Heavy Quarkonium at Finite Temperature from Lattice EFTs

Alexander Rothkopf
Institute for Theoretical Physics
Heidelberg University

References:

with Y. Burnier and O.Kaczmarek	JHEP 1512 (2015) 101
	JHEP 1610 (2016) 032
with S.Kim and P. Petreczky	PRD91 (2015) 054511, NPA956 (2016) 713
	in preparation
with J. Pawlowski	arXiv:1610.09531

VIITH WORKSHOP OF THE APS TOPICAL GROUP ON HADRONIC PHYSICS – WASHINGTON D.C., USA – FEBRUARY 1ST 2017

Physics Motivation

- From run1 and ongoing run2 at LHC: unprecedented amount of precision data

Bound states of $c\bar{c}$ or $b\bar{b}$: **Heavy quarkonium** $M_Q >> T_{med}$

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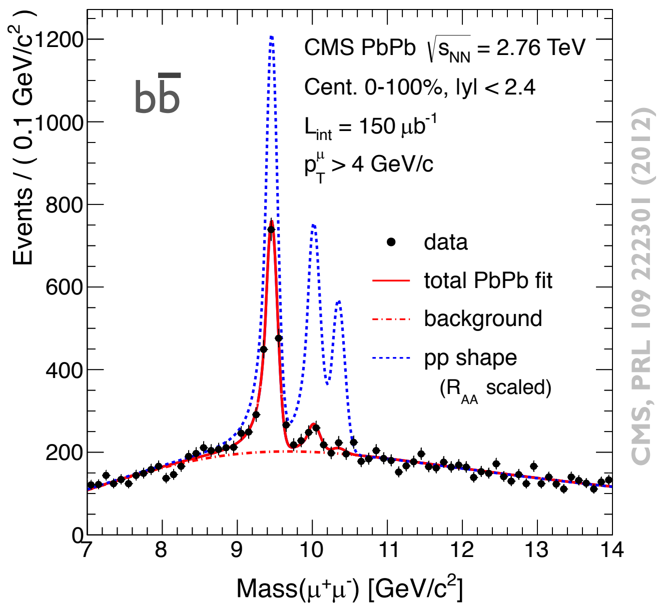

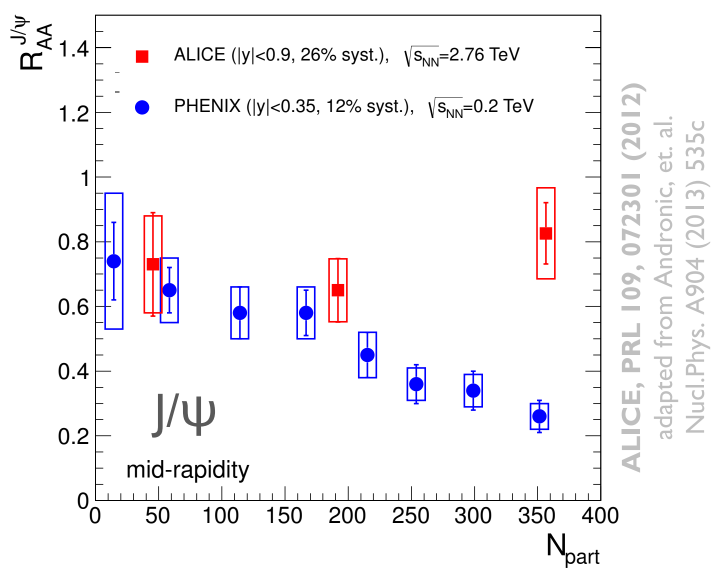

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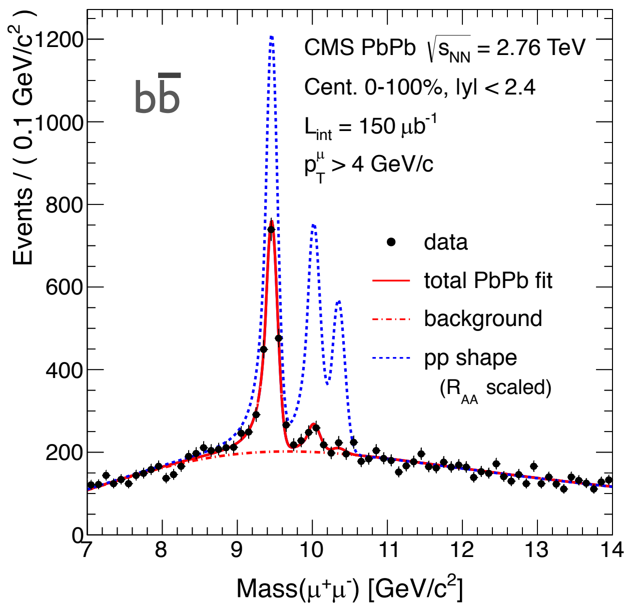
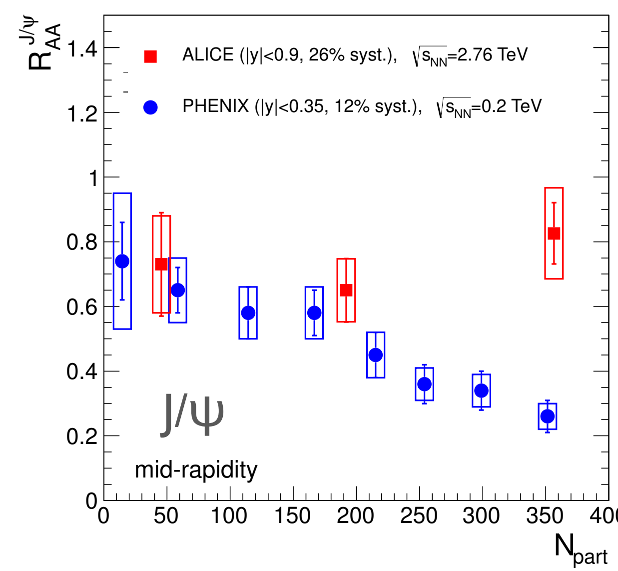


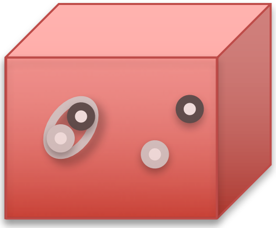
CMS, PRL 109 222301 (2012)



ALICE, PRL 109, 072301 (2012)
adapted from Andronic, et. al.
Nucl.Phys. A904 (2013) 535c

- Theory goal: 1^{st} principles insight into in-medium $Q\bar{Q}$ in heavy-ion collisions

A two-pronged approach to $Q\bar{Q}$



Assume full kinetic themalization of QQ
&
Static medium from lattice QCD

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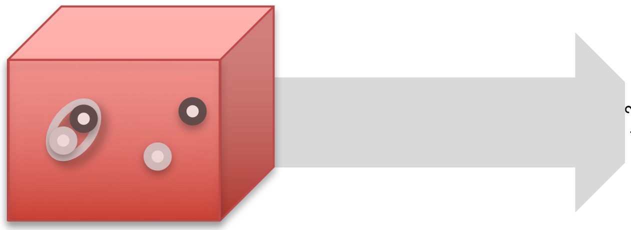

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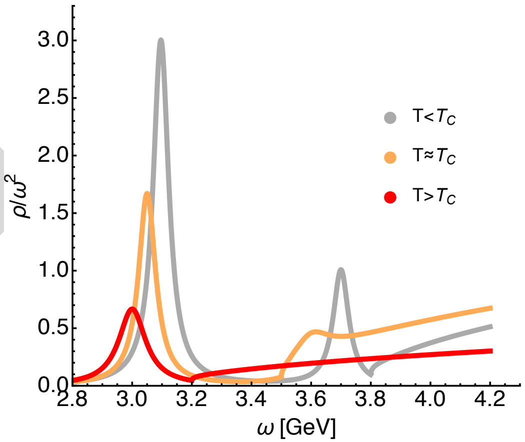
In-medium meson spectra



schematic depiction

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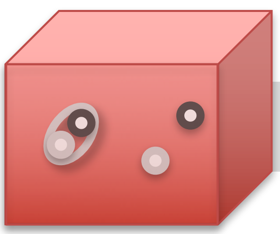

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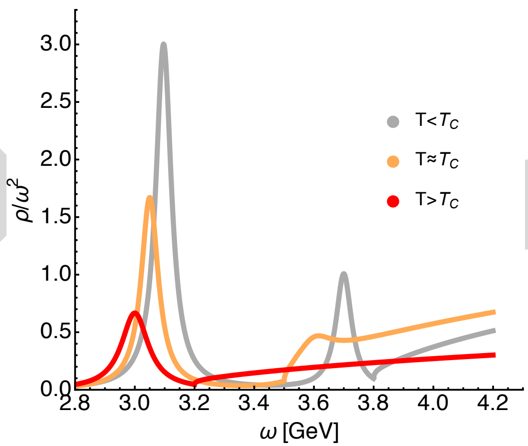

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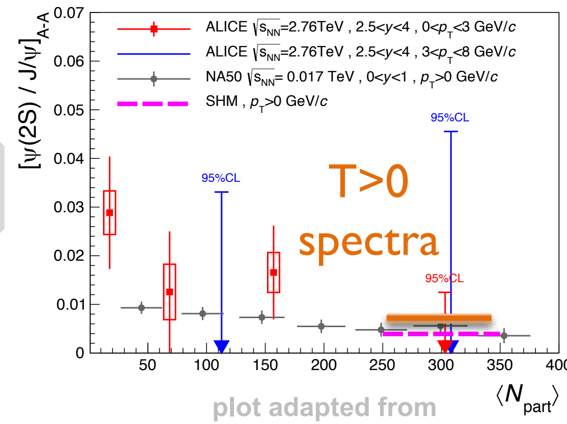
Observables
e.g. ψ’ / J/ψ ratio



plot adapted from
ALICE Collaboration JHEP05(2016)179

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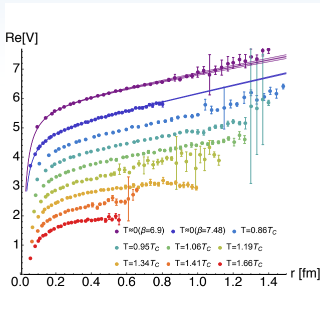
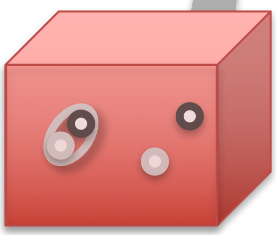
I. Via QQ potential from the lattice QCD Wilson loop

(currently static potential only)





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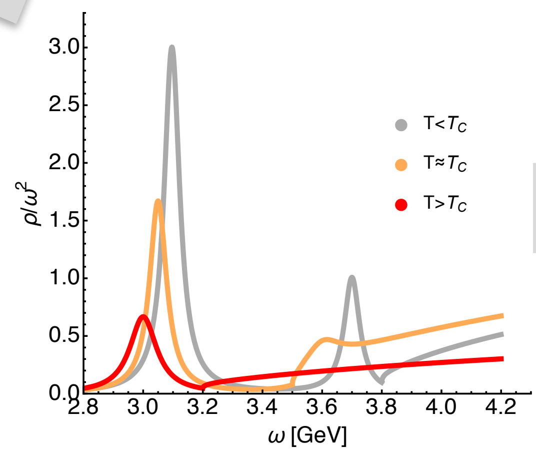

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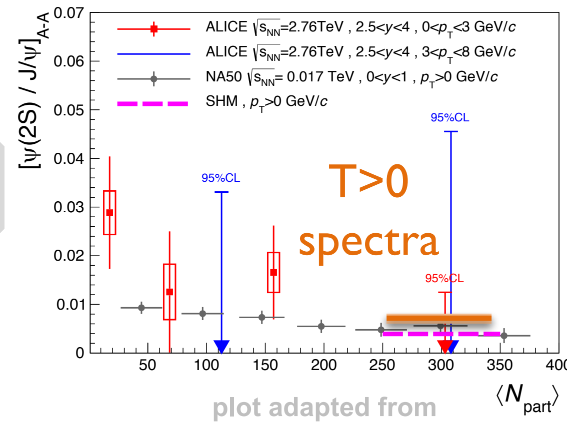

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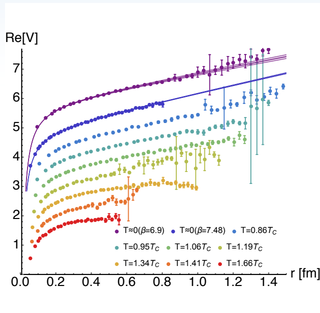

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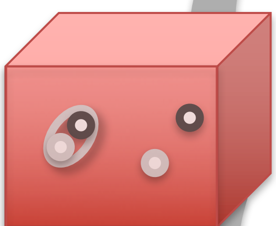

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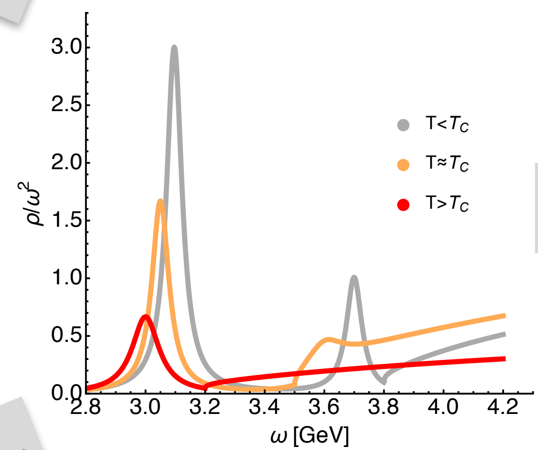

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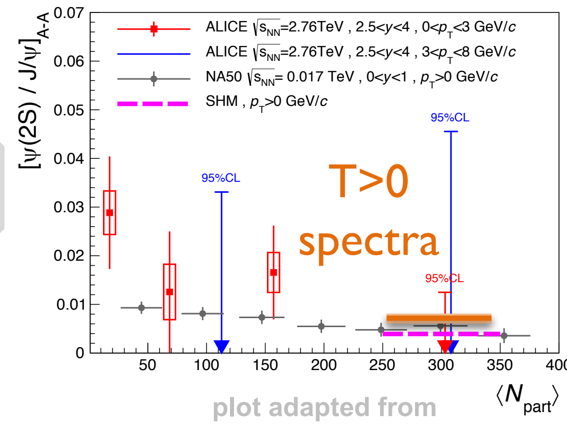

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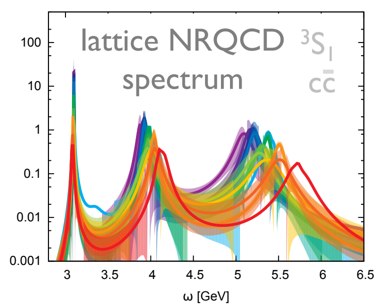

S. Kim, P. Petreczky, A.R.
in progress

II. Direct reconstruction of lattice meson spectra in NRQCD
(limited resolution)

A common challenge

- Dynamical information e.g. spectral functions not directly accessible on the lattice

