The momentum fractions for proton mass decomposition

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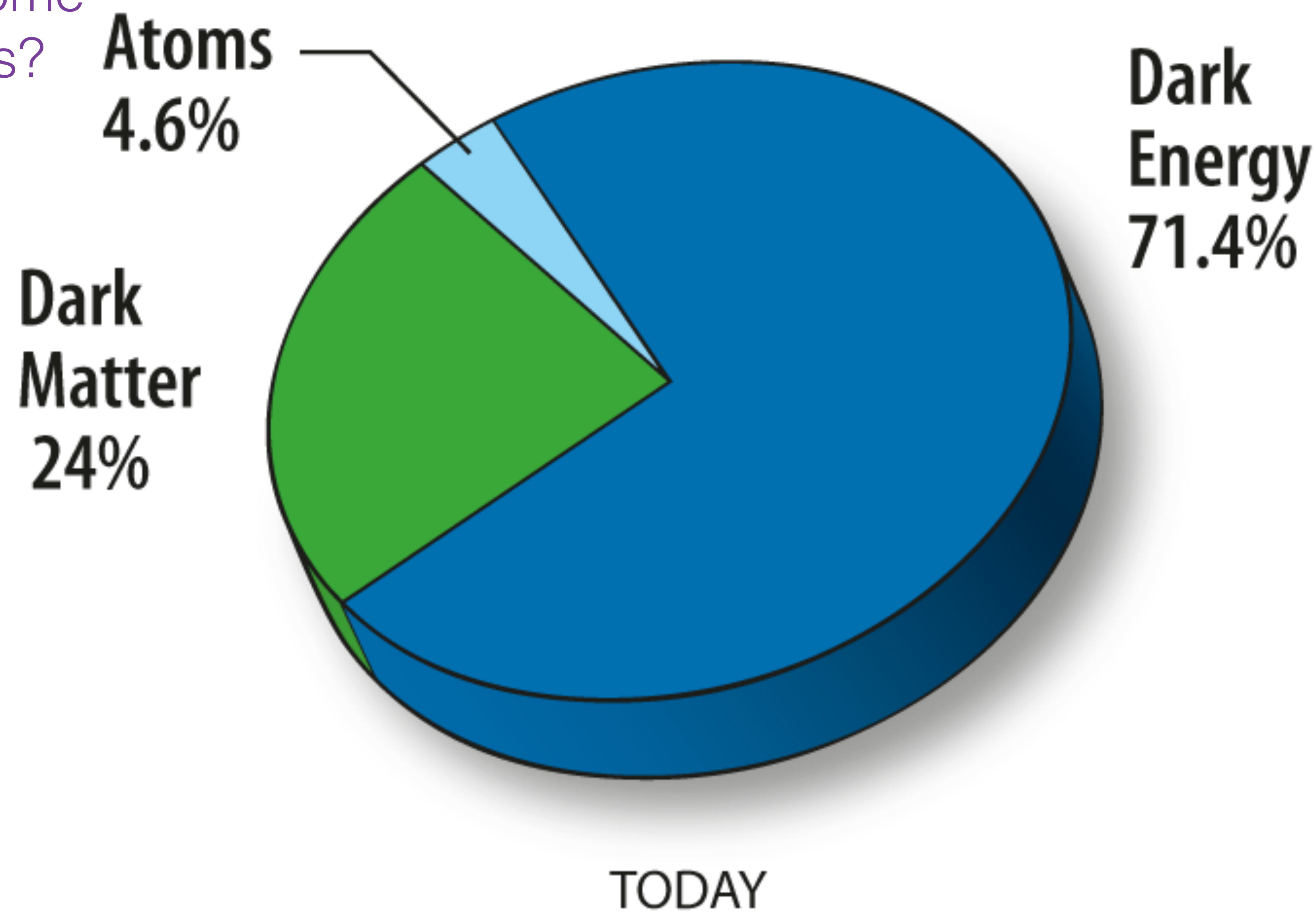
Motivation

Where does this observable 4.6% come from, due to Higgs?



