Low-energy effective action for pions and a dilatonic meson

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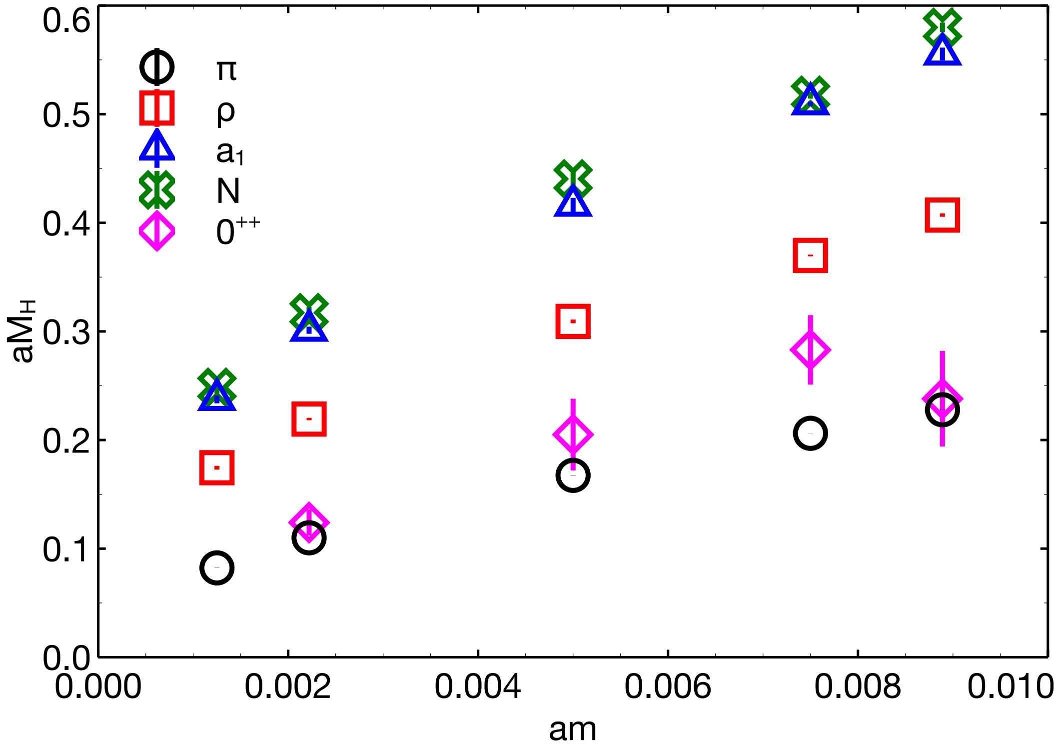
A light, narrow flavor-singlet scalar — seen on the lattice(?)

- $SU(3), N_f = 8$ fund. [LatKMI, LSD,..]



LSD collaboration, PRD 93 (2016) 114514

Consistent low-energy theory must contain both pions and the flavor-singlet scalar

- $SU(3), N_f = 2$ sextet [Fodor et al.]

Phases of $SU(N_c)$ with N_f fundamental-rep Dirac fermions

- running slows down when N_f is increased

$$\frac{\partial g^2}{\partial \log \mu} = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6$$