$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega)$$

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$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
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- Bayes theorem: Regularize the naïve χ^2 functional $P[D|\rho]$ through a prior $P[\rho|I]$

$$P[\rho|D,I] \propto P[D|\rho]\; P[\rho|I] \quad \Rightarrow \quad \frac{\delta P[\rho|D,I]}{\delta \rho_l} \stackrel{!}{=} 0$$

M. Jarrell, J. Gubernatis, Phys. Rep. 269 (3) (1996)

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  - Differ in the regulator functional P[ρ|I] and how to find the most probable spectrum

$P[\rho|I]=\mathrm{Exp}[S_{BR}]$ $\quad S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log\left[\frac{\rho}{m}\right]\right)$

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  - Systematic errors different: MEM extra smoothing, BR prone to ringing artifacts

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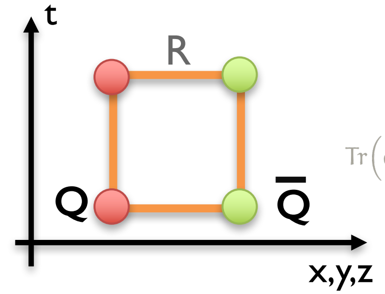

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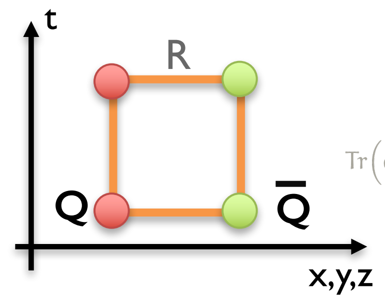

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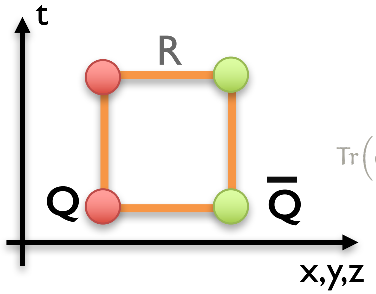

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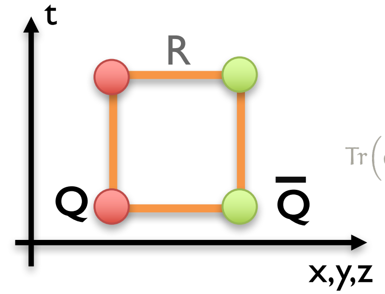

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- Spectral functions as bridge between the Euclidean and real-time Wilson loop

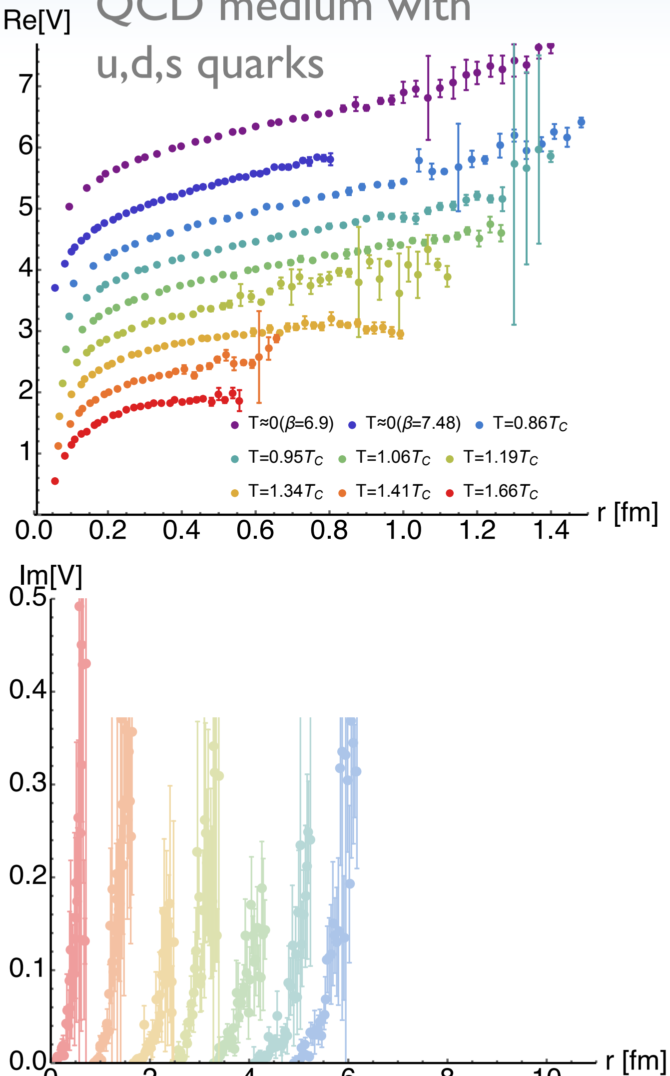
$$W_\square(R,t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \rho_\square(R,\omega) \quad \Longleftrightarrow \quad W_\square(R,\tau) = \int_{-\infty}^{\infty} d\omega\, e^{-\omega\tau} \rho_\square(R,\omega)$$

see A.R., T.Hatsuda & S.Sasaki , PRL 108 (2012) 162001, Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

T>0 static potential from the lattice

QCD medium with u,d,s quarks

Y.Burnier, O.Kaczmarek, A.R. PRL 114 (2015) 082001



Nf=2+1, 48^3x12, asqtad action, m_π~300MeV

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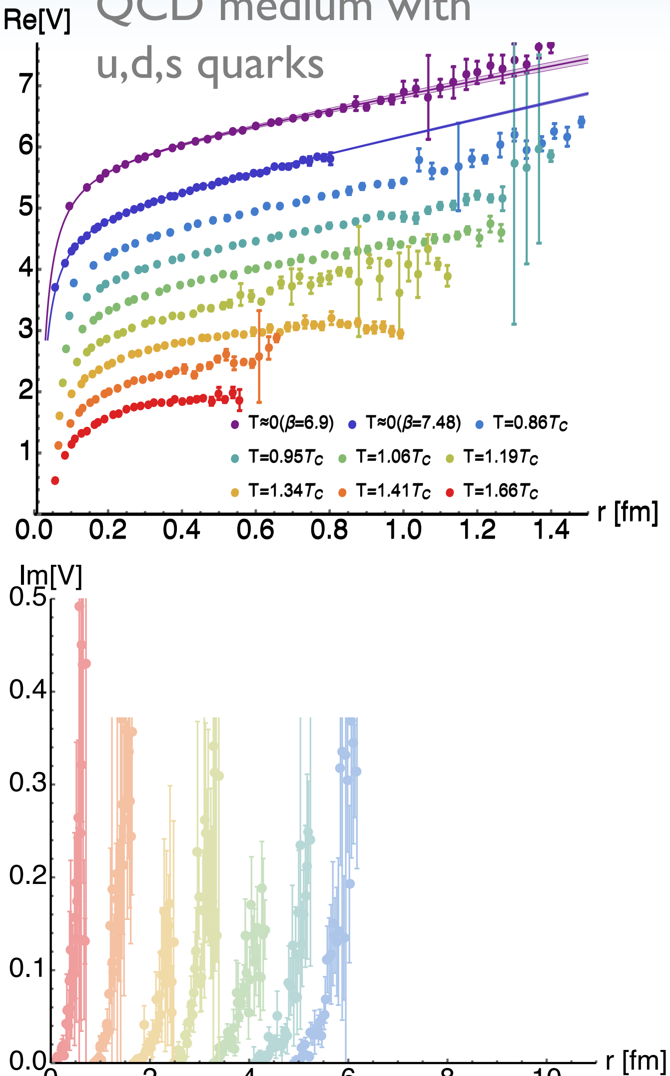

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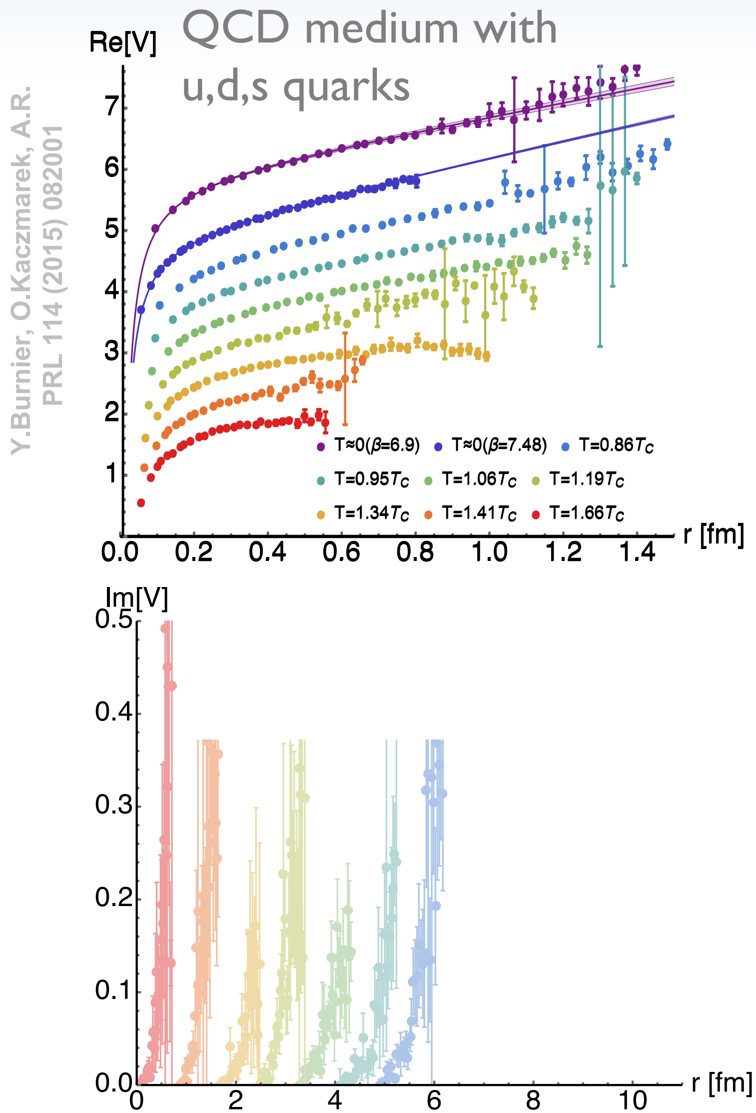

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Y. Burnier, A.R. PLB753 (2016) 232

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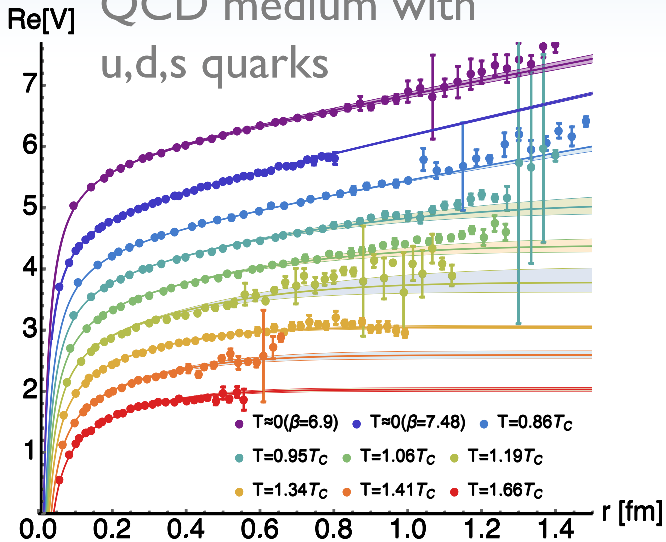

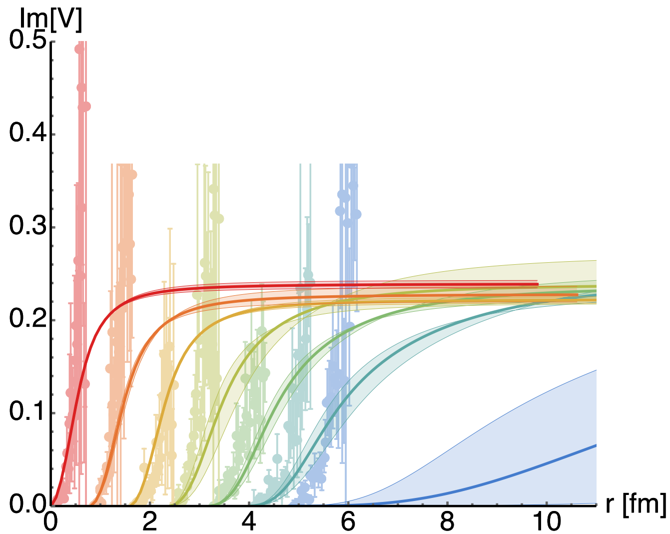

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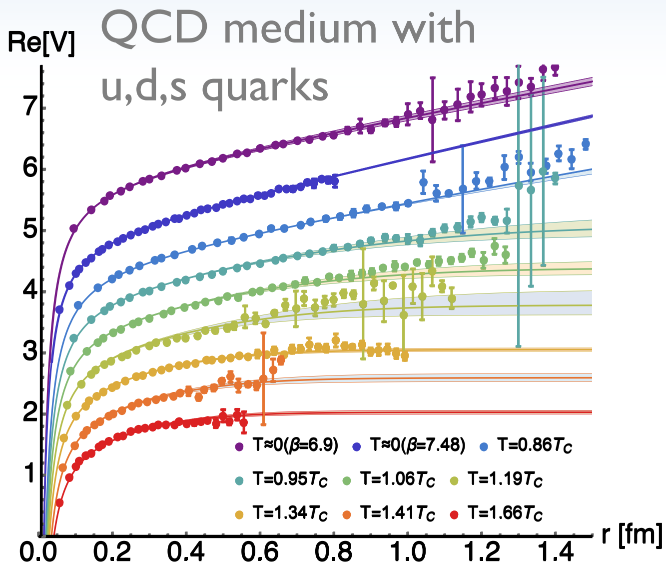

