Recent results on quasi parton distributions

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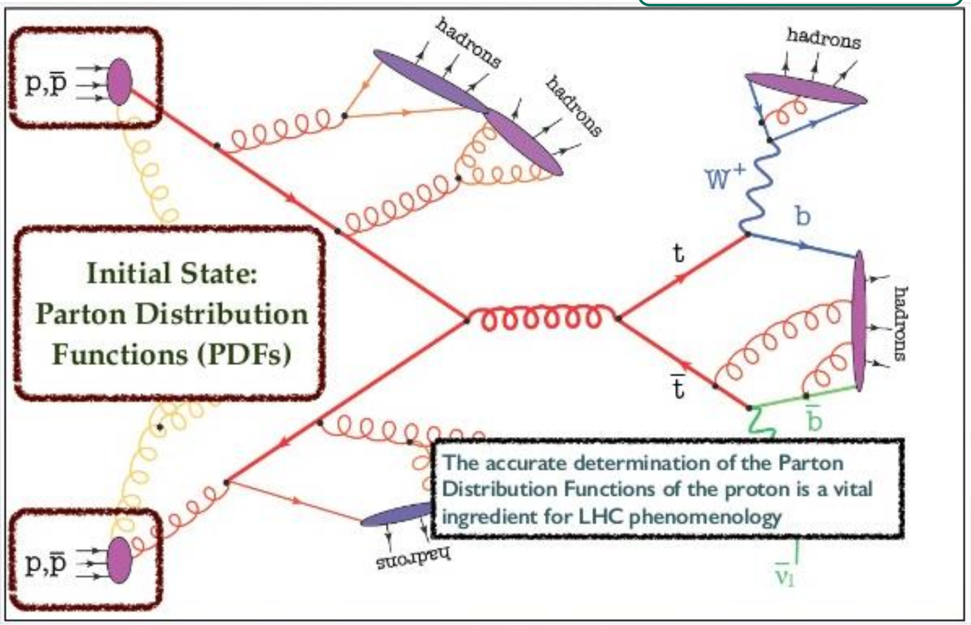
Parton distribution functions

J. Rojo's at SLAC, 2015

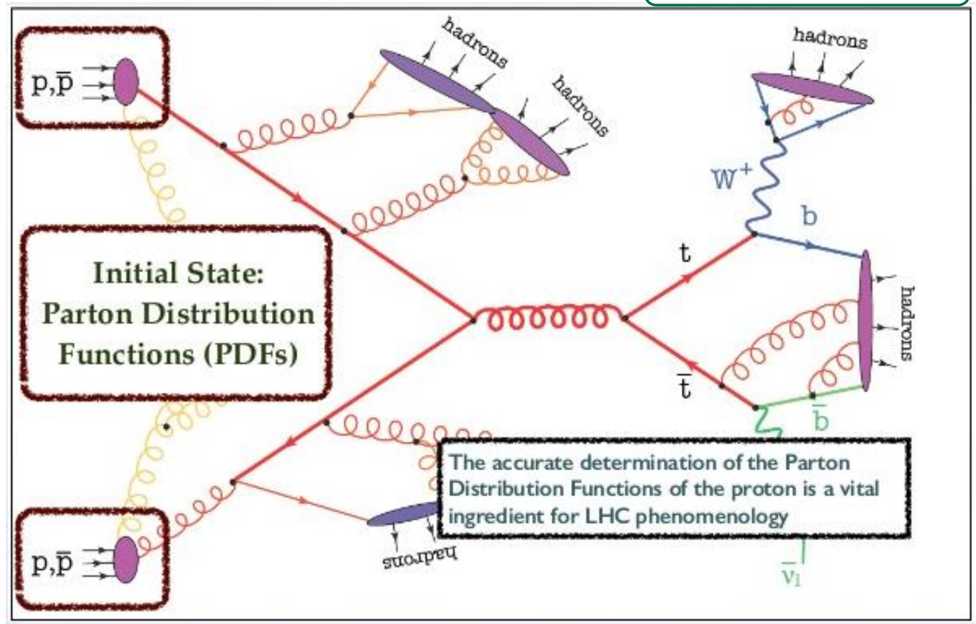
- Characterize momentum distribution of quark and gluon partons inside the hadron
- Important inputs for high-energy hadron colliders



Parton distribution functions

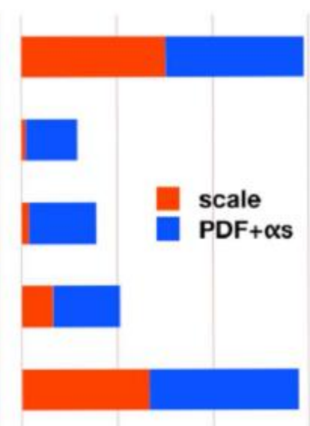
J. Rojo's at SLAC, 2015

- Characterize momentum distribution of quark and gluon partons inside the hadron
- Important inputs for high-energy hadron colliders
- Induce large uncertainties in Higgs production processes at the LHC and also in other processes helping understand SM and disentangle new physics effects
- A better understanding and precise determination desirable



HIGGS PRODUCTION

	Process	σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
NNLO QCD +NLO EW	VBF	1.56 pb	2.9%
	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%



