Renormalization Issues on Long-Link Operators

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IN THIS TALK

A. **Motivation**

B. **Introduction to quasi-PDFs**

C. **Perturbative Renormalization**

D. **Linear Divergence**

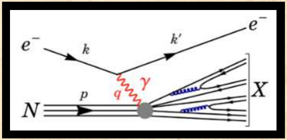
E. **Non-perturbative Renormalization**

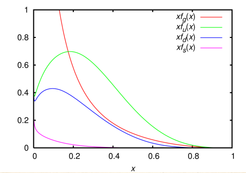
F. **Summary and Prospects**

A

MOTIVATION

Probing Nucleon Structure





CTEQ6 PDFs

Generalized Parton Distribution Functions

- **Comprehensive description of hadron structure**
- **Deep inelastic scattering (DIS) of leptons off nucleons**
- **necessary for the analysis of scattering data**
- **Parametrization of off-forward matrix of a bilocal quark operator (light-like)**

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda}\langle p'|\bar{\psi}(-\lambda n/2)\,\mathcal{O}\,\underbrace{\mathcal{P}e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}}\psi(\lambda n/2)|p\rangle$$

$q = p' - p$, $\bar{P} = (p' + p)/2$, n: **light-cone vector** ($\bar{P}.n = 1$), $\xi = -n \cdot \Delta/2$

PDFs:

powerful tool to describe the structure of a nucleon

Thus:

- ★ **first principle calculations of quark distributions are necessary**
  - **crucial test of QCD**
- ★ **Until recently direct lattice calculation inaccessible**
- ★ **On the lattice: moments of PDFs**

$$\boxed{f^n = \int_{-1}^{1} dx\, x^n f(x)}$$

★ **moments related to local operators**

A Unpolarized

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar{q}\,\gamma^{\{\mu_1}\, iD^{\mu_2}\ldots iD^{\mu_n\}}q$$

DIS, Drell-Yan, W-asymmetry, γ^+ jet, ...

B Helicity (polarized)

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar{q}\,\gamma_5\gamma^{\{\mu_1}\, iD^{\mu_2}\ldots iD^{\mu_n\}}q$$

polarized DIS, SIDIS, pp collisions,photo/electro production, ...

C Transversity

$$\mathcal{O}^{\mu_1\cdots\mu_{n+1}} = \bar{q}\,\sigma^{\mu_1\,\{\mu_2}\, iD^{\mu_3}\ldots iD^{\mu_{n+1}\}}q$$

single-spin asymmetry in SIDIS, ...

★ **rely on OPE to reconstruct the PDFs**

However, **reconstruction of PDFs seems unfeasible:**

- ★ **signal-to-noise is bad for higher moments**
- ★ **n $>$ 3: operator mixing (unavoidable!)**
- ★ **gluon moments: limited progress**

  (discon. diagram, signal quality, operator mixing)

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Novel direct approach [X.Ji, arXiv:1305.1539]

★ **compute a P*quasi*-DF (accessible on the lattice)**

★ **contact with physical PDFs via a matching procedure**

Talk by J. Zhang, this session

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Currently exploratory studies:

- ● [H-W. Lin et al., arXiv:1402.1462], [Jiunn-Wei Chen et al., arXiv:1603.06664]
- ● [C.Alexandrou et al., arXiv:1504.07455], [C.Alexandrou et al., arXiv:1610.03689]

B

INTRODUCTION

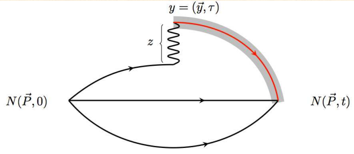
TO QUASI-PDFS

Access of PDFs on Euclidean lattice

- ★ **rest frame: parton physics correspond to light-cone correlation BUT:**
- ★ **same physics obtained from t-independent spatial correlation in the IMF**
- ★ **P*quasi*-DF ($\tilde{q}$) purely spatial for nucleons with finite momentum**

$$\boxed{\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)|\bar{\Psi}(z)\,\gamma^z\,\mathcal{A}(z,0)\Psi(0)|N(P_3)\rangle_{\mu^2}}$$

