Recent results on QCD thermodynamics from lattice

Sayantan Sharma

BROOKHAVEN NATIONAL LABORATORY

February 1, 2017

Outline

1. The QCD phase diagram: outstanding issues
2. Symmetries
3. Degrees of freedom in QCD
4. Realistic modelling of the heavy ion experiments and lattice

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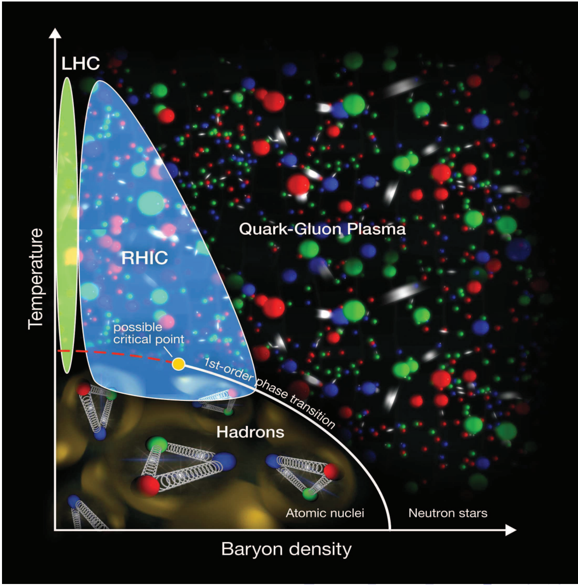
The QCD phase diagram: outstanding issues

- Understanding QCD phase diagram is one of the most challenging problems in the recent years.
- The underlying physics of confinement and chiral symmetry breaking is not yet completely understood.
  [Schaefer and Shuryak, 96]
- Challenges bring in new opportunities!
- Major new insights from Lattice QCD in last two years.
- It is also important in view of the BESII program at RHIC in 2019-2020.



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  - QCD thermodynamics at finite density.
  - Understanding freezeout conditions in HIC.
- The exciting news about developments in heavy quark potentials and color screening would be covered in the following talks:
  [See A. Rothkopf, Wed 16.25 PM, J. H. Weber, Wed 16.50 PM.]

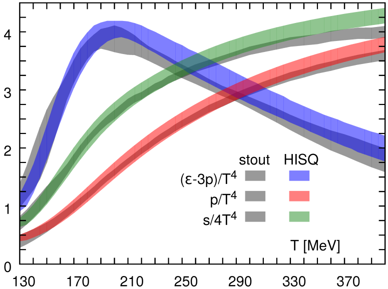
Outline

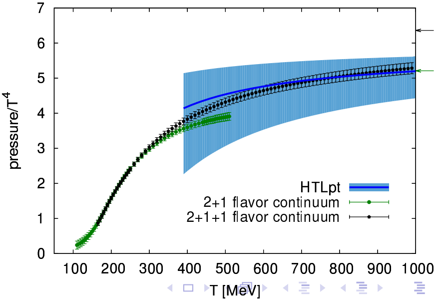
The phase diagram at $\mu_B = 0$

- For finite quark masses, no unique order parameter.
- It is now well established that $\mu_B = 0$ chiral symmetry restoration occurs via crossover transition with a $T_c = 154(9)$ MeV.

  [Budapest-Wuppertal collaboration, 1309.5258, HotQCD collaboration, Bazavov et. al, 1407.6387]
- The EoS for $2+1$ QCD is measured in the continuum and different lattice groups agree.[See Alexie Bazavov's talk, Wed 11:25AM].
- The dynamical effects of charm quarks included till 1 GeV $\rightarrow$ important as degrees of freedom and EoS during cosmological evolution.

