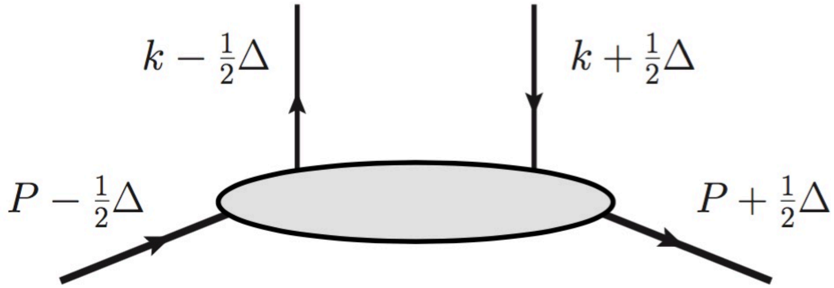
Generalized TMDs

(A. Metz, Temple University)

- Introduction and Motivation
- Recent Work on Observables for gluon GTMDs
- Quark GTMDs in Exclusive Double Drell-Yan Process
  (S. Bhattacharya, A. M., J. Zhou)
  - Leading-order diagrams and kinematics
  - Amplitude
  - Relevant quark GTMDs
  - Observables
- Summary and Outlook

Definition of GTMDs

- GTMD correlator: graphical representation



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- GTMD correlator: definition

$$W^{q\,[\Gamma]}_{\lambda,\lambda'}(P, \Delta, x, \vec{k}_\perp) = \int \frac{dz^- \, d^2\vec{z}_\perp}{2\,(2\pi)^3} \, e^{ik\cdot z} \, \langle p', \lambda' | \, \bar{q}(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \, q(\tfrac{z}{2}) \, | p, \lambda \rangle \Big|_{z^+=0}$$

– $W^{q\,[\Gamma]}_{\lambda,\lambda'}$ parameterized through GTMDs $X^q(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad \vec{k}_\perp \qquad \vec{\Delta}_\perp = \vec{p}\,'_\perp - \vec{p}_\perp$$

– proper definition and evolution of GTMDs very similar to TMD case
(Echevarria et al, 2016)

- Leading-twist chiral-even case (notation of Meissner, A. M., Schlegel, 2009)

$$W^{[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+}\,F_{1,2} + \frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}\,F_{1,4}\right]u(p,\lambda)$$

$$W^{[\gamma^+\gamma_5]}_{\lambda,\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[c_1\,G_{1,1} + c_2\,G_{1,2} + c_3\,G_{1,3} + c_4\,G_{1,4}\right]u(p,\lambda)$$

- Relation to GPDs and TMDs: examples

$$H(x,\xi=0,t) = \int d^2\vec{k}_\perp \mathrm{Re}\,F_{1,1}$$

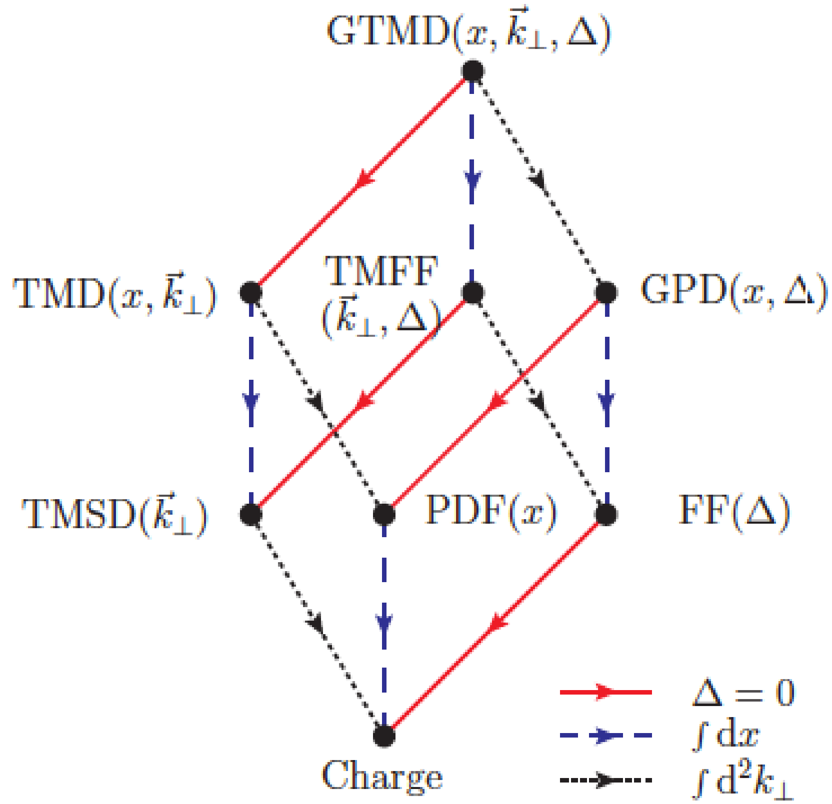
$$\tilde{H}(x,\xi=0,t) = \int d^2\vec{k}_\perp \mathrm{Re}\,G_{1,4}$$