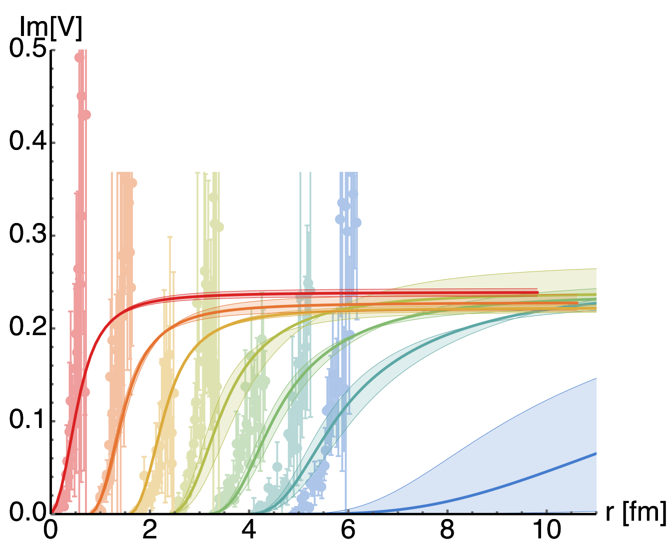

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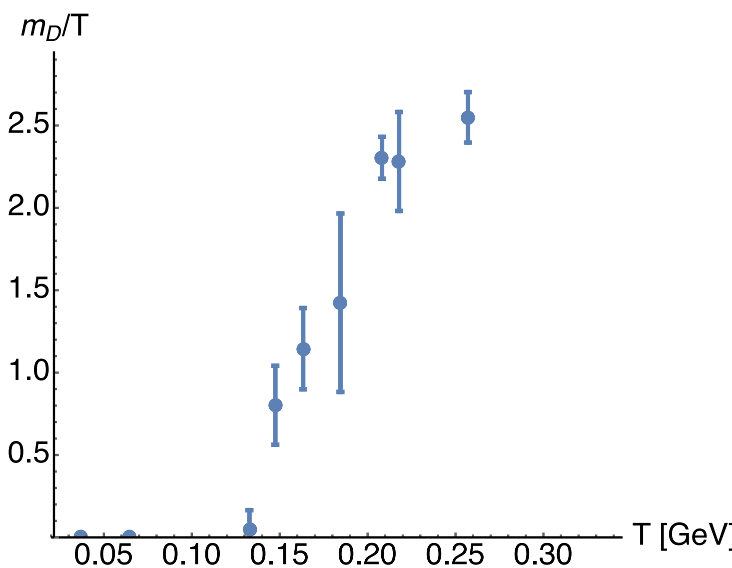

Y.Burnier, O.Kaczmarek, A.R., JHEP 1512 (2015) 101

S-wave spectral functions

- In absence of a true continuum extrapolation of the potential, combine: phenomenological T=0 Cornell potential & T>0 lattice QCD m_D

S-wave spectral functions

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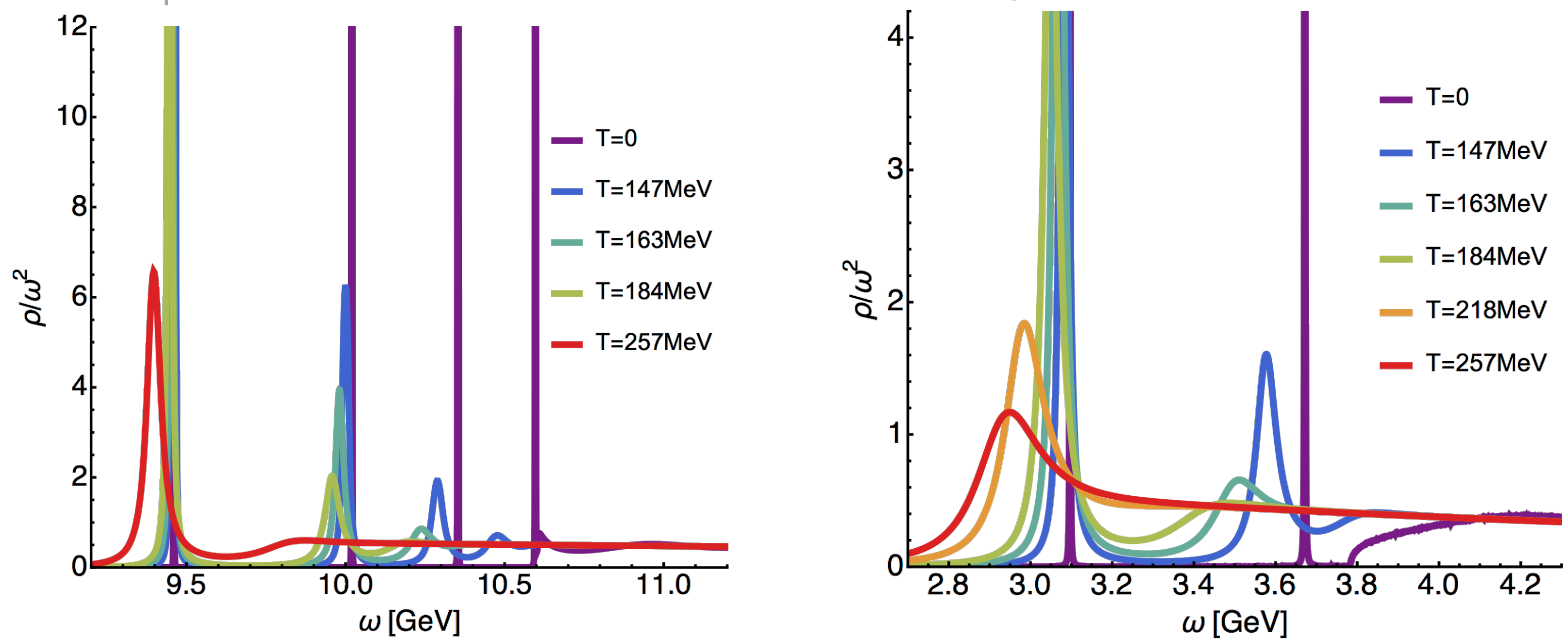
$^3S_1$ Charmonium J/ψ channel



Y.Burnier, O.Kaczmarek, A.R. JHEP 1512 (2015) 101

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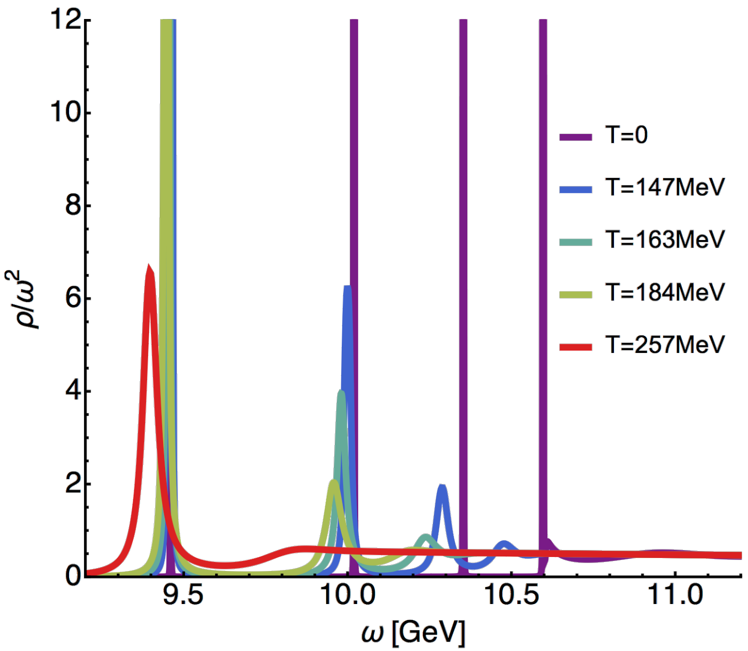

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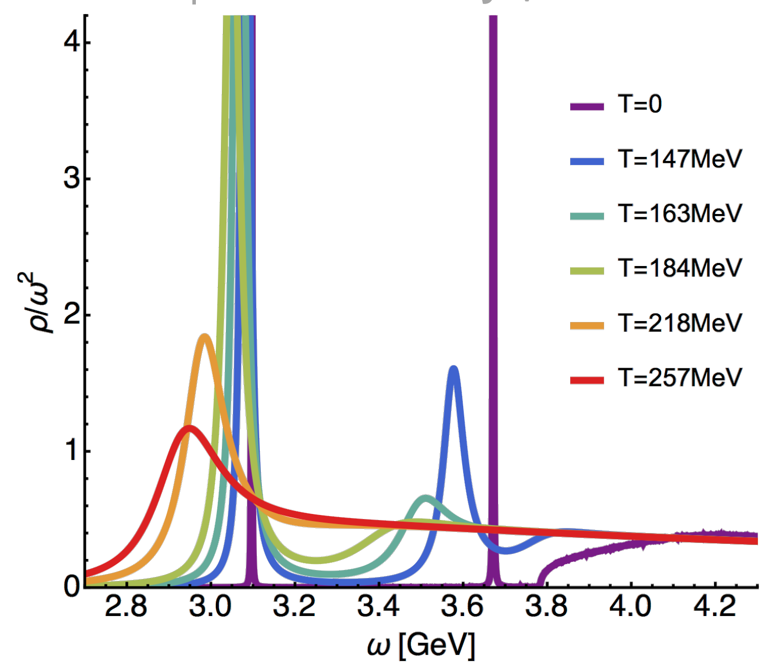
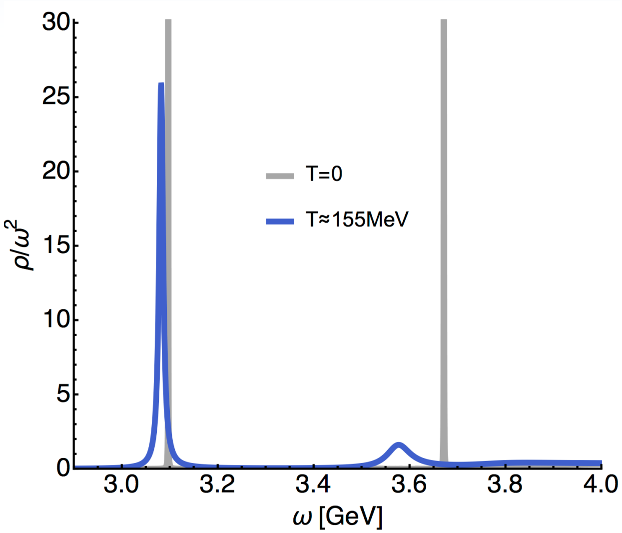


- Quarkonium melting is a gradual process, peaks do not suddenly disappear

ψ‘ to J/ψ ratio from T>0 spectra



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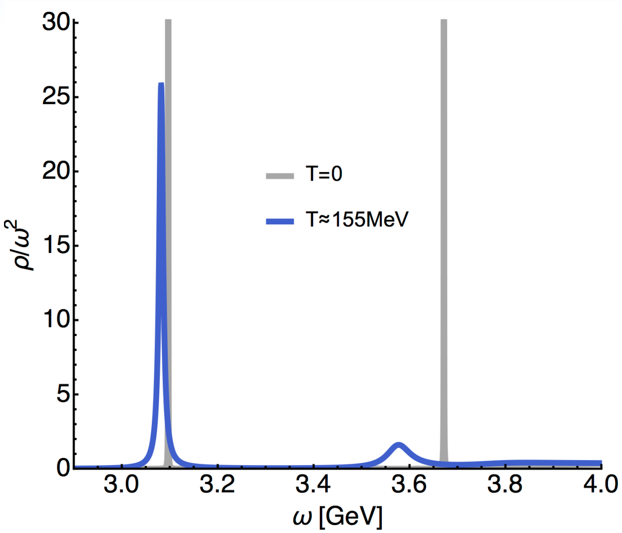

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(to leading order $\rho(P) = \rho(p_0^2 - \mathbf{p}^2)$)

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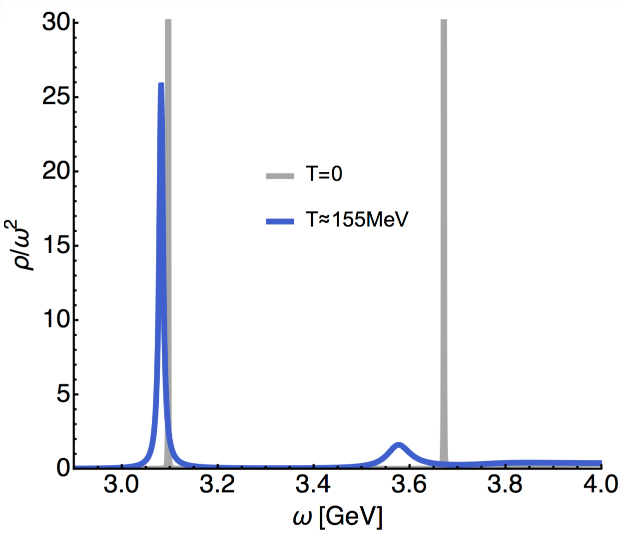

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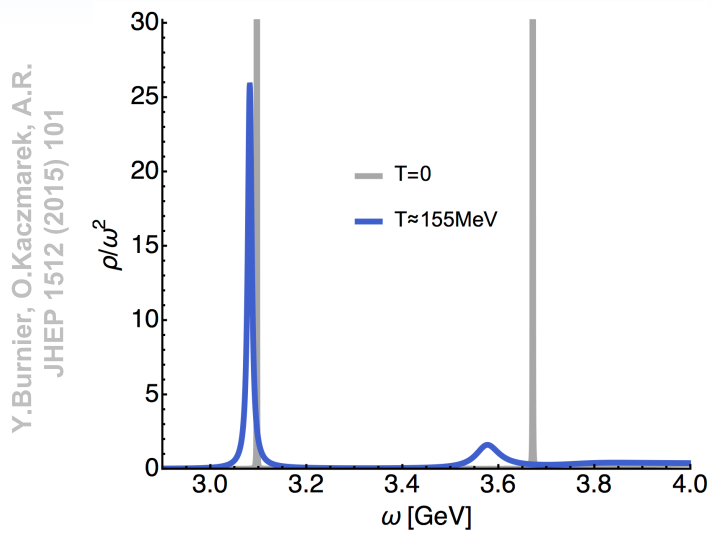

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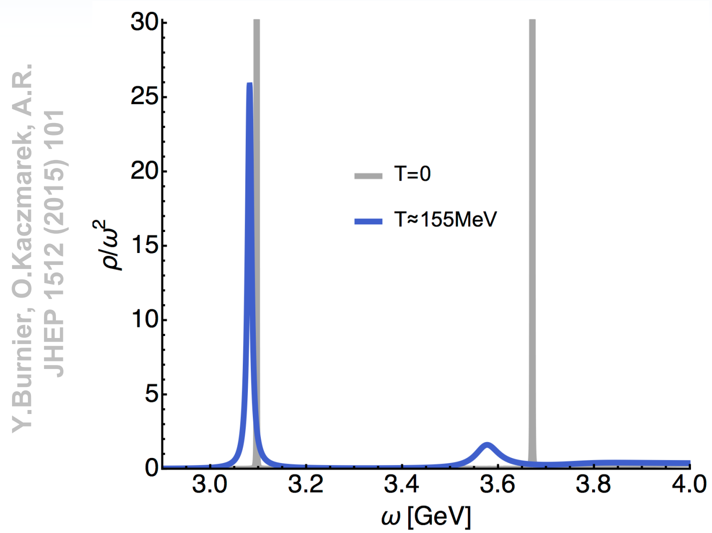

