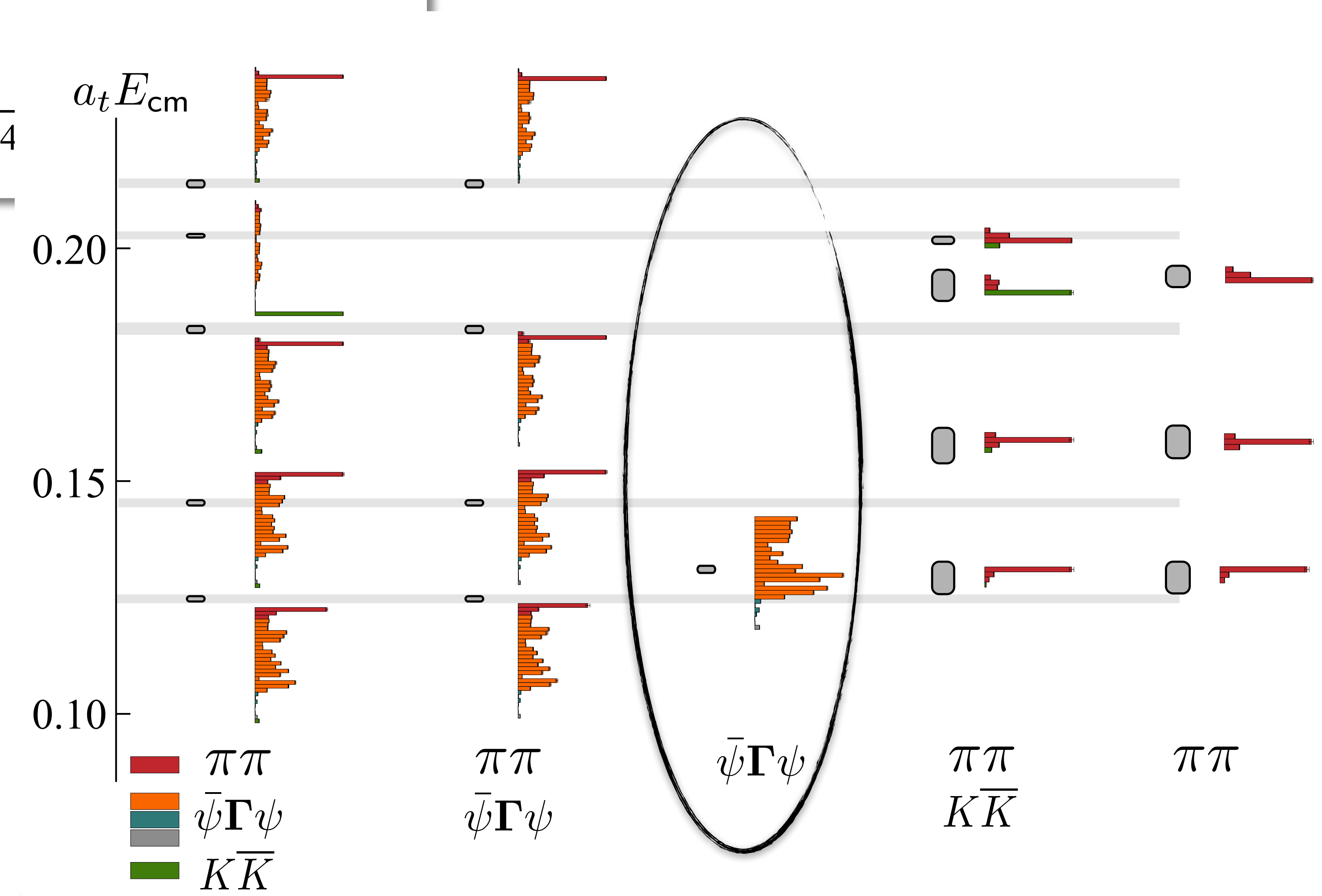


HadSpec Collaboration

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

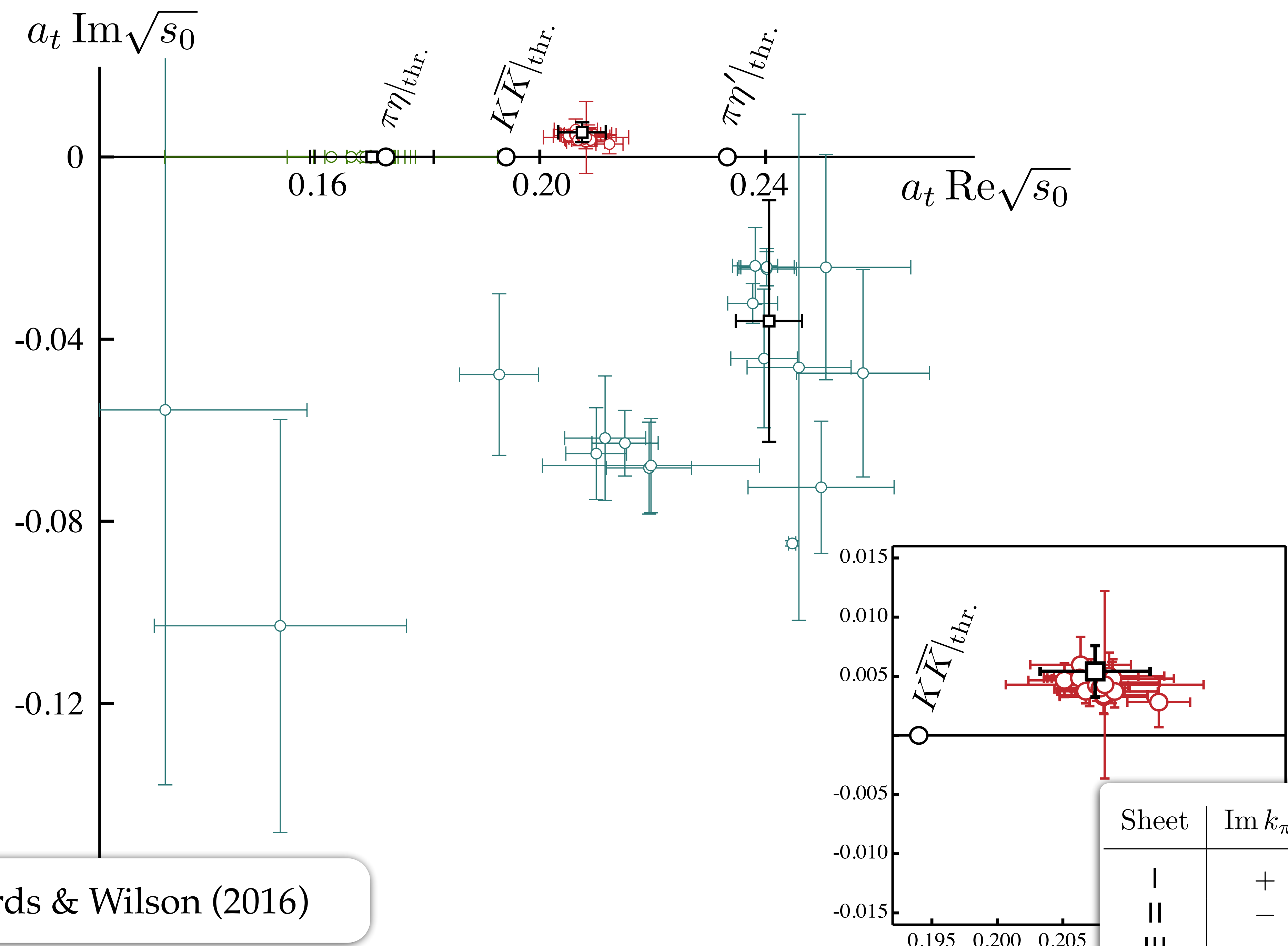
The incorrect answer





$a_0(980)$ poles

πη-KK-πη′ in I=1, m_π=391MeV



Sheet	Im $k_{\pi\eta}$	Im $k_{K\overline{K}}$	
I	+	+	
II	−	+	
III	−	−	[blue]
IV	+	−	[red]

Dudek, Edwards & Wilson (2016)

RB

Unitarized χPT

$$\mathcal{M}_{\mathrm{U}\chi\mathrm{PT}} = \mathcal{M}_{\mathrm{LO}} \frac{1}{\mathcal{M}_{\mathrm{LO}} - \mathcal{M}_{\mathrm{NLO}}} \mathcal{M}_{\mathrm{LO}}$$

$$\begin{aligned}
S &= 1 + 2i\sigma\mathcal{M} \\
\mathcal{M} &= (\mathrm{Re}(\mathcal{M}^{-1}) - i\sigma)^{-1} \\
\mathcal{M}^{-1} &= \mathcal{M}_{\mathrm{LO}}^{-1} \frac{1}{1 + \mathcal{M}_{\mathrm{LO}}^{-1}\mathcal{M}_{\mathrm{NLO}} + \ldots} = \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\mathcal{M}_{\mathrm{NLO}} + \ldots\right) \\