$$f_1(x,\vec{k}_\perp^{\,2}) = \mathrm{Re}\,F_{1,1}\big|_{\Delta=0} \qquad g_1(x,\vec{k}_\perp^{\,2}) = \mathrm{Re}\,\mathrm{G}_{1,4}\big|_{\Delta=0}$$

$$f^\perp_{1T}(x,\vec{k}_\perp^{\,2}) = -\,\mathrm{Im}\,F_{1,2}\big|_{\Delta=0} \qquad g_{1T}(x,\vec{k}_\perp^{\,2}) = \mathrm{Re}\,G_{1,2}\big|_{\Delta=0}$$

  - GTMDs have real and imaginary part
  - $F_{1,1}$ and $G_{1,4}$ presumably large
  - later on, mainly relevant are: $F_{1,1},\ \ F_{1,4},\ \ G_{1,1},\ \ G_{1,4}$

GTMDs as Mother Functions



(diagram from Lorcé, Pasquini, Vanderhaeghen, 2011)

- GTMDs describe the most general two-parton structure of hadrons
- In particular, modeling of GTMDs might be very useful

Further Aspects/Applications of GTMDs

- Parton orbital angular momentum in longitudinally polarized nucleon
  (Lorcé, Pasquini, 2011 / Hatta, 2011 / Kanazawa et al, 2014 / Hägler, Mukherjee, Schäfer, 2003)

$$L^{q,g} = -\int dx\, d^2\vec{k}_\perp \, \frac{\vec{k}_\perp^2}{M^2} \, F_{1,4}^{q,g}(x, \vec{k}_\perp^{\,2})\big|_{\Delta=0}$$

  – same equation for both $L_{\rm JM}$ and $L_{\rm Ji}$
    (Ji, Xiong, Yuan, 2012 / Lorcé, 2013)

  – $L_{\rm JM}$ can be computed in Lattice QCD
    (Engelhardt, 2017 / Rajan, Courtoy, Engelhardt, Liuti, 2016)

- Spin-orbit couplings (Lorcé, Pasquini, 2011 / Lorcé, 2014)

$$F_{1,4} \longleftrightarrow \vec{S}_N \cdot \vec{L}_q \qquad\qquad G_{1,1} \longleftrightarrow \vec{S}_q \cdot \vec{L}_q$$

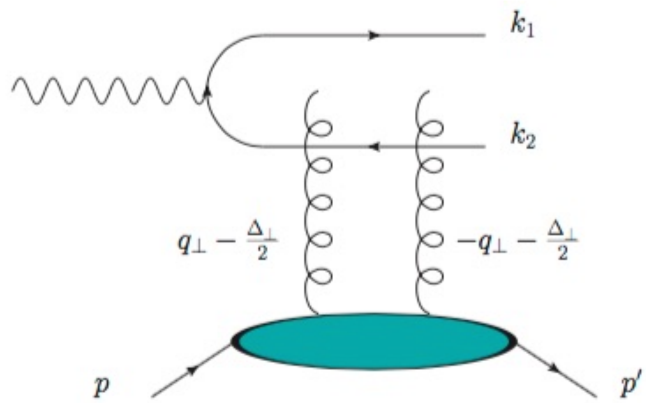
- Relation to Wigner phase space (quasi) distributions
  (Ji, 2003 / Belitsky, Ji, Yuan, 2003 / Lorcé, Pasquini, 2011 / ...)

$$\mathrm{WD}(x, \vec{k}_\perp, \vec{b}_\perp) \sim \int d^2\vec{\Delta}_\perp \, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \, \mathrm{GTMD}(x, \vec{k}_\perp, \vec{\Delta}_\perp)$$

  – in principle, 5-D imaging (but Wigner functions can become negative)

Recent Work on Observables for GTMDs

- Gluon GTMDs at small x through di-jet production in eA collisions
  (Hatta, Xiao, Yuan, 2016)