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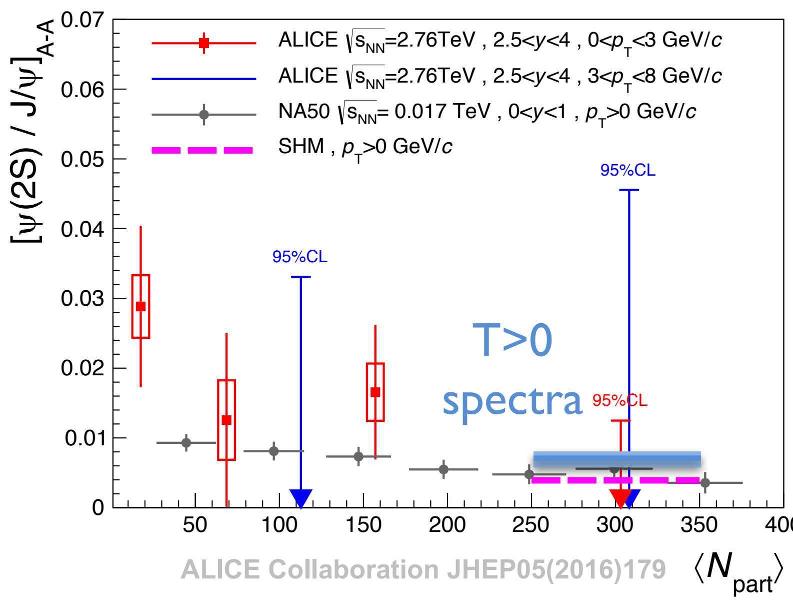

ALICE Collaboration JHEP05(2016)179

II. Direct determination: NRQCD

Relativistic treatment of light
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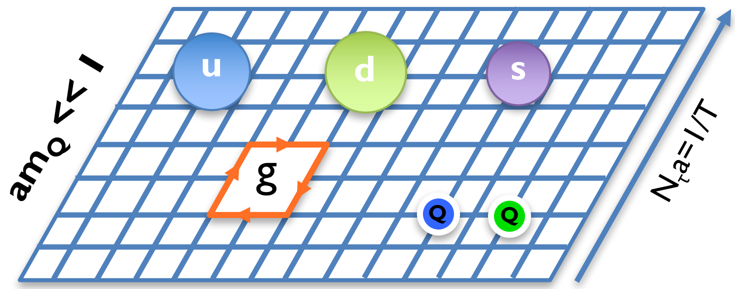

Full Lattice QCD simulation incl. $Q\bar{Q}$
(still too costly)

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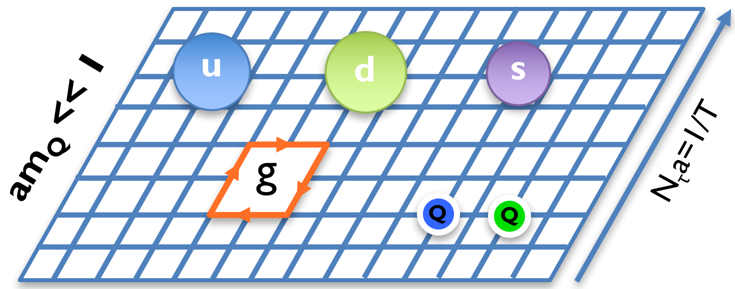

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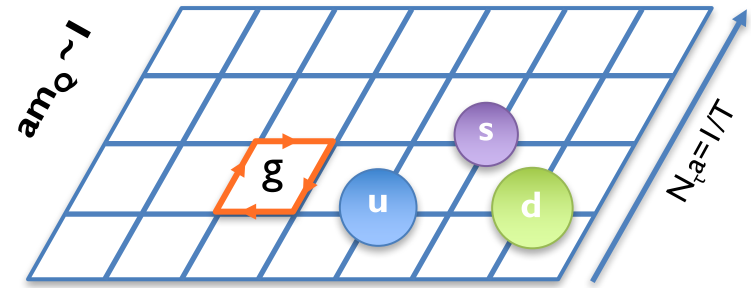

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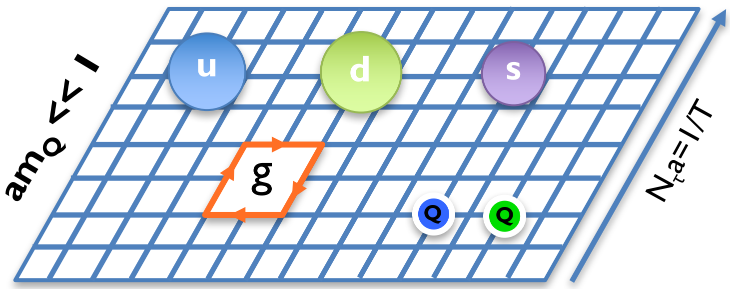

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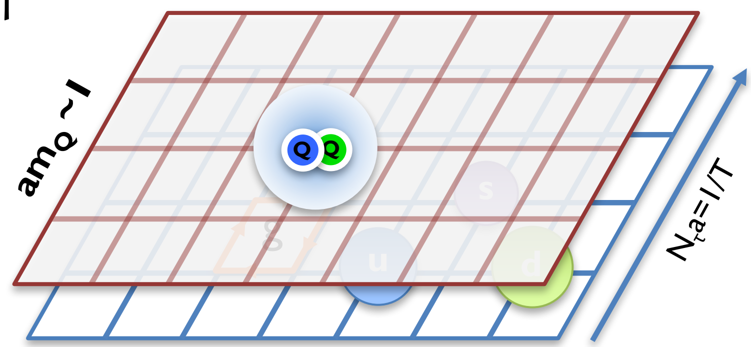

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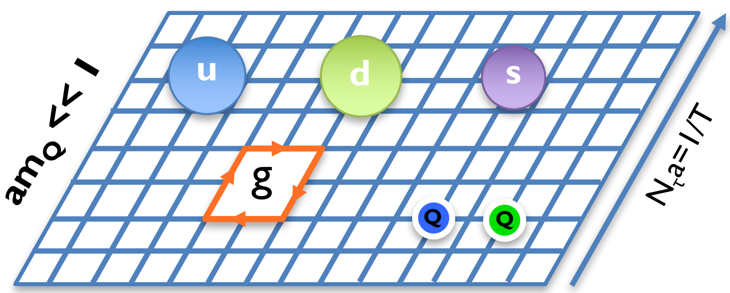

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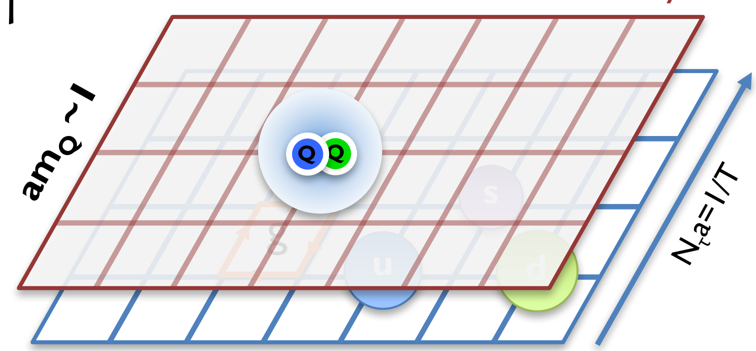

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Thacker, Lepage Phys.Rev. D43 (1991) 196-208

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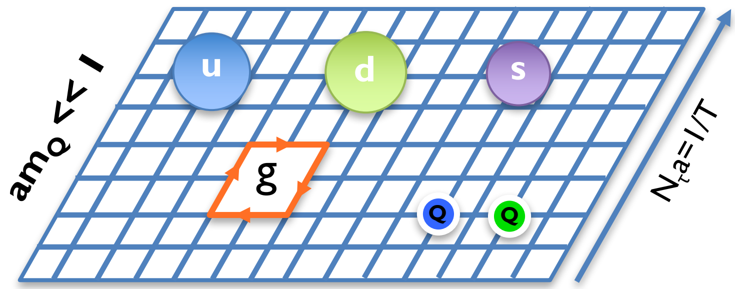

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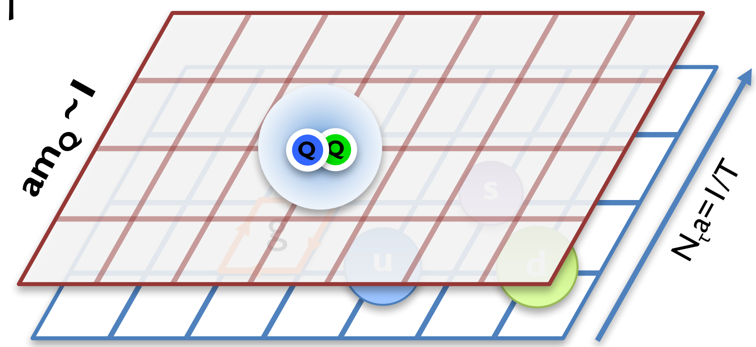

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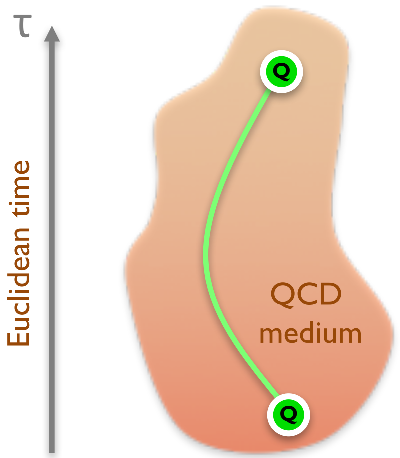
- State-of-the-art: realistic simulations of the QCD medium by the HotQCD collab.
  HotQCD PRD85 (2012) 054503, PRD90 (2014) 094503
  - 48^3x12 N_f=2+1 HISQ action m_π=161MeV T= [140 – 407] MeV $m_b a$= [2.8 – 0.96]

Correlation functions in NRQCD



Non-rel. propagator of
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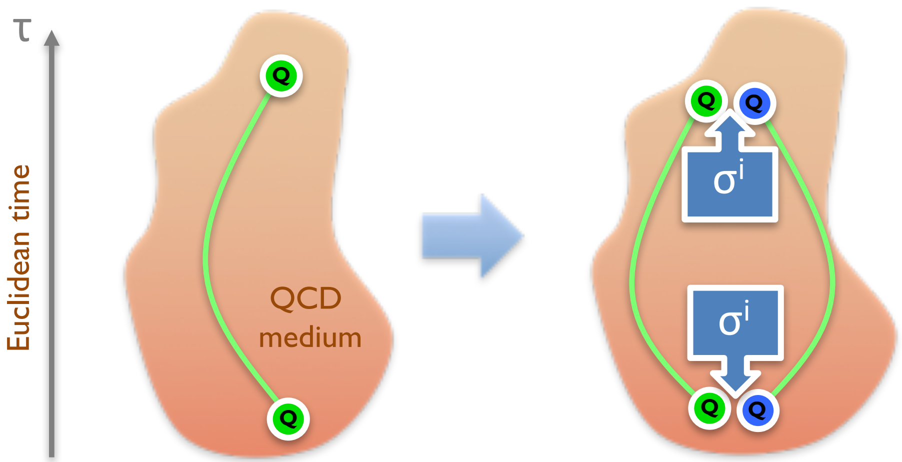

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Brambilla et. al. Rev.Mod.Phys. 77 (2005) 1423

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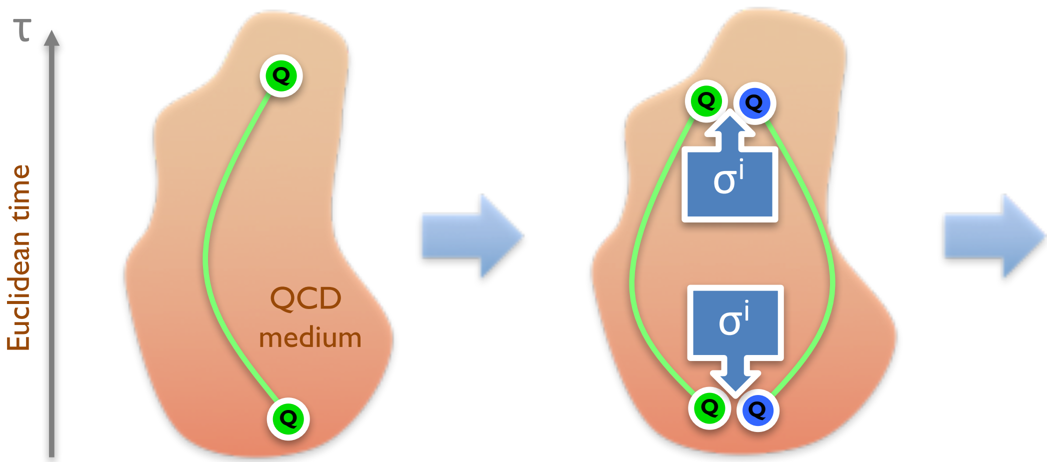

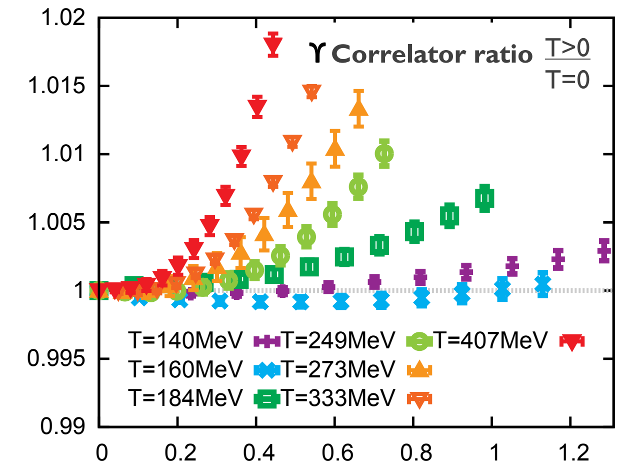

S.Kim, P.Petreczky, A.R. PRD91 (2015) 054511 and in preparation

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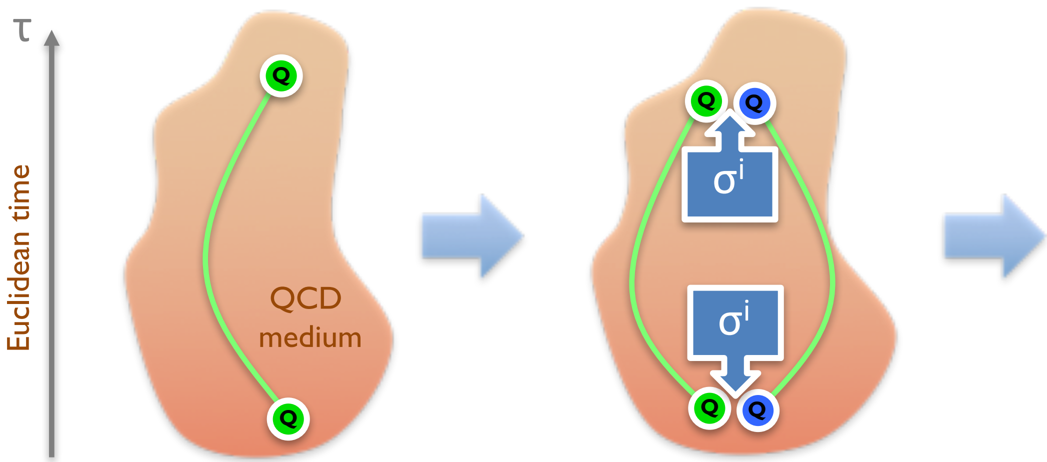