\mathrm{Re}(\mathcal{M}^{-1}) &= \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\mathrm{Re}(\mathcal{M}_{\mathrm{NLO}}) + \ldots\right)
\end{aligned}$$

Dobado and Pelaez (1997)

Oller, Oset, and Pelaez (1998)

Oller, Oset, and Pelaez (1999)

LL-factor

Relationship between amplitude and “form factor”:

$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E^\star_{\pi\pi},Q^2) = \left(\frac{F(E^\star_{\pi\pi},Q^2)}{\cot\delta_1(E^\star_{\pi\pi}) - i}\right)\sqrt{\frac{16\pi}{q^\star_{\pi\pi}\,\Gamma(E^\star_{\pi\pi})}}$$

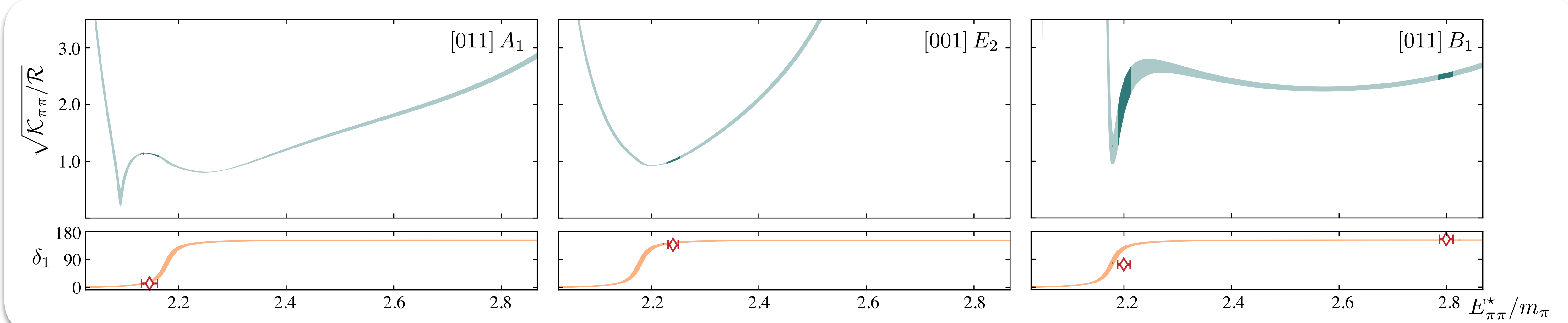
$$F(E^\star_{\pi\pi},Q^2) = \tilde{\mathcal{A}}(E^\star_{\pi\pi},Q^2;L)\sqrt{\frac{\mathcal{K}_{\pi\pi}}{\mathcal{R}}},$$

$$\frac{1}{\sqrt{2E^\star_{\pi\pi}\,\mathcal{K}_{\pi\pi}(E^\star_{\pi\pi})}} = \sin\delta_1(E^\star_{\pi\pi})\sqrt{\frac{16\pi}{q^\star_{\pi\pi}\,\Gamma(E^\star_{\pi\pi})}}$$

