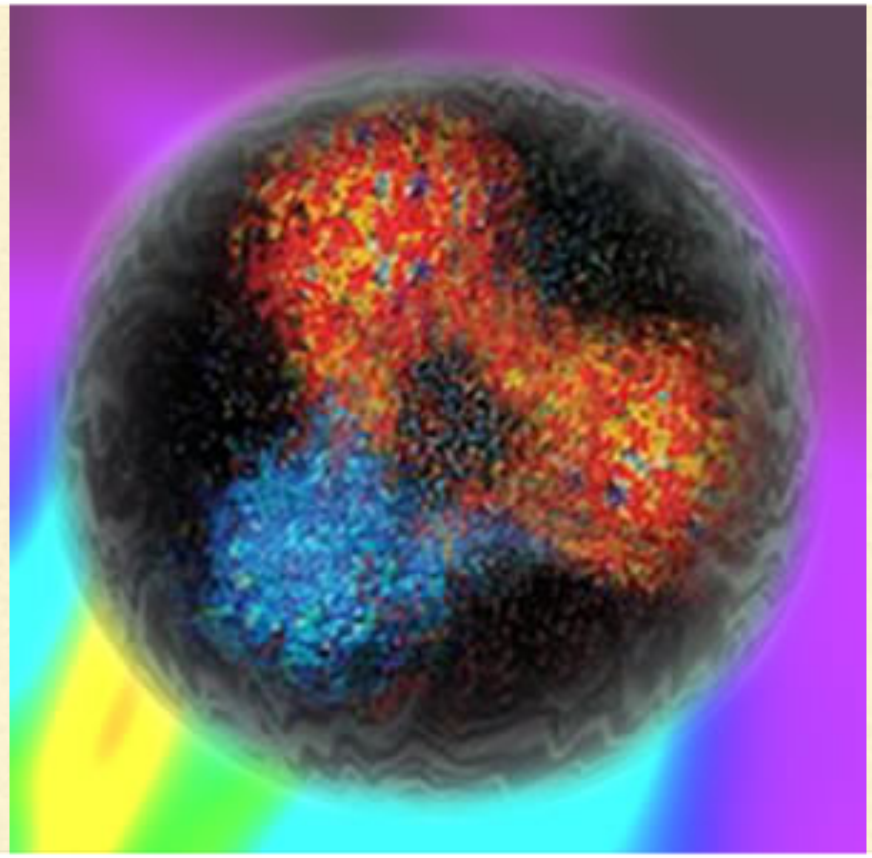


Quasi-Parton distributions and the Gradient Flow

Kostas Orginos (W&M/JLab)

Chris Monahan (Rutgers)

INTRODUCTION

- Goal: Compute properties of hadrons from first principles
  - Parton distribution functions (PDFs)
- Lattice QCD calculations is a first principles method
  - For many years calculations focused on Mellin moments
  - Can be obtained from local matrix elements of the proton in Euclidean space
    - Breaking of rotational symmetry –> power divergences
    - only first few moments can be computed
- Recently direct calculations of PDFs in Lattice QCD are proposed
- First lattice Calculations already available

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

QUASI-PARTON DISTRIBUTIONS

- Defined as non-local (space), equal time matrix elements in Euclidean space
- Equal time: rotation to Minkowski space is trivial
- PDFs are obtained in the limit of infinite proton momentum
- Matching to the infinite momentum limit can be obtained through perturbative calculations

X. Xiong, X. Ji, J. H. Zhang, Y. Zhao, Phys. Rev. D 90, no. 1, 014051 (2014)
T. Ishikawa et al. arXiv:1609.02018 (2016)

QPDFS: DEFINITION

Light-cone PDFs:

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+\omega^-} \left\langle P \left| T\,\overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2}\psi(0) \right| P \right\rangle_\mathrm{C} .$$

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0 \int_0^{\omega^-} \mathrm{d}y^- A_\alpha^+(0,y^-,\mathbf{0}_\mathrm{T})T_\alpha\right]$$

$$\langle P'|P\rangle = (2\pi)^3 2P^+\delta\left(P^+ - P'^+\right)\delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