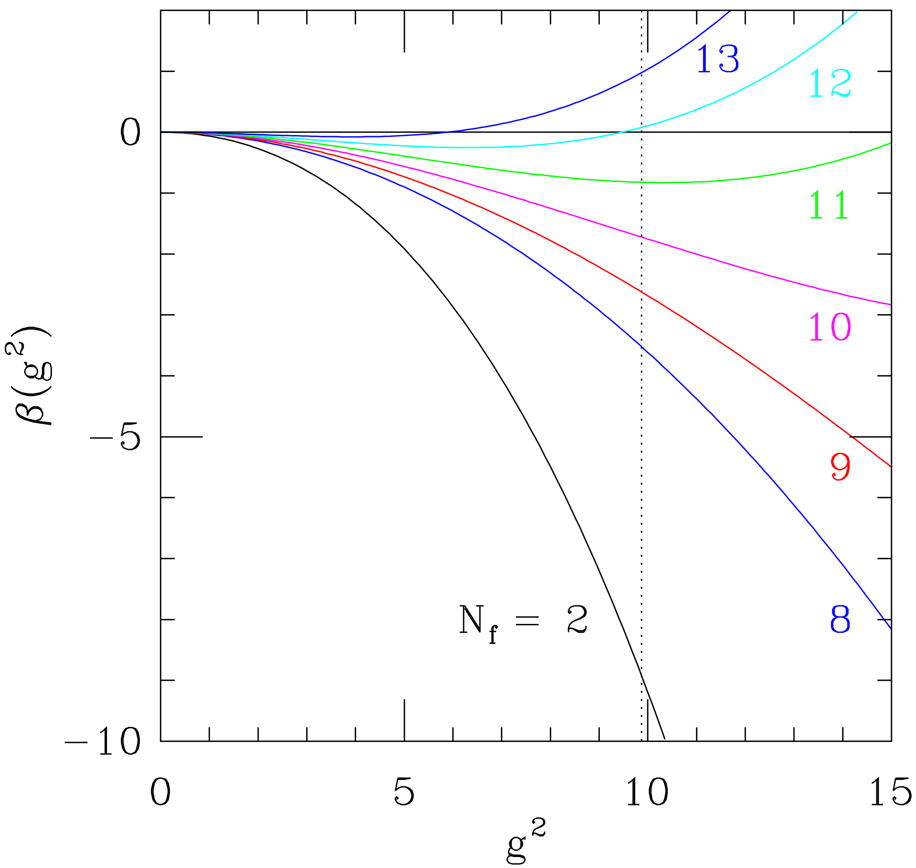
- two-loop IRFP g_*^2 develops when $b_1 > 0 > b_2$

- Gap equation $\Rightarrow$

  ChSB when $g^2(\mu) = g_c^2 = \dfrac{4\pi^2}{3C_2}$

- $SU(3)$, fund. rep: $g_c^2 = \pi^2 \simeq 9.87$

- chirally broken if $g_c < g_*(N_f)$



- emergent conformal symm. (IRFP) if $g_c > g_*(N_f)$

- sill of conformal window: $g_*(N_f^*) = g_c$ (note: N_f^* not an integer)

Pseudo Nambu-Goldstone boson of approx dilatation symmetry?

- dilatations: $\Phi_i(x) \to \lambda^{\Delta_i}\,\Phi_i(\lambda x)$, Δ_i scaling dimension of field $\Phi_i(x)$
- dilatation current $S_\mu = x_\nu T_{\mu\nu}$ is classically conserved for $m = 0$
- Trace anomaly [Collins, Duncan, Joglekar, PRD **16**, 438 (1977)]

$$\begin{aligned} \partial_\mu S_\mu &= T_{\mu\mu} = -T_{cl} - T_{an} \\ T_{cl} &= m\overline{\psi}\psi \qquad\qquad T_{an} = \frac{\beta(g^2)}{4g^2} F^2 + \gamma_m\, m\,\overline{\psi}\psi \end{aligned}$$

- below conformal sill, expect at ChSB scale $\beta(g_c^2) \propto N_f - N_f^*$
  hence, increasing N_f towards N_f^* $\Rightarrow$ smaller $\beta(g_c)$ at ChSB scale
  $\Rightarrow$ better scale invariance $\Rightarrow$ "dilatonic" pNG boson, τ, gets lighter
- use $N_f - N_f^*$ as small parameter (issue: N_f takes discrete values)

Low-energy EFT with dilatonic meson: power counting

- standard ChPT: fermion mass m is a parameter of the microscopic theory that can be tuned continuously towards zero

$\Rightarrow$ Systematic expansion in $m \sim m_\pi^2$ and p^2; massless pions for $m \to 0$

- issue: cannot turn off trace anomaly at fixed N_c, N_f
- similar: cannot turn off $U(1)_A$ anomaly; but it vanishes for $N_c \to \infty$

$\Rightarrow$ Systematic expansion in m, $1/N_c$, and p^2; massless η' for $m, 1/N_c \to 0$

- Here: Veneziano limit $\quad N_f, N_c \to \infty$ with $n_f = N_f/N_c$ fixed
  n_f becomes a continuous parameter; theory depends only on $g^2 N_c$ and n_f
  $n_f^* \; = \; \lim_{N_c\to\infty} N_f^*(N_c)/N_c \; = \;$ sill of conformal window for $N_c \to \infty$.
- Assume: $\quad T_{an} \sim (n_f - n_f^*)^\eta$ at the ChSB scale $\qquad$ [$\eta = 1$ in this talk]

$\Rightarrow$ Systematic expansion in m, $1/N_c$, $n_f - n_f^*$, and p^2;
massless dilatonic meson τ for $m, 1/N_c, |n_f - n_f^*| \to 0$