Motivation

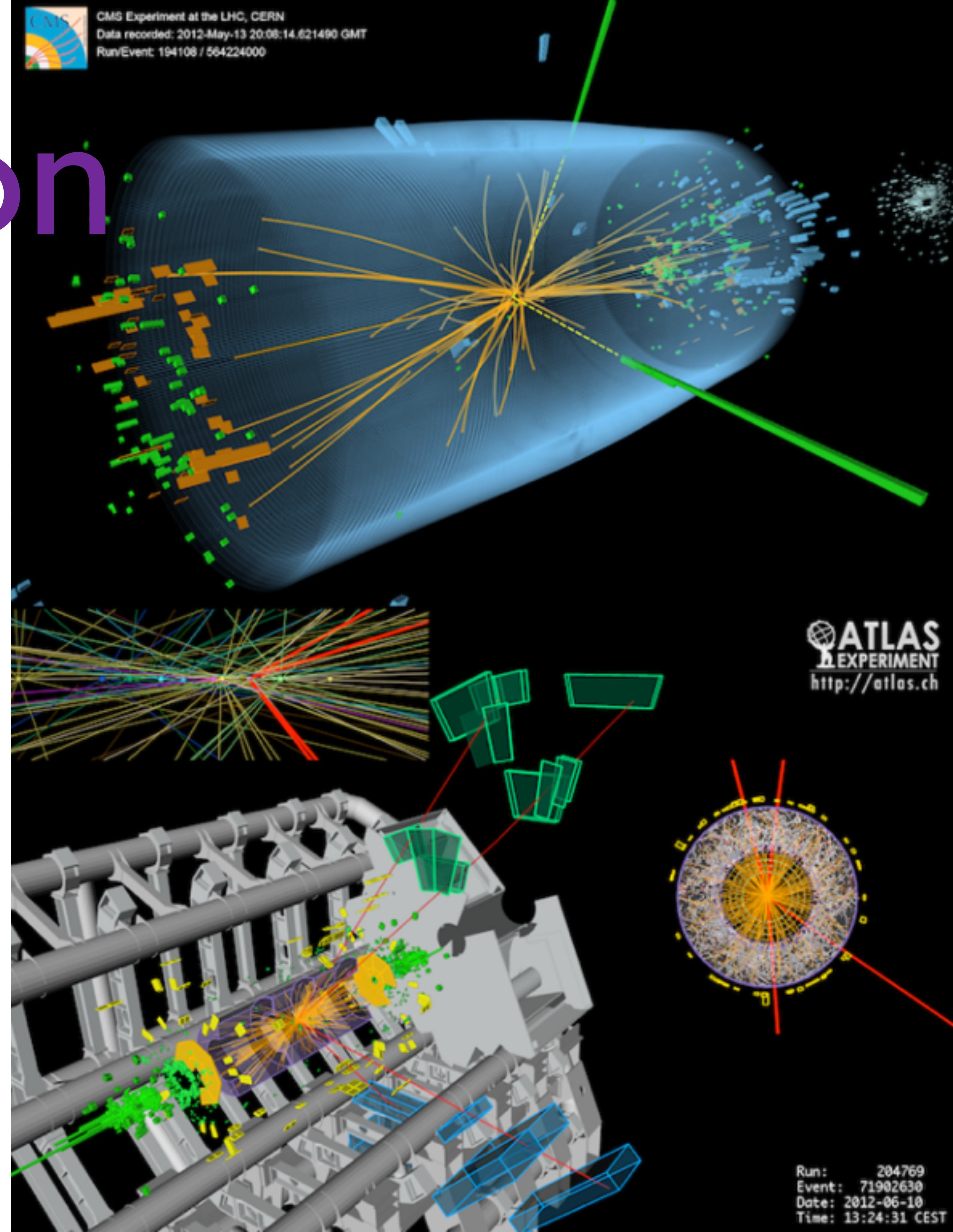
The Higgs boson makes the u/d quark having masses:

$m_u = 2.08(9)$ MeV

$m_d = 4.73(12)$ MeV

But the mass of the proton is *938.272046(21) MeV.* ~100 times of the sum of the quark masses.

