Color screening in high temperature quark-gluon-plasma

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in collaboration with
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(**TUMQCD** collaboration)

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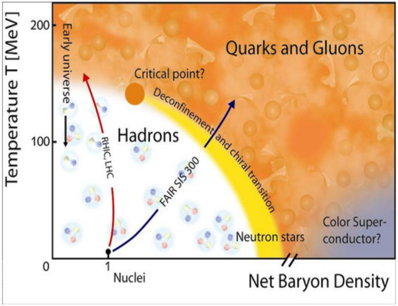
PRD 93 114502 (2016); **MPL A31** no.35, 1630040 (2016); arXiv:1601:08001

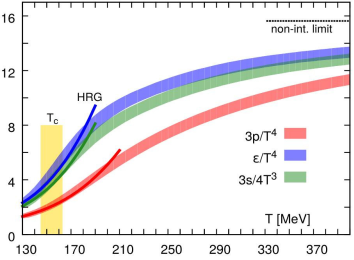
Quark-Gluon-Plasma - the high-temperature phase of QCD

QCD Phase diagram



- Smooth crossover region
- **Accidental symmetries are broken/restored in crossover.**

QCD Equation of state



A. Bazavov et al., **PRD 90** 094503 (2014) [HotQCD]

- **Increase of particle number –** HRG too low for $T > 150\,\mathrm{MeV}$!
- $\mu > 0$ also available by now

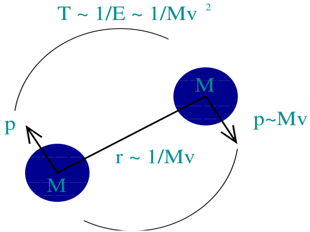
A. Bazavov et al., 1701.04325 (2017) [HotQCD]
J. Günther et al., 1607.02493 (2016) [BW coll.]

Color screening in a high temperature quark-gluon-plasma

- Introduction & Overview
- Effective field theories for heavy quarks
- Polyakov loops – color screening for a single quark
- Polyakov loop correlators – color screening for a quark-antiquark pair
- Summary

Heavy quarkonium – the positronium of QCD

Upsilon (**Υ**) and Psi (ψ) states are typical examples of quarkonia.



(figure by A. Vairo)

- Quarkonia are **non-relativistic bound states** of a **heavy** quark $Q = \{b, c\}$ and a heavy anti-quark $\bar{Q} = \{\bar{b}, \bar{c}\}$.
- **Hierarchy of scales** due to slow motion of quarks: $M \gg Mv \gg Mv^2$
- **Systematic expansion** in the quark mass is possible ($\frac{\Lambda_{\rm qcd}}{M} \ll 1$).

Static potential at finite temperature

- *Melting of quarkonia* is controlled by the **screened, complex static potential** $V_S(r,T)$, which has been calculated at next-to-leading order:

$$V_S(r,T) = -C_F\alpha_s\left(\frac{e^{-rm_D}}{r} + m_D + iT - \frac{2iT}{rm_D}\int_0^\infty \mathrm{d}x\,\frac{\sin(rm_D x)}{(x^2+1)^2}\right) + \mathcal{O}(g^4).$$

M. Laine et al., **JHEP 0703** 054 (2007)

- $\mathrm{Im}\,V_S \gg \mathrm{Re}\,V_S \Rightarrow$ **color screening is not effective** for melting at all!

P. Petreczky et al., **NPA 855** 125 (2011)

- No non-perturbative determination of $V_S(r, T>0)$ with a controlled error budget to date – *real-time properties such as complex potentials* at $T>0$ from *imaginary-time simulations* are *extremely difficult* at best.

$\Rightarrow$ obtain constraints for *complex quantities* from purely *real quantities*.

- **Singlet free energy** and real part of the **potential** appear to be related:

$$F_S(r,T) = \mathrm{Re}\,V_S(r,T) + \mathcal{O}(g^4) = -C_F\alpha_s\left(\frac{e^{-rm_D}}{r} + m_D\right) + \mathcal{O}(g^4).$$

N. Brambilla et al., **PRD 82** (2010)

Lattice gauge theory at finite temperature

Lattice QCD at finite temperature

- A *finite imaginary time direction* acts as an **inverse temperature**:

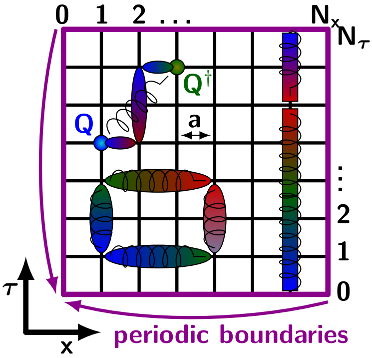
$$aN_\tau = \frac{1}{T}$$

- Always $N_\tau < \infty$ at finite a:

⇒ *always at a finite temperature* before continuum extrapolation.

- The **continuum limit** ($a \to 0$) **at fixed temperature** T is reached via *concurrent modification of* a *and* N_τ: continuum at $N_\tau \to \infty$.

Gauge-invariant operators on a euclidean space-time grid



Polyakov loops and free energies of static quark states

- The *Polyakov loop L* is the gauge-invariant expectation value of the traced propagator of a static quark (P) and related to its **free energy**: $L(T) = \langle P \rangle_T = \langle \mathrm{Tr}\, S_Q(x,x) \rangle_T = e^{-F_Q^{\mathrm{b}}/T}$. L needs renormalization.

A. M. Polyakov, **PL 72B** (1978); L. McLerran, B. Svetitsky, **PRD 24** (1981)

- The *Polyakov loop correlator* is related to *singlet* & *octet free energies*

$$C_P(r,T) = e^{-F^{\mathrm{b}}_{Q\bar{Q}}(r,T)} = \frac{1}{9} e^{-F_S^{\mathrm{b}}/T} + \frac{8}{9} e^{-F_A^{\mathrm{b}}/T} = \frac{1}{9} C_S(r,T) + \frac{8}{9} C_A(r,T).$$

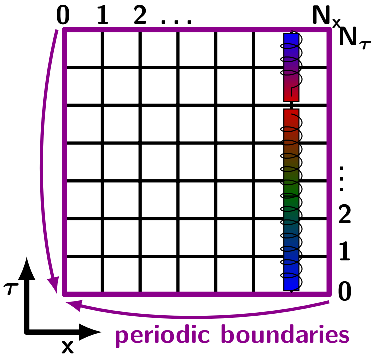
S. Nadkarni, **PRD 33, 34** (1986)

- The *Polyakov loop correlator* is related to the **potentials** of **pNRQCD**

$$C_P(r,T) = e^{-F^{\mathrm{b}}_{Q\bar{Q}}(r,T)} = \frac{1}{9} e^{-V_S^{\mathrm{b}}/T} + \frac{8}{9} L_A^{\mathrm{b}}\, e^{-V_A^{\mathrm{b}}/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1.$$

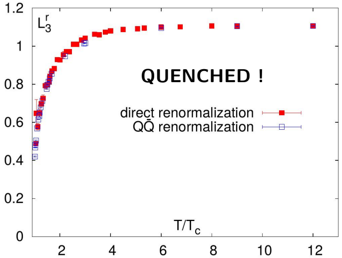
N. Brambilla et al., **PRD 82** (2010)

Color screening for a single static quark



The Polyakov loop as an order parameter

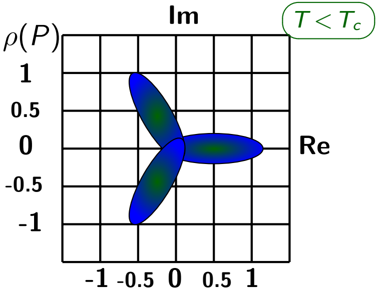
L^{r} in SU(3) pure gauge theory



S. Gupta et al., **PRD 77** 034503 (2008)

The *Polyakov loop* is *an order parameter* in *pure gauge theory* due to breaking **Z(3) center symmetry**.

