Classical-statistical simulations and the Chiral Magnetic Effect

Niklas Mueller

Heidelberg University

based on work together with: J. Berges, M. Mace, S. Schlichting, S. Sharma, N. Tanji, R. Venugopalan

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7th Workshop of the APS Topical Group on Hadronic Physics

Washington DC - ***2017 / 02 / 02***

Topology in Heavy Ion Collisions

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The Chiral Magnetic Effect

(Kharzeev, McLerran, Warringa 2007; Kharzeev, Fukushima, Warringa 2008)

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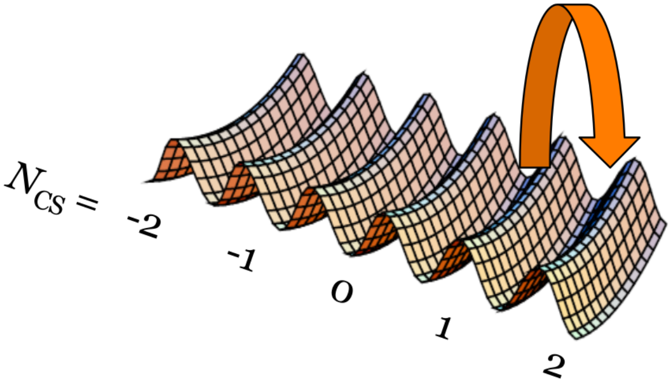
Topological Fluctuations (sphalerons)

– analogous to proposed 'electroweak baryogenesis'



Review: Kharzeev et al, Prog. Nucl. Part. Phys 88 (2016) 1

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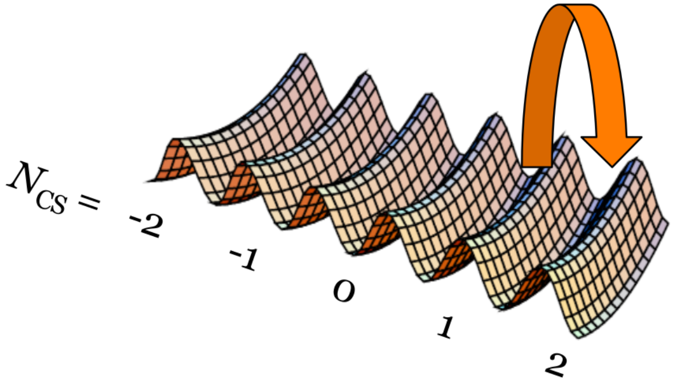

The axial anomaly

– axial non-conservation vs. topological density

$$\partial_\mu j_a^\mu(x) = 2m\eta_a(x) - \frac{g^2}{8\pi^2}\mathrm{Tr}F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

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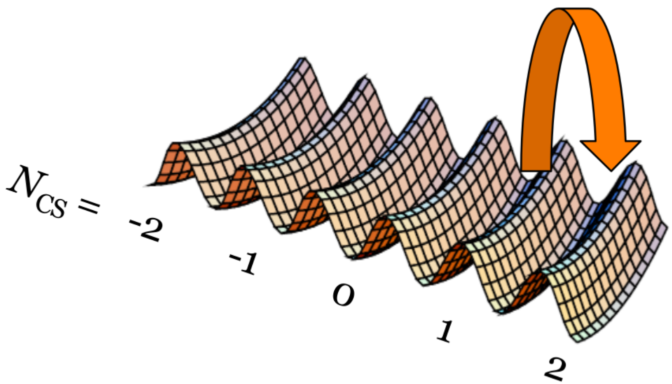
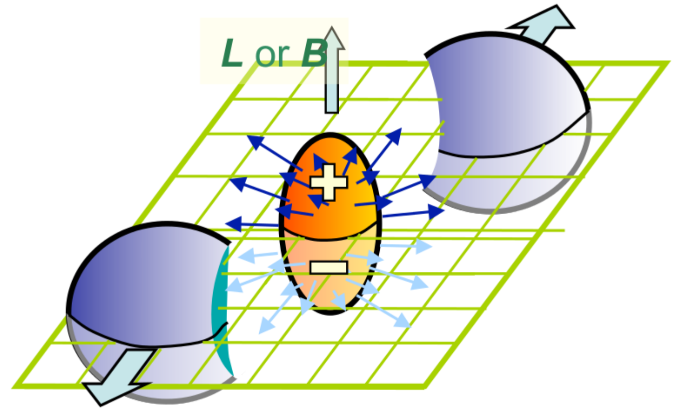


External Magnetic Fields
– 10^{18} Gauss, strongest ever produced in Lab!



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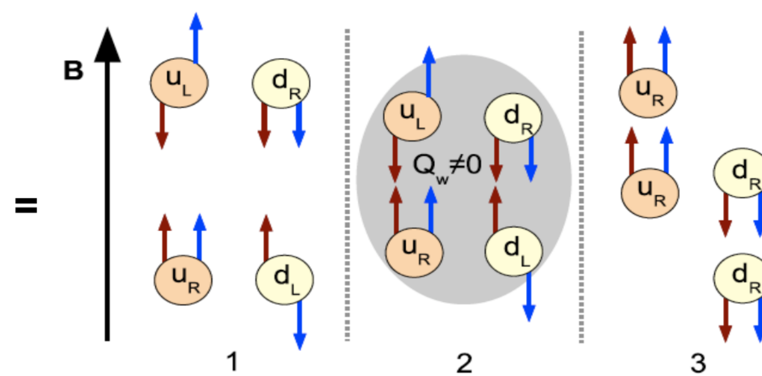

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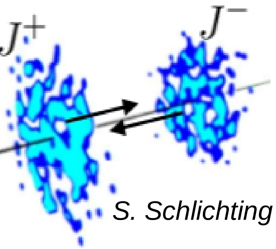
Outline

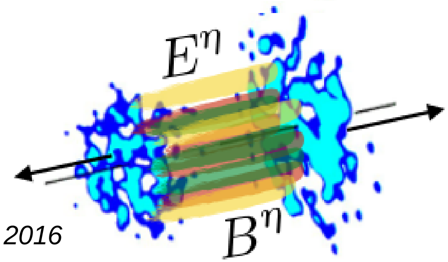
1. Anomalous phenomena in heavy ion collisions
2. Why are early times so important?
3. Classical-statistical simulations
4. Towards kinetic theory and the bigger picture
5. Conclusions

1. Anomalous Phenomena in Heavy Ion Collisions

CGC
colliding nuclei

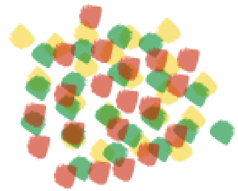


flux tubes



S. Schlichting 2016

over-occupied
plasma



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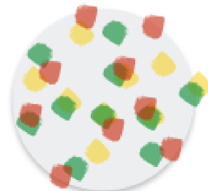

hydrodynamic
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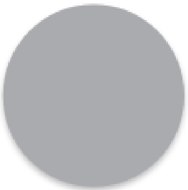

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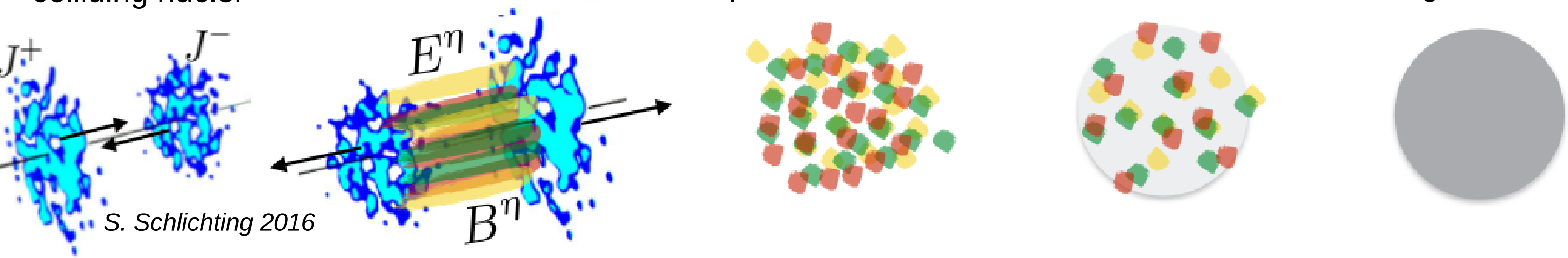
CGC colliding nuclei | flux tubes | over-occupied plasma | kinetic regime | hydrodynamic regime



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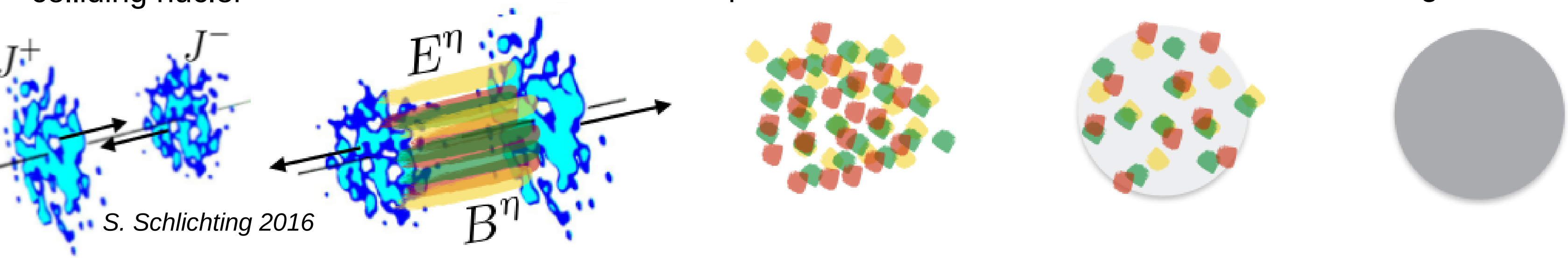

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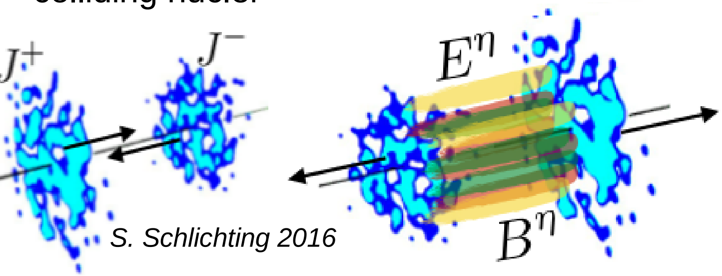

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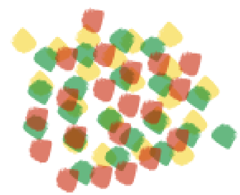

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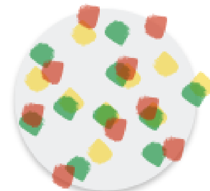

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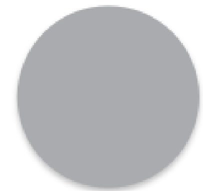

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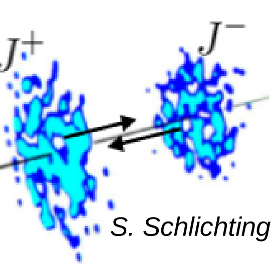

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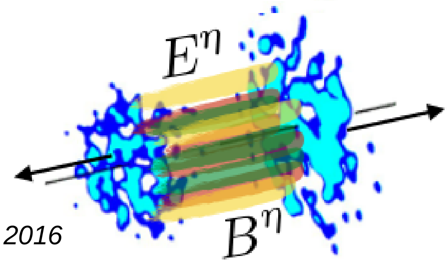

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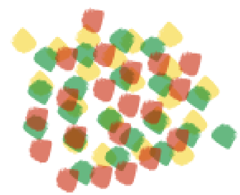

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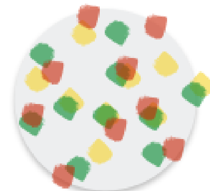

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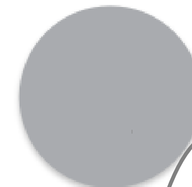

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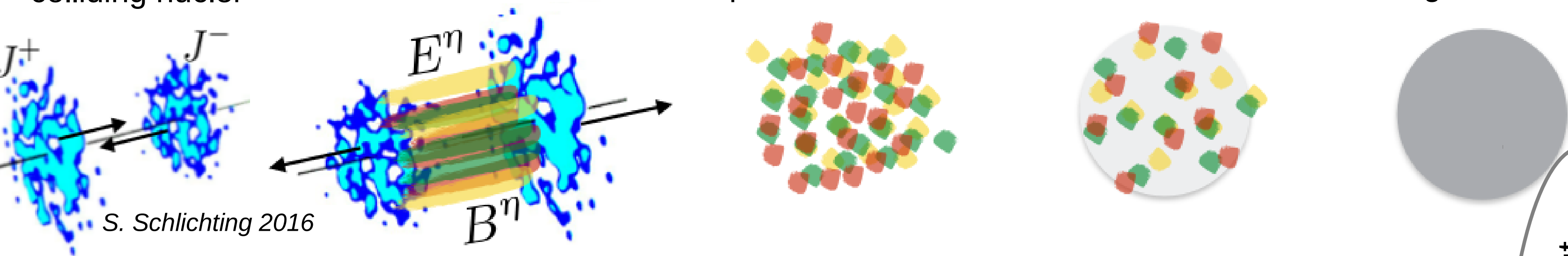

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Theory:

classical statistical simulations + fermions

chiral kinetic theory

anomalous hydrodynamics

1. Anomalous Phenomena in Heavy Ion Collisions

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- condensed matter systems (Li, Kharzeev et al, Nature 2015) ✓
- heavy ions (charged particle correlations: RHIC 2009, LHC) ?

