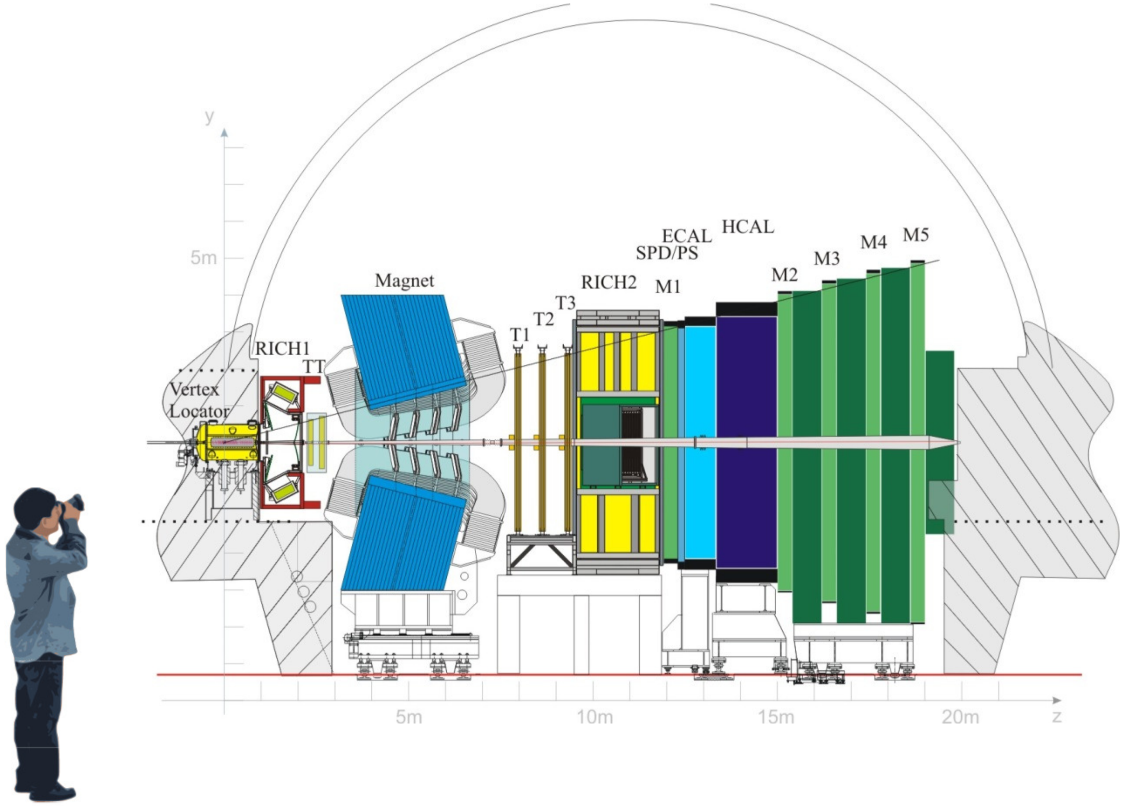
The Heavy-Quark Exotics:
A Snapshot from February 2017



Richard Lebed

ARIZONA STATE UNIVERSITY

7th Workshop of
the APS
Topical Group on
Hadronic Physics
February, 2017

Outline

1) A (very brief) survey of the exotic hadrons X,Y,Z, P_c

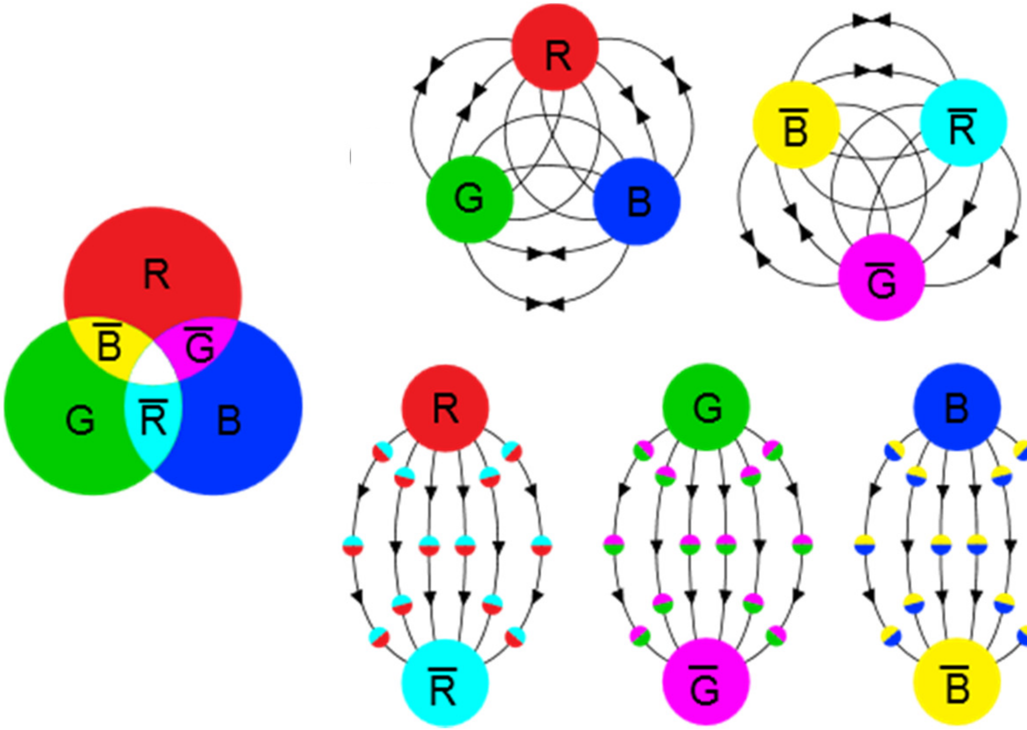
2) How are the tetraquarks X,Y,Z and pentaquarks P_c assembled?

3) A new dynamical picture for the X,Y,Z, P_c

4) Conclusions

For nearly 40 years after the inception of the quark model…

…Every single confirmed hadron took advantage of only two of the many possible color-neutral combinations: either quark-antiquark *mesons*, or quark-triple *baryons*



But many other alternatives are possible

The possible color-neutral ***exotics*** are: (g = gluon, q = quark)

- gg, ggg, … (*glueball*)
- $q\bar{q}g$, $q\bar{q}gg$, … (*hybrid meson*)
- $q\bar{q}q\bar{q}$, $q\bar{q}q\bar{q}q\bar{q}$, … (*tetraquark, hexaquark, …*)
- $qqqq\bar{q}$, $qqqqqqq\bar{q}$, … (*pentaquark*, *octoquark*, …)

i.e., (# of q) – (# of $\bar{q}$) = 0 mod 3, any number of g except one by itself

- Gell-Mann and Zweig actually both mentioned the multiquark options already in their seminal 1964 papers!

So why did it take so long to find them?

- Exotics can mix with ordinary hadrons with the same quantum numbers
  - Especially true for hadrons made solely from the light quark flavors, ***u***, ***d***, ***s***

- Weak experimental signals often either disappear with higher statistics, or are never confirmed by other experiments

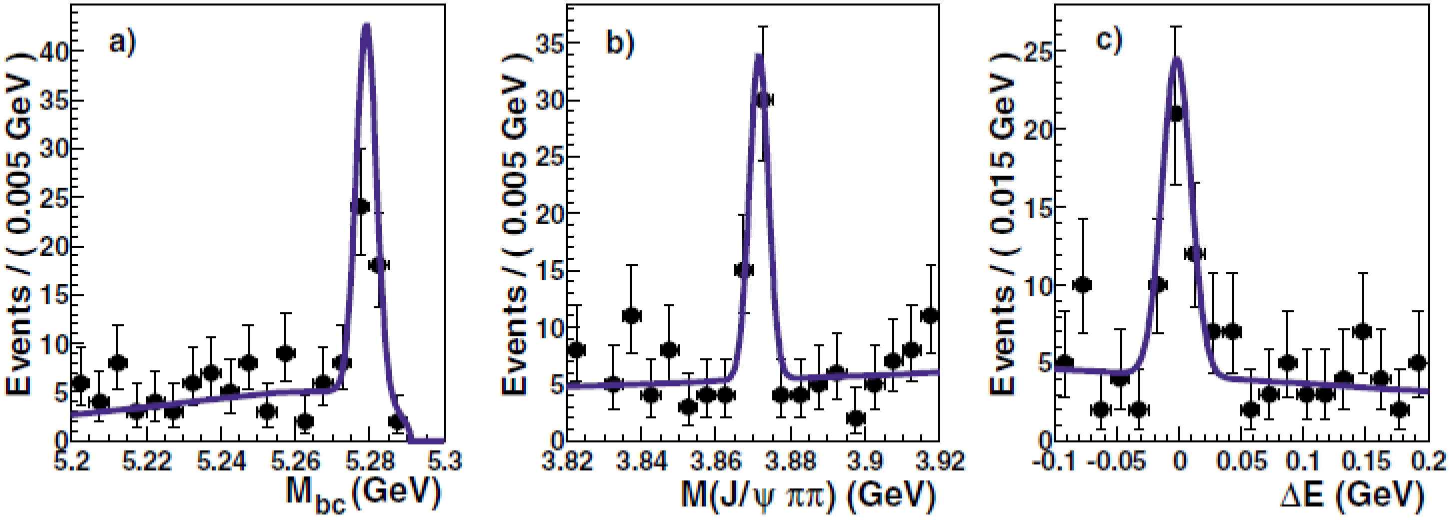
- A seemingly strong signal for a new particle, even one confirmed by multiple experiments, can turn out to be due to entirely different physics
  - *e.g.*, in the early 2000's, a famous pentaquark candidate $\Theta^+(1540)$ turned out not to be an *s*-channel *K*-*N* compound resonance, but the result of an unfortunate choice of kinematical *cuts* on the data and *t*-channel exchanges

- …So when the breakthrough finally came in **2003**, it was not instantly accepted by everyone

In 2003…

The Belle Collaboration (KEK) found evidence for a new particle at mass 3872 MeV

S.K. Choi *et al.*, Phys. Rev. Lett. **91** (2003) 262001



A nice windfall for an experiment whose primary physics goal was to observe *CP* violation in the *B* system

X = Unknown

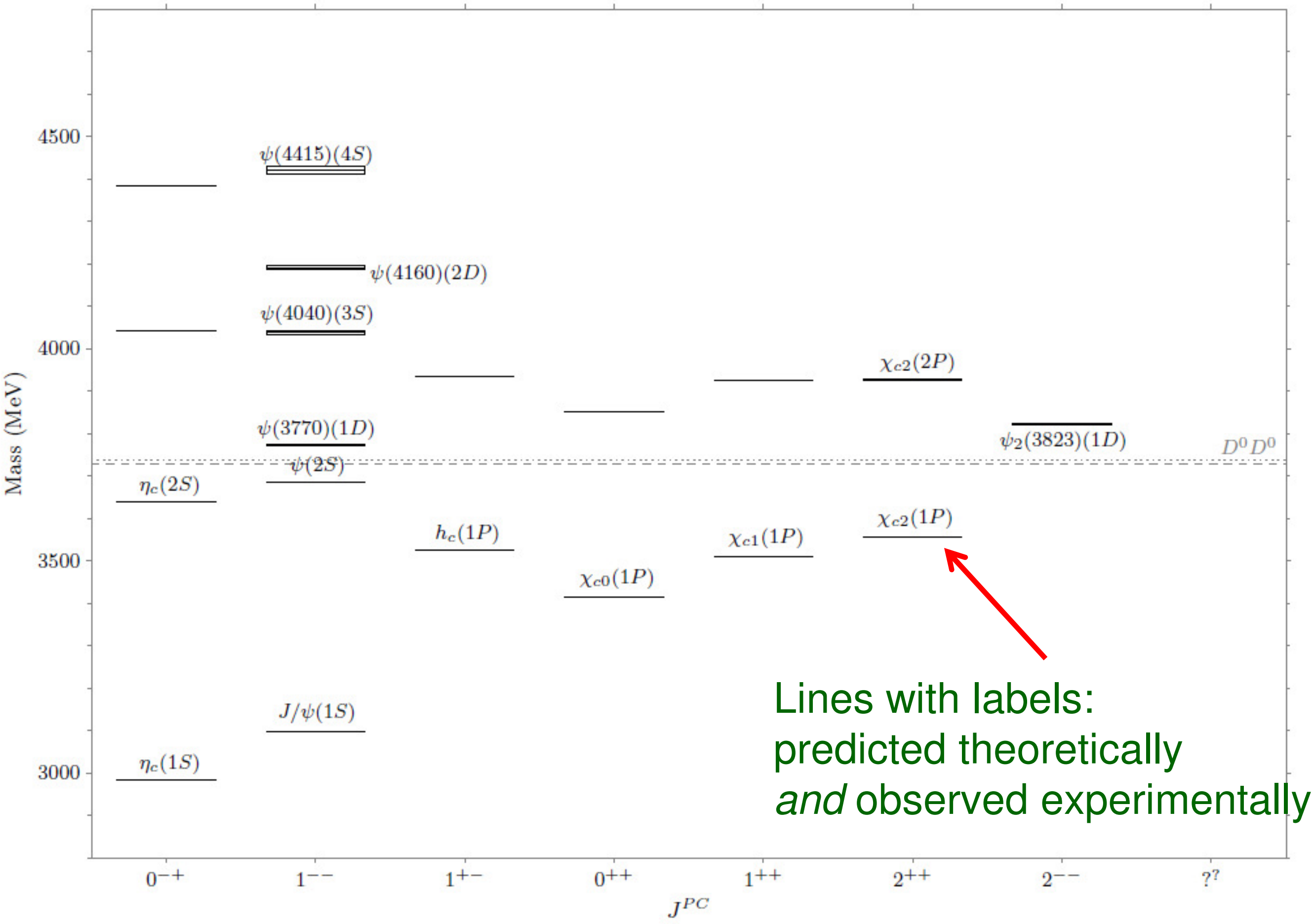
- Belle found a new charmoniumlike (containing $\bar{c}c$) resonance appearing in

  $$B \to K\,(J/\psi\,\pi^+\pi^-)$$

  - Believed to contain $c\bar{c}$ because it is in the same mass range as charmonium, and always decays into a final state containing $c\bar{c}$
- Has been confirmed at BaBar, CDF, DØ, LHCb, CMS
- $J^{PC} = 1^{++}$, but not believed to be ordinary $c\bar{c}$: Mass is many 10's of MeV below nearest $\bar{c}c$ candidate with these quantum numbers, $\chi_{c1}(2P)$
- Now called $\boldsymbol{X(3872)}$ [and believed to be a $(c\bar{c}u\bar{u})$ state]
  - $m_{X(3872)} = 3871.69 \pm 0.17\ \mathrm{MeV}$
  - Note: $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = +0.01 \pm 0.18\ \mathrm{MeV}$

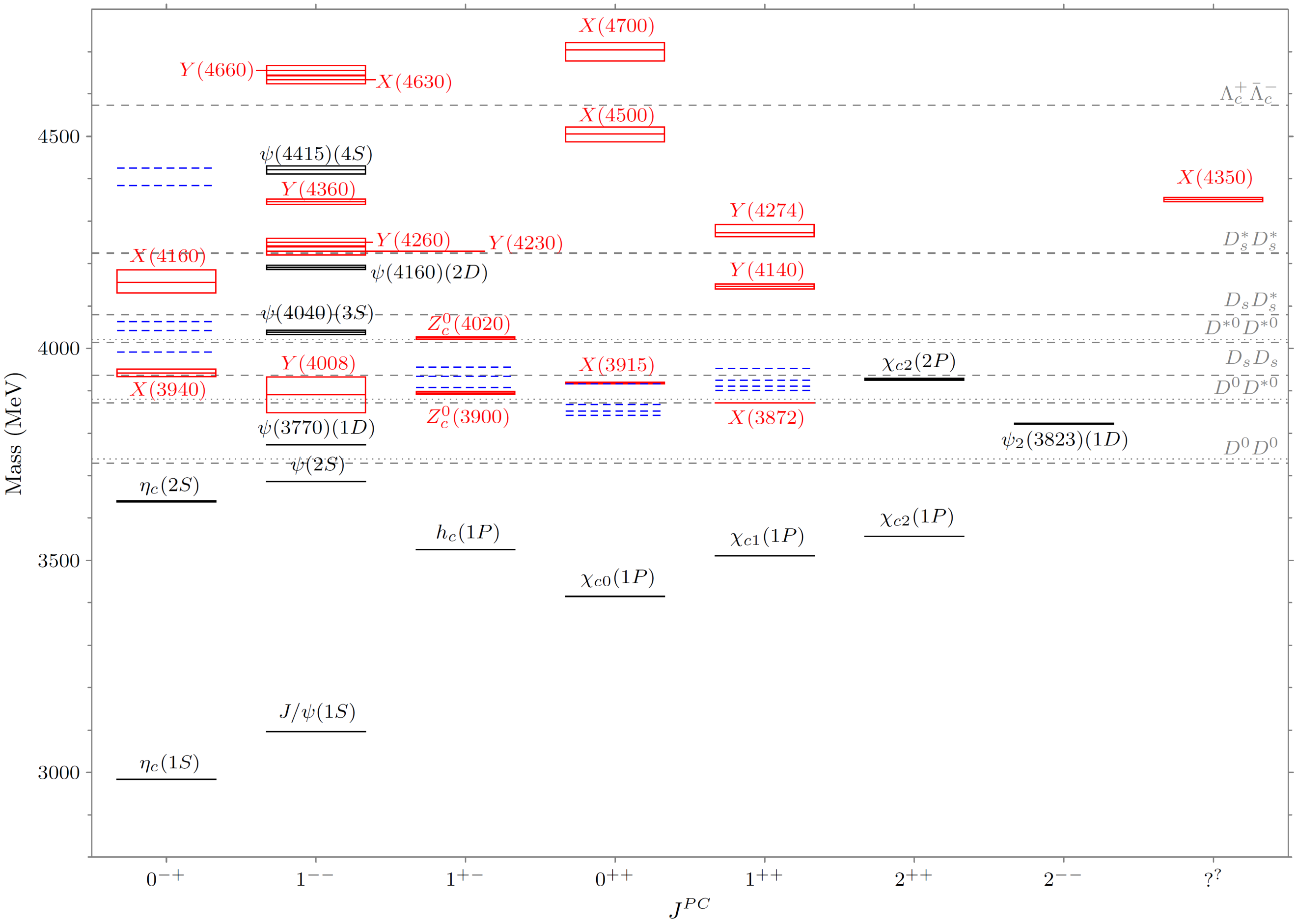
    Leads to endless speculation that *X*(3872) is a $D\bar{D}^*$ hadronic molecule $(D^0 = c\bar{u}, \bar{D}^{*0} = \bar{c}u)$
  - Width: $\Gamma_{X(3872)} < 1.2$ MeV