Where does the mass of proton come from, and ***how*** ?



Formalism

$$T_{\mu\nu} = \frac{1}{4}\overline{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2,$$

The energy momentum tensor
in the classic level

$$\overline{T}_{\mu\nu} = \frac{1}{4}\overline{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu}\psi - \frac{1}{16}g_{\mu\nu}\overline{\psi}\gamma_{(\rho}\overleftrightarrow{D}_{\rho)}\psi + F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$

The traceless part of the energy momentum tensor

$$T_{\mu\mu} = -m\bar{\psi}\psi - \gamma_m m\bar{\psi}\psi + \frac{\beta(g)}{2g}F^2$$

The trace part of the energy momentum tensor with equation of motion (EOM) applied, add the quantum trace anomalies.

Xiangdong Ji, PRL 74, 1071-1074 (1995)

Xiangdong Ji, PRL 74, 1071-1074 (1995)
Y. Yang, et.al. [χQCD], Phys. Rev. D 91, 074516 (2015)

Formalism

Then we have

$$
\begin{aligned}
M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\
&= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\
\frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,
\end{aligned}
$$

With

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

The quark mass

Gauge Invariant and scale independent combinations.

The QCD anomaly

$$
\begin{aligned}
H_a &= H_g^a + H_m^\gamma, \\
H_g^a &= \int d^3x\, \frac{-\beta(g)}{4g}(E^2 + B^2), \\
H_m^\gamma &= \sum_{u,d,s\cdots} \int d^3x\, \frac{1}{4}\gamma_m m\bar{\psi}\psi.
\end{aligned}
$$

The glue anomaly

The quark mass anomaly

The total energy

$$
\begin{aligned}
H_E &= \sum_{u,d,s\ldots} \int d^3x\, \bar{\psi}(\vec{D}\cdot\vec{\gamma})\psi, \\
H_g &= \int d^3x\, \frac{1}{2}(B^2 - E^2),
\end{aligned}
$$

The quark energy

The glue field energy

The quark mass term

Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$
$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