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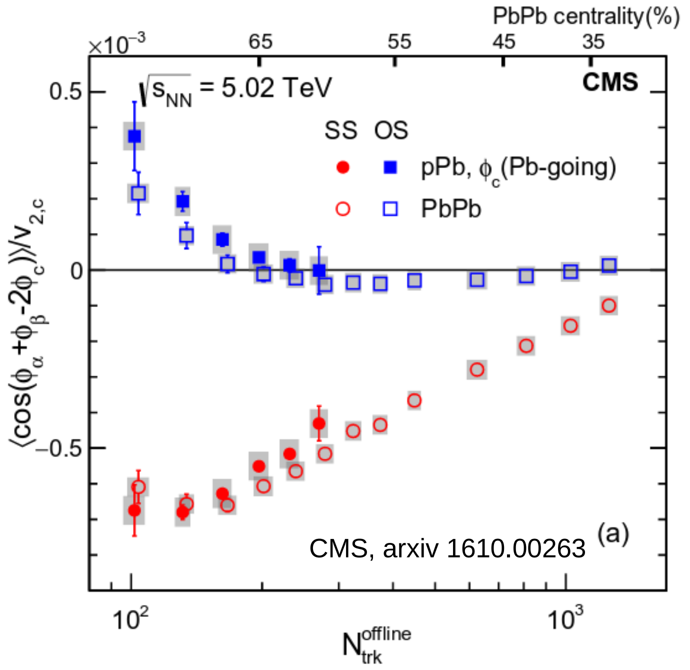

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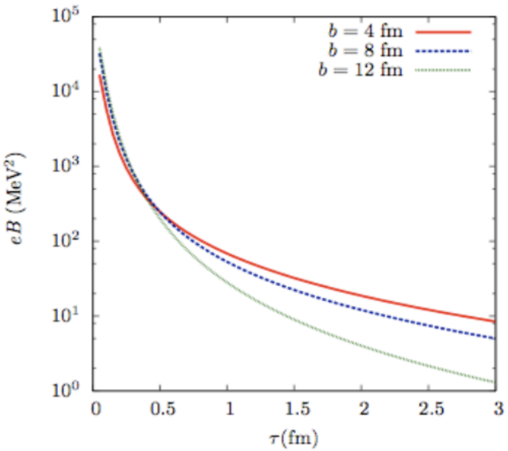
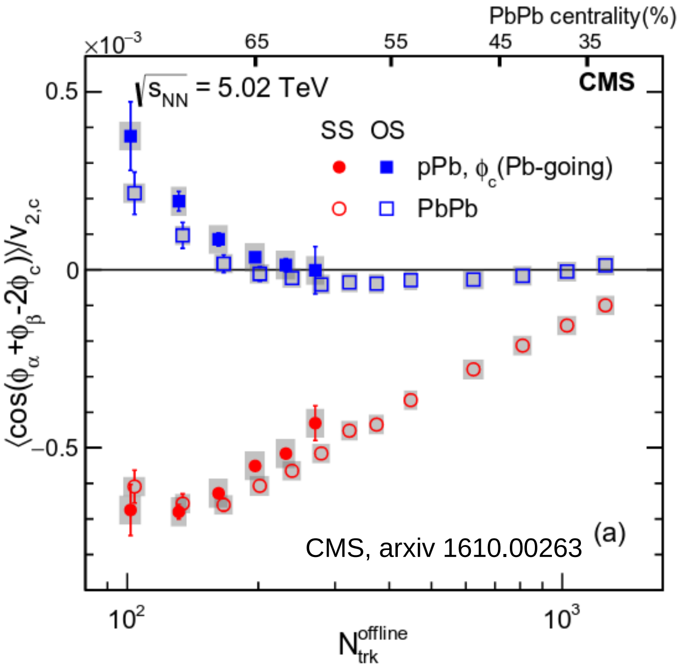
– *How long does the magnetic field live?*





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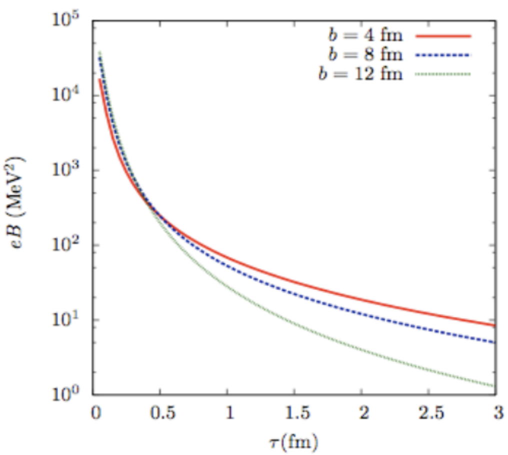

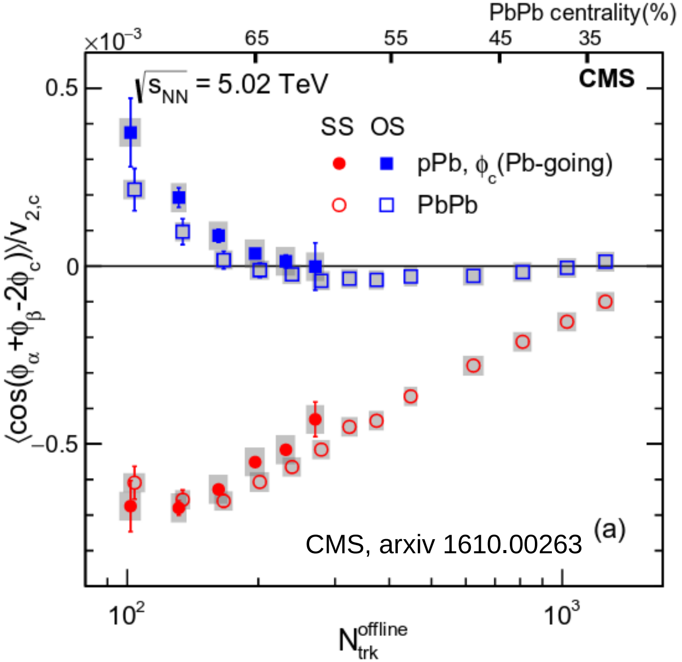

Challenge for theorists:

Quantitative Predictions needed at various stages of the collision:
Production at early time --- chiral transport --- hydrodynamics and observables

Clarification soon: Isobar run at RHIC!

2. Why are early times so important?

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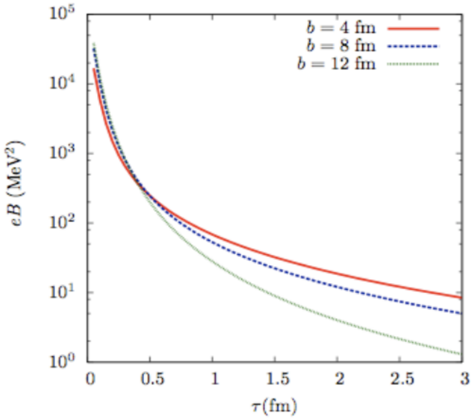
Most important direct contribution to CME at early times!

- (Abelian) **magnetic field** 'still alive'



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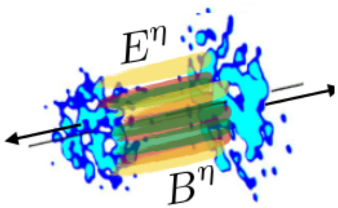

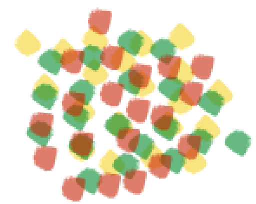

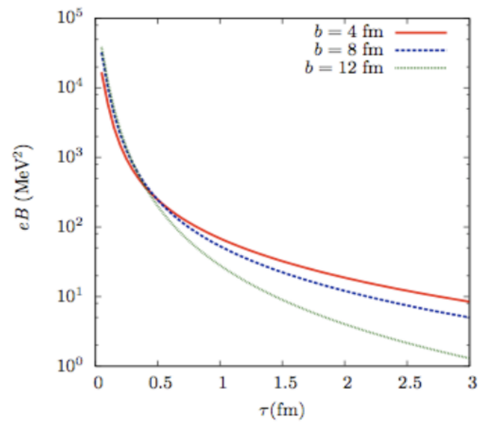

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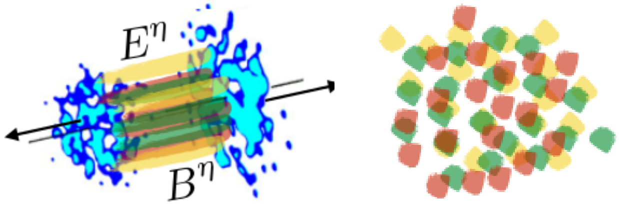

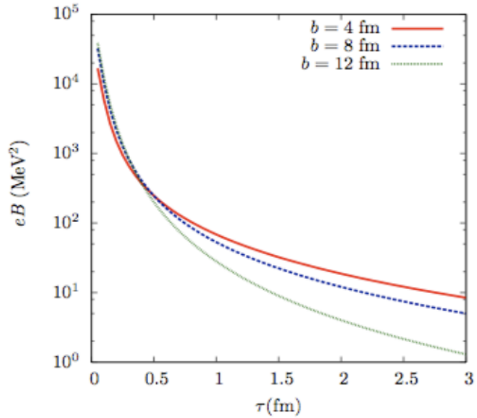

- **Quark production large**
  occupation numbers of gluons ~ 1/g → quark production O(1)

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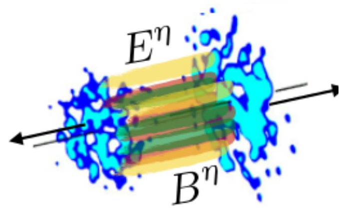

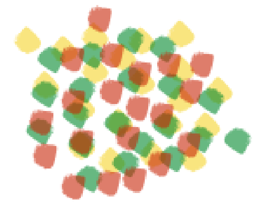

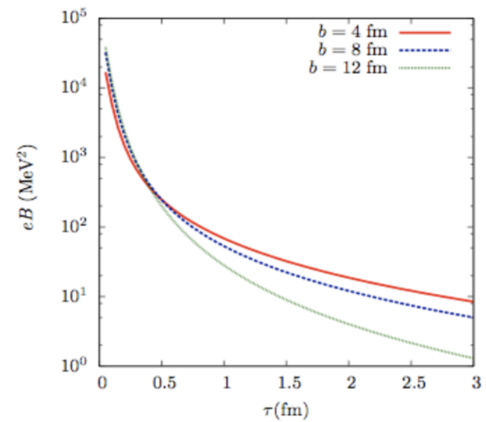

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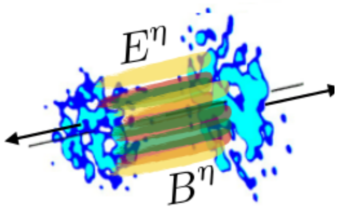

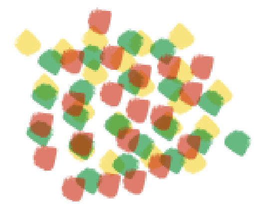

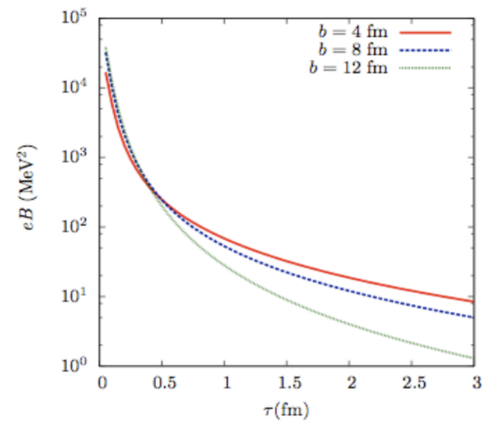

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Classical-Statistical Simulations

(see Kasper 2015 for an pedagogical introduction)

Can be systematically derived within the **Color Glass Framework** (Mclerran, Venugopalan 1994)

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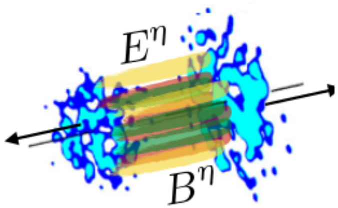

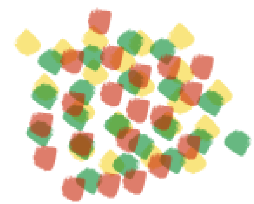

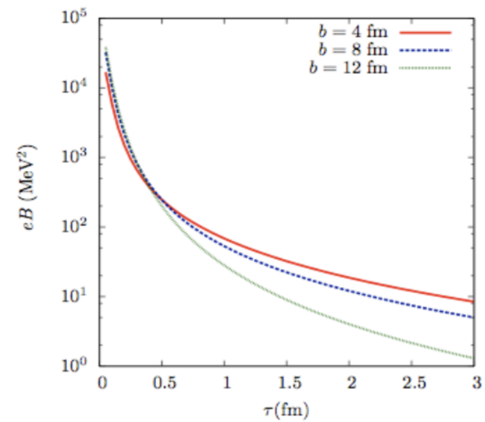

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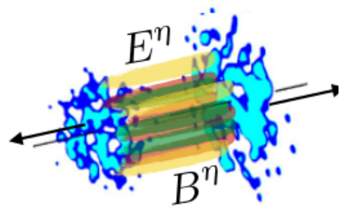