Moments:

$$a_0^{(n)} = \int_0^1 \mathrm{d}\xi\, \xi^{n-1}\left[f^{(0)}(\xi) + (-1)^n \overline{f}^{(0)}(\xi)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f(\xi)$$

Local matrix elements:

$$\left\langle P|\mathcal{O}_0^{\{\mu_1\ldots\mu_n\}}|P\right\rangle = 2a_0^{(n)}\left(P^{\mu_1}\cdots P^{\mu_n} - \text{traces}\right)$$

$$\mathcal{O}_0^{\{\mu_1\cdots\mu_n\}} = i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1}D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi(0) - \text{traces}$$

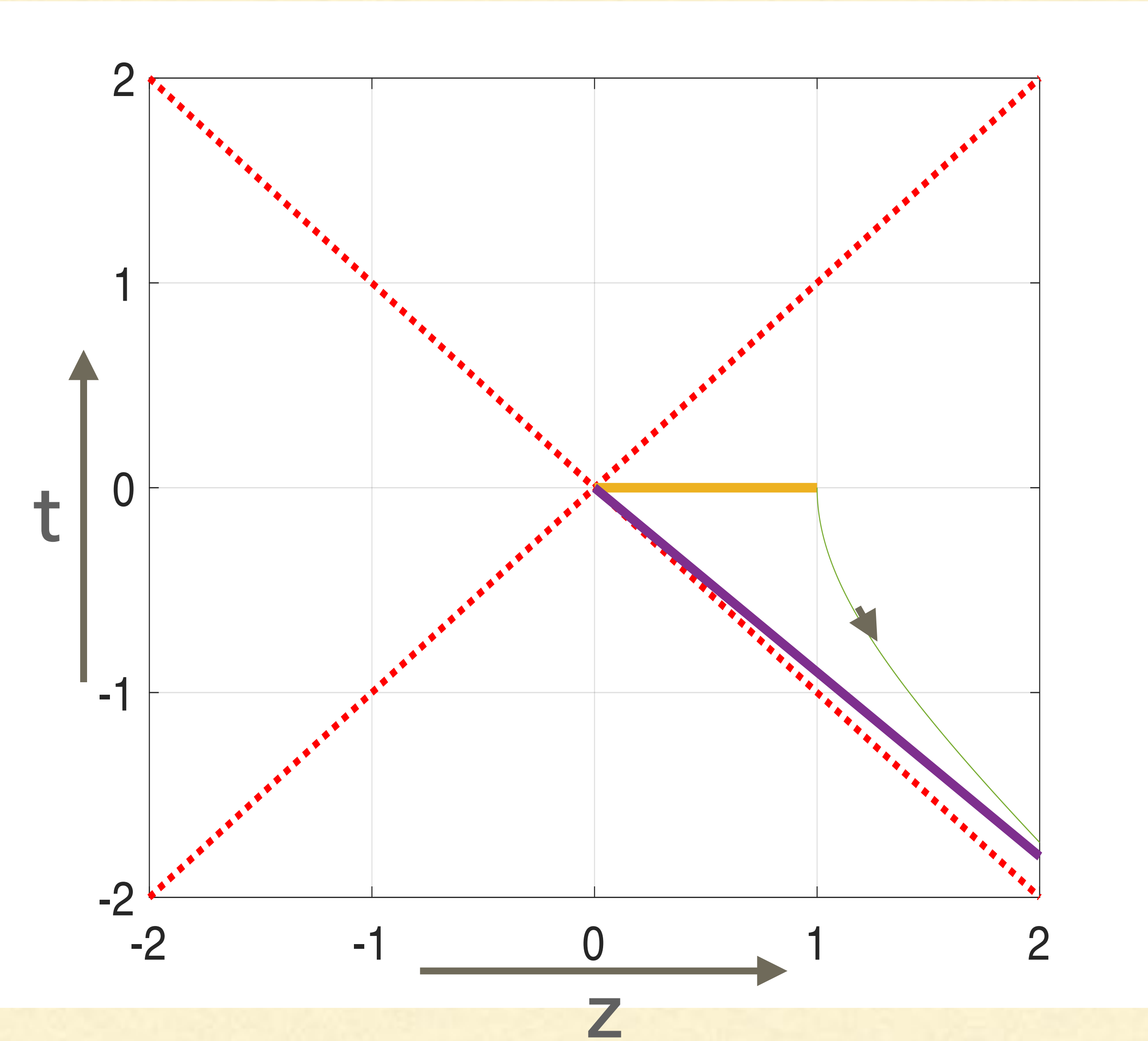
QPDFS: DEFINITION

$$h^0\left(z, P_z\right) = \frac{1}{2P_z}\left\langle P_z \left| \overline{\psi}(z)\mathbf{W}(0, z; \tau)\gamma_z \frac{\lambda^a}{2}\psi(0)\right| P_z \right\rangle_{\mathrm{C}}$$