Spurions in the microscopic theory (abelian symmetries)

axial $U(1)_A$ symmetry: $\mathcal{L}^{\rm MIC}(\theta) = \frac{1}{4}F^2 + \overline{\psi}\not{D}\psi + \theta i c g^2 F\tilde{F}$

- $\delta\mathcal{L}^{\rm MIC}(\theta) = 0$ if $U(1)_A$ transformation of axionic spurion is $\theta \to \theta + \alpha$
  similar $U(1)_A$ transformation for singlet meson $\eta' \to \eta' + \alpha$
- now set $\langle\theta\rangle = \theta_0 \Rightarrow$ explicit breaking: $\delta\mathcal{L}^{\rm MIC}(\theta_0) = -icg^2 F\tilde{F}$
- $\langle\theta\rangle = 0$ not special! $\mathcal{L}^{\rm MIC}(\theta_0 = 0)$ not invariant!

dilatations:

$$S^{\rm MIC}(\sigma) = \int d^dx \, \frac{e^{\sigma(d-4)}}{g_0^2}\left(\frac{1}{4}F^2 + \cdots\right) = \int d^4x \left(\mathcal{L}^{\rm MIC}(0) + \sigma T_{an} + \cdots\right)$$

- $S^{\rm MIC}(\sigma)$ invariant if σ transforms as dilatonic spurion $e^{\sigma(x)} \to \lambda e^{\sigma(\lambda x)}$
- again, explicit breaking: $S^{\rm MIC}$ not invariant for any $\langle\sigma\rangle$

EFT with pions $\Sigma(x) = e^{2i\pi(x)/f_\pi}$ and dilatonic meson $\tau(x)$

• scale transformation: (χ is fermion mass spurion, $\langle\chi\rangle = m$)

source fields: $\sigma(x) \to \sigma(\lambda x) + \log\lambda$, $\chi(x) \to \lambda^{1+\gamma_m}\chi(\lambda x)$

effective fields: $\tau(x) \to \tau(\lambda x) + \log\lambda$, $\Sigma(x) \to \Sigma(\lambda x)$

• invariant low-energy theory: $\tilde{\mathcal{L}}^{\rm EFT} = \tilde{\mathcal{L}}_\pi + \tilde{\mathcal{L}}_\tau + \tilde{\mathcal{L}}_m + \tilde{\mathcal{L}}_d \to \lambda^4\tilde{\mathcal{L}}^{\rm EFT}$
where

$$\begin{aligned}
\tilde{\mathcal{L}}_\pi &= V_\pi(\tau-\sigma)\,(f_\pi^2/4)\,e^{2\tau}{\rm tr}\,(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma) \\
\tilde{\mathcal{L}}_\tau &= V_\tau(\tau-\sigma)\,(f_\tau^2/2)\,e^{2\tau}(\partial_\mu\tau)^2 \\
\tilde{\mathcal{L}}_m &= -V_M(\tau-\sigma)\,(f_\pi^2 B_\pi/2)\,e^{(3-\gamma_m)\tau}{\rm tr}\left(\chi^\dagger\Sigma + \Sigma^\dagger\chi\right) \\
\tilde{\mathcal{L}}_d &= V_d(\tau-\sigma)\,f_\tau^2 B_\tau\,e^{4\tau}
\end{aligned}$$

with invariant potentials: $V(\tau(x) - \sigma(x)) \;\to\; V(\tau(\lambda x) - \sigma(\lambda x))$

$\Rightarrow$ No predictability without power counting!

Power counting hierarchy from matching correlation functions

- recall microscopic theory

$$\frac{\partial}{\partial\sigma(x)}\mathcal{L}^{\rm MIC}\Big|_{\sigma=\chi=0} = T_{an}(x)\Big|_{\chi=0} = \frac{\beta(g^2)}{4g^2}[F^2(x)] \sim n_f - n_f^*$$

- effective theory

$$\left(-\frac{\partial}{\partial\sigma(x)}\right)^n \tilde{\mathcal{L}}^{\rm EFT}\Big|_{\sigma=\chi=0} = V_d^{(n)}\big(\tau(x)\big)\, f_\tau^2 B_\tau\, e^{4\tau(x)} + \cdots$$

$$\Rightarrow \quad V(\tau-\sigma) = \sum_{n=0}^{\infty} c_n(\tau-\sigma)^n \quad \text{where} \quad c_n = O((n_f - n_f^*)^n)$$

$\Rightarrow$ Only a finite number of LECs at each order!