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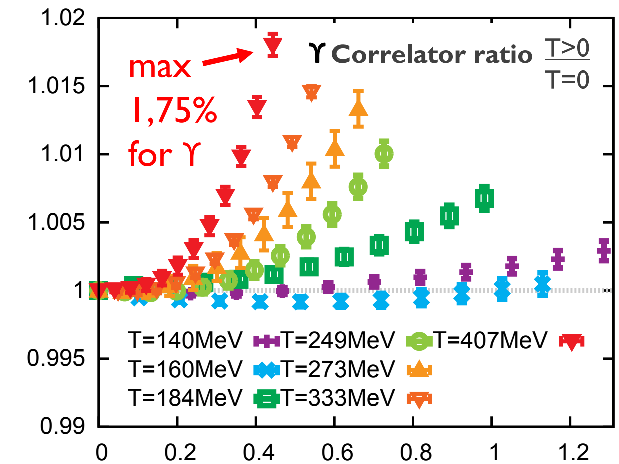

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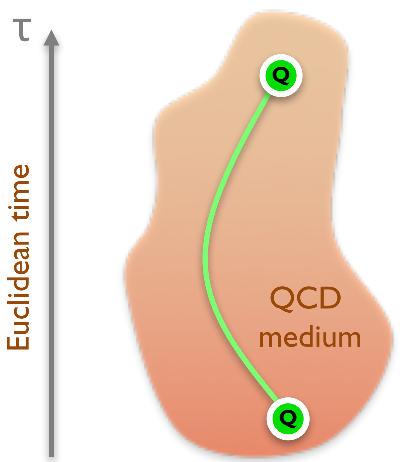

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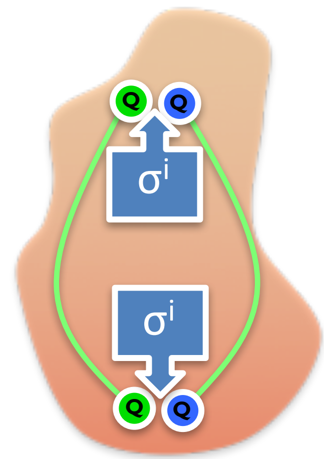

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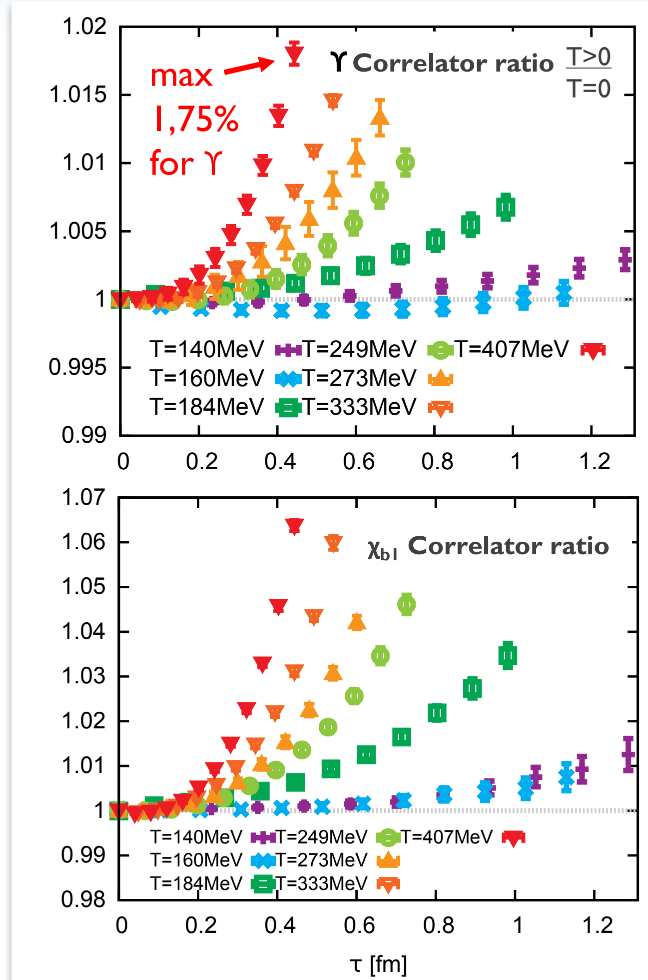

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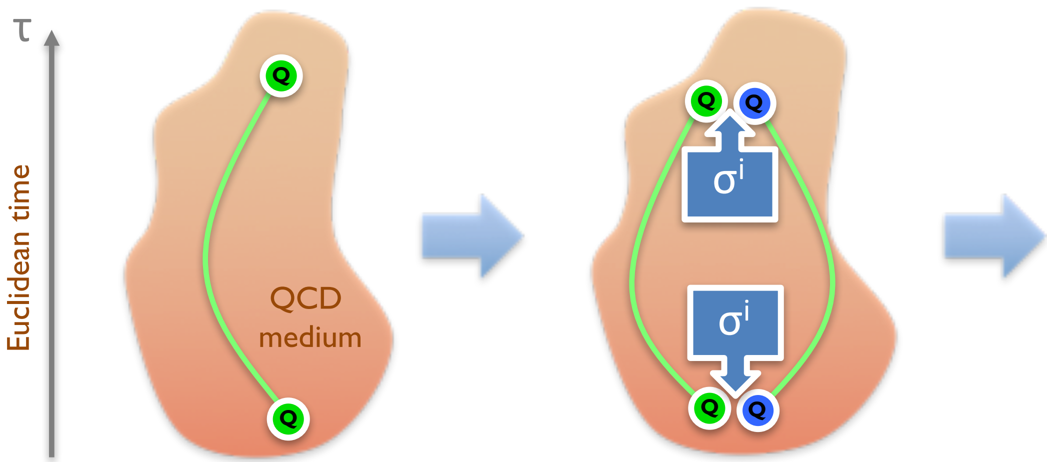

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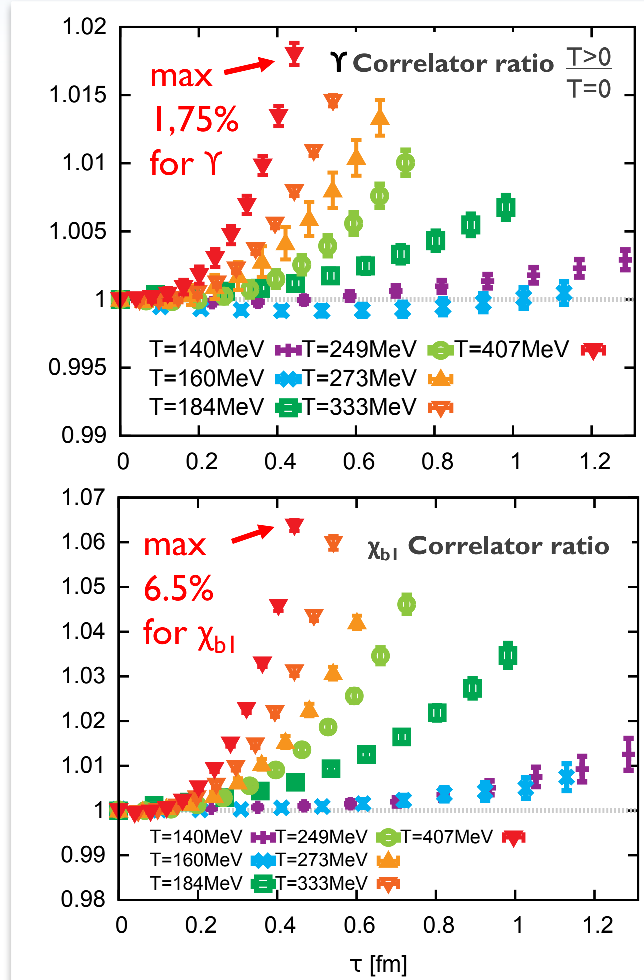

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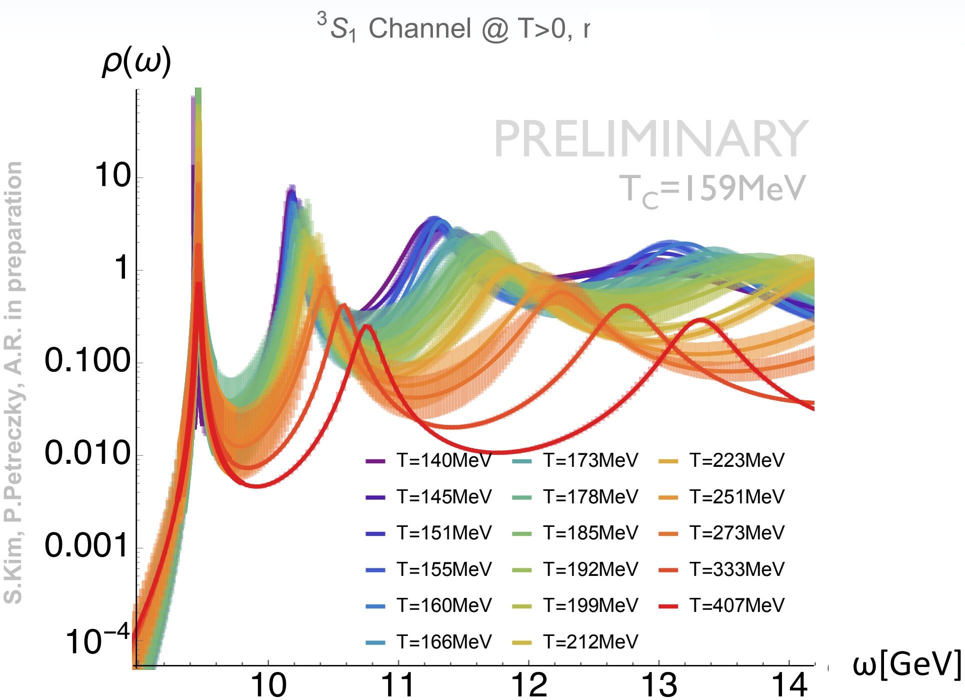

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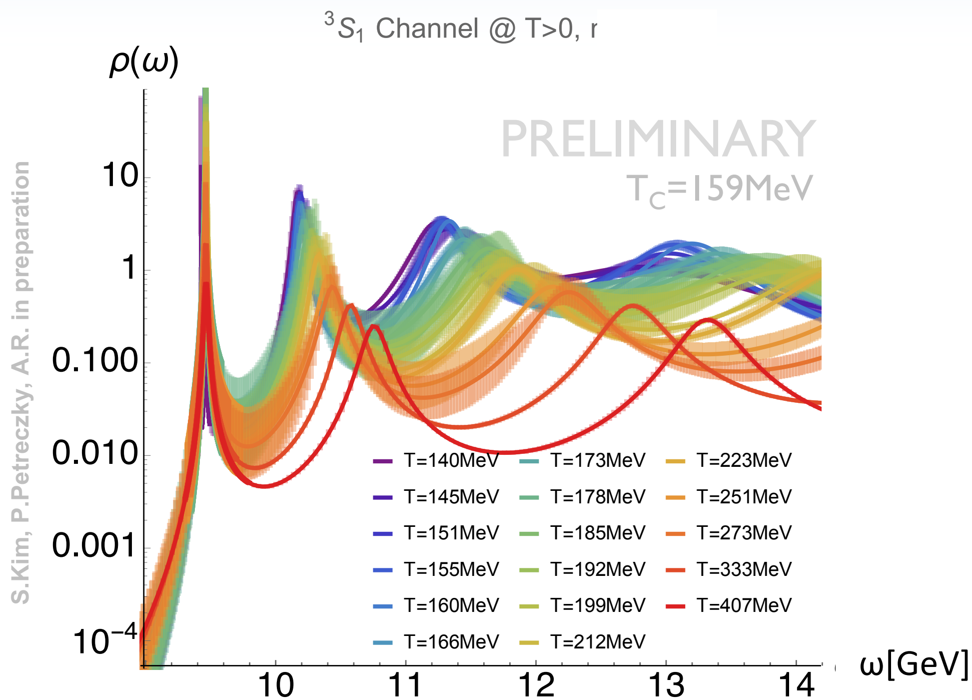

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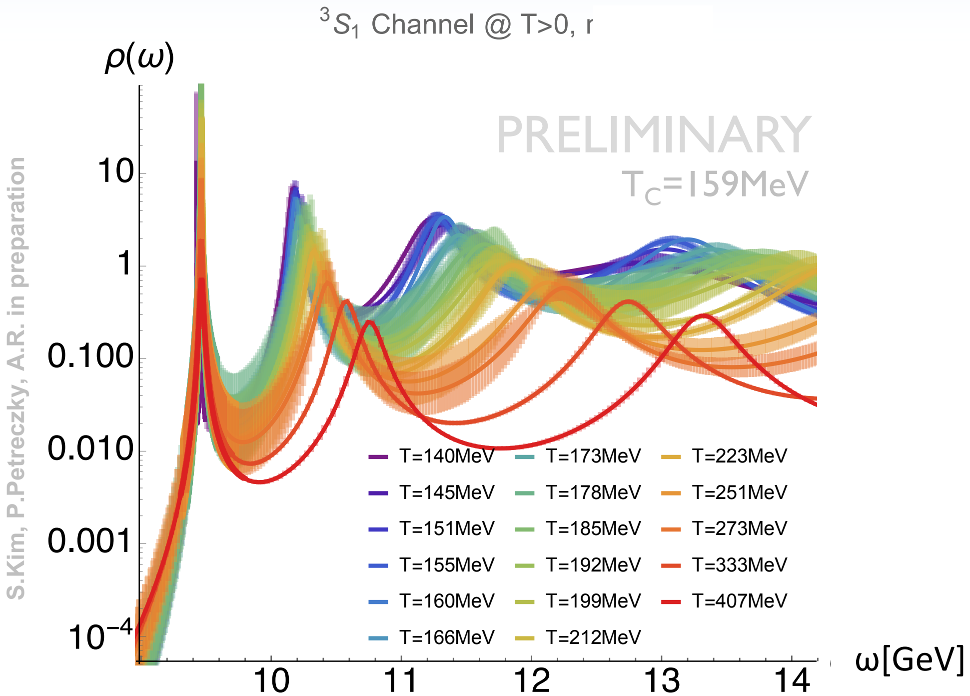