$$\mathbf{W}(z,0) = \mathcal{P}\exp\left[-ig_0\int_0^z \mathrm{d}z'\, A^3_\alpha(z'\mathbf{v})\mathbf{T}_\alpha\right], \quad \mathbf{v} = (0,0,1,0)$$

$$q^{(0)}\left(\xi, P_z\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}z\, e^{i\xi z P_z} h^{(0)}\left(z, P_z\right)$$

QPDFS: MAIN IDEA



$$\lim_{P_z \to \infty} q^{(0)}(x, P_z) = f(x)$$

X. Ji, Phys.Rev.Lett. 110, (2013)

Euclidean space time local matrix element is equal to the same matrix element in Minkowski space

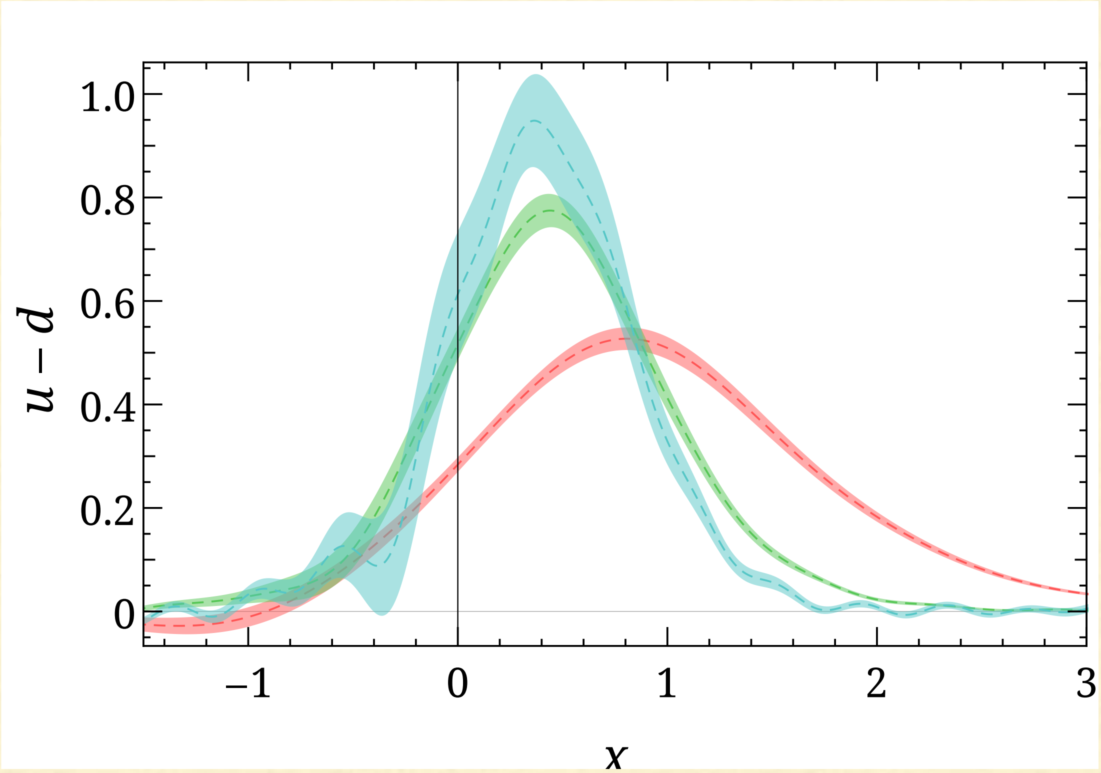
$$q\left(x,P_z\right)=\int_{-1}^{1}\frac{d\xi}{\xi}\,\widetilde{Z}\left(\frac{x}{\xi},\frac{\mu}{P_z}\right)f(\xi,\mu)+\mathcal{O}(\Lambda_{\mathrm{QCD}}/P_z,M_N/P_z)$$