Leading order lagrangian:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_\pi + \mathcal{L}_\tau + \mathcal{L}_m + \mathcal{L}_d \\
\mathcal{L}_\pi &= (f_\pi^2/4)\, e^{2\tau} \mathrm{tr}\,(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) \\
\mathcal{L}_\tau &= (f_\tau^2/2)\, e^{2\tau} (\partial_\mu \tau)^2 \\
\mathcal{L}_m &= -(m f_\pi^2 B_\pi/2)\, e^{(3-\gamma_m^*)\tau} \mathrm{tr}\left(\Sigma + \Sigma^\dagger\right) \\
\mathcal{L}_d &= [\tilde{c}_{00} + (n_f - n_f^*)(\tilde{c}_{01} + \tilde{c}_{11}\tau)]\, f_\tau^2 B_\tau\, e^{4\tau}
\end{aligned}
$$

- use τ shift and redefine LECs to get $\mathcal{L}_d = \tilde{c}_{11}(n_f - n_f^*)(\tau - 1/4)\, \hat{f}_\tau^2 \hat{B}_\tau\, e^{4\tau}$

- m = renormalized mass at ChSB scale

$\Rightarrow$ choose $\gamma_m(g) = \gamma_m(g_*) = \gamma_m^*$, where g_* is IRFP at the sill of the conformal window

- corrections are accounted for by expansion in $n_f - n_f^*$

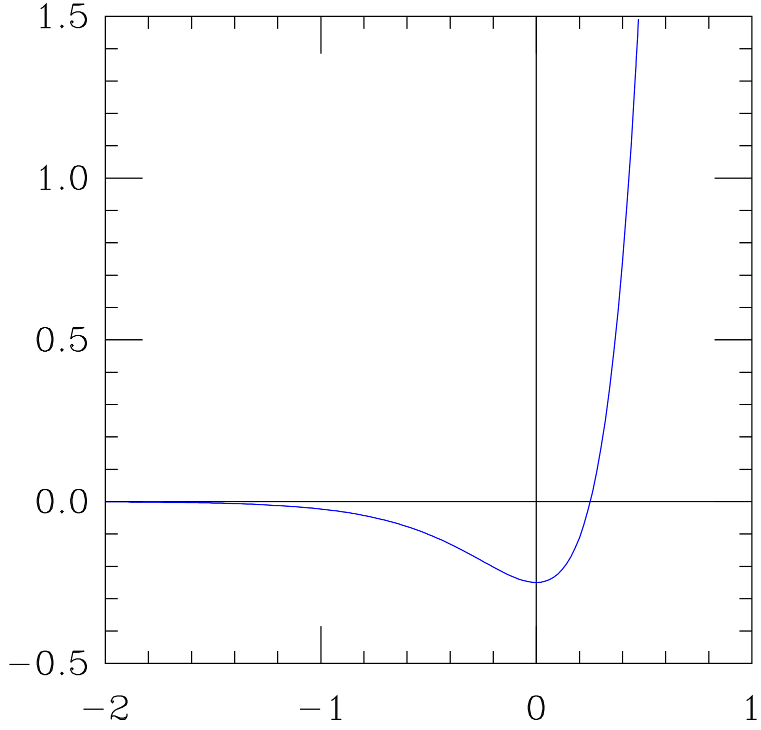
Classical vacuum in the chiral limit

- Dilatonic meson's potential: $V_{\rm cl}(\tau) \propto V_d(\tau) e^{4\tau} = \tilde{c}_{11}(n_f - n_f^*)(\tau - 1/4)\, e^{4\tau}$
- Self-consistency: $\tilde{c}_{11} < 0$ (recall $n_f < n_f^*$) $\Rightarrow$ $V_{\rm cl}(\tau)$ bounded from below
- Effective theory at leading order seems "almost" scale invariant

◦ But: linear term in $V_d(\tau)$ crucial; reflects breaking of scale invariance by running in microscopic theory

◦ Going to $n_f > n_f^*$, EFT classical potential becomes unbounded from below

$\Rightarrow$ EFT "knows" it cannot be used inside conformal window (where EFT has the wrong degrees of freedom)!