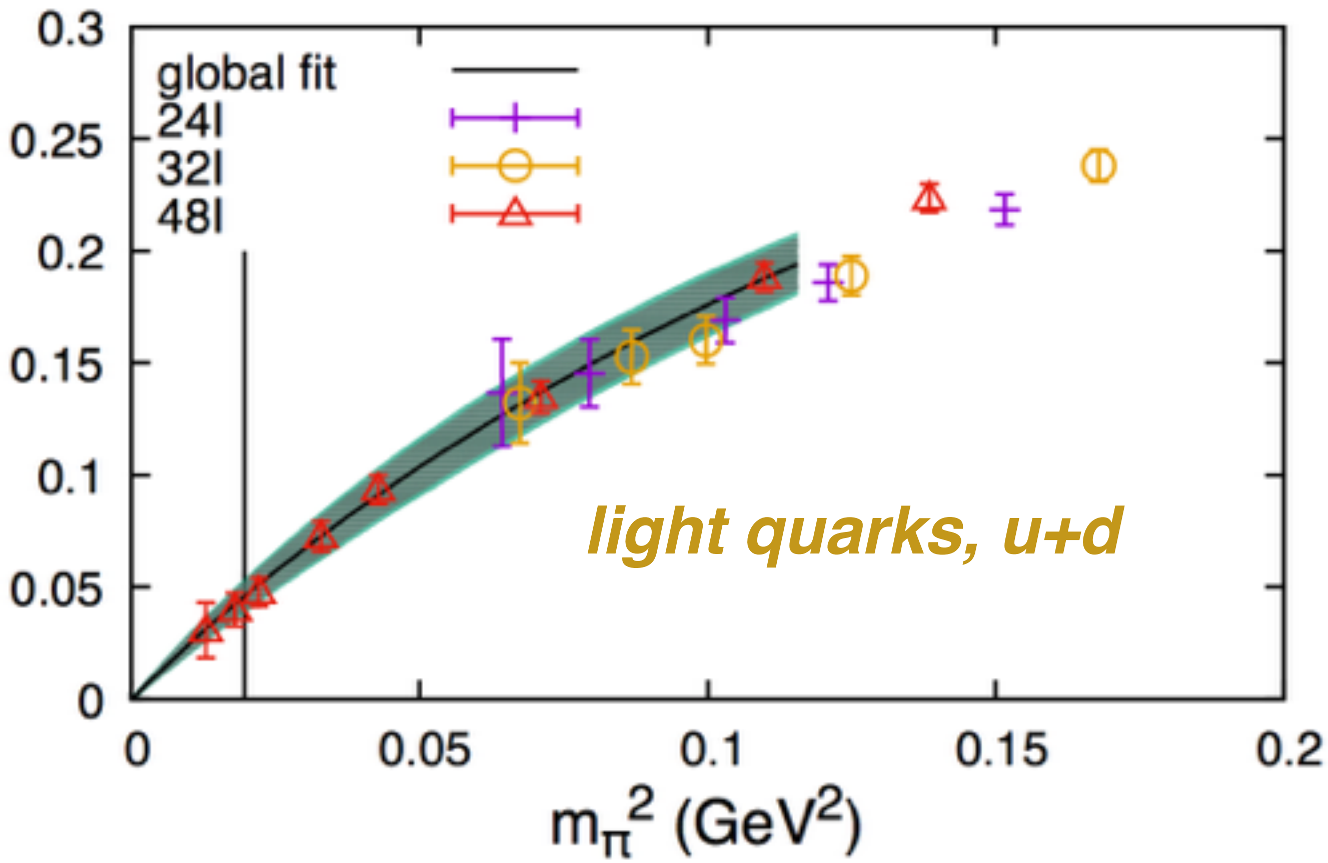
$$H_m = \sum_{u,d,s\cdots} \int d^3x \, m\bar{\psi}\psi,$$