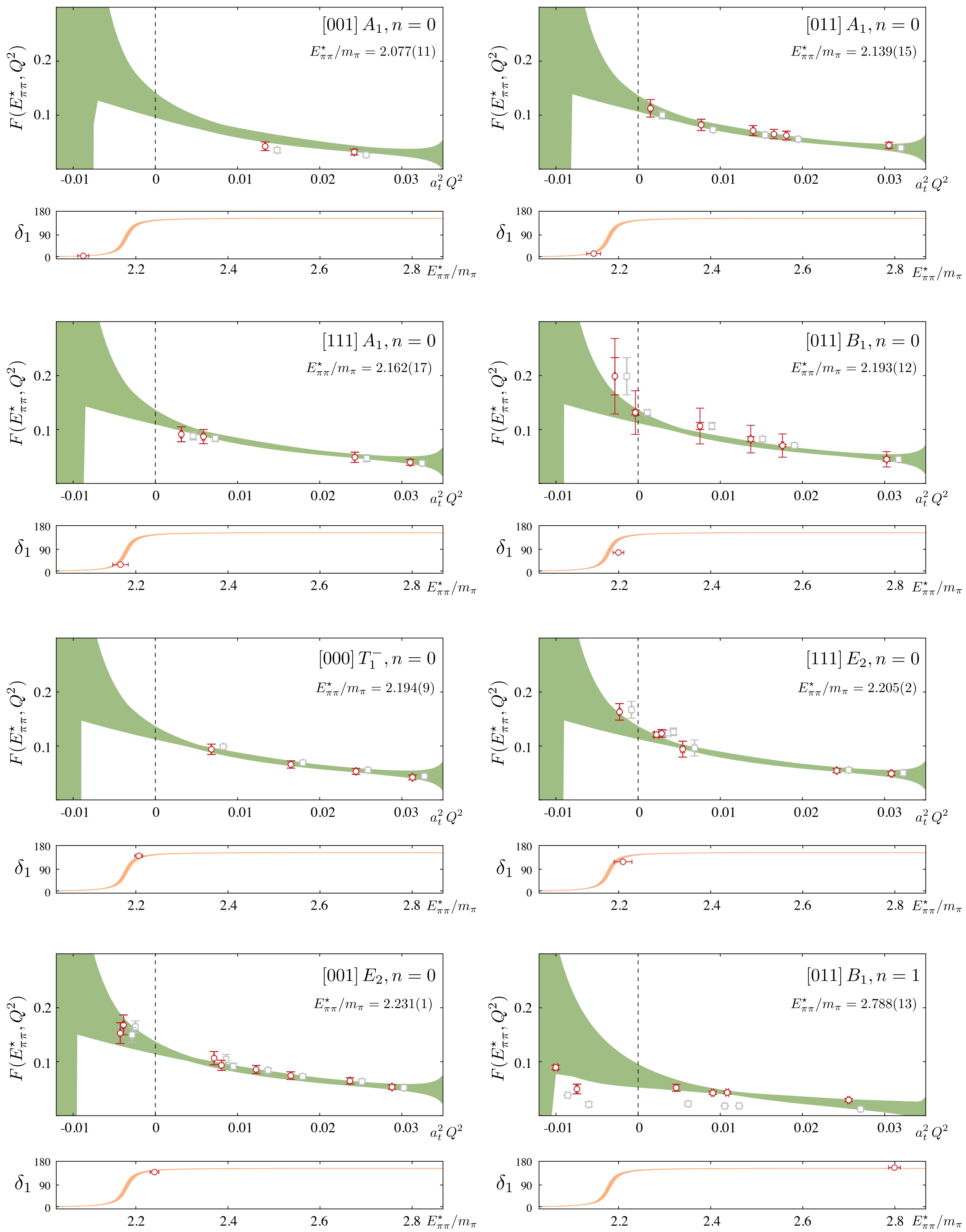
LL factor:

$$\frac{2E_\pi}{\mathcal{R}} = 32\pi\,\frac{E_\pi E_{\pi\pi}}{q^\star_{\pi\pi}}\,\cos^2\delta_1\,\frac{\partial}{\partial P^\star_{0,\pi\pi}}\left(\tan\delta_1 + \tan\phi^{\mathbf{P}_{\pi\pi},\Lambda_{\pi\pi}}\right)\Bigg|_{P^\star_{0,\pi\pi}=E^\star_{\pi\pi}}$$

$$= 32\pi\,\frac{E_\pi E_{\pi\pi}}{q^\star_{\pi\pi}}\left(\delta_1' + r\phi'\right),$$



“Form factor”