(J. Campbell, HCP2012)

Parton distribution functions

- PDFs are intrinsically non-perturbative quantities defined on the light-cone

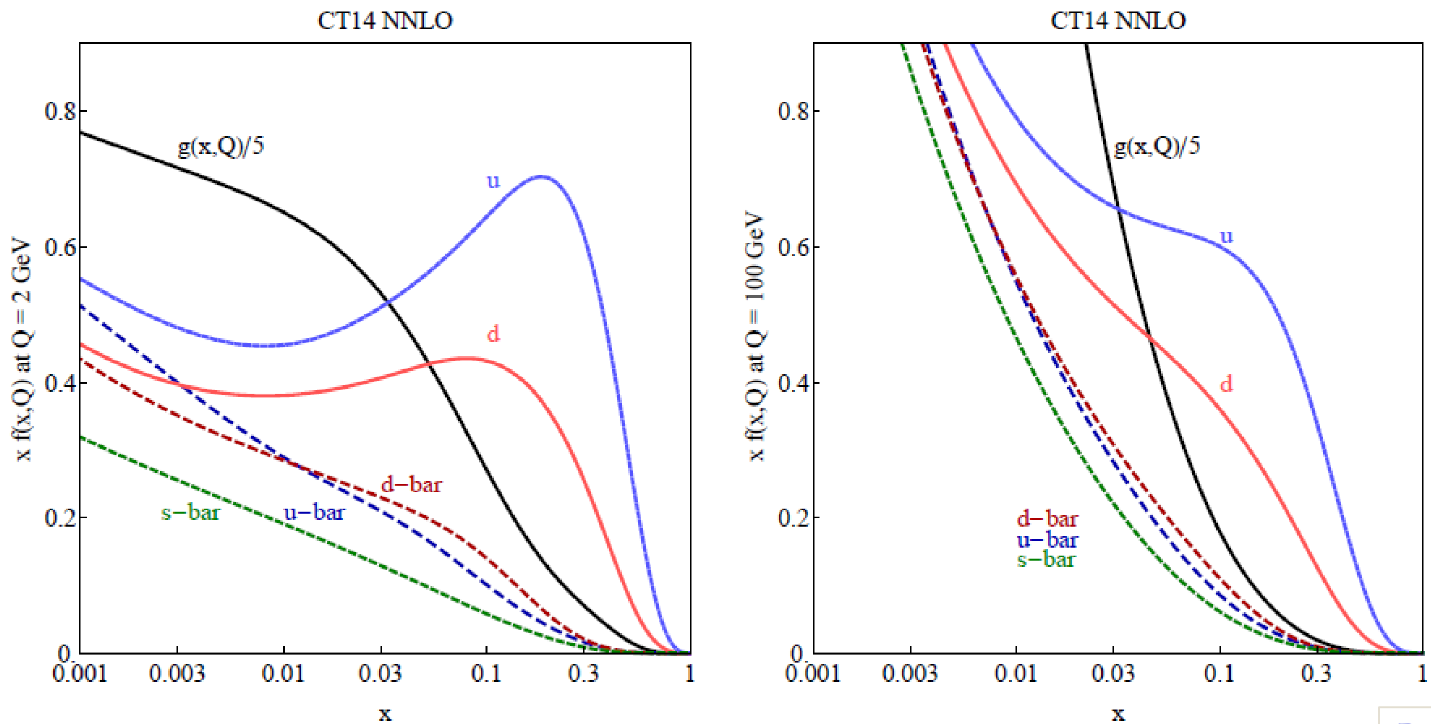
  - Hadrons can be viewed as constituted by point-like partons in high energy collisions
  - Operator definition (unpolarized quark distribution)

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

  - $P^\mu = (P^0, 0, 0, P^z), \xi^\pm = (t \pm z)/\sqrt{2}$
  - Light-cone correlation at equal light-front time, expectation of light-front quark number operator in the light-cone gauge

  - There has been long term effort on determining PDFs
    - CTEQ, NNPDF, MSTW…
    - Using a large variety of experimental data from DIS, Drell-Yan to jet production