The matching kernel can be computed in perturbation theory

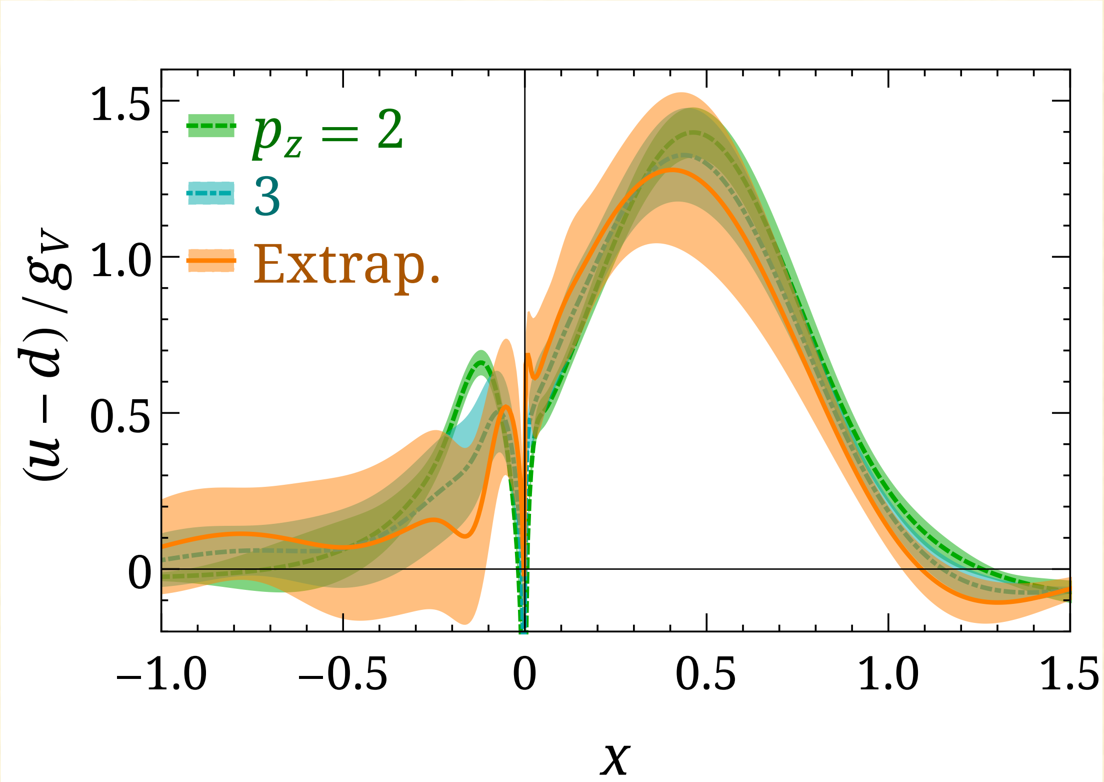
X. Xiong, X. Ji, J. H. Zhang, Y. Zhao, Phys. Rev. D 90, no. 1, 014051 (2014)
T. Ishikawa et al. arXiv:1609.02018 (2016)

- Practical calculations require a regulator (Lattice)
- Continuum limit has to be taken
  - renormalization
- Momentum has to be large compared to hadronic scales to suppress higher twist effects
- Practical issue with LQCD calculations at large momentum … signal to noise ratio

First Lattice results (Chen et. al)



Convergence with momentum extrapolation



Including the 1-loop matching kernel

Plots taken from: Chen et al. arXiv:1603.06664

Similar results have been achieved by Alexandrou et. al (ETMC)