- $\mathcal{A}(z,0)$: **Wilson line from $0 \rightarrow z$**
- z: **distance in any spatial direction (momentum boost in z direction)**



- ★ **At finite but feasibly large momenta on the lattice:**

  a large momentum EFT can relate Euclidean $\tilde{q}$ to PDFs through a factorization theorem
- ★ **use of Perturbation Theory for the matching**
- ★ **Computation is difficult and costly**

C

PERTURBATIVE RENORMALIZATION

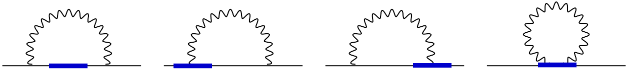
Perturbative Calculation

★ **Definition of Operator**

$$\mathcal{O}^{\mu}_{\Gamma} \equiv \overline{\psi}(x)\,\Gamma\,\mathcal{P}\,e^{i\,g\,\int_0^z A(\zeta)d\zeta}\,\psi(x+z\hat{\mu})$$

$$\Gamma = \hat{1}, \quad \gamma^5, \quad \gamma^i, \quad \gamma^i\,\gamma^5, \quad \gamma^5\,\sigma^{ij}, \quad \sigma^{ij}$$

★ **Feynman Diagrams**



★ **Strategy:**

Perturbative Calculation

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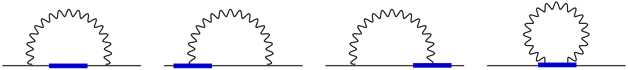

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A **Perform calculation in Dimensional Regularization (DR)**

Perturbative Calculation

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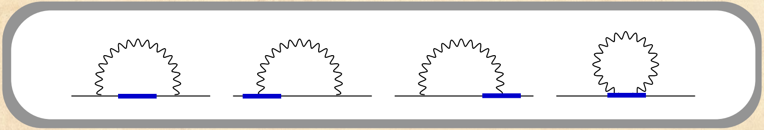

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- **Compute $Z^{DR,\overline{\rm MS}}$ from the poles (& anomalous dimension)**

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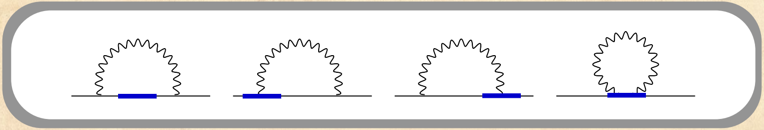

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B **Compute Feynman diagrams in Lattice Regularization (LR)**

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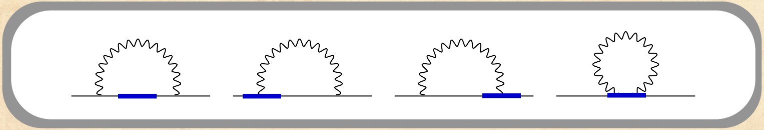


★ Strategy:

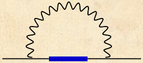
A **Perform calculation in Dimensional Regularization (DR)**

- **Compute $Z^{DR,\overline{\rm MS}}$ from the poles (& anomalous dimension)**

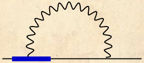
B **Compute Feynman diagrams in Lattice Regularization (LR)**

- **Extract $Z^{LR,\overline{\rm MS}}$ using the difference between DR and LR**

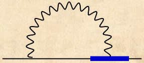
A. Dimensional Regularization



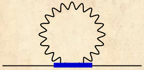
d1



d2



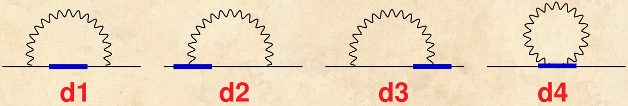
d3



d4

- **No linear divergence**

A. Dimensional Regularization



- **No linear divergence**
- **Poles:** $\Lambda_{\mathcal{O}}^{d1}\Big|_{\frac{1}{\epsilon}}=0$ $\quad \Lambda_{\mathcal{O}}^{d2+d3+d4}\Big|_{\frac{1}{\epsilon}}=\frac{g^2\,C_f}{16\,\pi^2}\,\frac{1}{\epsilon}\,(4-\beta)\,\Lambda_{\mathcal{O}}^{tree}$
- **Thus, $\overline{\rm MS}$-scheme:**