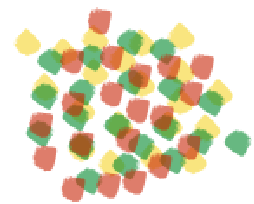

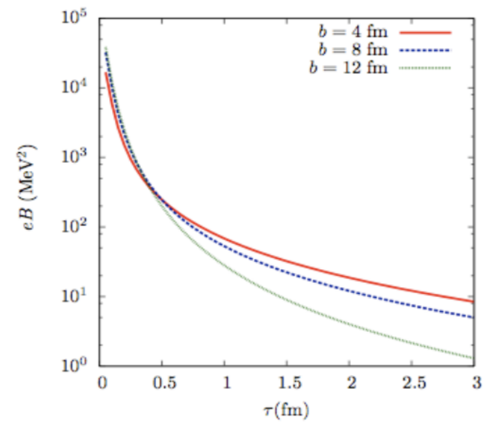

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→ **Fermions:** solve (operator-) Dirac equation

2. Real-time simulations

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Remarkable progress at early times – classical statistical simulations

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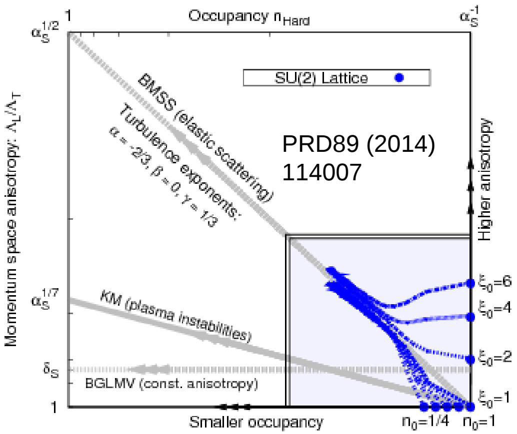
- Identification of **thermalization scenario**

(Berges, Boguslavski, Schlichting, Venugopalan 2014)



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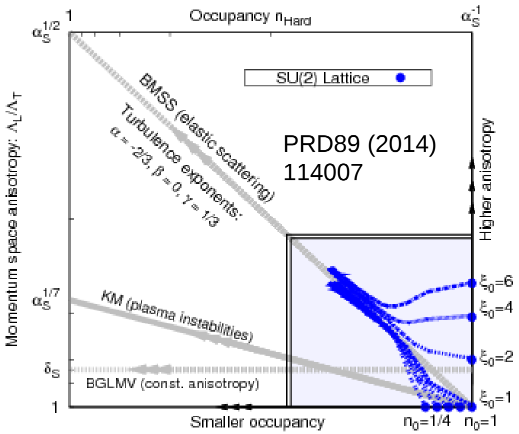
→ **Instabilities and Turbulence**

→ **Universal** Physics far-from-equilibrium

(recent connection to ultra-cold atoms PRL. 114 (2015) 061601)



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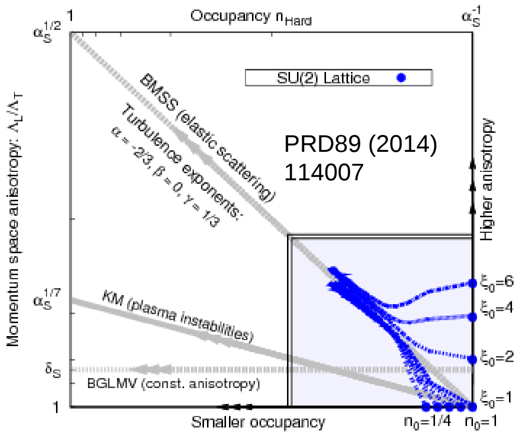
- Identification of **thermalization scenario**

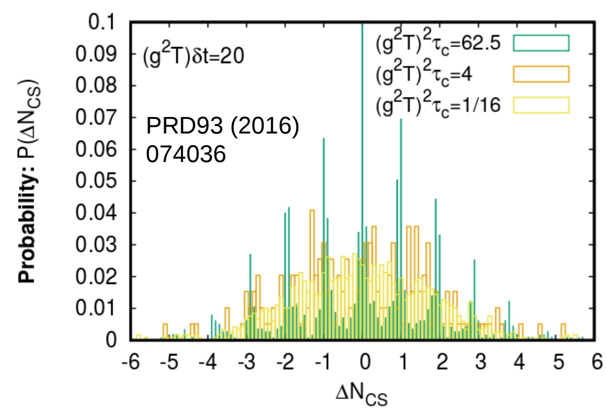
  (Berges, Boguslavski, Schlichting, Venugopalan 2014)

  → **Instabilities and Turbulence**
  → **Universal** Physics far-from-equilibrium
  (recent connection to ultra-cold atoms PRL. 114 (2015) 061601)

- **Topological Transitions** far-from-equilibrium

  (Mace, Schlichting, Venugopalan 2016, Moore 2011)





2. Real-time simulations

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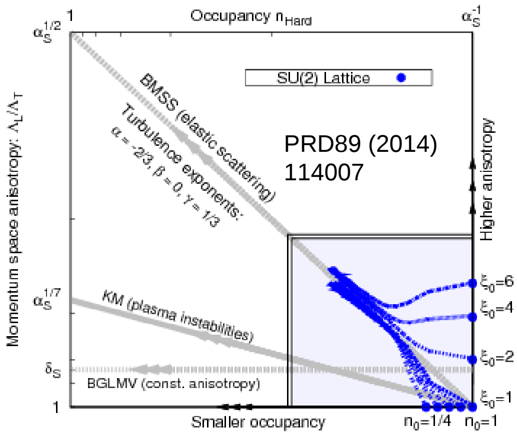

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- Anomalous Effects in **laser physics**

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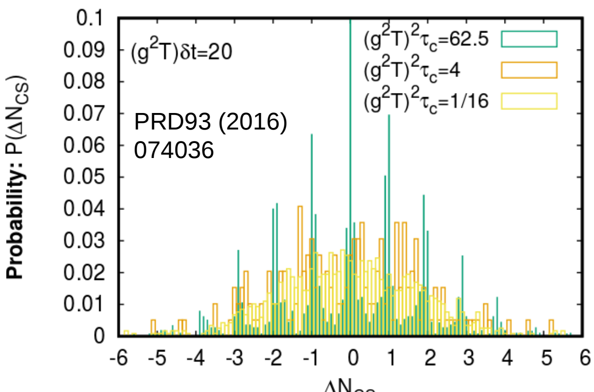

Many important contributions left out here --- my personal bias --- sorry!

2. Real-time simulations

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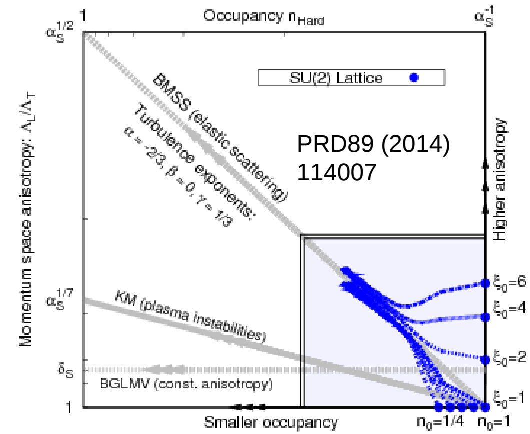

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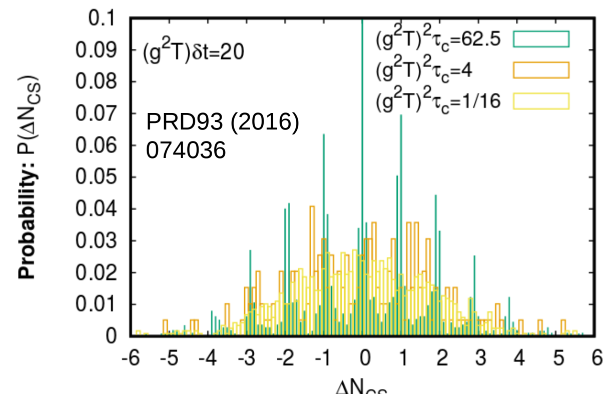

- Real-time simulations of the **Chiral Magnetic Effect and Anomalous Dynamics (QCD+QED)**

  (Tanji, NM, Berges PRD93 (2016) 074507; Mace, NM, Schlichting, Sharma PRL 117 (2016) 061601 & arXiv:1612.02477)

Many important contributions left out here --- my personal bias --- sorry!

Will show you today:

Far-from-equilibrium anomalous and topological dynamics in HI collisions can be understood quantitative and from first principles, using real-time lattice simulations in the classical statistical regime

Anomalous fermion dynamics during a topologicial transition

→ Clean test case for the development of our tools.

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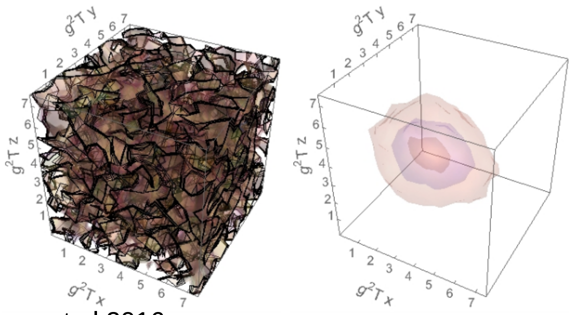
simplified situation: setting up an isolated sphaleron transition



Mace et al 2016

3. Real-time simulations

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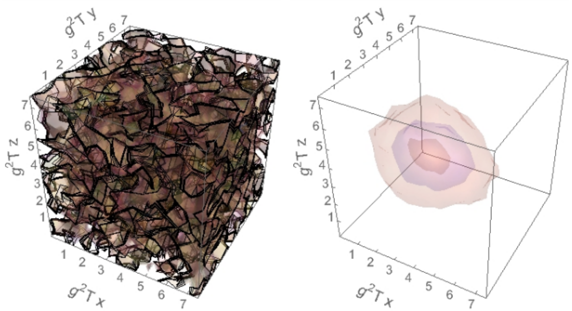

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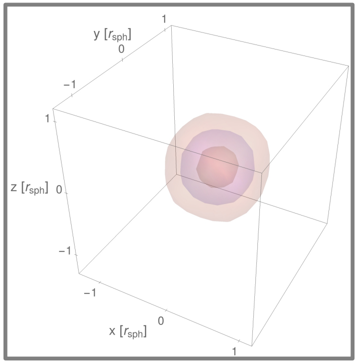

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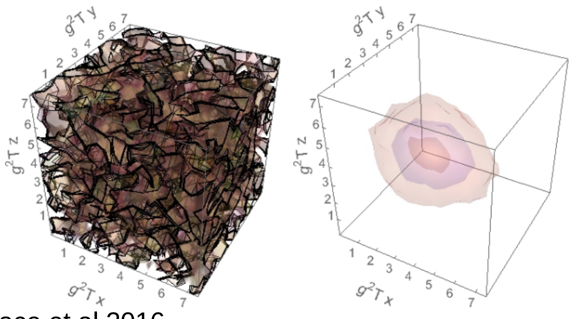

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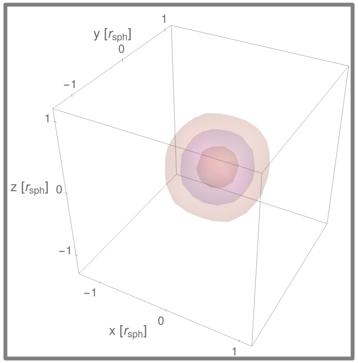

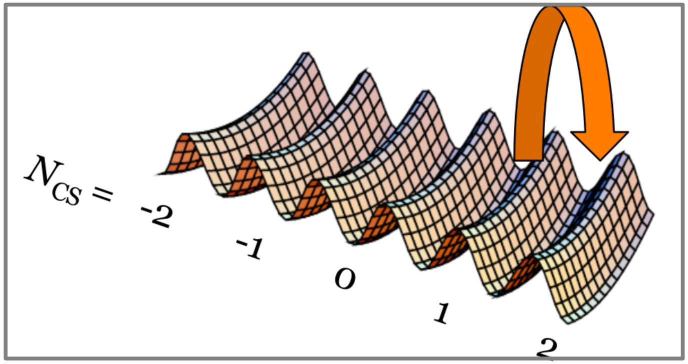

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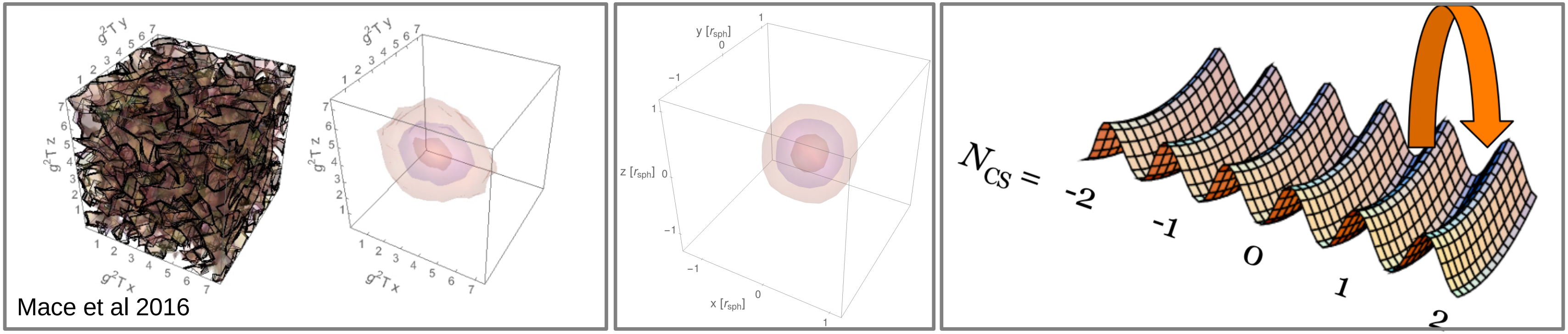

Fermions: Challenging!