A more general point of view:

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

$$\sigma(x, a, P_z) \xrightarrow{a \to 0} \tilde{\sigma}(x, \tilde{\mu}^2, P_z)$$

Minkowski space factorization:

$$\widetilde{\sigma}(x, \widetilde{\mu}^2, P_z) = \sum_{\alpha=\{q,\overline{q},g\}} H_\alpha \left(x, \frac{\widetilde{\mu}}{P_z}, \frac{\widetilde{\mu}}{\mu} \right) \otimes f_\alpha(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\widetilde{\mu}^2} \right)$$

H_α computable in perturbation theory

Related ideas see:

K-F Liu Phys.Rev. D62 (2000) 074501

Detmold and Lin Phys.Rev.D73:014501,2006

PROCEDURE OUTLINE

- Compute equal time matrix elements in Euclidean space using Lattice QCD at sufficiently large momentum in order to suppress higher twist effects
- Take the continuum limit (renormalization)
- Equal time: Minkowski – Euclidean equivalence
- Perform the matching Kernel calculation in the continuum

GRADIENT FLOW SMEARING

Is a way to obtain a finite matrix element in the continuum that can then be used to obtain the light cone PDFs

Issues related to the continuum limit
have been also discussed in :

T. Ishikawa et al. arXiv:1609.02018 (2016)

Chen et al arXiv:1609.08102 (2016)

GRADIENT FLOW

Luscher ['10,'13]

It is a suitably chosen map of the fields to new fields that have UV fluctuations suppressed

$$A_\mu \to B_\mu[A_\mu]$$

$$\bar{q} \to \bar{\Psi}[\bar{q}, q, A_\mu] \qquad q \to \Psi[\bar{q}, q, A_\mu]$$

Can be defined in the continuum as well as on the lattice

Correlation functions of the new fields can be studied perturbatively or non-perturbatively and used as probes of the underlying QFT.

Diffusion equations (lattice version):

$$\partial_s V_\mu(x,s) = -g_0^2 \partial_{V_\mu(x,s)} S_w V_\mu(x,s)$$
$$\partial_s \psi(x,s) = \overrightarrow{\Delta} \psi(x,s)$$
$$\partial_s \overline{\psi}(x,s) = \overline{\psi}(x,s) \overleftarrow{\Delta}$$

S_w is the Wilson gauge action

Δ is the lattice covariant laplacian

s is the flow time

s=0 the fields take the value of the original fields in the path integral

Integrate these equations for some time s resulting damping of the UV fluctuations down to scale

$$\mu = 1/\sqrt{s}$$

Solutions to the flow equations (leading order in coupling constant)

$$\psi(x,s) = \int K(x-y,s)q(y)d^4y + \cdots$$

$$B_\mu(x,s) = \int K(x-y,s)A_\mu(y)d^4y + \cdots$$

The “heat kernel” K is:

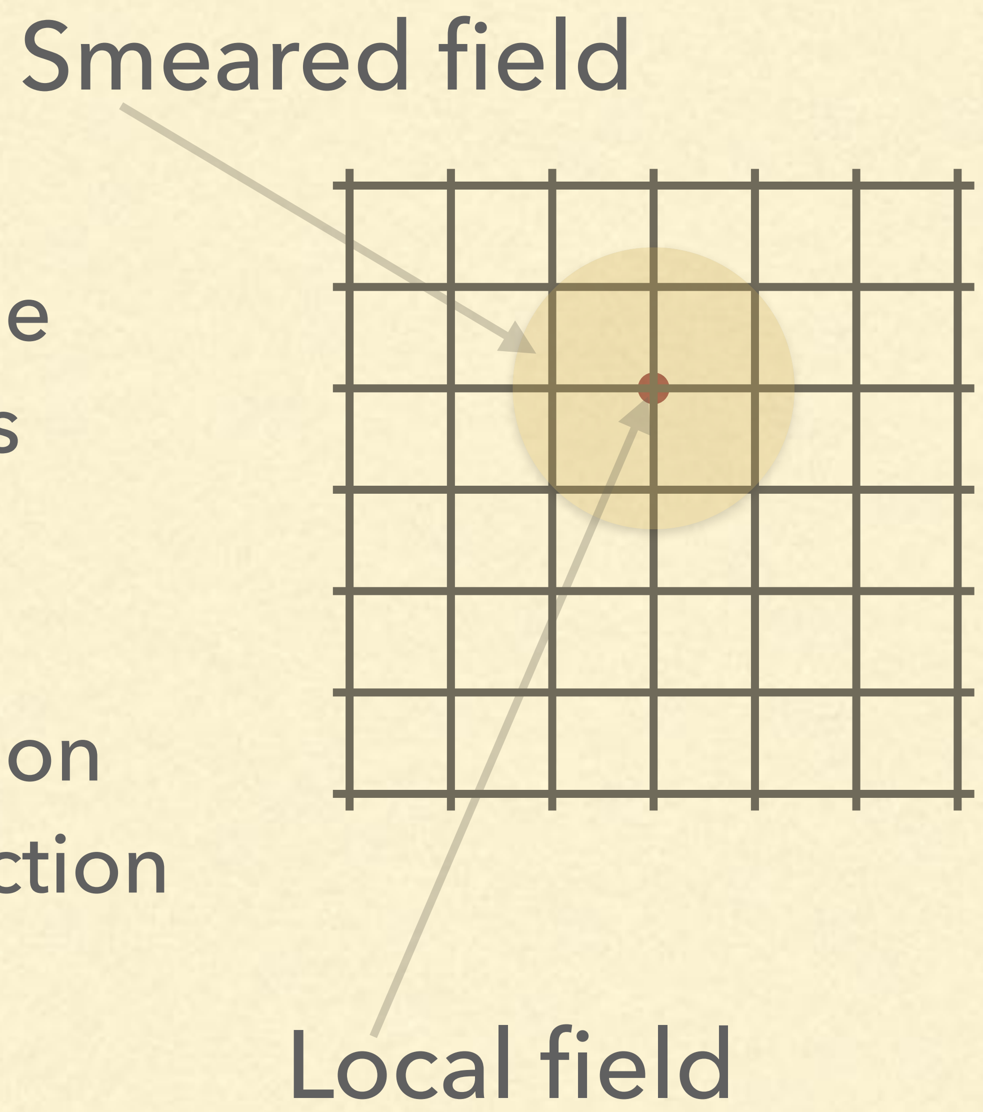
$$K(x,s) = \frac{e^{-x^2/4s}}{(4\pi s)^2}$$

Hence the exponential damping of UV fluctuations to scales:

$$\mu = 1/\sqrt{s}$$

Notable results (Luscher):

- Correlation functions of "smeared" gauge fields are finite if the underlying theory is renormalized (BRST symmetry)
- Correlation functions of "smeared" fermion fields are finite if an additional wave function renormalization is included
- Fermion wave function renormalization can be removed using the "ringed" smeared fermion fields



. H. Makino and H. Suzuki, PTEP 2014, 063B02 (2014), 1403.4772.

. K. Hieda and H. Suzuki (2016), 1606.04193

Ringed smeared fermions

$$\mathring{\chi}(\tau, x) = \sqrt{\frac{-2\dim(R)N_{\mathrm{f}}}{(4\pi)^2\tau^2\left\langle\bar{\chi}(\tau, x)\overleftrightarrow{\not{D}}\chi(\tau, x)\right\rangle}}\,\chi(\tau, x),$$

$$\mathring{\bar{\chi}}(\tau, x) = \sqrt{\frac{-2\dim(R)N_{\mathrm{f}}}{(4\pi)^2\tau^2\left\langle\bar{\chi}(\tau, x)\overleftrightarrow{\not{D}}\chi(\tau, x)\right\rangle}}\,\bar{\chi}(\tau, x)$$

Ringed fermion correlation functions require no additional renormalization

. H. Makino and H. Suzuki, PTEP 2014, 063B02 (2014), 1403.4772.