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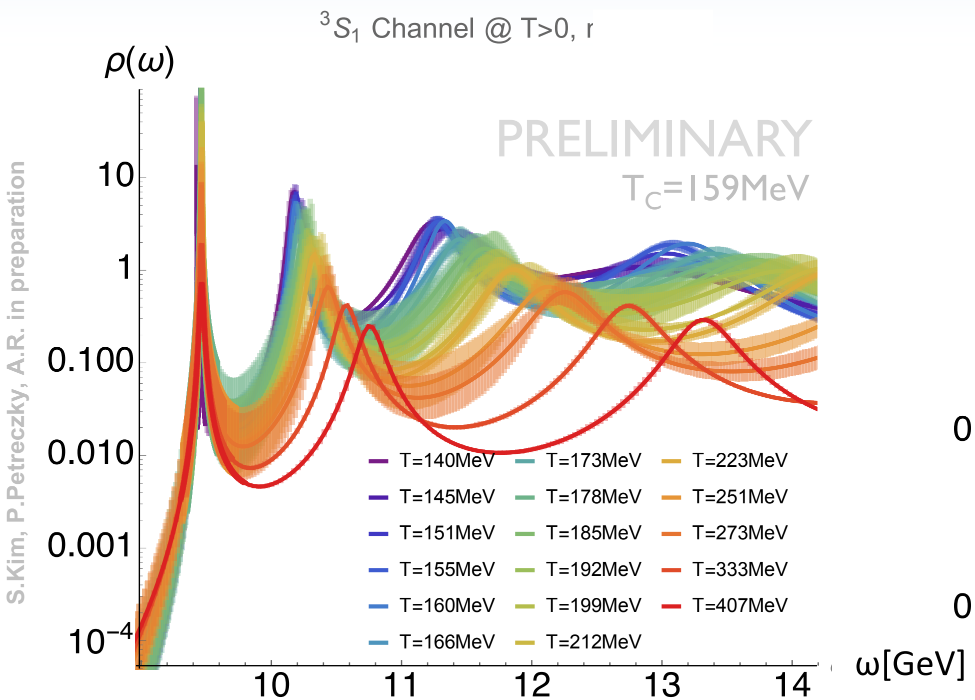

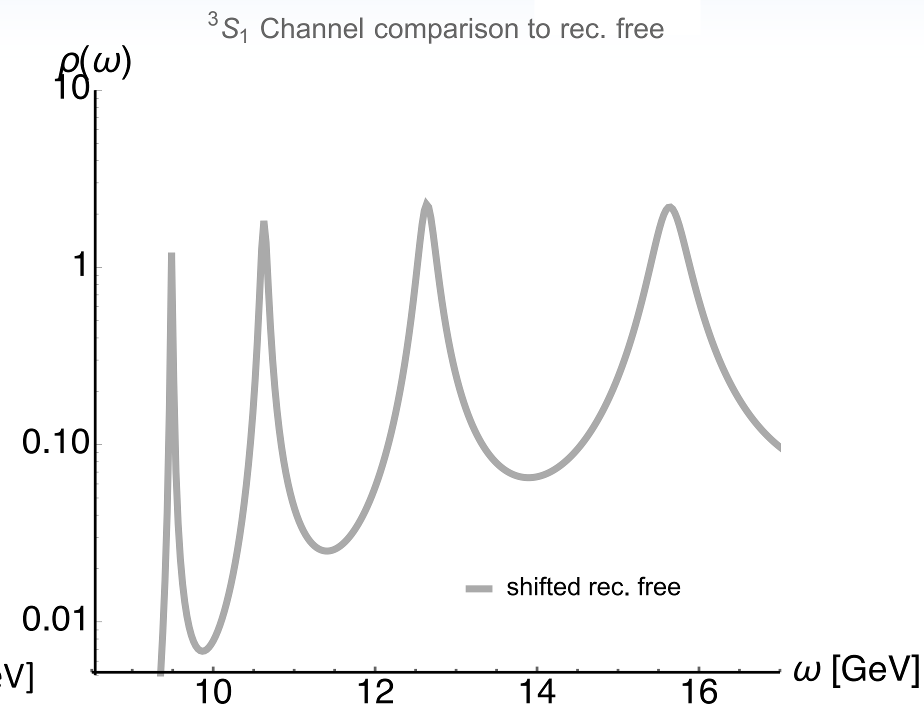

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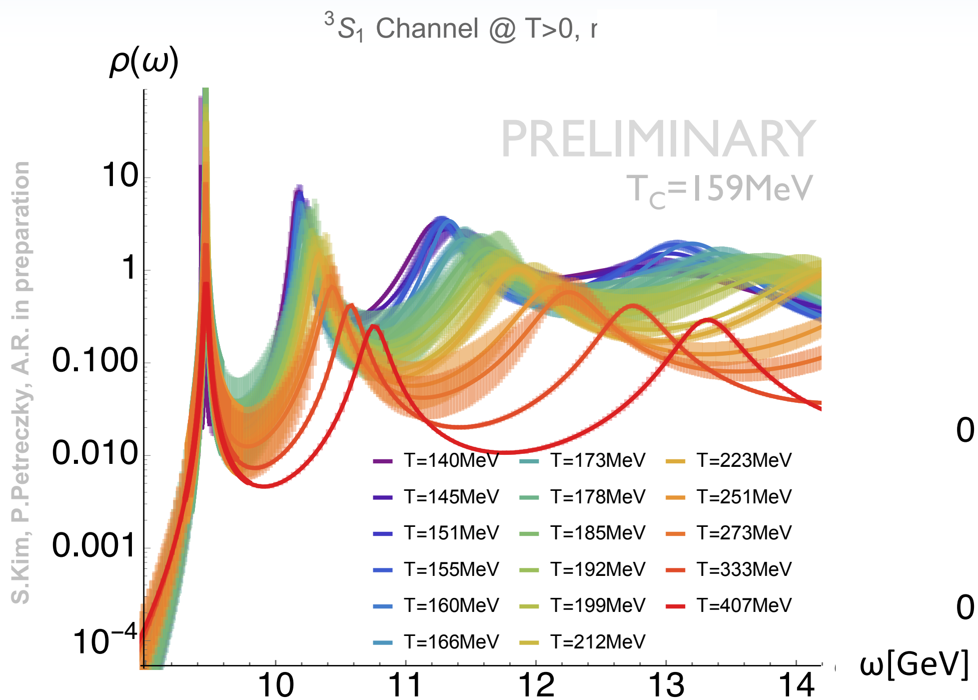

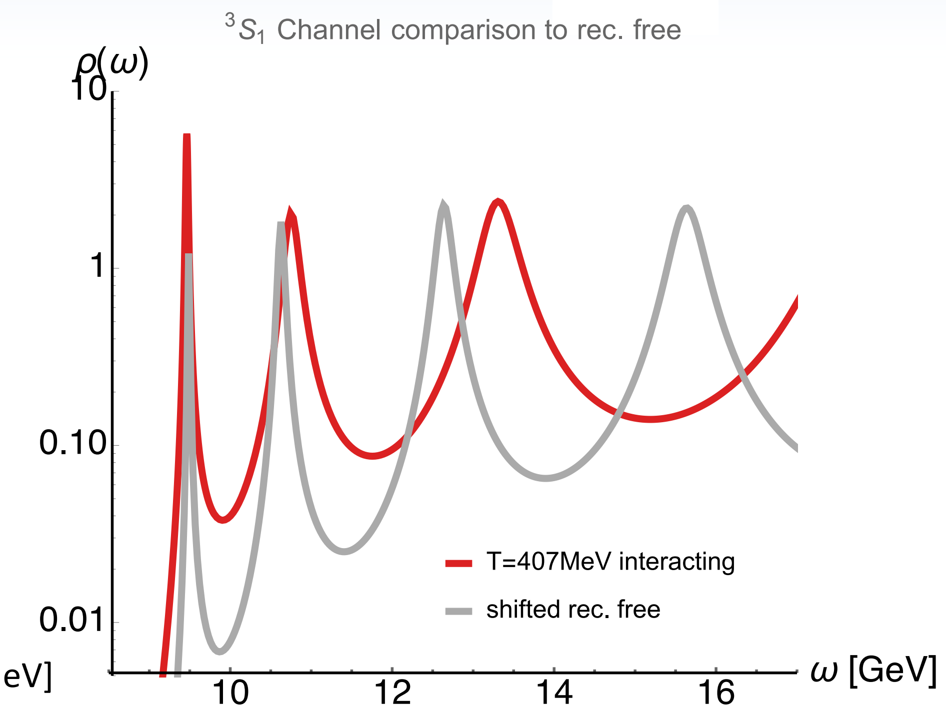

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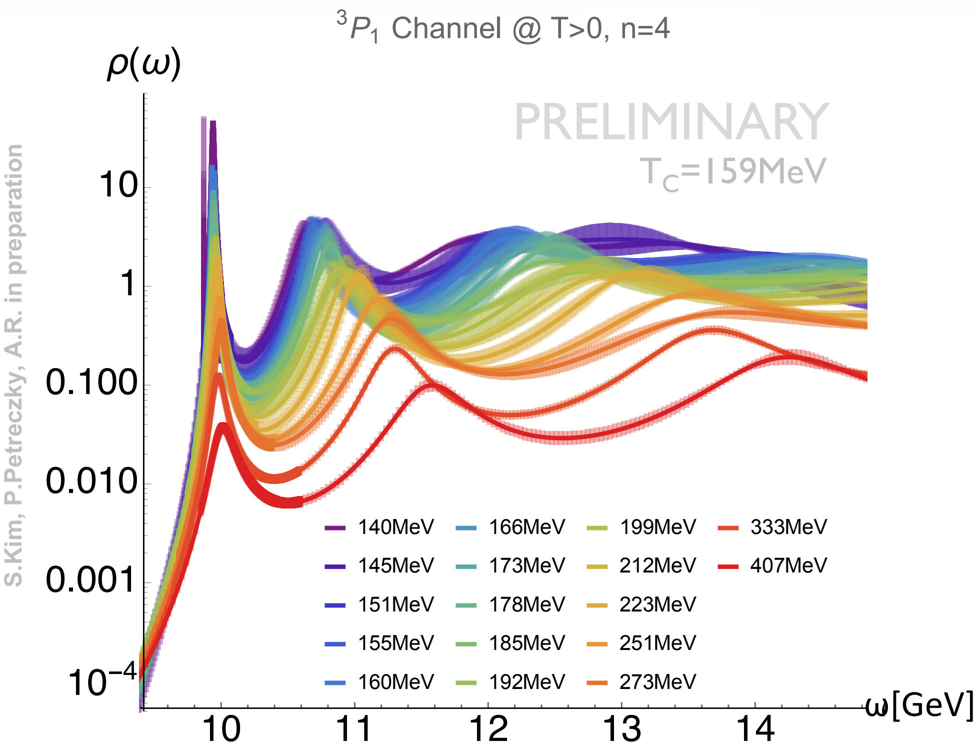

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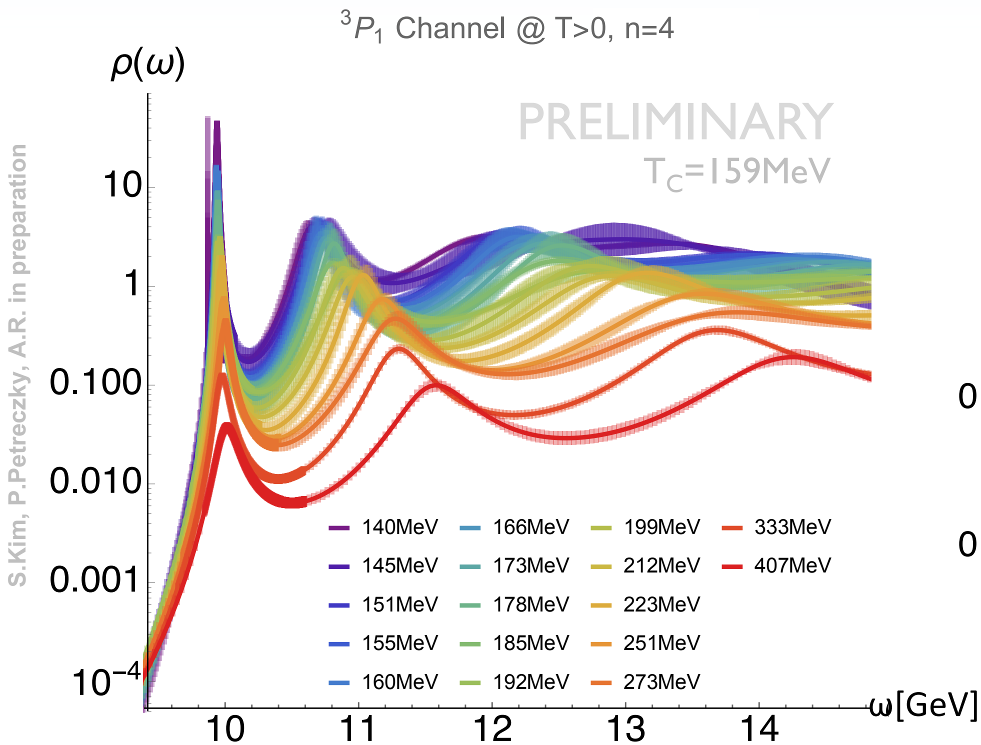

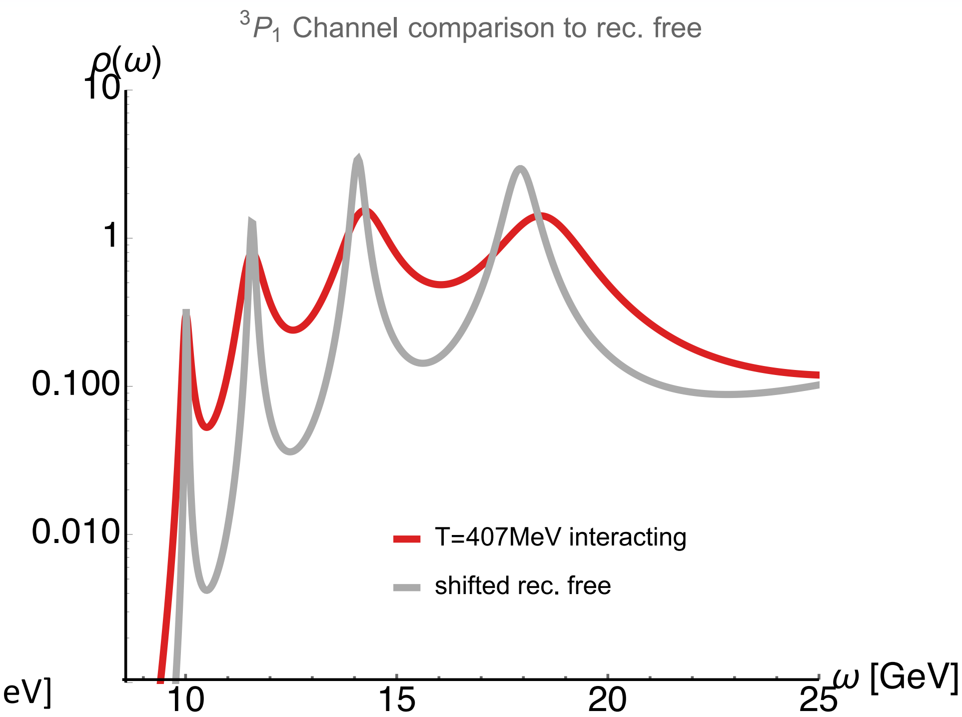

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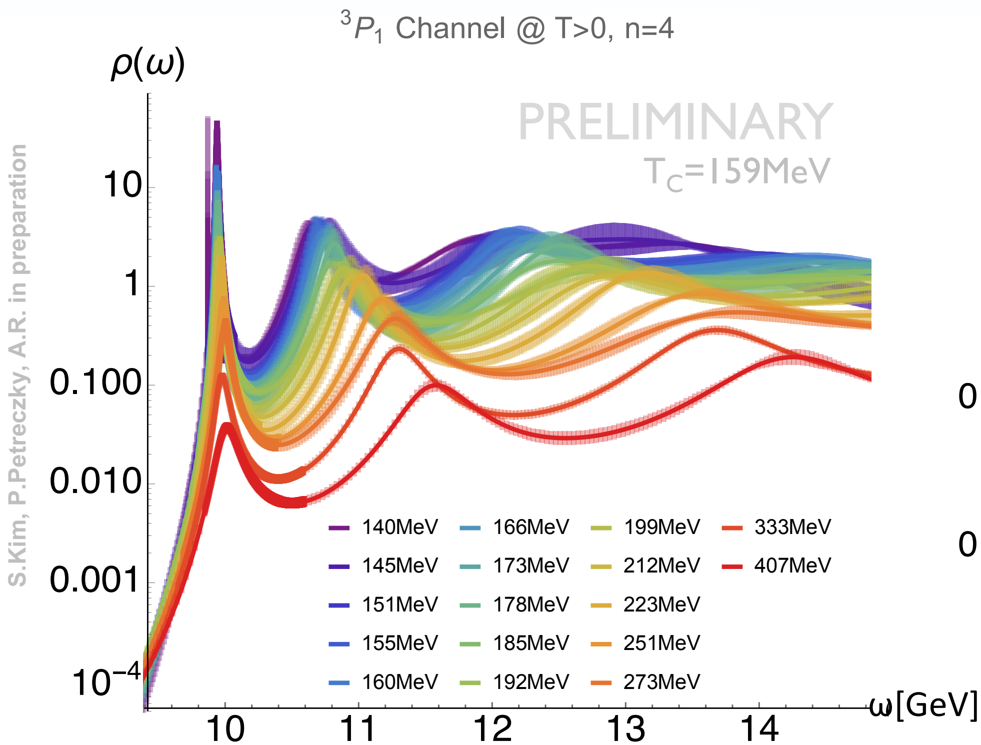