Z(3) center symmetry as a cartoon

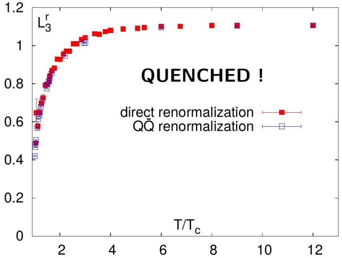


$L = 0 \Leftrightarrow F_Q = \infty$

Confinement in pure gauge theory

The Polyakov loop as an order parameter

L^{r} in SU(3) pure gauge theory



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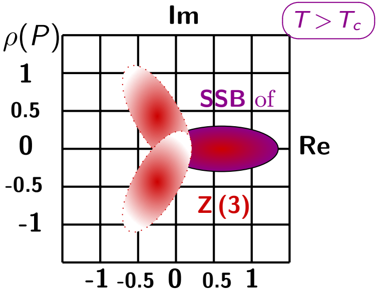
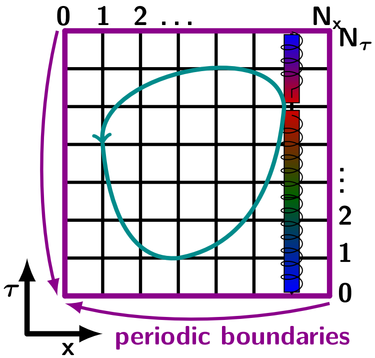


$L > 0 \Leftrightarrow F_Q < \infty$ (**color screening**)

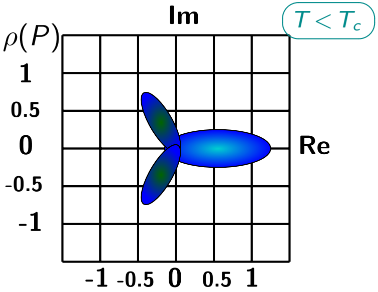
Deconfinement in pure gauge theory

The Polyakov loop as an order parameter

Polyakov loop with sea quarks



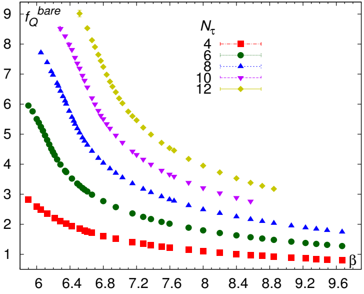
Z(3) center symmetry as a cartoon



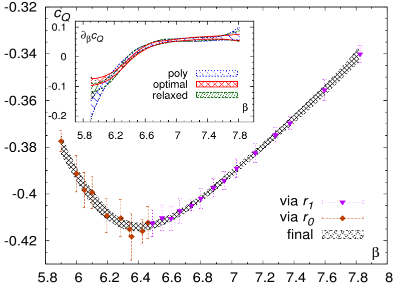
$L > 0 \Leftrightarrow F_Q < \infty$ (**string breaking**)

$F_Q \simeq \sum_i E_i$ due to **static hadrons**

Bare Polyakov loop, bare free energy and renormalization



TUMQCD collaboration, **PRD 93** 114502 (2016)



$Q\bar{Q}$ procedure ($T=0$)

The **free energy** $f_Q^{\rm b} \equiv \frac{F_Q^{\rm b}}{T} = -\log L$ is **UV divergent**. Renormalization as $L^{\rm r} = L\, e^{-\frac{C_Q}{T}} \quad \leftrightarrow \quad f_Q = f_Q^{\rm b} + \frac{C_Q}{T}$ with $C_Q = \frac{c}{a} + b + \mathcal{O}(a^2)$ leads to a **scheme dependence:** $b + \mathcal{O}(a^2)$

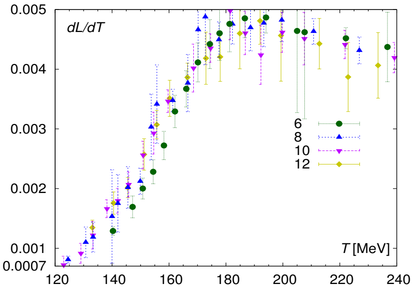
Fix $V_S(r_i) = V_S^{\rm b}(r_i) + 2C_Q(\beta) \overset{!}{=} \frac{V_i}{r_i}$ for $V_S'(r_i) = r_i^2 C_i$, where $\{C_i, r_i\}$ are known from quark models.

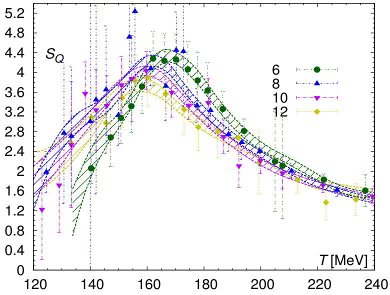
Large $T=0$ lattices are needed!

A. Bazavov et al., **PRD 85** 054503 (2012) [HotQCD]

A. Bazavov et al., **PRD 90** 094503 (2014) [HotQCD]

Critical behavior in renormalized Polyakov loop and free energy





Temperature derivative of L

$\frac{dL}{dT}$ peaks at $T \sim 190$ MeV

$\frac{dL}{dT}$ is explicitly **scheme dependent**,

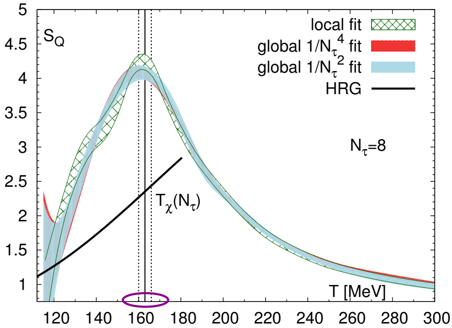
Temperature derivative of F_Q

$S_Q = -\frac{dF_Q}{dT}$ peaks at $T \sim 160$ MeV

though S_Q is a **measurable quantity**.

JHW, **MPL A31** no.35, 1630040 (2016)

T_c from chiral observables vs the peak of the entropy

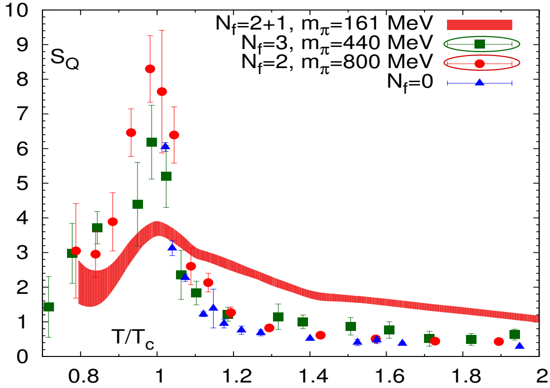


- T_χ defined via $O(2)$ scaling of $\chi_{m,l}$ ($O(4)$: $1-3.5$ MeV lower T_χ)

A. Bazavov et al., **PRD 85** 054503 (2012) [HotQCD]

- $T_S(N_\tau) \simeq T_\chi(N_\tau)$ for any N_τ despite *different cutoff effects* suggests a **close connection of chiral symmetry and deconfinement**.
- Hadron resonance gas (HRG) describes data below $T \sim 125$ MeV.