Solving Dirac operator equation

in mode-function expansion

$$i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}$$

$$\hat{\psi}_{\mathbf{x}}(t) = \frac{1}{\sqrt{V}}\sum_\lambda \left(\hat{b}_\lambda(0)\phi^u_\lambda(t,\mathbf{x}) + \hat{d}^\dagger_\lambda(0)\phi^v_\lambda(t,\mathbf{x})\right)$$

3. Real-time simulations

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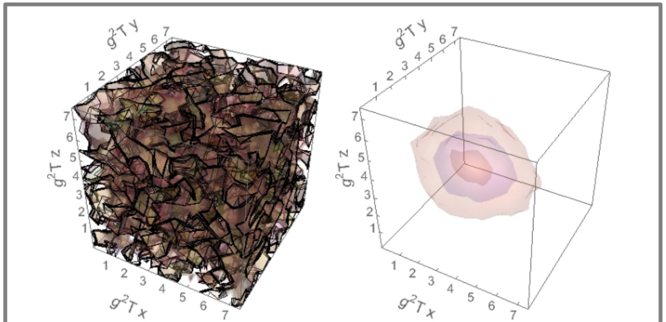
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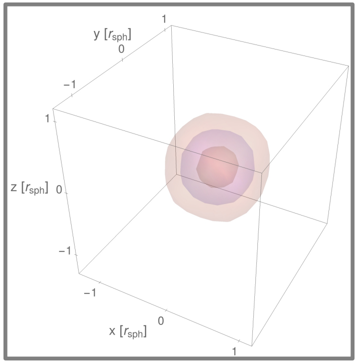

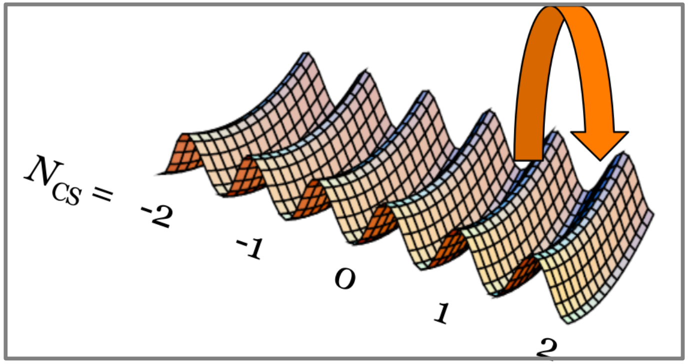

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→ extremely costly (~N^6)
→ big obstacle so far and many attempts at reducing price (e.g. 'low-cost' techniques)

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Chiral Magnetic and Chiral Separation Effect – Chiral Magnetic Waves

NM, Schlichting, Sharma, PRL 117 (2016) 142301; Mace, NM, Schlichting, Sharma, arXiv:1612.02477

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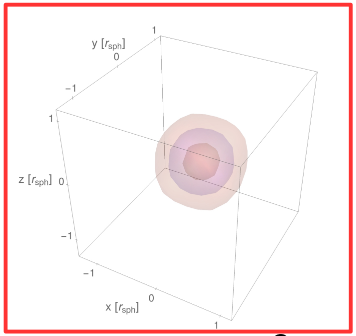
lattice size:24x24x64



3. Real-time simulations

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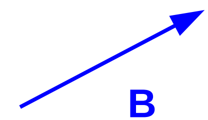
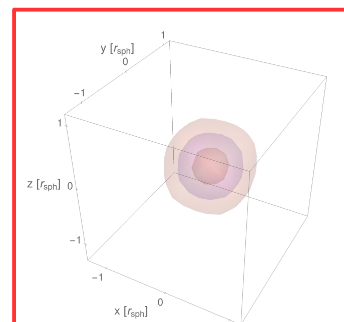


abelian magnetic field along z

Initially: Vacuum (no fermions, no axial charge)



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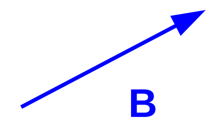
sphaleron transition (time)



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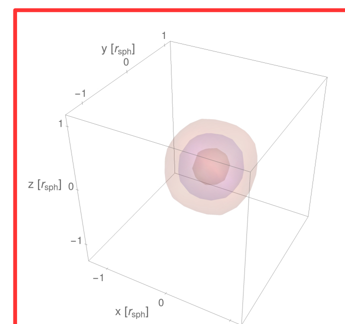

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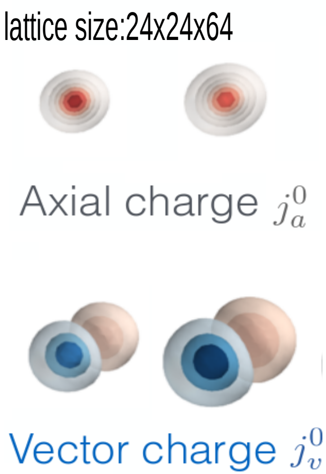

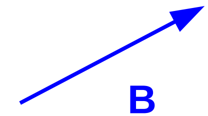

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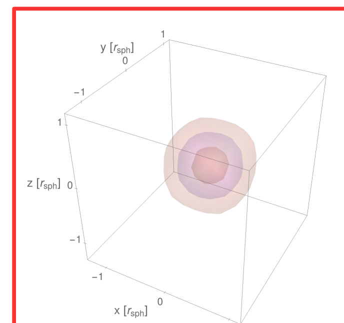

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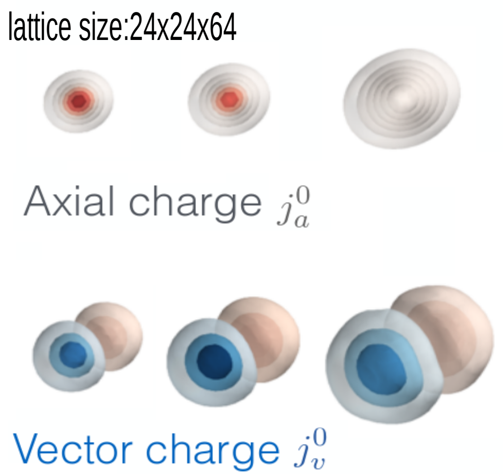

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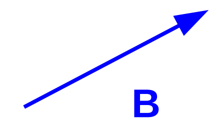

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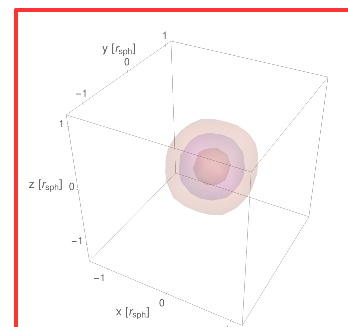

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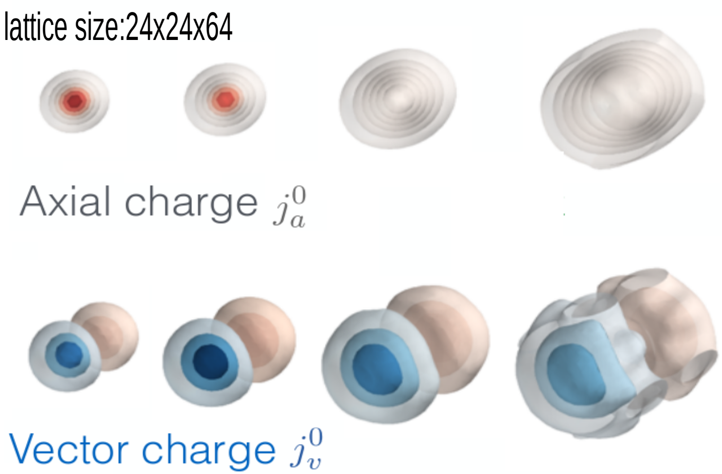

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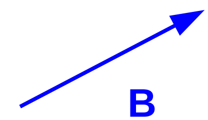

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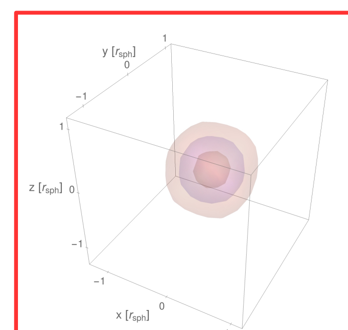

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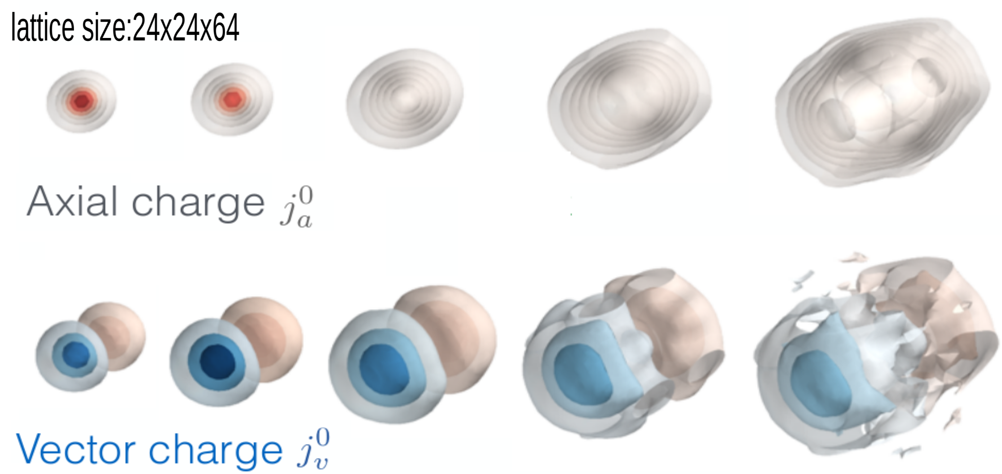

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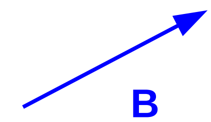

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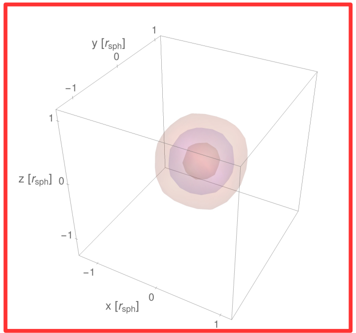

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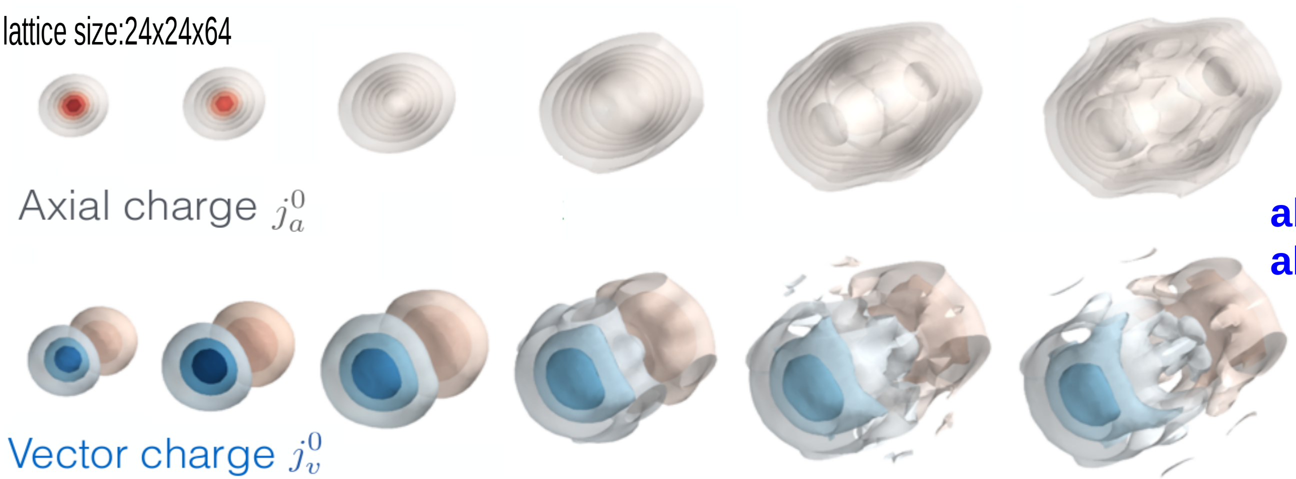

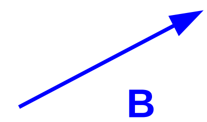

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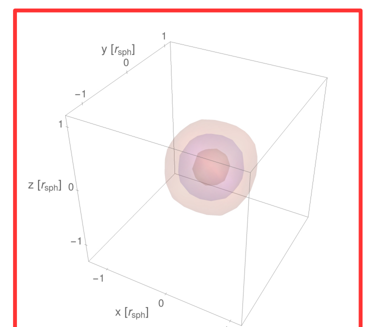