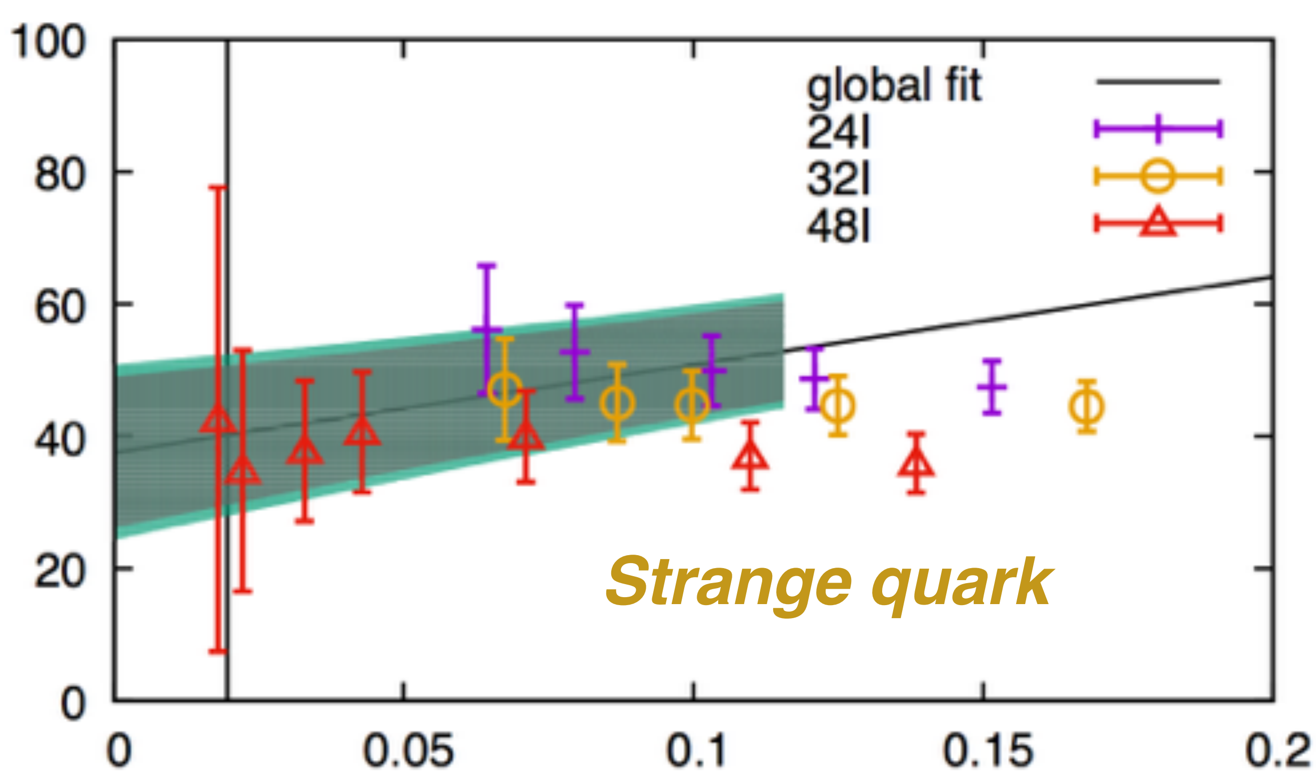
The quark mass

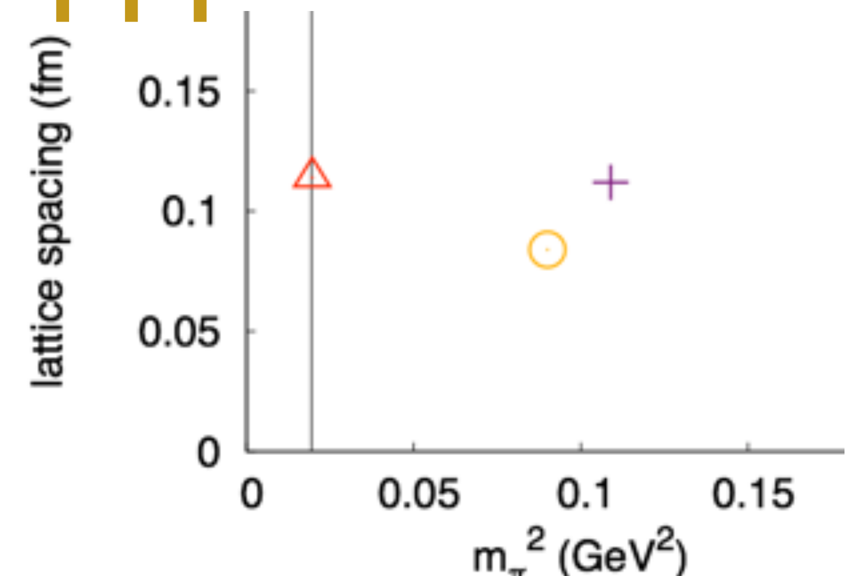
- Renormalization scheme/scale independent in continuum; also in discrete case when the chiral fermion is used.
- The term where the Higgs boson contributes.
- **Can be calculated directly in the lattice simulation.**