What the Charmonium System Should Look Like

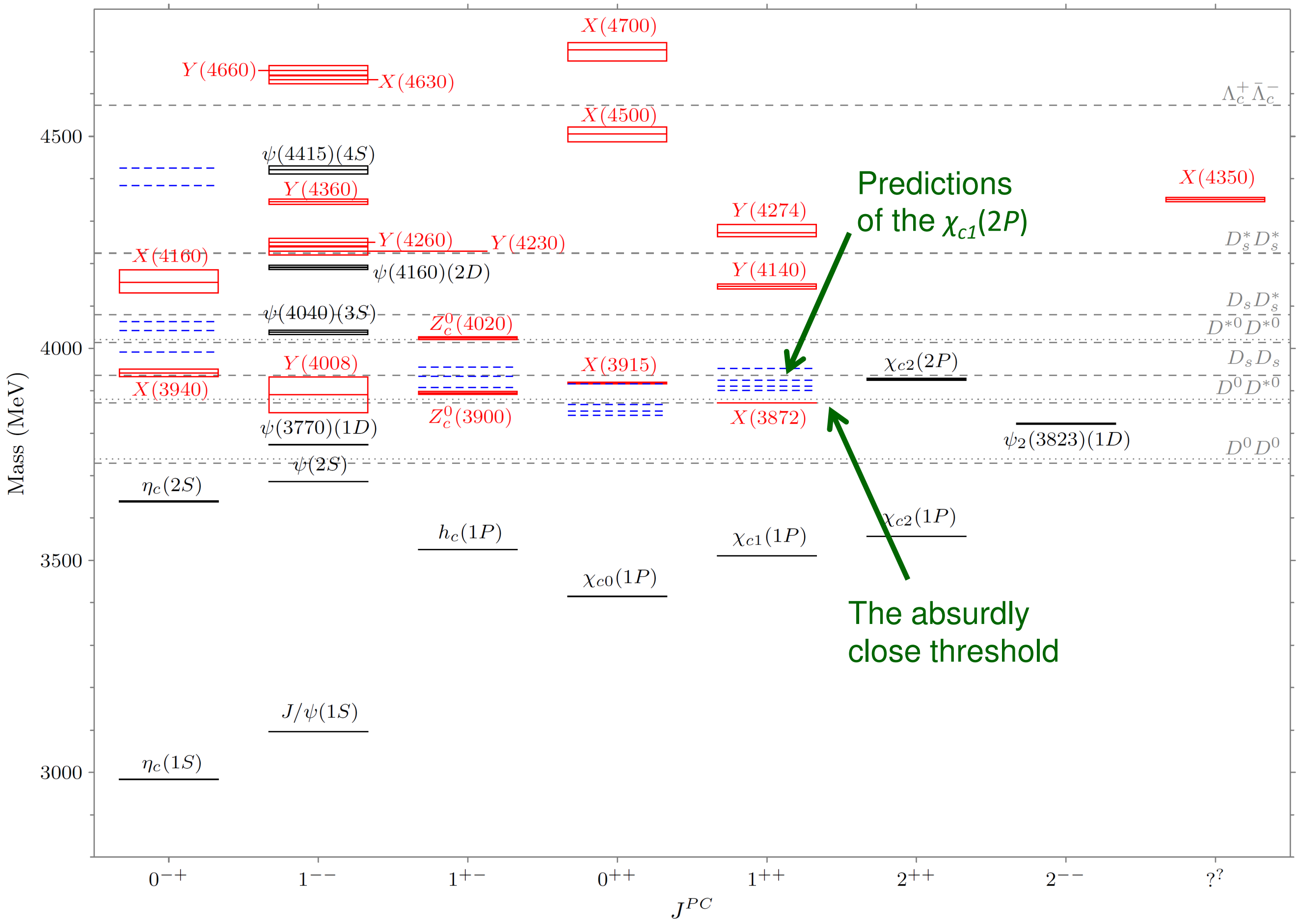


Lines with labels:
predicted theoretically
and observed experimentally

What the Charmonium System Really Looks Like February 2017



The Peculiar *X*(3872), the first *tetraquark*



Predictions of the $\chi_{c1}(2P)$

The absurdly close threshold

…And in 2005: ***Y***

BABAR Collaboration (B. Aubert *et al.*, PRL **95**, 142001 [2005])

Charmoniumlike states started to show up
in initial-state radiation (ISR) e^+e^- annihilation:

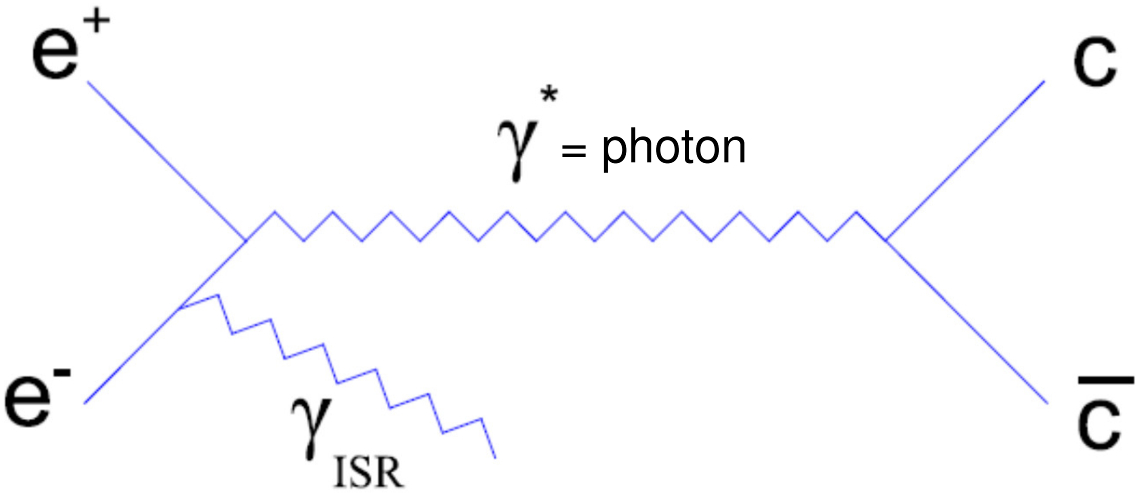


Figure from Nielsen *et al.*,
Phys. Rept. **497** (2010) 41

Such states necessarily have $J^{PC} = 1^{--}$
(same quantum numbers as the photon), and are called "*Y*"

This first-discovered one is named ***Y*(4260)**

…And in 2013: *Z*

BESIII Collaboration [Beijing] (M. Ablikim *et al.*, PRL **110**, 252001 [2013]),
Belle Collaboration (Z. Liu *et al.*, PRL **110**, 252002 [2013])

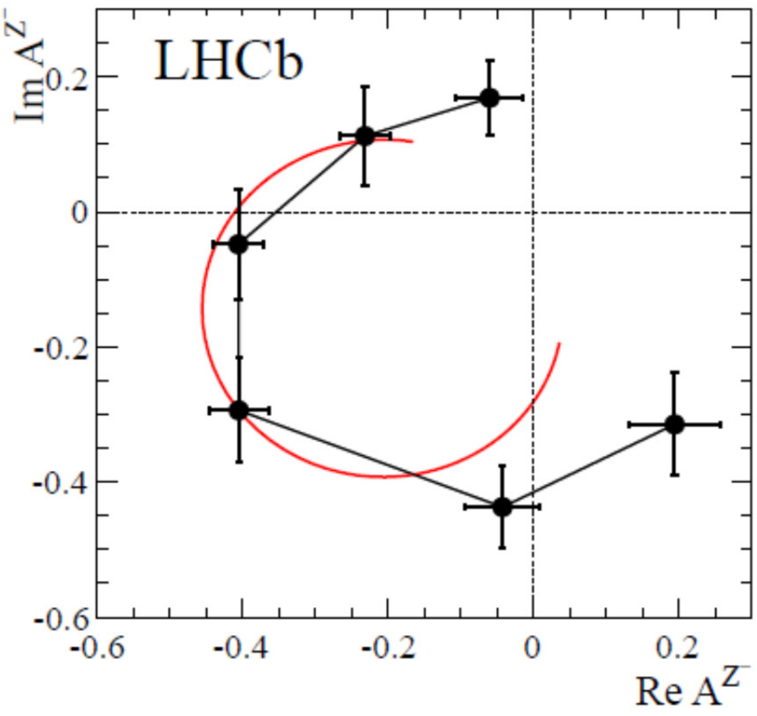
- A charged charmoniumlike resonance is observed in
  $$Y(4260) \rightarrow \pi^{-}(\pi^{+}J/\psi)$$
- Minimal possible flavor content: $c\bar{c}u\bar{d}$:
  No question that it has four valence quarks
- Now called $\boldsymbol{Z_c^{+}}$**(3900)**, $J^P = 1^{+}$
- *The first manifestly exotic state ever confirmed beyond 5σ by two experiments*

  [not counting the Θ^{+}(1540)]
- What if all these states are not really states, but rather brilliant forgeries, like the Θ^{+}(1540)?

…And in 2014: **Resonance**

LHCb Collaboration (R. Aaij *et al.*, PRL **112**, 222002 [2014])

- The first charged charmoniumlike exotic was actually first seen by Belle in 2008 (PRL **100**, 142001 [2008])
  and confirmed by them in papers from 2009 and 2013
- LHCb not only confirmed the state at 13.9σ, now called ***Z*⁺(4430)**, $J^P = 1^+$ but for the first time plotted the full complex production amplitude and showed that it obeys the proper phase-shift looping behavior of a Breit-Wigner **resonance**

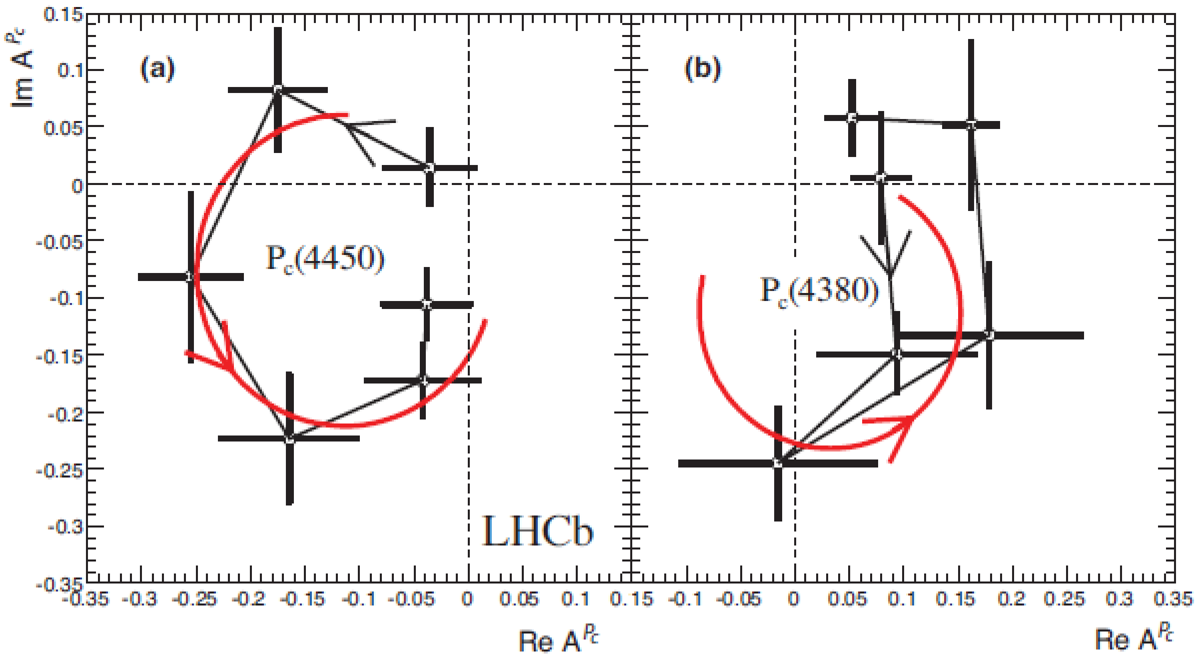


- **Welcome to the Age of the Third Hadron**

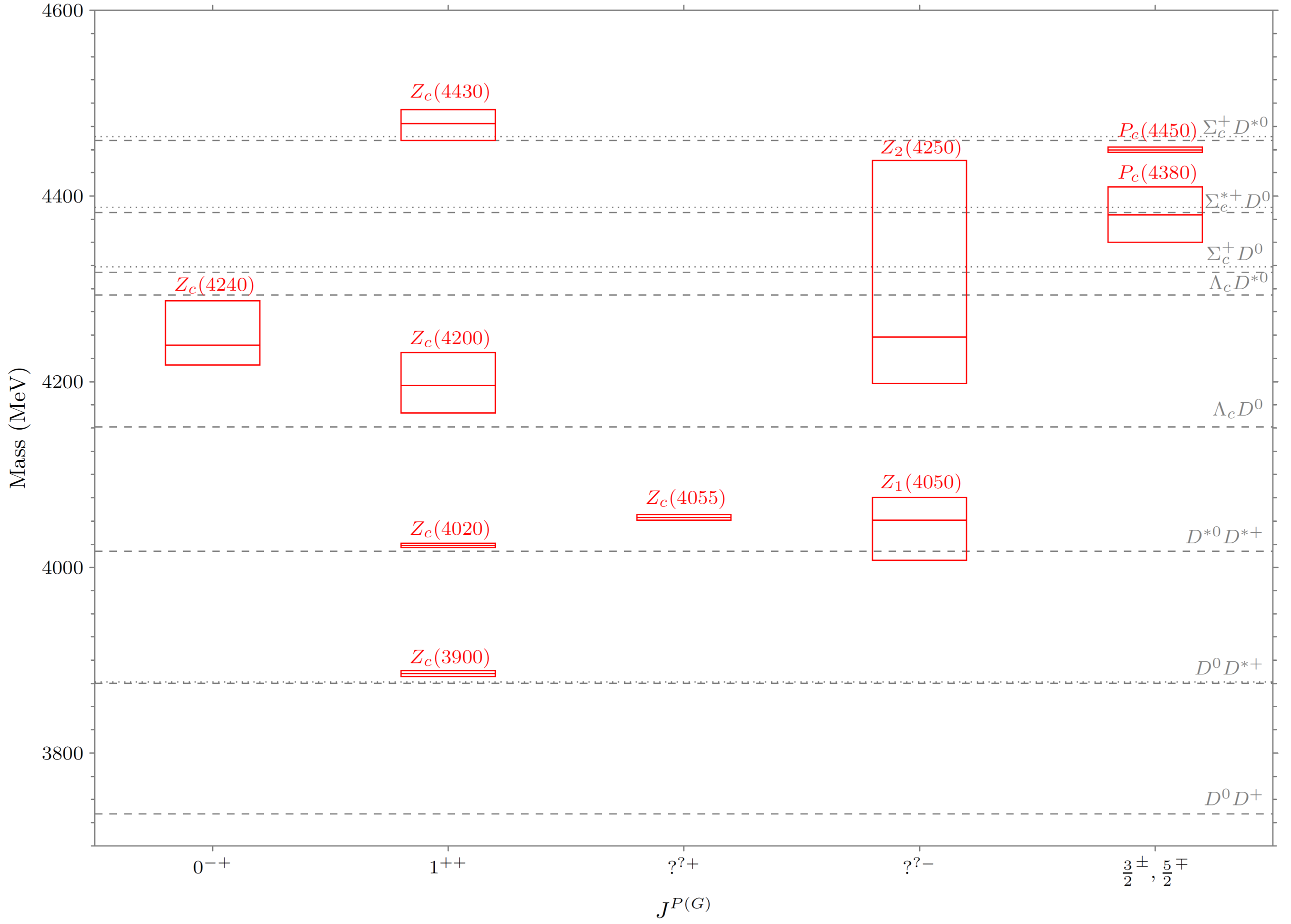
…And in 2015: $\boldsymbol{P_c}$

LHCb Collaboration [R. Aaij *et al.*, PRL **115** (2015) 072001]

- The first two *baryonic* charmoniumlike exotics, $P_c^+(4450)$, $P_c^+(4380)$
- Decay to $J/\psi + p \longrightarrow$Valence structure $c\bar{c}uud$: **Pentaquarks**!
- $m_1 = 4380 \quad \pm 8 \quad \pm 29$ MeV, $\Gamma_1 = 205 \pm 18 \pm 86$ MeV, **9σ significance**
- $m_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV, $\Gamma_2 = \ \ 39 \ \pm 5 \pm \ 19$ MeV, **12σ significance**
- Preferred J^P assignments:
  $(3/2^-, 5/2^+) >$
  $(3/2^+, 5/2^-) >$
  $(5/2^+, 3/2^-)$
- **Welcome to the Age of the Fourth Hadron**

