Transverse Momentum Dependent Fragmenting Jet Functions with Applications to Quarkonium Production

Thomas Mehen
Duke University

7th Workshop of the APS Topical Group on Hadronic Physics
Washington, DC
2/2/2017

Fragmenting Jet Functions (FJFs)

NRQCD and Quarkonium Production

Heavy Quarkonium FJFs

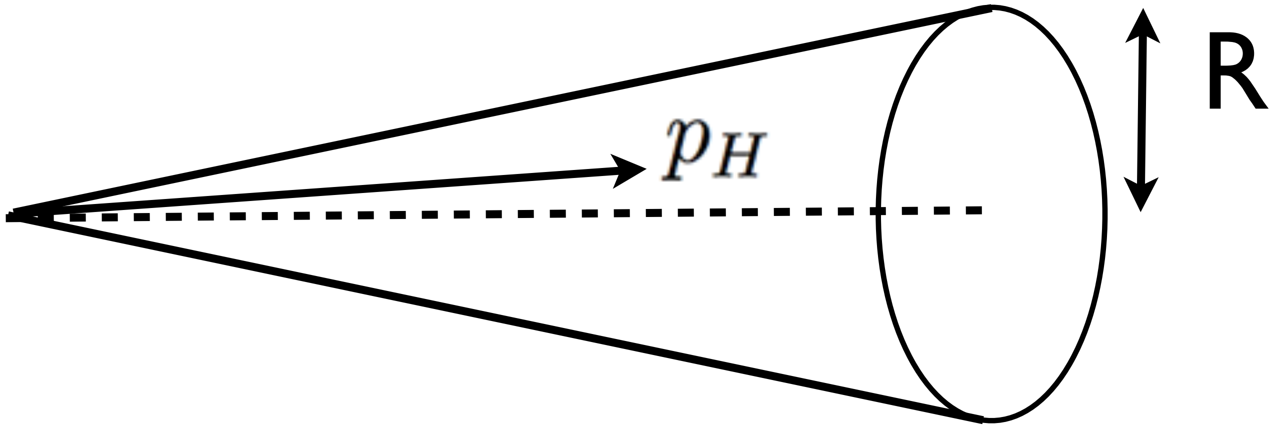
TMD-dependent FJFs and Heavy Quarkonium

Recent Data on Quarkonia in Jets (LHCb)

Fragmenting Jet Functions

M. Procura, I. Stewart, PRD 81 (2010) 074009
A. Jain, M. Procura, W. Waalewijn, JHEP 1105 (2011) 035
A. Procura, W. Waalewijn, PRD 85 (2012) 114041

jets with identified hadrons



Jet Energy: E

$$p_H^+ = z p_{\text{jet}}^+$$

cross sections determined by **fragmenting jet function (FJF):**

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: fragmentation functions

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dz}(e^+e^- \to h\,X) = \sum_i \int_z^1 \frac{dx}{x}\, C_i(E_{\rm cm}, x, \mu)\; D_i^h(z/x, \mu)$$

jet cross sections: jet functions

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}z}(E,R) = \int \mathrm{d}\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell$$

$$\boxed{\mathcal{G}_g^h(E,R,\mu,z) \longrightarrow D_i^h(z/x,\mu)\ ,\ J_\ell}$$

relationship to jet function:

$$\sum_h \int_0^1 \mathrm{d}z\, z\, D_j^h(z,\mu) = 1$$

$$J_i(E,R,z,\mu) = \frac{1}{2}\sum_h \int \frac{dz}{(2\pi)^3}\, z\, \mathcal{G}_i^h(E,R,z,\mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma}{dE} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell J_i(E,R,\mu)$$

$$\frac{d\sigma}{dE\,dz} = \int d\Phi_N \mathrm{tr}[H_N S_N] \prod_\ell J_\ell\, \mathcal{G}_i^h(E,R,z,\mu)$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E,R,z,\mu) = \sum_i \int_z^1 \frac{\mathrm{d}z'}{z'} \mathcal{J}_{ij}(E,R,z',\mu) D_j^h\left(\frac{z}{z'},\mu\right)\left[1+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{4E^2\tan^2(R/2)}\right)\right]$$

matching coefficients calculable in perturbation theory

$$\frac{\mathcal{J}_{gg}(E,R,z,\mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu)C_A}{\pi}\left[\left(L^2-\frac{\pi^2}{24}\right)\delta(1-z)+\hat{P}_{gg}(z)L+\hat{\mathcal{J}}_{gg}(z)\right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z)\ln z & z \le 1/2 \\ \frac{2(1-z+z^2)^2}{z}\left(\frac{\ln(1-z)}{1-z}\right)_+ & z \ge 1/2. \end{cases}$$

$$L = \ln[2E\tan(R/2)/\mu],$$

scale for $\mathcal{J}_{ij}(E,R,z,\mu)$

sum rule for matching coefficients

$$\sum_j \int_0^1 \mathrm{d}z\, z\, \mathcal{J}_{ij}(R,z,\mu) = 2(2\pi)^3\, J_i(R,\mu)$$

Non-Relativistic QCD (NRQCD) Factorization Formalism

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$$

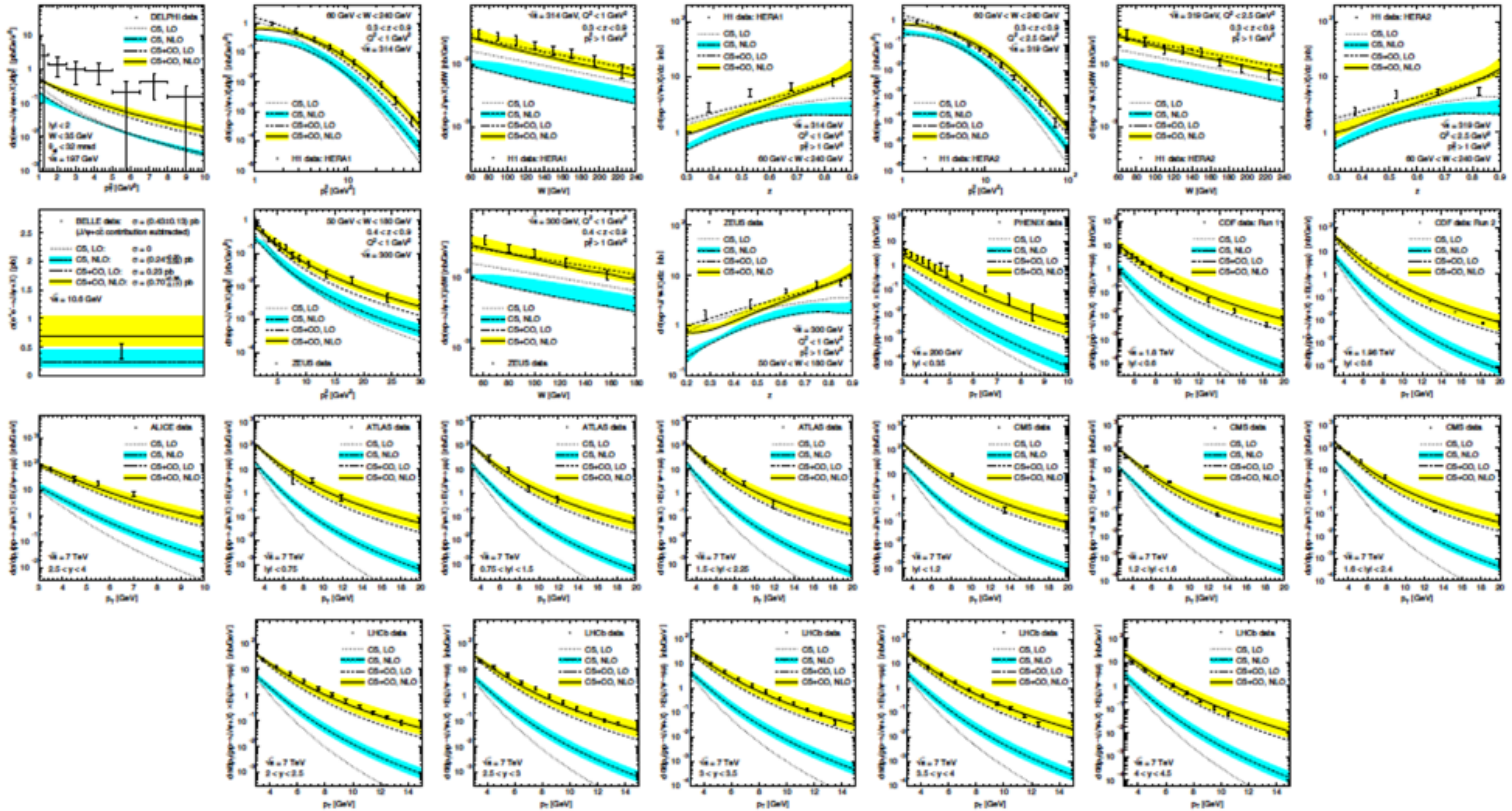
CSM - lowest order in v

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$$

color-octet mechanisms

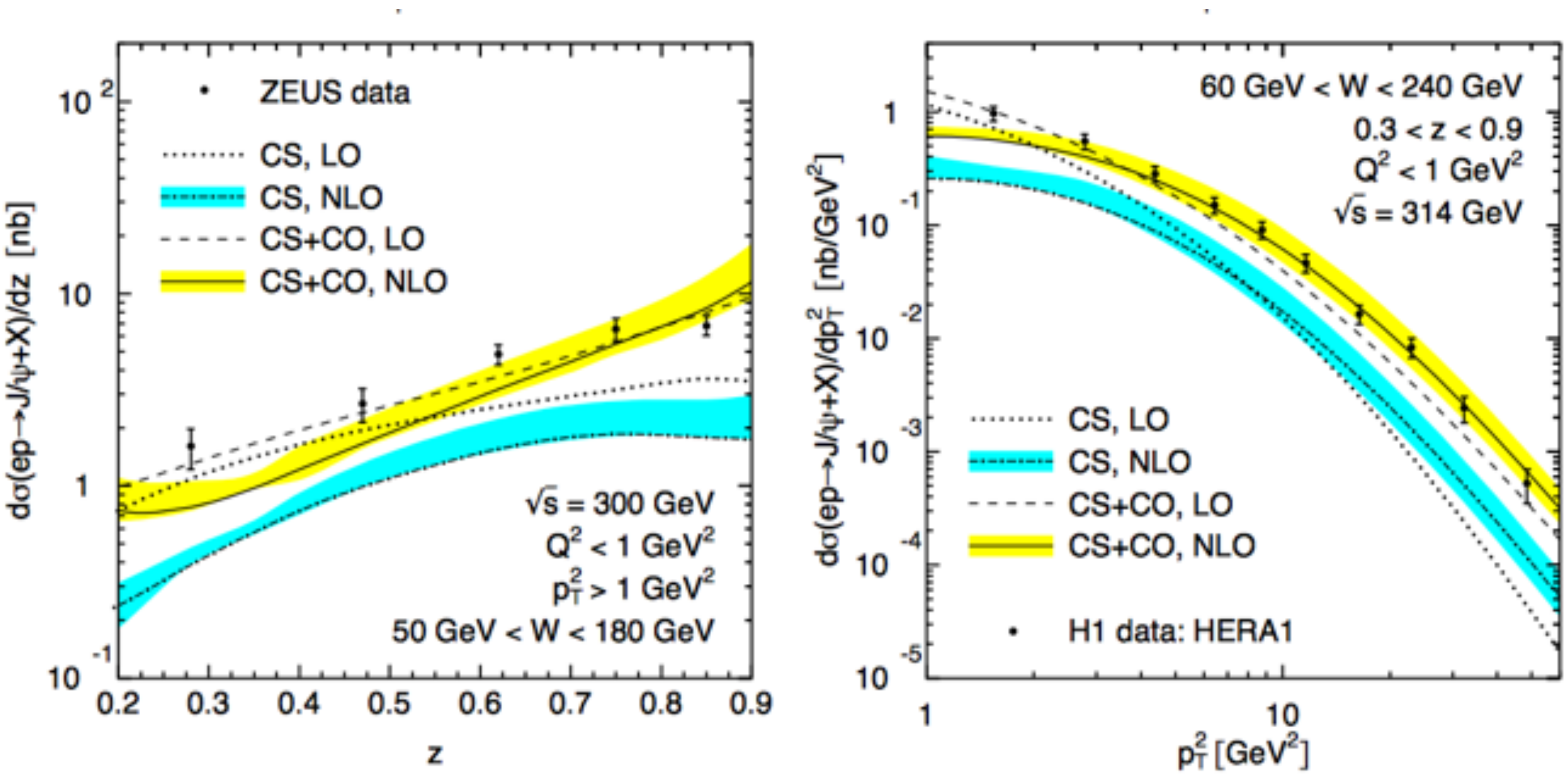
Global Fits with NLO CSM + COM

Butenschoen and Kniehl, PRD 84 (2011) 051501

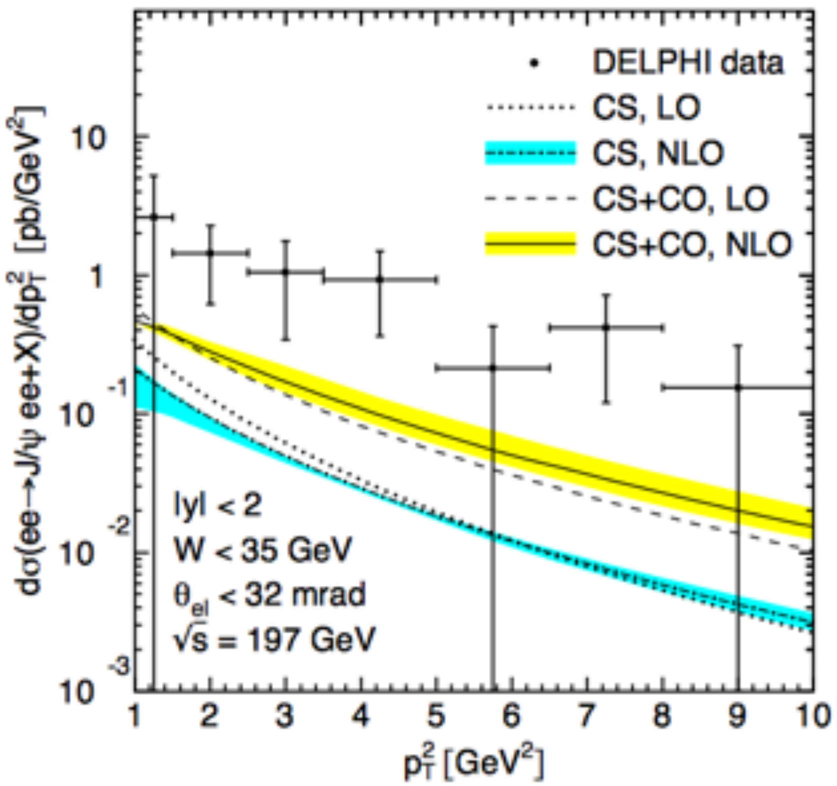


$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \rightarrow J/\psi + X$ fit to 194 data points, 26 data sets

NLO: CSM + COM Required to Fit Data



$$ep \to J/\psi + X$$



$$\gamma^*\gamma^* \to J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

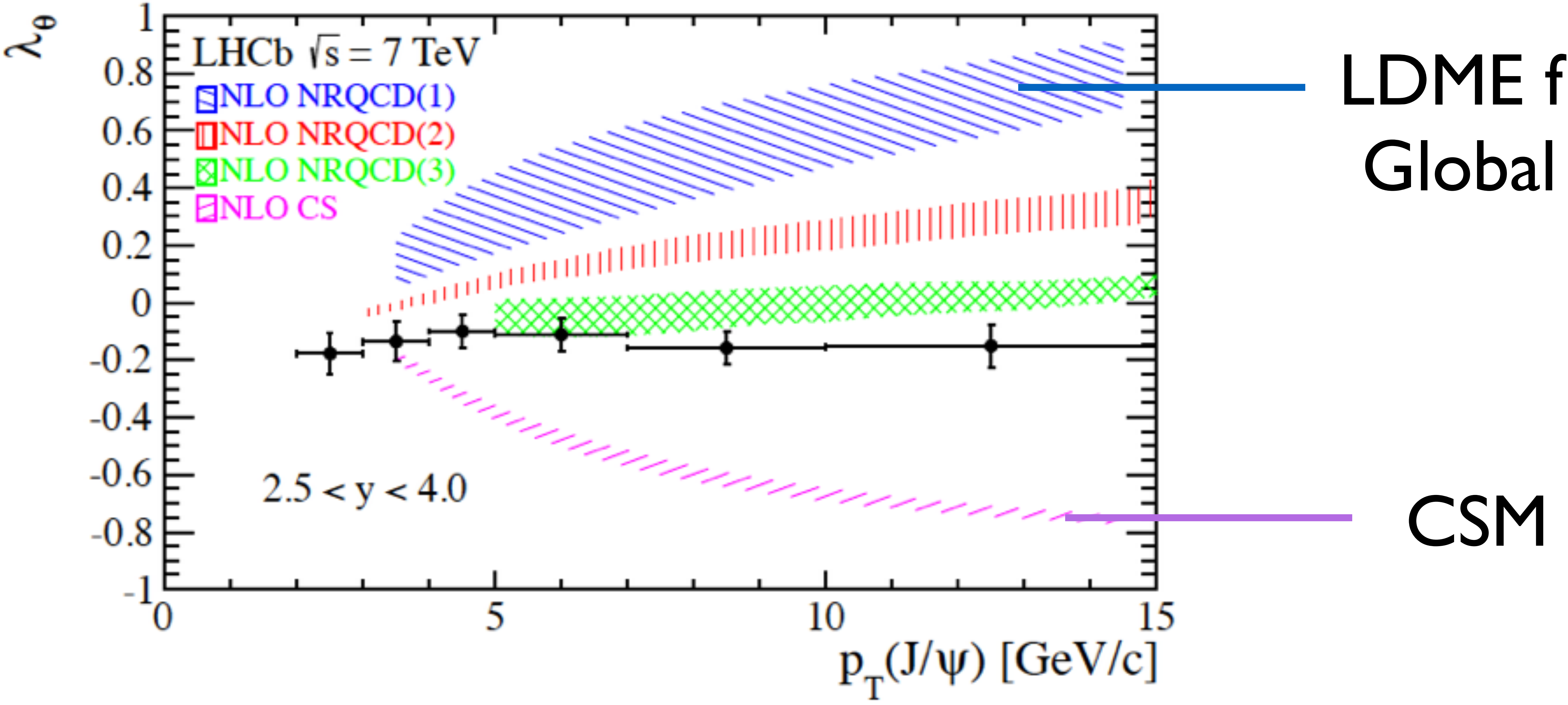
extracted LDME satisfy NRQCD v-scaling

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle = 1.32\ \mathrm{GeV}^3$$

$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}\ \mathrm{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}\ \mathrm{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}\ \mathrm{GeV}^5$