Y. Yang, et.al. [χQCD], Phys. Rev. D 94, 054503 (2016)

The quark mass term







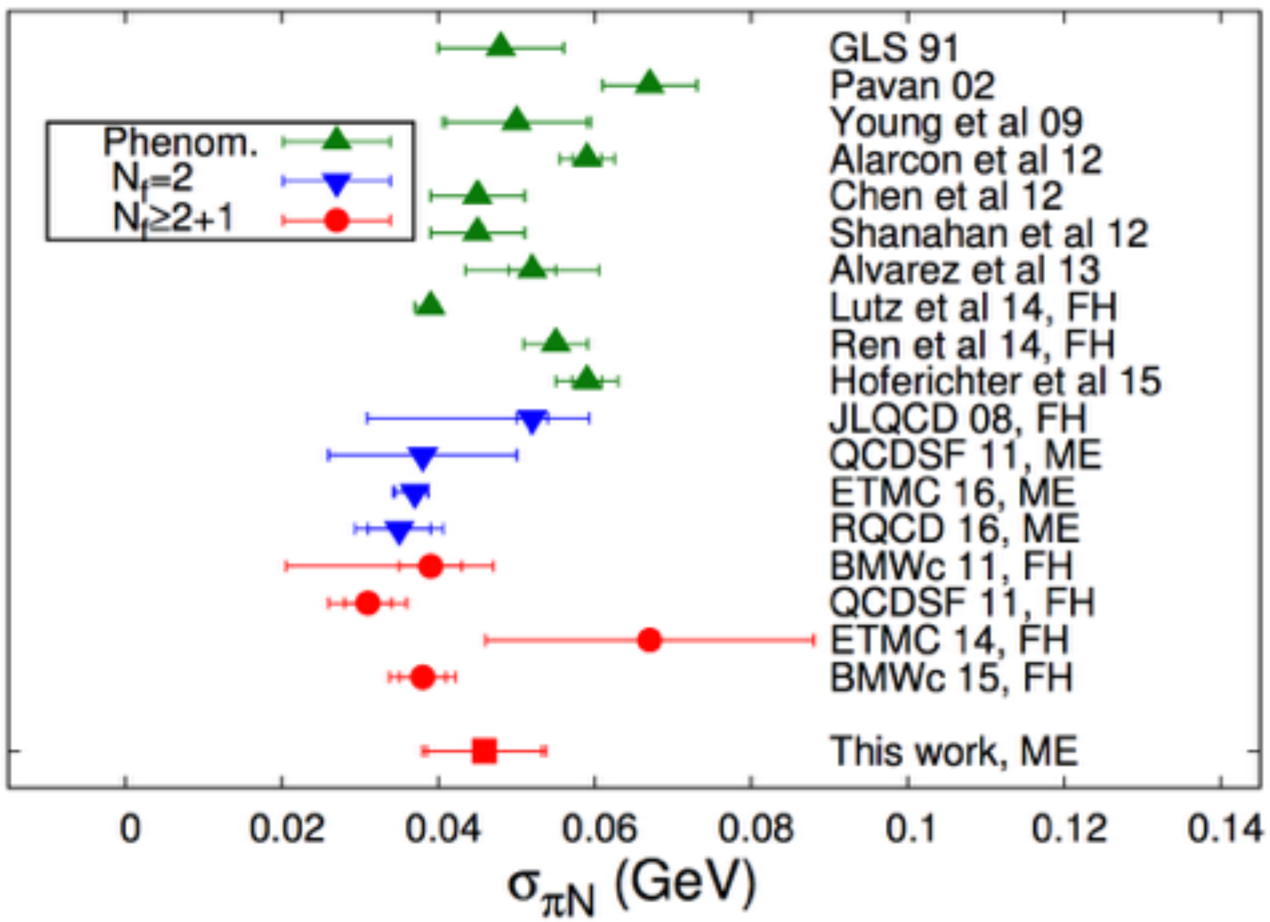
- Three 2+1 Domain wall fermion ensembles with different lattice spacing, volumes and sea quark masses;