  [Borsanyi et. al, 1606.07494]





The phase diagram at $\mu_B = 0$

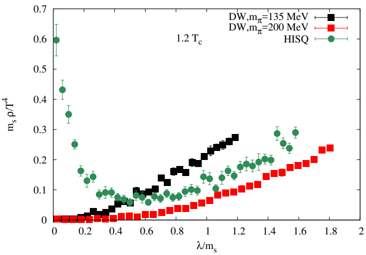
- However since $m_u, m_d << \Lambda_{QCD}$ there is an approximate $U_L(2) \times U_R(2)$ symmetry of QCD Lagrangian.
- $U_L(2) \times U_R(2) \rightarrow SU(2)_V \times SU(2)_A \times U_B(1) \times U_A(1)$
- At chiral crossover transition:
  $SU(2)_V \times SU(2)_A \times U_B(1) \rightarrow SU(2)_V \times U_B(1)$.
- Is $U_A(1)$ effectively restored at T_c? $\rightarrow$ can change the universality class of the second order phase transition at $\mu_B = 0$.
  Either O(4) or $U_L(2) \times U_R(2)/U_V(2)$

  [Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03, 13, Nakayama & Ohtsuki, 15]

The phase diagram at $\mu_B = 0$

- Not an exact symmetry$\rightarrow$ what observables to look for? Degeneracy of the 2-point correlators [Shuryak, 94] $\rightarrow$ higher point correlation functions imp.

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$



[V. Dick, et. al, 1602.02197]

- Observables non-analyticities+analytic part of the eigenvalue spectrum.

  [Aoki, Fukaya & Taniguchi, 1209.2061, HotQCD collaboration, 1205.3535, V. Dick et. al. 1502.06190]

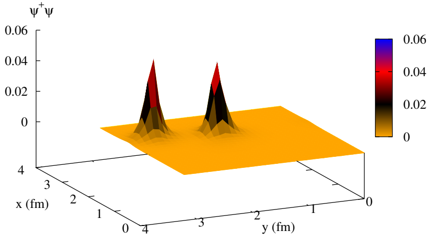
- Independent hints from study of screening masses (excitations) for π, η.

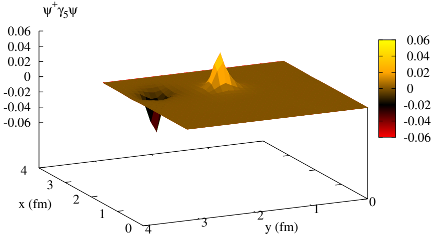
  [Y. Maezawa et. al. 1411.3018, B. Brandt et. al. 1608.06882]

- Non-analytic part still needs careful study. Analytic part of the spectrum strongly suggest that $U_A(1)$ is broken!

  [V. Dick, et. al, 1502.06190, 1602.02197, G. Cossu et. al., 1510.07395].

Microscopic origin of $U_A(1)$ breaking?





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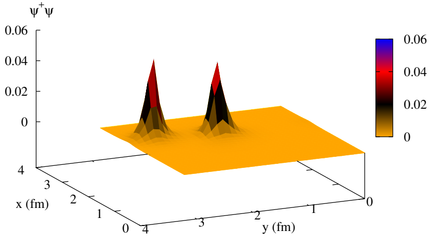
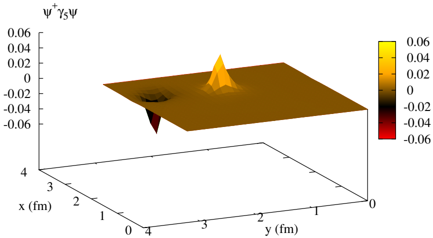


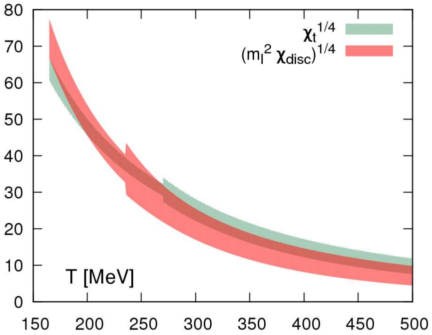


- Near-zero modes of QCD Dirac operator at $1.5\ T_c$ due to a weakly interacting instanton-antiinstanton pair!
- The density $\simeq 0.147(7) fm^{-4}$. This is much more dilute than an instanton liquid with density $1 fm^{-4}$.

[V. Dick, F. Karsch, E. Laermann, S. Mukherjee and S.S, 1502.06190].