$$\chi^2_{\mathrm{d.o.f.}} = 857/194 = 4.42$$

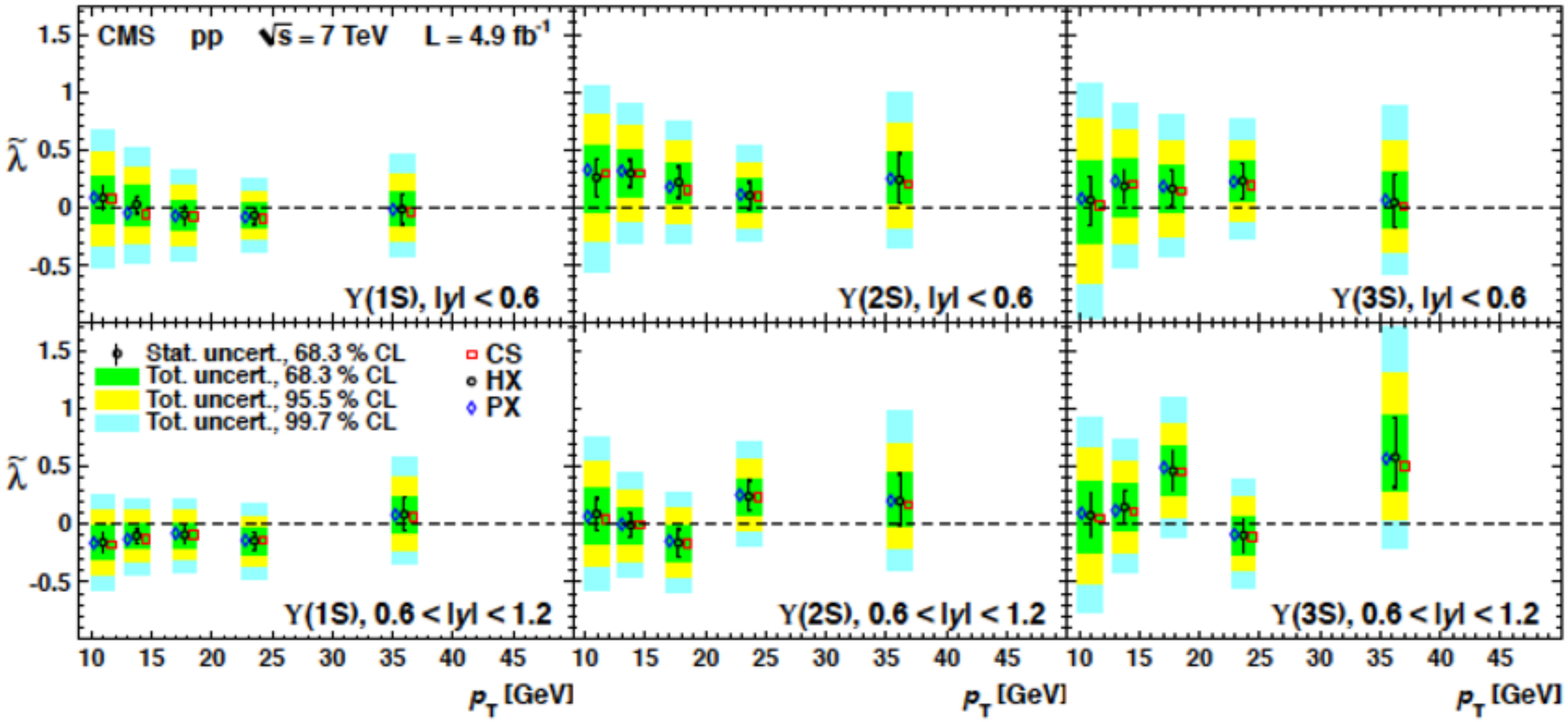
Polarization of J/ψ at LHCb



LDME from Global fits

CSM

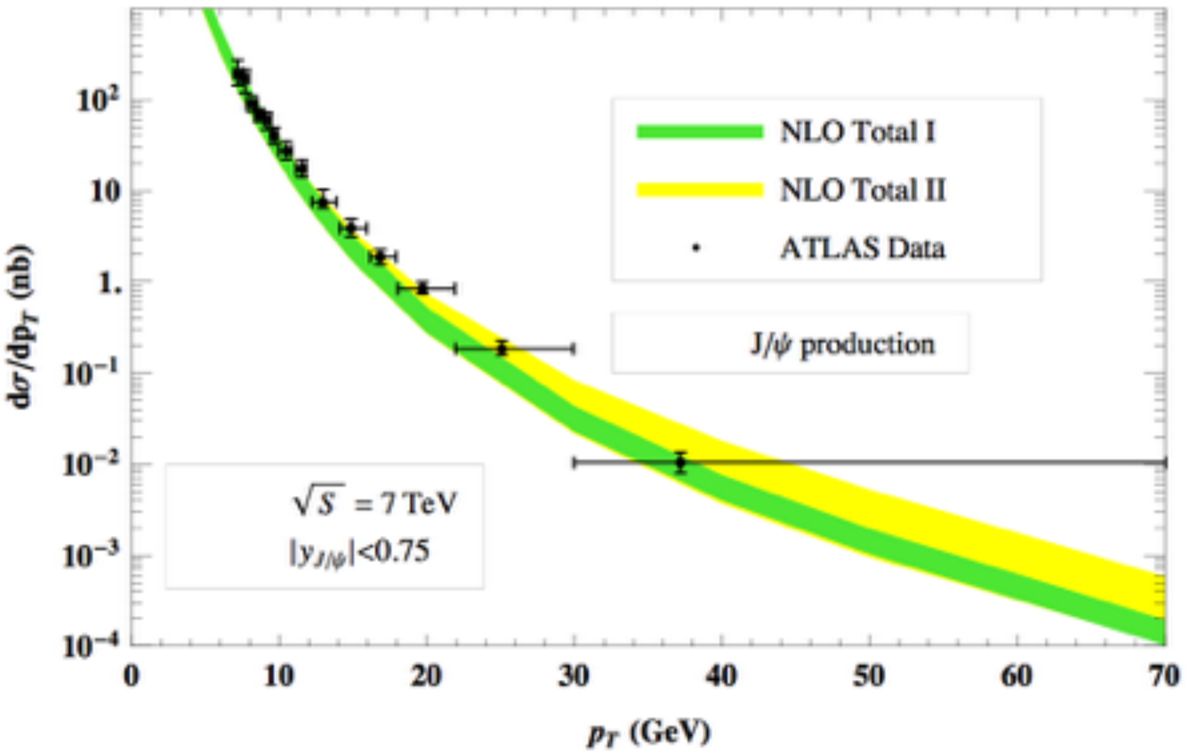
Polarization of Υ(nS) at CMS

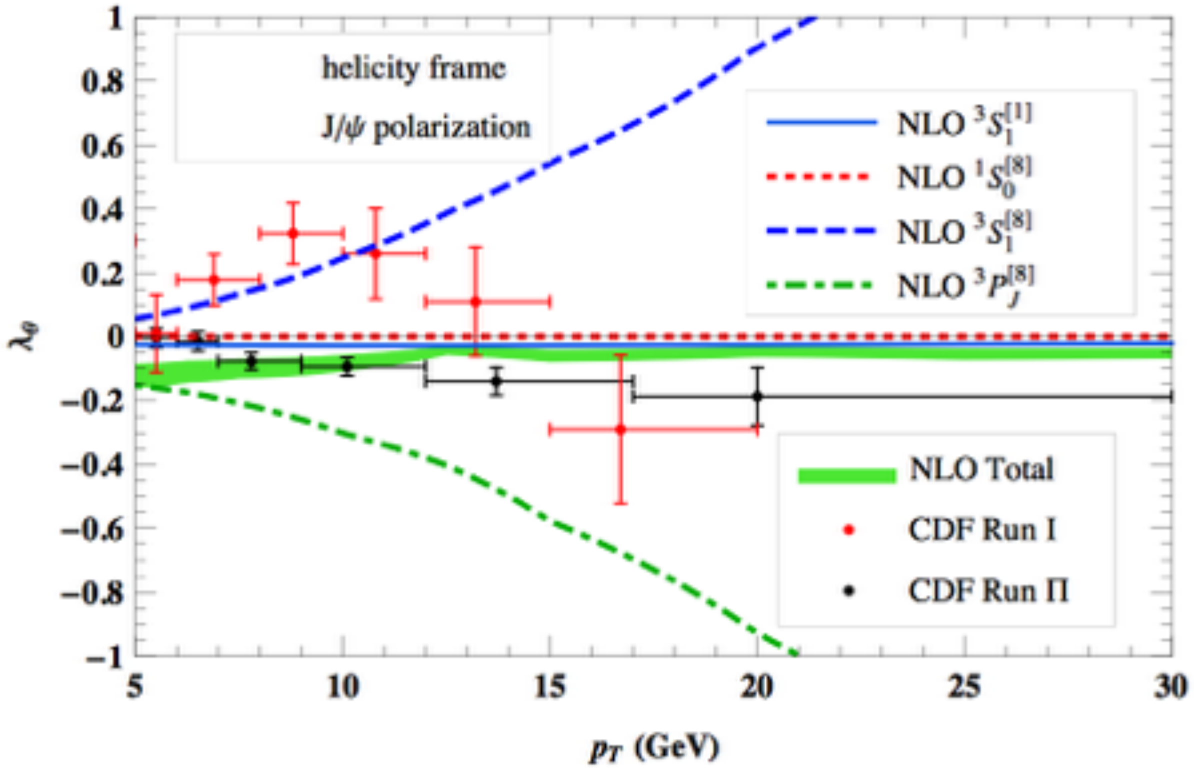


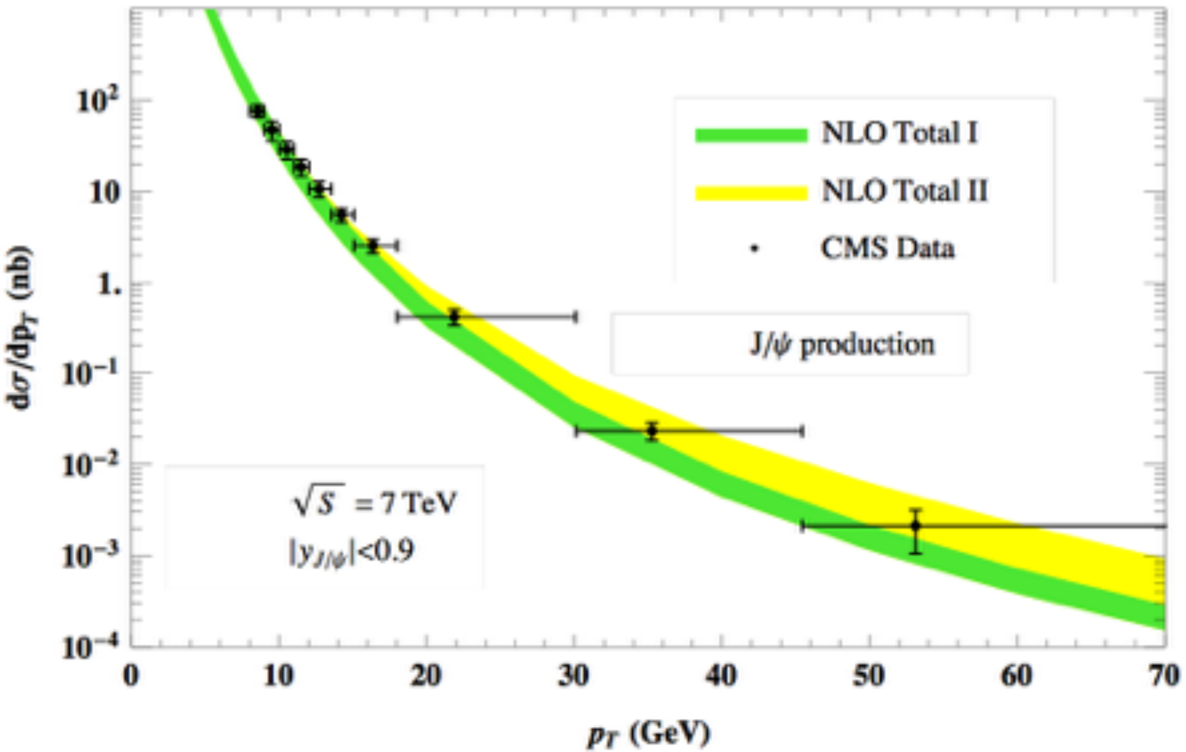
Recent Attempts to Resolve J/ψ Polarization Puzzle

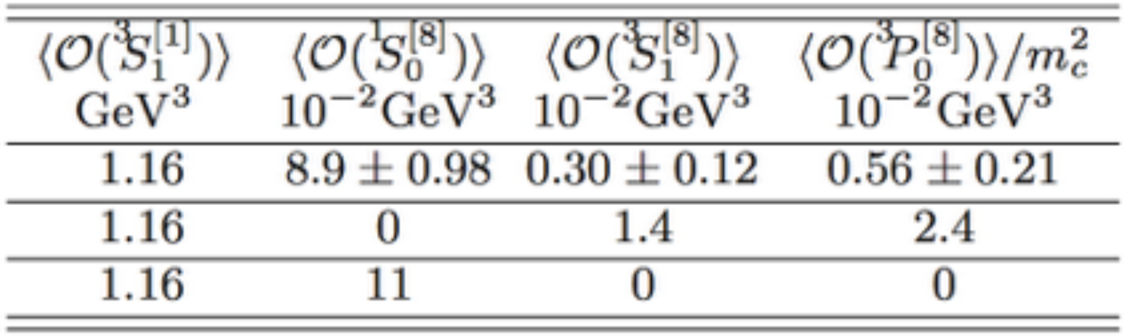
simultaneous NLO fit to CMS, ATLAS high p_t production, polarization

Chao, et. al. PRL 108, 242004 (2012)









$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ GeV^3	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ 10^{-2}GeV^3	$\langle\mathcal{O}(^3P_0^{[8]})\rangle/m_c^2$ 10^{-2}GeV^3
1.16	8.9 ± 0.98	0.30 ± 0.12	0.56 ± 0.21
1.16	0	1.4	2.4
1.16	11	0	0

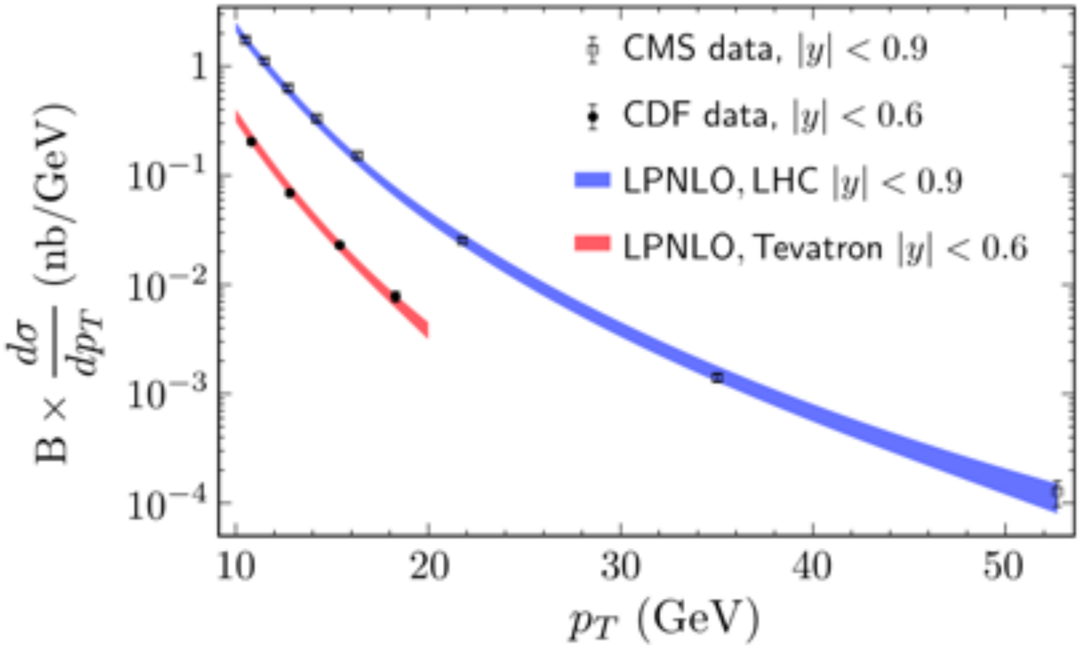
Recent Attempts to Resolve J/ψ Polarization Puzzle

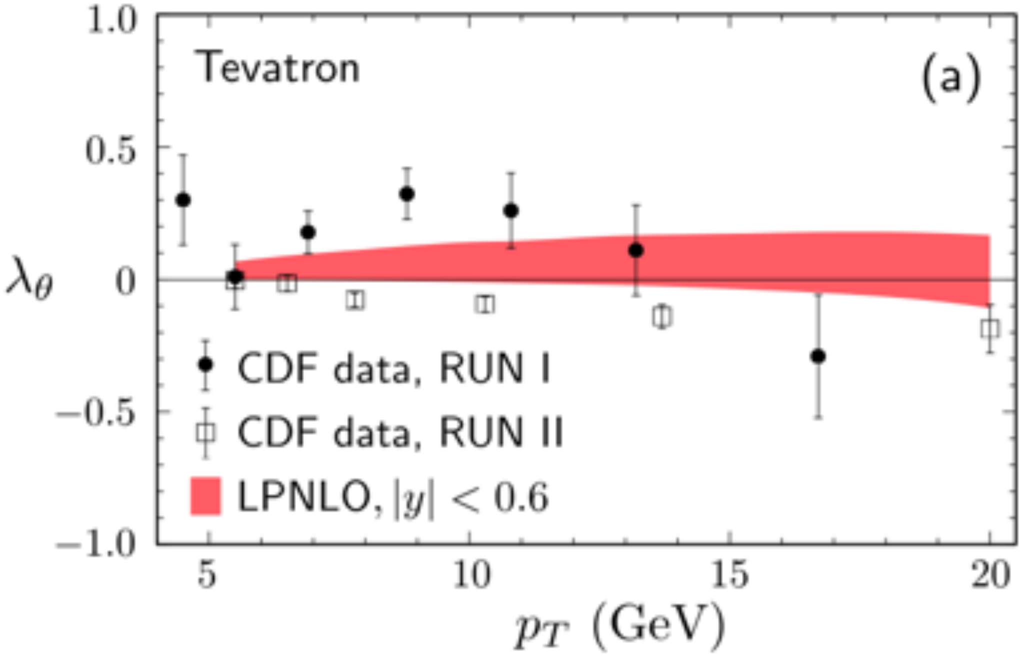
i) large p_t production at CDF

Bodwin, et. al., PRL 113, 022001(2014)

ii) resum logs of p_t/m_c using AP evolution

iii) fit COME to p_t spectrum, predict basically no polarization





Extracted COME **inconsistent** with global fits

$$\langle \mathcal{O}^{J/\psi}({}^1S_0^{(8)}) \rangle = 0.099 \pm 0.022\,\mathrm{GeV}^3$$

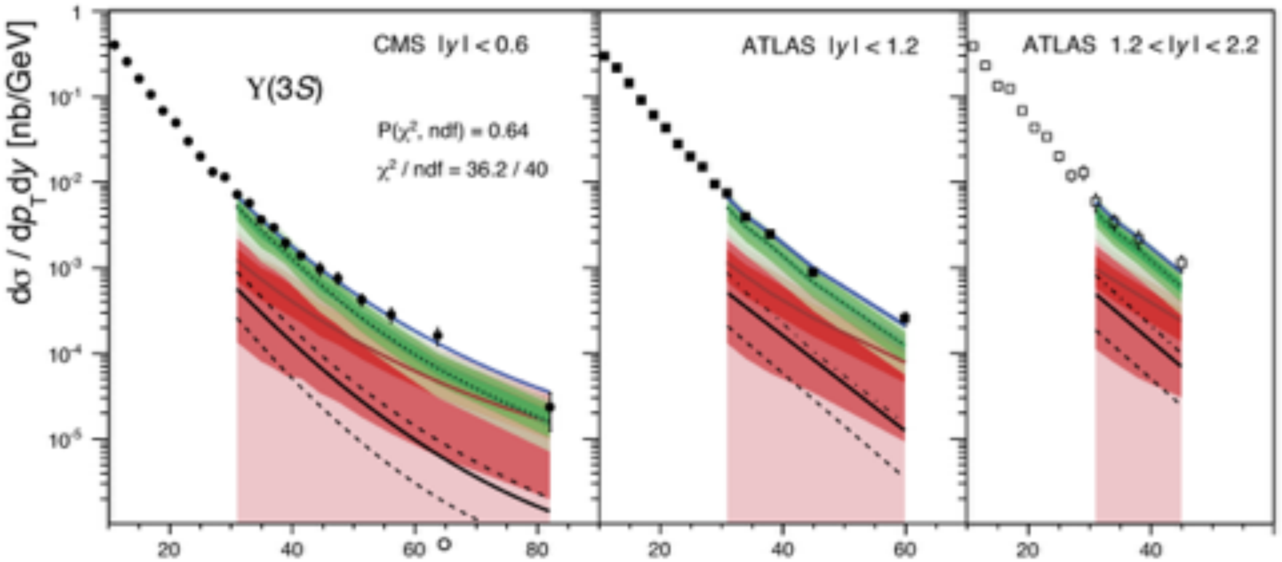
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{(8)}) \rangle = 0.011 \pm 0.010\,\mathrm{GeV}^3$$