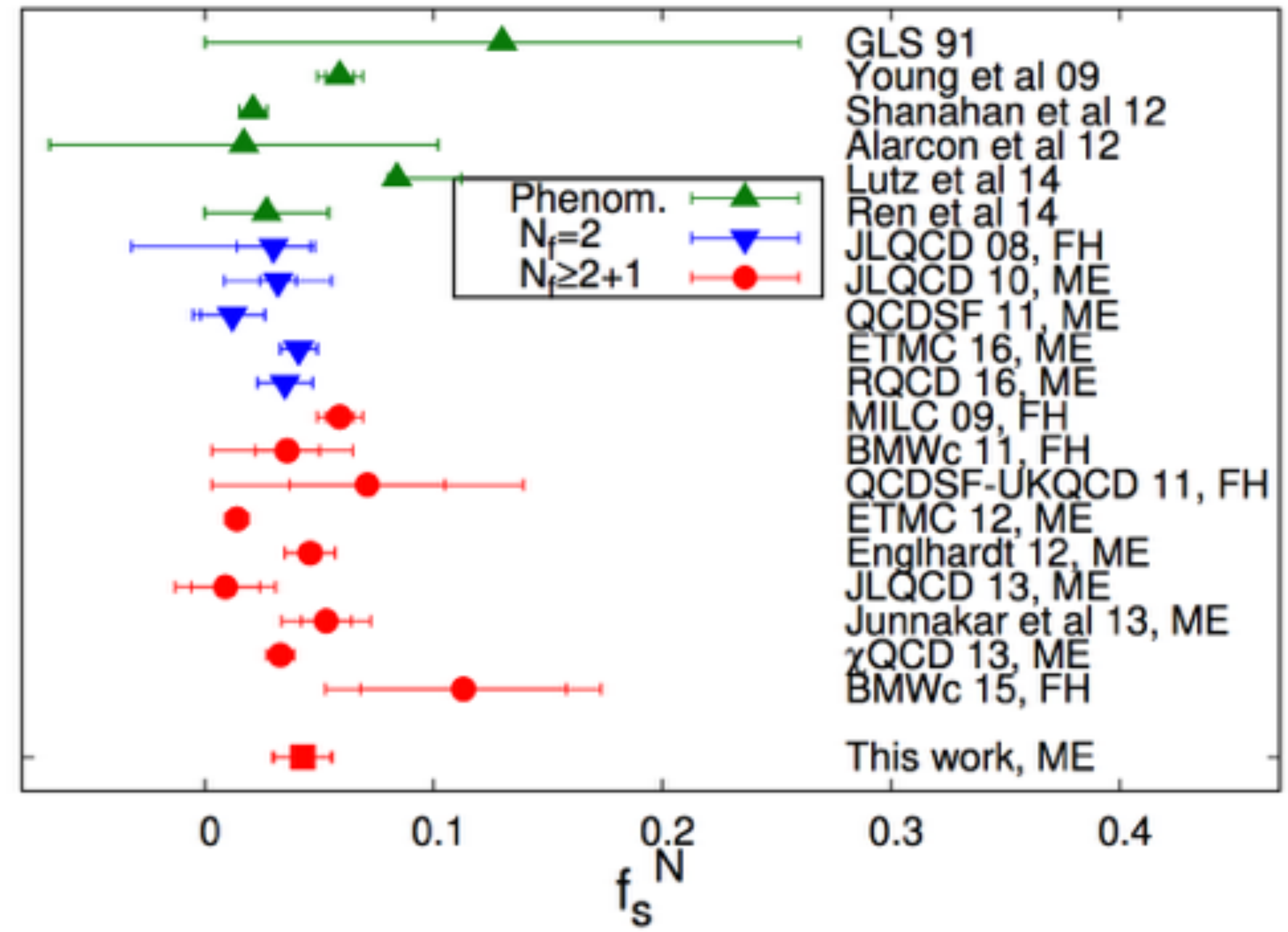
- Chiral fermion action for the valence quark without explicit breaking;

- The global fit with multiple valence quark masses on all the three ensembles to control the systematic uncertainties

Y. Yang, et.al. [χQCD], Phys. Rev. D 94, 054503 (2016)

The quark mass term





$\sigma_{\pi N}$ = 45.9(7.4)(2.8) MeV

σ_{sN} = 40.2(11.7)(3.5) MeV

$\langle H_m(u,d,s)\rangle / M_N$ = 9(2)%

The best result without the systematic uncertainty from the explicit breaking

The QCD anomaly

Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$
$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x\, \frac{-\beta(g)}{4g}(E^2 + B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x\, \frac{1}{4}\gamma_m m \bar{\psi}\psi.$$