$$\boxed{Z_{\mathcal{O}}^{DR,\,\overline{\rm MS}}=1-\frac{3}{\epsilon}\,\frac{g^2\,C_f}{16\,\pi^2}}$$

real function

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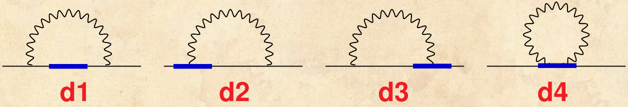

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- **RI-scheme:**

$$Z_{\mathcal{O}}^{DR,RI}\;Z_{\psi}^{-1}\,\frac{1}{12}\,\mathrm{Tr}\Big[\Lambda_{\mathcal{O}}^{1-loop}\,\Lambda_{\mathcal{O}}^{tree}\Big] = 1 \qquad Z_{\psi} = \frac{1}{12}\,\mathrm{Tr}\Big[S^{1-loop}\,\big(S^{tree}\big)^{-1}\Big]$$

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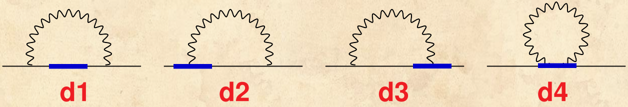

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- RI**-scheme:**

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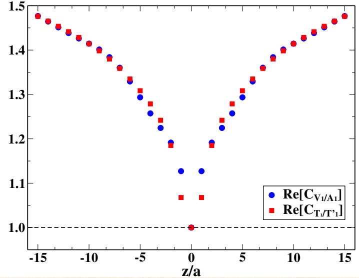
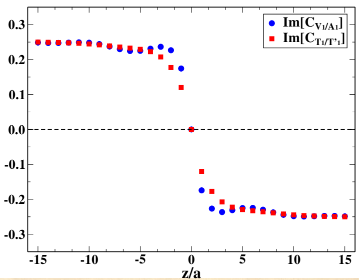
- **Complex Matrix Element** $\Longrightarrow Z_{\mathcal{O}}^{DR,RI}$: **Complex function**

Conversion Factor: RI to $\overline{MS}$

$$\mathcal{C}_{\mathcal{O}} = \frac{Z_{\mathcal{O}}^{DR,RI}}{Z_{\mathcal{O}}^{DR,\overline{\mathrm{MS}}}}$$





- $C_{\mathcal{O}}^{RI,\overline{\mathrm{MS}}}$: **Complex function**

★ **Gauge dependent (Non-gauge invariant external states)**

★ **Necessary ingredient for non-perturbative renormalization**

- **Applicable for alternative definitions of RI-type schemes**

Renormalized Green's functions

- **do not depend on the regularization choice**
- **dependence on $\bar{\mu}$ matches $\bar{\mu}$-dependence of the Z-function**

$$\boxed{\Lambda_{\mathcal{O}}^{1-\mathrm{loop}}\Big|_{q=q_\mu} = \Lambda_{\mathcal{O}}^{\mathrm{tree}} \left(\frac{\bar{\mu}^2}{q^2}\right)^{\left((4-\beta)\frac{g^2\,C_f}{16\,\pi^2}\right)} \left[1 + \frac{g^2\,C_f}{16\,\pi^2} F_{\mathcal{O}}(qz)\right]}$$