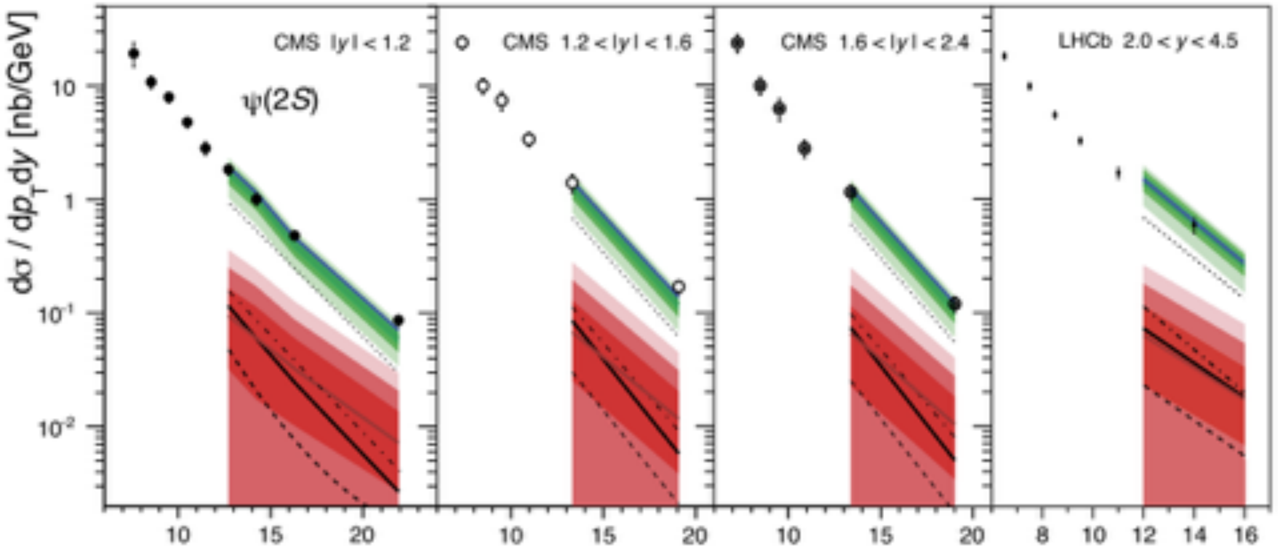
$$\langle \mathcal{O}^{J/\psi}({}^3P_0^{(8)}) \rangle = 0.011 \pm 0.010\,\mathrm{GeV}^5$$

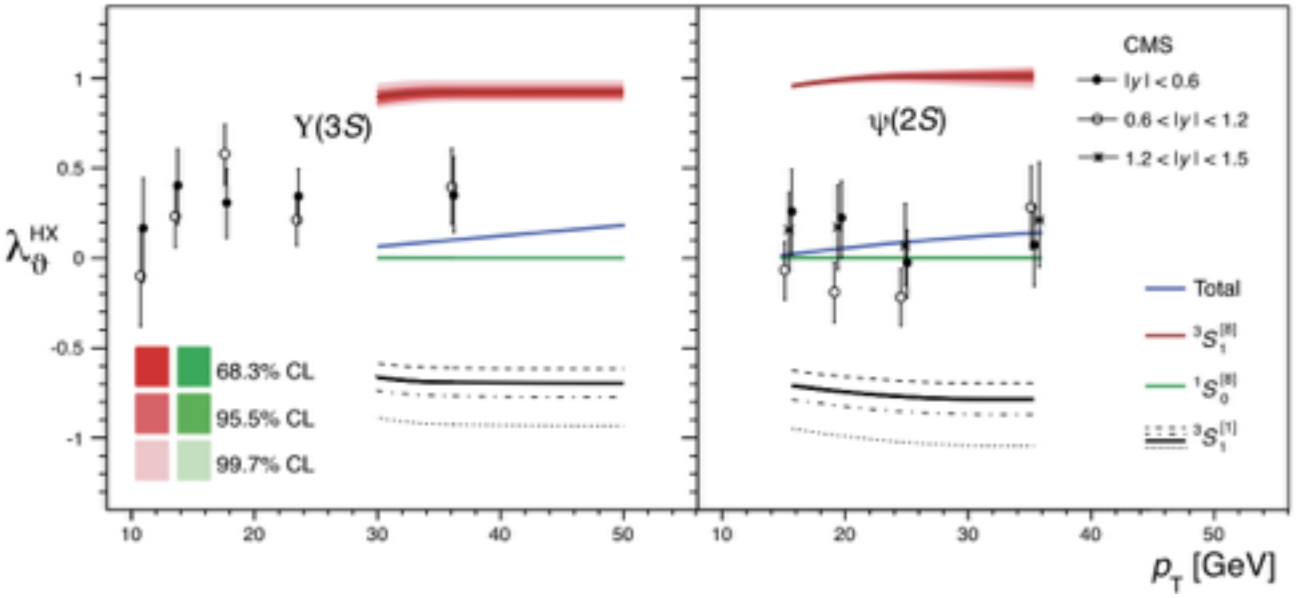
Recent Attempts to Resolve J/ψ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press







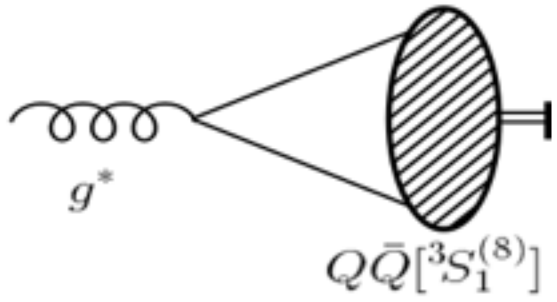
argue for $^1S_0^{(8)}$ dominance in both ψ(2S) & ϒ(3S) production

NRQCD fragmentation functions

Braaten, Yuan, PRD 48 (1993) 4230
Braaten, Chen, PRD 54 (1996) 3216
Braaten, Fleming, PRL 74 (1995) 3327

Perturbatively calculable **at the scale 2m$_c$**

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi \alpha_s(2m_c)}{3M_\psi^3} \langle O^\psi({}^3S_1^{(8)}) \rangle \delta(1-z)$$



$$D_g^{\psi(1)}(z, 2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2} \frac{\langle O^\psi({}^3S_1^{(1)}) \rangle}{M_\psi^3} \int_0^z dr \int_{(r+z^2)/2z}^{(1+r)/2} dy \frac{1}{(1-y)^2(y-r)^2(y^2-r)^2}$$
$$\sum_{i=0}^{2} z^i \left(f_i(r,y) + g_i(r,y) \frac{1+r-2y}{2(y-r)\sqrt{y^2-r}} \ln \frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}} \right),$$

Altarelli-Parisi evolution: 2m$_c$ to 2E tan(R/2)

FJF in terms of fragmentation function

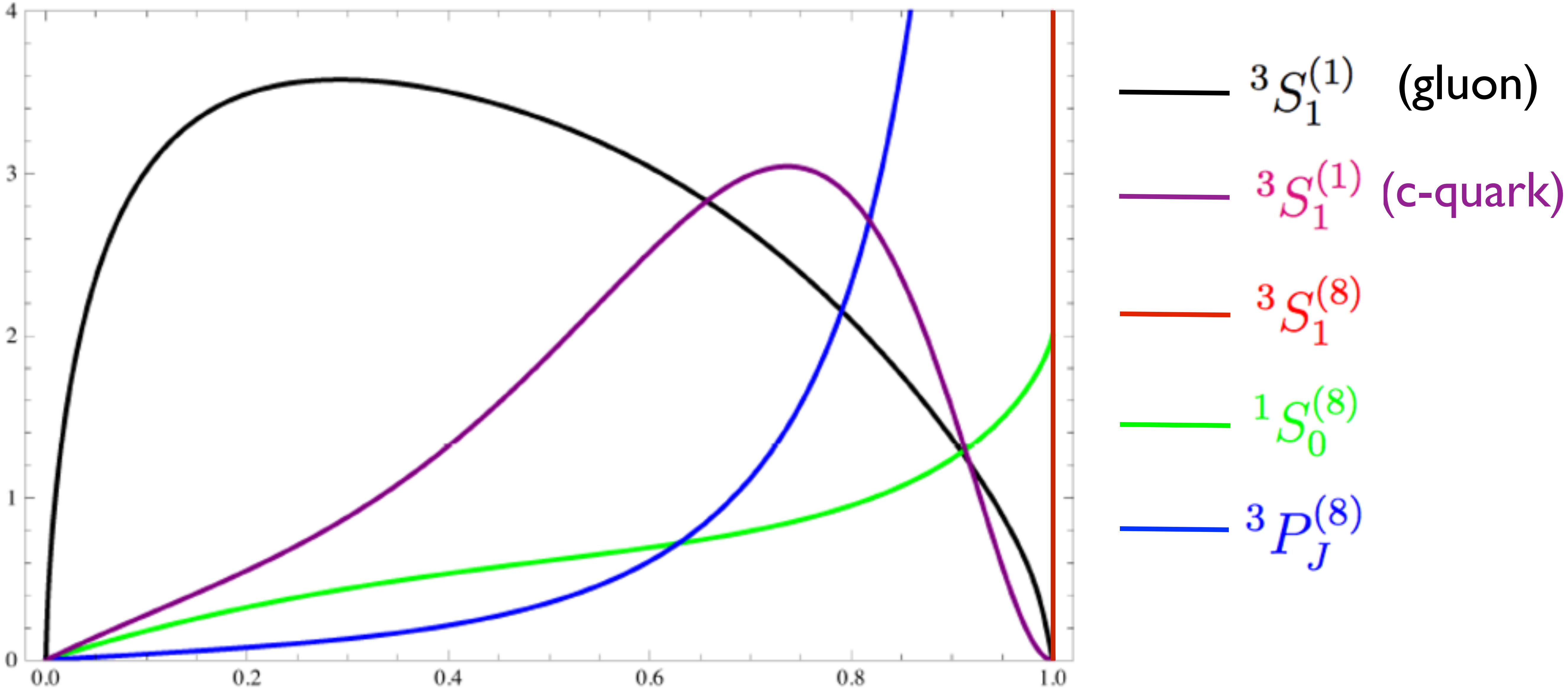
$$
\begin{aligned}
\mathcal{G}_g^{\psi}(E,R,z,\mu) = {} & D_{g\to\psi}(z,\mu)\left(1+\frac{C_A\alpha_s}{\pi}\left(L_{1-z}^2-\frac{\pi^2}{24}\right)\right) \\
& +\frac{C_A\alpha_s}{\pi}\left[\int_z^1 \frac{dy}{y}\tilde{P}_{gg}(y)L_{1-y}D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right. \\
& +2\int_z^1 dy\frac{D_{g\to\psi}(z/y,\mu)-D_{g\to\psi}(z,\mu)}{1-y}L_{1-y} \\
& \left.+\theta\left(\frac{1}{2}-z\right)\int_z^{1/2}\frac{dy}{y}\hat{P}_{gg}(y)\ln\left(\frac{y}{1-y}\right)D_{g\to\psi}\left(\frac{z}{y},\mu\right)\right]
\end{aligned}
$$

$$
L_{1-z}=\ln\left(\frac{2E\tan(R/2)(1-z)}{\mu}\right)
$$

For large E, FJF ~ NRQCD frag. function (at scale 2E tan(R/2))

$$
\boxed{\mathcal{G}_g^{h}(E,R,\mu=2E\tan(R/2),z)\rightarrow D_g^{\psi}(z,2E\tan(R/2))+O(\alpha_s)}
$$

NRQCD FF's (at scale 2m$_c$)

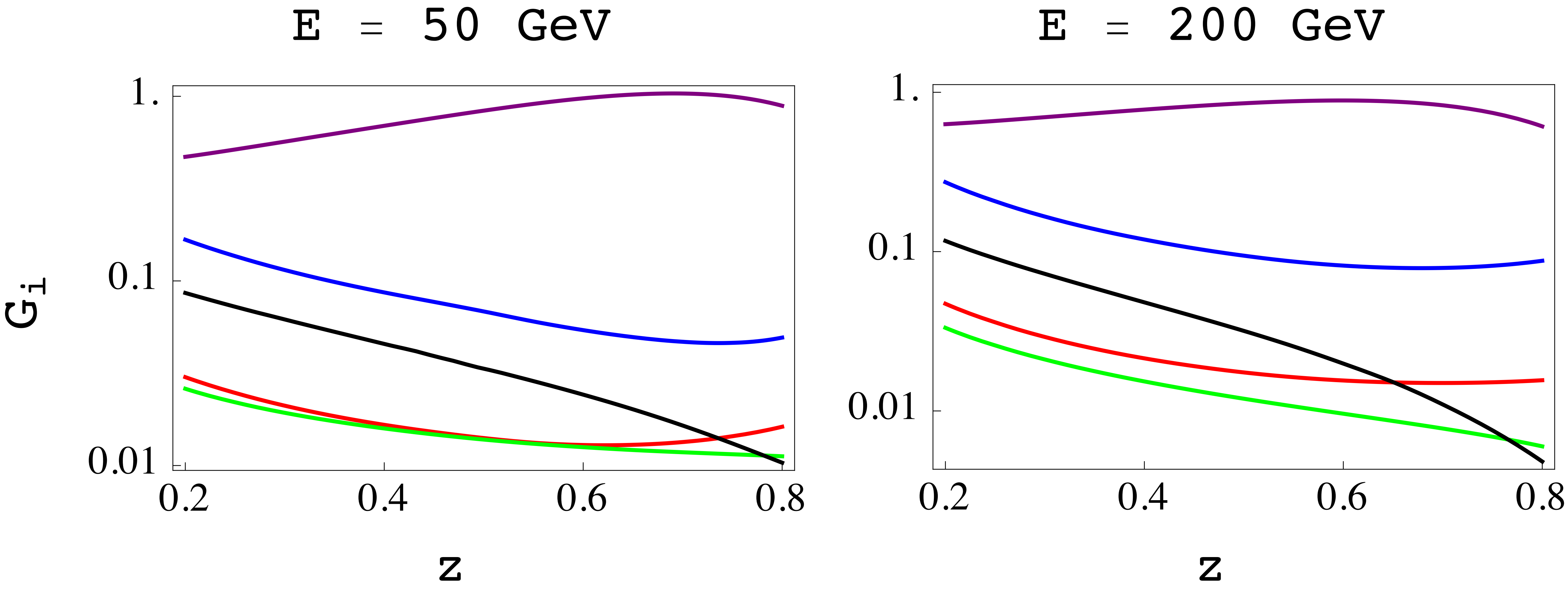


(normalization arbitrary)

