Unitarized amplitudes for diffractive production of three pion resonances

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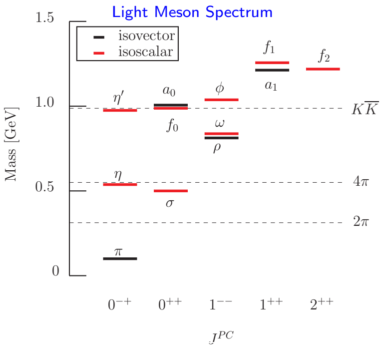
7th Workshop of the APS Topical Group on Hadronic Physics
February 1st – 3rd, 2017

Light Meson Spectroscopy

- Motivation: Study meson spectroscopy through peripheral resonance production of 3π systems e.g.

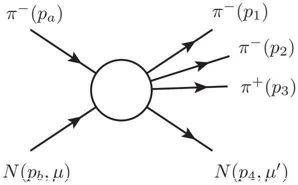
$$\pi N \to (3\pi)N, \quad \gamma N \to (3\pi)N$$

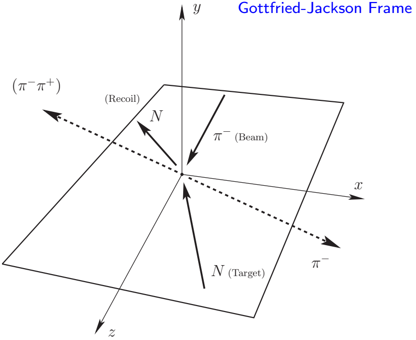
- Goal: Construct reaction amplitudes for such processes
- Outcome: Resonance content in various J^{PC} (Exotics?)
- Focus discussion on π beam, use COMPASS as template



3π Reaction

- Amplitude for $\pi^- N \to \pi^-\pi^-\pi^+ N$ denoted $A_{\mu'\mu}$
- High-energy behavior, $s \to \infty$ (190 GeV/c π^- beam at COMPASS) $\implies$ Exchange process dominated by pomeron
- Assume some isobar structure in model $\implies$ quasi-two-body process



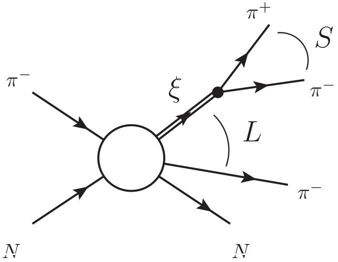


Partial Wave Decomposition

- Expand amplitude $A_{\mu'\mu}$ into partial waves

$$A_{\mu'\mu} = \sum_{JMLS\epsilon} F^{JM\epsilon}_{LS,\mu'\mu} \sum_{\lambda} \langle J\lambda | L0S\lambda \rangle \left(\frac{2J+1}{4\pi}\right)^{1/2} D^{J\epsilon\,*}_{M\lambda}(\Omega) \left(\frac{2S+1}{4\pi}\right)^{1/2} D^{S\,*}_{\lambda 0}(\Omega')$$

- Model partial wave amplitudes $F^{JM\epsilon}_{LS,\mu'\mu}$
- S-matrix principles constrain model
  - S-matrix is unitary ($S^\dagger S = 1$)
  - Amplitudes are analytic functions of momenta
- Fit model to COMPASS 'data' and determine resonances



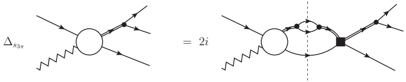
Amplitude Model: Unitarity and Analyticity

- Assume elastic rescattering only in unitarity equation

$$S = 1 + iT \ , \ S^\dagger S = 1 \implies T - T^\dagger = iT^\dagger T$$

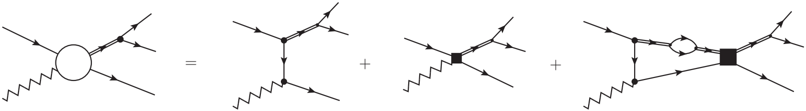
- Unitarity condition on partial wave amplitudes

$$\text{Disc}\, F_i(s_{3\pi}) = 2i \sum_j t_{ij}^*(s_{3\pi}) \rho_j(s_{3\pi}) F_j(s_{3\pi})$$