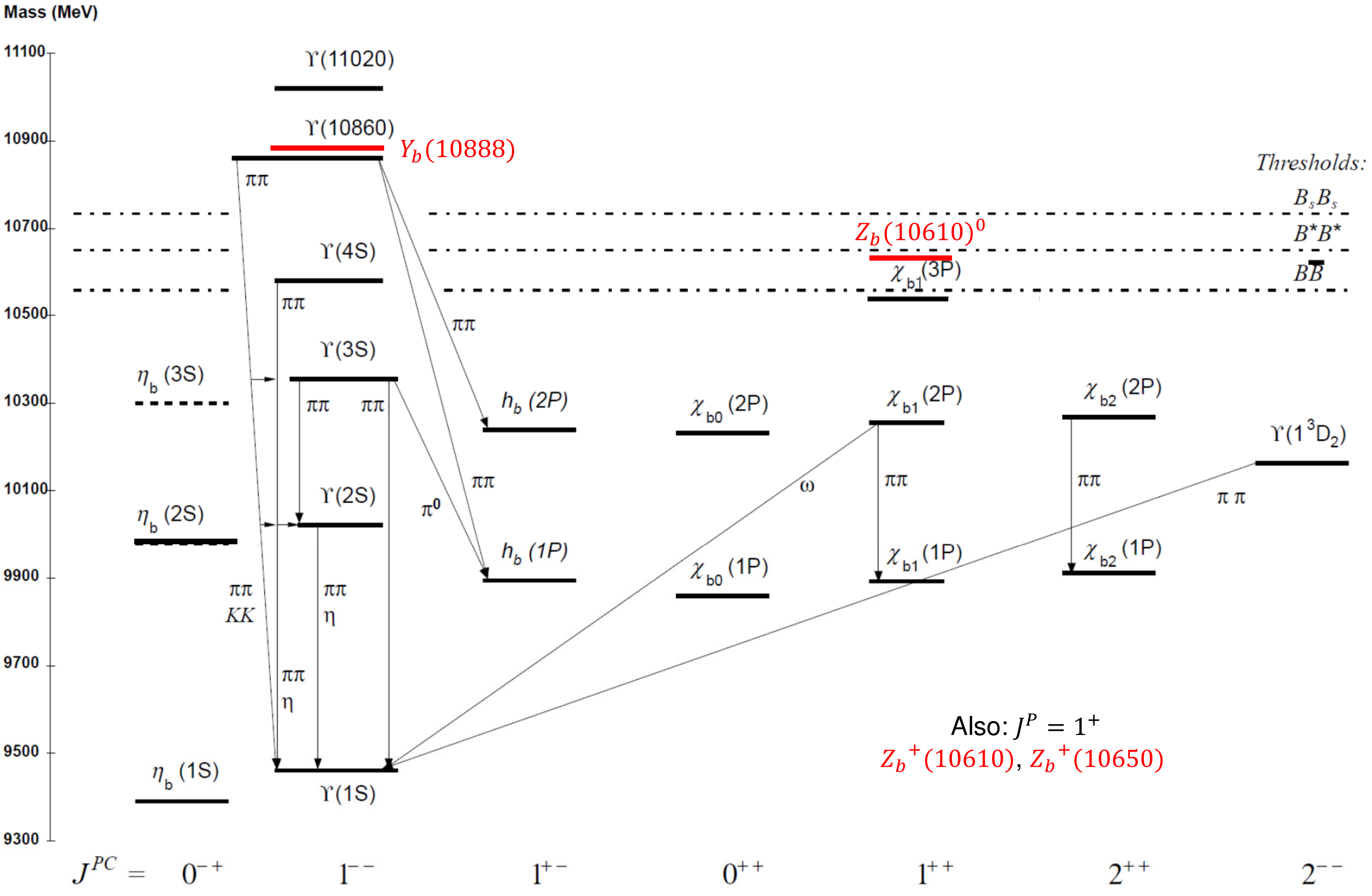


Charmonium: February 2017
Charged sector



THE BOTTOMONIUM SYSTEM

from the Particle Data Group, http://pdglive.lbl.gov/



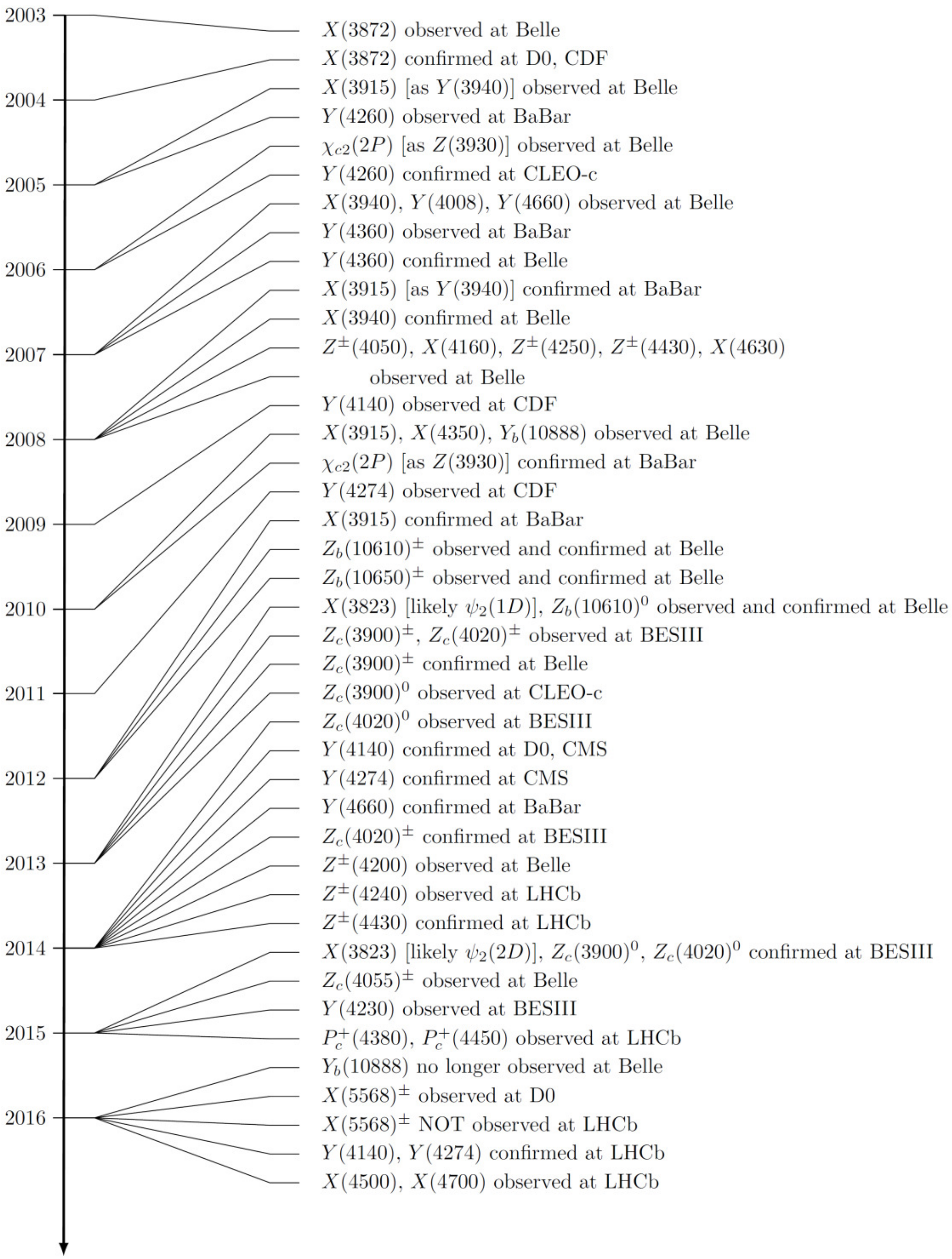
Also: $J^P = 1^+$

$Z_b{}^+(10610)$, $Z_b{}^+(10650)$

The exotics scorecard: February 2017

- **32** observed exotics
  - 27 in the charmonium sector
  - 4 in the (much less explored) bottomonium sector
  - 1 with a single *b* quark (and an *s*, a *u*, and a *d*)
- **15** confirmed (& none of the other 17 disproved) at >5σ in more than one experiment, mode, or both

New discoveries every single year



2003

2004

2005

2006

2007

2008

2009

2010

2011

2012

2013

2014

2015

2016

$X(3872)$ observed at Belle

$X(3872)$ confirmed at D0, CDF

$X(3915)$ [as $Y(3940)$] observed at Belle

$Y(4260)$ observed at BaBar

$\chi_{c2}(2P)$ [as $Z(3930)$] observed at Belle

$Y(4260)$ confirmed at CLEO-c

$X(3940)$, $Y(4008)$, $Y(4660)$ observed at Belle

$Y(4360)$ observed at BaBar

$Y(4360)$ confirmed at Belle

$X(3915)$ [as $Y(3940)$] confirmed at BaBar

$X(3940)$ confirmed at Belle

$Z^{\pm}(4050)$, $X(4160)$, $Z^{\pm}(4250)$, $Z^{\pm}(4430)$, $X(4630)$ observed at Belle

$Y(4140)$ observed at CDF

$X(3915)$, $X(4350)$, $Y_b(10888)$ observed at Belle

$\chi_{c2}(2P)$ [as $Z(3930)$] confirmed at BaBar

$Y(4274)$ observed at CDF

$X(3915)$ confirmed at BaBar

$Z_b(10610)^{\pm}$ observed and confirmed at Belle

$Z_b(10650)^{\pm}$ observed and confirmed at Belle

$X(3823)$ [likely $\psi_2(1D)$], $Z_b(10610)^0$ observed and confirmed at Belle

$Z_c(3900)^{\pm}$, $Z_c(4020)^{\pm}$ observed at BESIII

$Z_c(3900)^{\pm}$ confirmed at Belle

$Z_c(3900)^0$ observed at CLEO-c

$Z_c(4020)^0$ observed at BESIII

$Y(4140)$ confirmed at D0, CMS

$Y(4274)$ confirmed at CMS

$Y(4660)$ confirmed at BaBar

$Z_c(4020)^{\pm}$ confirmed at BESIII

$Z^{\pm}(4200)$ observed at Belle

$Z^{\pm}(4240)$ observed at LHCb

$Z^{\pm}(4430)$ confirmed at LHCb

$X(3823)$ [likely $\psi_2(2D)$], $Z_c(3900)^0$, $Z_c(4020)^0$ confirmed at BESIII

$Z_c(4055)^{\pm}$ observed at Belle

$Y(4230)$ observed at BESIII

$P_c^+(4380)$, $P_c^+(4450)$ observed at LHCb

$Y_b(10888)$ no longer observed at Belle

$X(5568)^{\pm}$ observed at D0

$X(5568)^{\pm}$ NOT observed at LHCb

$Y(4140)$, $Y(4274)$ confirmed at LHCb

$X(4500)$, $X(4700)$ observed at LHCb

Shameless Self-Promotion

1610.04528

Contents lists available at ScienceDirect

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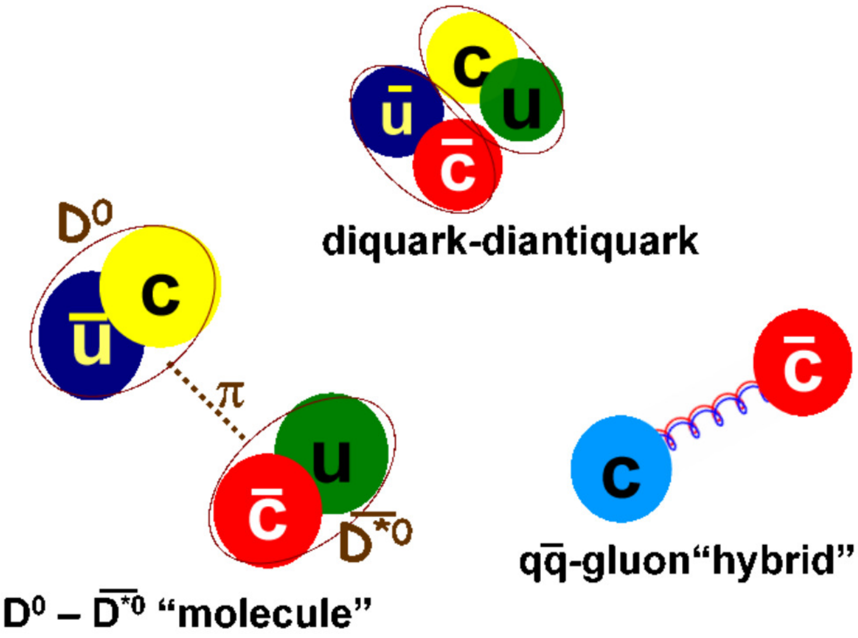
Review

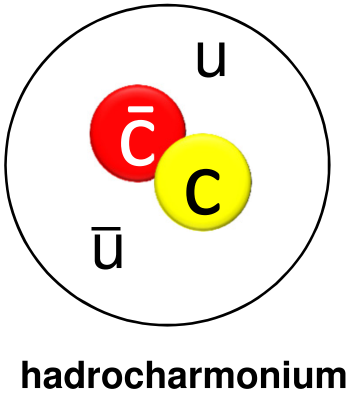
Heavy-quark QCD exotica

Richard F. Lebed [a,*], Ryan E. Mitchell [b], Eric S. Swanson [c]

…to learn in detail about the history of the discoveries
and the various theoretical interpretations attempted

How are tetraquarks assembled?





cusp effect:
Resonance created by rapid
opening of meson-meson threshold

Image from Godfrey & Olsen,
Ann. Rev. Nucl. Part. Sci. **58** (2008) 51

Trouble with the dynamical pictures

- Hybrids
  - Only usable for neutral states; then what are the *Z*'s?
  - Only produces certain quantum numbers (like $J^{PC} = 1^{++}$) easily
- Diquark and hadrocharmonium pictures
  - What stabilizes the states against instantly segregating into meson pairs?
  - Diquark models tend to overpredict the number of bound states
  - Why wouldn't hadrocharmonium *always* decay into charmonium, instead of $D\bar{D}$?
- Cusp effect
  - Might be able to generate some resonances on its own, but >20 of them? And certainly not ones as narrow as $X(3872)$ ($\Gamma < 1.2$ MeV)

The hadron molecular picture

- A number of *XYZ* states are *suspiciously* close to hadron thresholds
  - *e.g.*, recall $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = +0.01 \pm 0.18 \text{ MeV}$
- So we theorists have *hundreds* of papers analyzing the *XYZ* states as dimeson molecules
- But not all of them are!
  - *e.g.*, *Z*(4430) is a prime example
- Moreover, some *XYZ* states lie slightly *above* a hadronic threshold
  - *e.g.*, Y(4260) lies about 30 MeV above the $D_s^* \overline{D_s^*}$ threshold
  - How can one have a bound state with *positive* binding energy?

Prompt production

- If hadronic molecules are really formed, they must be very weakly bound, with very low relative momentum between their mesonic components
- They might appear in *B* decays, but would almost always be blown apart in collider experiments
- But CDF (Fermilab) & CMS (CERN) saw many!
  [Prompt *X*(3872) production, σ≈30 nb]
  - CDF Collaboration (A. Abulencia *et al.*), PRL **98**, 132002 (2007)
  - CMS Collaboration (S. Chatrchyan *et al.*), JHEP **1304**, 154 (2013)

➢ Hadronic molecules may exist,
but *X*(3872) does not seem to fit the expected profile

It is entirely possible…

- …that no single structure accommodates all of these exotic states
- Some could be molecules, some could be hybrids,
  some could be kinematical effects,
  or quantum-mechanical mixtures of these…
- But what none of these pictures take into account
  is the full complexity of QCD dynamics
  for rather short-lived states
- Here, then, is my suggestion:

Amazing (well-known) fact about color:

- The short-distance color attraction of combining two color-$\mathbf{3}$ quarks ($\mathbf{3}$ = red, blue, green) into a color-$\overline{\mathbf{3}}$ diquark is *fully half as strong* as that of combining a $\mathbf{3}$ and a $\overline{\mathbf{3}}$ into a color-neutral singlet (*i.e.*, **diquark attraction** is nearly as strong as the **confining attraction**)

- Just as one computes a spin-spin coupling,
$$\vec{s_1}\cdot\vec{s_2}=\frac{1}{2}\left[(\vec{s_1}+\vec{s_2})^2-\vec{s_1}^{\,2}-\vec{s_2}^{\,2}\right],$$
from two particles in representations 1 and 2 combined into representation 1+2:

- If s_1, s_2 = spin $\frac{1}{2}$, and $\vec{s_1}+\vec{s_2}$ = spin 0, get $-\frac{3}{4}$; if spin 1, get $+\frac{1}{4}$

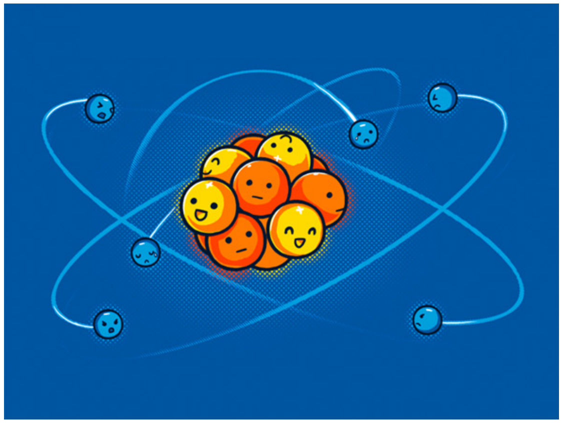
- The exact analogue formula for color charges gives the result stated above

A new tetraquark picture

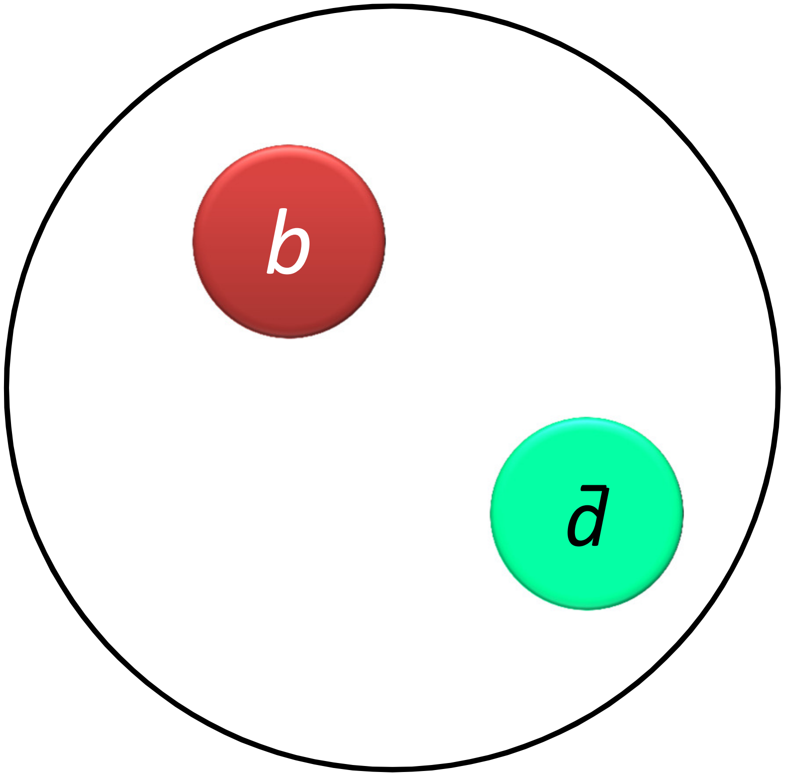
Stanley J. Brodsky, Dae Sung Hwang, RFL

Physical Review Letters **113**, 112001 (2014)

- CLAIM: At least some of the observed tetraquark states are bound states of diquark-antidiquark pairs
- BUT the pairs are not in a static configuration; they are created with a lot of relative energy, and rapidly separate from each other
- Diquarks are not color neutral!
  They cannot, by confinement, separate asymptotically far
- They must hadronize via large-*r* tails of mesonic wave functions, which suppresses decay widths to make them observably narrow
- Want to see this in action? Time for some cartoons!