Evolution to 2E tan(R/2) will soften discrepancies

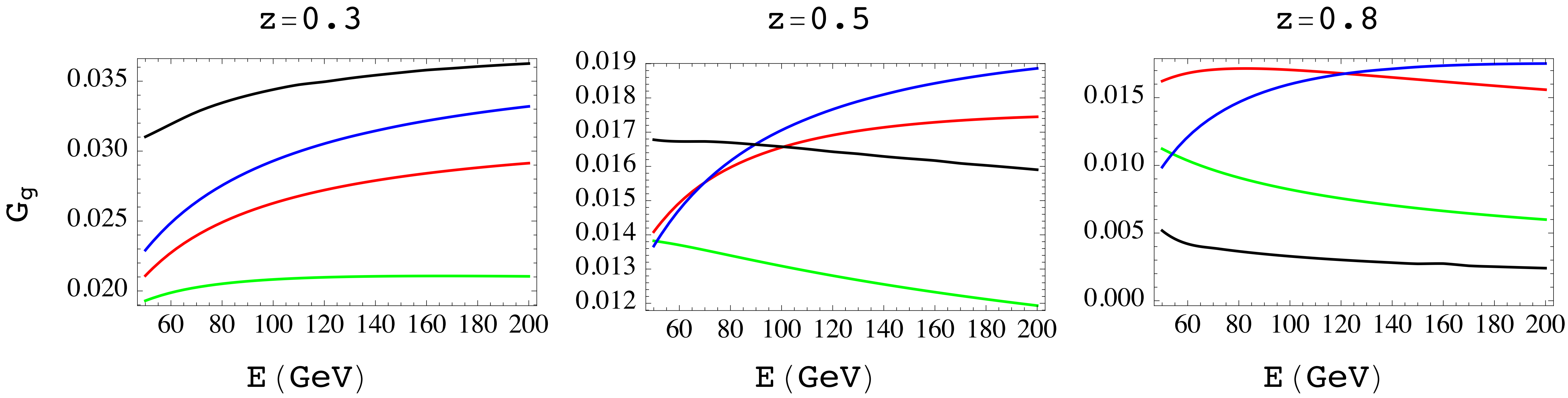
FJF's at Fixed Energy vs. z

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



FJF's at Fixed z vs. Energy

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

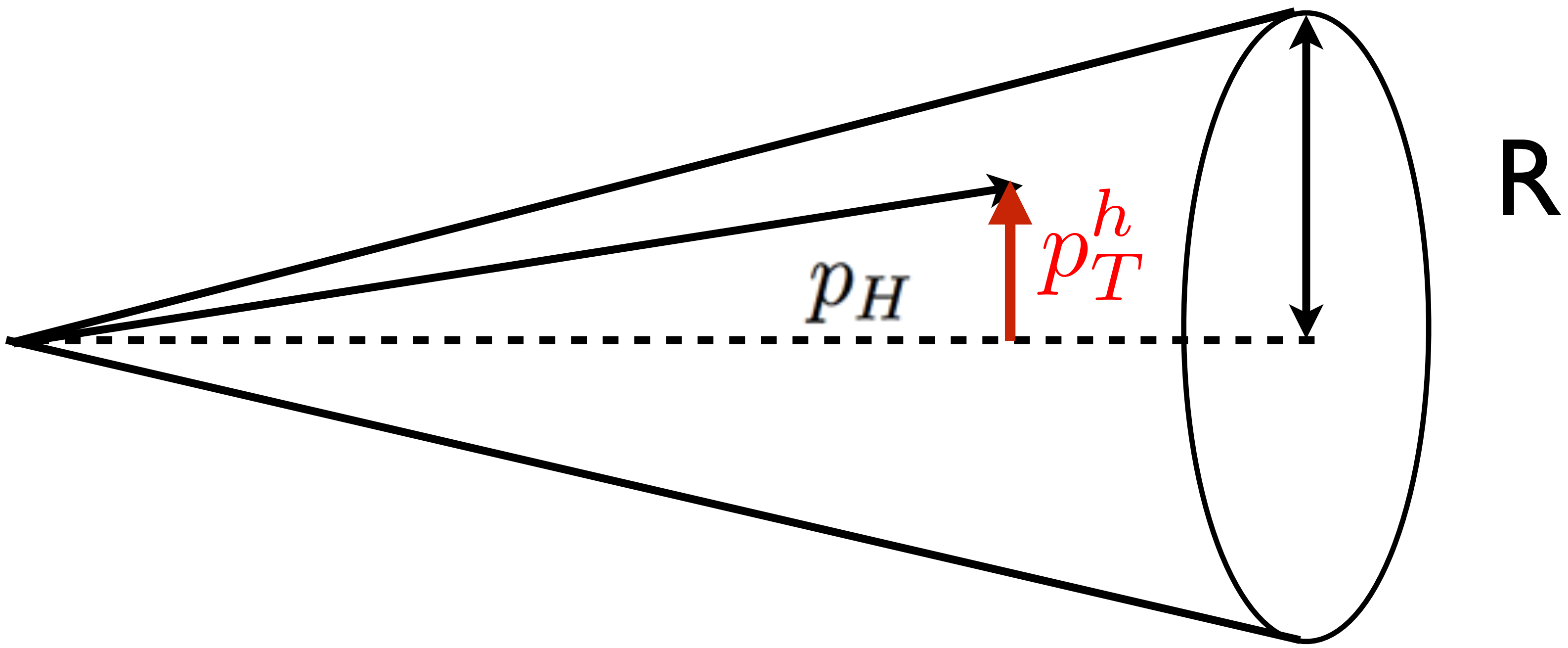


$^1S_0^{(8)}$ dominance predicts negative slope for z vs. E if z > 0.5

Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

jets with identified hadron: hadron z, p_T are both measured



Jet Energy: E

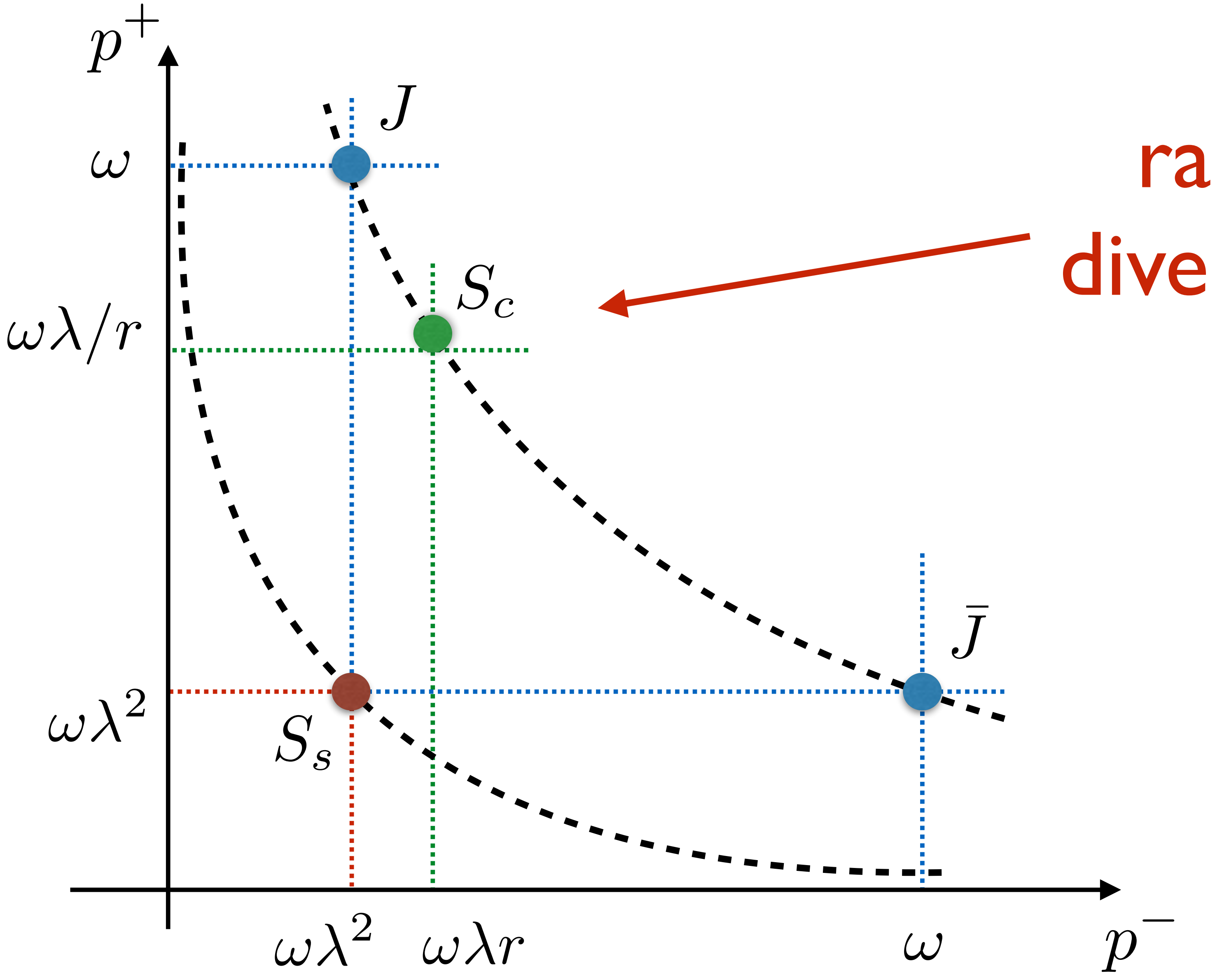
$$p_H^+ = z p_{\rm jet}^+$$

transverse momentum measured w/ rspt. to jet axis

jet axis ~ parton initiating jet if out of jet radiation is ultrasoft

$$\omega \gg p_T^h \gg \Lambda \gg \Lambda_{\rm QCD}$$

Scales in TMDFJF



rapidity divergences

$$p_c \sim \omega(\lambda^2, 1, \lambda) \qquad p_{cs} \sim p_h^\perp(r, 1/r, 1) \qquad p_{us} \sim \Lambda(1,1,1)$$

$$\lambda = p_h^\perp/\omega$$

Factorization Theorem

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = H_+(\mu) \times \left[\mathcal{D}_{q/h} \otimes_\perp S_C\right](\mathbf{p}_\perp, z, \mu)$$

$$H_+(\mu) = (2\pi)^2 N_c\, C_+^\dagger(\mu) C_+(\mu)$$

$$\begin{aligned}\mathcal{D}_{q/h}(\mathbf{p}_\perp^{\mathcal{D}}, z) \equiv \frac{1}{z} \sum_{X_n} \frac{1}{2N_c} \delta(p^-_{Xh;r}) \delta^{(2)}(p^\perp_{Xh;r})\, \mathrm{Tr}\Big[\frac{\bar{n}\!\!\!/}{2} \langle 0| \delta_{\omega,\overline{\mathcal{P}}} \chi_n(0) \delta^{(2)}(\mathcal{P}_\perp^{X_n} + \mathbf{p}_\perp^{\mathcal{D}}) |X_n h\rangle \\ \times \langle X_n h| \bar{\chi}_n(0) |0\rangle\Big]\end{aligned}$$

$$\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu, \nu) = \int_z^1 \frac{dx}{x}\, \mathcal{J}_{i/j}(\mathbf{p}_\perp, x, \mu, \nu) D_{j/h}\left(\frac{z}{x}, \mu\right) \;+\; \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{|\mathbf{p}_\perp|^2}\right)$$

$$S_C(\mathbf{p}_\perp^S) \equiv \frac{1}{N_c} \sum_{X_{cs}} \mathrm{Tr}\left[\langle 0| V_n^\dagger(0) U_n(0) \delta^{(2)}(\mathcal{P}_\perp + \mathbf{p}_\perp^S) |X_{cs}\rangle \langle X_{cs}| U_n^\dagger(0) V_n(0) |0\rangle\right]$$

Anomalous Dimensions for RGE, RRGE

RGE

$$\gamma_\mu^{S_C}(\nu) = \frac{\alpha_s C_i}{\pi} \ln\left(\frac{\mu^2}{r^2\nu^2}\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) = \frac{\alpha_s C_i}{\pi} \left(\ln\left(\frac{\nu^2}{\omega^2}\right) + \bar{\gamma}_i\right)$$

$$\gamma_\mu^{\mathcal{D}}(\nu) + \gamma_\mu^{S_C}(\nu) = \gamma_\mu^J = \frac{\alpha_s C_i}{\pi} \left(\ln\left(\frac{\mu^2}{r^2\omega^2}\right) + \bar{\gamma}_i\right)$$

Rapidity Renormalization Group

$$\gamma_\nu^{S_C}(p_\perp, \mu) = +(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

$$\gamma_\nu^{\mathcal{D}}(p_\perp, \mu) = -(8\pi)\alpha_s C_i \, \mathcal{L}_0(\mathbf{p}_\perp, \mu^2)$$

$$\gamma_\nu^{\mathcal{D}}(\mathbf{p}_\perp, \mu) + \gamma_\nu^{S}(\mathbf{p}_\perp, \mu) = 0$$

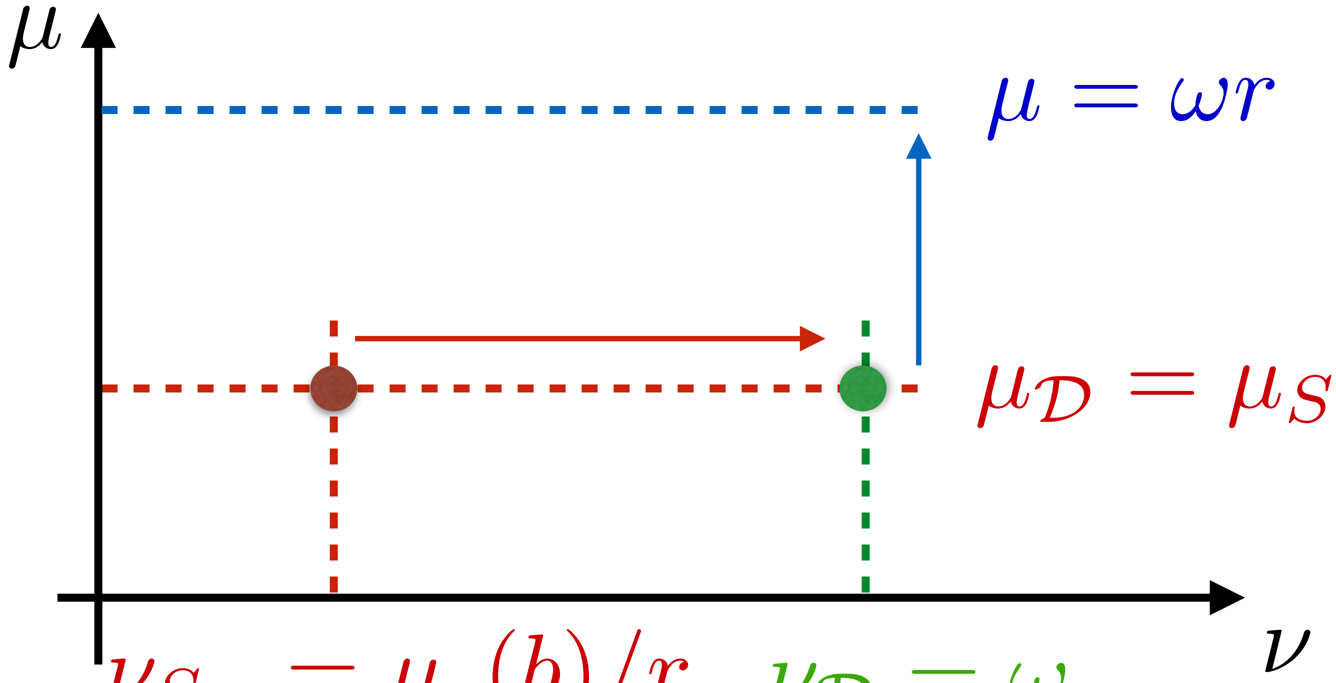
J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, PRL108 (2012) 151601

J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, JHEP1205 (2012) 084

Solution to Evolution Eqs. in Fourier Space

$$D_{i/h}(p_\perp, z, \mu) = (2\pi)^2\, p_\perp \int_0^\infty db\, b J_0(b p_\perp)\, \mathcal{U}_{S_C}(\mu, \mu_{S_C}, m_{S_C})\, \mathcal{U}_{\mathcal{D}}(\mu, \mu_{\mathcal{D}}, 1)$$

$$\times\, \mathcal{V}_{S_C}(b, \mu_{S_C}, \nu_{\mathcal{D}}, \nu_{S_C})\, \mathcal{FT}\Big[\mathcal{D}_{i/h}(\mathbf{p}_\perp, z, \mu_{\mathcal{D}}, \nu_{\mathcal{D}}) \otimes_\perp S^i_C(\mathbf{p}_\perp, \mu_{S_C}, \nu_{S_C})\Big]$$

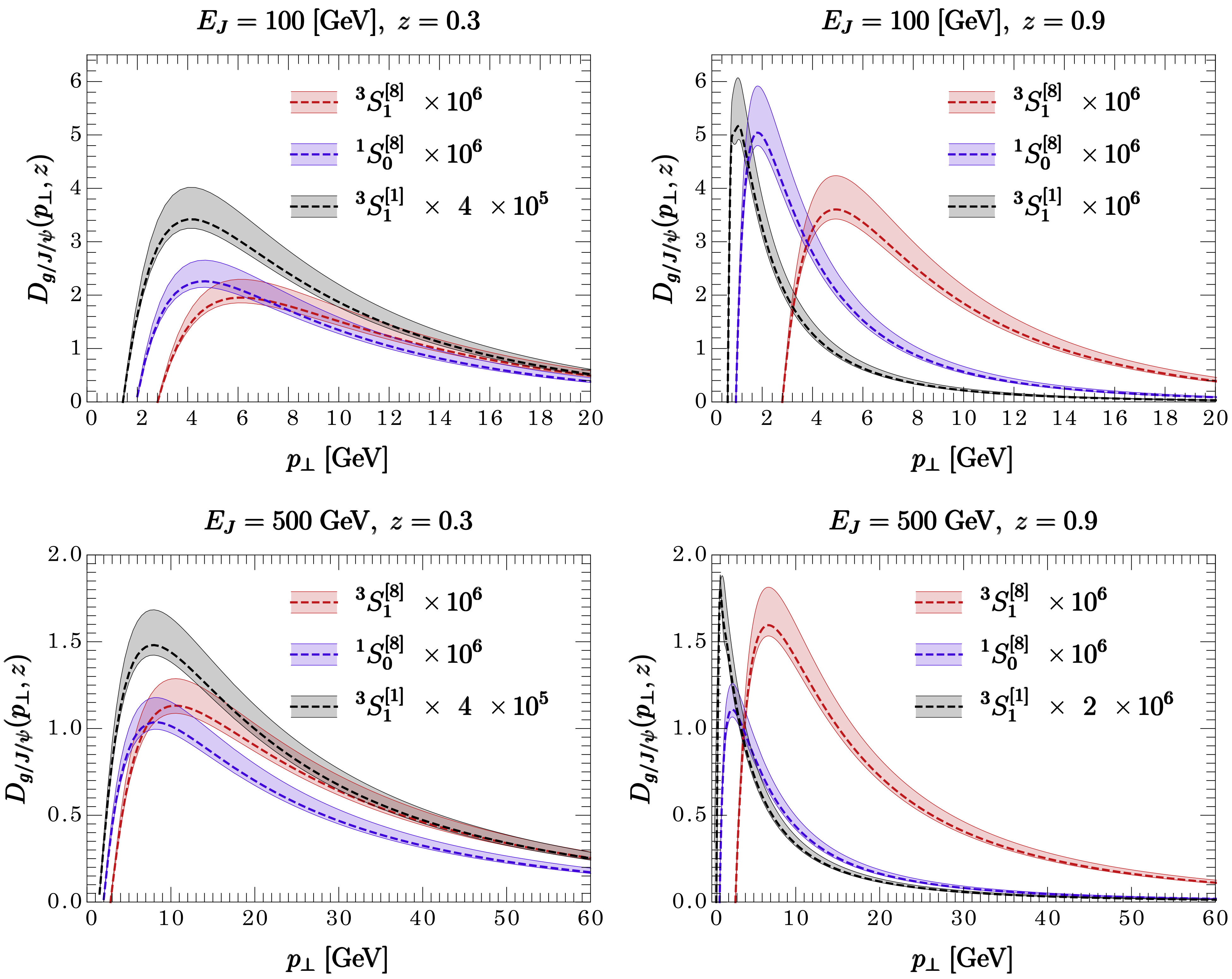


$$\mu = \omega r$$

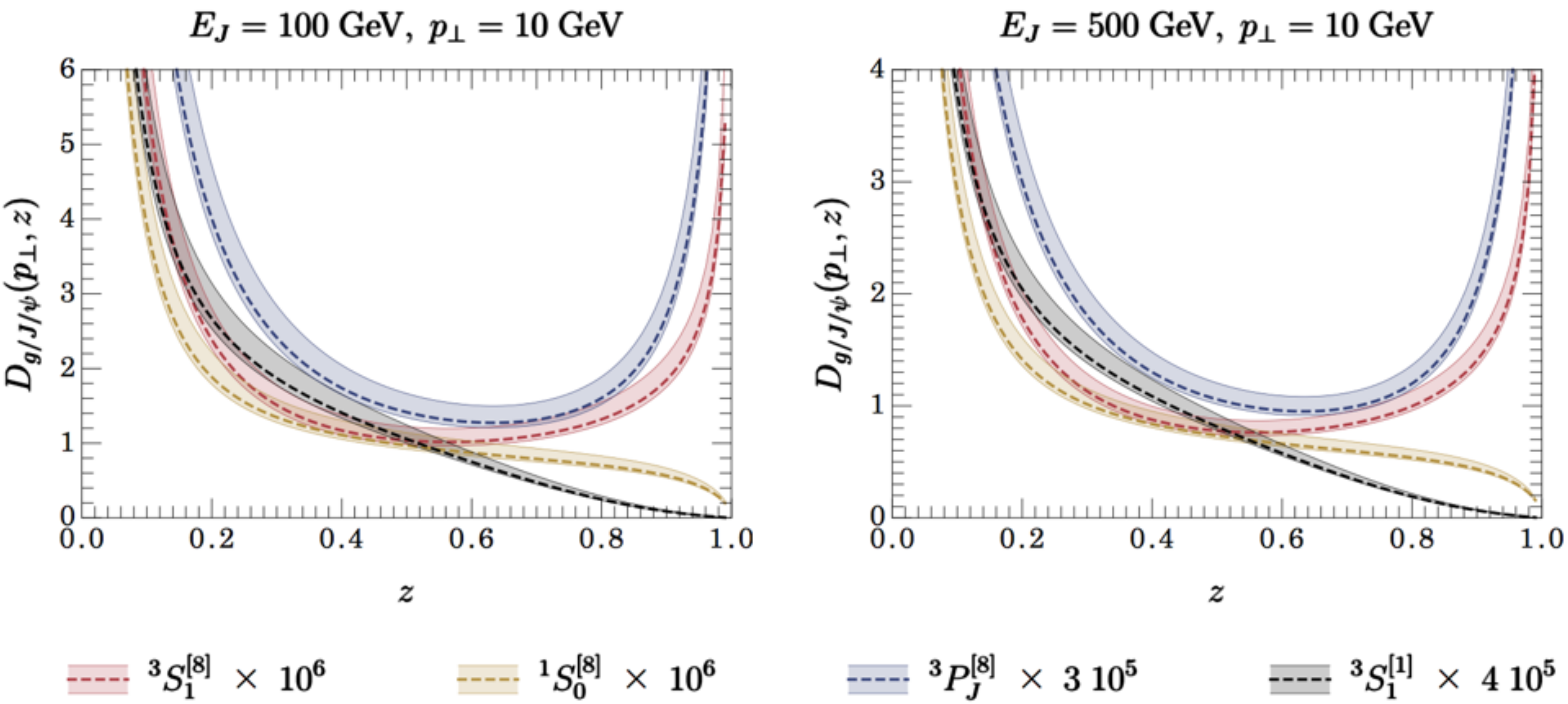
$$\mu_{\mathcal{D}} = \mu_{S_C} = \mu_c(b) = 2e^{-\gamma_E}/b$$