  – small-x formalism compatible with TMD factorization using GTMDs

- Longitudinal SSA in same process may give access to gluon OAM at small x
  (Hatta, Nakagawa, Yuan, Zhao, 2016)

- Longitudinal SSA in same process may give access to gluon OAM at intermediate x
  (Ji, Yuan, Zhao, 2016)
  – weighted cross section and collinear factorization

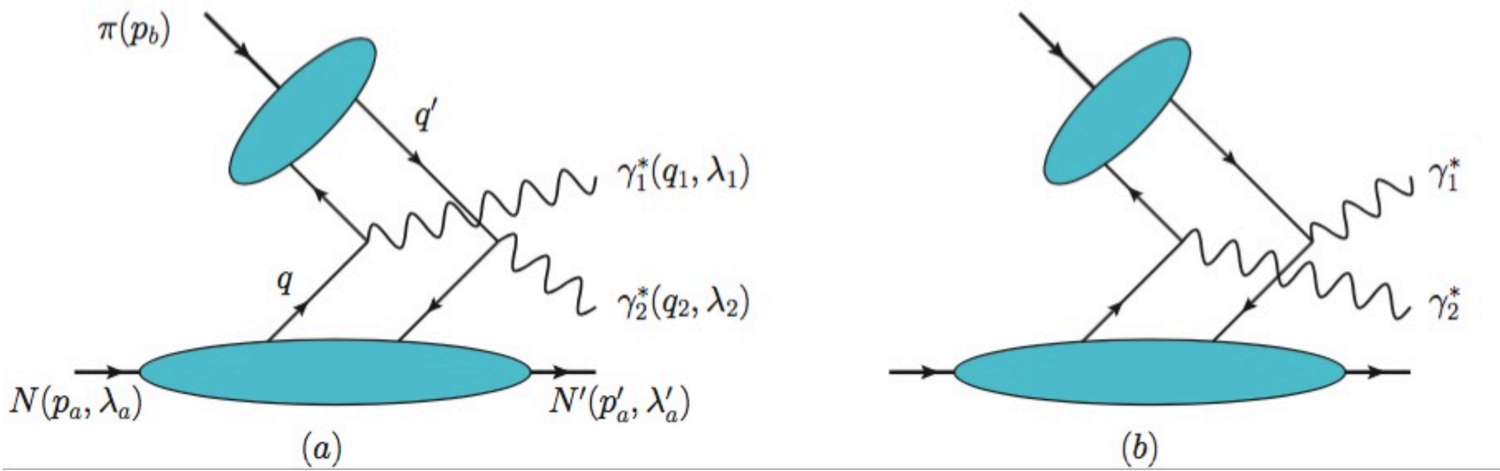
- Gluon GTMDs at small x in pA collisions exploiting double-parton scattering
  (Hagiwara, Hatta, Xiao, Yuan, 2017)

- Earlier work using GTMDs in spirit of Generalized Parton Model

Exclusive double Drell-Yan: $\pi\, N \to (\ell_1^- \ell_1^+)\,(\ell_2^- \ell_2^+)\, N'$

(Bhattacharya, AM, Zhou)

1. Leading-order diagrams and kinematics



- Consider all possible charge states
- Two graphs: amplitude symmetric under exchange $\gamma_1^* \longleftrightarrow \gamma_2^*$
- Kinematics of interest (TMD-type)

$$s = (p_a + p_b)^2 \ \text{large} \qquad q_1^2,\ q_2^2 \ \text{large} \qquad |\vec{q}_{i\perp}^{\,2}| \ll q_i^2$$

$$\xi_a = \frac{q_1^+ + q_2^+}{2P_a^+} \ \text{cannot be too small}$$

2. Amplitude

$$\mathcal{T}^{\lambda_1,\lambda_2}_{\lambda_a,\lambda_a'} = \mathcal{T}^{\mu\nu}_{\lambda_a,\lambda_a'}\, \varepsilon^*_\mu(\lambda_1)\, \varepsilon^*_\mu(\lambda_2)$$

$$\mathcal{T}^{\mu\nu}_{\lambda_a,\lambda_a'} \sim \alpha_{\rm em} \sum_{q,q'} e_q e_q' \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp}\, \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right) \Phi_\pi^{q'q}(x_b, \vec{k}^2_{b\perp})$$
$$\left[-i\varepsilon_\perp^{\mu\nu}\left(W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp}) - W^{qq'\,[\gamma^+]}_{\lambda_a,\lambda_a'}(-x_a,-\vec{k}_{a\perp})\right)\right.$$
$$\left. -g_\perp^{\mu\nu}\left(W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda_a'}(x_a,\vec{k}_{a\perp}) + W^{qq'\,[\gamma^+\gamma_5]}_{\lambda_a,\lambda_a'}(-x_a,-\vec{k}_{a\perp})\right)\right]$$