Parton distribution functions



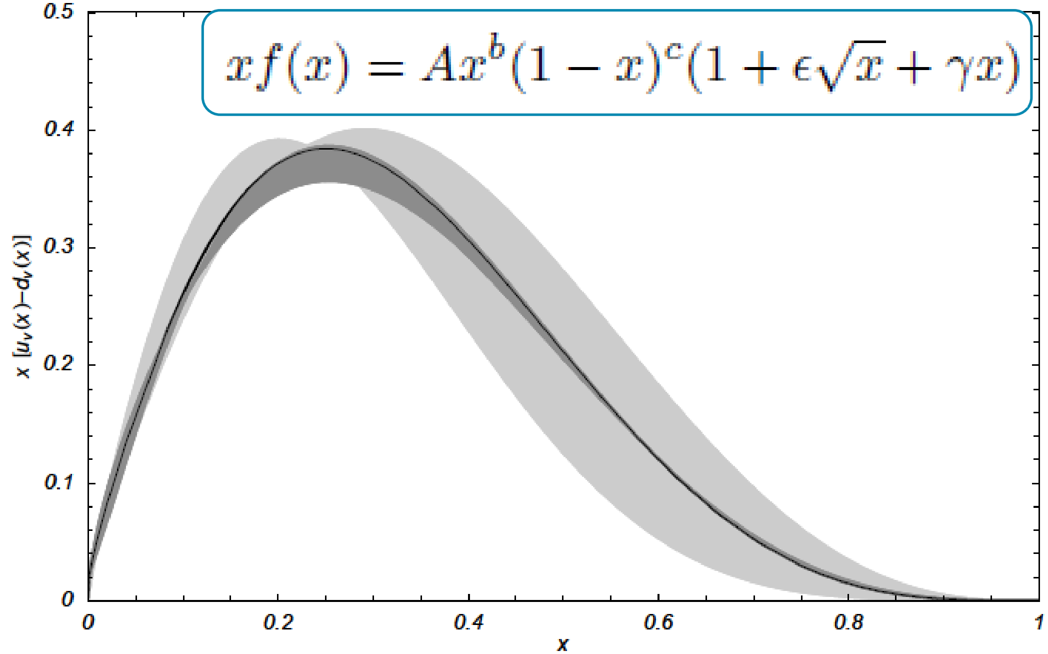
Dulat et. al.
arxiv: 1506.07443

FIG. 4: The CT14 parton distribution functions at $Q = 2$ GeV and $Q = 100$ GeV for $u, \overline{u}, d, \overline{d}, s = \overline{s}$, and g.

$$x\, f_a(x, Q_0) \;=\; x^{a_1}\,(1-x)^{a_2}\,P_a(x)$$

- CTEQ
  - Parametrize x-dependence of PDFs at a low scale
  - Fitting parameters to experimental data
  - DGLAP scale evolution

Parton distribution functions



Detmold et. al.
Mod.Phys.Lett.A, 03'

Fig. 5. Reconstructed isovector valence quark distribution $x(u_v - d_v)$ in the proton at $Q^2 = 4\ \text{GeV}^2$. The central fit curve (solid line) and error band (lightly shaded) are compared with the envelope of the phenomenological distributions[20] (darkly shaded).

- Lattice QCD (Euclidean approach, cannot directly access light-cone quantities such as the PDFs)
  - Compute their moments, which are local operator matrix elements
  - Parametrize PDFs with a smooth functional form and fit unknown parameters to moments
  - # of calculable moments limited due to mixing

Large momentum effective theory

- An effective theory framework that allows to compute light-cone or parton observables from Euclidean quantities [Ji 14′]

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- For a light-cone observable, e.g. the PDFs or the gluon helicity, construct a Euclidean quasi observable, which in general is frame-dependent, but approaches the light-cone observable in the infinite momentum limit

$$\tilde{O}(P,\mu) \stackrel{P\to\infty}{\longrightarrow} O(\mu)$$

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  - The choice for quasi observable is not unique, for example

$$A^0 \stackrel{P\to\infty}{\longrightarrow} A^+, \quad A^3 \stackrel{P\to\infty}{\longrightarrow} A^+$$

  - Instead of computing the light-cone observable directly, one can compute the quasi observable at a finite hadron momentum P. The difference between quasi and light-cone observables is in finite or infinite momentum, hence they shall have the same IR physics

Large momentum effective theory

- In general, both quasi and light-cone observables suffer from UV divergences. They have different UV behavior because of different order of limits (the infinite momentum limit and the UV regularization are not exchangeable)
  - Taking inf. mom. limit first $\rightarrow$ physical case, light-cone
  - Taking UV regularization first $\rightarrow$ frame dependent, but practically calculable

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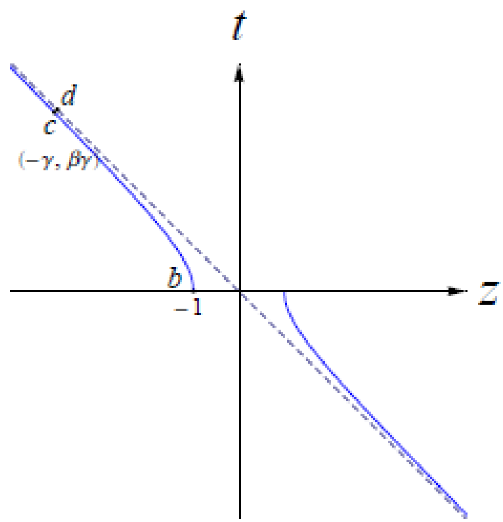
- Summary:
- Euclidean quasi observable is potentially computable on the lattice
- Perturbative matching allows to extract light-cone observable from the quasi observable
- A good approximation of the light-cone observable can be achieved at a moderately large momentum

Quasi parton distribution

$$\tilde{q}(x,\Lambda,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0,0_\perp,z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

- Equal time correlation, quark fields separated along z-direction, no time dependence and can be accessed on lattice, $x = k^z/P^z$ [Ji 13′]