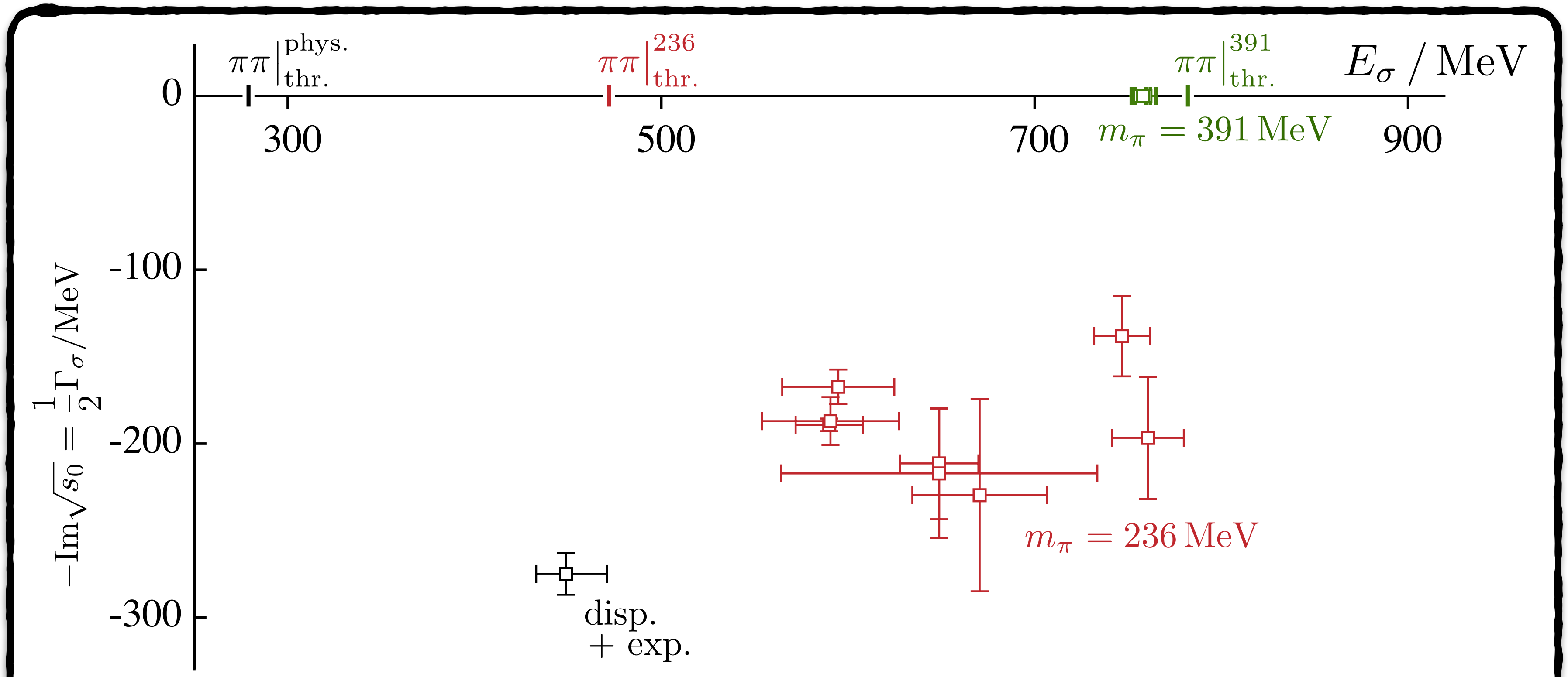


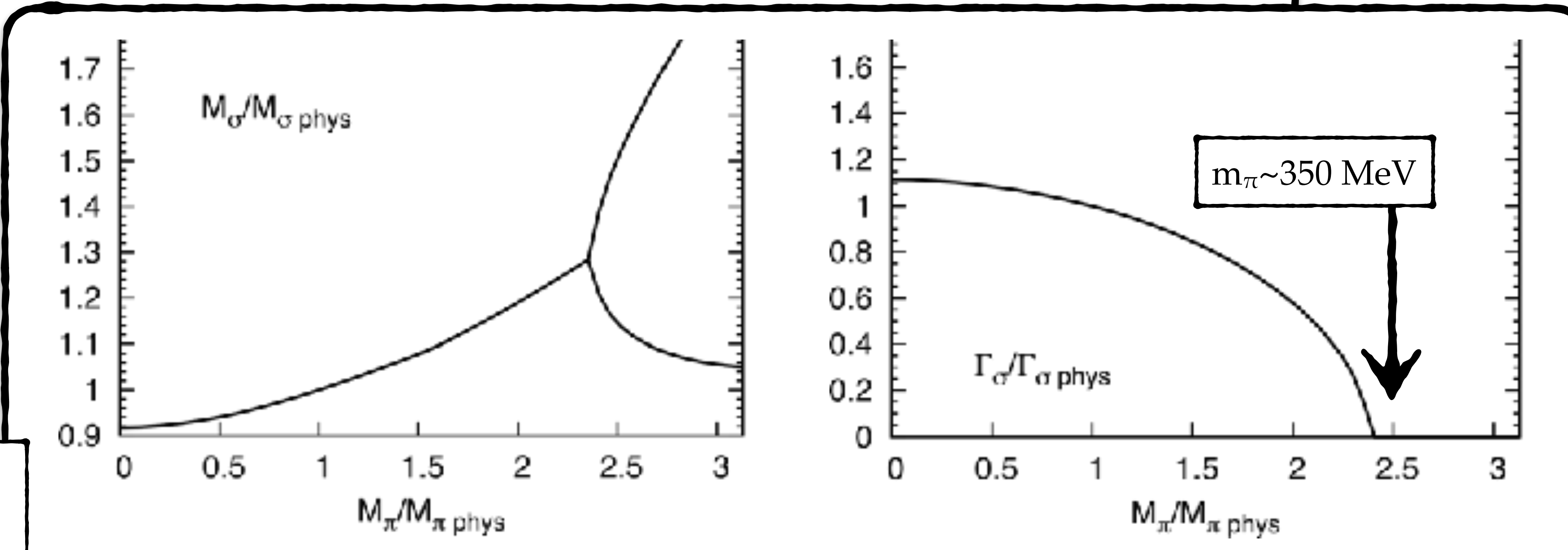
Fit parametrization:

$$h^{[\{\alpha,\beta\}]}(E^\star_{\pi\pi}, Q^2) = \frac{\alpha_1}{1+\alpha_2 Q^2 + \beta_1(E^{\star 2}_{\pi\pi} - m_0^2)} + \alpha_3 Q^2 + \alpha_4 Q^4 + \alpha_5 \exp\left[-\alpha_6 Q^2 - \beta_2(E^{\star 2}_{\pi\pi} - m_0^2)\right] + \beta_3(E^{\star 2}_{\pi\pi} - m_0^2) + \beta_4(E^{\star 4}_{\pi\pi} - m_0^4),$$