Independent confirmation: Topological susceptibility

- Topological susceptibility measurement at high T on the lattice suffers from rare topological tunneling, lattice artifacts.
- Going towards continuum limit difficult due to freezing of topology.
- New fermionic observables developed to crosscheck the standard definition of $\chi_t = \int d^4x \langle F\tilde{F}(x) F\tilde{F}(0) \rangle$. [P. Petreczky, H-P Schadler, SS, 1606.03145].
- Continuum extrapolated results now available for QCD!



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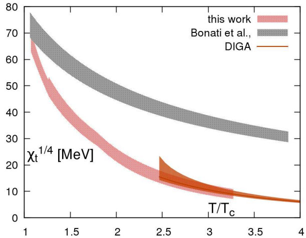
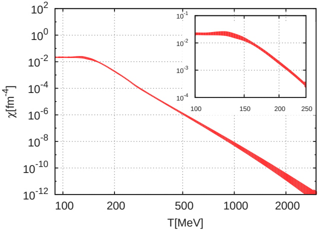


[P. Petreczky, H-P Schadler, SS, 1606.03145].

- Dilute gas prediction: $b = 4 - \frac{11N_c}{3} - \frac{2N_f}{3}$.

- Fit ansatz: $\chi_t^{1/4} = AT^{-b}$.
- $b = 0.9 - 1.2$ for $T < 250$ MeV. Agrees well with [Bonati et. al, 1512.06746]
- $T > 300$ MeV: Continuum extrapolated $b = 1.85(15)$ in agreement with dilute instanton gas.
  confirmed also in an independent study with improved topological tunneling techniques at high temperatures [Borsanyi et. al, 1606.07494]



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- Most lattice studies hint that $U_A(1)$ is broken significantly near and even beyond T_c.
- Underlying microscopic origin is being studied in quite detail. $\rightarrow$ hints to interplay between topology in QCD and chiral phase transition as suggested from studies by Shuryak and his collaborators [Shuryak, 82, Ilgenfritz & Shuryak, 88, Shuryak and Schaefer, 96].

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  [Shuryak, 82, Ilgenfritz & Shuryak, 88, Shuryak and Schaefer, 96].
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- Topological susceptibility has been measured in lattice QCD $\rightarrow$ suggests non-trivial top-fluctuations in hot QCD medium even at 1 GeV, consequences for axion cosmology.

- **Challenges** Is is possible to understand the intricate connection between chiral symmetry breaking and confinement through a detailed study of the topological constituents of QCD near T_c? What are the topological constituents near T_c?

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Basic observables: Fluctuations of conserved charges

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- One of the methods to circumvent sign problem:
  Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T
  [Bielefeld-Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0, T)}{T^4} + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \chi_2^B(0, T) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \ldots$$

Different orders of fluctuations appear as Taylor coefficients

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- Higher derivatives $\rightarrow$ more inversions
  Inversion is the most expensive step on the lattice !
- Why extending to higher orders so difficult?
  - Matrix inversions increasing with the order
  - Delicate cancellation between a large number of terms for higher order QNS.

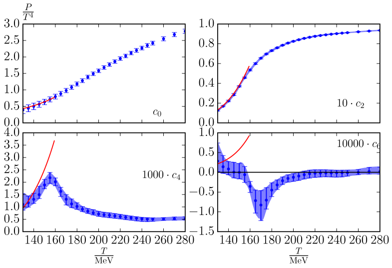
Possible ways out

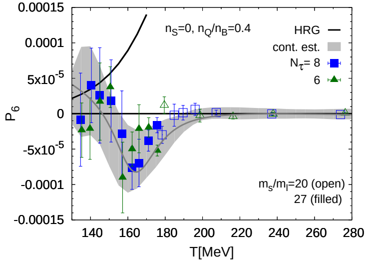
- Introducing μ such that it appears as a linear term multiplying the conserved number [Gavai & Sharma, 1406.0474] as in the continuum instead of conventional e^{μ}.

$$D(0)_{xy} - \frac{\mu a}{2}\eta_4(x)\left[U_4^{\dagger}(y)\delta_{x,y+\hat{4}} + U_4(x)\delta_{x,y-\hat{4}}\right] .$$