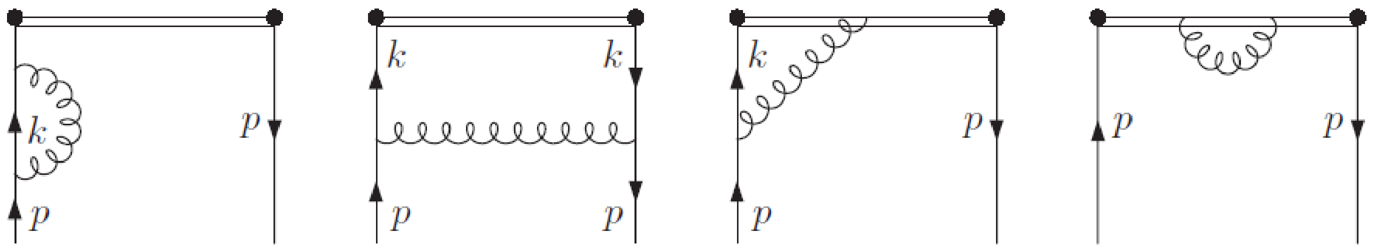
$$\begin{array}{lll} \text{(a)} & P_z = \text{finite}, & \lambda = (0,0,0,-1) \\ \text{(b)} & P_z \to \infty, & \lambda = (0,0,0,-1) \\ \text{(c)} & P_z = 0, & \lambda = (\beta\gamma,0,0,-\gamma) \text{ with } \beta \to 1 \\ \text{(d)} & P_z = 0, & \lambda^2 = 0 \\ & \text{or } P_z \to \infty, & \lambda^2 = 0. \end{array}$$

Quasi parton distribution

- Matching relation [Ji 13', Ji, Xiong, Zhang and Zhao, 13']]

$$\tilde{q}(x,\Lambda,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) q(y,\mu)$$

  - @NLO



  - The quasi distribution does not vanish outside [-1,1]
  - Both distributions have soft and coll. div., soft div. cancels in themselves, coll. div. identical

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, \qquad \xi > 1$$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln\left[4\xi(1-\xi)\right] - \frac{2\xi}{1-\xi} + 1 + \frac{\Lambda}{(1-\xi)^2 P^z}, \qquad 0 < \xi < 1$$

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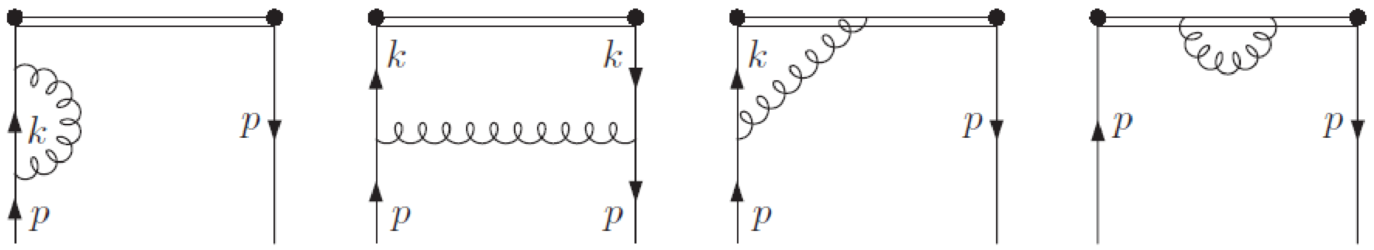

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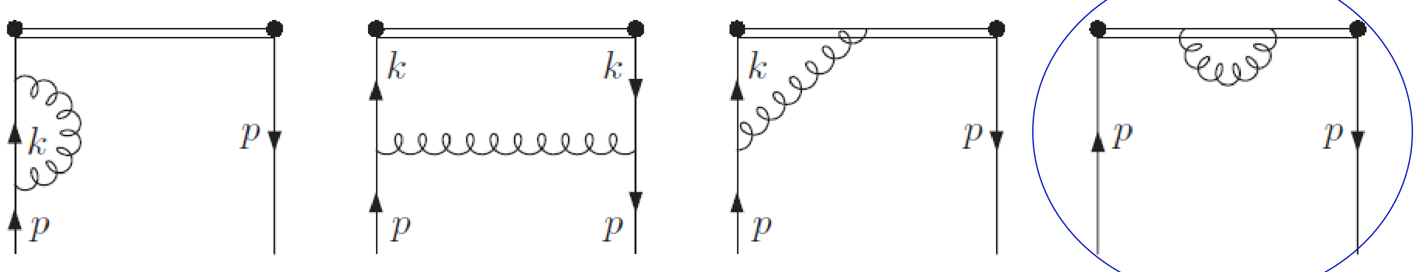