Critical behavior of the entropy



O. Kaczmarek, F. Zantow, **hep-lat/0506019 (2005)**

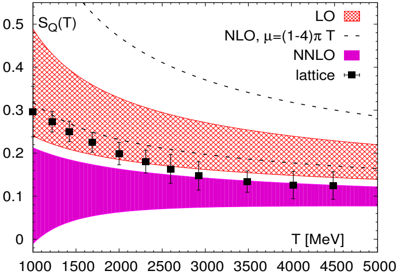
P. Petreczky, K. Petrov **PRD 70** 054503 (2004)

- The **peak decreases for lower quark masses** and for finer lattices.

→ interpret critical behavior as **melting of the static-light mesons**.

- The entropy peaks at $T_S = 153^{+6.5}_{-5}$ MeV in the continuum limit.

Confronting weak-coupling predictions at high temperatures

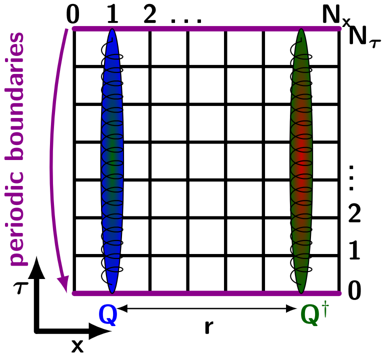


- Discretization effects are very mild for $T > 500\,\mathrm{MeV}$.
- We compare $S_Q(T,4)$ with a weak-coupling calculation for 3 flavors.

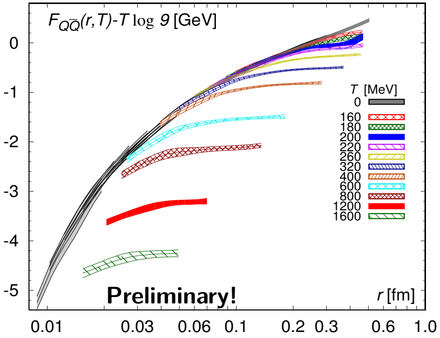
M. Berwein et al., **PRD 93** 034010 (2016)

- For $T \gtrsim 3\,\mathrm{GeV}$, $S_Q(T,4)$ agrees with NNLO.
- Higher temperature than for quark number susceptibilities ($T_{\mathrm{qns}} \sim 300\,\mathrm{MeV}$) due to static Matsubara mode contribution to S_Q.

Color screening for a static quark-antiquark pair



Polyakov loop correlator and $Q\bar{Q}$ free energy

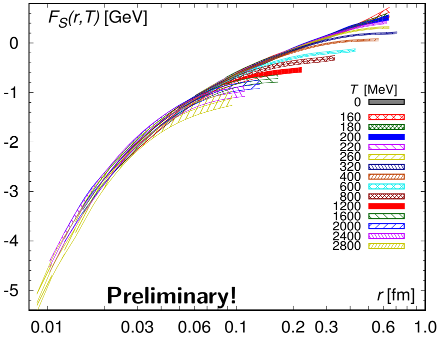


- **Free energy of a $Q\bar{Q}$ pair**, $F_{Q\bar{Q}}$, is also called *color-averaged potential*:

$$C_P(r,T) = \langle P(0)P^\dagger(\boldsymbol{r})\rangle_T = e^{-\frac{F_{Q\bar{Q}}(r,T)}{T}} = \frac{1}{9}e^{-\frac{F_S(r,T)}{T}} + \frac{8}{9}e^{-\frac{F_A(r,T)}{T}}.$$

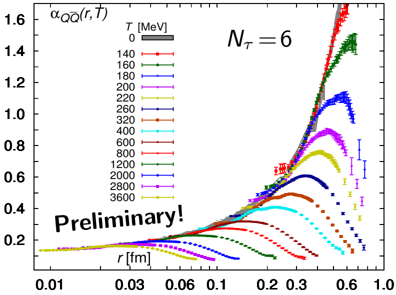
- $F_{Q\bar{Q}} - T\log 9$ is rather close to $T=0$ **static energy** V_S up to $rT \sim 0.15$.

Singlet free energy in Coulomb gauge



- **Singlet free energy:** $C_S(r,T) = \frac{1}{3}\left\langle \sum_{a=1}^{3} W_a(0) W_a^\dagger(\boldsymbol{r}) \right\rangle_T = e^{-F_S(r,T)/T}$
- Wilson line correlator requires explicit **gauge fixing** (Coulomb gauge)
- F_S is rather consistent with $T=0$ **static energy** $V_S(r)$ up to $rT \sim 0.35$.

Effective coupling: confining and screening regimes

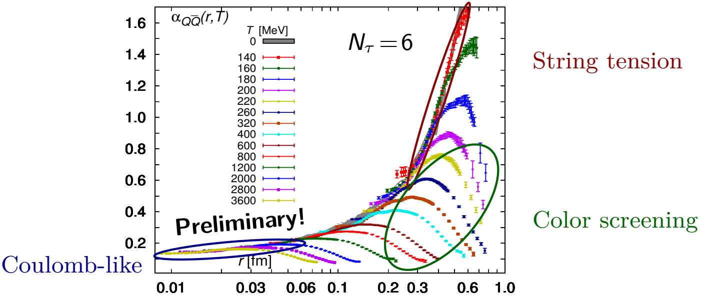


- **Effective coupling** $\alpha_{Q\bar{Q}}(r,T)$ is a proxy for the **force** between Q and $\bar{Q}$.

$$\alpha_{Q\bar{Q}}(r,T) = \frac{r^2}{C_F}\frac{\partial E(r,T)}{\partial r}, \quad E = \{F_S(r,T), V_S(r)\}$$

- $\alpha_{Q\bar{Q}}$ clearly distinguishes two different regimes at small and large r.

Effective coupling: confining and screening regimes



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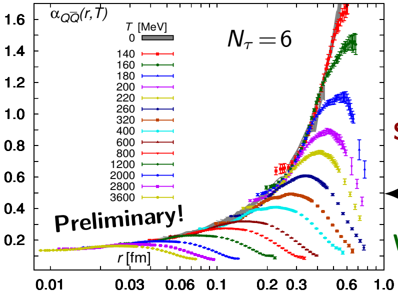
Effective coupling: confining and screening regimes



Strongly-coupled QGP

← $\max(\alpha_{Q\bar{Q}}) \sim 0.5$

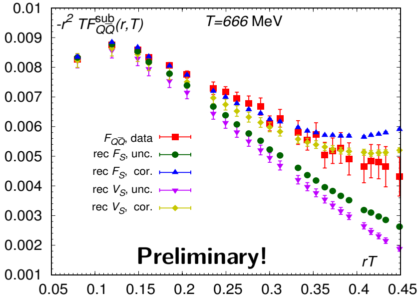
Weakly-coupled QGP

- **Effective coupling** $\alpha_{Q\bar{Q}}(r, T)$ is a proxy for the **force** between Q and $\bar{Q}$.

$$\alpha_{Q\bar{Q}}(r, T) = \frac{r^2}{C_F} \frac{\partial E(r,T)}{\partial r}, \quad E = \{F_S(r, T), V_S(r)\}$$

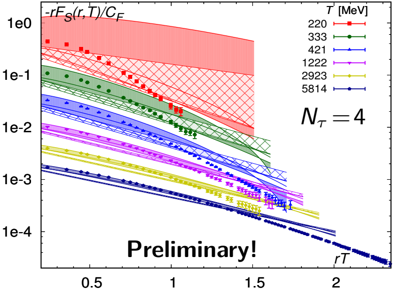
- $\alpha_{Q\bar{Q}}$ clearly distinguishes two different regimes at small and large r.