- Given rescattering t_{ij}, can write dispersive solution

$$F_i(s_{3\pi}) = b_i(s_{3\pi}) + \sum_j t_{ij}(s_{3\pi}) c_j + \frac{1}{\pi} \sum_j t_{ij}(s_{3\pi}) \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_j(s')}{s' - s_{3\pi}}$$

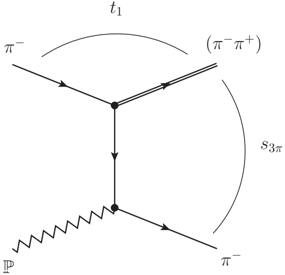


Amplitude Model: Production Mechanism

- For production process, use Deck exchange amplitude

$$A^{Deck}_{\mu'\mu} = A^{\pi\pi}\frac{1}{t_1 - m_\pi^2}\mathcal{M}_{\mu'\mu}$$

- π-exchange is closest cut to physical region $\implies$ dominant effect



- Partial wave projection of Deck is input to model

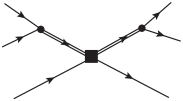
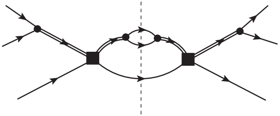
Amplitude Model: Rescattering and K-Matrix

- Model requires unitarity of rescattering $(\pi^-\pi^-\pi^+ \to \pi^-\pi^-\pi^+)$ amplitude
- Work in quasi-two body limit: Isobars are quasi-stable particles

$$t_{ij}(s_1, s_{3\pi}, s_1') = f_i(s_1) M_{ij}(s_{3\pi}) f_j(s_1')$$

- Quasi-two body unitarity condition on rescattering amplitudes

$$\Delta\, M_{ij}(s_{3\pi}) = 2i \sum_k M^*_{ik}(s_{3\pi}) \rho_k(s_{3\pi}) M_{kj}(s_{3\pi})$$

$\Delta_{s_{3\pi}}$  $= 2i$ 

Amplitude Model: Rescattering and K-Matrix

- The rescattering amplitude is parameterized by the K-matrix

$$\mathbf{M}^{-1}(s_{3\pi}) = \mathbf{K}^{-1}(s_{3\pi}) - \frac{s_{3\pi}}{\pi} \int ds' \frac{\rho(s_{3\pi})}{s'(s' - s_{3\pi})}$$

- Parameters of the K-matrix are fit parameters
- Construct K-matrix to satisfies unitarity and causal requirements

$$K_{ij}(s_{3\pi}) = \sum_r \frac{g_i^r g_j^r}{m_r^2 - s_{3\pi}} + \sum_k \gamma_{ij}^k s_{3\pi}^k$$

- This K-matrix parameterization could lead to spurious poles on the first sheet $\implies$ Need CDD prescription

$$\mathbf{K}^{-1}(s_{3\pi}) = \mathbf{C}_0 - \mathbf{C}_1\, s_{3\pi} - \sum_{j=1}^{N_j} \frac{\mathbf{C}_2^j}{\mathbf{C}_3^j - s_{3\pi}}$$

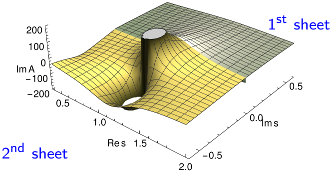
Pole Hunting - Analytic Continuation

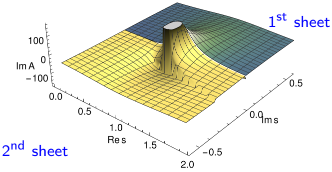
- Resonance are poles in the complex energy plane
- Analytically continue amplitude into complex *s*-plane

$$\mathbf{M}^{\mathrm{II}} = \mathbf{M}[1 + i\rho\,\mathbf{M}]^{-1} \implies \det[1 + i\rho\,\mathbf{M}] = 0$$