. K. Hieda and H. Suzuki (2016), 1606.04193

SMEARED QUASI-PDFS

$$h^{(s)}\left(\frac{z}{\sqrt{\tau}}, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \frac{1}{2P_z}\left\langle P_z \left| \overline{\chi}(z;\tau)\mathcal{W}(0,z;\tau)\gamma_z \frac{\lambda^a}{2}\chi(0;\tau) \right| P_z \right\rangle_{\mathrm{C}}$$

τ is the flow time

χ is the ringed smeared quark field

$\mathcal{W}$ is the smeared gauge link

$$q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{2\pi} e^{i\xi z P_z} P_z\, h^{(s)}(\sqrt{\tau}z, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}),$$

At fixed flow time the quasi-PDF is finite in the continuum limit

Using the previous definitions we have

$$\left(\frac{i}{P_z}\frac{\partial}{\partial z}\right)^{n-1} h^{(s)}\left(\frac{z}{\sqrt{\tau}}, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^{n-1} e^{-i\xi z P_z} q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right)$$

By introducing the moments

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \frac{\Lambda_{\mathrm{QCD}}}{P_z}, \frac{M_{\mathrm{N}}}{P_z}\right) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^{n-1} q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right)$$

Taking the limit of z going to 0 we obtain:

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \frac{\Lambda_{\rm QCD}}{P_z}, \frac{M_{\rm N}}{P_z}\right) = \frac{c_n^{(s)}(\sqrt{\tau}P_z)}{2P_z^n}\left\langle P_z \left| \left[\overline{\chi}(z;\tau)\gamma_z(i\overleftarrow{D}_z)^{(n-1)}\frac{\lambda^a}{2}\chi(0;\tau)\right]_{z=0}\right| P_z\right\rangle_{\rm C}.$$

i.e. the moments of the quasi-PDF are related to local matrix elements of the smeared fields

These matrix elements are not twist-2. Higher twist effects enter as corrections that scale as powers of

$$\frac{\Lambda_{\rm QCD}}{P_z}, \quad \frac{M_N}{P_z}$$

after removing M_N/P_z effects

[H.-W. Lin, et. al Phys.Rev. D91, 054510 (2015)]

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}\right) = c^{(s)}(\sqrt{\tau}P_z)b_n^{(s,{\rm twist}-2)}\left(\sqrt{\tau}\Lambda_{\rm QCD}\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

Small flow time expansion: Luscher ['10,'13]

$$b_n^{(s,\mathrm{twist}-2)}\big(\sqrt{\tau}\Lambda_{\mathrm{QCD}}\big) = \widetilde{C}_n^{(0)}(\sqrt{\tau}\mu)a^{(n)}(\mu) + \mathcal{O}(\sqrt{\tau}\Lambda_{\mathrm{QCD}})$$

$a^{(n)}(\mu)$ are the moments of the PDFs

The quasi-PDF moments then are:

$$b_n^{(s)}\big(\sqrt{\tau}\Lambda_{\mathrm{QCD}}\big) = C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)a^{(n)}(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

$$\Lambda_{\mathrm{QCD}}, M_N \ll P_z \ll \tau^{-1/2}$$

Introducing a kernel function such that:

$$C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z) = \int_{-\infty}^{\infty} dx\, x^{n-1} \widetilde{Z}(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z)$$

We can undo the Mellin transform:

$$q^{(s)}\left(x, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}P_z\right) = \int_{-1}^{1} \frac{d\xi}{\xi} \widetilde{Z}\left(\frac{x}{\xi}, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) f(\xi, \mu) + \mathcal{O}(\sqrt{\tau}\Lambda_{\mathrm{QCD}})$$

Therefore smeared quasi-PDFs are related to PDFs if

$$\Lambda_{\mathrm{QCD}}, M_N \ll P_z \ll \tau^{-1/2}.$$

CONCLUSIONS

- Quasi-PDFs provide a novel way to study hadron structure in Lattice QCD
- Lattice calculations from several groups are on the way
- Several ideas for dealing with the continuum limit are now developing
- Here I presented gradient flow as a tool that allows us to obtain continuum quasi-PDFs that can then be related to PDFs via a convolution to a perturbatively calculable kernel function.
- Lattice calculations to understand the effectiveness of this approach are underway
- Analytic calculations for obtaining the matching kernel are also being developed