$$\nu_{S_C} = \mu_c(b)/r \qquad \nu_{\mathcal{D}} = \omega$$

Application to Quarkonium Production

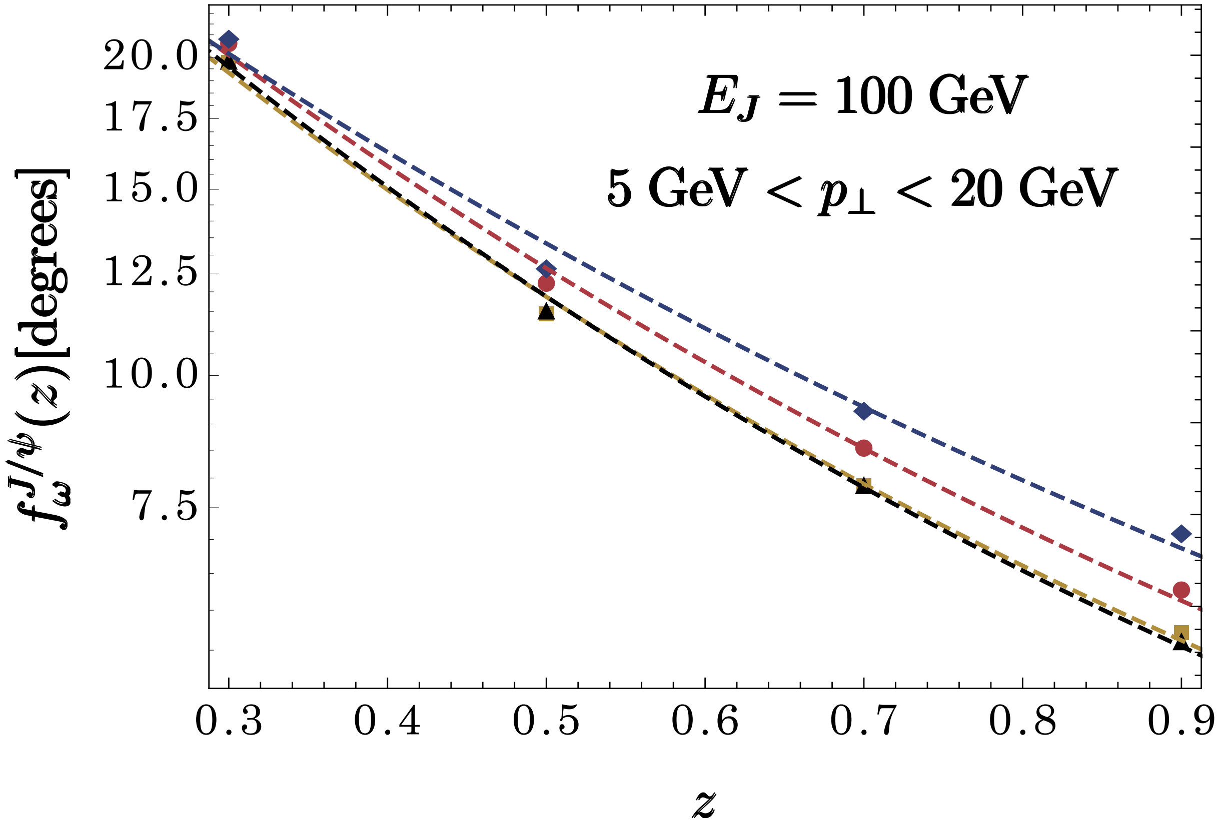


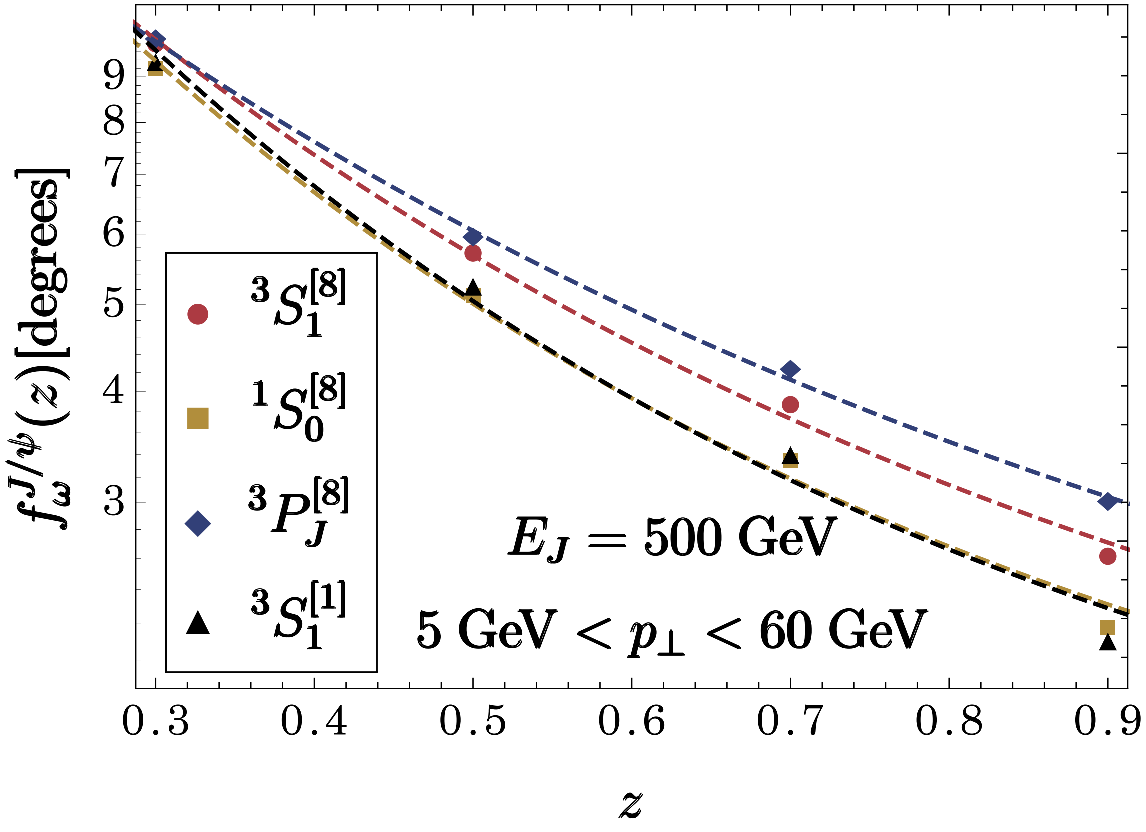
Application to Quarkonium Production

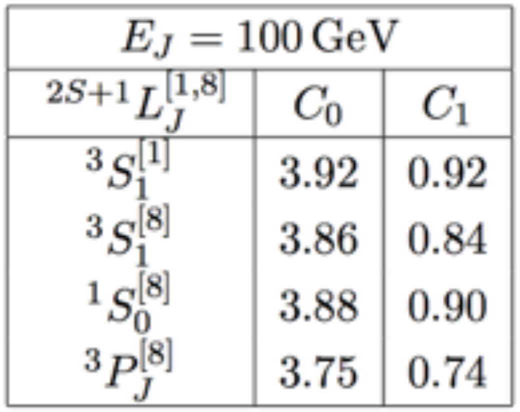


Application to Quarkonium Production

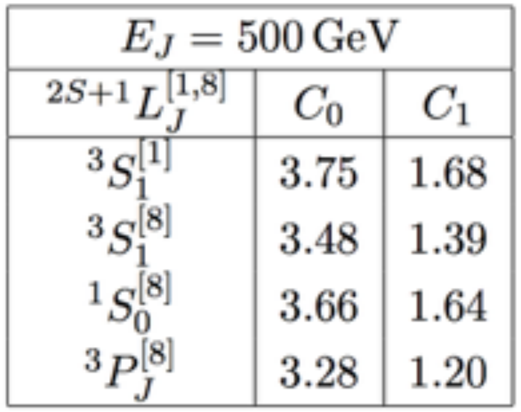
$$\langle \theta \rangle (z) \sim \frac{2\int dp_\perp\, p_\perp D_{g/h}(p_\perp, z, \mu)}{z\omega \int dp_\perp\, D_{g/h}(p_\perp, z, \mu)} \equiv f^h_\omega(z)$$







$E_J = 100$ GeV		
${}^{2S+1}L_J^{[1,8]}$	C_0	C_1
${}^3S_1^{[1]}$	3.92	0.92
${}^3S_1^{[8]}$	3.86	0.84
${}^1S_0^{[8]}$	3.88	0.90
${}^3P_J^{[8]}$	3.75	0.74



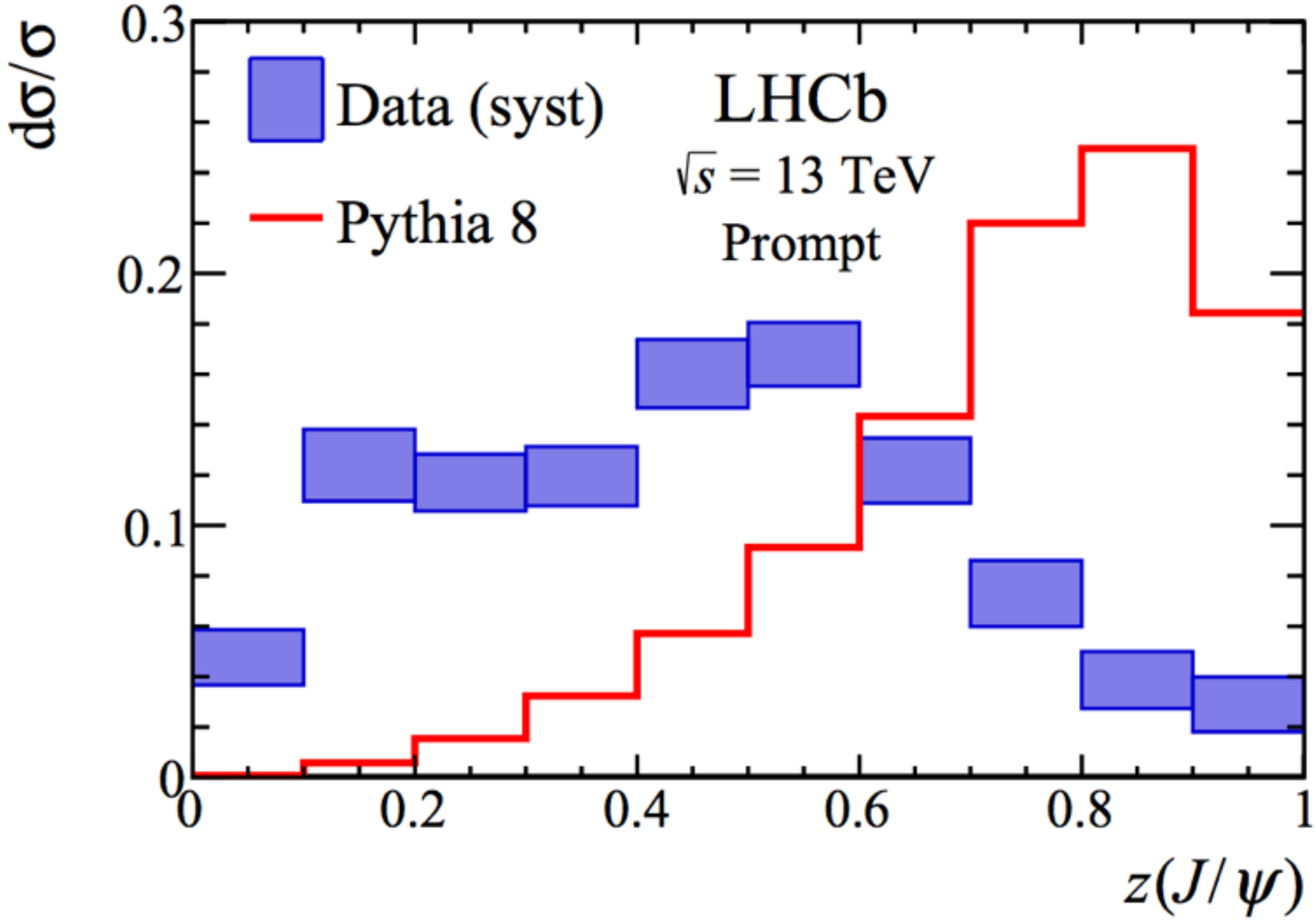
$E_J = 500$ GeV		
${}^{2S+1}L_J^{[1,8]}$	C_0	C_1
${}^3S_1^{[1]}$	3.75	1.68
${}^3S_1^{[8]}$	3.48	1.39
${}^1S_0^{[8]}$	3.66	1.64
${}^3P_J^{[8]}$	3.28	1.20

$$\ln(f(x)) = g(x; C_0, C_1) \text{ s.t. } g(x=0) = C_0$$

$$g_2(x) = C_0 \exp(-C_1 x)$$

Recent Observations of Quarkonia within Jets

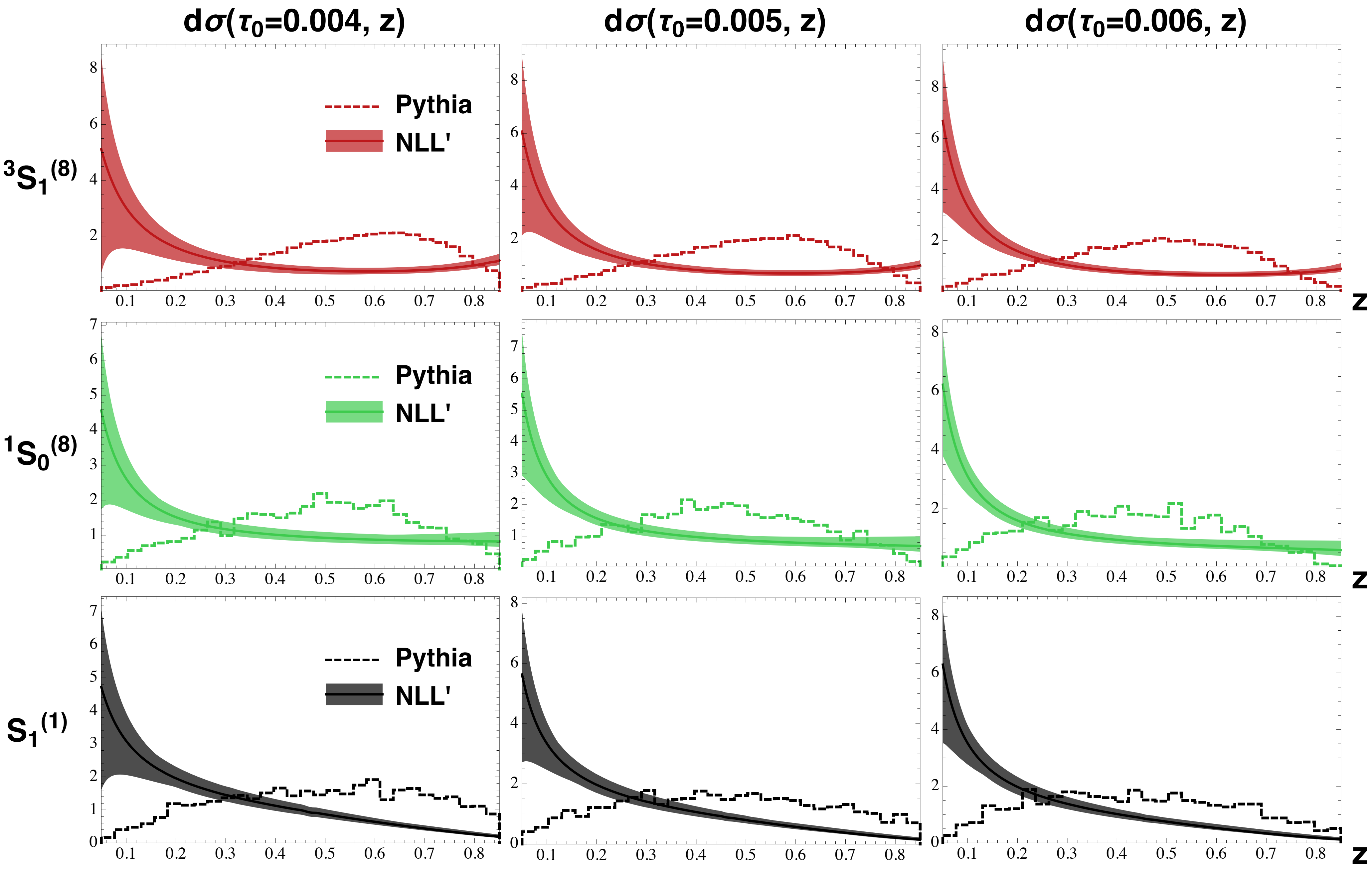
LHCb collaboration, arXiv:1701.05116



cuts: $2.5 < \eta_{\mathrm{jet}} < 4.0 \quad p_{T,jet} > 20\,\mathrm{GeV} \quad p(\mu) > 5\,\mathrm{GeV}$

NLL’ FJF vs. Pythia

R. Bain, L. Dai, A. Hornig, A. K. Leibovich, Y. Makris, T. Mehen JHEP 1606 (2016) 121

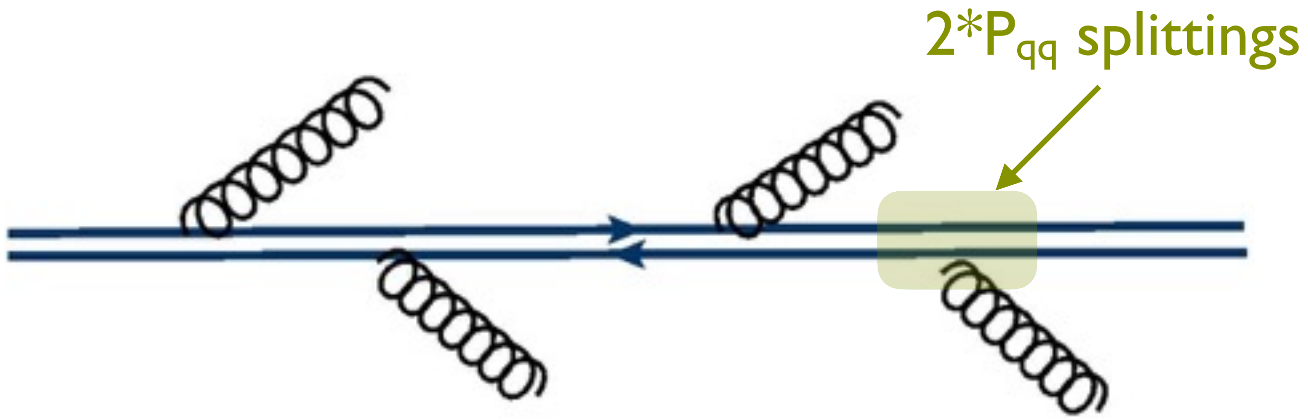


$e^+e^- \to \bar{q}qg$ $\quad E_{CM} = 250\,\mathrm{GeV} \quad \tau_0 = s/\omega^2$