- No divergences exist for sixth order susceptibilities and beyond. [Gavai & Sharma, 1406.0474]
- Number of inversions significantly reduced for $6th$ and higher orders. For 8th order QNS the no. of matrix inversions reduced from 20 to 8.
- Calculate n_B in imaginary μ and extract higher order fluctuations. [See Szabolcs Borsanyi's talk, Fri, 14:25 PM].
- Current state of the art: 6th order fluctuations known with very good precision: [Gunther et. al, 1607.02493, D'Elia et. al., 1611.08285, Bielefeld-BNL-CCNU, 1701.04325]

Possible ways out

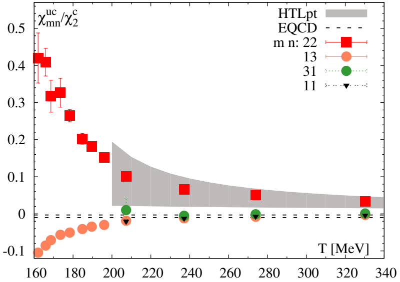




[Gunther et. al, 1607.02493, Bielefeld-BNL-CCNU, 1701.04325]

- Heavy-ion experiments at different collision energies sets non-trivial constraints: $n_s = 0$, n_B/n_Q=constant.
- Can be implemented easily within Taylor series method. [Bielefeld-BNL-CCNU, 1701.04325]. Also implemented in imaginary μ. [Gunther et. al, 1607.02493]
- χ_6^B has very distinct structure $\rightarrow$ deviates from Hadron Resonance gas picture for $T < T_c$. Weak coupling results cannot predict the dip at $T > T_c$ $\rightarrow$ signatures of a strongly coupled medium?

Can we understand degrees of freedom in hot QCD medium?



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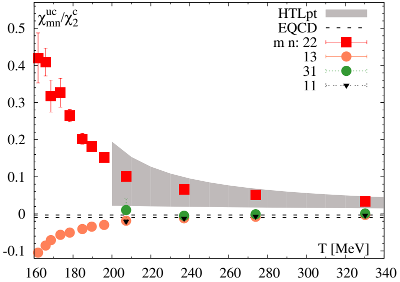

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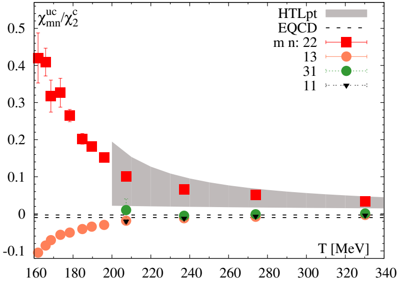

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- Deviation from Hard Thermal Loop results between 160 – 200 MeV.

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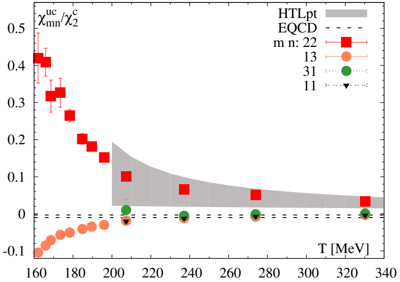

- Can the hot QCD medium be described in terms of quasi-particles?
- Look at a simple system: correlation between charm and light quarks
- Deviation from Hard Thermal Loop results between 160 – 200 MeV.
- Charm quarks not a good quasi-particle below 200 MeV? What happens after charm hadron melts at T_c. [Mukherjee, Petreczky, SS, 1509.08887.].

Charm d.o.f at deconfinement

- What are the microscopic constituents beyond T_c?
  Model charm d.o.f in QCD medium as charm meson+baryon+quark-like excitations.

$$p_C(T,\mu_B,\mu_C) = p_M(T)\cosh\left(\frac{\mu_C}{T}\right) + p_{B,C=1}(T)\cosh\left(\frac{\mu_C+\mu_B}{T}\right) + p_q(T)\cosh\left(\frac{\mu_C+\mu_B/3}{T}\right).$$

- Considering fluctuations upto 4th order there are 6 measurements and thus 2 trivial constraints $\chi_4^C = \chi_2^C$, $\chi_{11}^{BC} = \chi_{13}^{BC}$.
- A more non-trivial constraint:
  $c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0$.
- Non-trivial check: LQCD data agree with the constraints in the model thus validating it. [Mukherjee, Petreczky, SS, 1509.08887].