The σ / f₀(500) vs m_π

$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0} (s_0 - s)\, t(s)$$

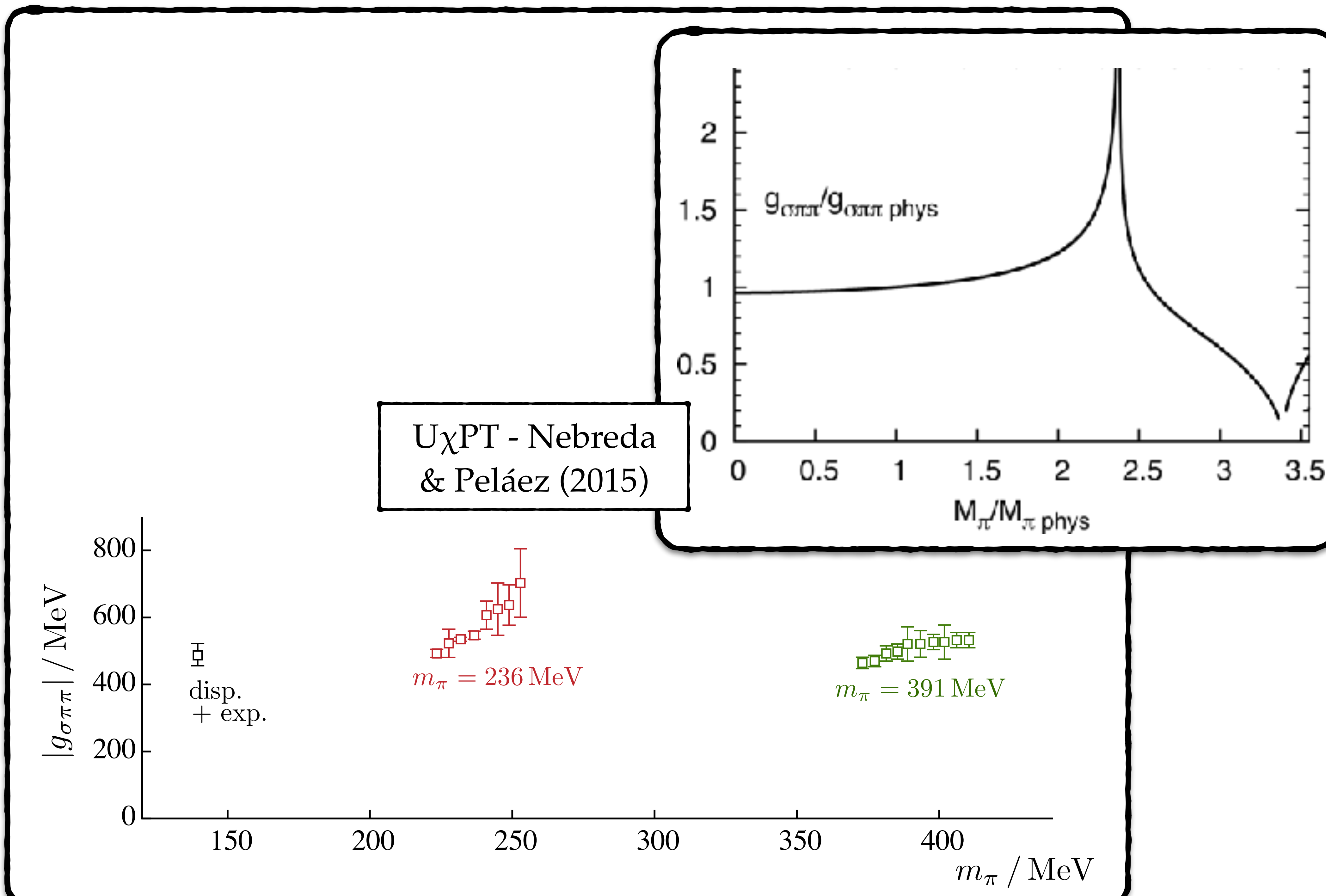




UχPT - Nebreda & Peláez (2015)