Tree-level masses

- $\underline{m = 0:}$ shifted classical vacuum: $v = \langle \tau \rangle = 0$

◦ dilatonic meson mass: $m_\tau^2 = 4\tilde{c}_{11}(n_f - n_f^*)\hat{B}_\tau$ $\hat{B}_\tau = e^{2v[\text{pre-shift}]} B_\tau$

$\Rightarrow$ m_τ vanishes for $n_f \to n_f^*$

- $\underline{m > 0:}$ $V_{\text{cl}}(\tau) = V_d(\tau)\, e^{4\tau} - \frac{m}{\mathcal{M}}\, e^{(3-\gamma_m^*)\tau}$

$\Rightarrow$ $v(m)$ increases monotonically with m (from $v(0) = 0$)

◦ dilatonic meson mass: $m_\tau^2 = 4\tilde{c}_{11}(n_f - n_f^*)\hat{B}_\tau e^{2v(m)}\big(1 + (1 + \gamma_m^*)v(m)\big)$

$\Rightarrow$ m_τ increases monotonically with m

◦ pion mass: $m_\pi^2 = 2\hat{B}_\pi m e^{(1-\gamma_m^*)v(m)}$

$\Rightarrow$ m_π increases with m faster than ordinary ChPT for $\gamma_m^* < 1$

Varying n_f towards n_f^*

- what happens at the conformal sill?

$$m^2_{\pi,\tau} = \left[O(n_f - n_f^*) + O(m/\Lambda)\right] \Lambda^2 \ll \Lambda^2 \sim m^2_{\mathrm{non-NGB}}$$

$\Rightarrow$ even though chiral symm. breaking scale $\Lambda \to 0$ when $n_f \to n_f^*$,

$m_{\pi,\tau}$ vanish faster, consistent with EFT framework

- condensate enhancement (needs $\tilde{c}_{00} > 0$)

$$\frac{\langle \overline{\psi}\psi \rangle}{\hat{f}_\pi^3} = -\frac{B_\pi}{f_\pi} e^{-\gamma_m^* v} \qquad v = -\frac{1}{4} - \frac{\tilde{c}_{00}}{\tilde{c}_{11}(n_f - n_f^*)}$$ ("gauge" choice $\tilde{c}_{01} = 0$)

Summary & further comments

- light scalar found in chirally broken "walking" theories
- crude dynamical model (2-loop + gap equation): $\beta(g_c^2) \propto n_f - n_f^* = n_f - 4$
- main assumption: $T_{an} \sim (n_f - n_f^*)$ at the onset of ChSB
- EFT allows for systematic treatment of both pions and the dilatonic meson

◦ for two-index (and higher) irreps, asymptotic freedom forbids $N_f \to \infty$
can try the EFT anyway, for fixed N_c, N_f assuming $N_f - N_f^*$ small
with a *non-integer* N_f^* close to (and above) integer N_f

◦ can add SM fermions as in other composite Higgs models
fermion-dilaton coupling $\propto$ fermion mass

Matching the trace anomaly

- dilatation current: $S_\mu = x_\nu \Theta_{\mu\nu} = x_\nu (T_{\mu\nu} + K_{\mu\nu}/3)$

$$\begin{aligned} \langle 0 | \, \Theta_{\mu\nu}(x) \, | \tau \rangle &= \frac{\hat{f}_\tau}{3} (-\delta_{\mu\nu} p^2 + p_\mu p_\nu) \, e^{ipx} \\ \langle 0 | \, S_\mu(x) \, | \tau \rangle &= i p_\mu \hat{f}_\tau \, e^{ipx} \end{aligned}$$

- anomalous divergence shows up at leading order in EFT:

$$\begin{aligned} \partial_\mu S_\mu &= \tilde{c}_{11} (n_f - n_f^*) f_\tau^2 B_\tau e^{4\tau} + (1 + \gamma_m^*) m \, \frac{f_\pi^2 B_\pi}{2} \, e^{(3-\gamma_m^*)\tau} \mathrm{tr}\,(\Sigma + \Sigma^\dagger) \\ &= -\frac{\beta(g^2)}{4g^2} \, F^2(\mathrm{EFT}) - (1 + \gamma_m^*) m \, \overline{\psi}\psi(\mathrm{EFT}) \end{aligned}$$

- ◦ GMOR relation for $m \to 0$: $-(2m/N_f) \, \langle \overline{\psi}\psi \rangle \; = \; \hat{f}_\pi^2 m_\pi^2$
- ◦ GMOR-like relation for dilatonic meson: $-(\beta(g^2)/g^2) \, \langle F^2 \rangle \; = \; \hat{f}_\tau^2 m_\tau^2$