Nonleptonic $\bar{B}^0$ meson decay



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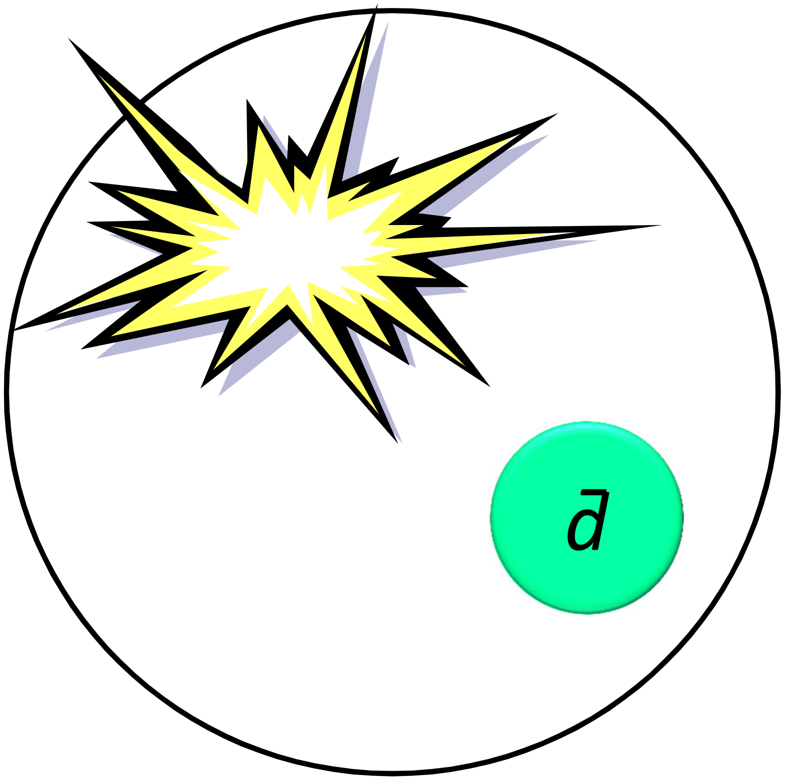
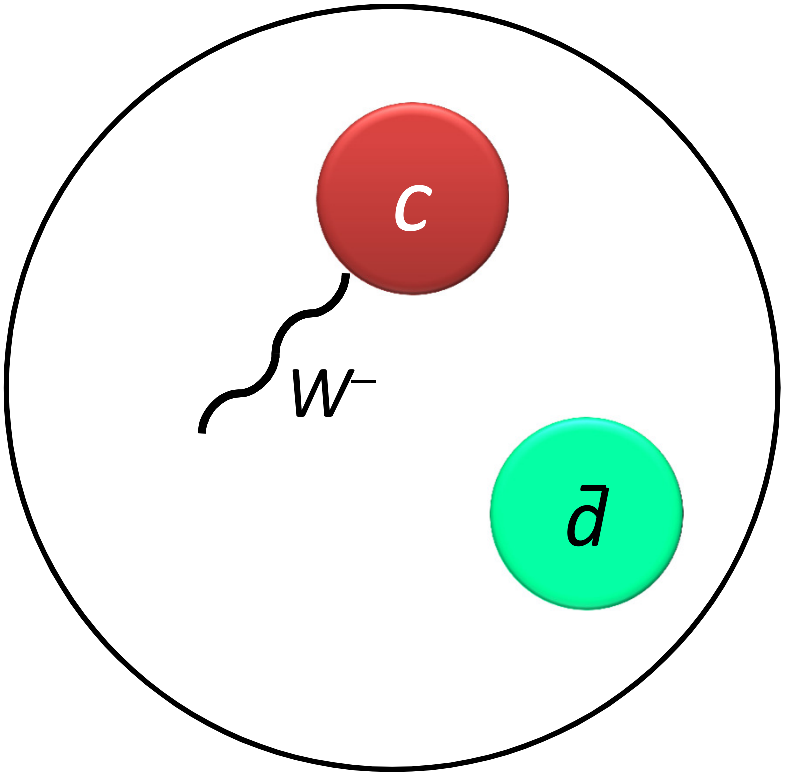


Nonleptonic $\bar{B}^0$ meson decay



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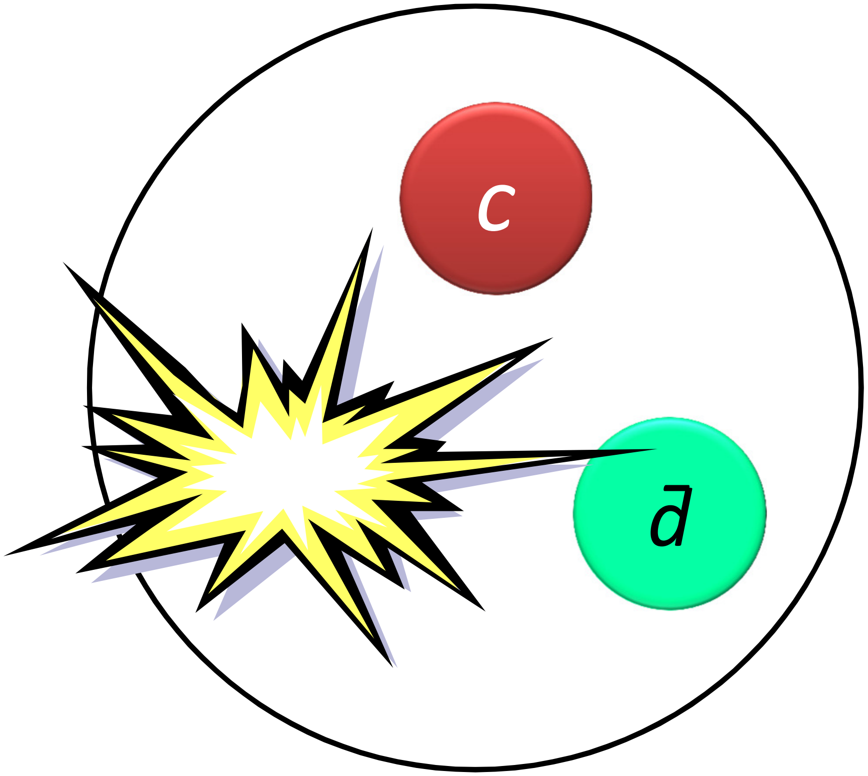
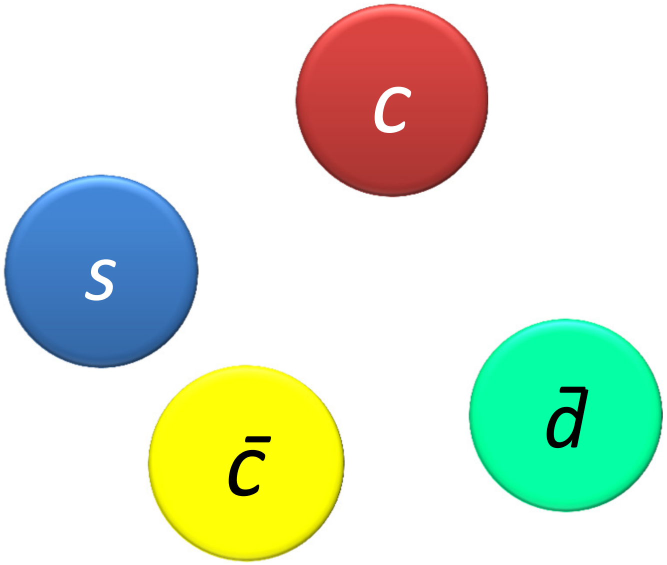


Nonleptonic $\bar{B}^0$ meson decay

B.R.~22%

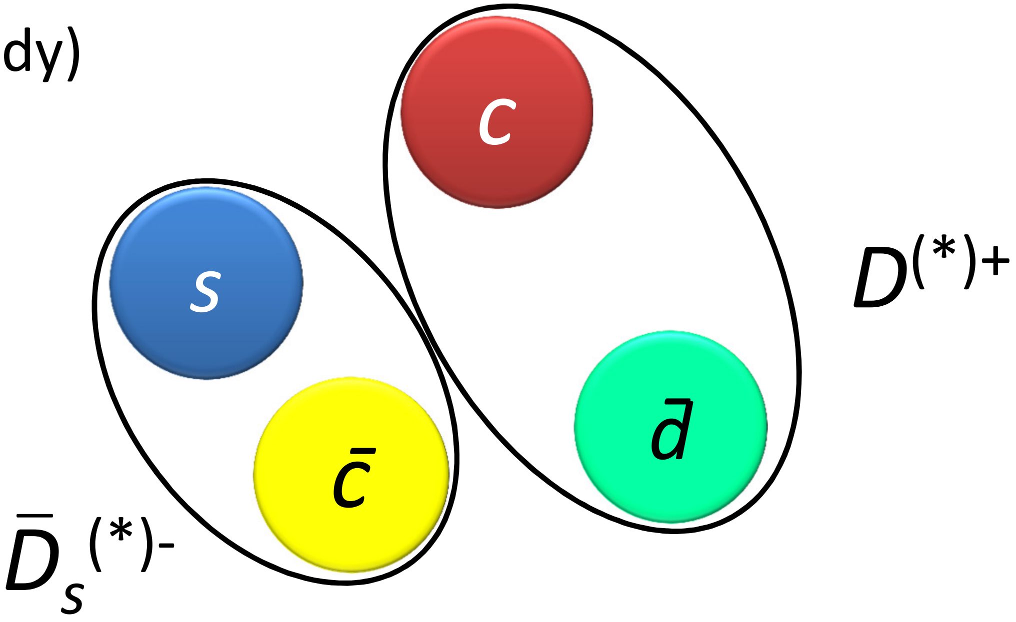
(Branching Ratio = probability)



What happens next?
Option: Color-allowed

B.R.~5%

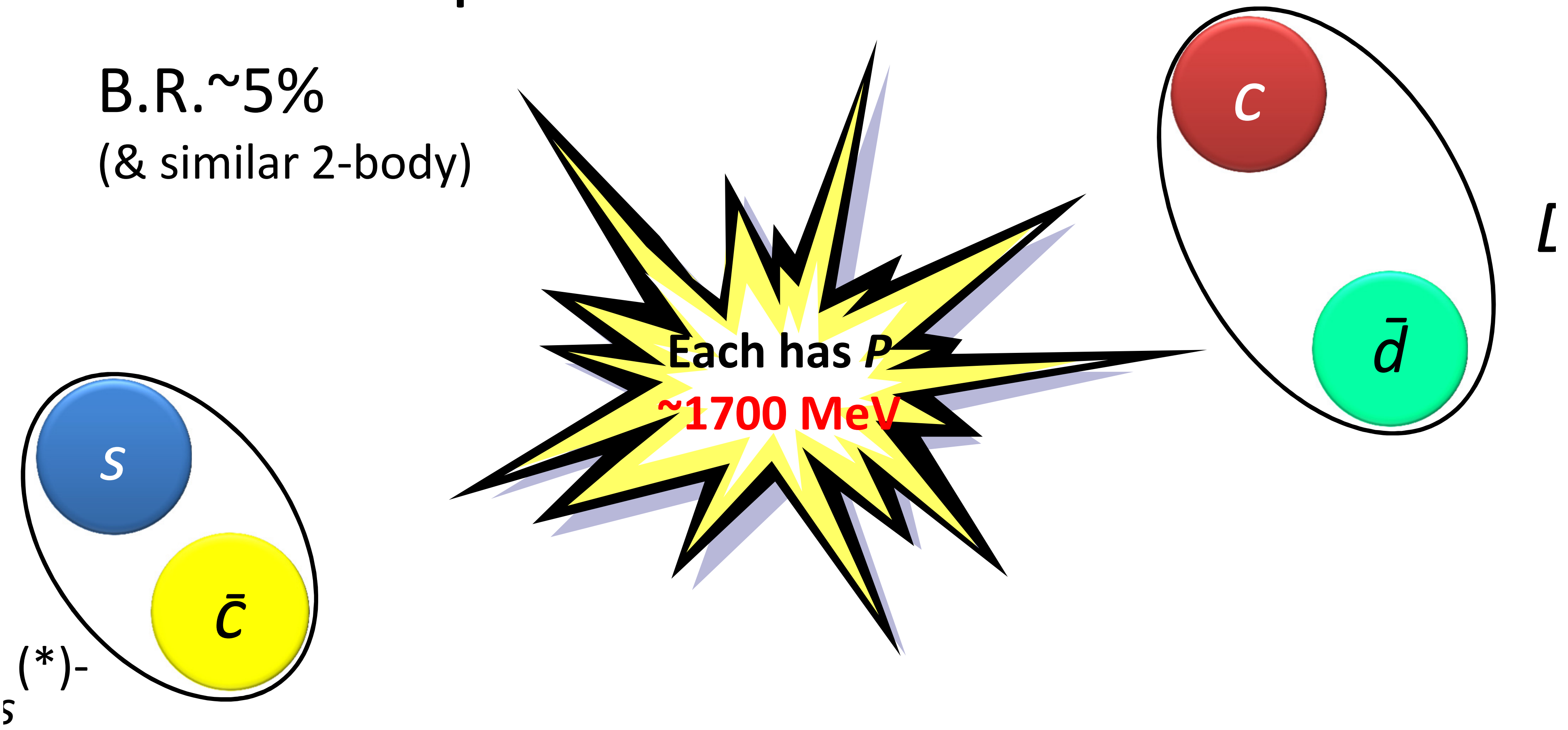
(& similar 2-body)



What happens next?
Option: Color-allowed

B.R.~5%

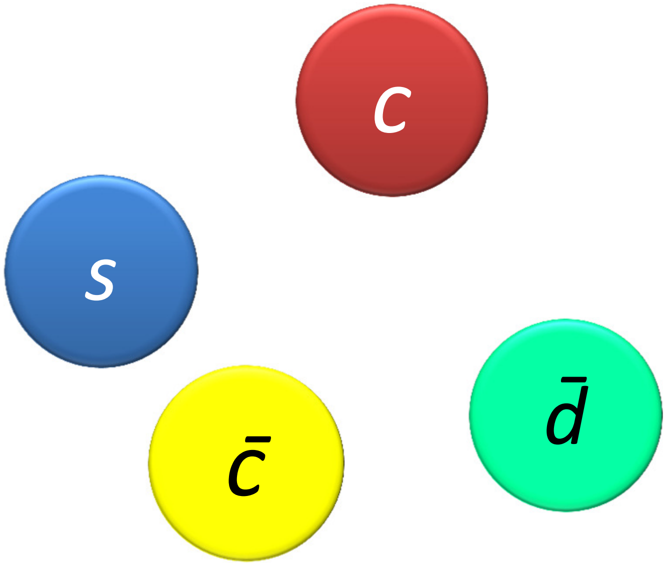
(& similar 2-body)



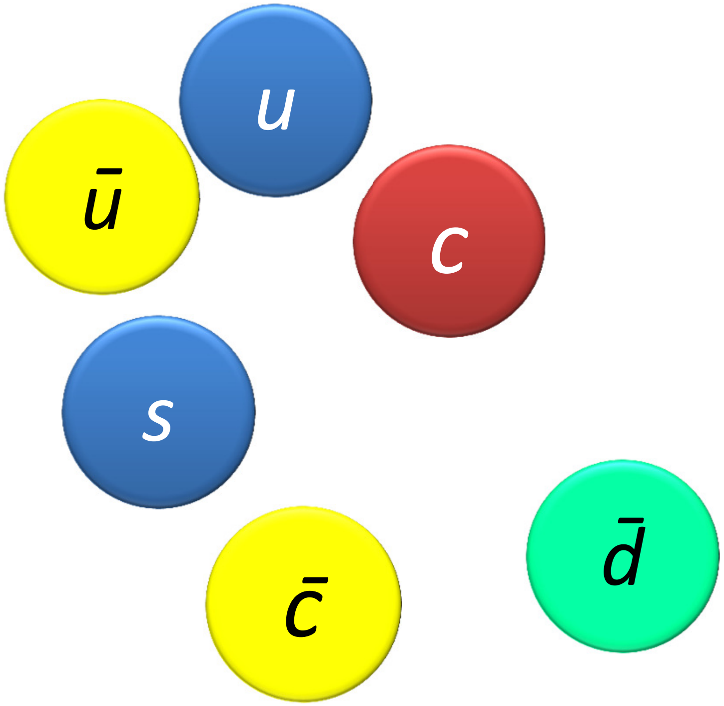
Each has *P*

~1700 MeV

What happens next?
Option: Diquark formation



What happens next?
Option: Diquark formation



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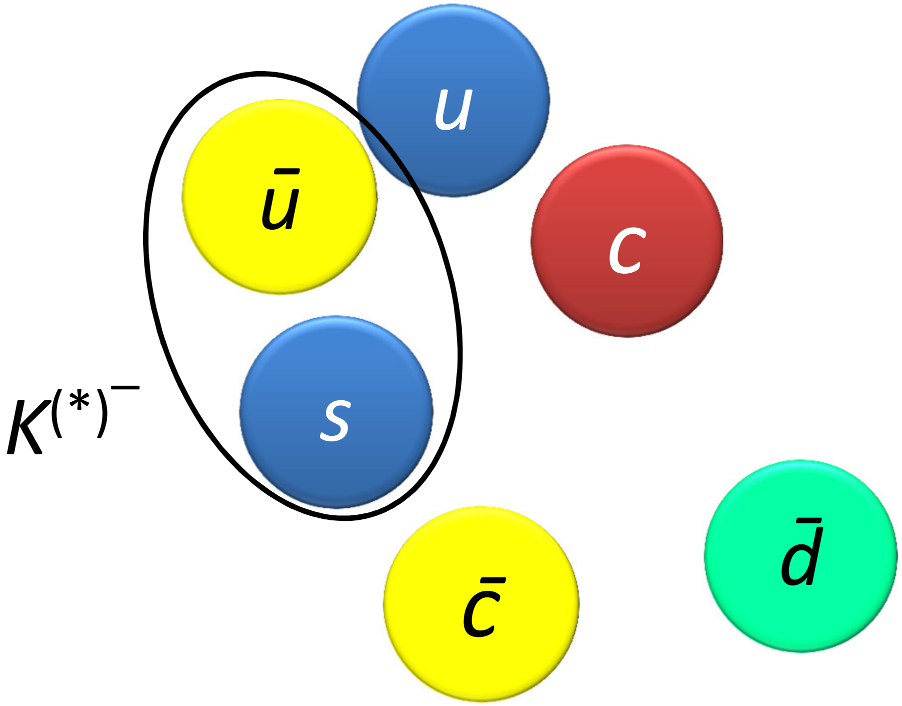

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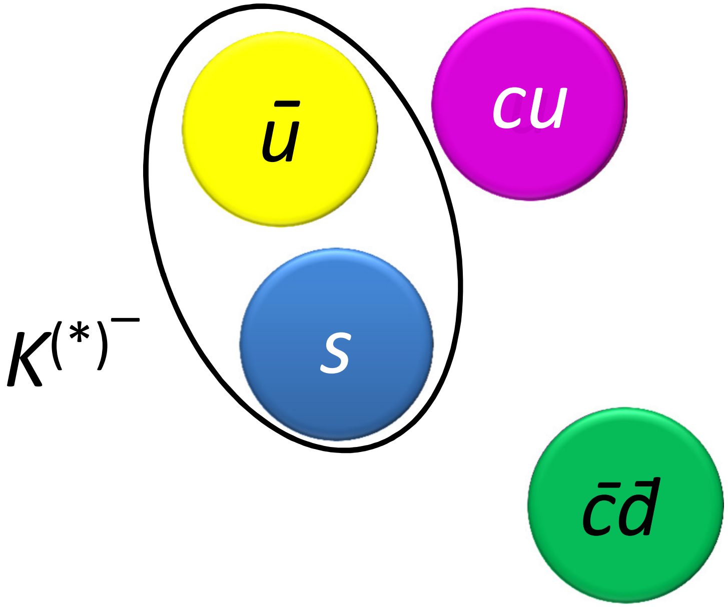