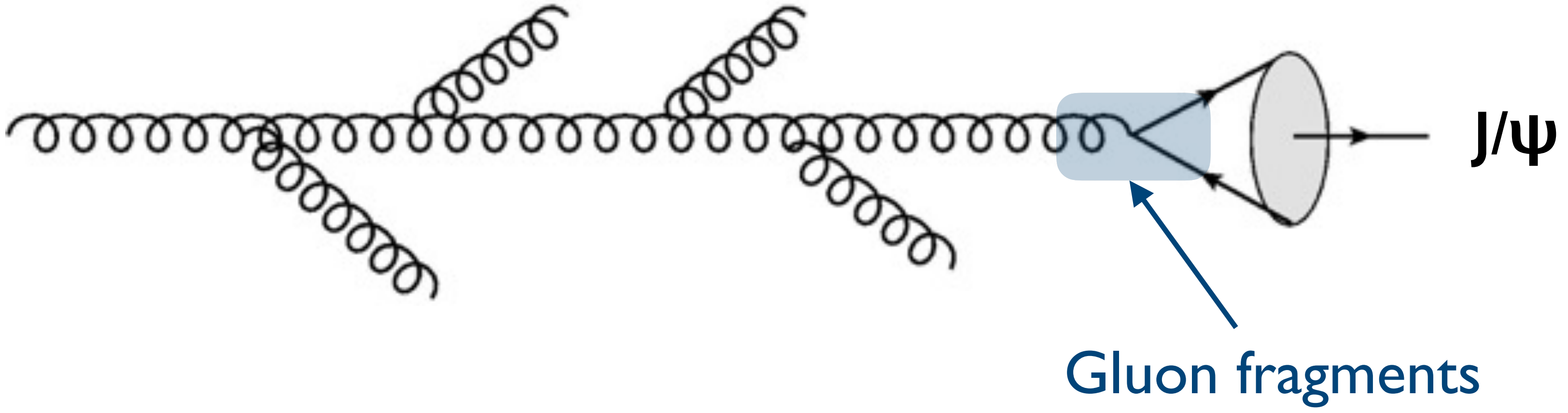
↳ jet w/ J/ψ

Explaining difference between NLL’ vs Pythia

PYTHIA’s model for showering color-octēt cc pairs:



Physical picture of analytical calculation

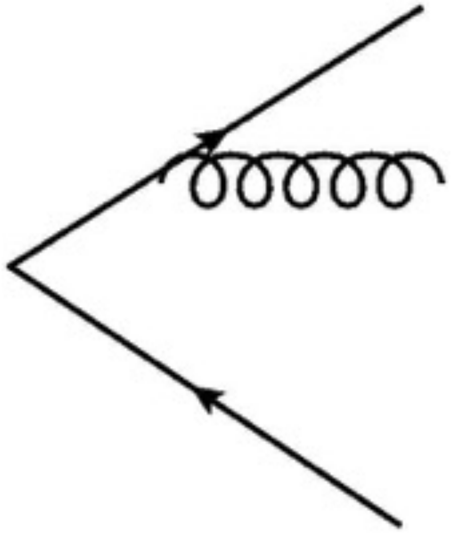


Pythia z distributions much harder than NLL’ calculations

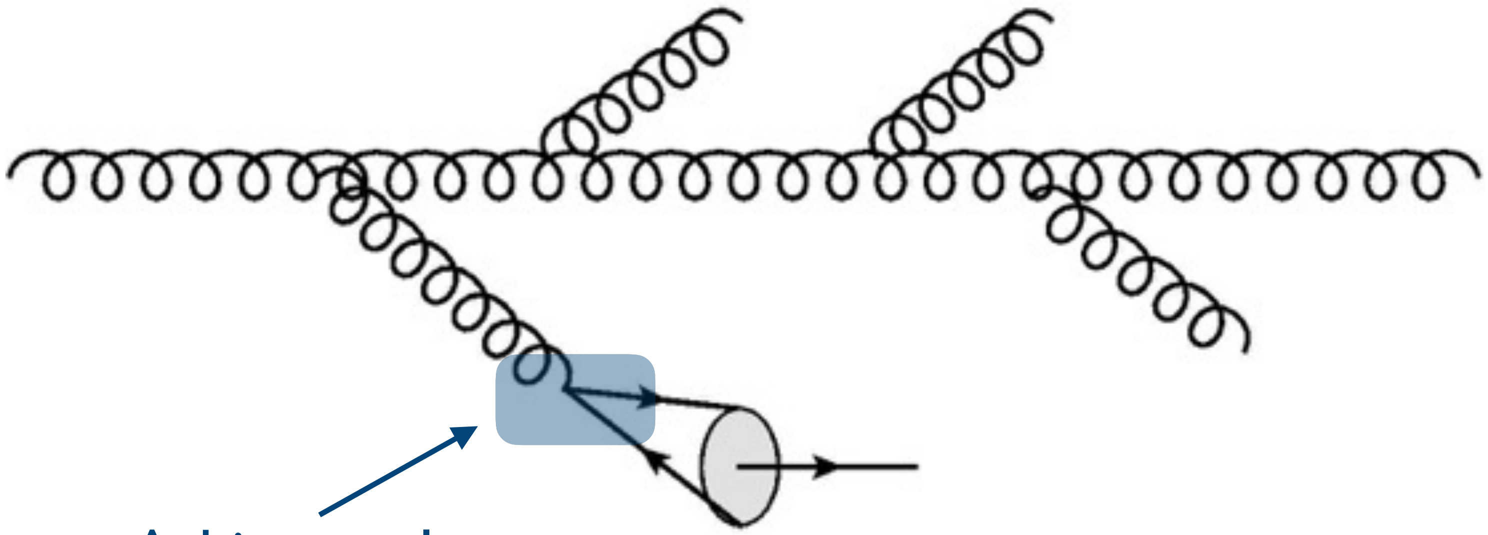
Gluon Fragmentation Improved PYTHIA (GFIP)

Madgraph 5

$$e^+e^- \to b\,\bar{b}\,g$$



PYTHIA + Convolution

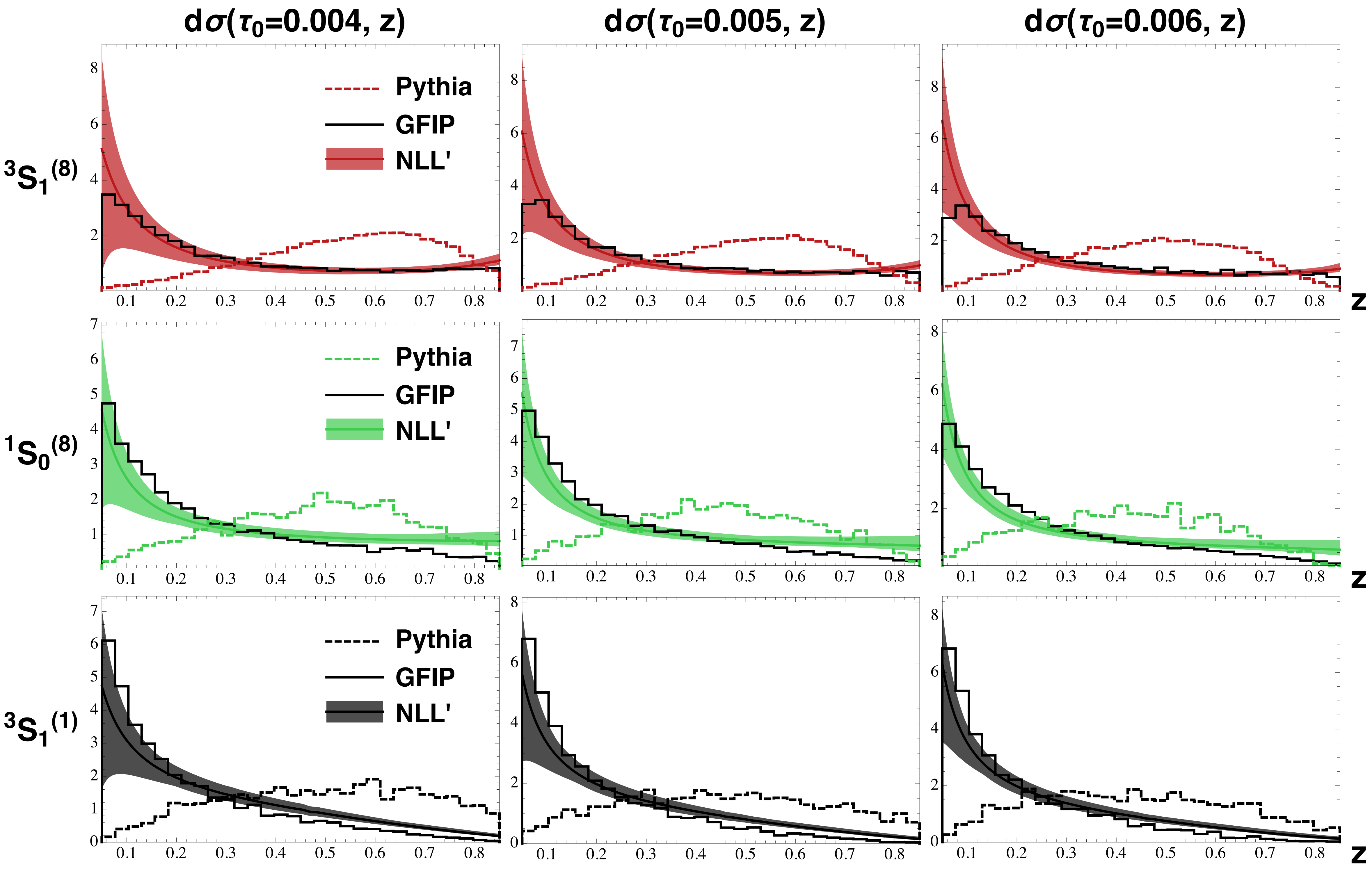


Arbitrary gluon fragments

shower gluon with PYTHIA down to scale ~2m_c, no hadronization

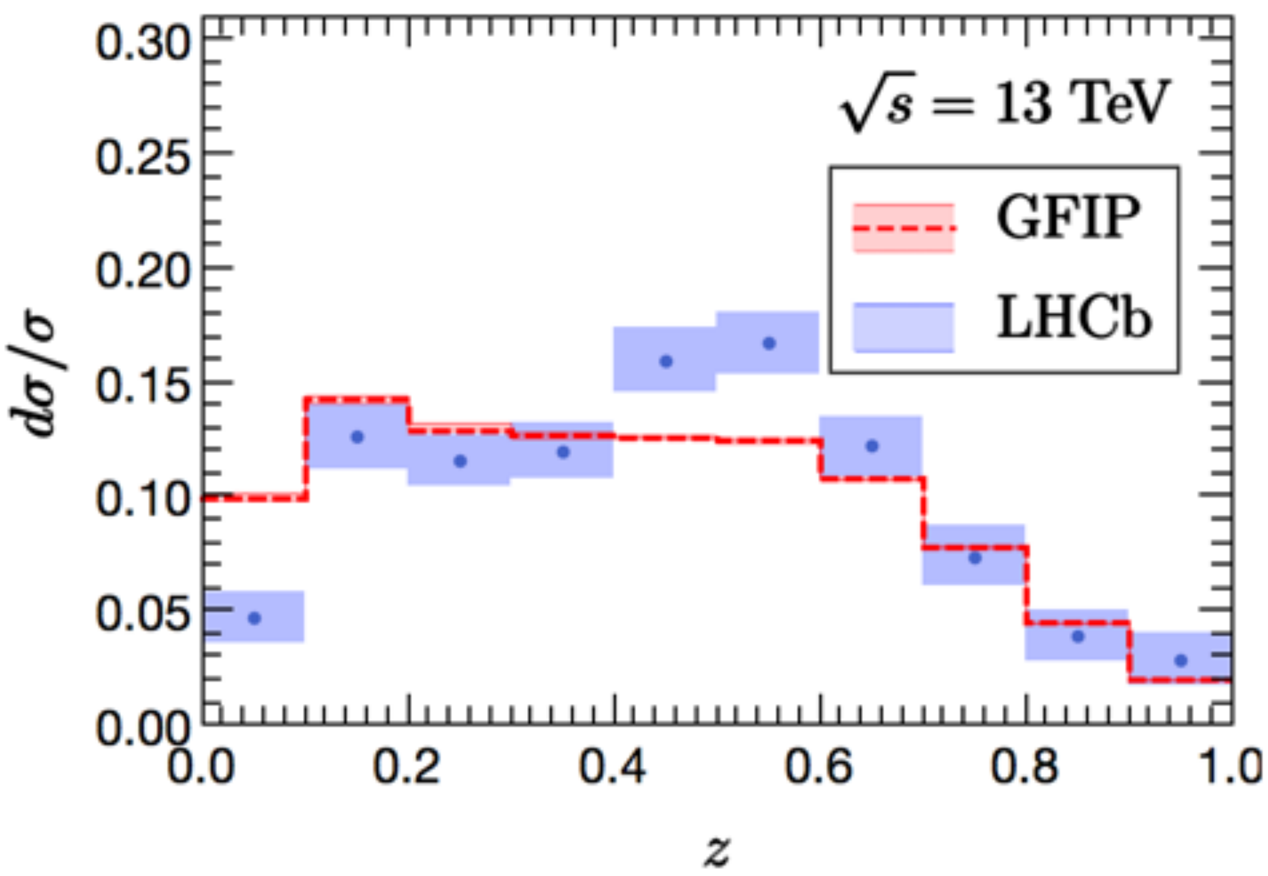
convolve final state gluon distribution w/ NRQCD FFs

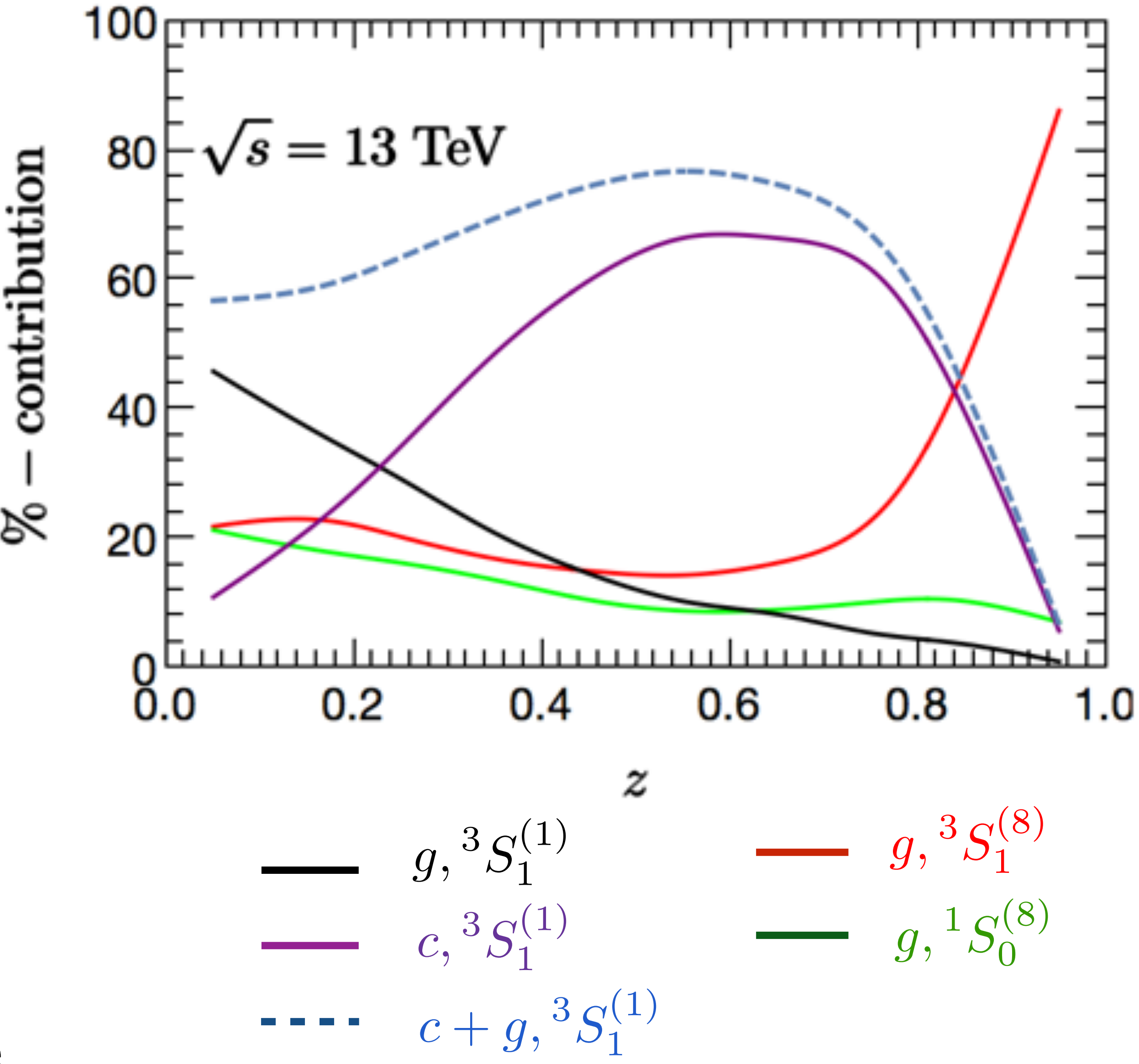
NLL', PYTHIA, and GFIP



GFIP and Recent LHCb Observations

R. Bain, L. Dai, A. K. Leibovich, Y. Makris, T. Mehen, to appear





LDME from Global fit (no P-wave)

color singlet g, c fragmentation dominate

weak sensitivity to color-octet

NRQCD: good agreement with data

Conclusions

measuring $Q\bar{Q}$ within jets, and using jet observables should provide insights into $Q\bar{Q}$ production

If ${}^1S_0^{(8)}$ mechanism dominates high p_T production FJF should have negative slope for z(E), for z>0.5

p_T-dependent quarkonium fragmenting jet functions (TMDFJFs)

p_T, theta of quarkonium in jet sensitive to NRQCD production mechanism

Preliminary analysis of recent LHCb data

Backup

fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{\mathrm{d}x^+}{4\pi}\, e^{ik^- x^+/2}\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \, \langle 0 | \not{\bar{n}}\, \Psi(x^+, 0, 0_\perp) | X h \rangle \langle X h | \bar{\Psi}(0) | 0 \rangle \Big|_{p_h^\perp = 0}$$

fragmentation function (SCET)

$$D_q^h\Big(\frac{p_h^-}{\omega}, \mu\Big) = \pi\omega \int dp_h^+\, \frac{1}{4N_c}\, \mathrm{Tr} \sum_X \not{\bar{n}}\, \langle 0 | [\delta_{\omega,\bar{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(0)] | X h \rangle \langle X h | \bar{\chi}_n(0) | 0 \rangle$$

Jet function (SCET)

$$J_u(k^+\omega) = -\frac{1}{\pi\,\omega}\, \mathrm{Im} \int d^4x\, e^{ik\cdot x}\, i\, \langle 0 \big| \mathrm{T}\, \bar{\chi}_{n,\omega,0_\perp}(0)\, \frac{\not{\bar{n}}}{4N_c}\, \chi_n(x) \big| 0 \rangle$$

fragmentation jet function (SCET)