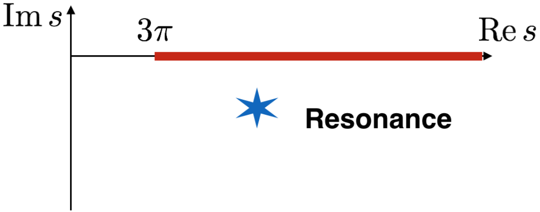
→ Search for poles
→ Determine resonance mass and width

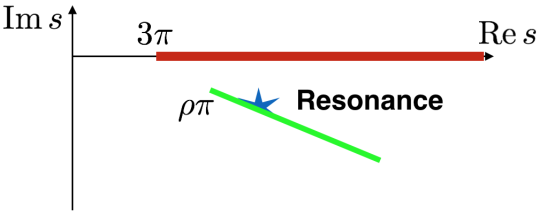
- Quasi-two-body phase space introduces new cuts (Woolly cuts)





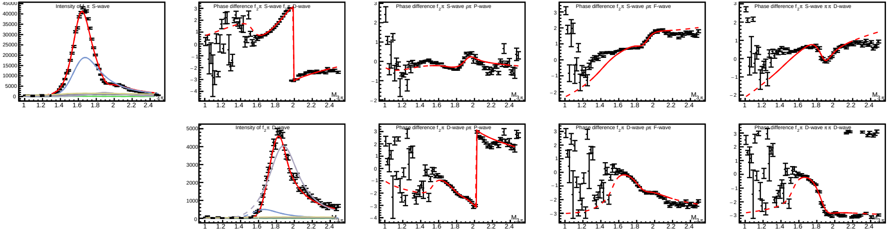
Pole Hunting - Analytic Continuation

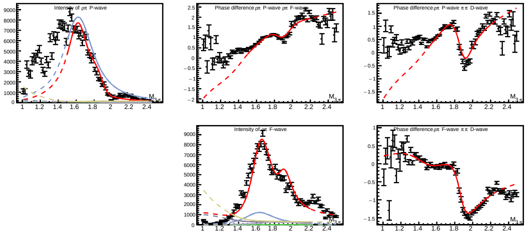


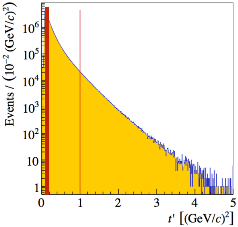


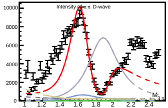
Application in 2^{-+}

In the fit	Partial wave	I_{max} at lowest t'-bin	I_{max} at highest t'-bin	Fit range (GeV)
	$2^{-+}0^{+}$			
$\star$	$f_2\pi$ S-wave	42955	2722	1.3 to 2.4
$\star$	$(\pi\pi)_S$ D-wave	9730.8	1341.6	1.2 to 2.2
$\star$	$\rho\pi$ F-wave	9135.2	943.7	1.3 to 2.2
$\star$	$\rho\pi$ P-wave	8778.4	2256.3	1.5 to 2.4
$\star$	$f_2\pi$ D-wave	4829.4	512.5	1.6 to 2.4
	$f_0\pi$ D-wave	1751.1	440.7	
	$\rho_3\pi$ P-wave	711.3	314.7	
	$f_2\pi$ G-wave	314.7	150.3	
	$2^{-+}1^{+}$			
	$\rho\pi$ P-wave	10820.2	3197.8	
	$f_2\pi$ S-wave	1808.7	3679.2	
	$\rho\pi$ F-wave	892.2	732.4	
	$(\pi\pi)_S$ D-wave	872	844.7	
	$\rho_3\pi$ P-wave	623.9	151.4	
	$f_2\pi$ D-wave	379.3	542.4	
	$2^{-+}2^{+}$			
	$\rho\pi$ P-wave	330.2	593.1	
	$f_2\pi$ S-wave	125.6	309.7	
	$f_2\pi$ D-wave	100.8	384.8	