The σ / $f_0(500)$ vs m_π

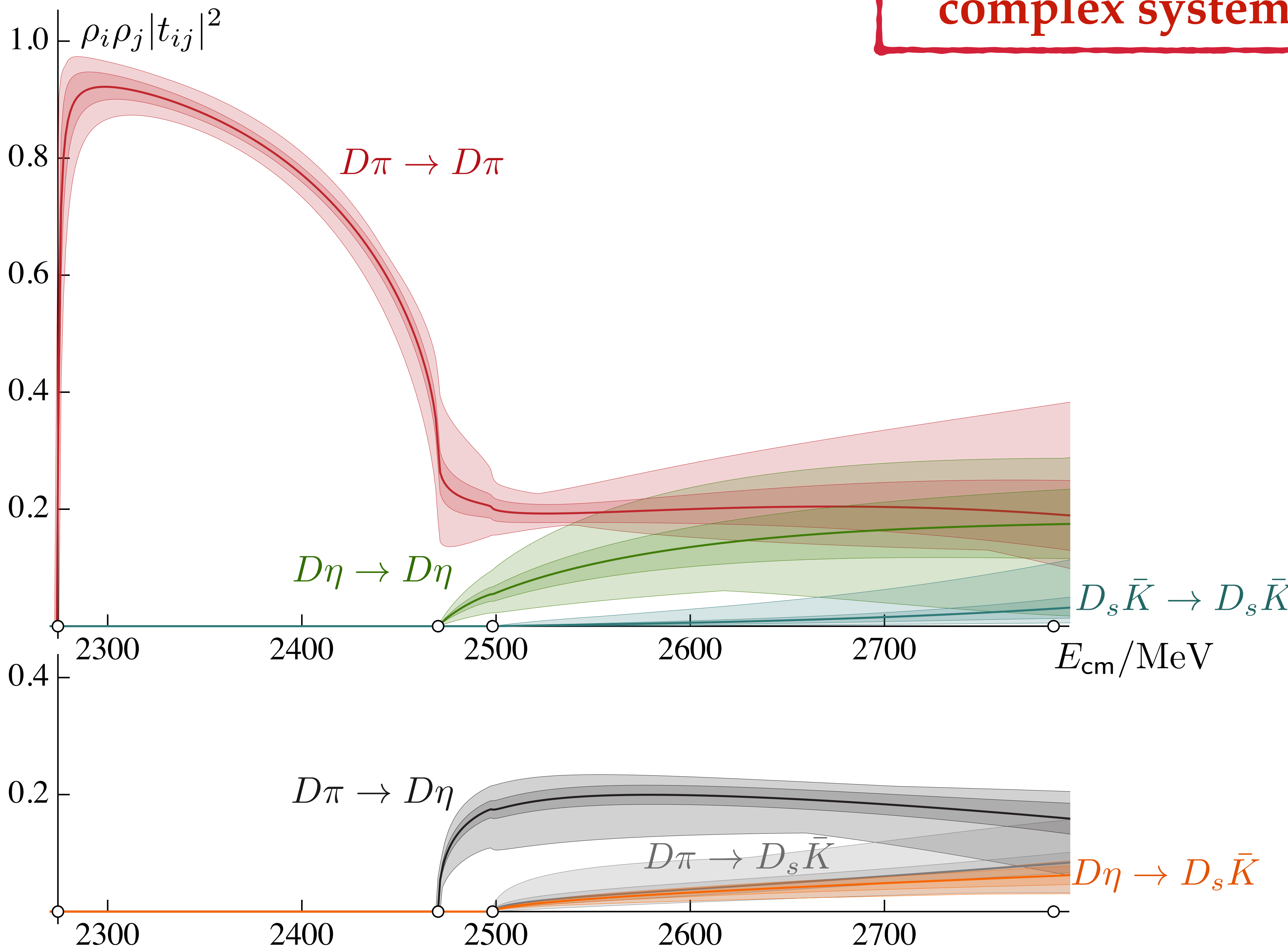
$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0} (s_0 - s)\, t(s)$$



$D\pi$–$D\eta$–$D_s\overline{K}$ scattering

(I=1/2 channel)