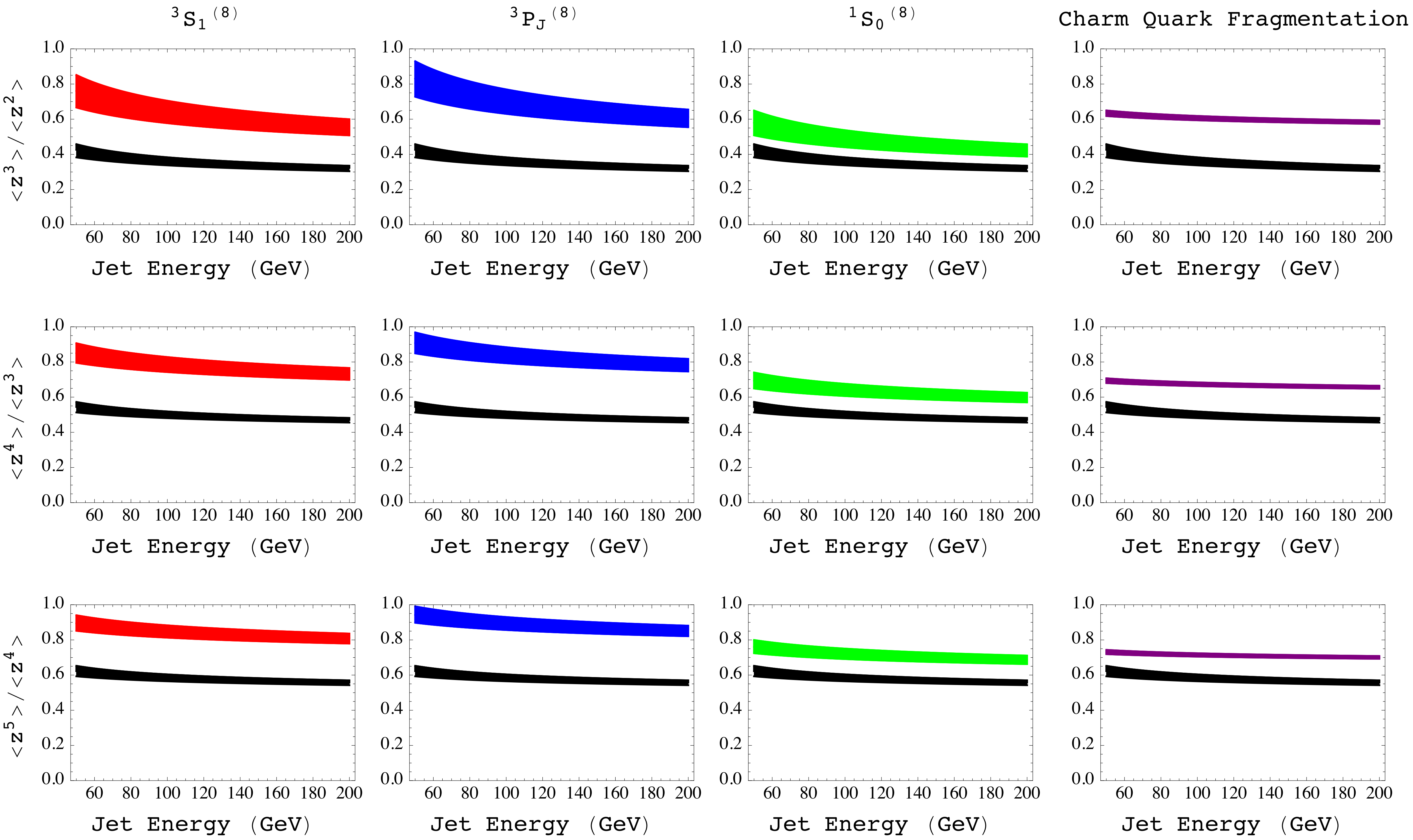
$$\mathcal{G}_{q,\mathrm{bare}}^h(s, z) = \int \mathrm{d}^4y\, e^{\mathrm{i}k^+ y^-/2} \int \mathrm{d}p_h^+ \sum_X \frac{1}{4N_c}\, \mathrm{tr}\left[\frac{\not{\bar{n}}}{2} \langle 0 | [\delta_{\omega,\overline{\mathcal{P}}}\, \delta_{0,\mathcal{P}_\perp}\, \chi_n(y)] \big| X h \rangle \langle X h | \bar{\chi}_n(0) | 0 \rangle\right]$$

$$\delta(p^+/z - P_H^+) \rightarrow \delta(p^+/z - P_H^+)\,\delta(p^- - s/p^+)$$

FF FJF

Ratios of Moments

$$E\tan(R/2) < \mu < 4E\tan(R/2)$$

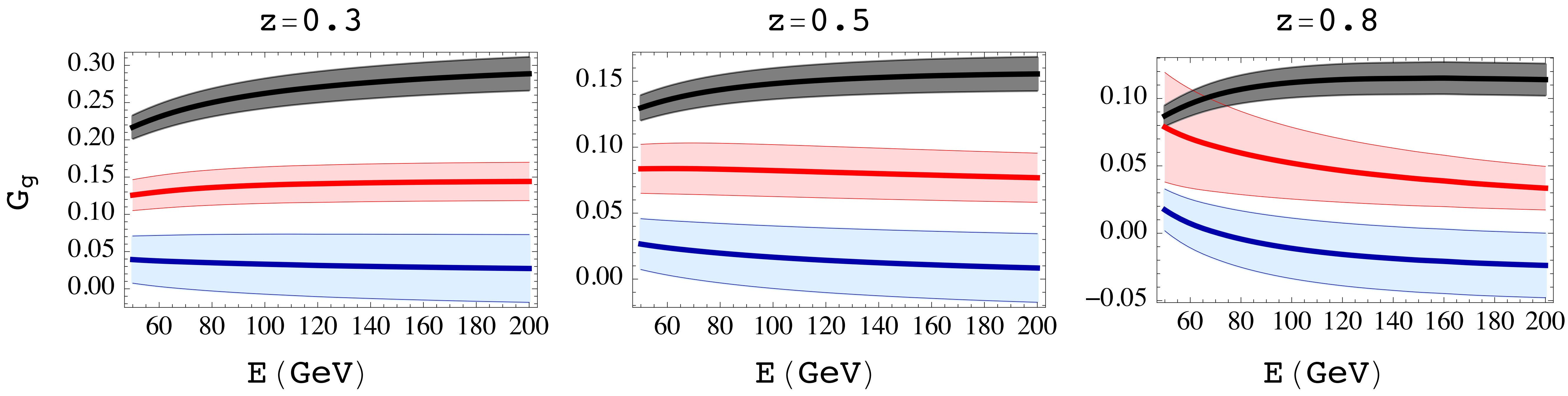


Ratios of Moments

$$\left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^3P_J^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^3S_1^{(8)}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^1S_0^{(8)}} \approx \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{\mathrm{c-quark}} > \left.\frac{\langle z^{n+1}\rangle}{\langle z^n\rangle}\right|_{^3S_1^{(1)}}$$

Gluon FJF for different extractions of LDME

fix z, vary energy



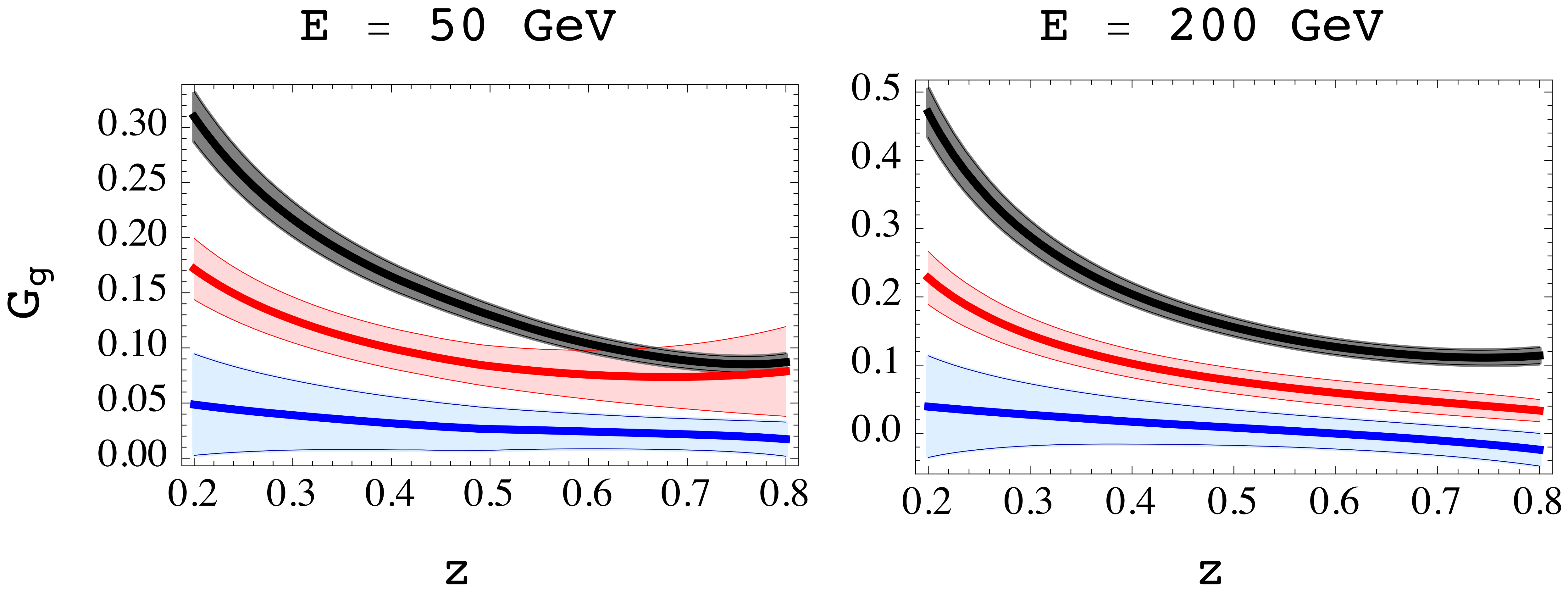
— Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822

— Bodwin, et. al. arXiv:1403.3612

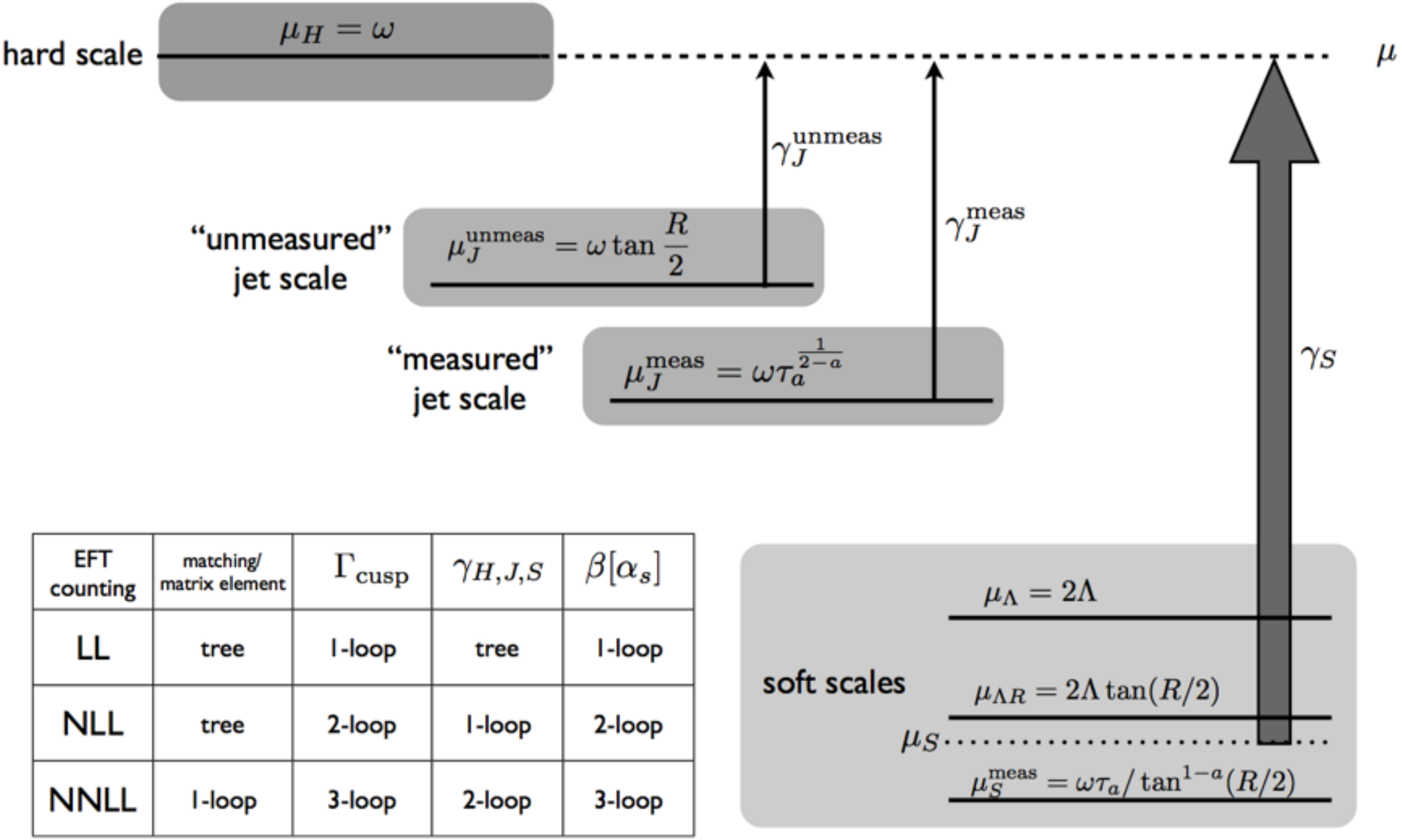
— Chao, et. al. PRL 108, 242004 (2012)

Gluon FJF for different extractions of LDME

fix energy, vary z



Scales in Jet Cross section



hard scale

$\mu_H = \omega$

μ

γ_J^{unmeas}

γ_J^{meas}

"unmeasured" jet scale

$\mu_J^{\text{unmeas}} = \omega \tan \frac{R}{2}$

"measured" jet scale

$\mu_J^{\text{meas}} = \omega \tau_a^{\frac{1}{2-a}}$

γ_S

soft scales

$\mu_\Lambda = 2\Lambda$

$\mu_{\Lambda R} = 2\Lambda \tan(R/2)$

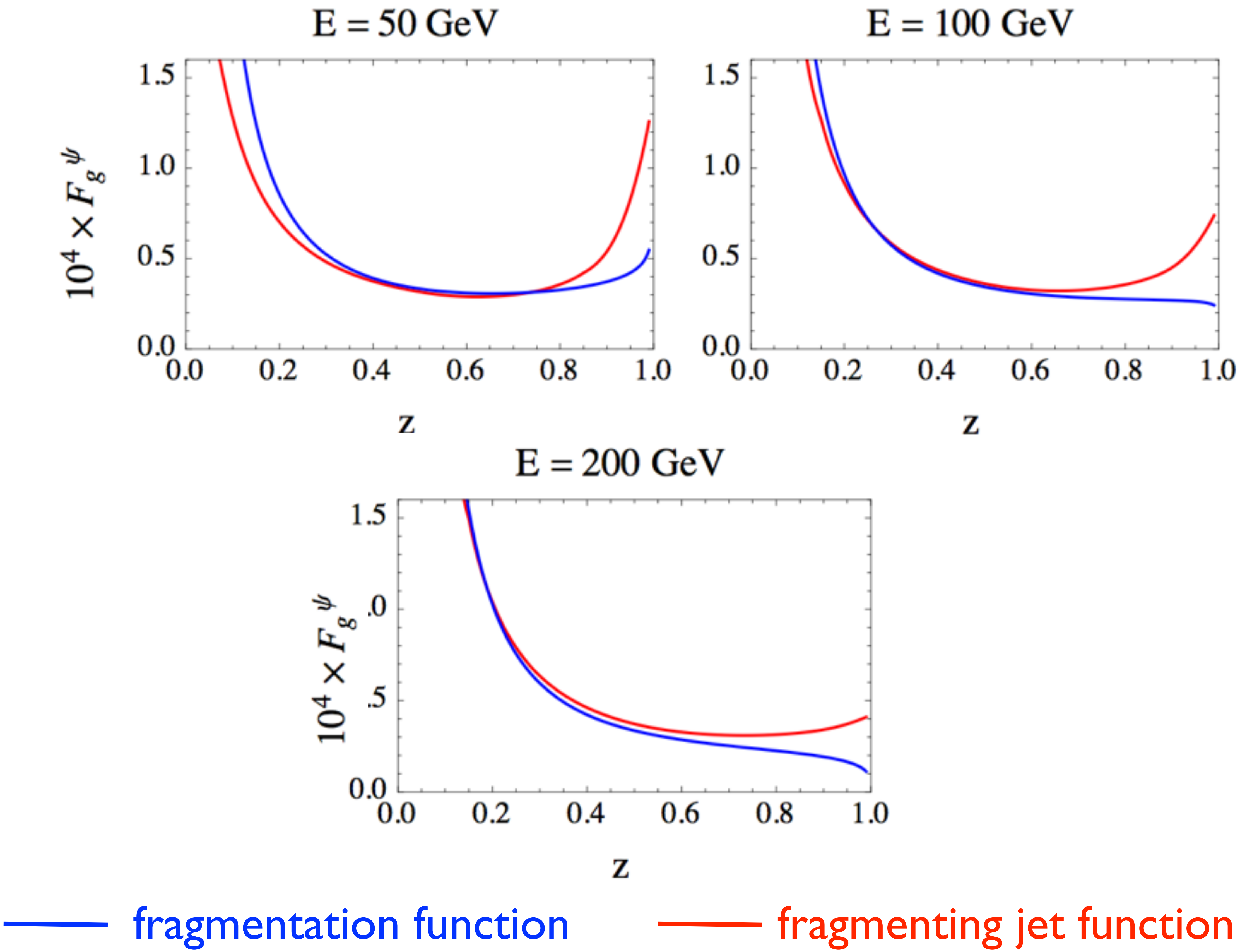
μ_S

$\mu_S^{\text{meas}} = \omega \tau_a / \tan^{1-a}(R/2)$

EFT counting	matching/ matrix element	Γ_{cusp}	$\gamma_{H,J,S}$	$\beta[\alpha_s]$
LL	tree	1-loop	tree	1-loop
NLL	tree	2-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	3-loop

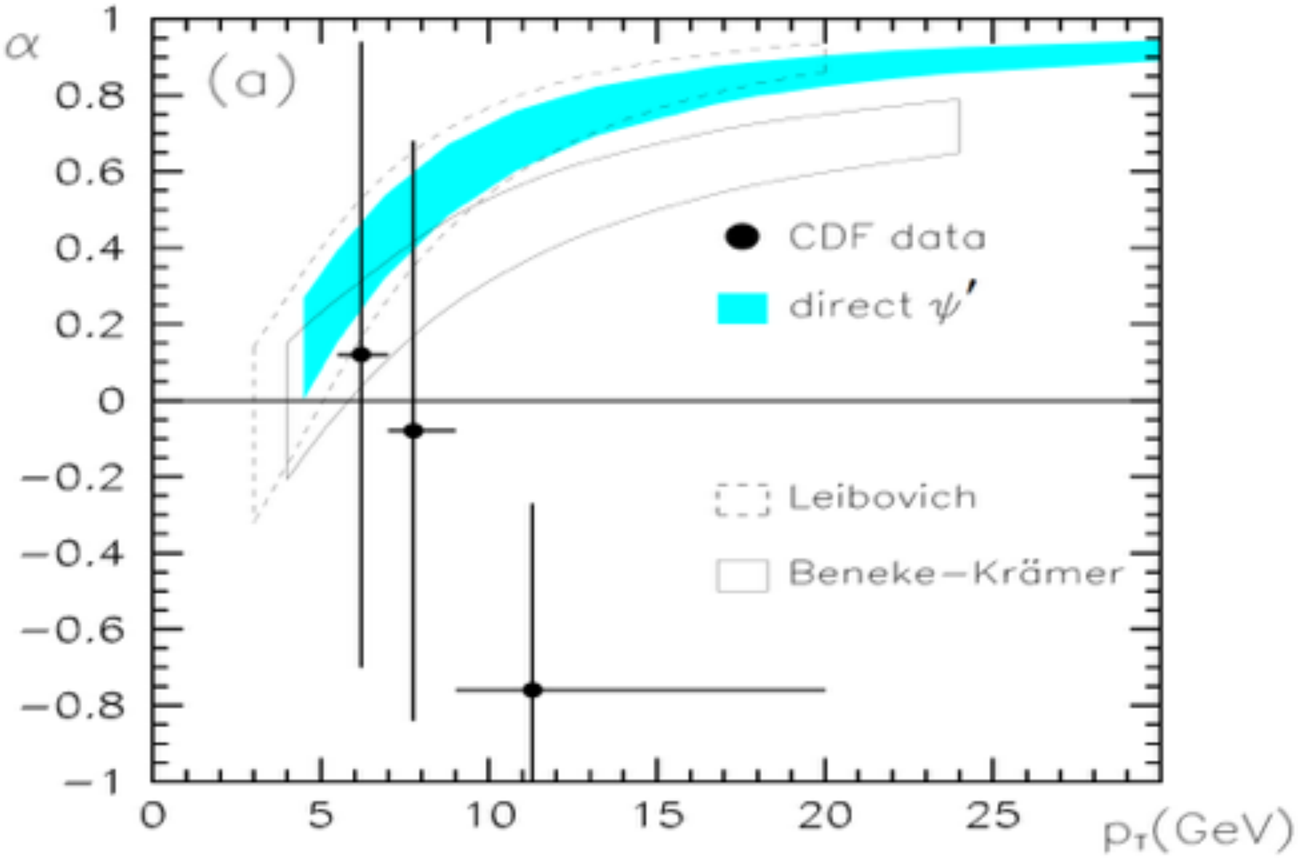
Color-Octet $^3S_1$ fragmentation function, FJF