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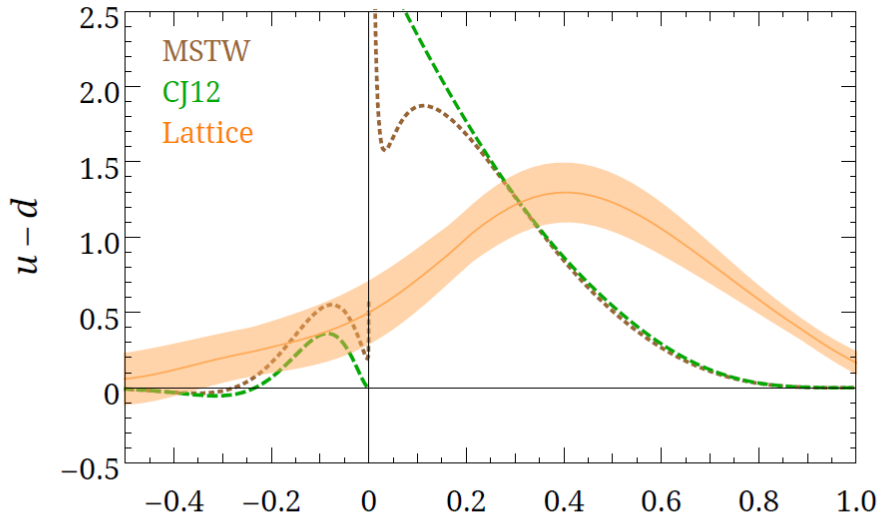
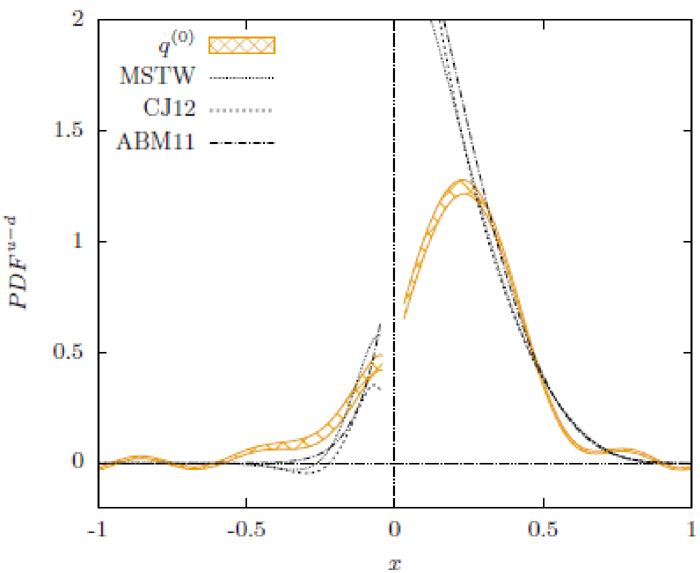
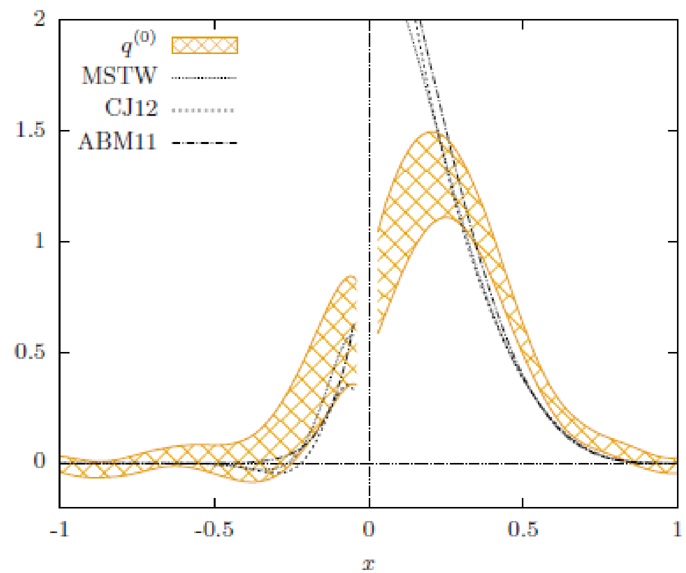
Exploratory studies on the lattice

- Unpolarized quark density
  - HYP smearing to smoothen Wilson line gauge links
  - One-loop matching and mass corrections