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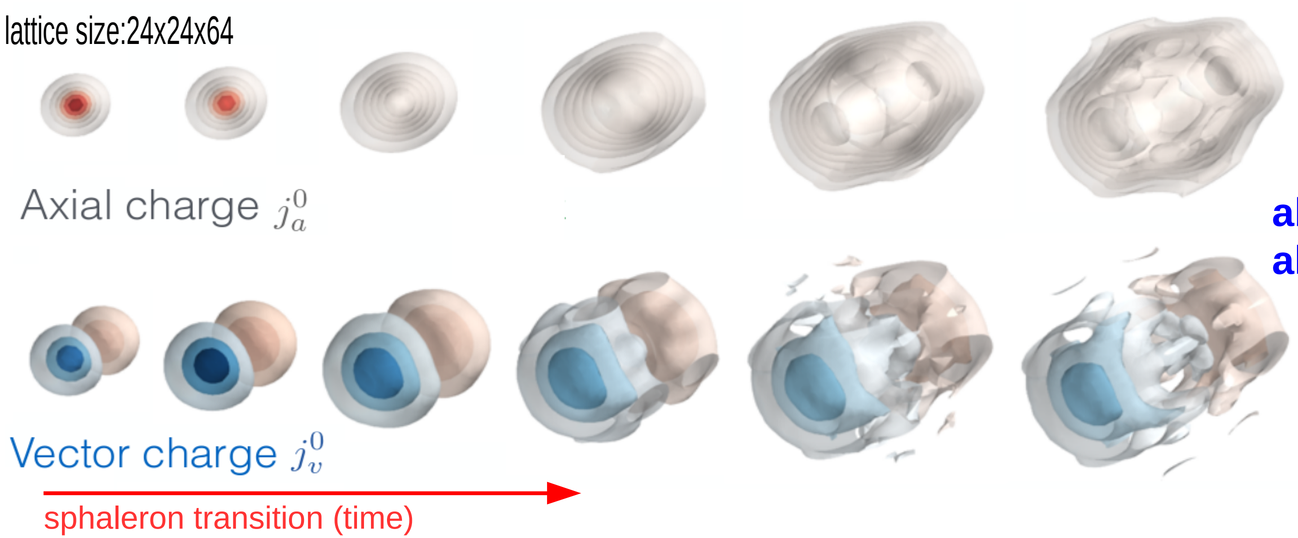

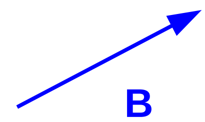

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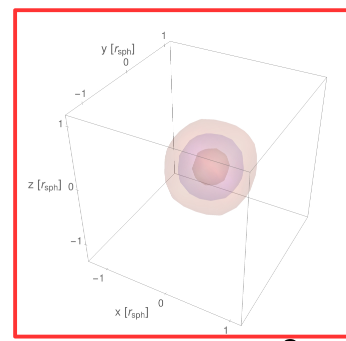
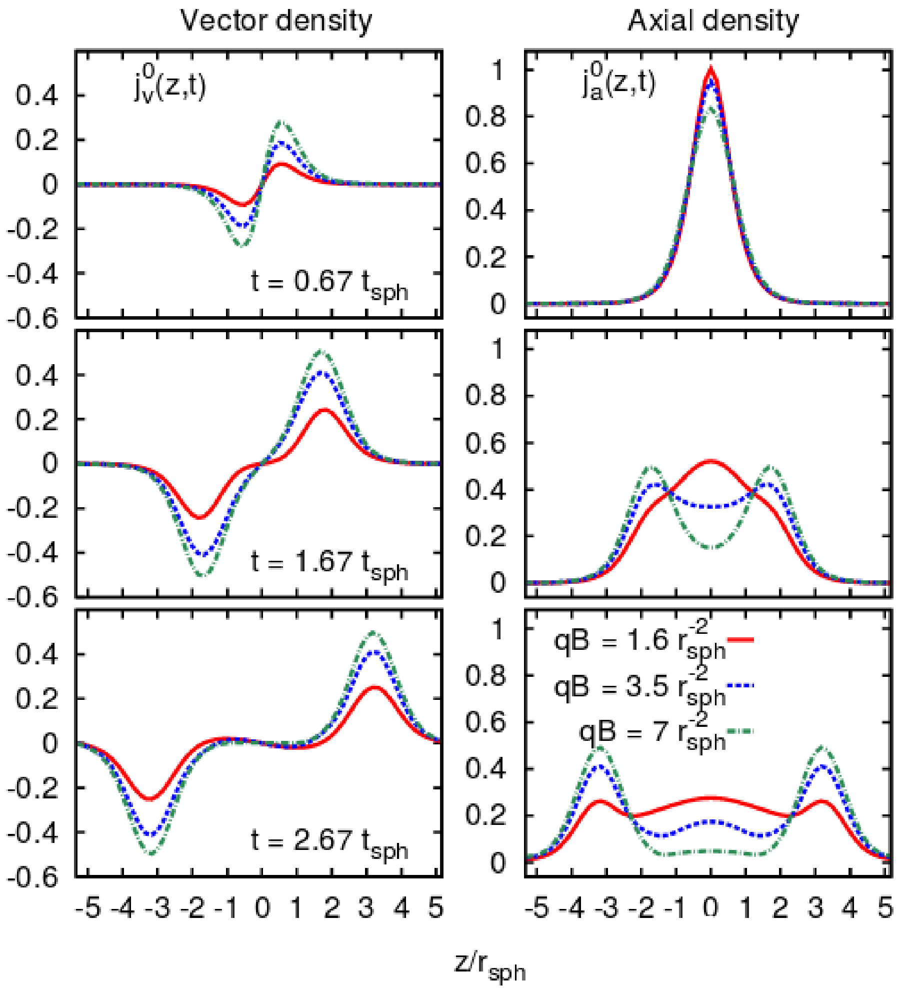


Chiral Magnetic Effect: Electric current generated due to axial charge produced

Chiral Separation Effect: Axial current generated due to electric charge

3. Real-time simulations



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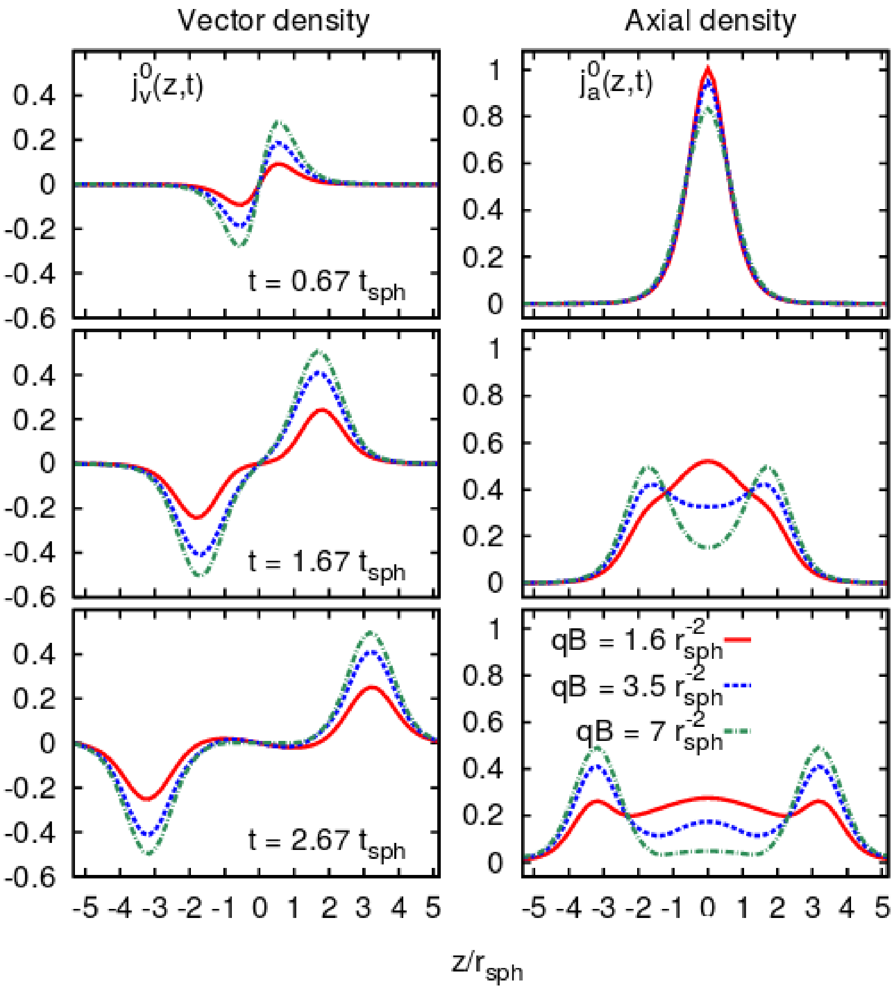

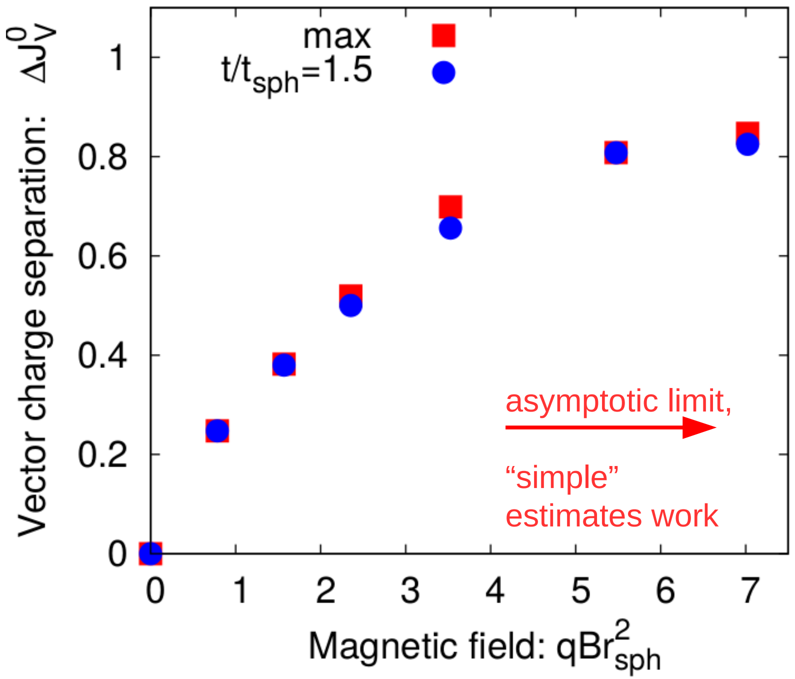

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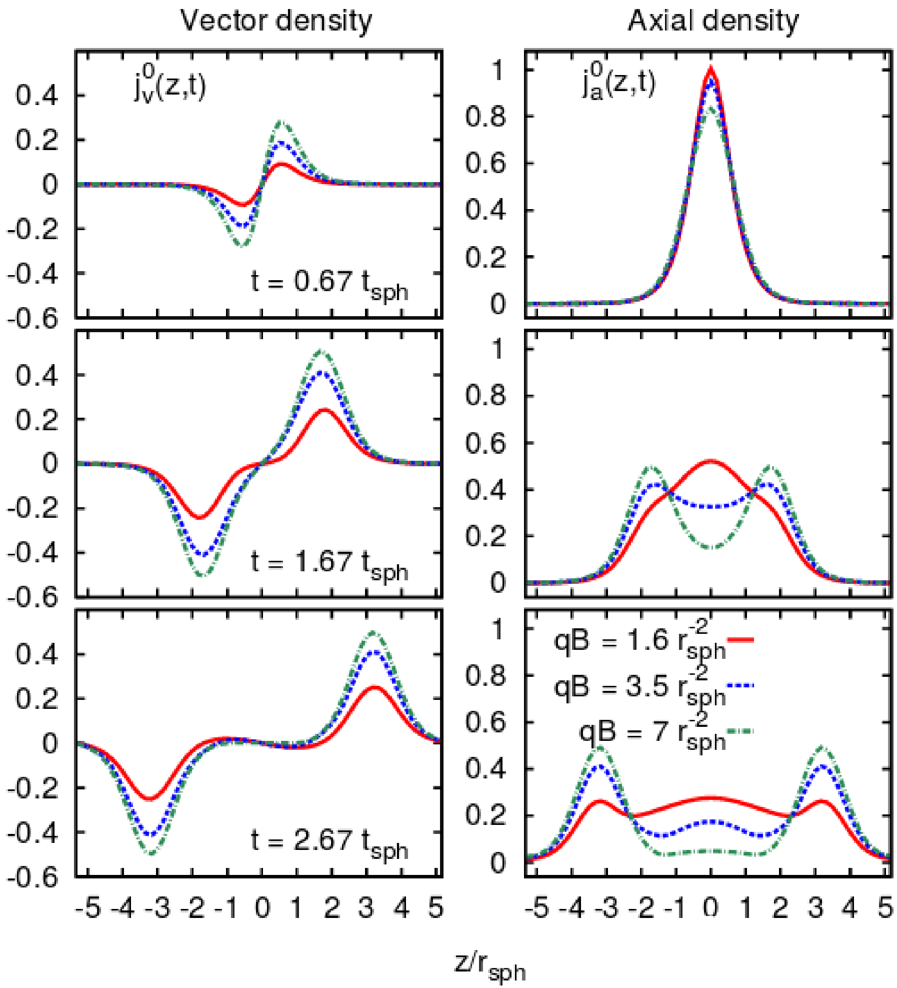