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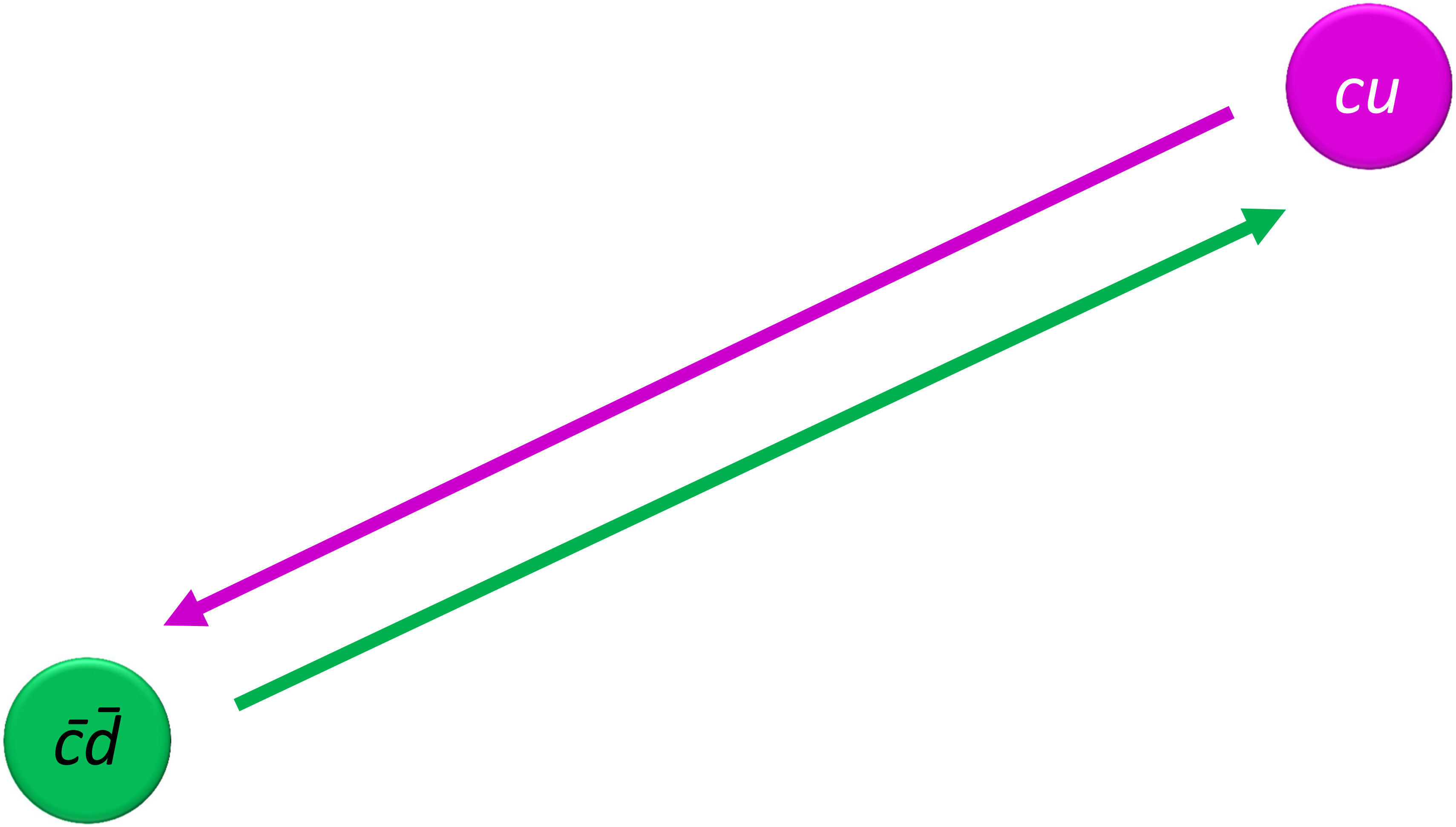
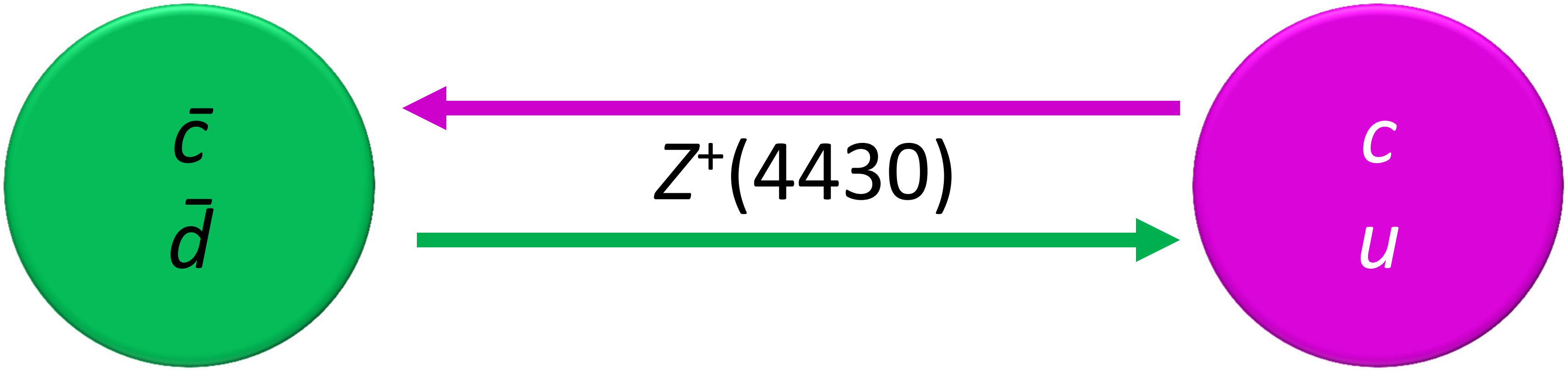


The diquarks are then forced to hadronize by the stretching of meson wave functions from one side to the other



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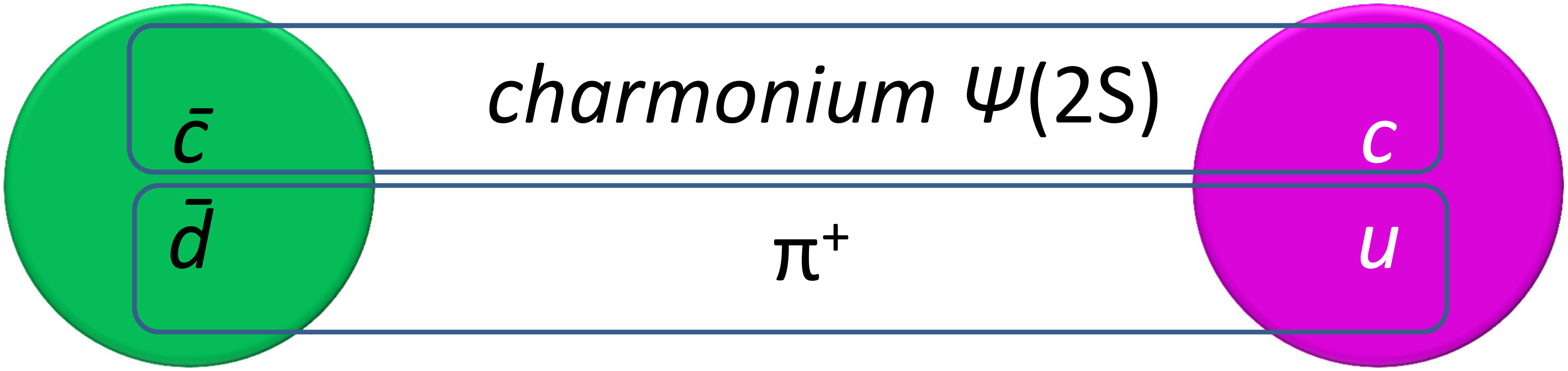
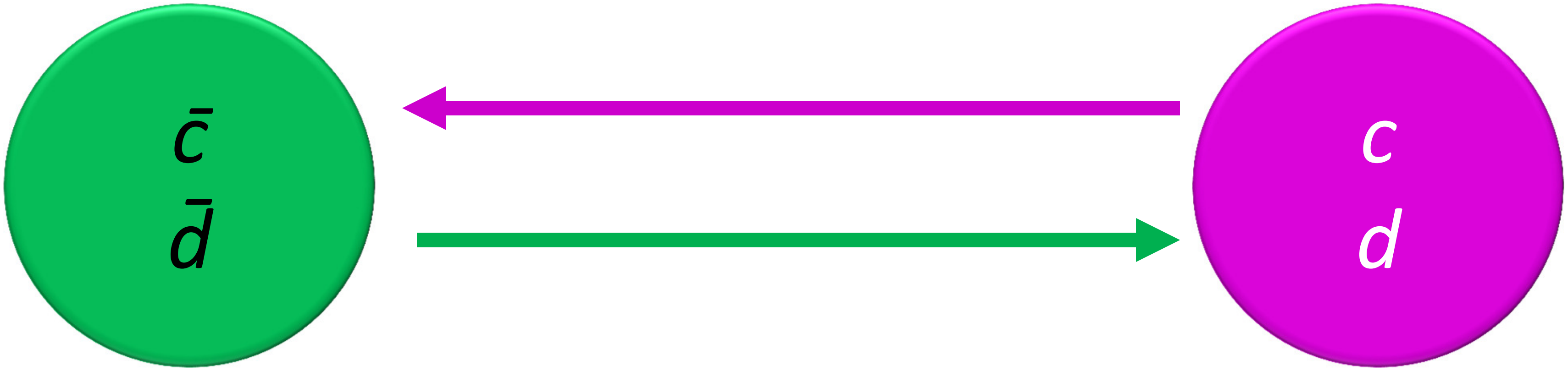


Why doesn’t this just happen?
It’s called *baryonium*



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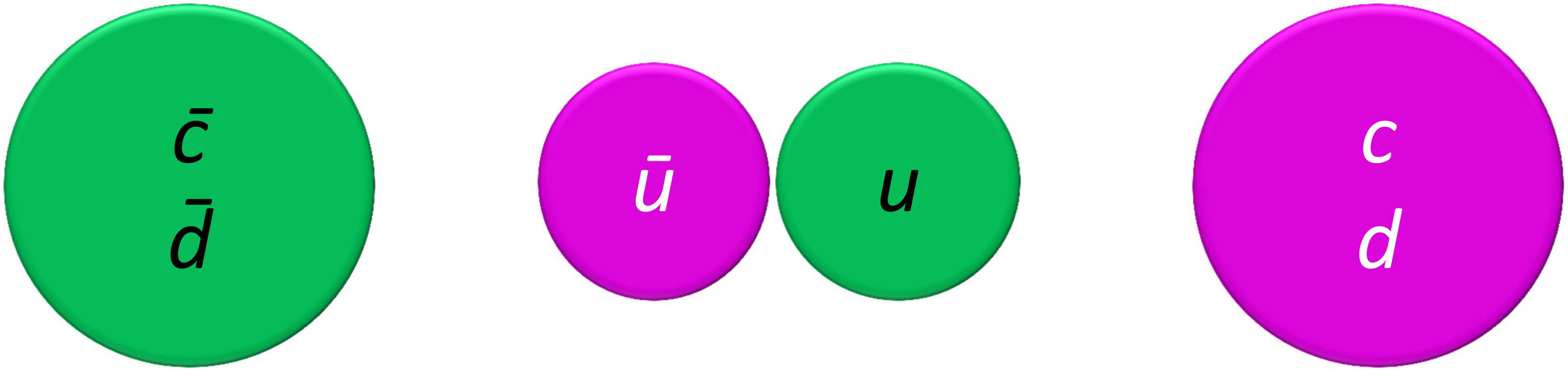


Why doesn’t this just happen?

It’s called *baryonium*

It *does* happen, as soon as the threshold $2M_{\Lambda_c} = 4573$ MeV is passed

The lightest exotic above this threshold, *X*(4632) , decays into $\Lambda_c + \bar{\Lambda}_c$

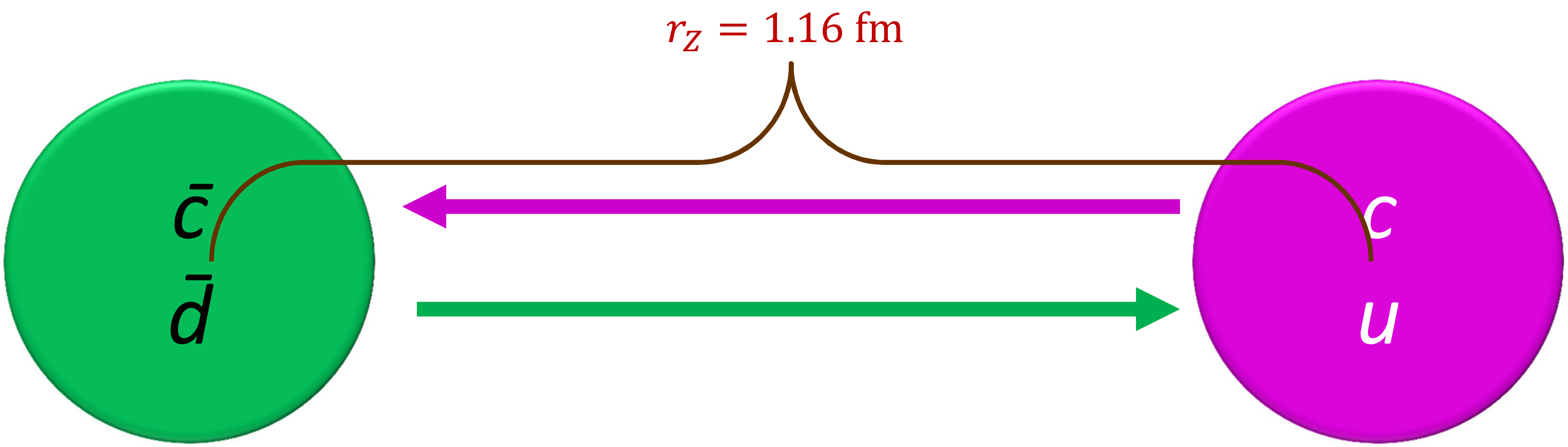
How far apart do the diquarks actually get?

- Since this is still a $\mathbf{3} \leftrightarrow \overline{\mathbf{3}}$ color interaction, just use the Cornell potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_{cq}^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}\mathbf{S}_{cq}\cdot\mathbf{S}_{\overline{cq}},$$

[This variant: Barnes et al., PRD **72**, 054026 (2005)]

- Use that the kinetic energy released in $\overline{B}^0 \longrightarrow K^- + Z^+(4430)$ converts into potential energy until the diquarks come to rest
- Decay transition most effective at this point (WKB turning point)

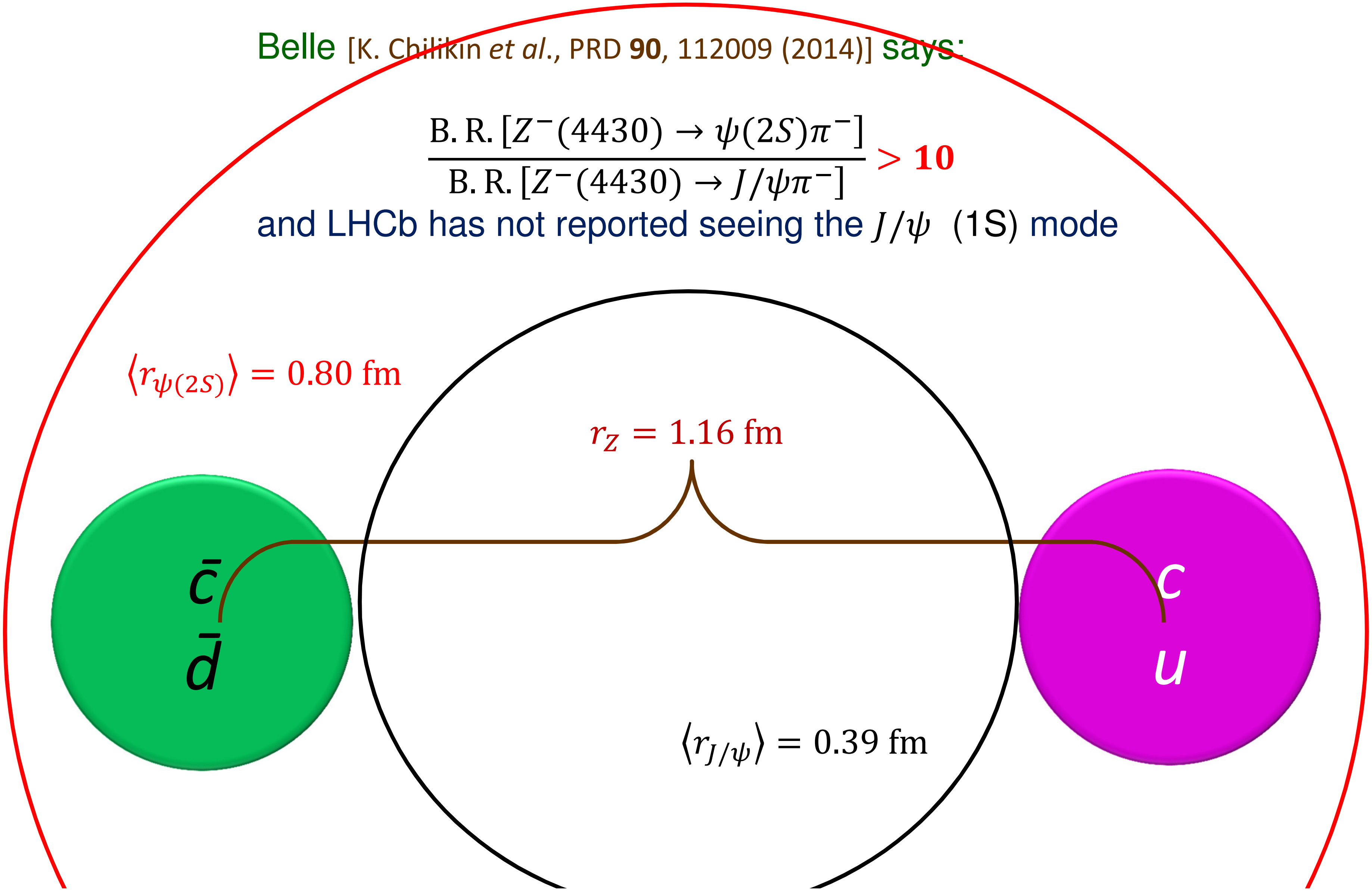


Fascinating *Z*(4430) fact:

Belle [K. Chilikin *et al.*, PRD **90**, 112009 (2014)] says:

$$\frac{\mathrm{B.R.}\,[Z^-(4430) \to \psi(2S)\pi^-]}{\mathrm{B.R.}\,[Z^-(4430) \to J/\psi\pi^-]} > \mathbf{10}$$

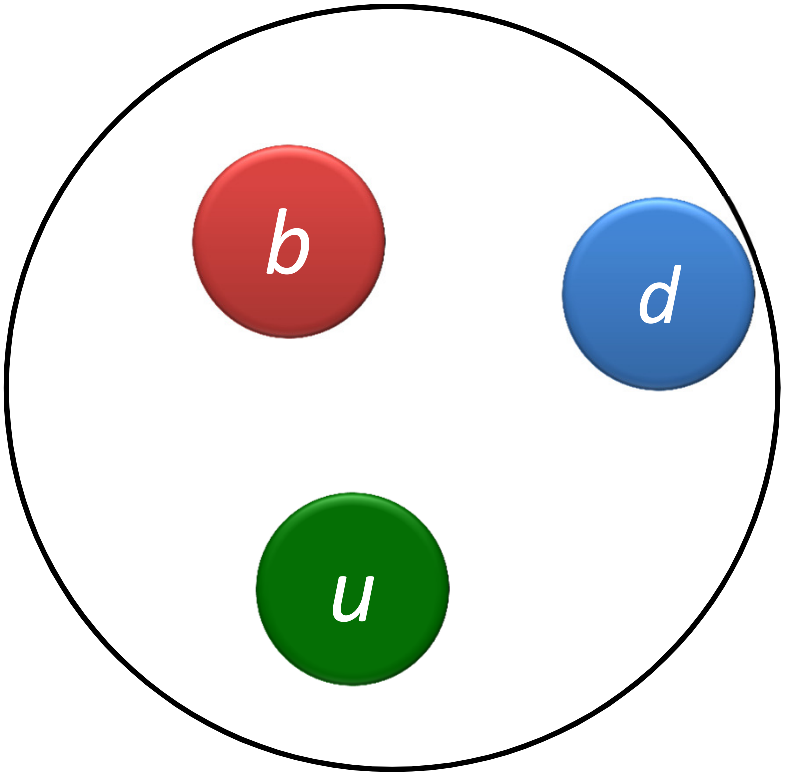
and LHCb has not reported seeing the J/ψ (1S) mode



Does the dynamical diquark picture have anything to say about the P_c states?