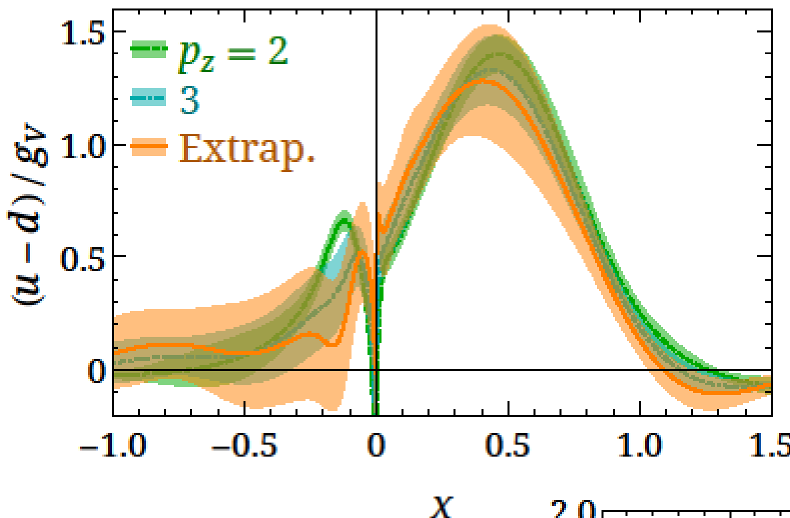


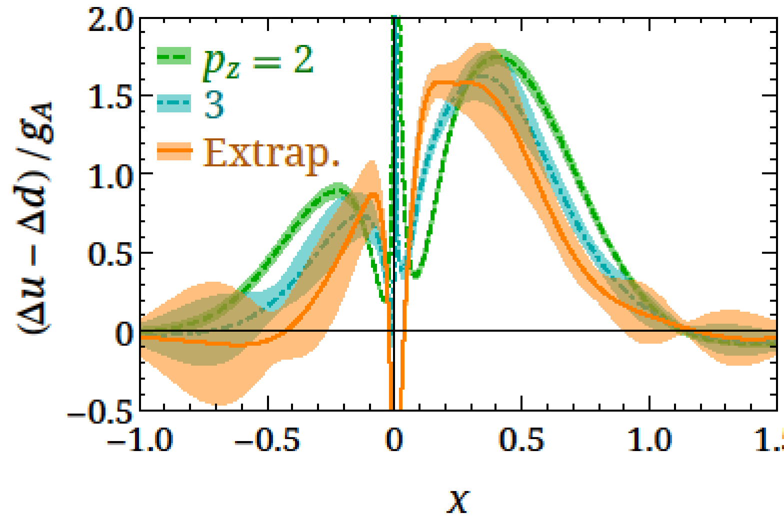
Lin et. al. 15'
$m_\pi = 310\ MeV, L = 3fm, a = 0.12fm$

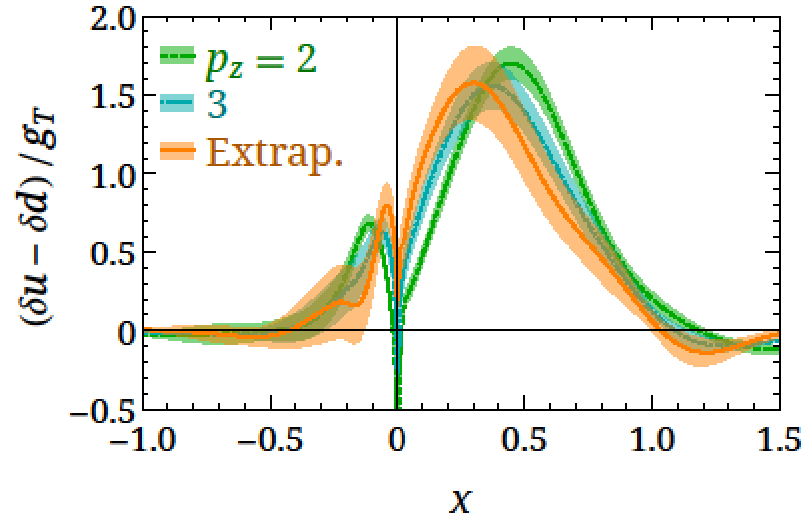
Alexandrou et. al. 15'
$m_\pi = 370\ MeV, L = 2.6fm, a = 0.08fm$

Exploratory studies on the lattice

- Unpolarized, helicity and transversity distributions
  - One-loop matching + mass corrections + higher-twist corrections







Chen et. al. 16'
$m_\pi = 310\ MeV, L = 3fm, a = 0.12fm$

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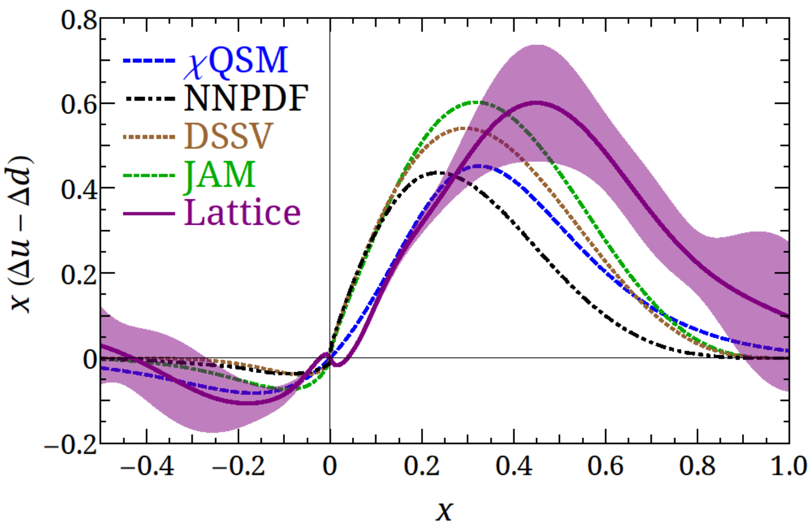
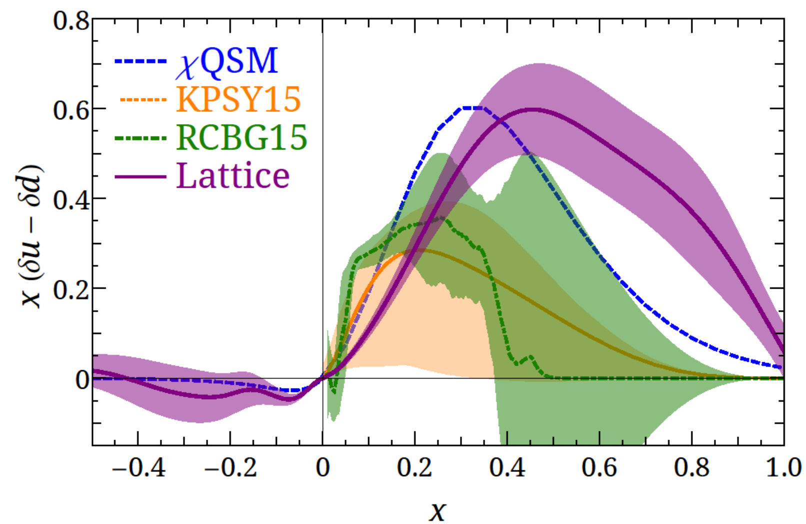