Confronting weak-coupling predictions at short distances



- *pNRQCD*: C_P is given in terms of **potentials** V_S **and** V_A **at** $T=0$ and of the *adjoint Polyakov loop* L_A *at* $T>0$: N. Brambilla et al., **PRD 82** (2010)
  $C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{9} e^{-V_S/T} + \frac{8}{9} L_A\, e^{-V_A/T} + \mathcal{O}(g^6)$ for $rT \ll 1$.
- We reconstruct V_A from V_S and L_A from L via **Casimir scaling** and include the **Casimir scaling violation**: $8V_A + V_S = 3\frac{\alpha_s^3}{r}[\frac{\pi^2}{4} - 3] + \mathcal{O}(\alpha_s^4)$.

Confronting weak-coupling predictions in the screening regime (I)

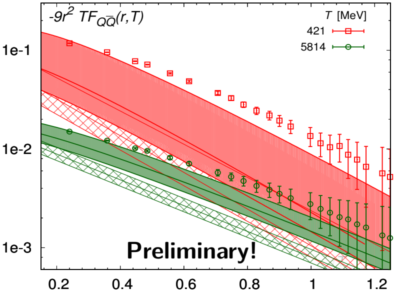


Hashed bands: LO
Solid bands: NLO

Scale uncertainty
$\mu = (1-4)\pi T$
due to resummation

- Leading order singlet free energy: $F_S(r,T) = -C_F\alpha_s\left[\frac{e^{-rm_D}}{r} + m_D\right]$.
- The singlet free energy in the electric screening regime was caluclated at NLO by Laine et al. M. Laine et al., **JHEP 0703** 054 (2007)
- Lattice and NLO results are compatible up to $rT \sim 0.8$.

Confronting weak-coupling predictions in the screening regime (II)

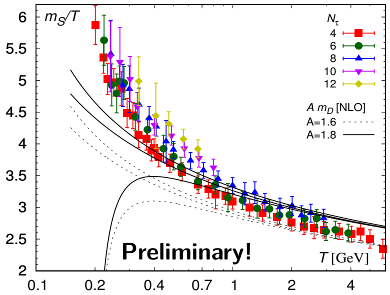


Hashed bands: LO
Solid bands: NLO

Scale uncertainty
$\mu = (1{-}4)\pi T$
due to resummation

- Leading order free energy: $F_{Q\bar{Q}}(r,T) = -\frac{\alpha_s^2}{9}\frac{e^{-2rm_D}}{r^2} + C_F\alpha_s m_D.$
- The **perturbation series of $F_{Q\bar{Q}}$ breaks down** in the screening regime: NLO exceeds LO, **NNLO is non-perturbative**! S. Nadkarni, **PRD 33** (1986)
- The NLO result is much closer to the lattice data for $rT \lesssim 0.4$.

Asymptotic singlet screening mass



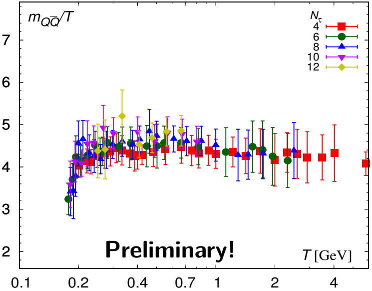
Triplets of lines:

Scale uncertainty $\mu = (1{-}4)\pi T$ due to resummation

- Color screening becomes stronger for larger distances and any free energies must reach asymptotic screening behavior: $F = -a\frac{e^{-rm}}{r} + c$.
- The asymptotic **singlet screening mass** m_S exceeds the NLO Debye mass (electric mass in Electrostatic QCD). E. Braaten, A. Nieto, **PRD 53** (1996).
- Asymptotic and rescaled NLO masses share similar T dependence.

O. Kaczmarek, **PoS CPOD07** (2007).

Asymptotic screening mass of $F_{Q\bar{Q}}$



- The screening mass $m_{Q\bar{Q}}$ is already **at $rT \sim 0.45$ asymptotic**.
- $\frac{m_{Q\bar{Q}}}{T}$ is **at most mildly temperature dependent** for $T > 200\,\mathrm{MeV}$.
- $m_{Q\bar{Q}}$ is compatible with the *magnetic mass* m_M from smeared Polyakov loop correlators and with the ground state of massless $N_f = 3$ EQCD.

S. Borsányi et al., **JHEP 1504** 138 (2015) [BW coll.]; A. Hart et al., **NPB 586** (2000)

- We study color screening and deconfinement using the renormalized Polyakov loop and related observables.
- We see in the entropy $S_Q = -\frac{dF_Q}{dT}$ and in the ratio of Polyakov susceptibilities $R_T = \frac{\chi_T}{\chi_L}$ crossover behavior at $T \sim T_c$.
- We extract $T_S = 153^{+6.5}_{-5}$ **MeV from the entropy, in agreement with** $T_\chi = 160(6)$ **MeV** (chiral susceptibilities, O(2) scaling fits, $\frac{m_l}{m_s} = \frac{1}{20}$).

N_τ	∞	12	10	8	6
T_S	$153^{+6.5}_{-5}$	157.5(6)	159(4.5)	162(4.5)	167.5(4.5)
T_χ	160(6)	161(2)	$[162(2)]^*$	164(2)	171(2)

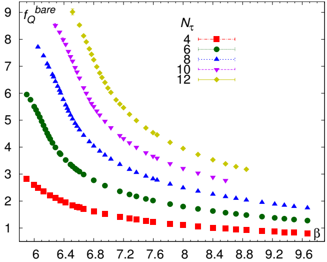
- **Weak-coupling behavior** of the Polaykov loop sets in for $T \sim 3$ GeV.

Color screening permits to precisely measure the onset of deconfinement.

- Continuum limit of static quark correlators in $N_f = 2{+}1$ QCD up to $T \sim 2.8\,\mathrm{GeV}$ and down to $r \sim 0.018\,\mathrm{fm}$.
- Static $Q\bar{Q}$ correlators show **remnants of confinement**, and up to $T \sim 300\,\mathrm{MeV}$ QGP is strongly coupled.
- Onset of thermal effects is much stronger if **color-octet states** contribute.
- The free energy Fqq is given in terms of $T = 0$ **potentials and the adjoint Polyakov loop at** $T > 0$ in line with weakly-coupled *pNRQCD*.
- We confirm **electric screening** in both $F_{Q\bar{Q}}$ and F_S at $rT \sim 0.25$.
- The screening mass of $m_{Q\bar{Q}}$ is consistent with **EQCD predictions for the lowest scalar glueball** and has a trivial temperature dependence.