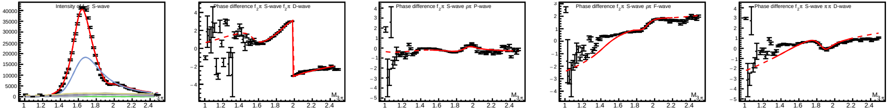


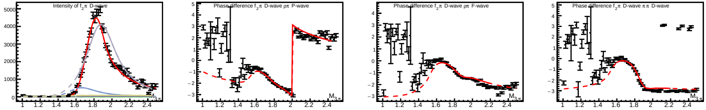


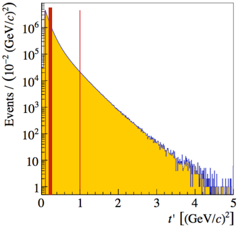


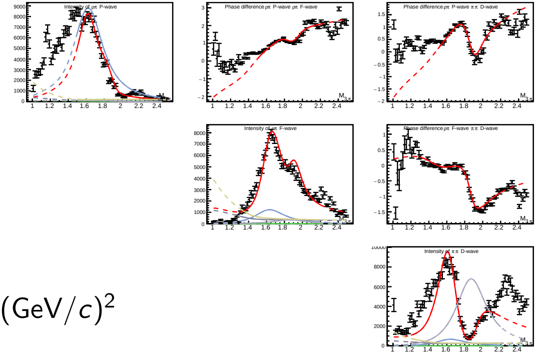


$$0.100000 < t' < 0.112853\,(\text{GeV}/c)^2$$

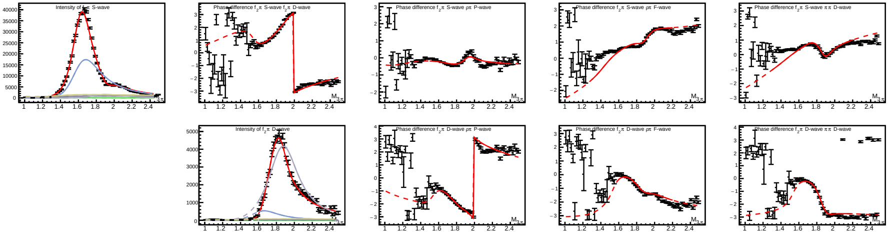


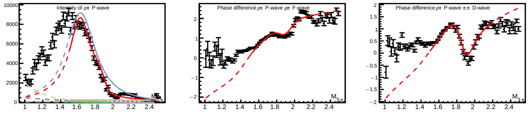


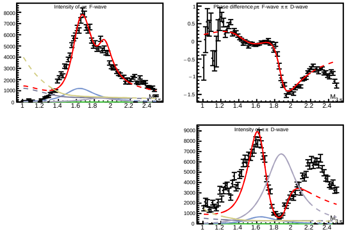


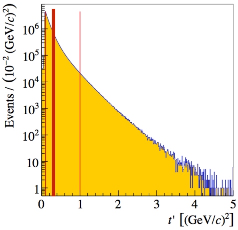


$$0.112853 < t' < 0.127471\,(\mathrm{GeV}/c)^2$$