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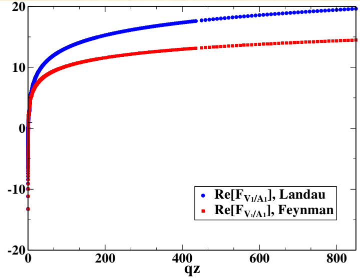
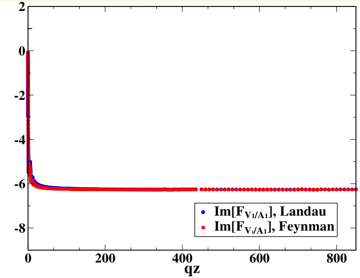
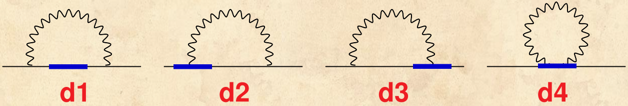




$$\lim_{q\to 0} F_{V_1(A_1)}(qz) = 3\left(1+\gamma_E+\log\left(\frac{qz}{2}\right)\right)$$

B. Lattice Regularization



★ **Linear divergence from tadpole diagram (d4):** $\propto |z|/a$

★ **Expected from Dotsenko & Vergeles:** [Nucl. Phys. B169 (1980) 527]

$$\Lambda_{\mathcal{O}_\Gamma} = e^{-c\,\frac{|z|}{a}}\Lambda^R\,, \quad c \sim 1$$

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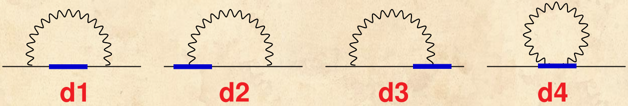

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★ **Condition to extract $Z_{\mathcal{O}}^{LR,\overline{\rm MS}}$ using DR results:**

$$\boxed{\langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{DR,\,\overline{\rm MS}}_{\rm amp} - \langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{LR}_{\rm amp} = \frac{g^2\,C_f}{16\,\pi^2}\,e^{i\,q_\mu\,z} \;\times\mathcal{F}}$$

- **Green's functions complicated functions of external momentum BUT**
- $\mathcal{F}$**: 0th degree polynomial in external momentum** $\Longrightarrow$
- **extraction of $Z_{\mathcal{O}}^{LR,\overline{\rm MS}}$ without intermediate RI-type scheme**

Results

$$\mathcal{F} = \Gamma\left(c_1 + c_2\,\beta + c_3\,\frac{|z|}{a} + \log\left(a^2\bar{\mu}^2\right)(4-\beta)\right) + (\Gamma\cdot\gamma_\mu + \gamma_\mu\cdot\Gamma)\,\left(c_4 + c_5\,c_{\rm SW}\right)$$

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linear divergence

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linear divergence

mixing term

Consequenses:

★ **Vector mixes with Scalar if γ^μ in same direction as Wilson line**

★ **HOWEVER for clover action at $c_{\mathrm{SW}} = -c_4/c_5$: mixing vanishes**

★ **Polarized $\&$ Transversity do not exhibit mixing (presence of γ^5)**

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★ **Polarized $\&$ Transversity do not exhibit mixing (presence of γ^5)**

Interestingly:

★ **Vector with γ^μ perpendicular to the Wilson line does not mix**

Note that all above are valid to 1-loop Perturbation Theory

Consequently:

$$Z_{\mathcal{O}}^{LR,\overline{\rm MS}} = 1 + \frac{g^2\,C_f}{16\,\pi^2}\,\Big(e_1 + e_2\,\frac{|z|}{a} + e_3\,c_{\rm SW} + e_4\,c_{\rm SW}^2 - 3\log\left(a^2\bar{\mu}^2\right)\Big)$$

$$\star\; Z_{mix}^{LR,\overline{\rm MS}} = 0 + \frac{g^2\,C_f}{16\,\pi^2}\,\left(e_5 + e_6\,c_{\rm SW}\right)$$

★ **Wherever mixing occurs**

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⋆ **Wherever mixing occurs**