Color screening plays essentially no role in sequential melting, which is a consequence of quarkonium dissociation.

Details of the ensembles



TUMQCD collaboration, **PRD 93** 114502 (2016)

- For each N_τ: 31 – 43 temperatures; T **range from** $0.72\,T_c$ **up to** $30\,T_c$
- **HISQ/Tree** action, errors: $\mathcal{O}(\alpha_s a^2, a^4)$; taste-breaking much reduced.
- Ensembles: $\frac{N_\sigma}{N_\tau} = 4$, $m_l = \frac{m_s}{20} \Leftrightarrow m_\pi = 161\,\text{MeV}$ (most from HotQCD)
  A. Bazavov et al., **PRD 85** 054503 (2012), **PRD 90** 094503 (2014) [HotQCD]
- Two ensembles: $\frac{N_\sigma}{N_\tau} = 6$ ensembles for $N_\tau = 4$
- Three ensembles each: $m_l = \frac{m_s}{5} \Leftrightarrow m_\pi = 322\,\text{MeV}$, for $N_\tau = 8, 10, 12$

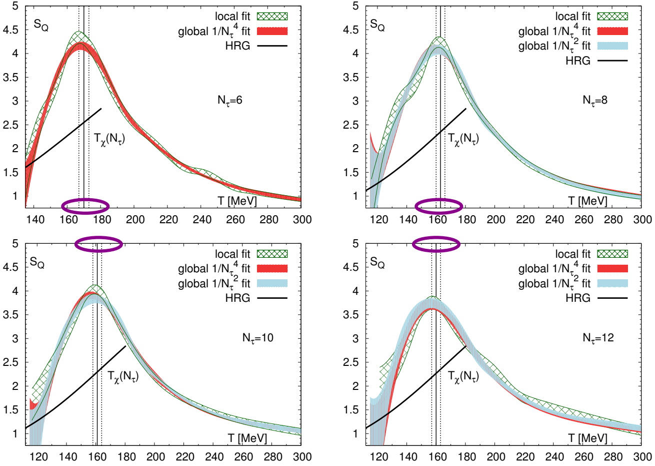
Scheme independence of the entropy

$$S_Q = -\frac{d(F_Q^{\mathrm{bare}} + C_Q)}{dT} = -\frac{\partial\left(F_Q^{\mathrm{bare}} + \frac{b}{a}\right)}{\partial T} + \frac{1}{T}\frac{\partial\left(c + \mathcal{O}(a^2)\right)}{\partial \log a} = -\frac{\partial F_Q}{\partial T} + \mathcal{O}(a^2).$$

Scheme dependence of the Polyakov loop

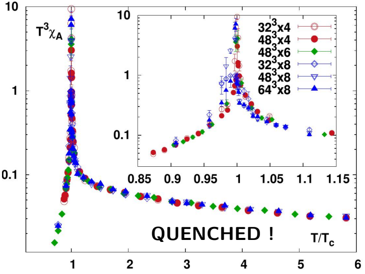
$$0 \stackrel{!}{=} \frac{1}{L}\frac{\partial^2 L}{\partial T_L^2} = \left[\frac{\partial f_Q}{\partial T_L}\right]^2 - \left[\frac{\partial^2 f_Q}{\partial T_L^2}\right] = \frac{F_Q^2 + 2[S_Q - 1]T_L F_Q}{T_L^4} + \frac{S_Q^2 - 2S_Q + T_L \frac{\partial S_Q}{\partial T_L}}{T_L^2}.$$

T_c from chiral observables vs the peak of the entropy

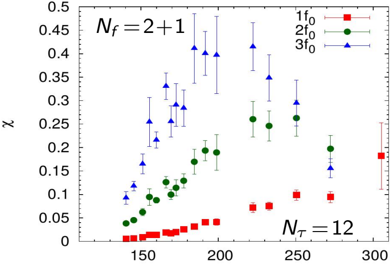


T_χ defined via $O(2)$ scaling fits to $\chi_{m,l}$ A. Bazavov et al., **PRD 85** 054503 (2012) [HotQCD]

Polyakov loop susceptibilities



P.M. Lo et al., **PRD 88** 074502 (2013)

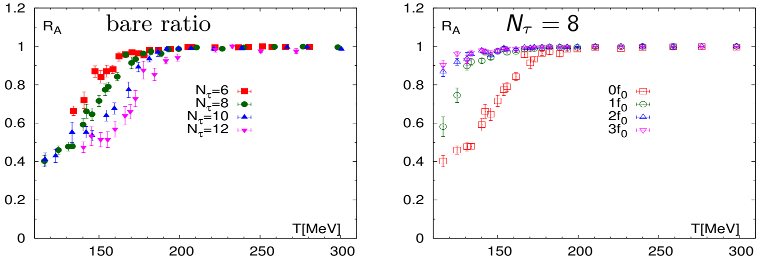


- **Polyakov loop susceptibility**: $\frac{\chi_A}{VT^3} = \left(\langle |P|^2 \rangle - \langle |P| \rangle^2 \right)$
- Mixes different representations: $9|P_3|^2 = 8P_8 - 1$
- **Casimir scaling violations** (P. Petreczky, H.-P. Schadler, **PRD 92** 094517 (2015))

$\rightarrow$ no $Q\bar{Q}$ scheme, renormalize 2+1 flavor HISQ data via gradient flow

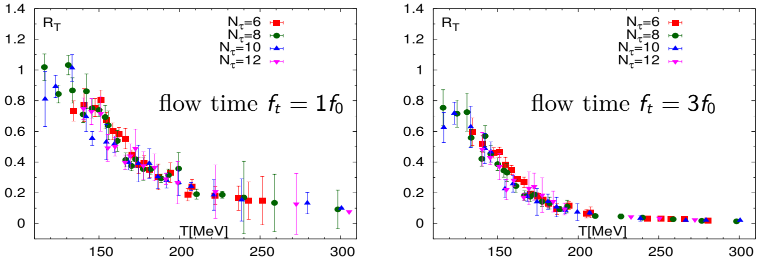
- χ_A strongly f_t dependent, no indication for critical behavior

Ratios of Polyakov loop susceptibilities



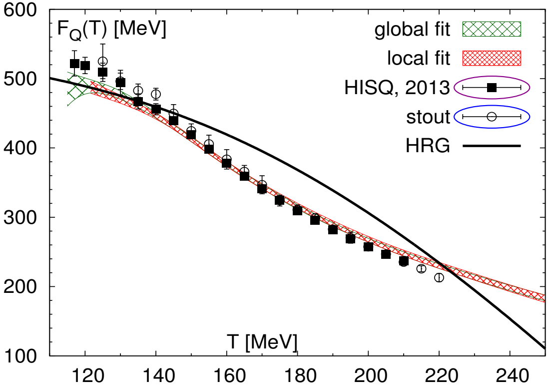
- **Longitudinal** and **transverse** Polyakov loop susceptibilities:
  $\frac{\chi_L}{VT^3} = \langle \mathrm{Re}\, P^2 \rangle - \langle \mathrm{Re}\, P \rangle^2, \quad \frac{\chi_T}{VT^3} = \langle \mathrm{Im}\, P^2 \rangle$
- $R_A = \chi_A / \chi_L$: step function behavior **cannot be related to crossover**.