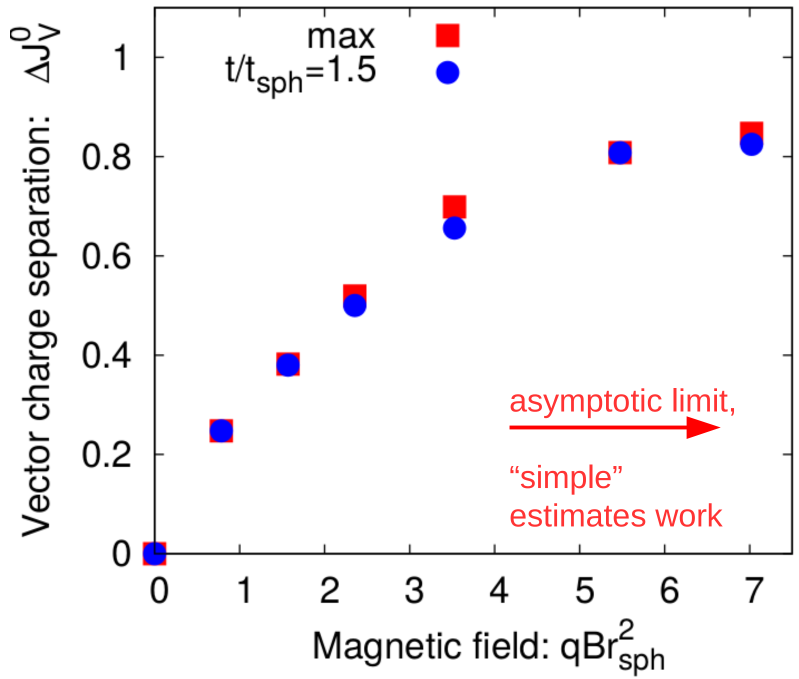

finite magnetic field:

→ important deviations from 'ideal' picture of CME

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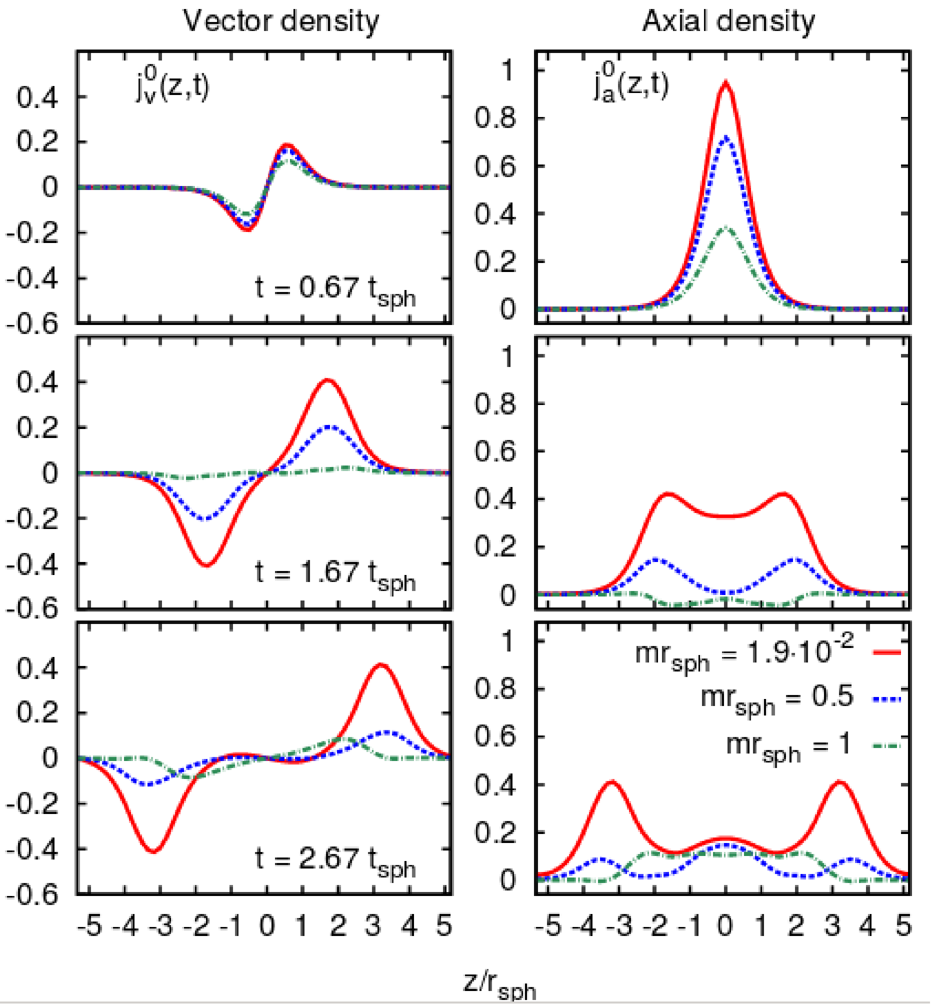
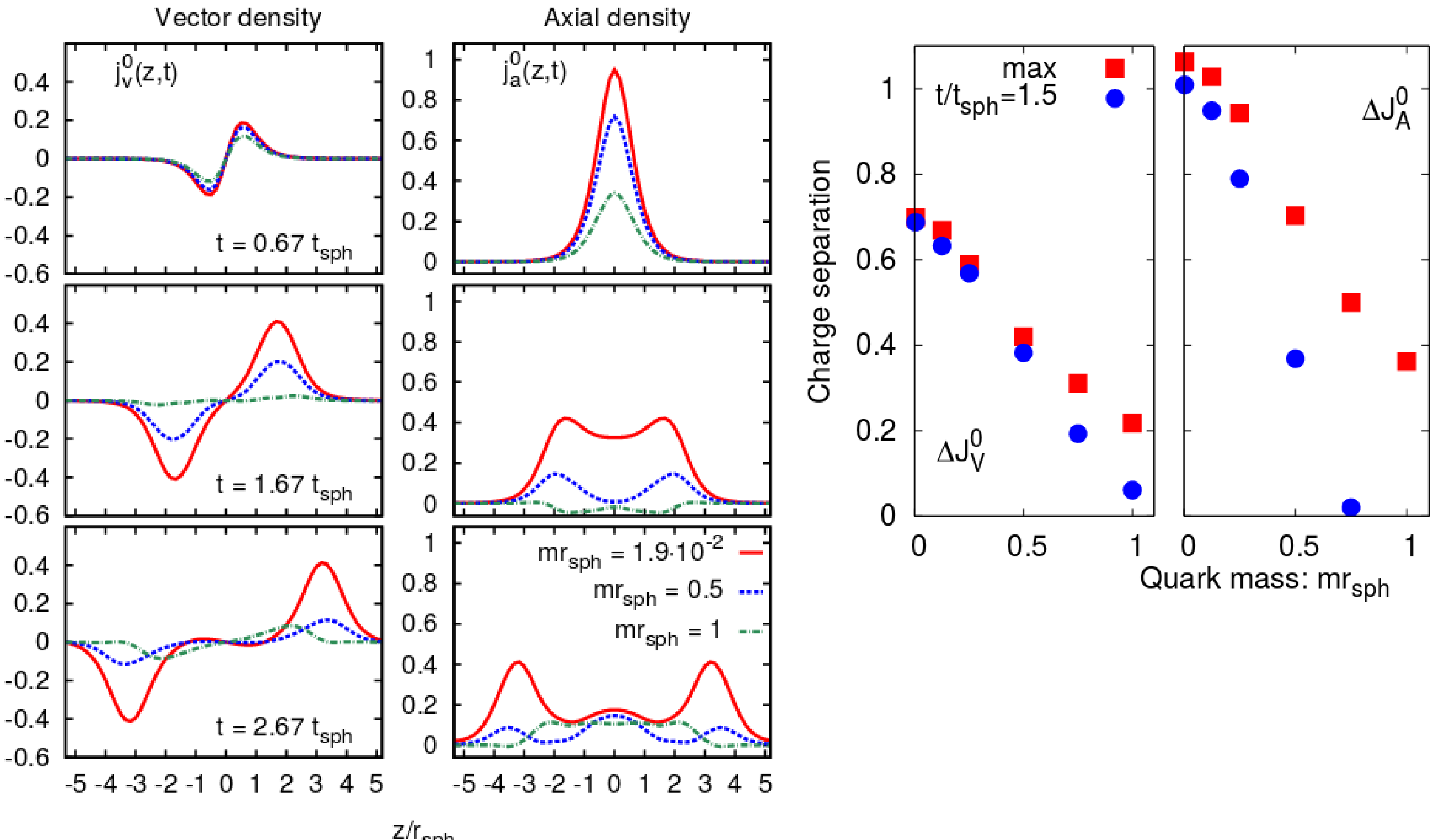
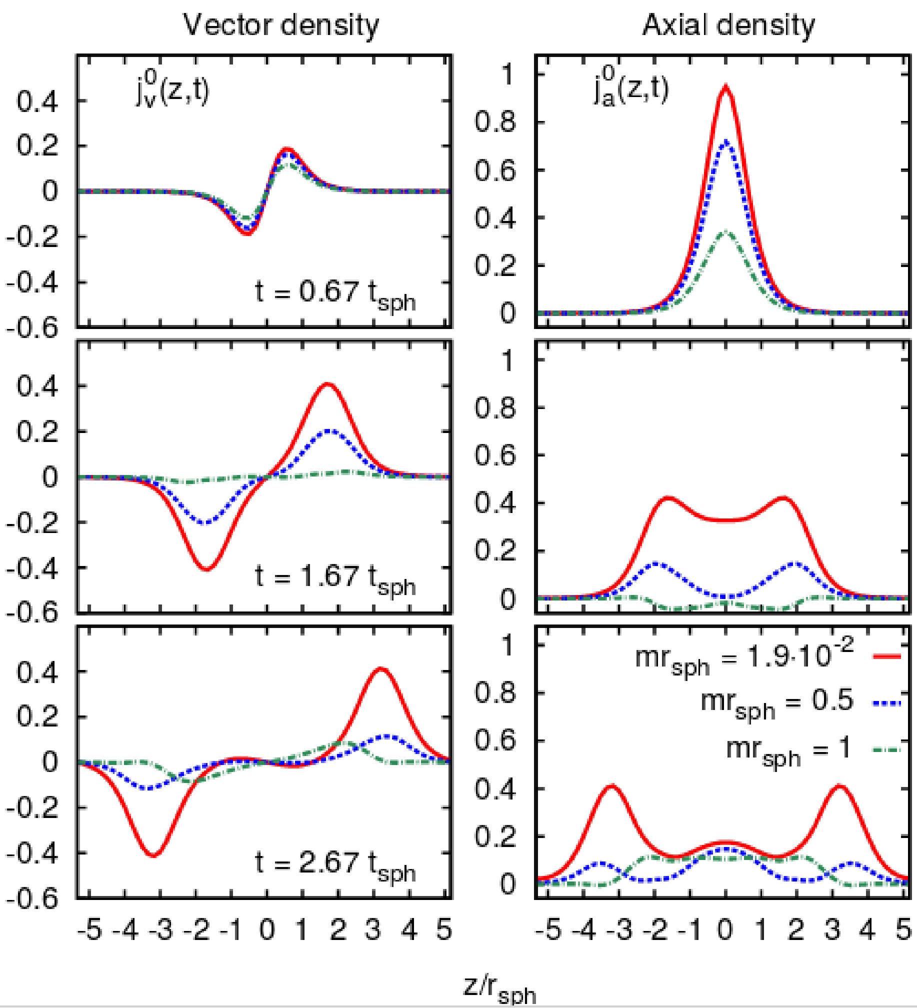
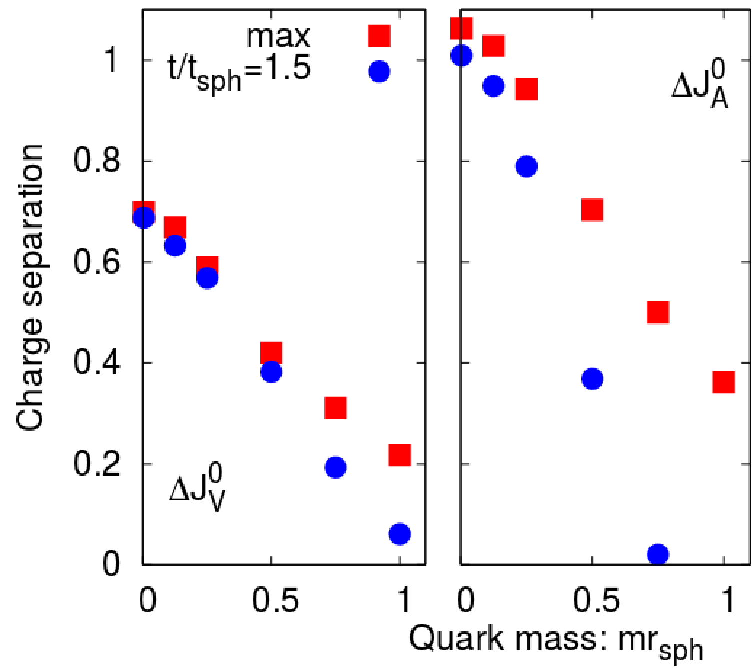



Vector density
Axial density
$j_V^0(z,t)$
$j_a^0(z,t)$
t = 0.67 t_{sph}
t = 1.67 t_{sph}
t = 2.67 t_{sph}
$mr_{sph} = 1.9 \cdot 10^{-2}$
$mr_{sph} = 0.5$
$mr_{sph} = 1$
z/r_{sph}
max
t/t_{sph}=1.5
Charge separation
ΔJ_V^0
ΔJ_A^0
Quark mass: mr_{sph}





finite quark mass effects
→ anomalous transport
suppressed for heavy quarks

Simulating chiral fermions in real-time: *Overlap fermions*

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Lattice simulations:
exactly chiral fermions challenging
(at least in strong coupling regime)

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Simulating chiral fermions in real-time: *Overlap fermions*

Lattice simulations:
exactly chiral fermions challenging
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- in the classical-statistical limit **mass renormalized perturbatively**
  → Wilson fermions arbitrary close to massless limit

Simulating <u>chiral</u> fermions in real-time: *Overlap fermions*

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Benchmark:

**Wilson-fermions vs.
Overlap fermions**

- mass matters! chiral instabilities
  (Yamamoto, Akamatsu, Kaplan, Reddy, Sen, Dvornikov...)