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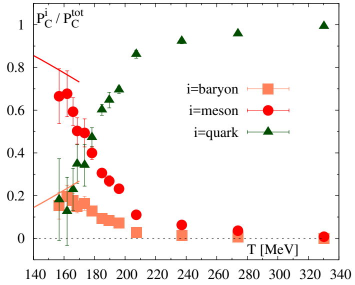

- Meson and baryon like excitations survive upto $1.2\,T_c$.
- Quark-quasiparticles start dominating the pressure beyond $T \gtrsim 200$ MeV $\Rightarrow$ hints of strongly coupled QGP [Mukherjee, Petreczky, SS, 1509.08887].
- Introduce more sophistications: it is now possible to rule out di-quark excitations atleast for the charm sector for $T > T_c$..
- **Challenge** : Understand microscopics in strange sector. Fate of Kaon fluctuations reported. [Noronha-Hostler et. al., 1607.02527]

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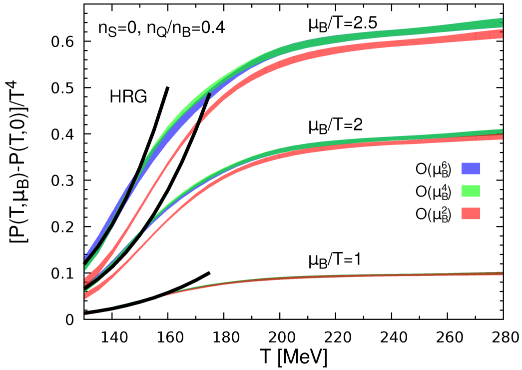
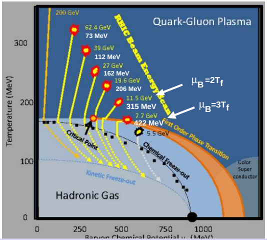
Basic issues and requirements

- Do the expanding fireball formed in most central heavy-ion collisions attain local thermal equilibrium? $\rightarrow$ can be modelled by viscous hydrodynamics.
- Equation of state from lattice QCD indispensable input for the hydrodynamic evolution.
- For most RHIC energies: $n_S = 0$, $n_Q/n_B = 0.4$ need to calculate EOS for the constrained case.
- In order to disentangle the thermal fluctuations from non-thermal ones, need to measure suitable fluctuations of conserved charges on the lattice $\rightarrow$ then perform dynamical evolution in rapidity and relate to experimental measurements. [Asakawa & Kitazawa 1512.05038].
  Dynamical evolution of fluctuations near critical point in model studies show interesting patterns Mukherjee, Venugopalan, Yin, 1605.09341

EoS away from criticality

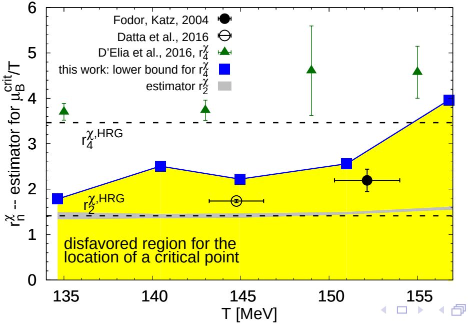
- The pressure already well determined by χ_B^6 for $\mu_B/T \leq 2.5$ [Bielefeld-BNL-CCNU, 1701.04325]. [See H. Ohno's talk, Fri 14:00 PM].
- Extension to $\mu_B/T \sim 3$ is in progress to cover all the allowed range for energies of heavy-ion collisions to be probed in Beam Energy Scan II experiments $\rightarrow$ need to measure χ_8^B? Control errors on such measurements.