Critical behavior of Polyakov loop susceptibilities



- Ratios of **longitudinal** and **transverse** Polyakov loop susceptibilities:
  $\frac{\chi_L}{VT^3} = \left[\langle \mathrm{Re}\, P^2\rangle - \langle \mathrm{Re}\, P\rangle^2\right], \ \frac{\chi_T}{VT^3} = \langle \mathrm{Im}\, P^2\rangle$ P. Lo et al., **PRD 88** 014506 (2013)
- We use gradient flow for renormalization. M. Lüscher, **JHEP 1008** 071 (2010)
- $R_T = \chi_T/\chi_L$: **crossover pattern** for $f_t \geq f_0$, exposes **critical behavior**.

Free energy and hadron resonance gas

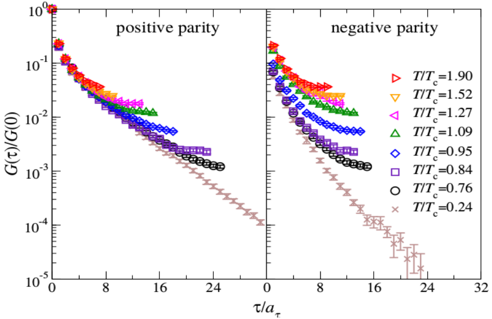


A. Bazavov, P. Petreczky, **PRD 87**, 094505 (2013)

S. Borsanyi et al., **JHEP 09**, 073 (2010) [BW]

- F_Q is for low T below and for high T above the older HISQ result, due to **better continuum limit and renormalization constant**.
- Hadron resonance gas agrees with our data up to $T \lesssim 135$ MeV.

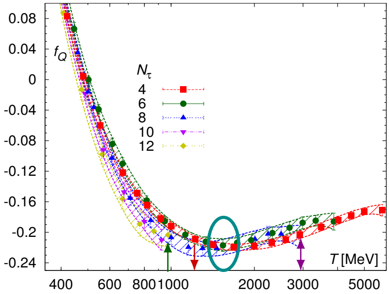
Thermal modification of nucleons

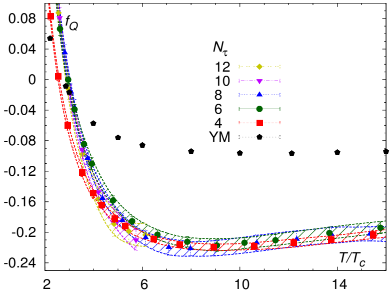


G. Aarts et al., **PRD 92** 014503 (2015)

- Correlation functions for nucleons of different parity become degenerate at finite temperature.
- It is not obvious whether this modification must be factored into hadron resonance gas models.

Free energy at high temperatures and quenching effects





Direct renormalization in **two steps**

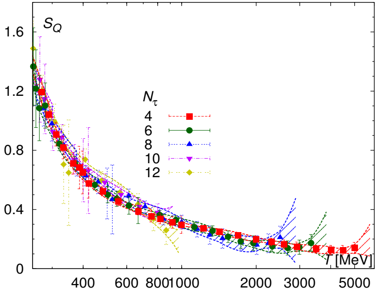
$(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}})$	$\max(T)$	$\to (\beta, N_\tau)$
(7.825,4)	1222 MeV	$\to$ (8.850,12)
(8.850,4)	2920 MeV	$\to$ (9.670,8)

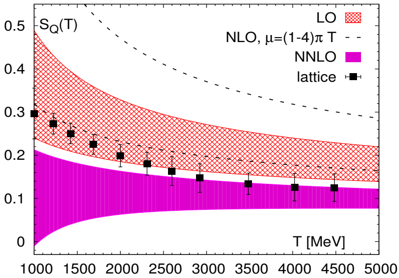
$\max(T) = 5814$ MeV for $N_\tau = 4$

Cutoff effects flip at $T \sim 1.6$ GeV.

- **Minimum** of f_Q at $T \sim 10\,T_c$ ($N_\tau = 4$) in pure gauge theory
  S. Gupta et al., **PRD 77** 034503 (2008)
- **Quark contribution** to f_Q for $T \gtrsim 4\,T_c$ is apparently $\sim 60\%$.
- Large **charm** contribution...?

Entropy at high temperature and onset of weak coupling





Static energies from **lattice and weak coupling approaches** differ by **unphysical additive divergences**.

Avoided when studying **derivatives**, i.e. **static $Q\bar{Q}$ force** or **entropy**

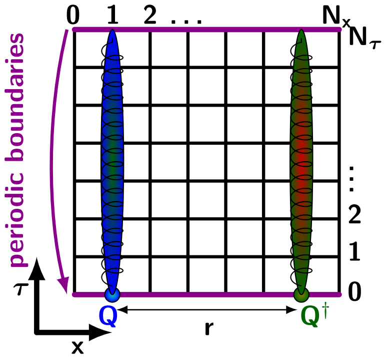
Cutoff effects in $S_Q(T)$ are **small**.

We compare $S_Q(T, 4)$ with weak coupling calculation for 3 flavors.

M. Berwein et al., **PRD 93** 034010 (2016)

For $T \gtrsim 3$ GeV, $S_Q(T, 4)$ agrees with NNLO. The continuum limit should agree for lower T already.

Static meson correlators at asymptotically LARGE distances

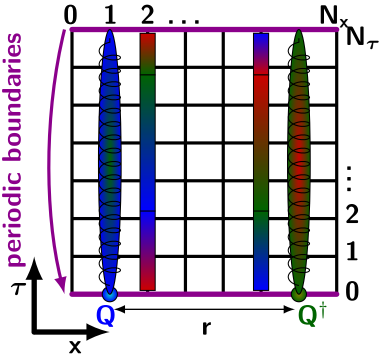


Free energy of static $Q\bar{Q}$ pair:

$$f_{Q\bar{Q}}(T,r) = F_{Q\bar{Q}}(T,r)/T$$
$$= -\log \langle P(0)P^\dagger(\mathbf{r})\rangle_T$$

Poylakov loop correlator $C_P(T,r)$

Static meson correlators at asymptotically LARGE distances



$r \gg 1/T$: static $Q\bar{Q}$ **decorrelate**

$$\lim_{r\to\infty} C_P(T,r) = L(T)^2$$

Apparent due to **color screening**

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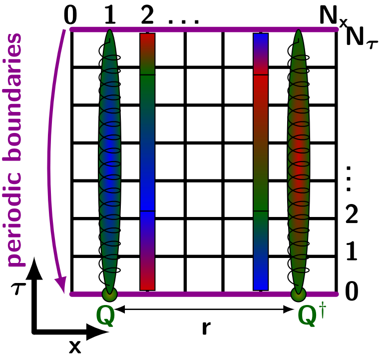

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Poylakov loop correlator $C_P(T,r)$