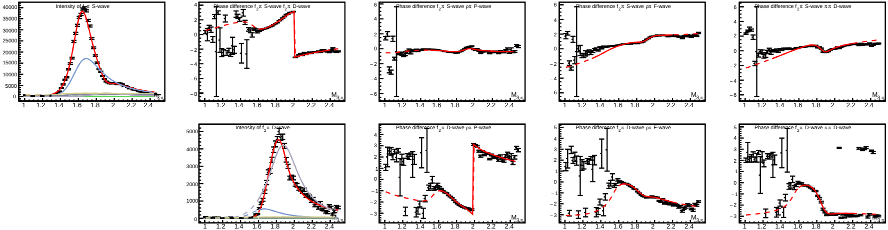


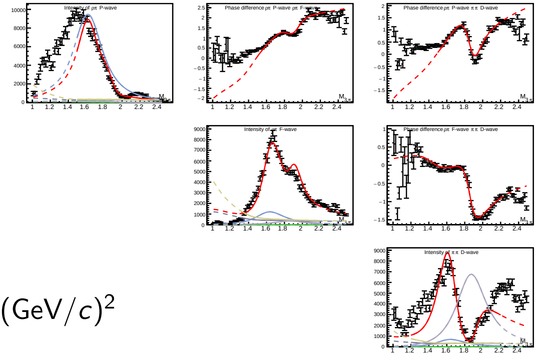


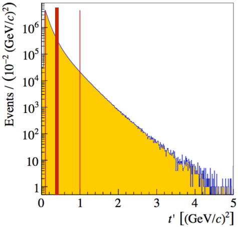




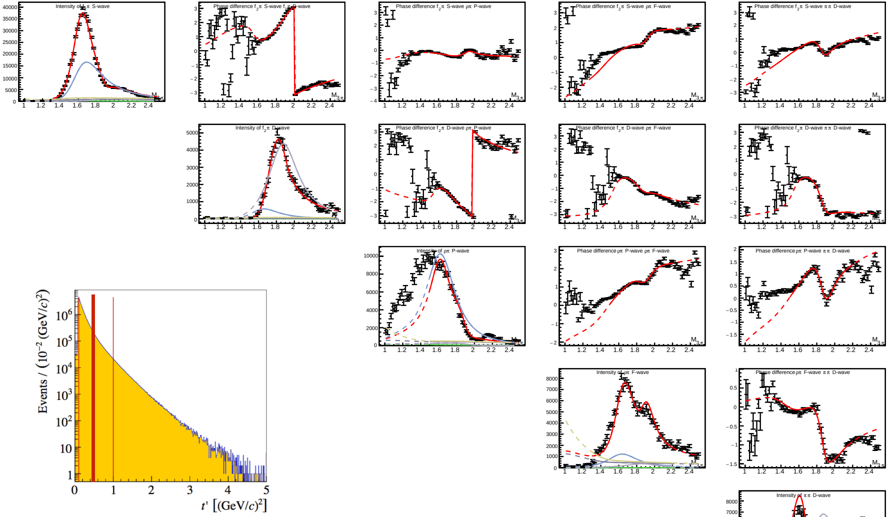
$$0.127471 < t' < 0.144385 \, (\text{GeV}/c)^2$$

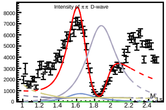




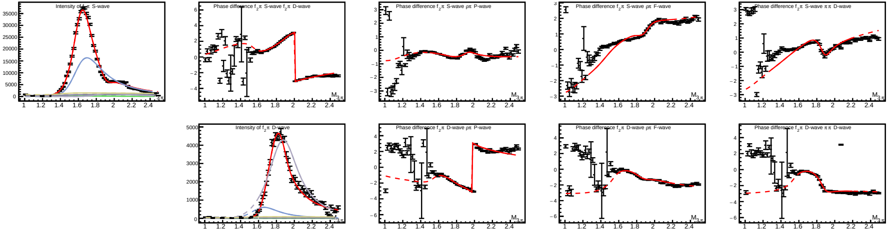


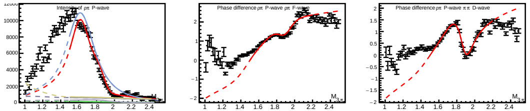
$$0.144385 < t' < 0.164401\,(\mathrm{GeV}/c)^2$$