- Yes. RFL, Phys. Lett. B **749** (2015) 454

Nonleptonic Λ_b baryon decay



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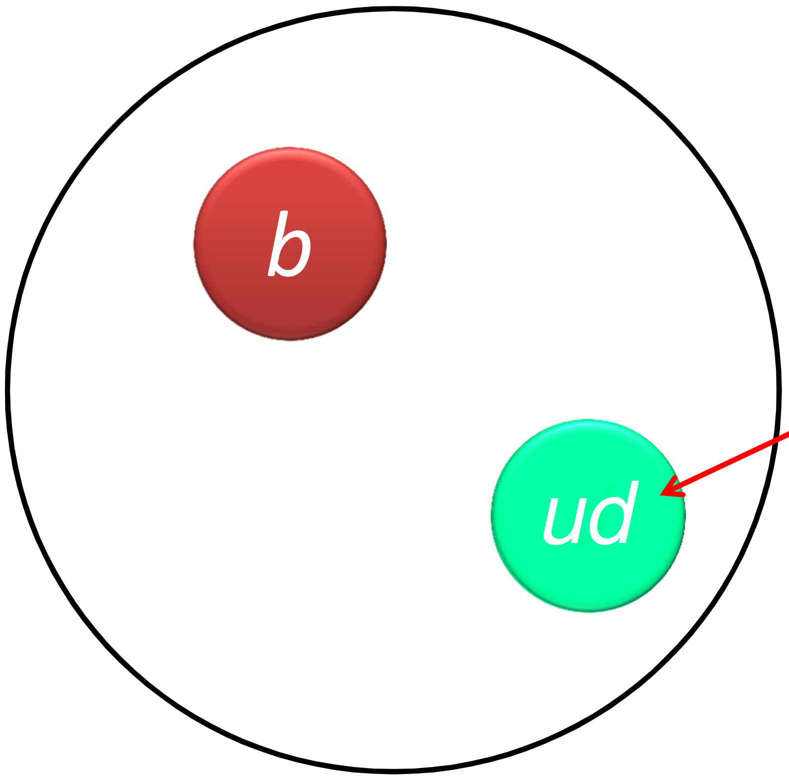
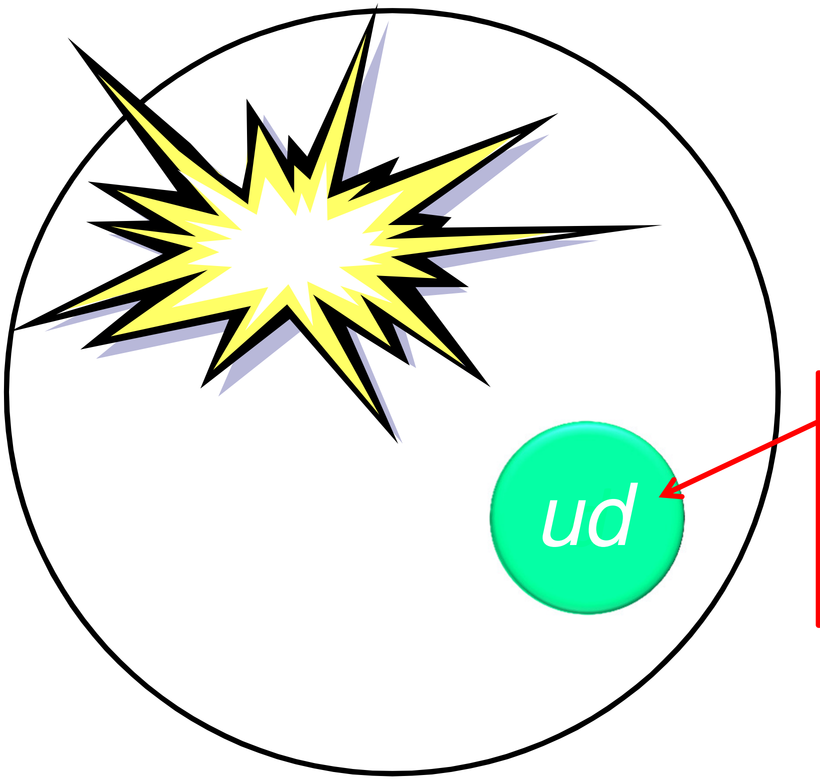


This is a diquark!

- Color $\overline{\mathbf{3}}$
- Isospin 0
- Spin 0

Nonleptonic Λ_b baryon decay



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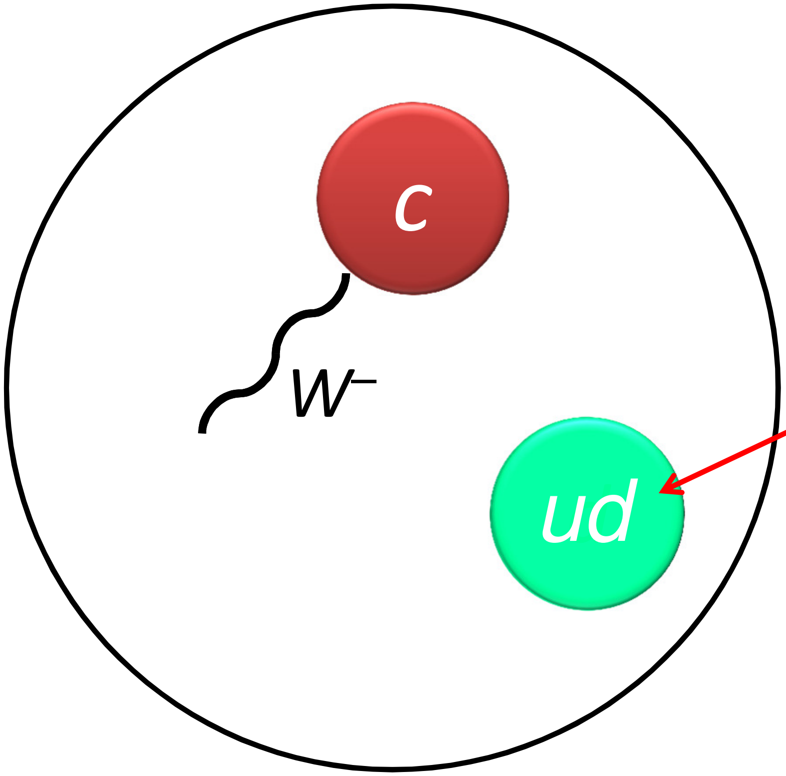

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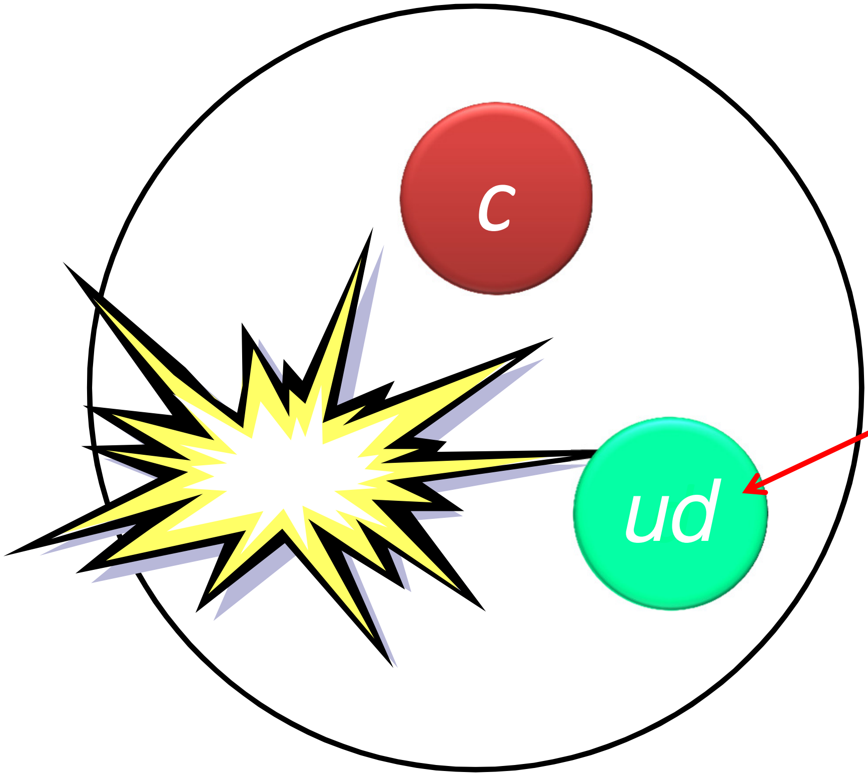

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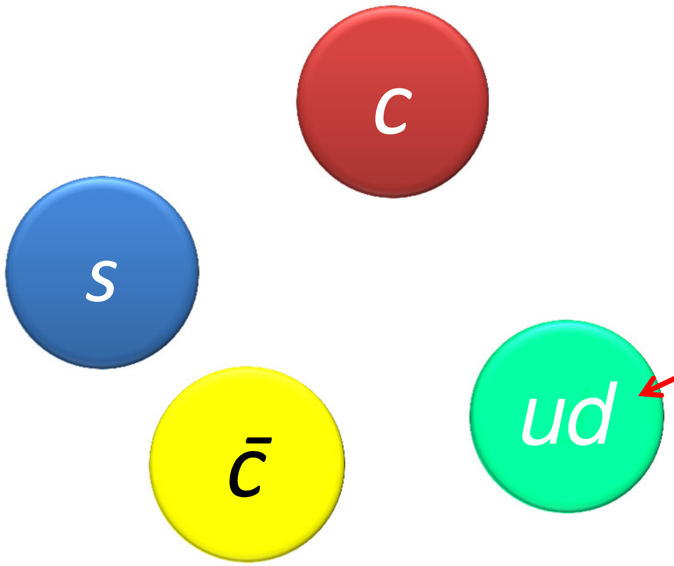

This is a diquark!

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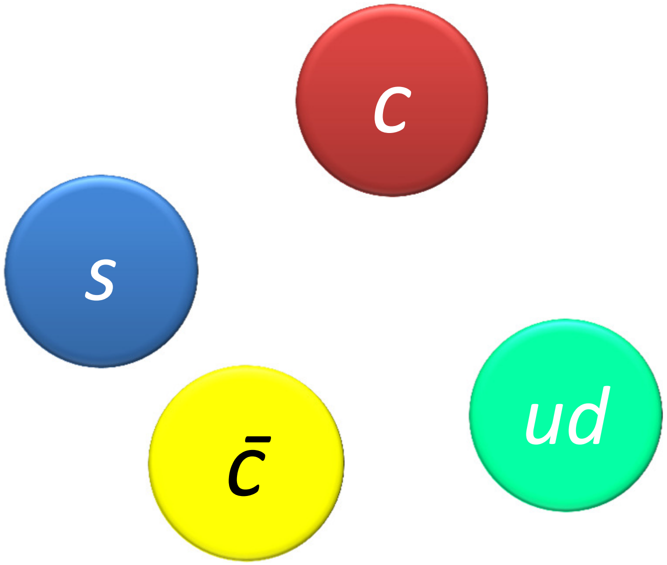
- Isospin 0
- Spin 0

What happens next?
Diquark *and triquark* formation



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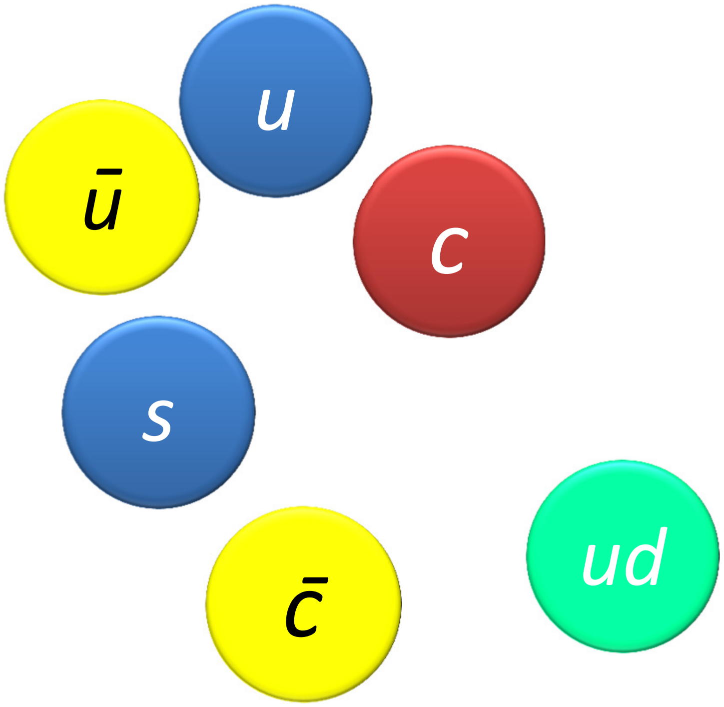

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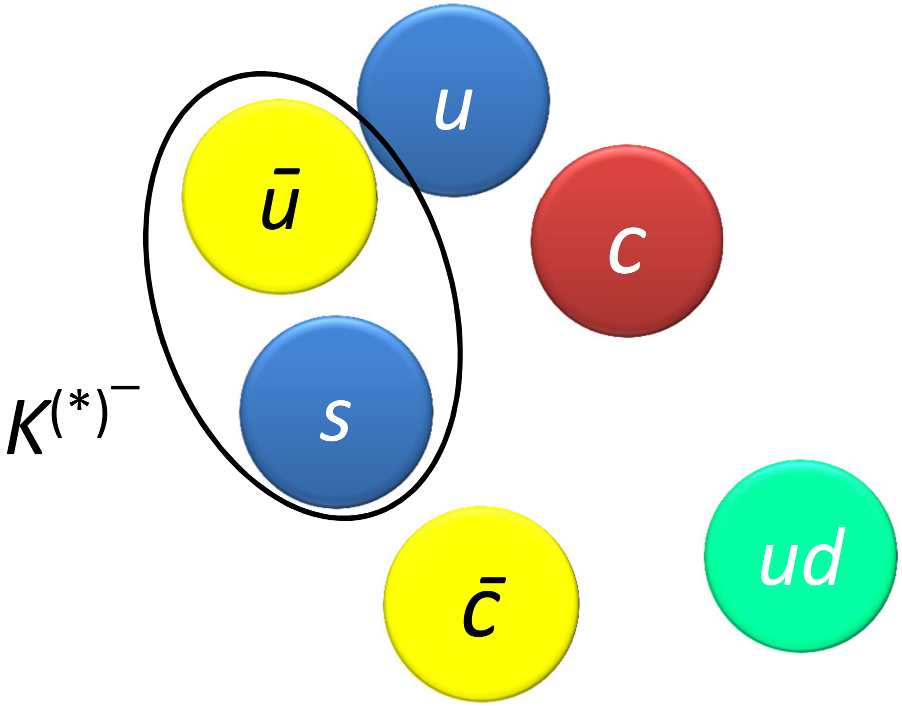

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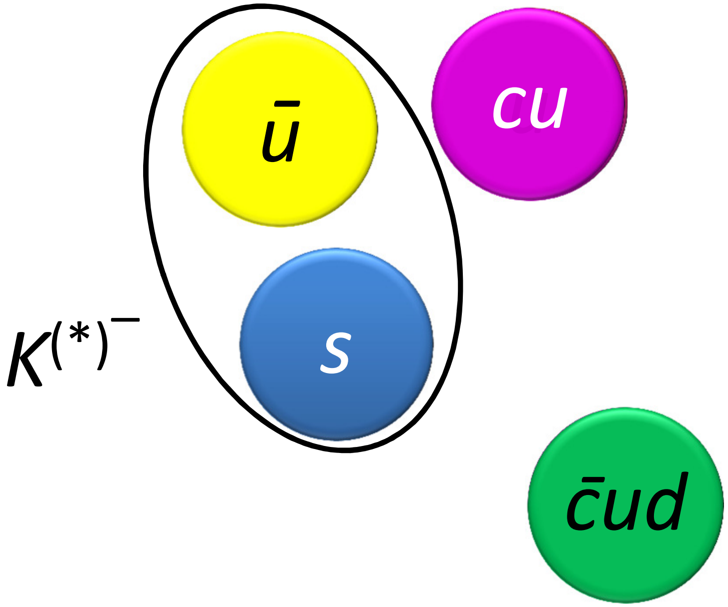