Increasingly complex systems

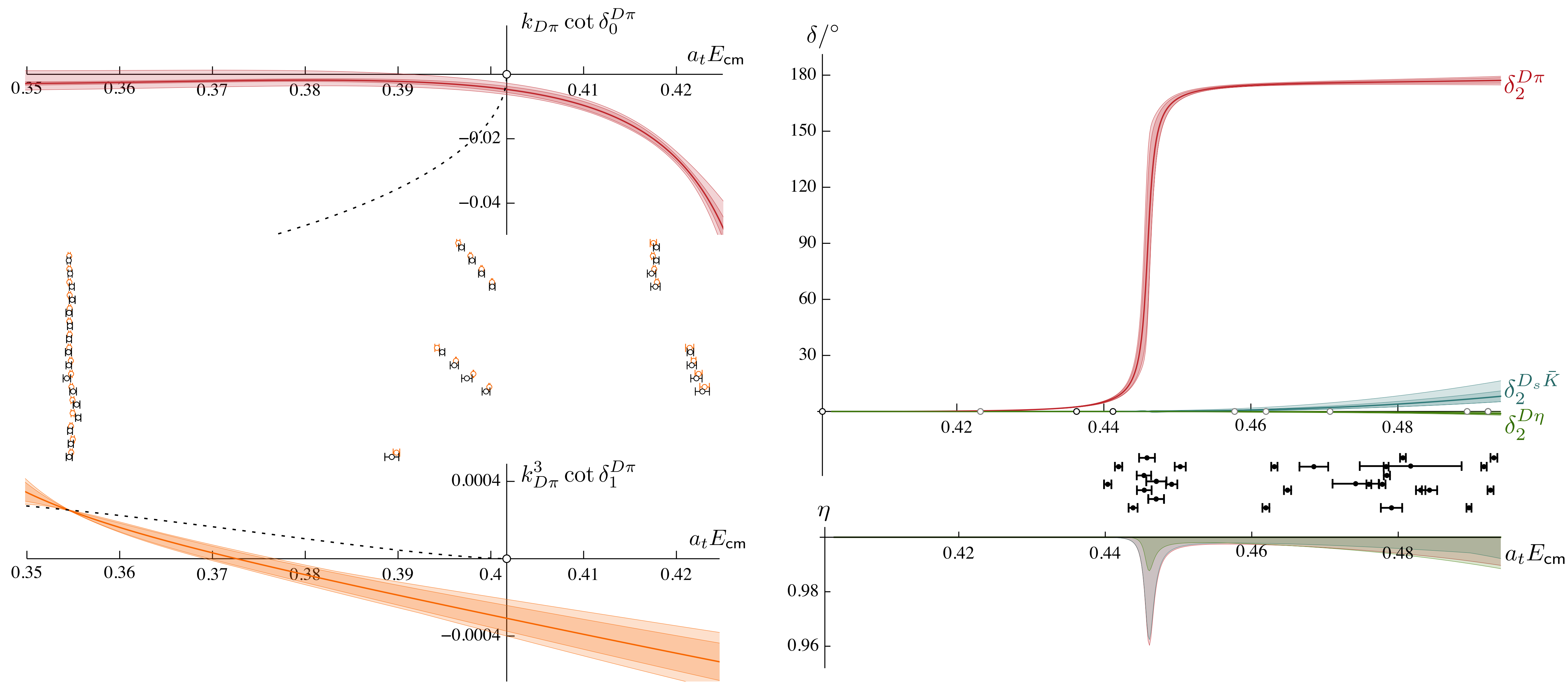


RB

Moir, Peardon, Ryan, Thomas, Wilson

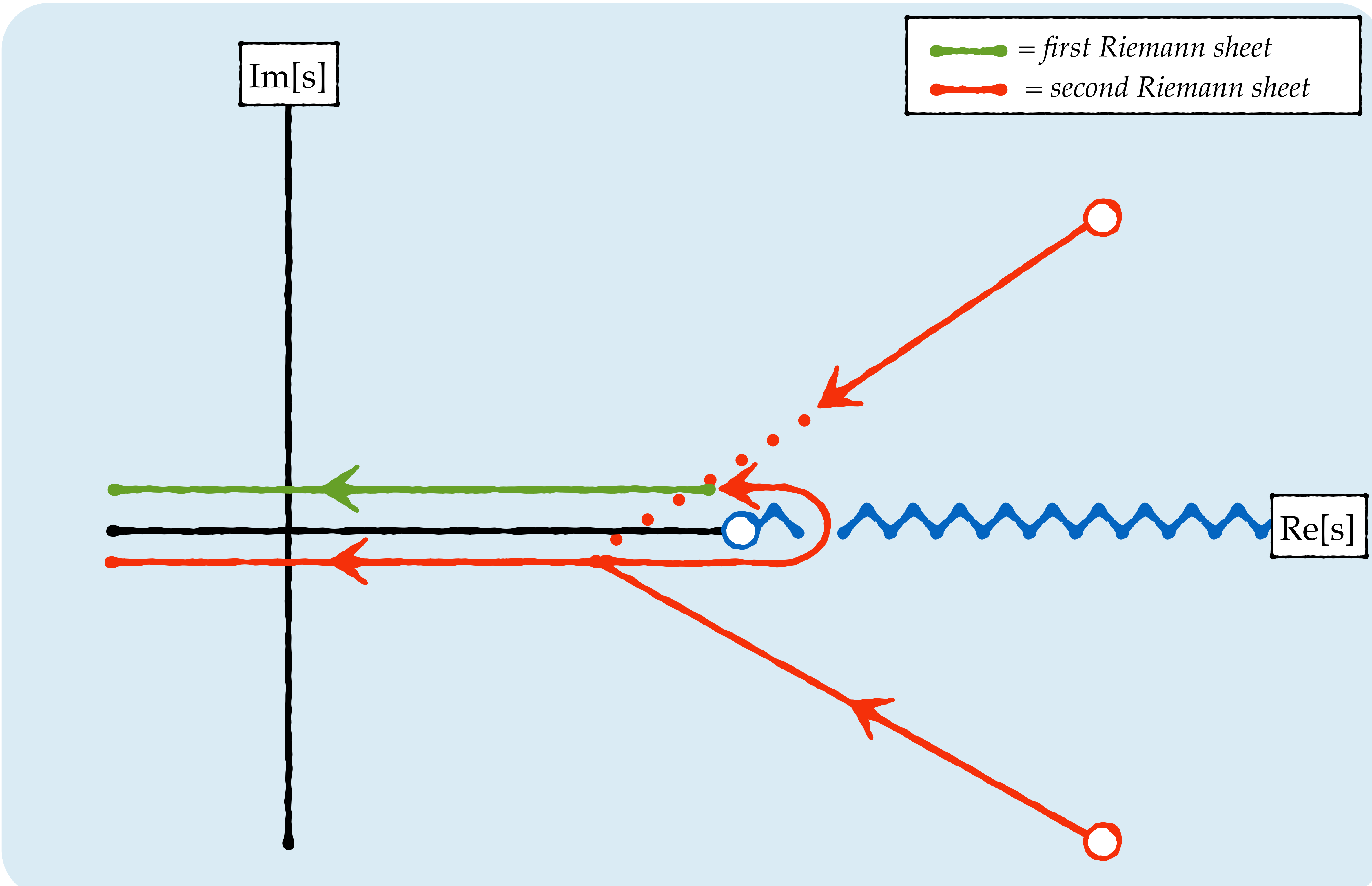
$D\pi$–$D\eta$–$D_s\overline{K}$ scattering

(I=1/2 channel)