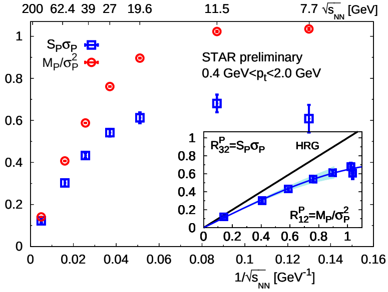


Critical-end point search from Lattice

- The series for χ_2^B should diverge at the critical point. On finite lattice ratios of Taylor coefficients equal, indep. of volume [Gavai& Gupta, 03]
- Radius of convergence from Taylor expansion: $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.
- Definition is true for $n \to \infty$. How large n could be on a finite lattice?
- New studies from Taylor expansions and imaginary μ sets a current bound for CEP to be $\mu_B/T > 2$ [Bielefeld-BNL-CCNU, 1701.04325, D'Elia et. al., 1611.08285] though some studies point to a lower bound. [Datta et. al., 1612.06673, Fodor and Katz, 04]



Fluctuations measured at freezeout: Are these thermalized?



- Ratios of cumulants are independent of the volume of the fireball
- First to second moment: $\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}\left(\frac{\mu_B}{T}\right)^3$
- $S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}\left(\frac{\mu_B}{T}\right)^3$
  μ_B is unknown parameter and model dependent.

Clear deviation from Hadron Resonance gas description in experimental data!

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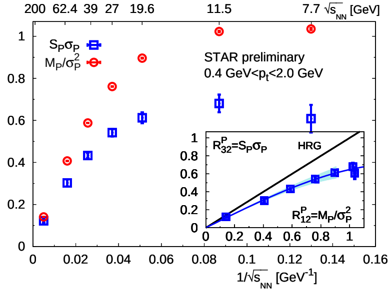

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  μ_B is unknown parameter and model dependent.
- Instead $S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \ldots$ removes model uncertainties!

[Karsch et. al., arxiv:1512.06987]

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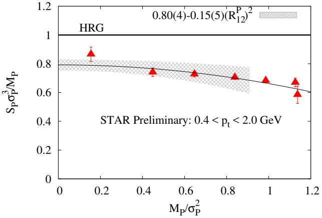
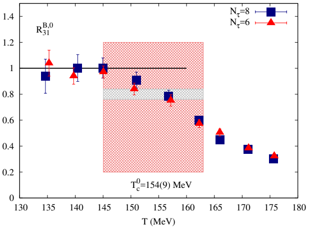
Fluctuations at freezeout and lattice

$$R_{31}^B = \frac{S_B \sigma_B^3}{M_B} = \frac{\chi_4^B}{\chi_2^B} + \frac{1}{6}\left[\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right]\left(\frac{M_B}{\sigma_B^2}\right)^2$$ [Karsch et. al., arxiv:1512.06987]

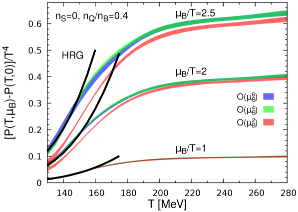
- Experimental data tantalizingly close to QCD prediction → Accidental coincidence or hints of thermalization?





- **Challenges** Need to perform dynamical evolution of the ratios of cumulants.
- Caveat: In experiments only charged baryons (protons) measured $n_P \neq n_B$!, take into account p_t cuts in the data. Look for suitable observables!

Summary and Outlook



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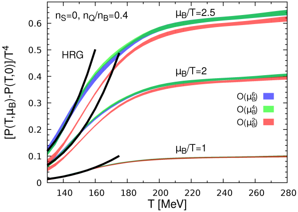

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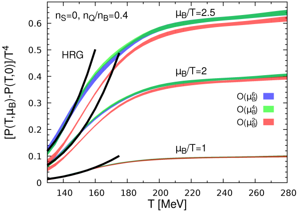

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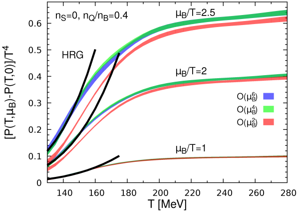

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- Fluctuations data suggest QCD medium beyond T_c non-perturbative. Quasi-particle picture valid $\sim 1.5\,T_c$ and beyond. Existence of broad resonance atleast in charm sector observed $\rightarrow$ crucial for dynamical modelling of hot QCD medium.