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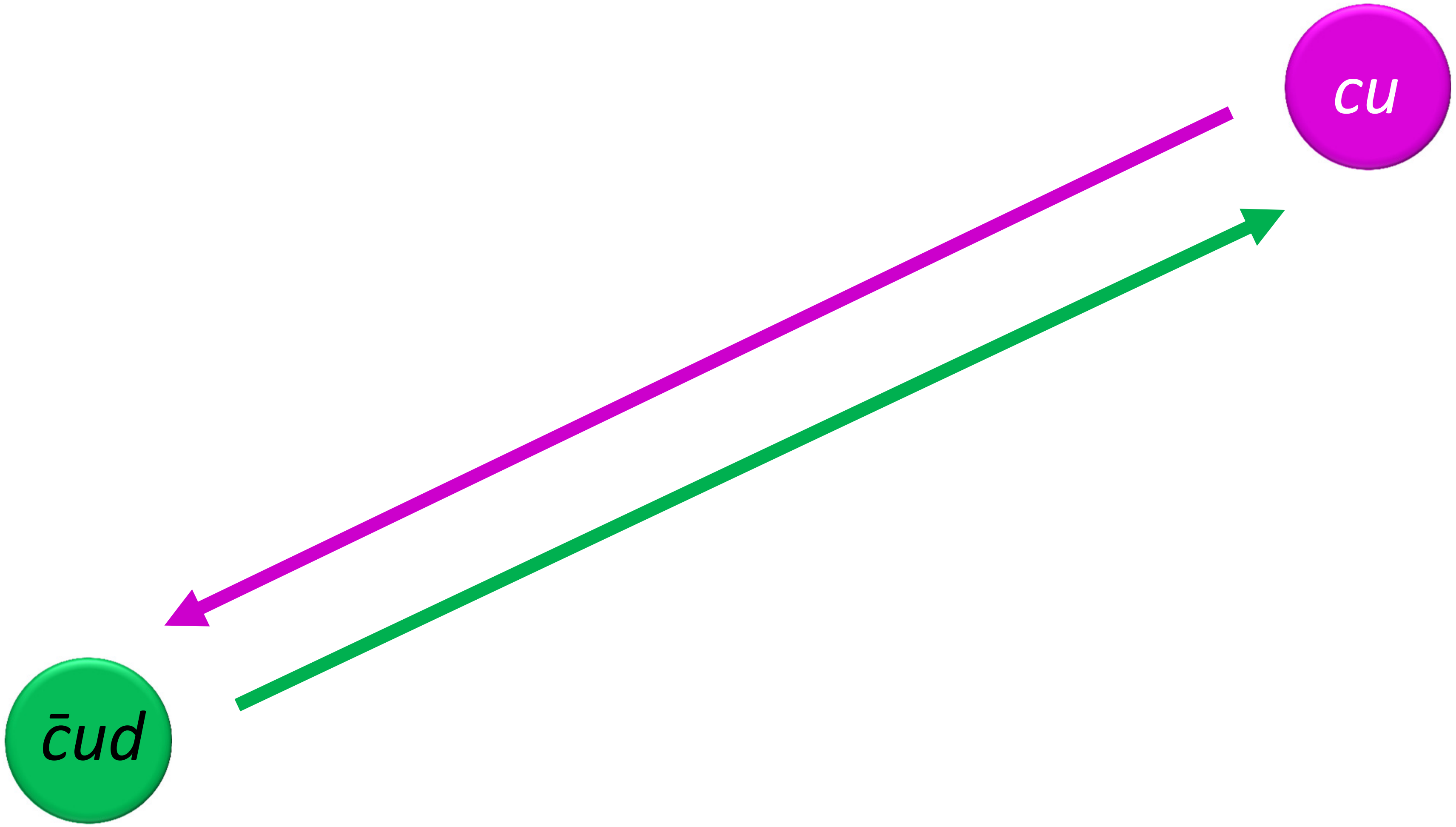
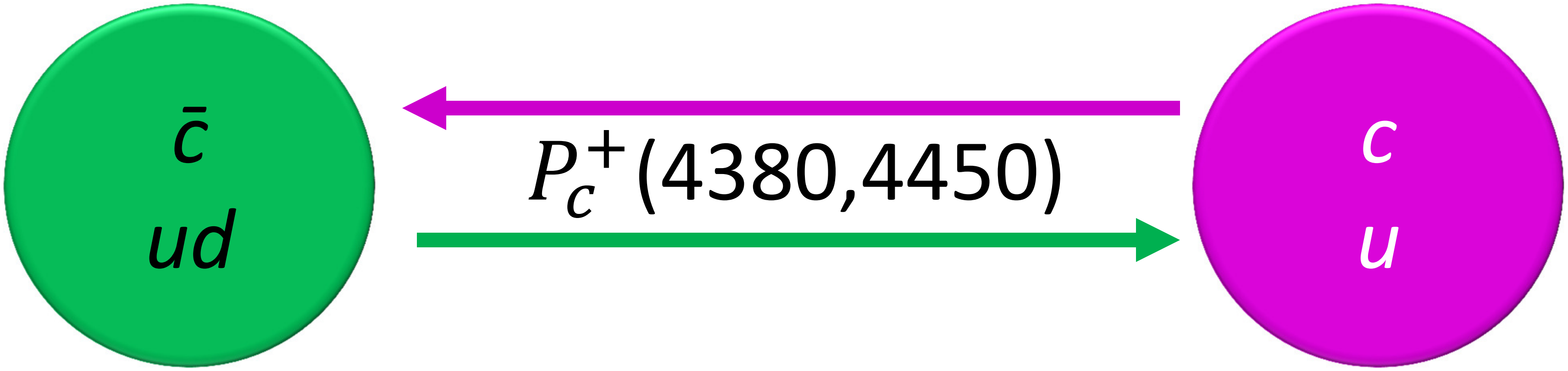


The same color triplet mechanism, supplemented with the fact that the *ud* in *Λ* baryons themselves act as diquarks, predicts a rich spectrum of *pentaquarks*



$\bar{c}$

ud

$P_c^+(4380,4450)$

c

u

$J^P = \frac{1}{2}^-$

$J^P = L^{(-1)^L}$

$J^P = 0^+, 1^+$

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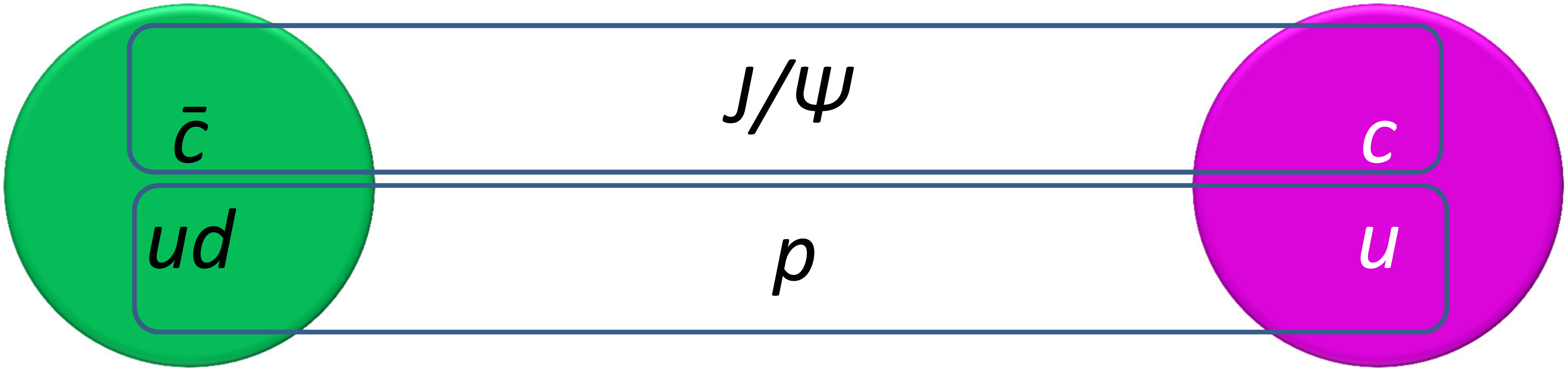


$J^P = \frac{1}{2}^-$

$J^P = L^{(-1)^L}$

$J^P = 0^+, 1^+$

Where else can these diquarks matter?

- Opening meson thresholds creates an effective attraction (***cusp effect***) that can pull diquark-antidiquark poles towards them, so $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.11 \pm 0.21\ \mathrm{MeV}$ is not so unnatural
  [S. Blitz & RFL, Phys. Rev. D **91** (2015) 094025]

- Any evidence for tightly-bound diquarks in high-energy processes?
  It would affect the *quark counting rules*
  [S. Brodsky & RFL, Phys. Rev. D **91** (2015) 114025]

- Could the dynamical diquarks have shown up in the $s\bar{s}$ system?
  Possibly in ϕ photoproduction [RFL, PRD **92** (2015) 114006],
  and may appear in $\Lambda_c \to \phi p \pi^0$ [RFL, PRD **92** (2015) 114030]

- Would diquarks form in an ideal gas of q and $\bar{q}$?
  Yes, perhaps 10's of % of the time [RFL, PRD **94** (2016) 034039]

Conclusions

- The past three years have provided confirmation of the existence of the *tetraquark* and observation of the *pentaquark*, the third and fourth classes of hadron
- Over 30 such states (X, Y, Z, P_C) have thus far been observed
- All of the popular physical pictures for describing their structure seem to suffer some imperfection
- We propose an entirely new dynamical picture based on a diquark-antidiquark (or triquark) pair rapidly separating until forced to hadronize due to confinement
- Then several mysteries, *e.g.*, which particles the X, Y, Z, P_C states like to decay into, have simple explanations
- Much new work has been done, but much more remains!

Backup slides

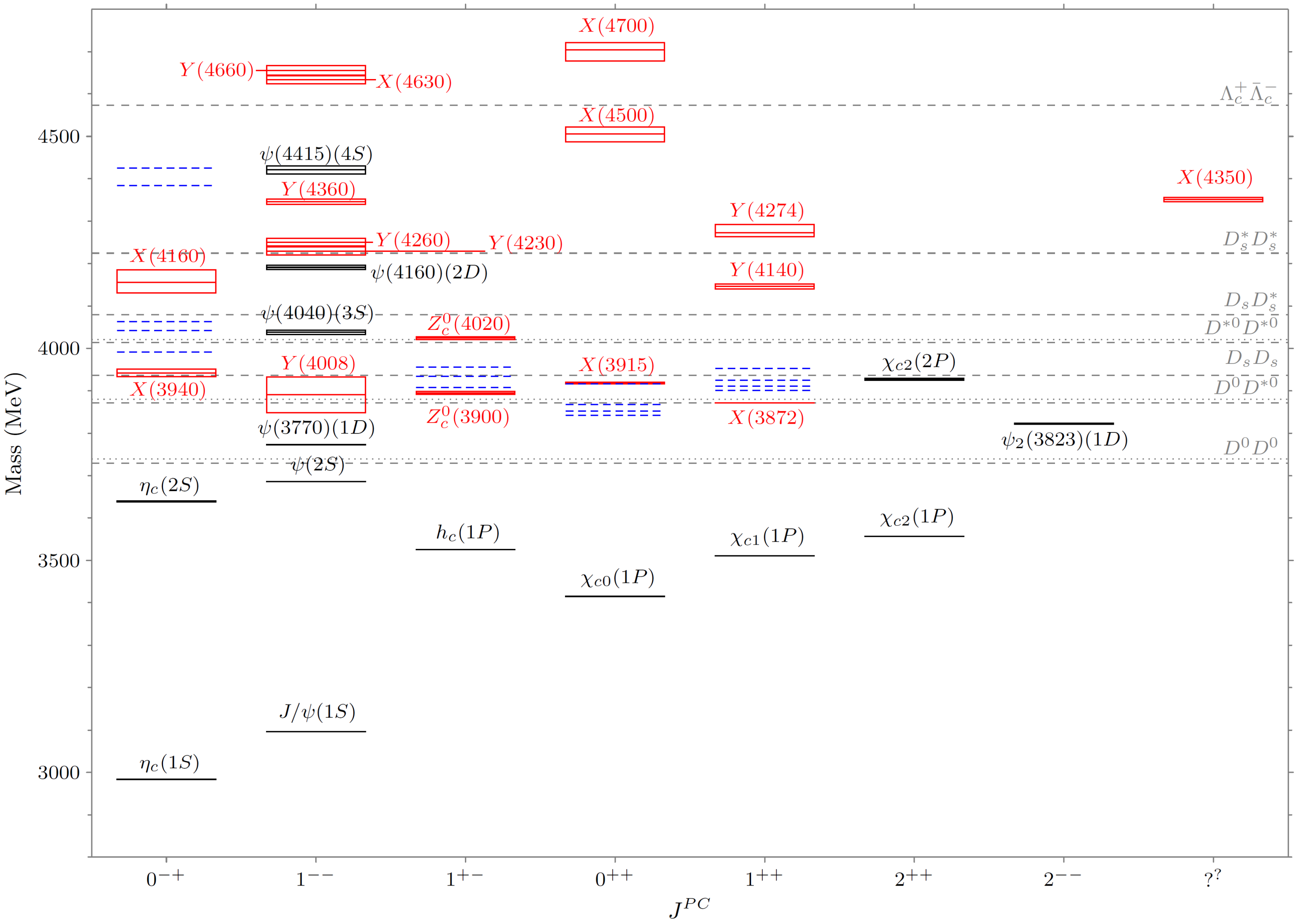
Quarkonium

- A number of the complications of light-quark systems are much less prominent for heavy quarks *c* and *b*:
  - By virtue of being heavy ($m_Q \gg \Lambda_{\mathrm{QCD}}$), heavy quarks Q act more like static, more localized color sources
  - They can be treated as nonrelativistic within their hadron
  - The running QCD coupling constant α_s is much smaller at the heavy-quark energy scale than at low energy, say, ≈ 0.3
- So model the interaction between a heavy quark-antiquark pair using a strong "Coulomb" force and a confining force, the famous **Cornell potential**

  [E. Eichten *et al.*, PRD **17** (1978) 3090; **21** (1980) 203],

$$V(r) = -\frac{a}{r} + br$$

Charmonium: February 2017
Neutral sector



Our limited nomenclature

- $\mathcal{X}$: A state with c+$\bar{c}$ decays that is produced from B decay
- $\mathcal{Y}$: A state with c+$\bar{c}$ decays that is produced in association with initial-state radiation in e^+e^- annihilation
- $\mathcal{Z}$: A state with c+$\bar{c}$ decays that is charged
- $\mathcal{P}_c$: A state with c+$\bar{c}$ decays and baryon number

Obvious problems lie ahead with this naming scheme:

- X states have also been produced in, say, $p\bar{p}$
- Y states have also been produced in B decays
- Z state neutral isospin partners are being discovered
- X, Y, Z states have observed transitions amongst themselves, strongly suggesting a common structure

The Breit-Wigner resonance

- All resonances in physics (for damped oscillators, LRC circuits, elementary particles with short lifetimes) mean essentially the same thing: a large enhancement (peak) of the amplitude in a particular range of energy input in the form of a *Lorentzian distribution*

$$|f(E)|^2 \propto \frac{1}{(E^2 - M^2)^2 + M^2\Gamma^2}$$

- In the case of quantum mechanics, the amplitude f is

$$f(E) \propto \frac{1}{(E^2 - M^2) + iM\Gamma}$$

(Breit-Wigner amplitude)

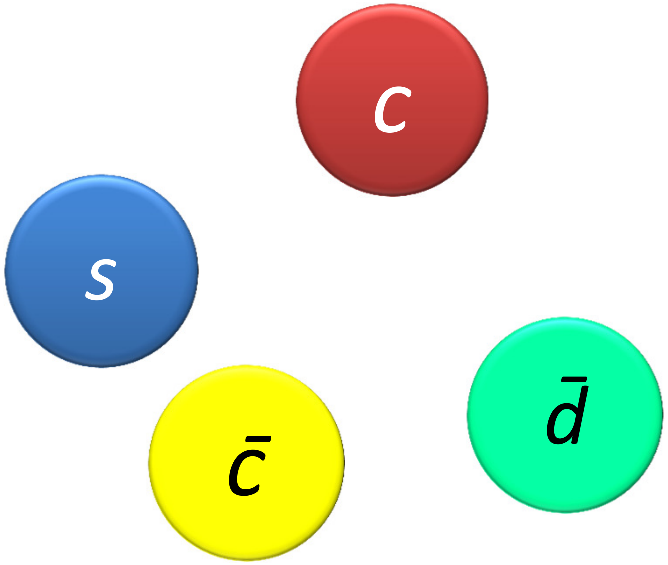
- One finds that in increasing E through the peak, the *phase angle* of $f(E)$ increases from 0 to 2π, *i.e.*, it forms a loop in the complex plane
→ evidence of a true resonance, not just a “bump” in the amplitude

Prompt production

- If hadronic molecules are really formed, they must be very weakly bound, with very low relative momentum between their mesonic components
- They might appear in *B* decays, but would almost always be blown apart in collider experiments
- But CDF & CMS (CERN) saw many! [Prompt *X*(3872) production, σ≈30 nb]
  - CDF Collaboration (A. Abulencia *et al*.), PRL **98**, 132002 (2007)
  - CMS Collaboration (S. Chatrchyan *et al*.), JHEP **1304**, 154 (2013)
- Perhaps strong final-state interactions, π exchanges between D^0 and $\overline{D^{*0}}$?
  - P. Artoisenet and E. Braaten, Phys. Rev. D **81**, 114018 (2010); D **83**, 014019 (2011)
- Such effects can be significant, but do not appear to be sufficient to explain the size of the prompt production
  - C. Bignamini *et al*., Phys.Lett. B **228** (2010); A. Esposito *et al.*, J. Mod. Phys. **4**, 1569 (2013); A. Guerrieri *et al*., Phys. Rev. D **90**, 034003 (2014)

➢ Hadronic molecules may exist, but *X*(3872) does not seem to fit the profile

What happens next?
Option 2: Color-suppressed



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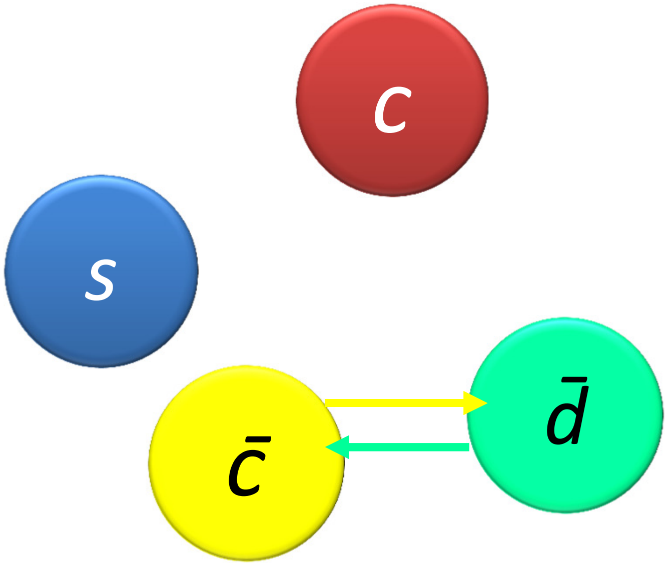