- $\Delta\vec{q}_\perp = \vec{q}_{1\perp} - \vec{q}_{2\perp}$
- $\vec{q}_{1\perp}$, $\vec{q}_{2\perp}$ can be expressed through $\Delta\vec{q}_\perp$, $\vec{\Delta}_{a\perp}$
- $\Phi_\pi^{q'q}(x_b, \vec{k}^2_{b\perp})$ is pion light-front wave function (modulo prefactors)
- Both $W^{[\gamma^+]}$ and $W^{[\gamma^+\gamma_5]}$ contribute
- Longitudinal parton momenta fixed

$$x_a = \frac{q_1^+ - q_2^+}{2P_a^+} \to \text{ERBL region } (-\xi_a \le x_a \le \xi_a) \qquad x_b = 1 - \frac{q_1^-}{p_b^-} = \frac{q_2^-}{p_b^-}$$

- Dominant amplitude for transversely polarized photons

3. Relevant quark GTMDs

$$W^{[\gamma^+]}_{\lambda,\lambda'} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+}\,F_{1,2} + \frac{i\,\sigma^{i+}\Delta^i_\perp}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}\,F_{1,4}\right]u(p,\lambda)$$

$$\sim \Big\{ \Big[M\delta_{\lambda,\lambda'} - \frac{1}{2}\Big(\lambda\Delta^1_\perp + i\Delta^2_\perp\Big)\delta_{\lambda,-\lambda'} \Big] F_{1,1}$$

$$+ \frac{i\varepsilon^{ij}_\perp k^i_\perp \Delta^j_\perp}{M^2}\Big[\lambda M\delta_{\lambda,\lambda'} - \frac{\xi}{2}\Big(\Delta^1_\perp + i\lambda\Delta^2_\perp\Big)\delta_{\lambda,-\lambda'}\Big] F_{1,4}$$

$$+ \text{ more helicity-flip terms} \Big\}$$

$$W^{[\gamma^+\gamma_5]}_{\lambda,\lambda'} \sim \Big\{ -\frac{i\varepsilon^{ij}_\perp k^i_\perp \Delta^j_\perp}{M^2}\Big[M\delta_{\lambda,\lambda'} - \frac{1}{2}\Big(\lambda\Delta^1_\perp + i\Delta^2_\perp\Big)\delta_{\lambda,-\lambda'} \Big] G_{1,1}$$

$$+ \Big[\lambda M\delta_{\lambda,\lambda'} - \frac{\xi}{2}\Big(\Delta^1_\perp + i\lambda\Delta^2_\perp\Big)\delta_{\lambda,-\lambda'}\Big] G_{1,4}$$

$$+ \text{ more helicity-flip terms} \Big\}$$

- Focus on $F_{1,4}$ and $G_{1,1}$
- Recall that $F_{1,1}$ and $G_{1,4}$ presumably large $\rightarrow$ interference might be promising

4. Observables

- Relation between amplitude and cross section

$$d\sigma^{\lambda_1,\lambda_2}_{\lambda_a,\lambda'_a} = \frac{\pi}{s^{3/2}} \frac{1+\xi_a}{1-\xi_a} \, |\mathcal{T}^{\lambda_1,\lambda_2}_{\lambda_a,\lambda'_a}|^2 \, \delta(p'^0_a + q^0_1 + q^0_2 - \sqrt{s}) \, \frac{d^4 q_1}{(2\pi)^4} \, \frac{d^4 q_2}{(2\pi)^4}$$

- Unpolarized, single-spin asymmetry, double-spin asymmetry

$$\begin{aligned}
\tau_{UU} &= \frac{1}{2} \sum_{\lambda,\lambda'} |\mathcal{T}_{\lambda,\lambda'}|^2 \\
\tau_{LU} &= \frac{1}{2} \sum_{\lambda'} \left(|\mathcal{T}_{+,\lambda'}|^2 - |\mathcal{T}_{-,\lambda}|^2 \right) \\
\tau_{LL} &= \frac{1}{2} \Big((|\mathcal{T}_{+,+}|^2 - |\mathcal{T}_{+,-}|^2) - (|\mathcal{T}_{-,+}|^2 - |\mathcal{T}_{-,-}|^2) \Big)
\end{aligned}$$