$r \gg 1/T$: static $Q\bar{Q}$ **decorrelate**

$$\lim_{r\to\infty} C_P(T,r) = L(T)^2$$

Apparent due to **color screening**

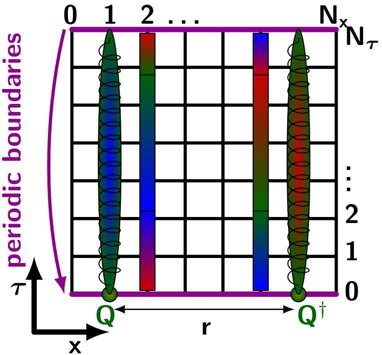
For **any color configuration** of $Q\bar{Q}$

$$\lim_{r\to\infty} C_S(T,r) = \langle L(T)\rangle^2$$

C_S is defined in **Coulomb gauge** as

$$C_S(T,r) = \frac{1}{3}\sum_{a=1}^{3} W_a(T,0)W_a^\dagger(T,\mathbf{r})$$

Static meson correlators at asymptotically LARGE distances



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Poylakov loop correlator $C_P(T,r)$

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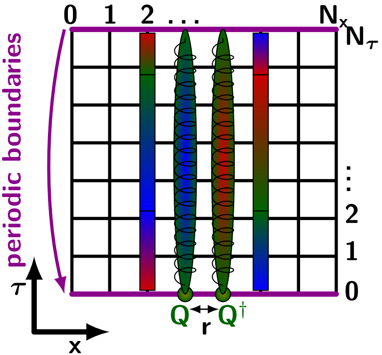
$$C_S(T,r) = \frac{1}{3}\sum_{a=1}^{3} W_a(T,0)W_a^\dagger(T,\mathbf{r})$$

$$\frac{C_S^r}{C_S^b} = \frac{C_P^r}{C_P^b} = \frac{(L^r)^2}{(L^b)^2} = \exp\left[-2N_\tau c_Q\right]$$

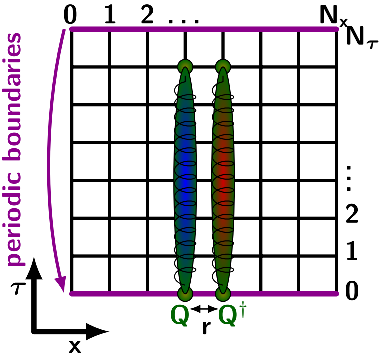
Static meson correlators at asymptotically SMALL distances



$r \ll \frac{1}{T}$: **small thermal effects** in $F_S(T,r) = -T \log C_S(T,r)$

For $r \ll \frac{1}{T}$: **vacuum-like** due to **asymptotic freedom**

Static meson correlators at asymptotically SMALL distances

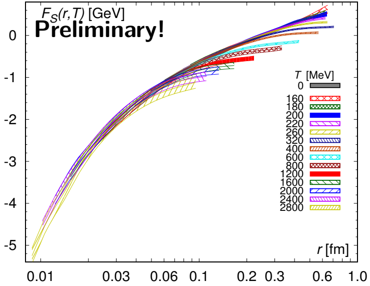


$r \ll \frac{1}{T}$: **small thermal effects** in $F_S(T,r) = -T \log C_S(T,r)$

For $r \ll \frac{1}{T}$: **vacuum-like** due to **asymptotic freedom**

$r \ll \frac{1}{T}$ is a **vacuum-like** regime

$$F_S(T,r) = V_S(r) + \mathcal{O}(rT)$$



$$\frac{F_S^{\mathrm{r}} - F_S^{\mathrm{b}}}{a} = \frac{V_S^{\mathrm{r}} - V_S^{\mathrm{b}}}{a} = -2c_Q$$

Renormalization scheme: $Q\bar{Q}$ procedure

Fix the static energy ($V_S \equiv V$)

$$V^r(\beta, r) = V^b(\beta, r) + 2c_Q(\beta)$$

for each β (β omitted below) to

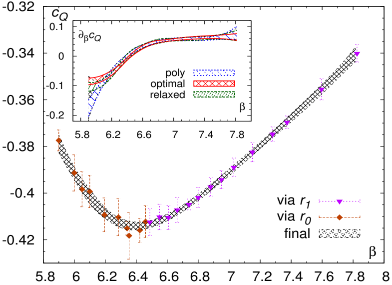
$$V^r(r) = \frac{V_i}{r_i}, \quad r^2 \frac{\partial V(r)}{\partial r}\bigg|_{r=r_i} = C_i,$$

with $V_0 = 0.954$, $V_1 = 0.2065$
and $C_0 = 1.65$, $C_1 = 1.0$

- Use HotQCD results for $2c_Q$

A. Bazavov et al., **PRD 90** 094503 (2014)

- Interpolate in β
- Add $N_\tau c_Q$ to $f_Q^{\text{bare}}(T[\beta, N_\tau])$.



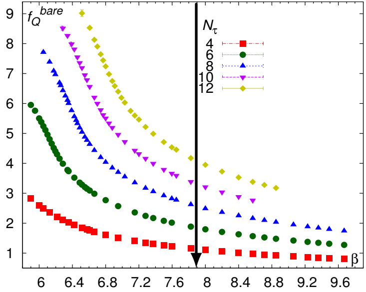
Drawbacks of $Q\bar{Q}$ procedure

- currently limited to $\beta \leq 7.825$

Advantages of $Q\bar{Q}$ procedure

- unambiguous procedure

Renormalization scheme: direct renormalization



$T(\beta, N_\tau) = T(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}})$ implies

$$c_Q(\beta) = \frac{1}{N_\tau}\Big\{N_\tau^{\mathrm{ref}} c_Q(\beta^{\mathrm{ref}}) + f_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - f_Q^{\mathrm{b}}(\beta, N_\tau)\Big\}$$

infer $c_Q(\beta)$ from $c_Q(\beta^{\mathrm{ref}})$

Essential caveat:
The approach is invalid if **cutoff effects persist after renormalization**.

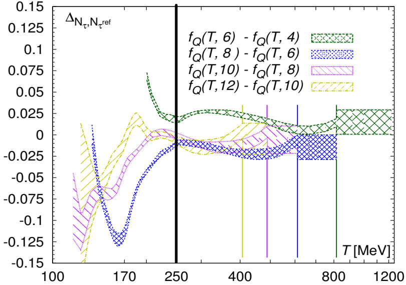
How to renormalize for $\beta > 7.825$?

Direct renormalization scheme

S. Gupta et al., **PRD 77** 034503 (2008)

$$f_Q(T(\beta, N_\tau), N_\tau) = f_Q^{\mathrm{b}}(\beta, N_\tau) + N_\tau c_Q(\beta)$$

Renormalization scheme: direct renormalization



$\Delta_{N_\tau, N_\tau^{\rm ref}} = f_Q^{\rm r}(\beta, N_\tau) - f_Q^{\rm r}(\beta^{\rm ref}, N_\tau^{\rm ref})$

$T < 250$ MeV: *large*, fluctuating
$T > 250$ MeV: *small*, **rather flat**
We estimate $\Delta_{N_\tau, N_\tau^{\rm ref}}$ for $\beta > 7.825$ as constant with conservative error.

$T(\beta, N_\tau) = T(\beta^{\rm ref}, N_\tau^{\rm ref})$ implies

$$c_Q(\beta) = \frac{1}{N_\tau}\Big\{N_\tau^{\rm ref} c_Q(\beta^{\rm ref}) + \Delta_{N_\tau, N_\tau^{\rm ref}} + f_Q^{\rm b}(\beta^{\rm ref}, N_\tau^{\rm ref}) - f_Q^{\rm b}(\beta, N_\tau)\Big\}$$

infer $c_Q(\beta)$ from $c_Q(\beta^{\rm ref})$

Essential caveat:
The approach is invalid if **cutoff effects persist after renormalization**.