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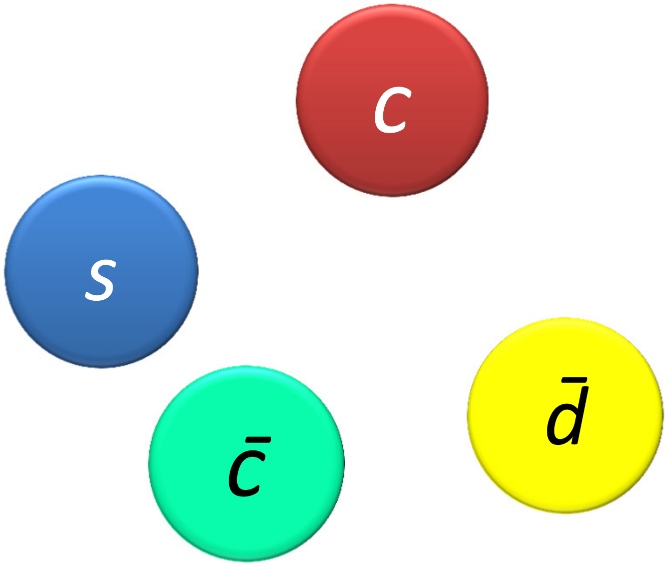

What happens next?
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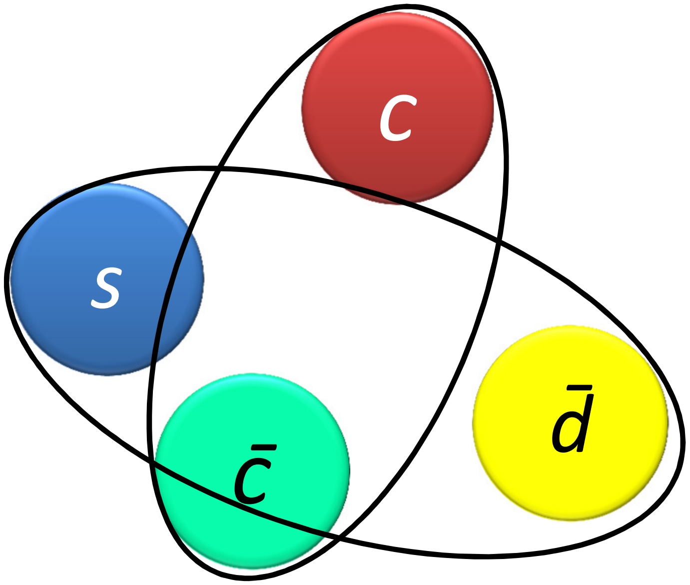
B.R.~2.3%



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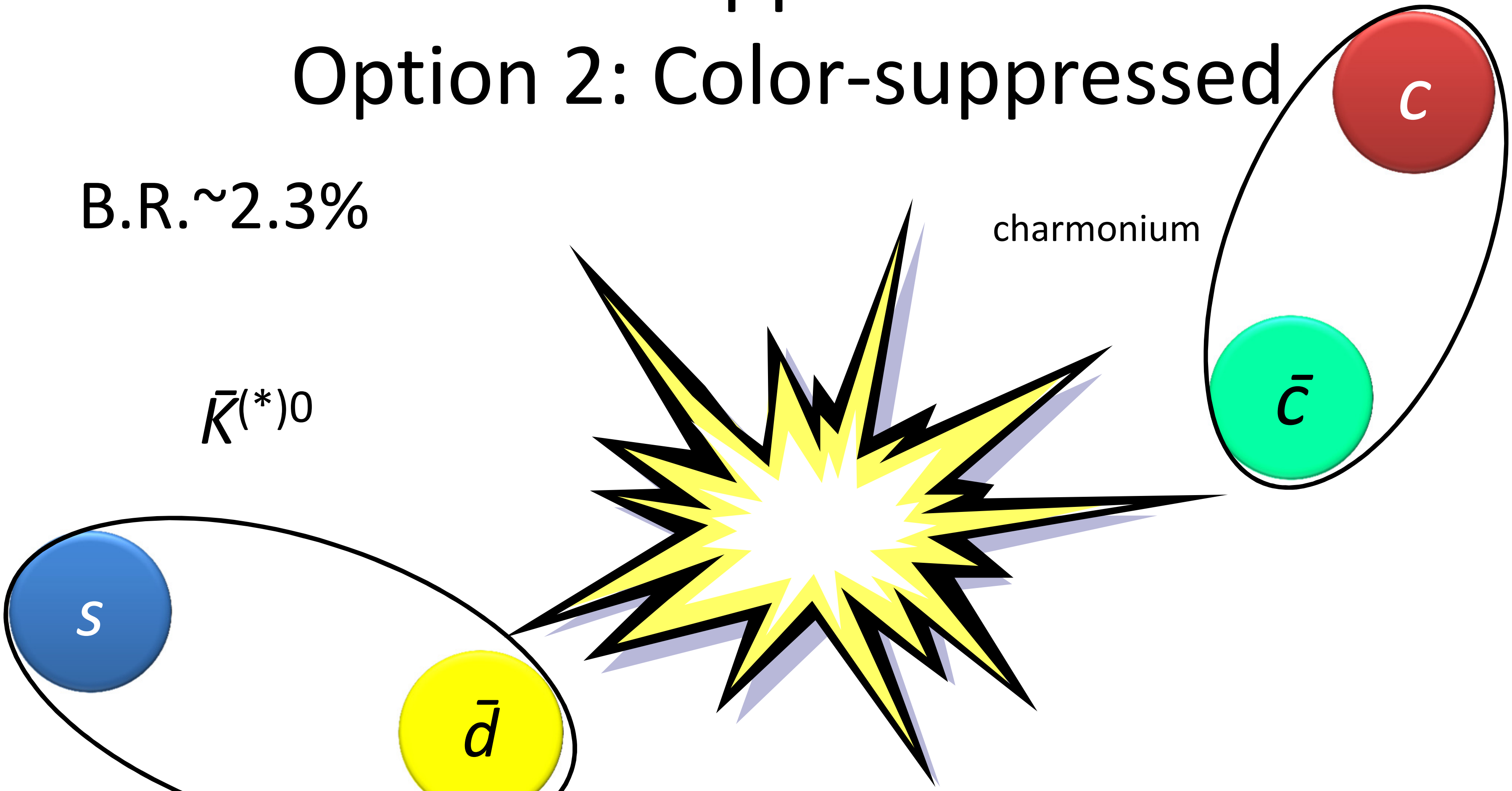
B.R.~2.3%



The large-*r* wave function tails and resonance widths

- The simple fact that the diquark-antidiquark pair is capable of separating further than the typical mean size of ordinary hadrons before coming to rest implies:
  - The decay transition overlap matrix elements are suppressed, SO
  - The decay transition rate is suppressed, SO
  - The width is smaller than predicted by generic dimensional analysis (*i.e.*, by phase space alone)

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  - ➢ The decay transition overlap matrix elements are suppressed, SO
  - ➢ The decay transition rate is suppressed, SO
  - ➢ The width is smaller than predicted by generic dimensional analysis (*i.e.*, by phase space alone)
- *e.g.*, $\Gamma[Z(4430)] = 180 \pm 31\ \mathrm{MeV}$
  (*cf.* $\Gamma[\rho(770)] = 150\ \mathrm{MeV}$)
- But why would these diquark-antidiquark states behave like resonances at all?

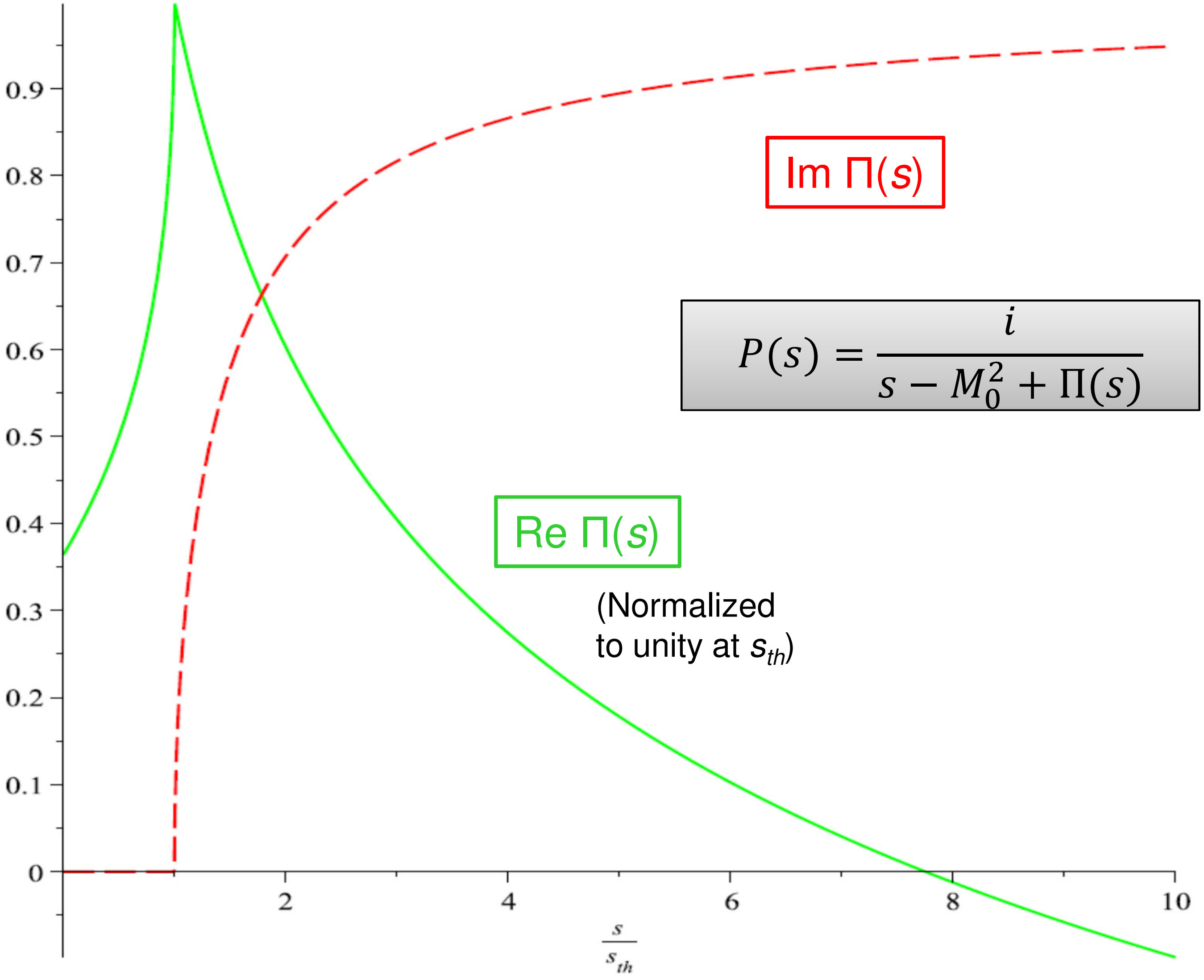
For one thing,

- Diquark-antidiquark pairs create their own bound-state spectroscopy [L. Maiani *et al.*, PRD **71** (2005) 014028]
  - Simple Hamiltonian with spin-spin interactions among the four quarks
  - Once one bound state is found, a whole multiplet arises
  - Then compare predicted spectrum to experiment
- Original version predicts states with quantum numbers and multiplicities not found to exist (*XYZ* phenomenology not very well developed then), but a new version of the model [L. Maiani *et al.*, PRD **89** (2014) 114010] appears to be much more successful
  - Crucial revision: Dominant spin-spin couplings are *within* each diquark
  - *e.g.*, *Z*(4430) is radial excitation of *Z*(3900);
    Y states are *L*=1 color flux tube excitations

And furthermore,

- The presence of nearby hadronic thresholds can attract nearby diquark resonances: *Cusp effect*
  - The complex amplitude $\Pi(s)$ that is a source for the tetraquarks in terms of total energy $\sqrt{s}$ develops a branch point at the threshold to produce on-shell hadrons (due to *unitarity*: the *optical theorem*)
  - But the full amplitude is analytic everywhere, except for resonant poles and cuts that start at the branch points (due to *causality*)
  - This fact allows for a *dispersion relation* (like Kramers-Kronig) that expresses $\mathrm{Re}\,\Pi(s)$ as an integral over $\mathrm{Im}\,\Pi(s)$
  - If $\mathrm{Im}\,\Pi(s)$ at suddenly shoots up from zero, then $\mathrm{Re}\,\Pi(s)$ must develop a sharp peak, or *cusp*
  - Since the self-energy $\Pi(s)$ appears in the resonance propagator Green's function, the cusp in $\mathrm{Re}\,\Pi(s)$ acts as a shift in the mass, effectively dragging the resonant pole toward threshold

The Cusp



$$P(s) = \frac{i}{s - M_0^2 + \Pi(s)}$$

(Normalized
to unity at s_{th})

How closely can cusps attract thresholds?

- Consider the *X*(3872), with $\Gamma < 1.2 \text{ MeV}$
  - We saw that $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = +0.01 \pm 0.18 \text{ MeV}$
  - But also that *X*(3872) is almost certainly not a $\overline{D^{*0}}D^0$molecule
  - Moreover,

$$m_{X(3872)} - m_{J/\psi} - m_{\rho^0_{peak}} = -0.50 \text{ MeV}$$

$$m_{X(3872)} - m_{J/\psi} - m_{\omega_{peak}} = -7.89 \text{ MeV}$$

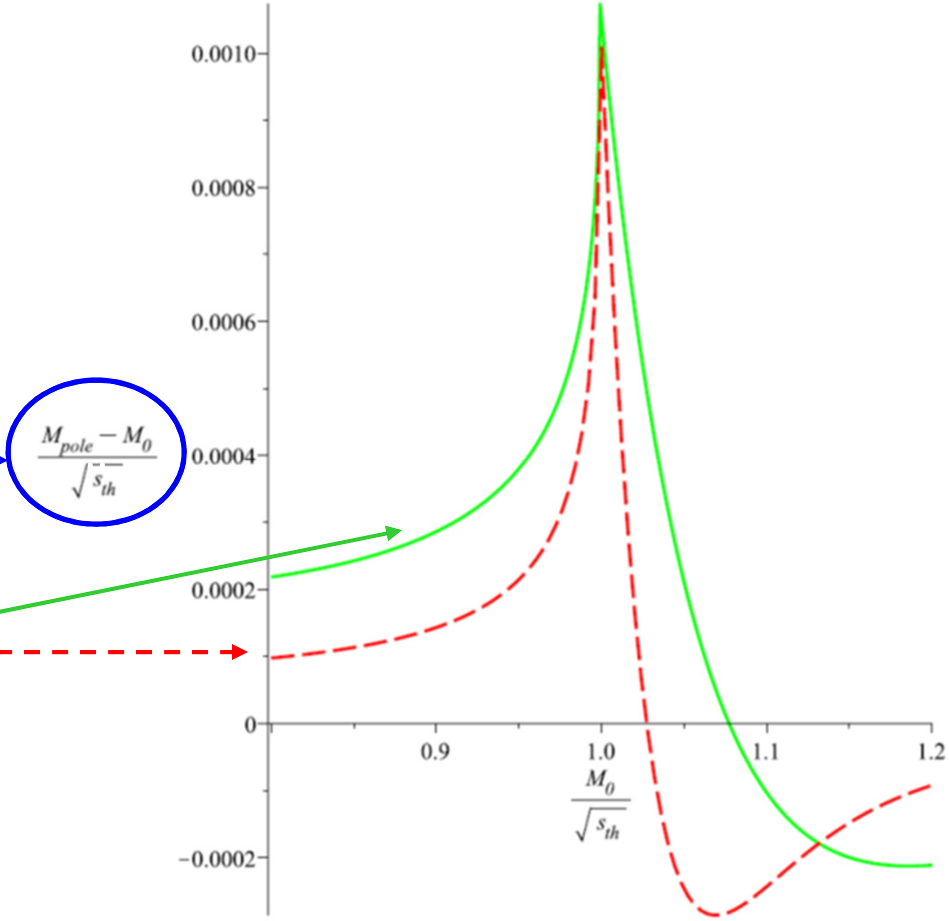
  - Bugg [J. Phys. G **35** (2008) 075005] showed that the *X*(3872) is far too narrow to be a cusp alone—Some sort of resonance must be present
  - But since several channels all open up very near 3.872 GeV, they all contribute to a big cusp that can drag, say, a diquark-antidiquark resonance from perhaps 10’s of MeV away to become the *X*(3872)

Example cusp effects

S. Blitz & RFL, Phys. Rev. D **91** (2015) 094025

M_0: Bare resonant pole mass
S_{th}: Threshold *s* value [here $(3.872\ \text{GeV})^2$]
M_{pole}: Shifted pole mass

Relative size of pole shift (about 0.12% near S_{th}, or 5 MeV)



At the charm scale,
a cusp from an opening
diquark pair threshold
is more effective than
one from a meson pair!

What determines cusp shapes?

- Traditionally, a phenomenologically-based exponential form factor is used in the case of meson pair production:

  $$F_{\text{mes}}^2(s) = \exp\left(-\frac{s-s_{th}}{\beta^2}\right),$$

  where β is a typical hadronic scale (~0.5-1.0 GeV)
- For processes at high energy (s), or when the high-s tails of form factors are important (as in dispersion relations), use *constituent counting rules* [Matveev *et al*., Lett. Nuovo Cim. **7**, 719 (1973); Brodsky & Farrar, PRL **31**, 1153 (1973)]
- In any hard process in which a constituent is diverted through a finite angle, there will be a factor of $1/s$ (or $1/t$) coming from a propagator of the virtual particle redirecting it
- Using this logic, the form factor $F(s)$ of a particle with 4 quark constituents can quickly be shown to scale as

$$F_{\text{diq}}(s) \sim \left(\frac{\alpha_s}{s}\right)^3 \to F_{\text{diq}}(s) = \left(\frac{s_{th}}{s}\right)^3$$

Can the counting rules be used for cross sections as well?

- *With ease*: S. Brodsky and RFL, Phys. Rev. D **91** (2015) 114025
- Exotic states can be produced in threshold regions in e^+e^- (BES, Belle), electroproduction (JLab 12), hadronic beam facilities (P̄ANDA at FAIR, AFTER@LHC) and are best characterized by cross section ratios
- Two examples:

$$1)\quad \frac{\sigma(e^+e^-\to Z^+(c\bar{c}u\bar{d})+\pi^-(\bar{u}d))}{\sigma(e^+e^-\to\mu^+\mu^-)} \propto \frac{1}{s^4} \text{ as } s\to\infty$$

$$2)\quad \frac{\sigma(e^+e^-\to Z^+(c\bar{c}u\bar{d})+\pi^-(\bar{u}d))}{\sigma(e^+e^-\to\Lambda_c(cud)+\bar{\Lambda}_c(\bar{c}\,\bar{u}\bar{d}))} \to const \text{ as } s\to\infty$$

Ratio numerically smaller if Z_c behaves like weakly-bound dimeson molecule instead of diquark-antidiquark bound state due to weaker meson color van der Waals forces