  - summation over photon helicities λ_1, λ_2 implied
  - consider polarization of nucleon in initial and final state
  - consider longitudinal and transverse nucleon polarization

- “Direct” access to $F_{1,4}$ and $G_{1,1}$

$$\tau_{UU} + \tau_{LL} - \tau_{XX} + \tau_{YY} \sim$$

$$\frac{1}{M^4}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j\right)^2 C^{(+)}\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, F_{1,4}\, \Phi_\pi\right] C^{(+)}\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, F_{1,4}^*\, \Phi_\pi\right]$$
$$+\, C^{(+)}\left[G_{1,4}\, \Phi_\pi\right] C^{(+)}\left[G_{1,4}^*\, \Phi_\pi\right]$$

$$C^{(\pm)}\left[w(\vec{k}_{a\perp}, \vec{k}_{b\perp}) F_{m,n} \Phi_\pi\right] =$$
$$\sum_{q,q'} e_q e_q' \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp}\, \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right) w(\vec{k}_{a\perp}, \vec{k}_{b\perp})$$
$$\left[F_{m,n}^{qq'}(x_a, \vec{k}_{a\perp}) \pm F_{m,n}^{qq'}(-x_a, -\vec{k}_{a\perp})\right] \Phi_\pi^{q'q}(x_b, \vec{k}_{b\perp}^2)$$

$$\vec{\beta}_\perp = \frac{\vec{\Delta}_{a\perp}^2\, \Delta\vec{q}_\perp - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)\, \vec{\Delta}_\perp}{\vec{\Delta}_{a\perp}^2\, \Delta\vec{q}_\perp^2 - (\vec{\Delta}_{a\perp} \cdot \Delta\vec{q}_\perp)^2}$$

  - when summing over λ_1, λ_2 no interference between $W^{[\gamma^+]}$ and $W^{[\gamma^+\gamma_5]}$
  - similar double-polarization observable for $G_{1,1}$ (in combination with $F_{1,1}$)
  - through measuring photon polarization $F_{m,n}$ and $G_{m,n}$ can be “disentangled”
  - observables may be challenging (cancellation of potentially large numbers)

- Access to $F_{1,4}$ and $G_{1,1}$ using interference

$$\tau_{LU} + \tau_{UL} \sim \left(|\mathcal{T}_{+,+}|^2 - |\mathcal{T}_{-,-}|^2\right) \sim$$

$$\frac{1}{M^2}\,\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j \,\mathrm{Im}\left\{ C^{(-)}\left[F_{1,1}\,\Phi_\pi\right] C^{(+)}\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, F_{1,4}^*\,\Phi_\pi\right] - C^{(+)}\left[G_{1,4}\,\Phi_\pi\right] C^{(-)}\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, G_{1,1}^*\,\Phi_\pi\right] \right\}$$

  - additional terms (GTMDs) in SSAs τ_{LU} and τ_{UL} alone
  - interference involving other GTMDs through other polarization observables
  - observable mostly sensitive to $\mathrm{Im}\, F_{1,4}$ and $\mathrm{Im}\, G_{1,1}$

$$\tau_{XY} + \tau_{YX} \sim \frac{1}{M^2}\,\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j \,\mathrm{Re}\left\{ C^{(-)}\left[F_{1,1}\,\Phi_\pi\right] C^{(+)}\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, F_{1,4}^*\,\Phi_\pi\right] - C^{(+)}\left[G_{1,4}\,\Phi_\pi\right] C^{(-)}\left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp}\, G_{1,1}^*\,\Phi_\pi\right] \right\}$$

  - better sensitivity to $\mathrm{Re}\, F_{1,4}$ and $\mathrm{Re}\, G_{1,1}$

Summary and Outlook

- GTMDs are the most general two-parton correlation functions

- GTMDs attracted considerable interest (relation to orbital angular momentum)

- Recent work on how gluon GTMDs could, in principle, be measured

- GTMDs in exclusive double Drell-Yan
  - access to quark GTMDs (in ERBL region)
  - focus on $F_{1,4}$ and $G_{1,1}$
  - GTMDs can be "disentangled" through suitable polarization observables
  - one might also consider nucleon-nucleon collisions (factorization ?)
  - one might also produce heavy gauge bosons
  - numerical estimates needed
  - measurement probably challenging at current facilities

- Calculation can be extended to other processes, like $p\,p \to p\,p\,\eta_c\,\eta_c$
  (work in progress)