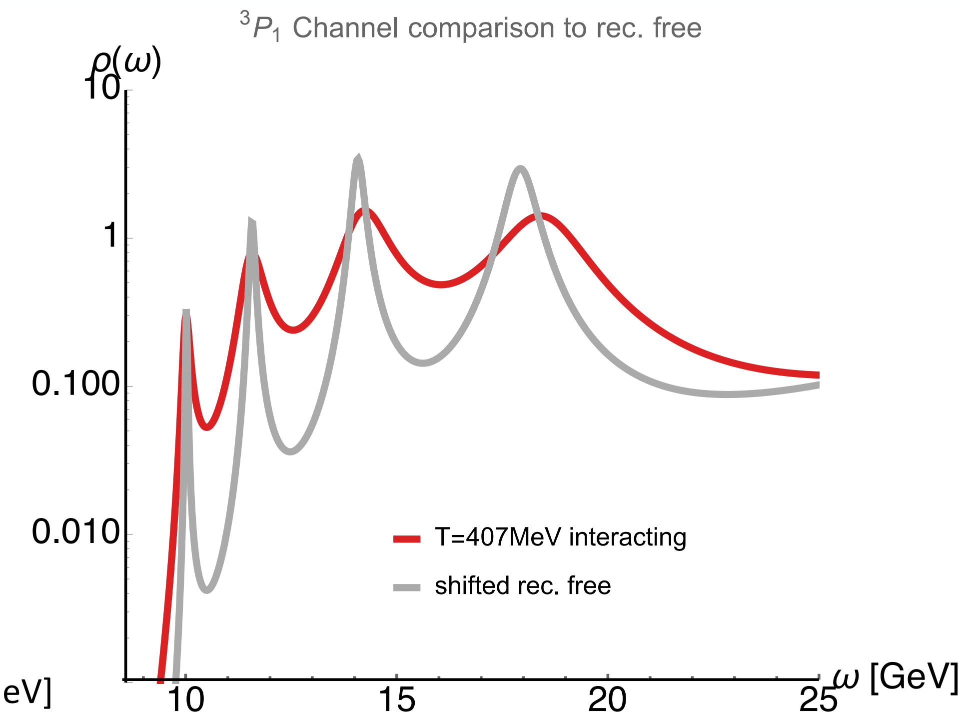

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- @QM2017: New Bottomonium results with reduced ringing due to improved Bayesian strategy and first T>0 results on Charmonium spectra.

How to improve spectral accuracy?

- Intrinsic problem of standard spectral reconstruction: exponential information loss

$$D(\tau) = \int_0^\infty d\omega \, \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh[\omega\beta/2]} \, \rho(\omega)$$

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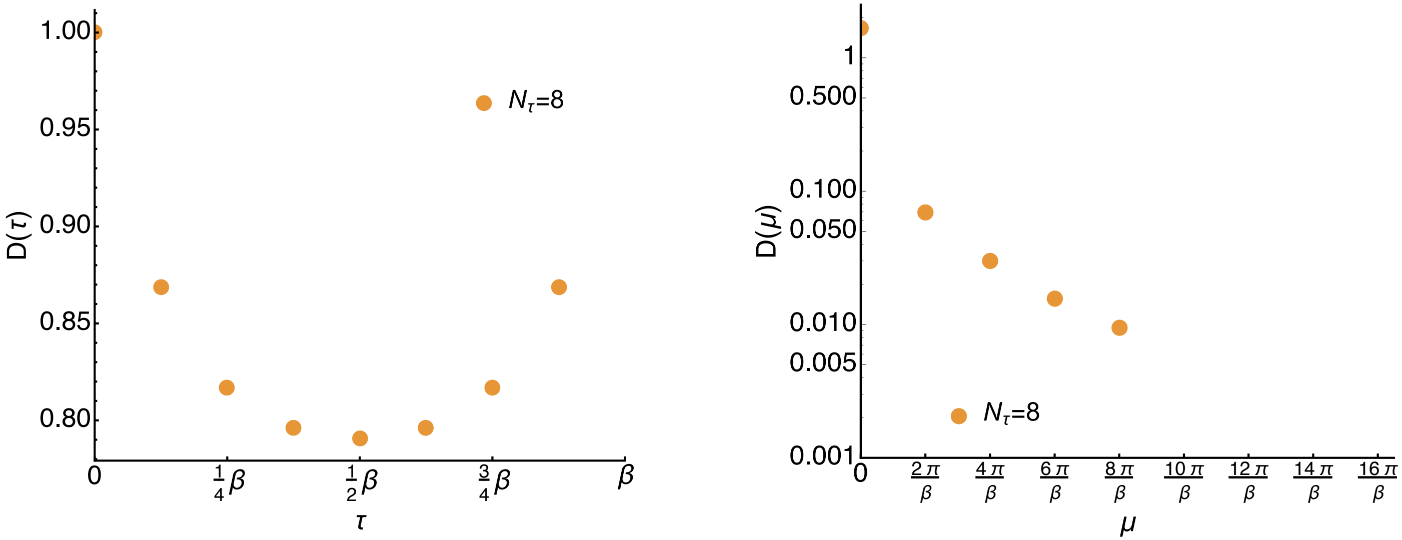

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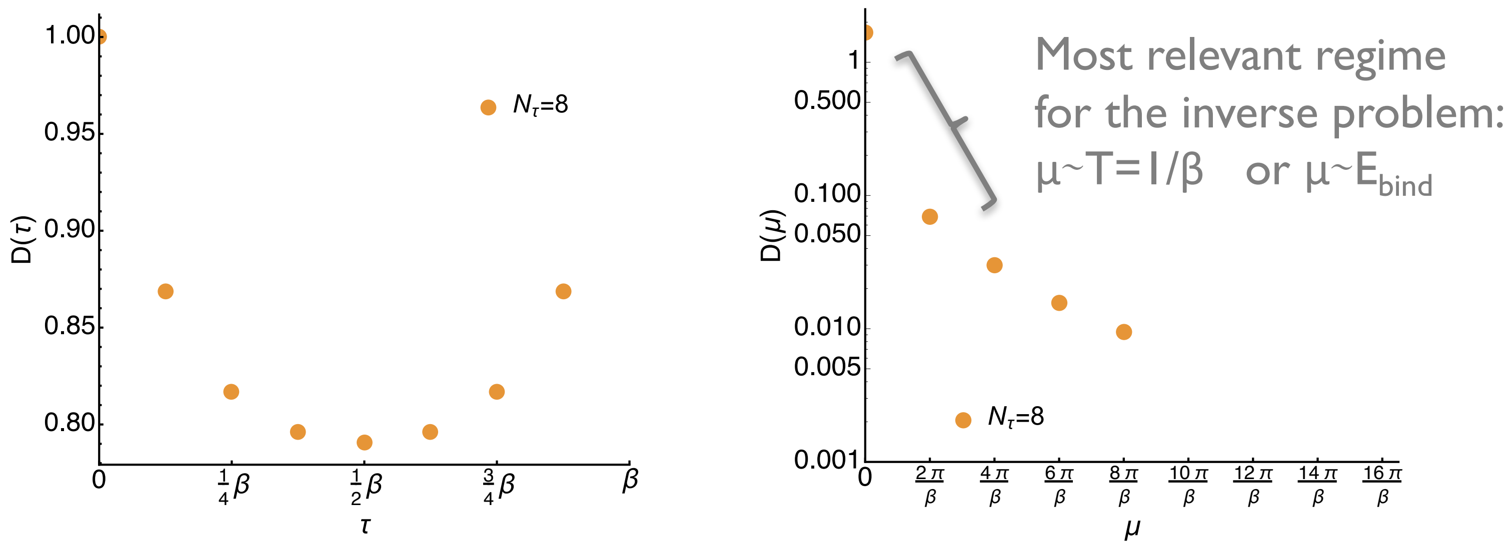

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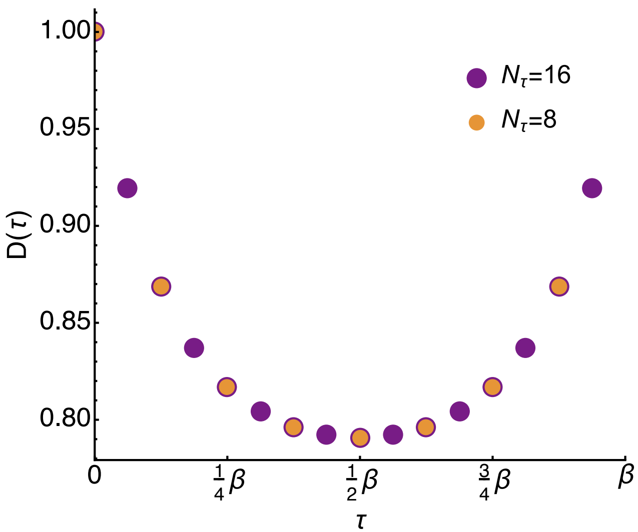

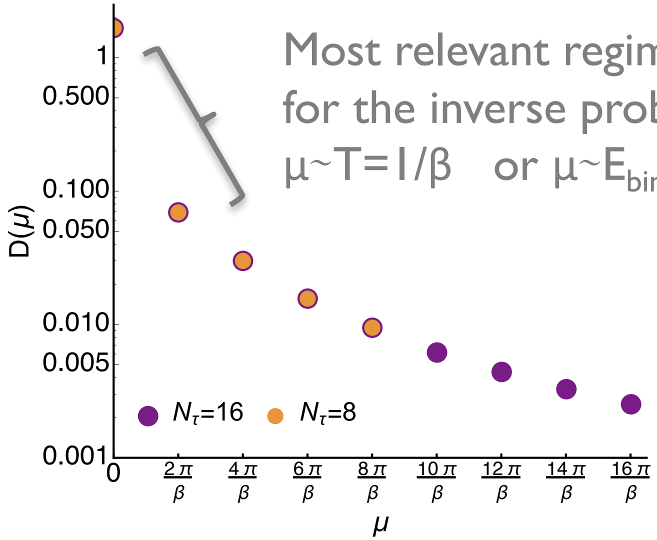

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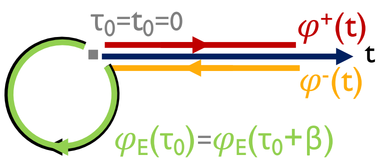

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arXiv:1610.09531

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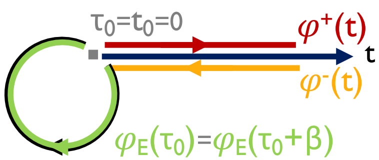

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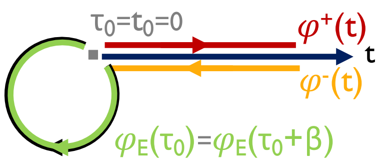

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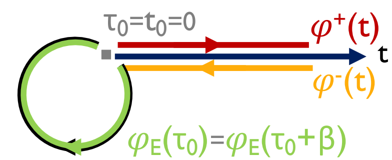

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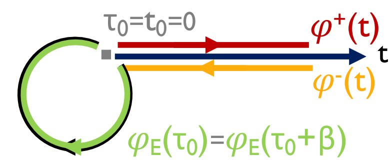

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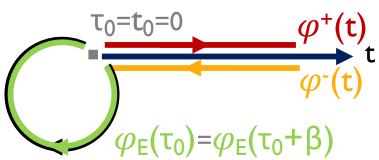

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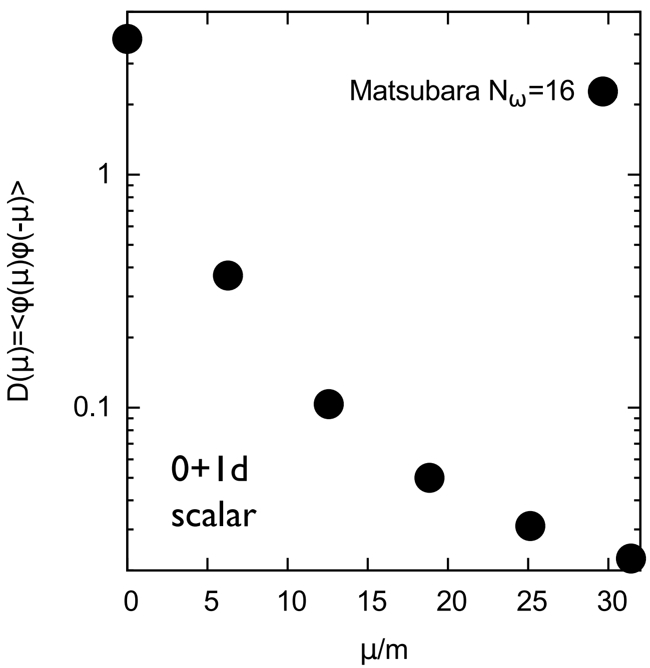

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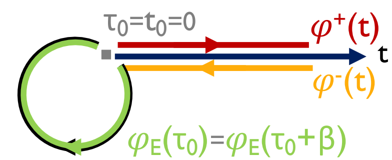