- real-time evolutio beyond early time

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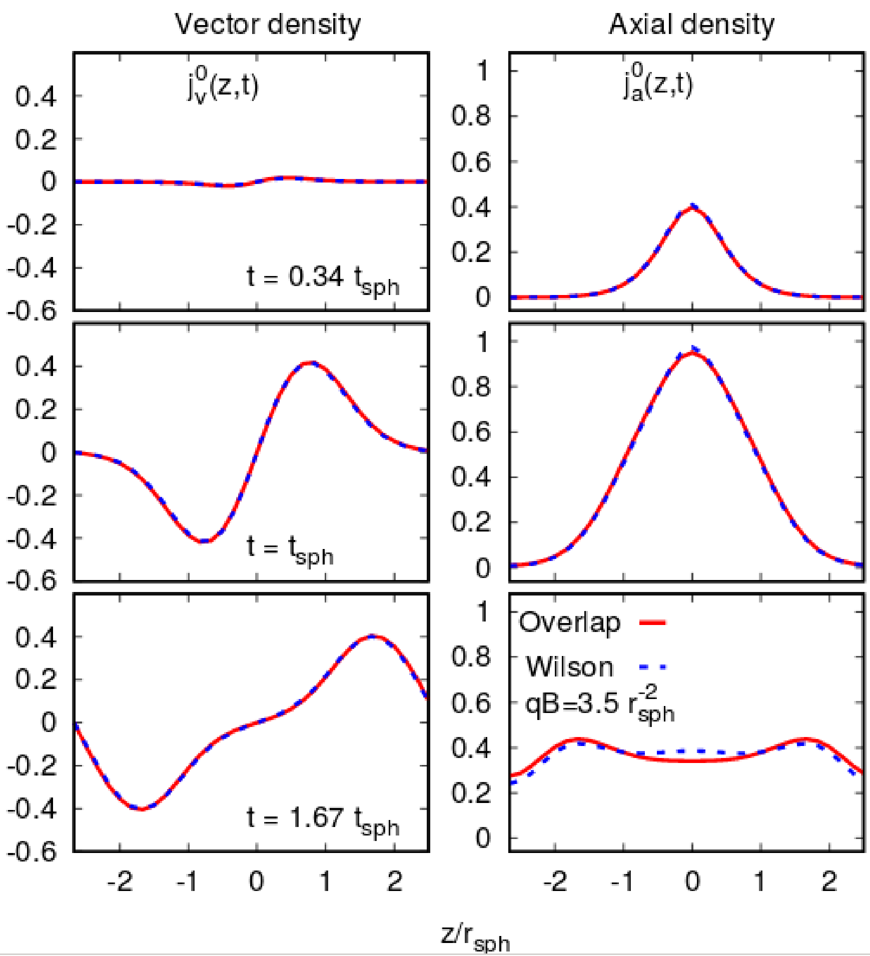

How this became possible – Technical breakthroughs

The challenge with fermions

How this became possible – Technical breakthroughs

The challenge with fermions

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Evolution of the fermion operators **extremely costly due to mode-function expansion** (cost with lattice size: N^6 for fermions vs. N^3 for 'YM only')

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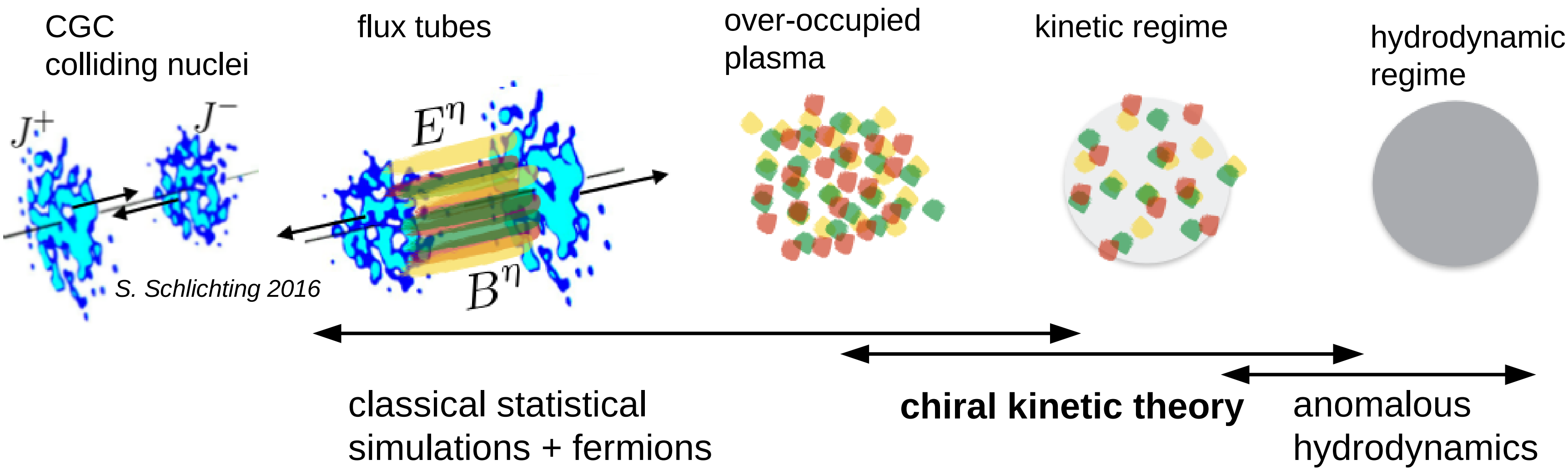
Computational Ressources: NERSC SCC

Check *arXiv:1612.02477* for the current state-of-art!

4. Relativistic Chiral Kinetic Theory

The limits of classical statistical simulations



4. Relativistic Chiral Kinetic Theory

The limits of classical statistical simulations

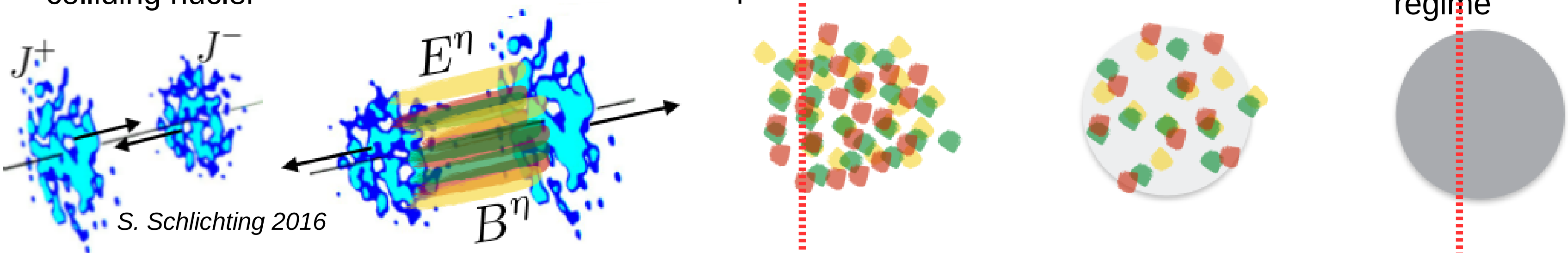
CGC
colliding nuclei

flux tubes

over-occupied
plasma

kinetic regime

hydrodynamic
regime



classical statistical
simulations + fermions

chiral kinetic theory

anomalous
hydrodynamics

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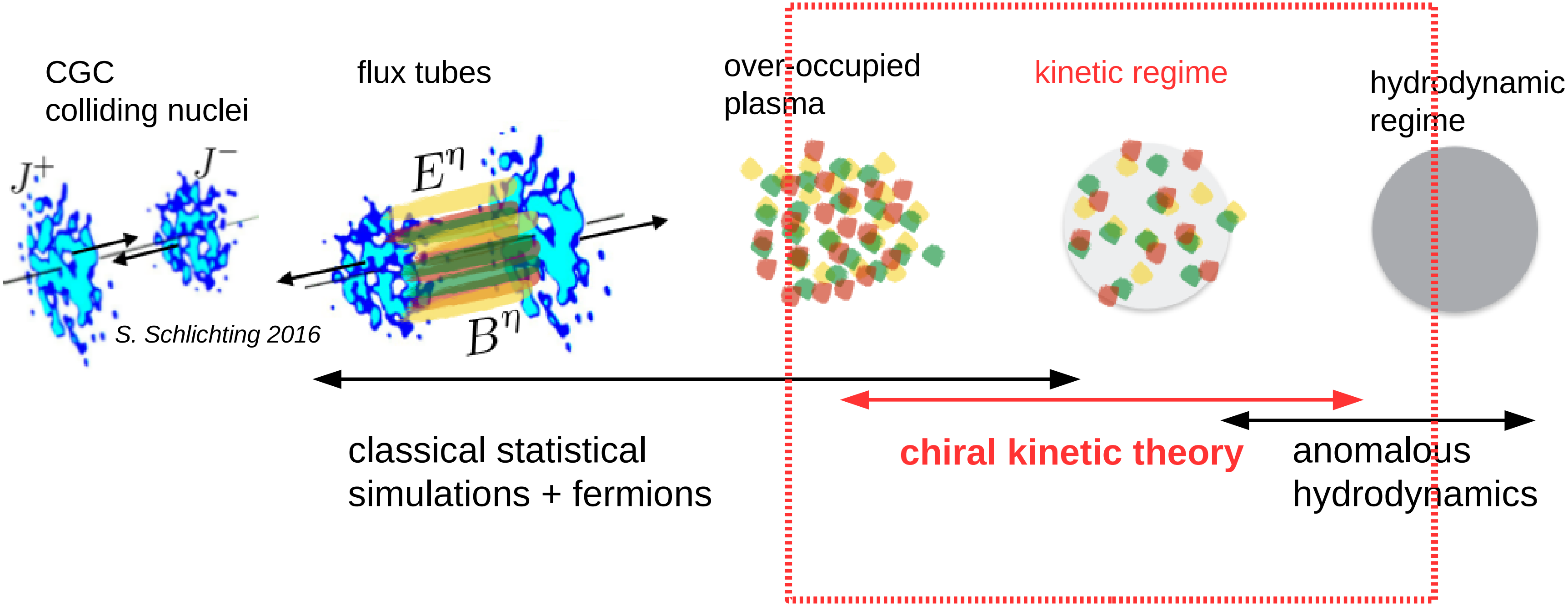
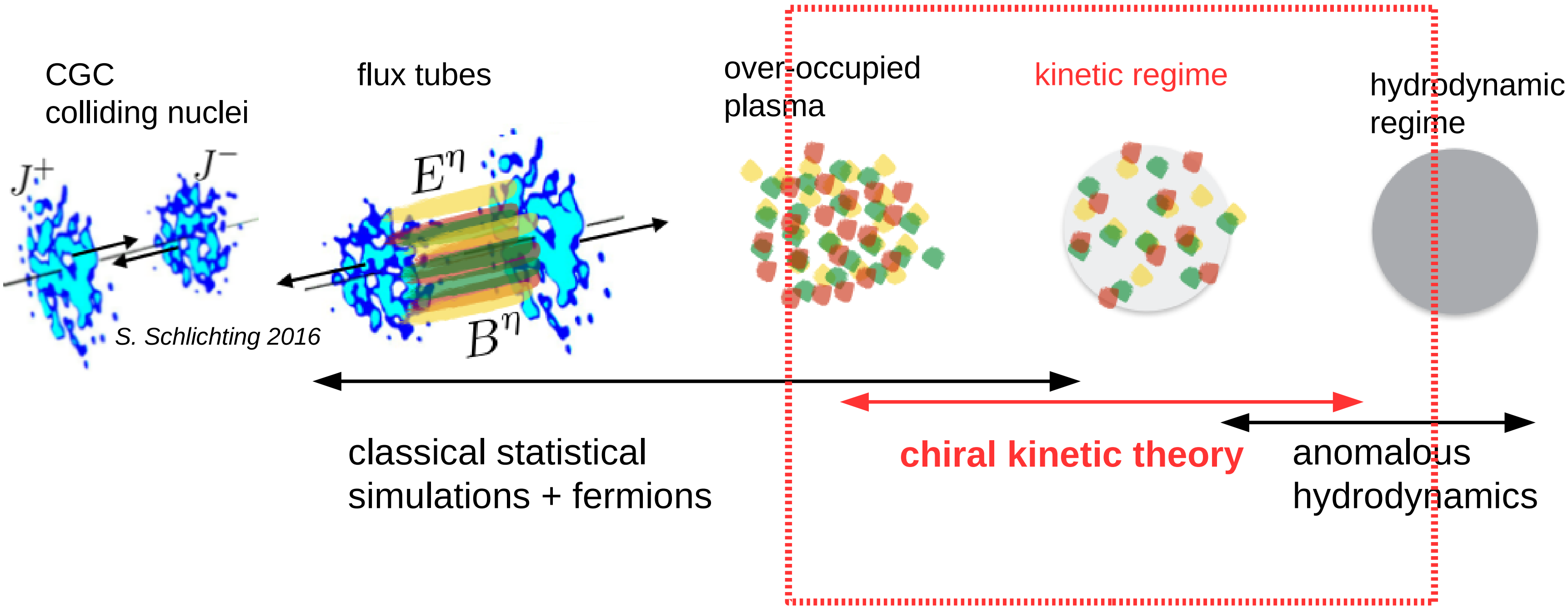
The limits of classical statistical simulations



classical-statistical simulations become inaccurate as occupation numbers drop
– **subsequent evolution of chiral and vector currents influenced by medium:**