- **Compute cutoff effects for low** β and include in relation.
- **Estimate cutoff effects for high** β and include as well.
- **Finally check consistency!** ✓

Renormalization scheme: gradient flow

Gradient flow approach

M. Lüscher, **JHEP 1008** 071 (2010)

Diffusion-type field evolution in an artificial **fifth dimension** t

$$\dot{V}_\mu = -g_0^2 \{\partial_\mu S[V]\} V_\mu$$

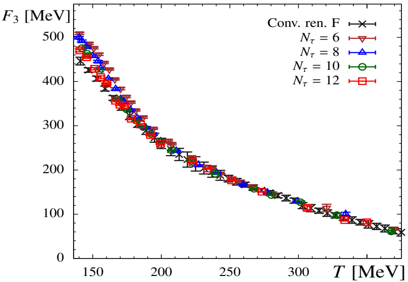
Fields V_μ at finite flow time

$$V_\mu \equiv V_\mu(x,t), \; V_\mu(x,0) = U_\mu(x)$$

are smeared out over length scale $f_t = \sqrt{8t}$, have no short distance singularities, **no UV divergences**

fixed flow time t defines a specific renormalization scheme, if

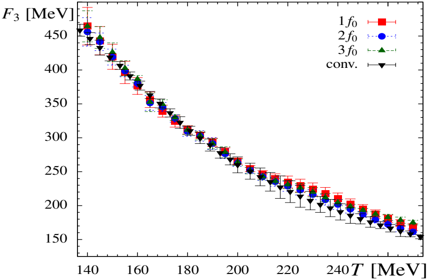
$$a \ll f_t = \sqrt{8t} \ll 1/T = aN_\tau$$



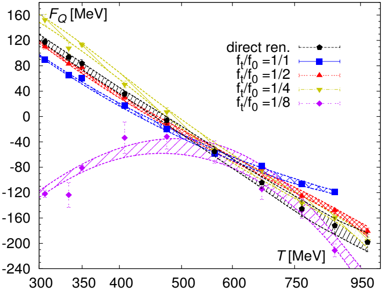
P. Petreczky, H.-P. Schadler, **PRD 92** 094517 (2015)

- $T \lesssim 400$ MeV: f_t **dependence** mild, constant differences.
- Cross-check of $Q\bar{Q}$ procedure with result at flow time f_t.

Gradient flow renormalization at high temperatures



P. Petreczky, H.-P. Schadler, **PRD 92** 094517 (2015)



The continuum limit at low T is within errors **independent of** f_t.

Higher T (smaller a): $0 < f_t \leq 1$

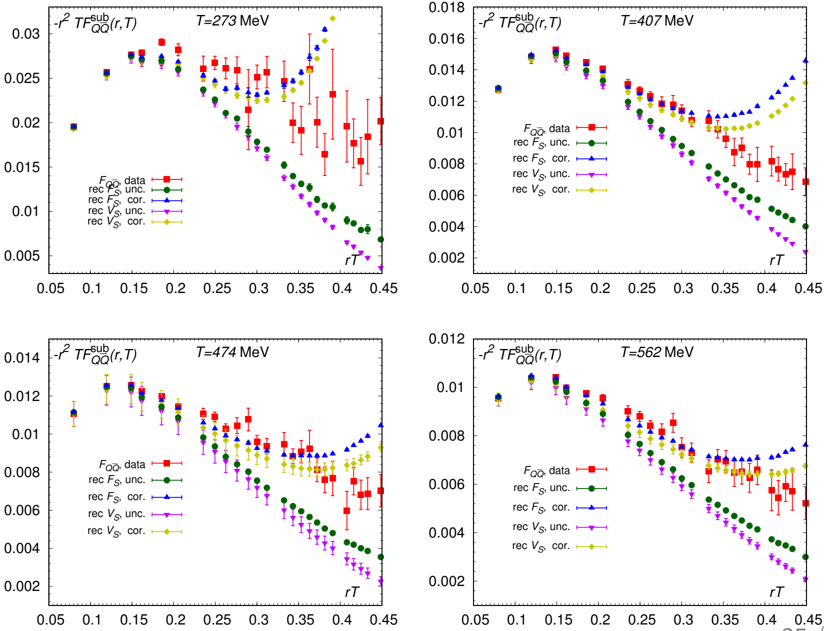
$F_Q(T, 12)$ via direct renormalization & gradient flow for different f_t.

Strong f_t **dependent cutoff effects**

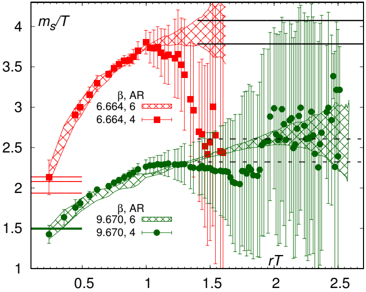
At LO: smaller S_Q for larger f_t.

Larger N_τ needed to afford smaller flow times at higher temperatures

Reconstruction of the Polyakov loop correlator

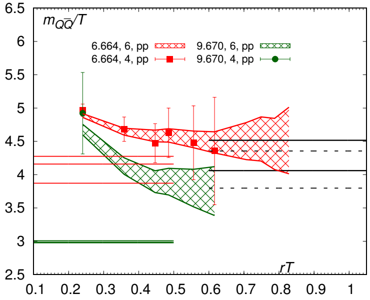


Asymptotic screening of the singlet free energy



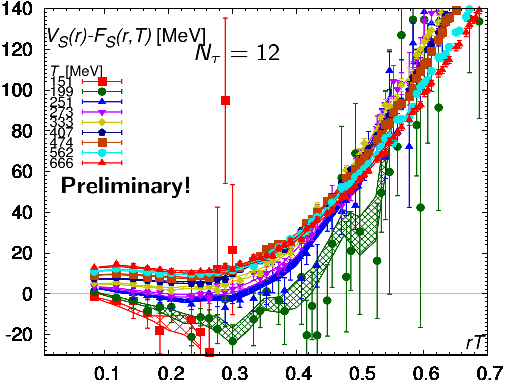
- The singlet screening mass is volume independent within errors.
- The screening mass reaches saturation at $rT \sim 1-1.5$.
- Huge ensemble sizes are required due to large noise contamination.
- We estimate the asymptotic screening and its error from its value at intermediate distances: $m_S(r \to \infty) \simeq m_S(r \sim 0.7) + 0.5 \pm 0.1$.

Asymptotic screening of the free energy



- The screening mass is volume independent within errors.
- The screening mass reaches saturation at $rT \sim 0.45$.
- Even huger ensemble sizes are required due to small signal.
- We estimate the asymptotic screening and its error from its value at intermediate distances: $m_{Q\bar{Q}}(r \to \infty) \simeq m_{Q\bar{Q}}(r \sim 0.24) - 0.6 \pm 0.2$.

Is F_S a good estimate for $\mathrm{Re}\, V_S$?



- Thermal modifications are small for $r \to 0 \to$ study $V_S(r) - F_S(r, T)$.
- $V_S(r)$ and $F_S(r, T)$ differ by up to 10 MeV for $rT \lesssim 0.27$.