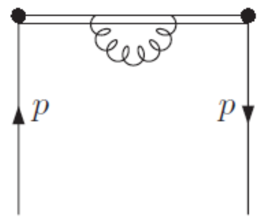


  - Sea flavor asymmetry

Chen et. al. 16'
$m_\pi = 310\ MeV, L = 3fm, a = 0.12fm$

Renormalization of power divergence

- Power divergence comes from Wilson line self energy [Ishikawa et al. 16′, Chen et al. 16′]
  - At one-loop, a linear div. is associated with



  - It is well-known that linear divergence associated with Wilson line can be removed by a mass renormalization (e.g. in auxiliary z-field formalism)
  - In a sense, the auxiliary field can be understood as a Wilson line extending between $[z, \infty]$

$$Z(z) = L(z, \infty) \ \text{ satisfies } \ [\partial_z - igA_z(z)]\, Z(z) = 0$$

  - Analogous to a heavy quark field
  - Non-local Wilson line can be interpreted as a two-point function of z-field

$$L(z,0) = Z(z) Z^\dagger(0)$$

  - Renormalizes analogously to a heavy quark two-point function [Dotsenko and Vergeles 80′, Dorn 86′]

$$L^{\text{ren}}(z,0) = \mathcal{Z}_Z^{-1} e^{-\delta m|z|} L(z,0)$$

Renormalization of power divergence

- One-loop illustration
  - The Wilson line self energy diagram gives

$$\lim_{\epsilon\to 0}\int dk_z \frac{\alpha_s C_F \Lambda}{2\pi}\frac{[\delta(k_z-\bar{x}p_z)-\delta(\bar{x}p_z)]p_z}{k_z^2+\epsilon^2}$$

  - Mass counterterm contributes

$$-\int \frac{dz}{2\pi}p_z\, e^{i(x-1)p_z z}|z|\,\delta m = -\lim_{\epsilon\to 0}\int \frac{dz}{2\pi}p_z\, e^{-i\bar{x}p_z z}\frac{1-e^{-\epsilon|z|}}{\epsilon}\delta m$$
$$= -\lim_{\epsilon\to 0}\int \frac{dk_z}{\pi}p_z\frac{\delta(\bar{x}p_z)-\delta(k_z-\bar{x}p_z)}{k_z^2+\epsilon^2}\delta m.$$

  - Therefore

$$\delta m = -\frac{\alpha_s C_F}{2\pi}(\pi\Lambda)$$

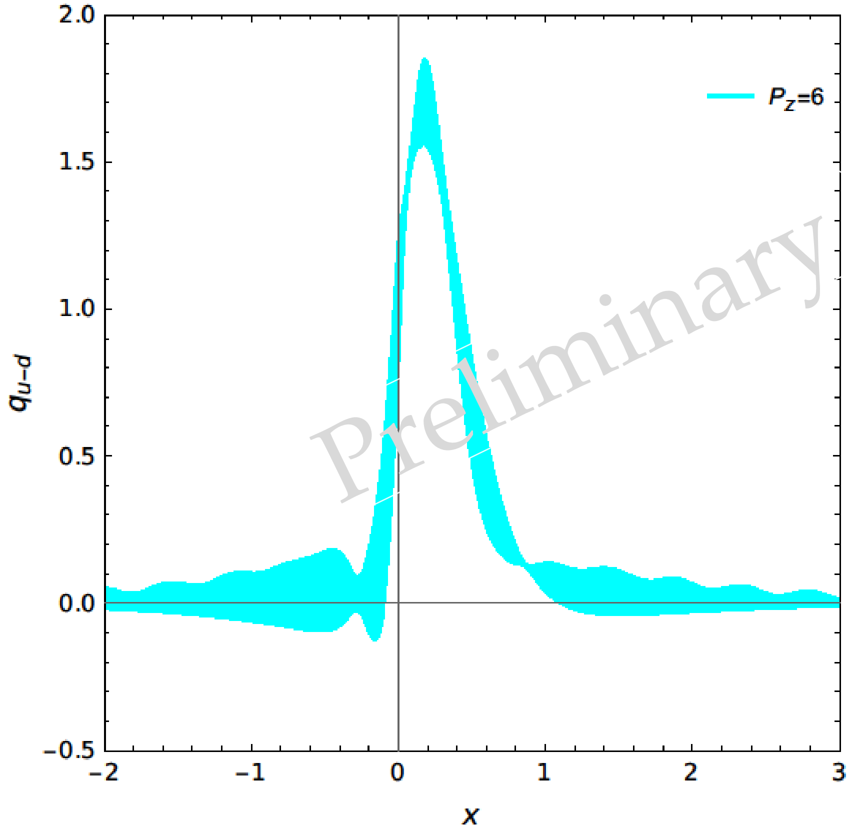
  - It is gauge-independent

- Can be extended to higher-loop orders

Improved quasi quark distribution

$$\tilde{q}_{\rm imp}(x,\Lambda,p^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z - \delta m|z|} \langle p|\overline{\psi}(0,0_\perp,z)\gamma^z L(z,0)\psi(0)|p\rangle$$