The limits of classical statistical simulations



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– **subsequent evolution of chiral and vector currents influenced by medium:**

→ scattering of topological domains
→ axial transport
→ E&M conductivity

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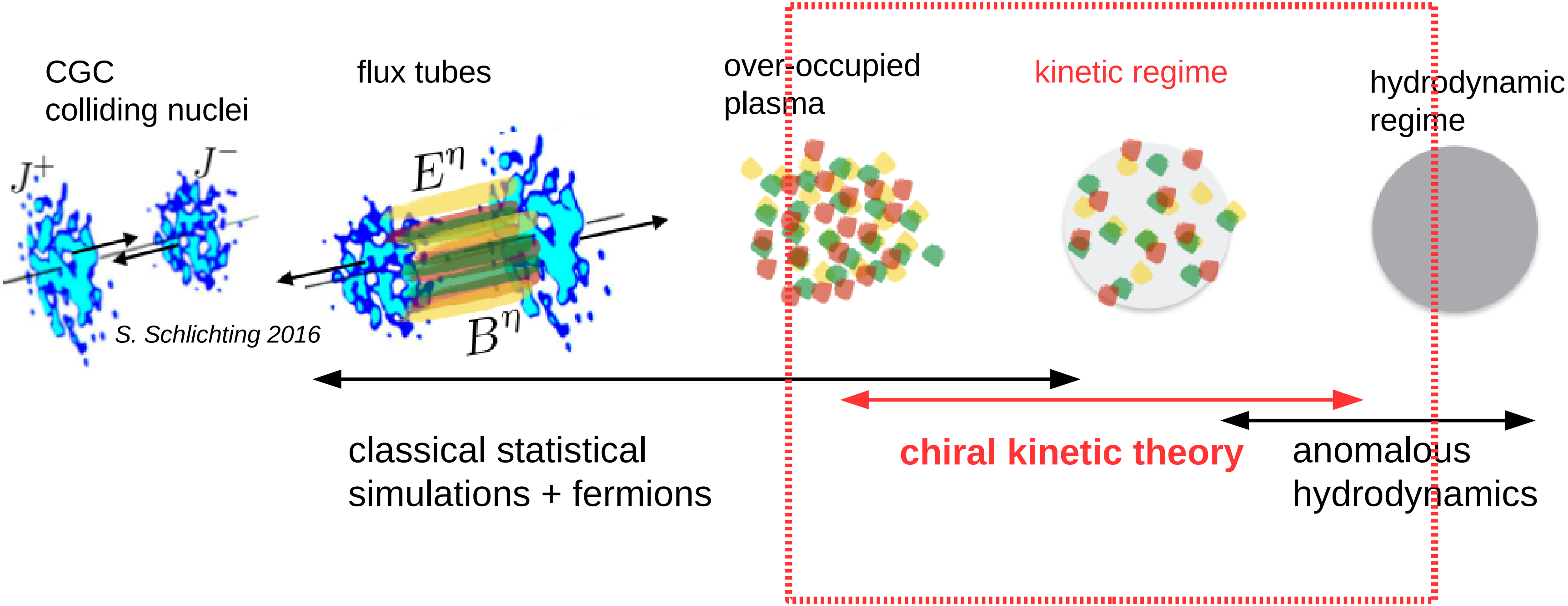

classical-statistical simulations become inaccurate as occupation numbers drop

– **subsequent evolution of chiral and vector currents influenced by medium:**

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Need to connect early time and hydrodynamic evolution!

→ talk by Jinfeng Liao

Chiral Kinetic Theory

Connection between Berry phase (adiabatic limit) and quantum anomalies

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Chiral Kinetic Theory

→ talk by Jinfeng Liao

Connection between Berry phase (adiabatic limit) and quantum anomalies

$$\dot{\mathbf{k}} = \mathbf{E} + \mathbf{v} \times \mathbf{B}$$

$$\dot{\mathbf{x}} = \frac{\partial \mathbf{H}}{\partial \mathbf{k}} - \dot{\mathbf{k}} \times \mathbf{\Omega}$$

massless fermions, large chemical potential

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4. Relativistic Chiral Kinetic Theory

→ talk by Jinfeng Liao

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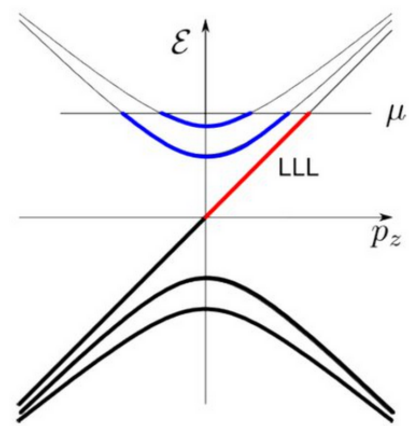
massless fermions, large chemical potential

- Absence of level crossings (adiabaticity) vs. anomaly?



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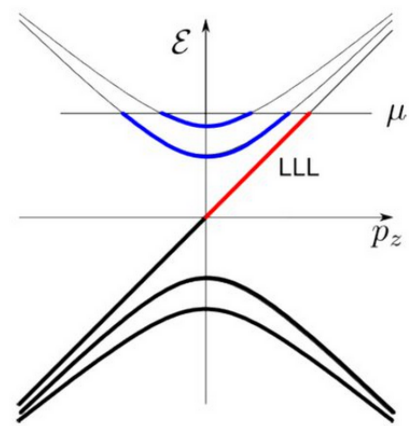
(Fujikawa 2005)



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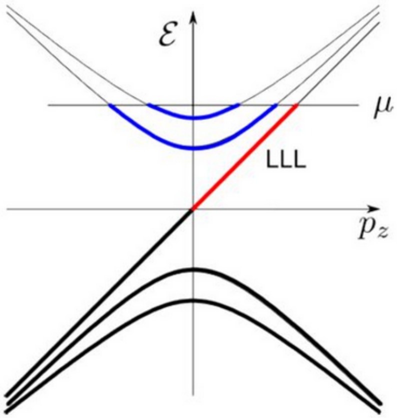

4. Relativistic Chiral Kinetic Theory

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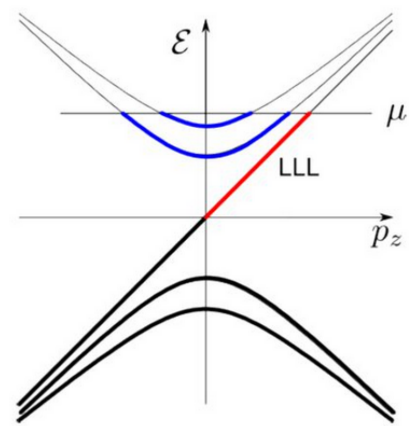
- Lorentz covariance (side jumps?)



Chiral Kinetic Theory – just a teaser

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How can this understanding be extended to first quantization ***(x,p,...)*** *– from first principles?*

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see NM, R. Venugopalan, arXiv:1701.03331 (+ longer paper very very soon);
and NM, R. Venugopalan, Y.Yin, in prep.

5. Conclusions

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- I have shown you real-time classical statistical simulations of fermion production during sphaleron transitions in background magnetic fields

- Axial anomaly realized in lattice simulations using Wilson fermions

- Chiral Magnetic and Chiral Separation Effect emerge dynamically

- Observation of the Chiral Magnetic Wave

- Have investigated finite mass and magnetic field dependence. Finite quark mass plays an important role: dissipation of anomalous currents

- Simulated chiral lattice fermions in real-time – overlap!

What's next?

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The techniques are established for the quantitative first principle simulation of far-from-equilibrium dynamics of fermions and gauge fields!

→ **Realistic heavy ion collision – CGC initial conditions & expansion**
(work in progress with J. Berges, M. Mace, S.Schlichting, S. Sharma, N. Tanji)

→ **chiral plasma instabilities**
(with M. Mace, S.Schlichting, S. Sharma)

→ provide **initial conditions for hydrodynamics**

→ **electromagnetic properties of the Glasma**, how long does the magnetic field live?

→ **connection to chiral kinetic theory & anomalous hydrodynamics**
(with R. Venugopalan, Y. Yin)

→ **non-thermal photons and the direct photon puzzle**
(with J. Berges, O. Garcia-Montero, N. Tanji)

Thanks for your attention
and thanks to

Backup

A. Classical Statistical Simulations

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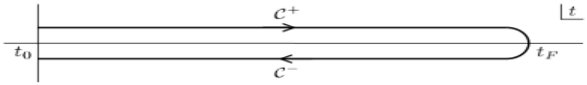
(see for example Kasper et al. Phys.Rev. D90 (2014) 2, 025016)



typical situation: large **coherent or highly occupied** gauge fields
→ correspondence principle

$$A^+_{\mu,n} = \bar{A}_{\mu,n} + \frac{1}{2}\tilde{A}_{\mu,n}\,,$$

$$A^-_{\mu,n} = \bar{A}_{\mu,\bar{n}} - \frac{1}{2}\tilde{A}_{\mu,\bar{n}}\,,$$

'classical' 'quantum'

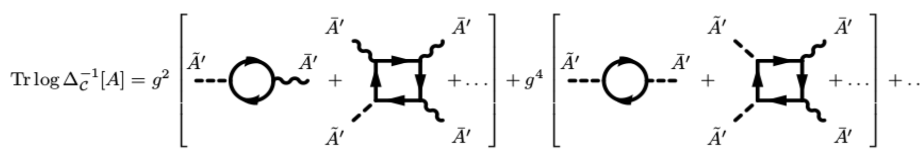
- initial stages of a heavy ion collision: A ~ 1/g
- colliding laser beams → large and coherent fields

The classical-statistical approximation is a systematic expansion of the 'quantum' fields around the 'classical' fields

Fermions are never "classical"

$$Z_\mathcal{C} = \int [dA]\,\rho_G(A)\,\exp\left(\mathrm{Tr}\log\Delta_\mathcal{C}[A]^{-1} + iS_G\right)$$

$$\boxed{i\gamma^0\partial_t\hat{\psi} = (-i\not{D}^s_W + m)\hat{\psi}}$$



Exact description via **modefunctions** up to **24x24x64** lattices

B. Backup
Algorithmic Improvements

Fermions: Exact description via **modefunctions** up to 24x24x64 lattices

Improved operators (NM, S.Schlichting, S.Sharma, **arXiv:1606.00342, arXiv:1612.02477**)

We use a tree-level improved version of the lattice Hamiltonian, which takes the form

$$H = \sum_x \psi_x^\dagger m \gamma^0 \psi \quad + \quad \frac{1}{2} \sum_{n,x,i} C_n \psi_x^\dagger \gamma^0 \Big[\Big(-i\gamma^i - n r_w \Big) U_{x,+ni} \psi_{x+ni} + 2 n r_w \psi_x - \Big(-i\gamma^i + n r_w \Big) U_{x,-ni} \psi_{x-ni} \Big]$$

where r_w denotes the Wilson coefficient, the coefficients C_n are chosen to optimize the convergence, and we introduce the following short hand notation for the connecting gauge links

$$U_{x,+ni} = \prod_{k=0}^{n-1} U_{x+ki,i} \;, \qquad U_{x,-ni} = \prod_{k=1}^{n} U^\dagger_{x-ki,i} \tag{1}$$

Wilson-averaging (M.Mace, NM, S.Schlichting, S.Sharma, *arXiv:1612.02477*)

→ improvement of chiral properties
→ extremely important for larger fermion masses
→ average fermionic observables over Wilson parameters with opposite sign
→ leading order errors in the anomaly equation cancel

Backup
C. Magnetic Fields on the lattice

Magnetic fields break translation invariance → magnetic translation group

- Magnetic fields on a torus very non-trivial
(see Al-Hashimi & Wiese "Accidental Symmetries", also Bali et al.)

$$U_{y,n} = \mathrm{e}^{\mathrm{i}a^2 qBn_x} \;;\; U_{x,N_x-1,n_y,n_z} = \mathrm{e}^{-\mathrm{i}a^2 qBN_x n_y}$$
$$U_{x,n} = \mathbf{1}, n_x \neq N_x - 1; U_{z,n} = \mathbf{1}$$

- Intriguing lattice artefacts!

→ spoil the low-cost method
--- while there probably are field configurations where low-cost works,
this is certainly not the case in magnetic fields

Backup
D. Anomaly Realization on the Lattice

Chiral Symmetry + Fermion doubling + Chiral Anomaly
= “one of the prettiest connections I have ever seen”

Lattice Fermions: Species Doubling, Chiral Invariance, and the Triangle Anomaly

Luuk H. Karsten (Stanford U., ITP) , Jan Smit (Amsterdam U.)

Sep 1980 - 38 pages

Nucl.Phys. B183 (1981) 103

In *Rebbi, C. (Ed.): Lattice Gauge Theories and Monte Carlo Simulations*, 495-532. (Nucl. Phys. B183 (1981) 103-140) and Stanford Univ. - ITP-677 (80,REC.NOV.) 71p (1981)

DOI: 10.1016/0550-3213(81)90549-6

Conference: C81-06-01, p.495-532

Proceedings

ITP-677-STANFORD

Contributions

- The axial anomaly and the fermion doubling problem are intimately related

- Lattice theory regularized on the basis of the action already

- Anomaly comes from the non-trivial continuum limit of any regulator you put in to remove doublers