Action	e_1	e_2	e_3	e_4	e_5	e_6
Wilson	**24.3063**	**-19.9548**	**-2.24887**	**-1.39727**	**14.4499**	**-8.28467**
Iwasaki	**12.5576**	**-12.9781**	**-1.60101**	**-0.97321**	**9.93653**	**-6.52764**

- **Results available for other gluonic actions**

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- **Results available for other gluonic actions**

★ e_5 & e_6 **have opposite signs** $\Longrightarrow$

Mixing vanishes at a positive value of c_{SW}^0

(realistic for simulations)

Wilson: $c_{\mathrm{SW}}^0 \sim 1.74$**,** **Iwasaki:** $c_{\mathrm{SW}}^0 \sim 1.52$

D

LINEAR DIVERGENCE

Linear Divergence

$$\frac{\Delta h(P_3,z)}{\Delta h(P'_3,z')}=\frac{e^{-c\frac{|z|}{a}}\,Z^{-1}_{\Delta h}(a\bar{\mu})\left(\frac{P_3}{\bar{\mu}}\right)^{2\gamma\Delta h}\,\Delta h^R(P_3 z)}{e^{-c\frac{|z'|}{a}}\,Z^{-1}_{\Delta h}(a\bar{\mu})\left(\frac{P'_3}{\bar{\mu}}\right)^{2\gamma\Delta h}\,\Delta h^R(P'_3 z')}=e^{-c\frac{(|z|-|z'|)}{a}}\left(\frac{P_3}{P'_3}\right)^{-6\frac{g^2C_f}{16\pi^2}}$$

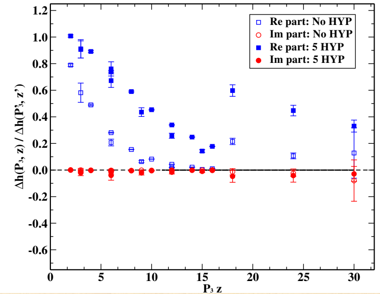
Ratio: real function

Linear Divergence

$$\frac{\Delta h(P_3,z)}{\Delta h(P'_3,z')} = \frac{e^{-c\frac{|z|}{a}}\ Z^{-1}_{\Delta h}(a\bar{\mu})\left(\frac{P_3}{\bar{\mu}}\right)^{2\gamma\Delta h}\ \Delta h^R(P_3 z)}{e^{-c\frac{|z'|}{a}}\ Z^{-1}_{\Delta h}(a\bar{\mu})\left(\frac{P'_3}{\bar{\mu}}\right)^{2\gamma\Delta h}\ \Delta h^R(P'_3 z')} = e^{-c\frac{(|z|-|z'|)}{a}}\left(\frac{P_3}{P'_3}\right)^{-6\frac{g^2 C_f}{16\pi^2}}$$

Ratio: real function

Polarized



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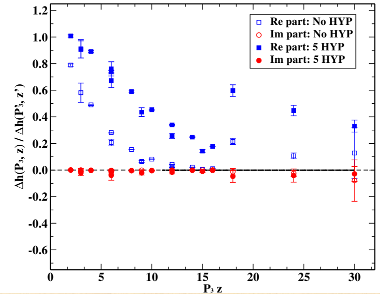
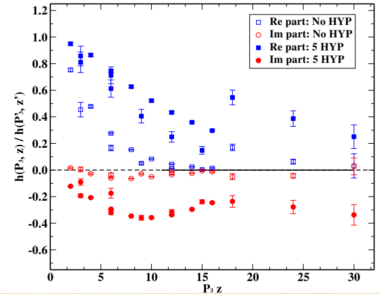