UχPT expectation for σ / $f_0(500)$

σ / $f_0(500)$ vs m_π

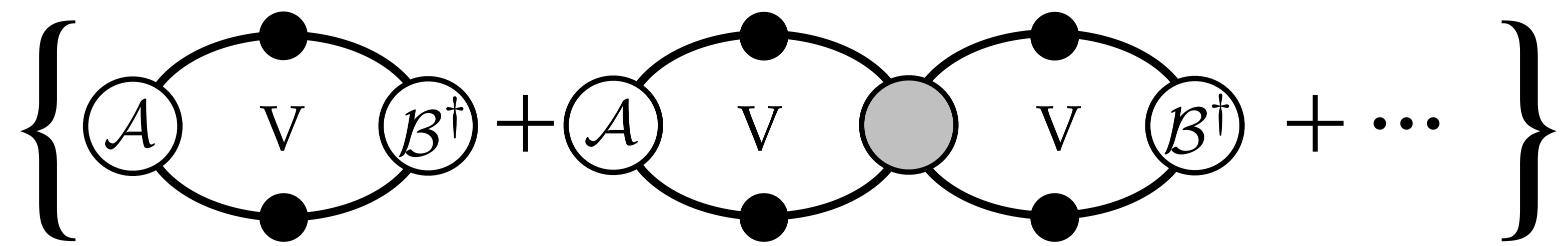


Sketch of Lüscher

Two particles in a box

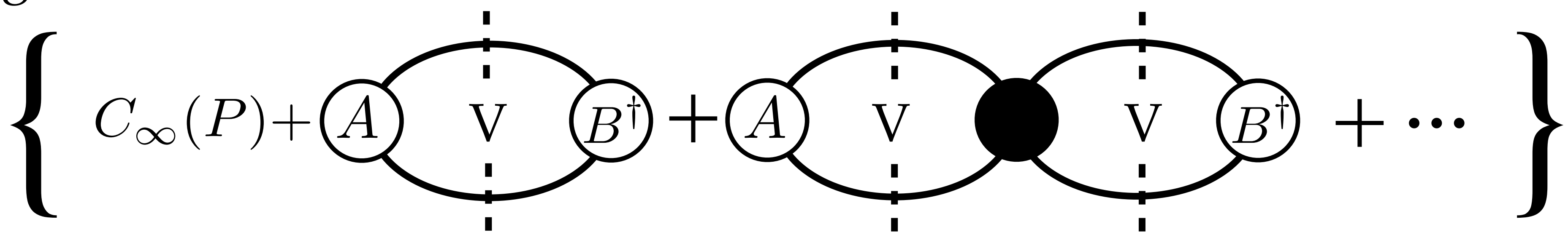
Onto two particles:

$$L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t}$$



After some massaging…

$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t}$$



$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \left\{ C_\infty(P) - A(P) \frac{1}{F^{-1}(P,L) + \mathcal{M}(P)} B^\dagger(P) \right\}$$

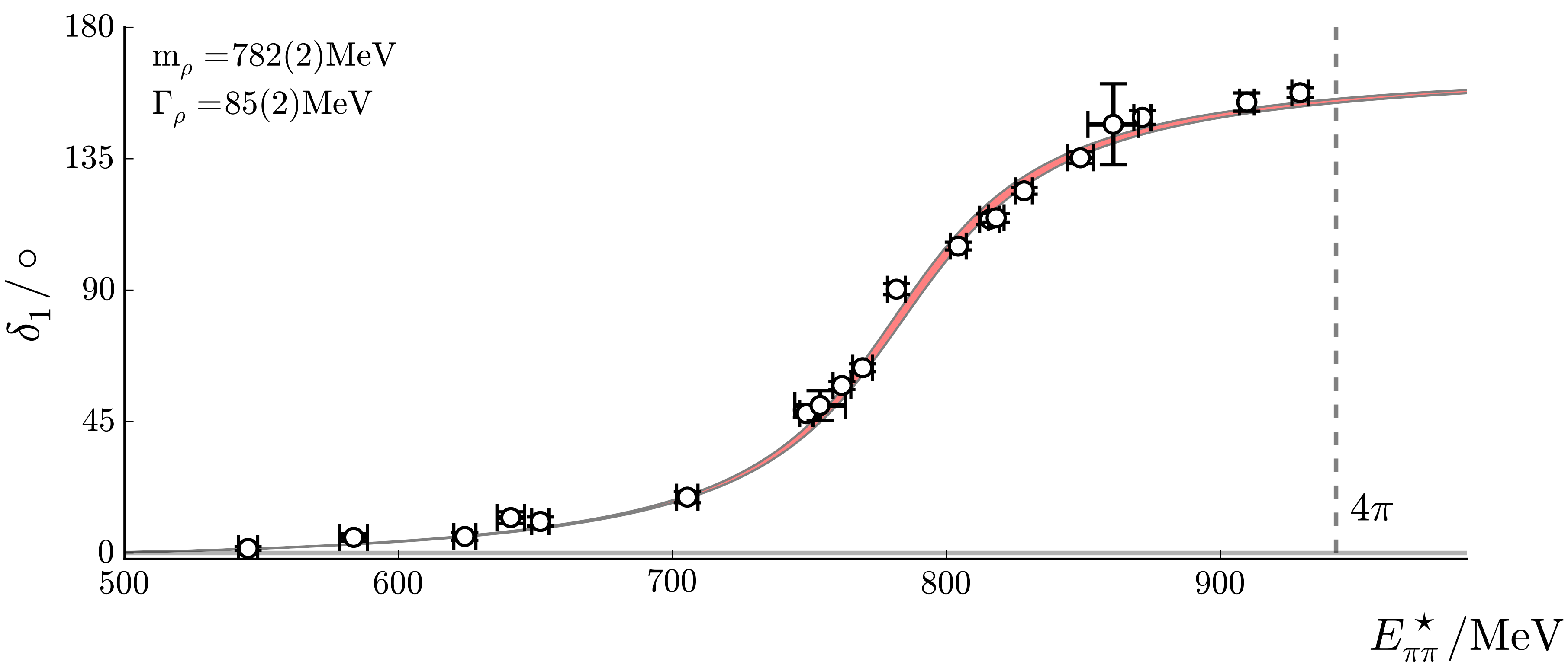
poles satisfy: $\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$

Chiral fits

Chiral fit

$$\alpha_1 \equiv -2\ell_1^r + \ell_2^r,\ \alpha_2 \equiv \ell_4^r$$

$$\alpha_1(770\ \text{MeV}) = 14.7(4)(2)(1) \times 10^{-3}$$
$$\alpha_2(770\ \text{MeV}) = -28(6)(3)\binom{01}{11} \times 10^{-3}$$



previos results:

$$\alpha_1(770\ \text{MeV}) \in [9, 13] \times 10^{-3}$$
$$\alpha_2(770\ \text{MeV}) \in [1, 12] \times 10^{-3}$$

m_π dependence