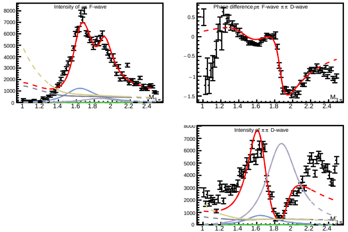


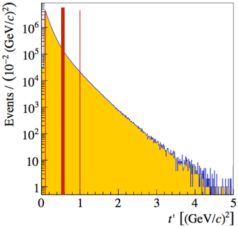


$$0.164401 < t' < 0.188816\,(\mathrm{GeV}/c)^2$$

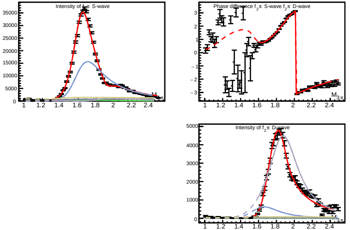


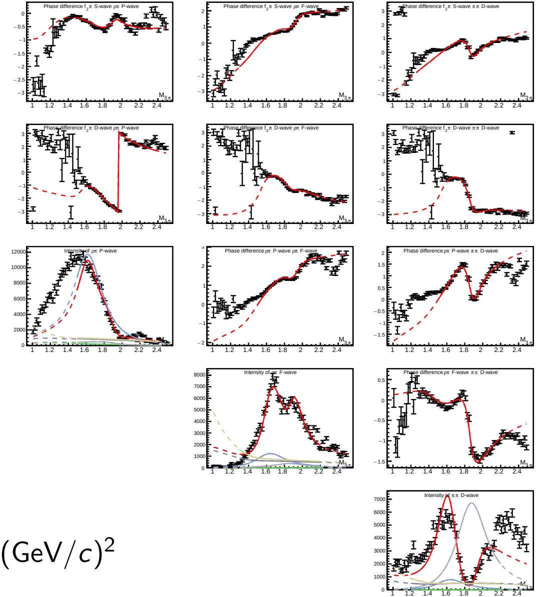


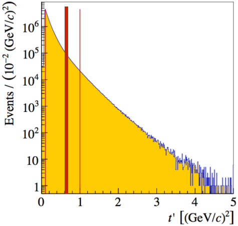




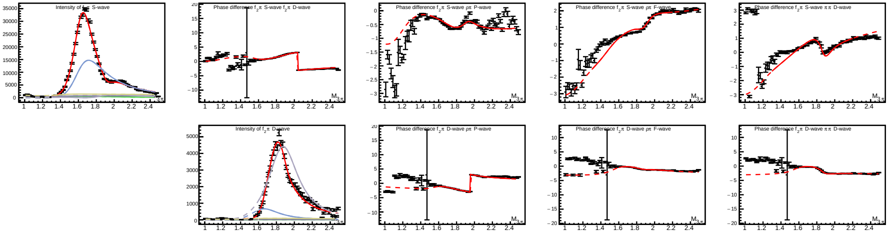
$$0.188816 < t' < 0.219907\,(\mathrm{GeV}/c)^2$$

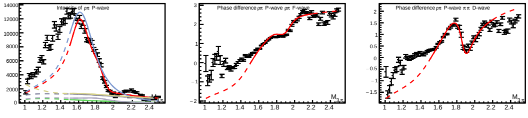


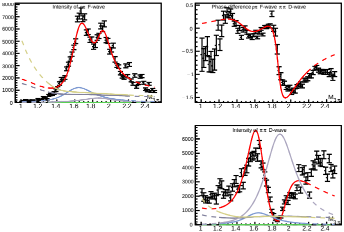


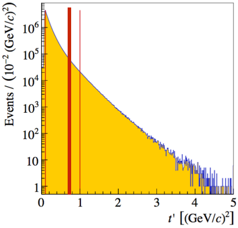


$$0.219907 < t' < 0.262177\,(\text{GeV}/c)^2$$

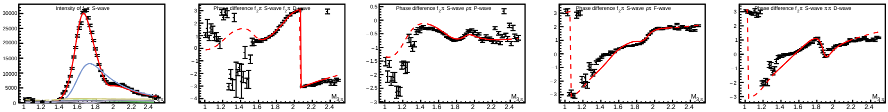


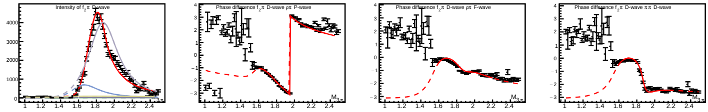


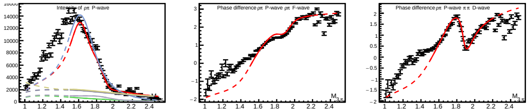


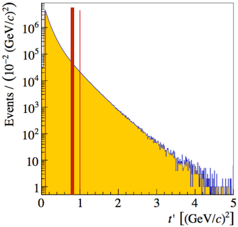


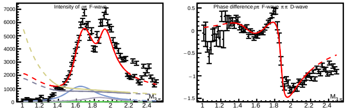
$$0.262177 < t' < 0.326380 \, (\text{GeV}/c)^2$$