Unpolarized



★ **Method not applicable for the unpolarized (mixing with scalar)**

★ **Transversity: similar behavior with the helicity (zero Im part)**

Linear Divergence

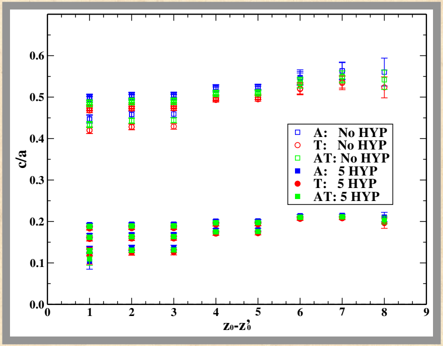
- Suggested ratio: a function of $z - z'$ and P_3/P'_3
- $N_f=2+1+1$ Twisted Mass fermions, $m_\pi=375$MeV
- Non-perturbative data for momenta: $P_3=\frac{2\pi}{L}n, \quad n=1,2,3$
- $32^3 \times 64$, $z\epsilon[0:15]$

$$\Downarrow$$

several fit options for c/a

- Data do not show dependence on operator (expected, 1-loop PT)

A: Axial
T: Tensor



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NON-PERTURBATIVE RENORMALIZATION

Non-perturbative Renormalization

- **Similar process as the renormalization of the local and covariant derivative currents**
- **Compute Z-factor on each value of z (length of Wilson Line)**

RI**-scheme:**

$$Z_{\mathcal{O}}^{DR,RI}\, Z_\psi^{-1}\, \frac{1}{12}\,\mathrm{Tr}\left[\mathcal{V}_{\mathcal{O}}^{1-loop}\, \mathcal{V}_{\mathcal{O}}^{tree}\right] = 1$$

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- ★ $Z_{\mathcal{O}}^{DR,RI}$ **includes the linear divergence**

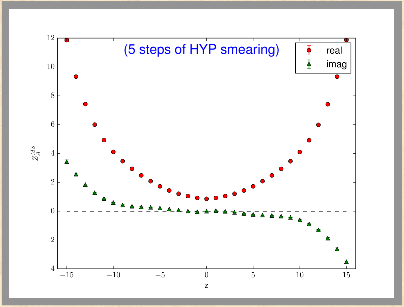
$$\boxed{Z_{\mathcal{O}}^{DR,RI} \equiv \mathcal{Z}_{\mathcal{O}}\, e^{-c\,\frac{|z|}{a}}}$$

  (The vertex function $\mathcal{V}$ has the same divergence as the nucleon matrix element)

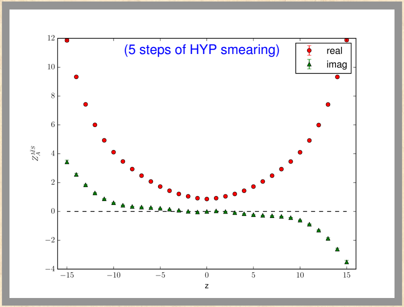
- ★ **Use conversion factor and anomalous dimension to convert to the $\overline{\mathrm{MS}}$ and evolve to 2 GeV.**

Numerical Results

For $z\!=\!0$ we get Z_A (local current)



Numerical Results



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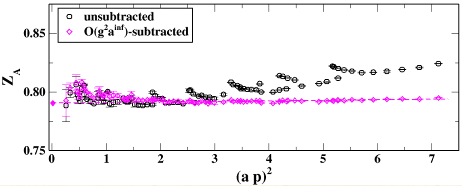
What's next?

- ★ In the process of renormalizing the nucleon matrix elements
- ★ Subtraction of lattice artifact (utilize our perturbative calculation)



[M. Constantinou et al., arXiv:1509.00213]

D

SUMMARY

PROSPECTS

SUMMARY & PROSPECTS

Progress in understanding the renormalization of quasi-PDFs

- ★ **Techniques to understand and remove linear divergence**
- ★ **Study of multiplicative renormalization**
  (perturbatively and non-perturbatively)
- ★ **Eliminate mixing where present using perturbation theory**

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Many more things to be done

- ★ **Apply smearing in perturbative calculation**
- ★ **Subtraction of lattice artifacts using perturbative results**
- ★ **Mixing elimination non-perturbatively**
- ★ **Investigation of cases with gamma matrix perpendicular to the Wilson line (to avoid mixing)**

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THANK YOU !