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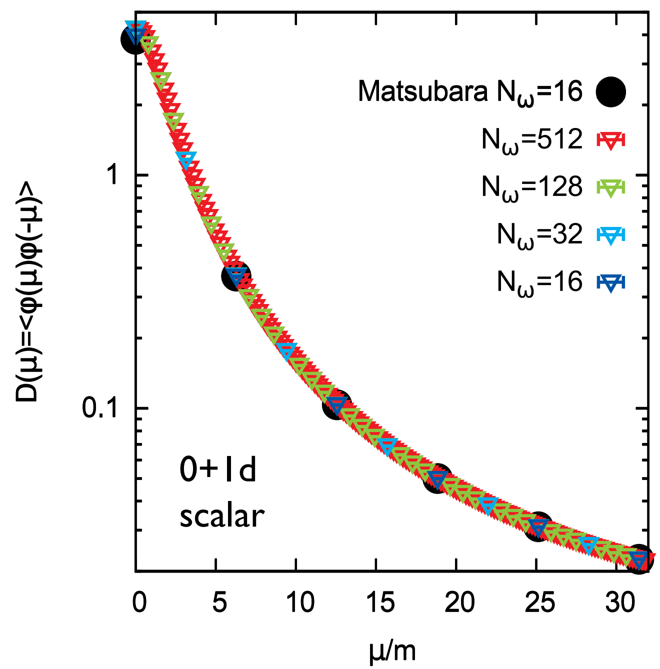

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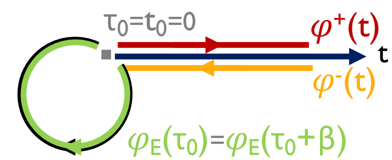

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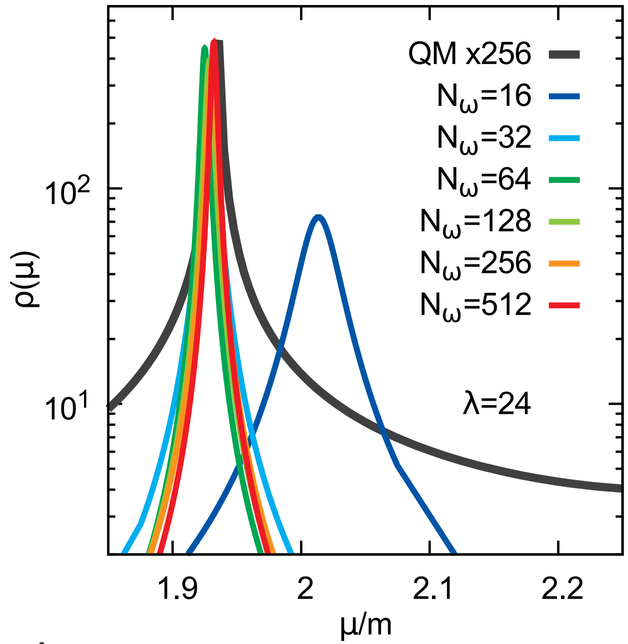

- 0+1d scalar: significantly improved spectral reconstructions

Summary

- Heavy quarkonium matured into a precision probe in heavy-ion collisions

- Direct and indirect lattice QCD approaches to in-medium quarkonium spectra

  - pNRQCD: V_{QQ} does not contain velocity corrections yet but spectra not resolution limited
    hierarchical modification of spectra: states broaden and shift to lower masses

    Spectra precise enough to estimate ψ‘ to J/ψ ratio assuming an instantaneous freezeout scenario Y.Burnier, O.Kaczmarek, A.R. JHEP 1512 (2015) 101, JHEP 1610 (2016) 032

  - NRQCD: includes finite velocity corrections but still limited by simulation data
    correlation functions show hierarchical in-medium modification
    spectra challenging but show reasonable disappearance of bound state features
    S.Kim, P.Petreczky, A.R. PRD91 (2015) 054511, NPA956 (2016) 713 and in preparation

- A new approach to tackling the exponential hardness of spectral reconstructions

  - Simulating directly in imaginary frequencies improves accuracy of spectral reconstruction

  - Generalization of the simulation method to gauge theories work in progress

  J. Pawlowski and A.R. arXiv:1610.09531 and in preparation with student F. Ziegler

Thank you for your attention

Extracting V^{QCD} from lattice QCD

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A.R., T.Hatsuda & S.Sasaki PRL 108 (2012) 162001

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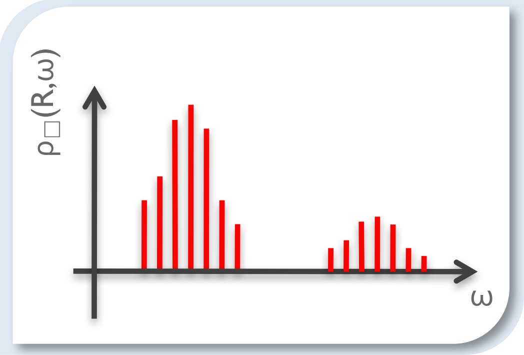

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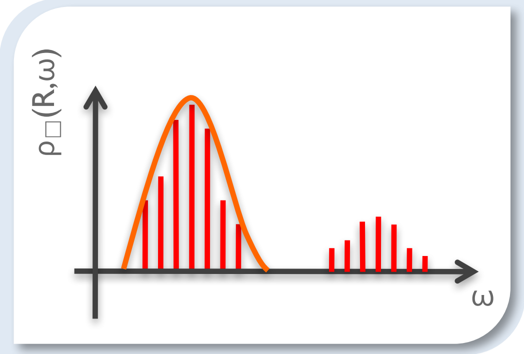

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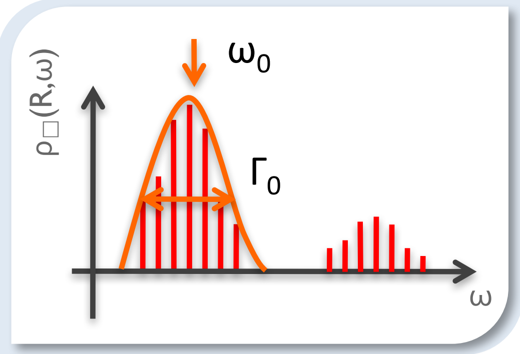

$$\rho_\square(R,\omega) = \frac{1}{\pi} e^{\gamma_1(R)} \frac{\Gamma_0(R)\cos[\gamma_2(R)] - (\omega_0(R)-\omega)\sin[\gamma_2(R)]}{\Gamma_0^2(R) + (\omega_0(R)-\omega)^2} + \kappa_0(R) + \kappa_1(R)(\omega_0(R)-\omega) + \ldots$$

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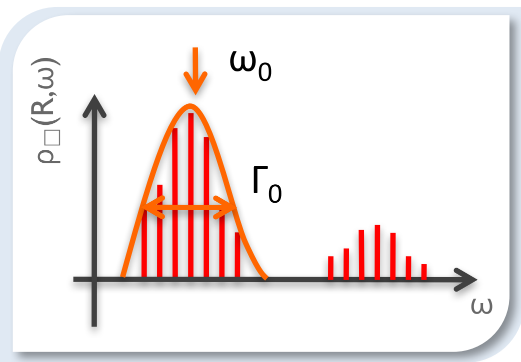

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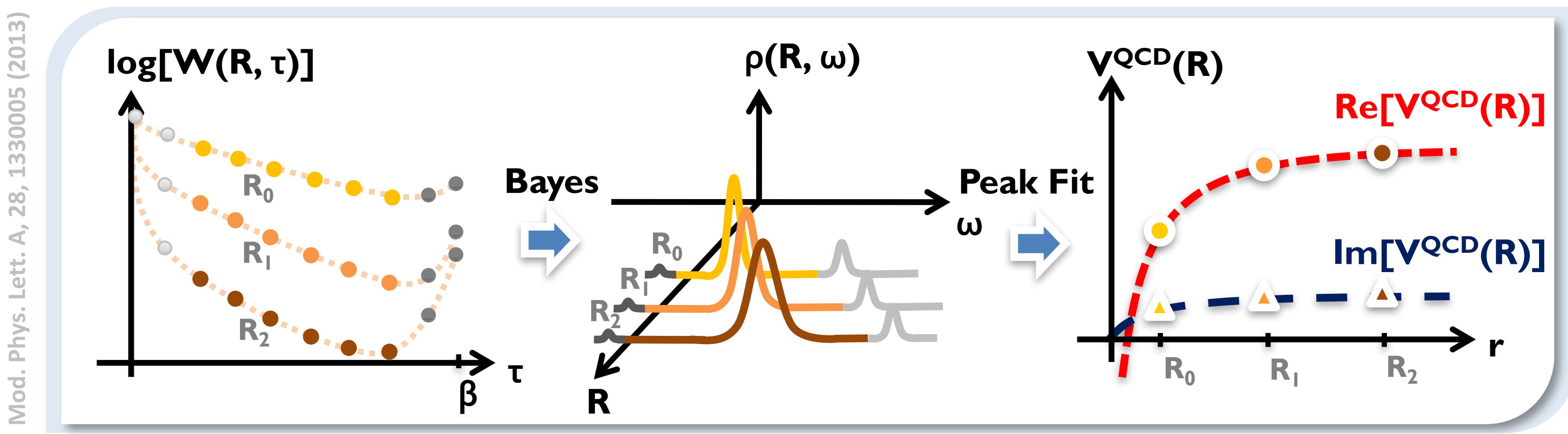
$$V^{QCD}(R) = \omega_0(R) + i\Gamma_0(R)$$

technical details: Y.Burnier, A.R. Phys.Rev. D86 (2012) 051503

Summary: V^{QCD} from the lattice

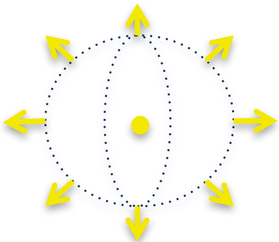
- From lattice QCD correlators to the complex heavy quark potential



- Technical detail: Wilson Line correlators in Coulomb gauge instead of Wilson loops
  Practical reason: Absence of cusp divergences, hence less suppression along τ

Generalized Gauss law and V^{QCD}

- Towards phenomenology: Analytic expression for Re[V^{QCD}] and Im[V^{QCD}] needed



$$V_{Q\bar{Q}}^{T=0} = V_C(r) + V_S(r) = -\frac{\alpha_S}{r} + \sigma r + c$$

Strategy:

α_s,σ and c are vacuum prop. and do not change with T

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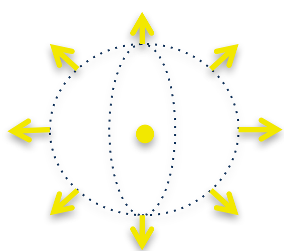

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$$\vec{\nabla}\left(\frac{\vec{\nabla}V(r)}{r^{a+1}}\right) = -4\pi\, q\, \delta(\vec{r})$$

$$V(r) = a\, q\, r^a$$

$$\vec{E} = -\vec{\nabla}V(r)$$

Strategy:

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Coulombic: a=-1 q=α_s

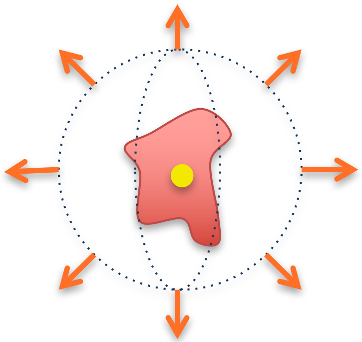
$$\vec{\nabla}\left(\vec{\nabla}V_C(r)\right) = -4\pi\, \alpha_S\, \delta(\vec{r})$$

String-like: a=+1 q=σ

$$\vec{\nabla}\left(\frac{\vec{\nabla}V_S(r)}{r^2}\right) = -4\pi\, \sigma\, \delta(\vec{r})$$

V. V. Dixit,
Mod. Phys. Lett. A 5, 227 (1990)

Introducing medium effects



In the classical theory of Debye: Boltzmann distr. backgr. charges <ρ>

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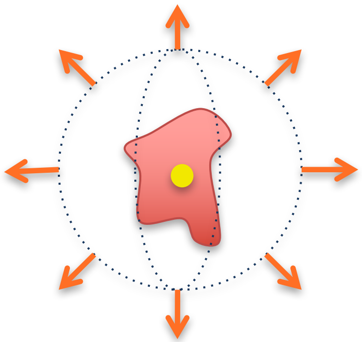

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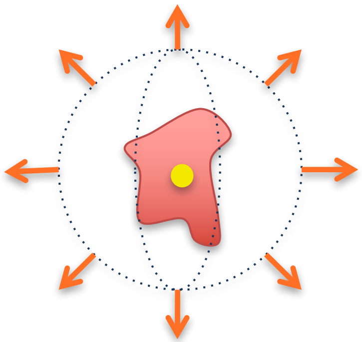

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linear response form where m_D->0 is possible

$$g(x) = 2\int_0^\infty dp\frac{\sin(px)}{px}\frac{p}{p^2+1}$$

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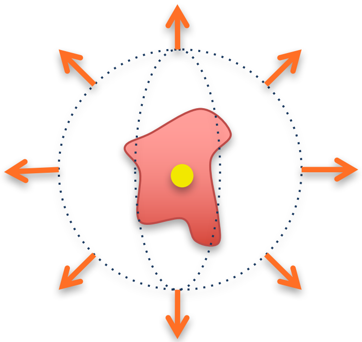
Y.Burnier, A.R.: arXiv:1506.08684

Introducing medium effects



In the classical theory of Debye: Boltzmann distr. backgr. charges <ρ>

$$\vec{\nabla}\left(\vec{\nabla}V_C(r)\right) = -4\pi\alpha\left(\delta(\vec{r}) + \langle\rho(\vec{r})\rangle\right)$$

P. Debye, H. Hückel, Phys.Z. 24, 185-206 (1923)

Here instead: Introduce medium via weak coupling HTL permittivity ε

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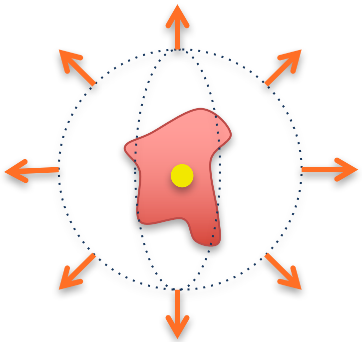
Introducing medium effects



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$V_S(r)$: Gauss Law operator not diagonal in Fourier space: assume validity of linear response

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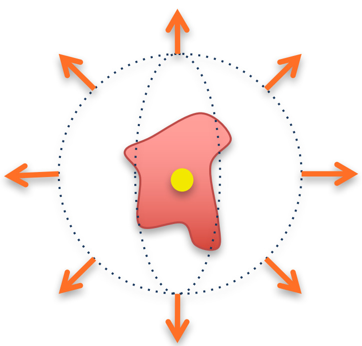
Introducing medium effects



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Im[V_S] as integral expression using Wronskian

D_ν parabolic cylinder function