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

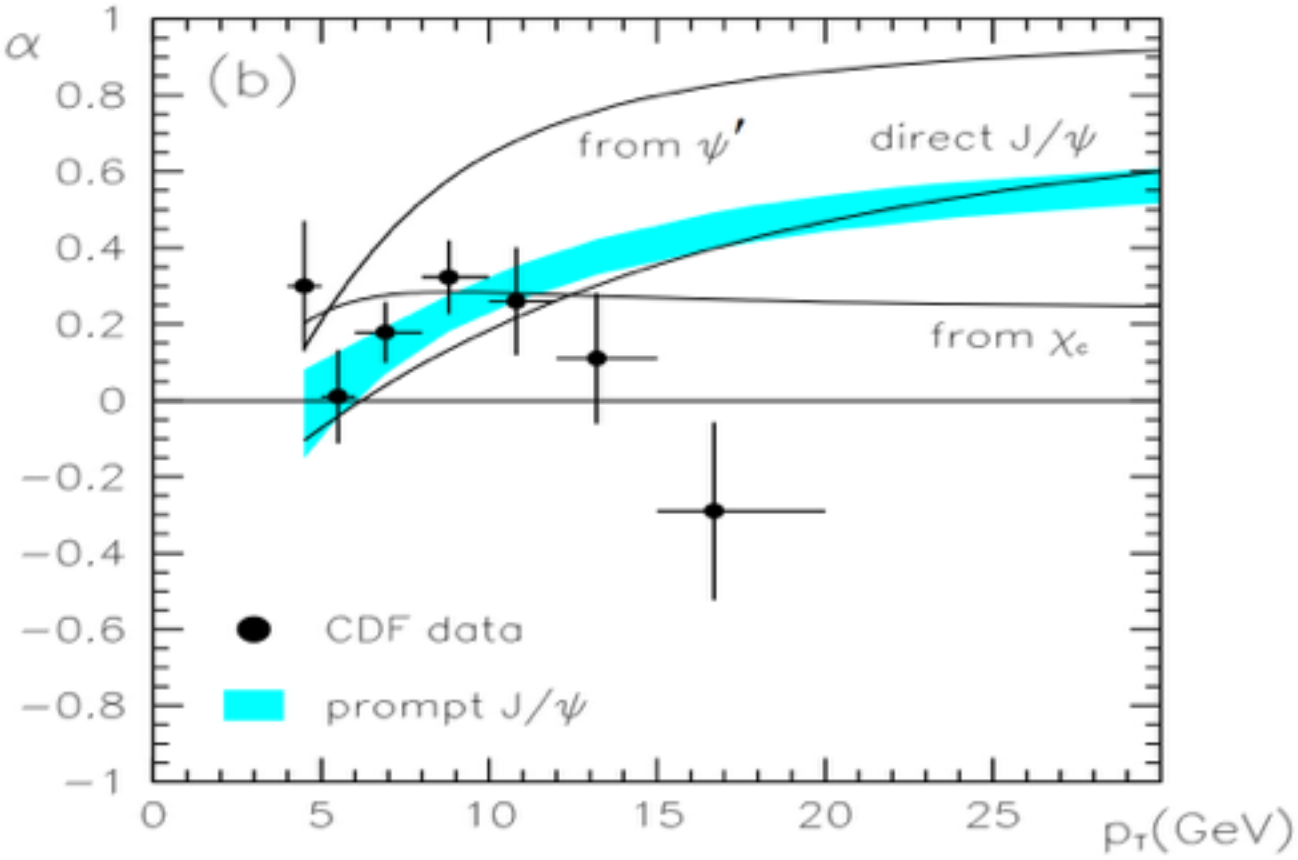


Polarization Puzzle

${}^3S_1^{[8]}$ fragmentation at large p_T predicts transversely polarized J/ψ, ψ’



ψ ′



J/ψ

Braaten, Kniehl, Lee, 1999

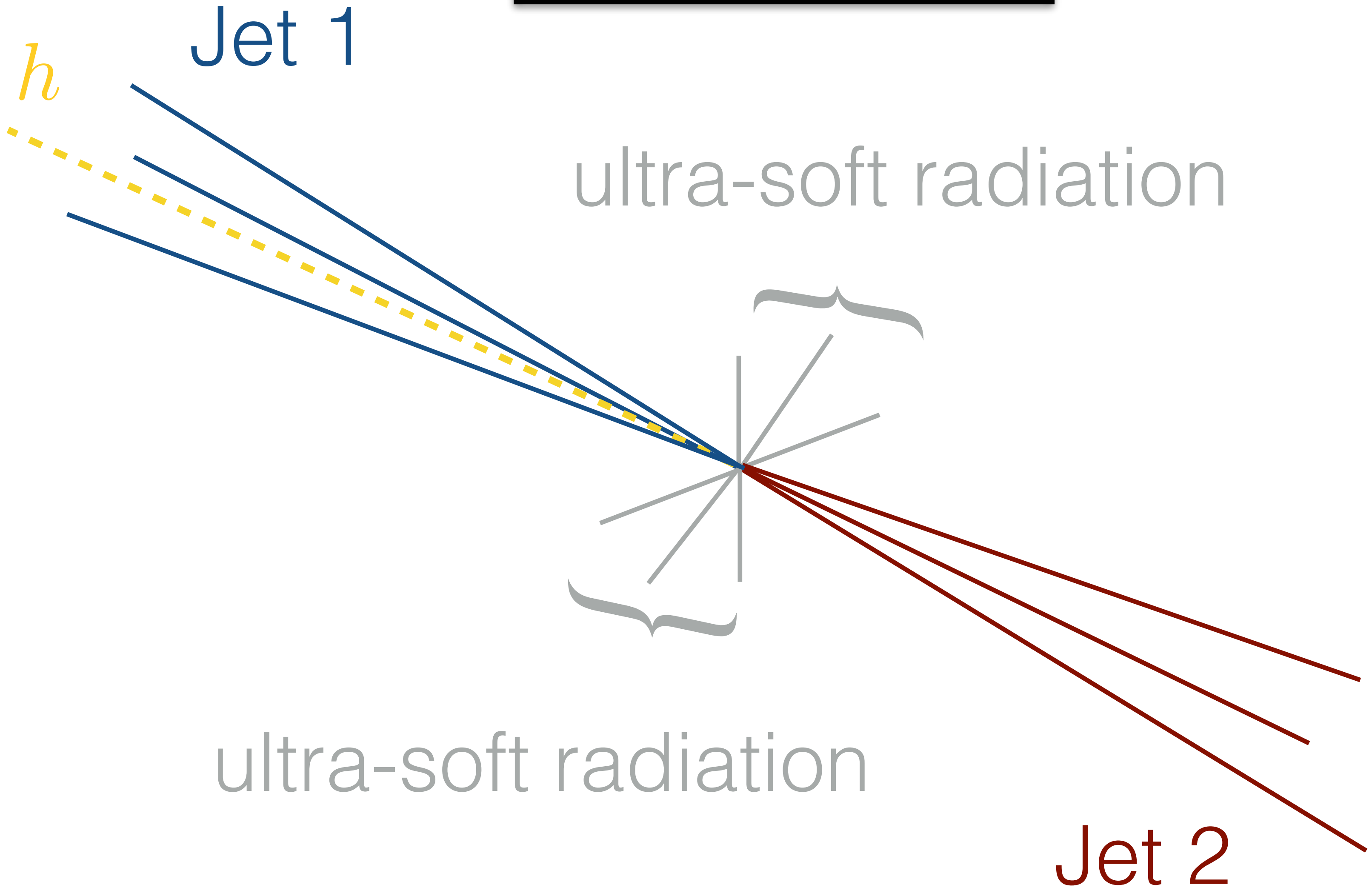
$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = \frac{1}{z} \sum_X \frac{1}{2N_c} \delta(p^-_{Xh;r}) \delta^{(2)}(\mathbf{p}_\perp + \mathbf{p}^X_\perp) \, \mathrm{Tr} \Big[\frac{\bar{\not{n}}}{2} \langle 0 | \delta_{\omega, \overline{\mathcal{P}}} \chi_n^{(0)}(0) | Xh \rangle \langle Xh | \bar{\chi}_n^{(0)}(0) | 0 \rangle \Big]$$

$$\int d^2\mathbf{p}^h_\perp \; D_{q/h}(\mathbf{p}^h_\perp, z, \mu) = D_{q/h}(z, \mu)$$

Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

$$\boxed{\omega \gg p_h^{\perp} \gg \Lambda}$$



$$D_{i/h}\left(z, p_h^{\perp}, \mu\right)$$

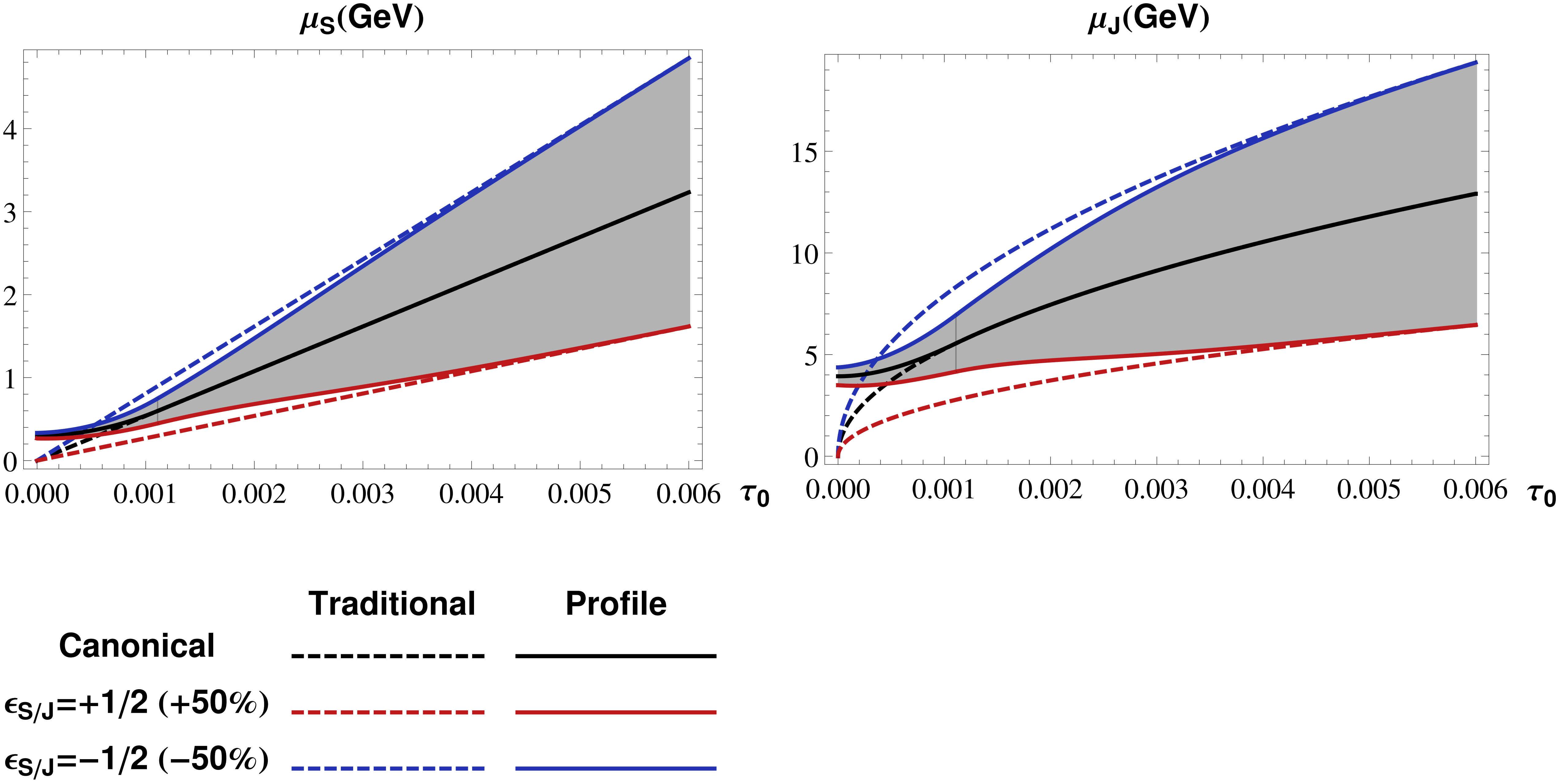
$$p_c \sim \omega(\lambda^2, 1, \lambda)$$

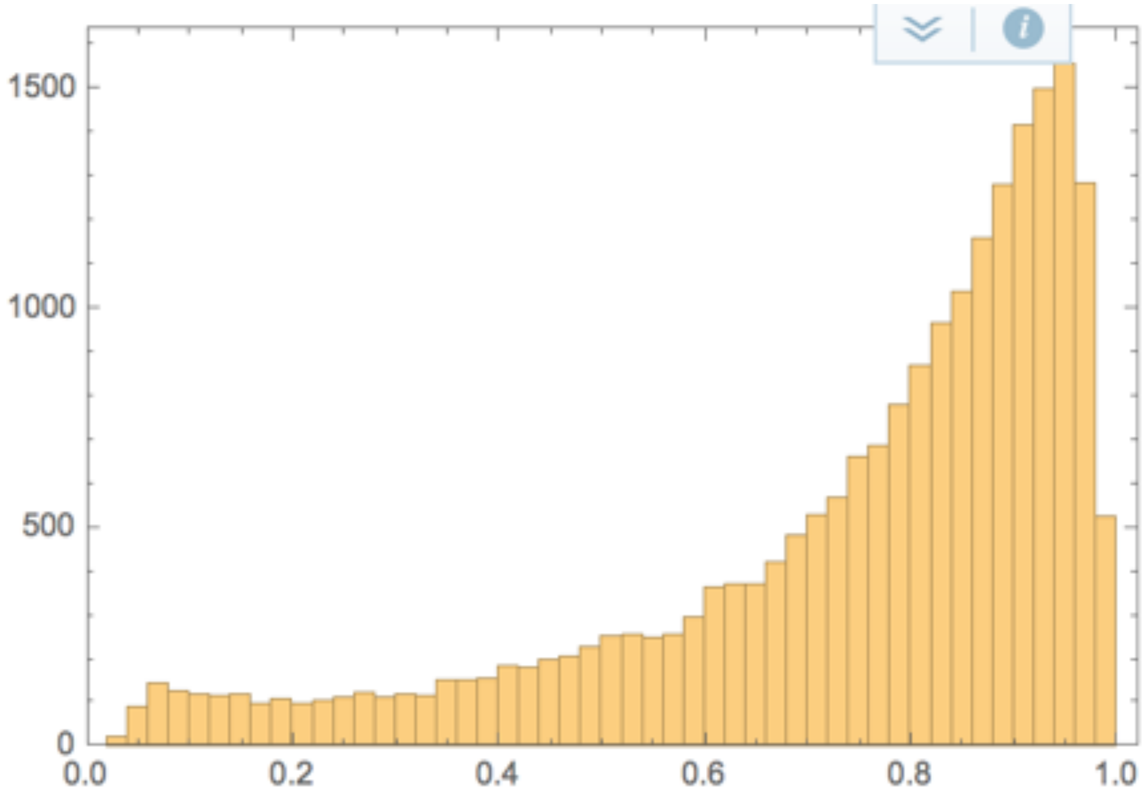
$$p_{cs} \sim p_h^{\perp}(r, 1/r, 1)$$

$$p_{us} \sim \Lambda(1, 1, 1)$$

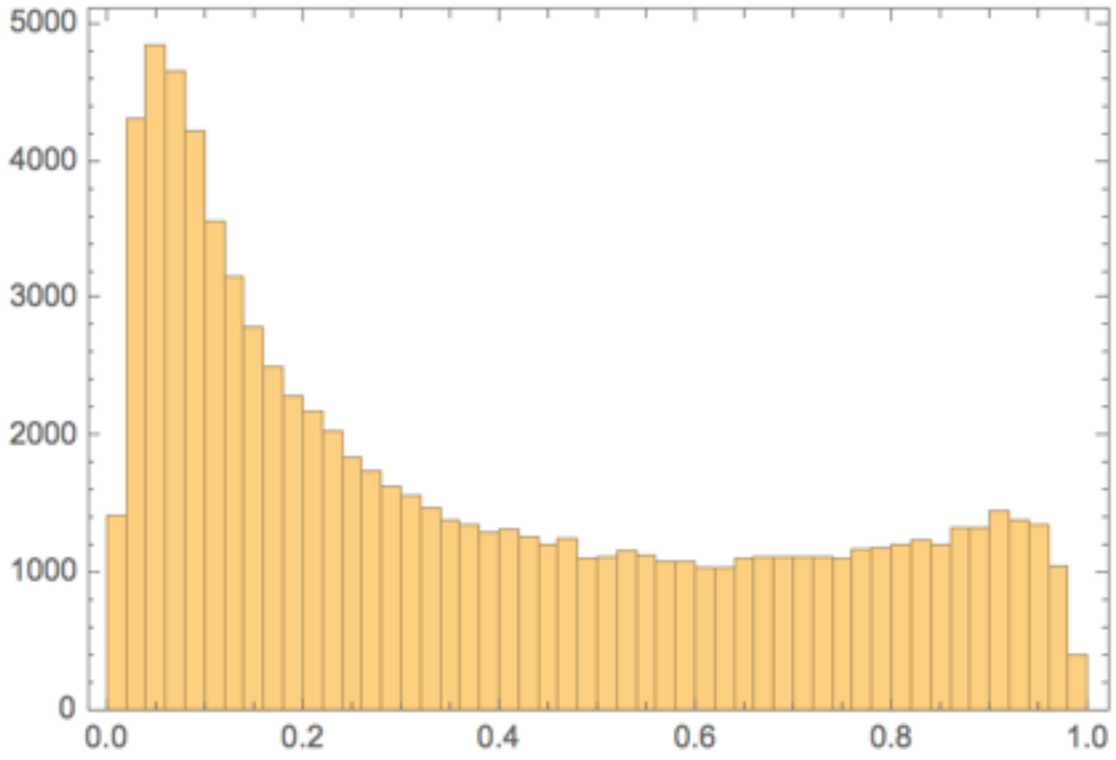
$$\lambda = p_h^{\perp}/\omega$$

Profile Functions





c distribution



g distribution