- Wilson line renormalization removes power div.



near physical pion mass, $L \approx 6fm, a = 0.09fm$

Summary and outlook

- PDFs are difficult to compute due to their non-perturbative and intrinsically Minkowskian nature
- Large momentum effective theory offers a practical possibility to directly compute light-cone quantities such as the PDFs from Euclidean lattice
  - Matching
  - Renormalization
  - Power corrections
    - Non-perturbative evolution for extrapolation from moderate to large momentum [Radyushkin 16', 17']
  - ……
- Exploratory results show encouraging features
- A lot more effort needed for lattice results to reach accuracy comparable with phenomenological fits

BACKUP SLIDES

Pion distribution amplitude

- Improved pion quasi DA

$$\tilde{\phi}_{\rm imp}(x, P_z) = \frac{i}{f_\pi}\int \frac{dz}{2\pi} e^{-i(x-1)P_z z - \delta m|z|}\langle \pi(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z)|0\rangle$$

- Mass counterterm [Musch et. al. 11']
  - Choose a Wilson loop long in t-direction such that higher excitations are sufficiently suppressed
  - Fit the quark potential

$$V(r) = -\frac{1}{a}\lim_{t\to\infty}\ln\frac{\langle {\rm Tr}[W(t,r)]\rangle}{\langle {\rm Tr}[W(t-a,r)]\rangle}$$

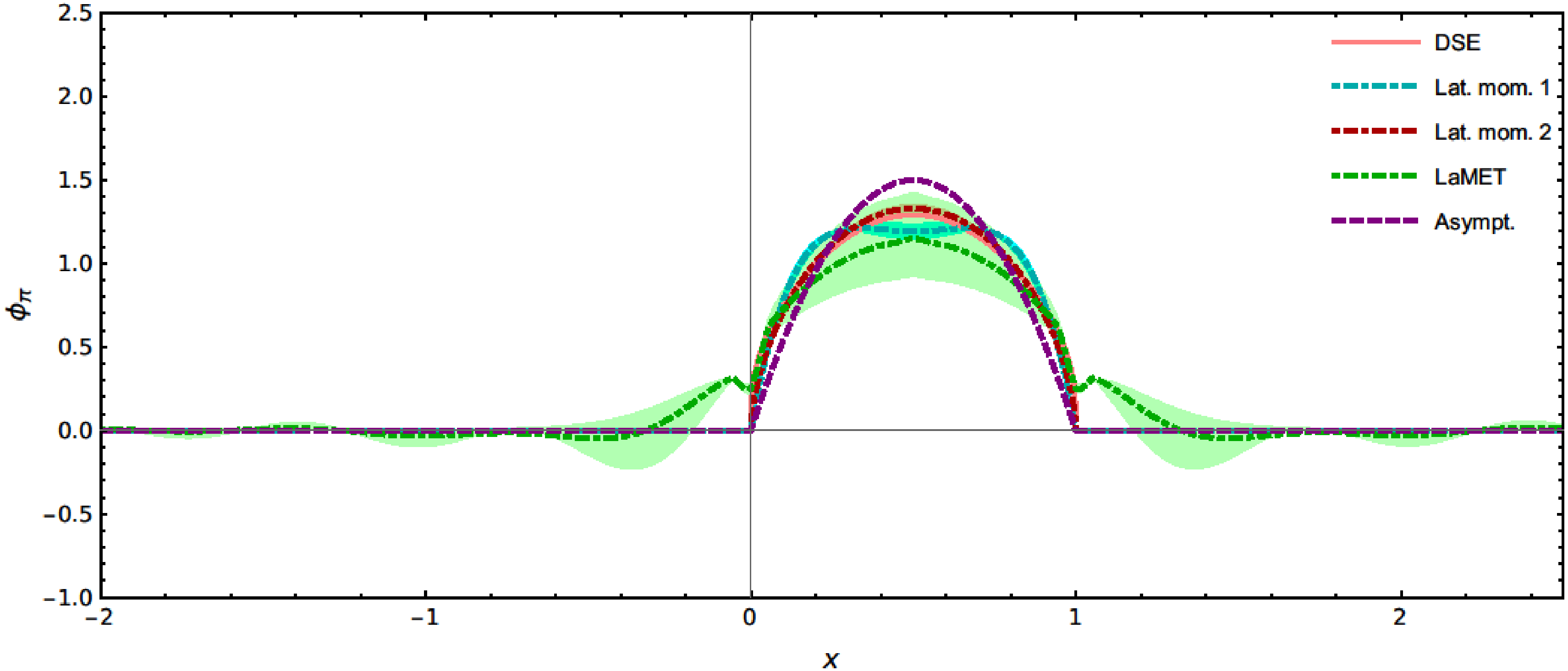
to

$$V(r) = \frac{c_1}{r} + c_2 + c_3 r$$

gives

$$\delta m \simeq -260 \pm 200 \text{ MeV}$$

Pion distribution amplitude



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