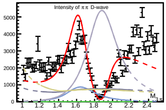




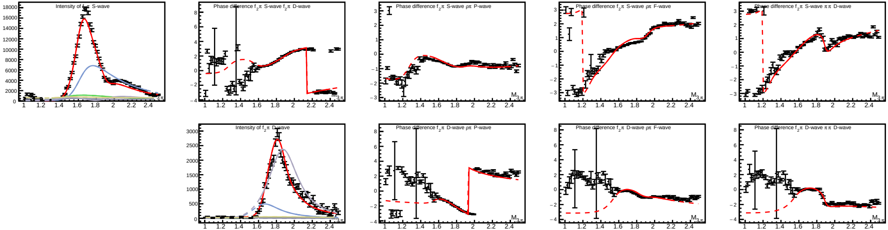


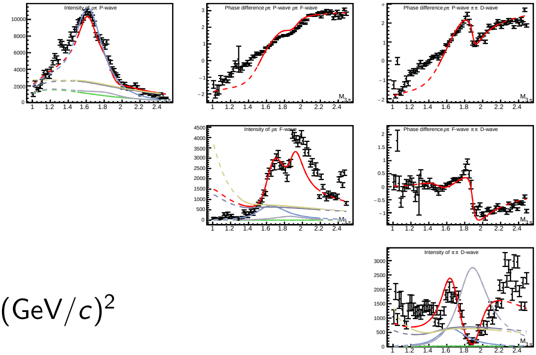


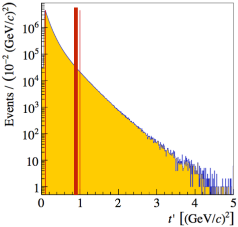




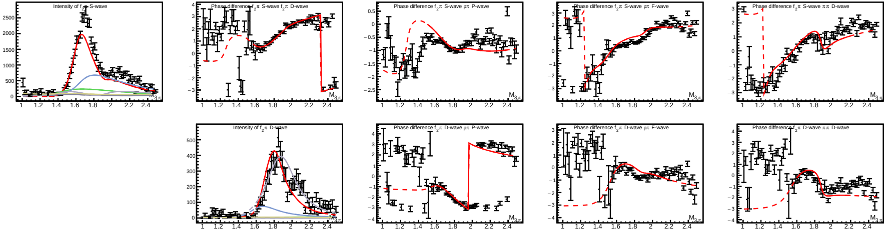
$$0.326380 < t' < 0.448588\,(\mathrm{GeV}/c)^2$$

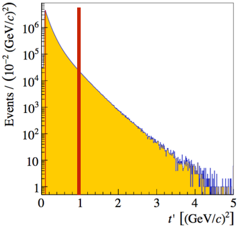


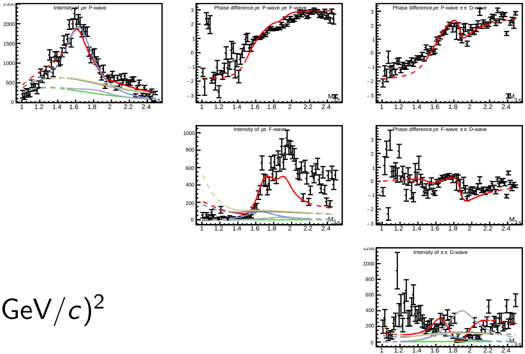




$$0.448588 < t' < 0.724294\,(\mathrm{GeV}/c)^2$$







$$0.724294 < t' < 1.000000\,(\mathrm{GeV}/c)^2$$

Status and Future Work

- We have developed a model for analysis of COMPASS 3π
- Status of 2^{-+} analysis
  - Performing cross-checks on codes
  - Performing preliminary fits
  - Preparing systematic analysis
- Perform mass-dependent analysis in 1^{++} sector
  - Investigate a_1-puzzle
  - Possible strong $\pi K \overline{K}$ component in sector (coupled channel analysis?)
- Can extend analysis for neutral 3π sector, opportunity for coupled channel analysis
- Extensions for photon beams for JLab 12 GeV upgrade