The quark mass anomaly

- **The joint contribution of the QCD anomaly can be deduced from the quark mass term, with the sum rule above.**
- The total QCD anomaly is renormalization scheme/scale independent.
- ***H_a/M_N = 23(1)%***

The quark/gluon energy

Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$
$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle,$$

- The quark/glue energy can be deduced from the momentum fraction,

$$\langle H_E\rangle = \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle,$$
$$\langle H_q\rangle = \frac{3}{4}\langle x\rangle_q M + \frac{1}{4}\langle H_m\rangle.$$

$$\langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M.$$

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.

- **It is more straightforward to obtain the quark/glue momentum fraction first, and convert it to the quark/glue energy.**

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

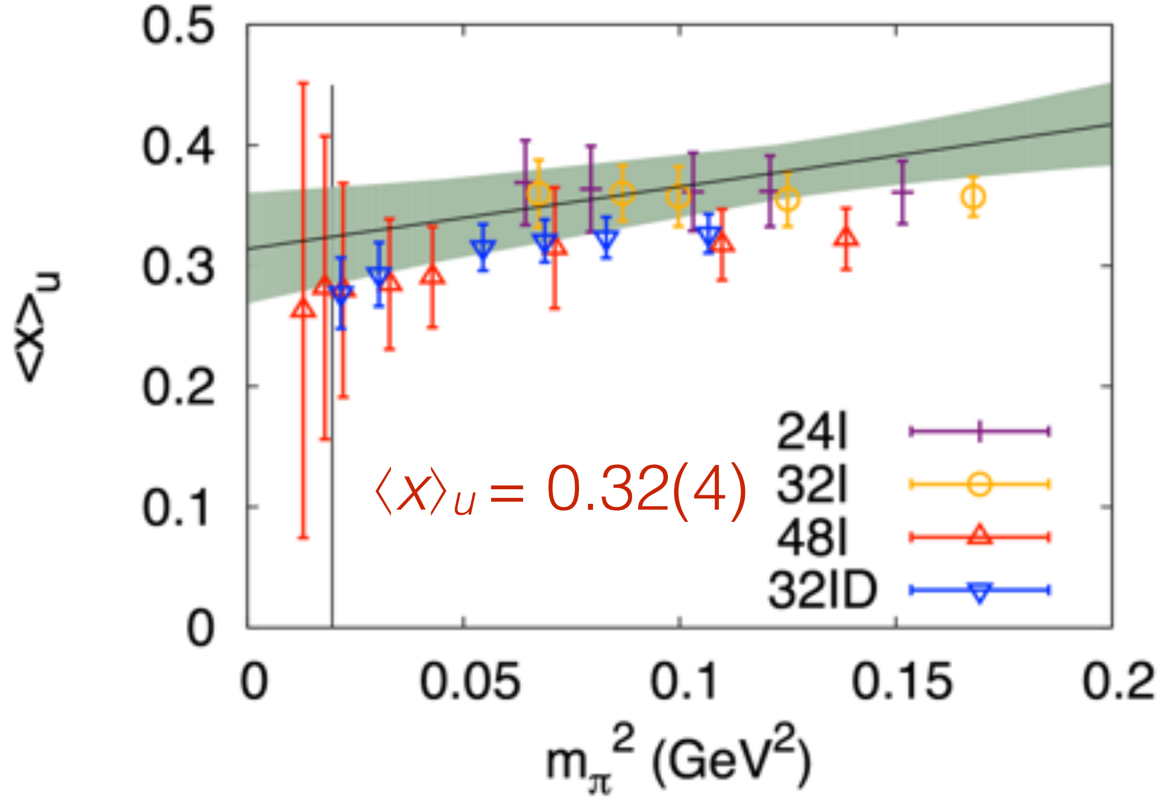
$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

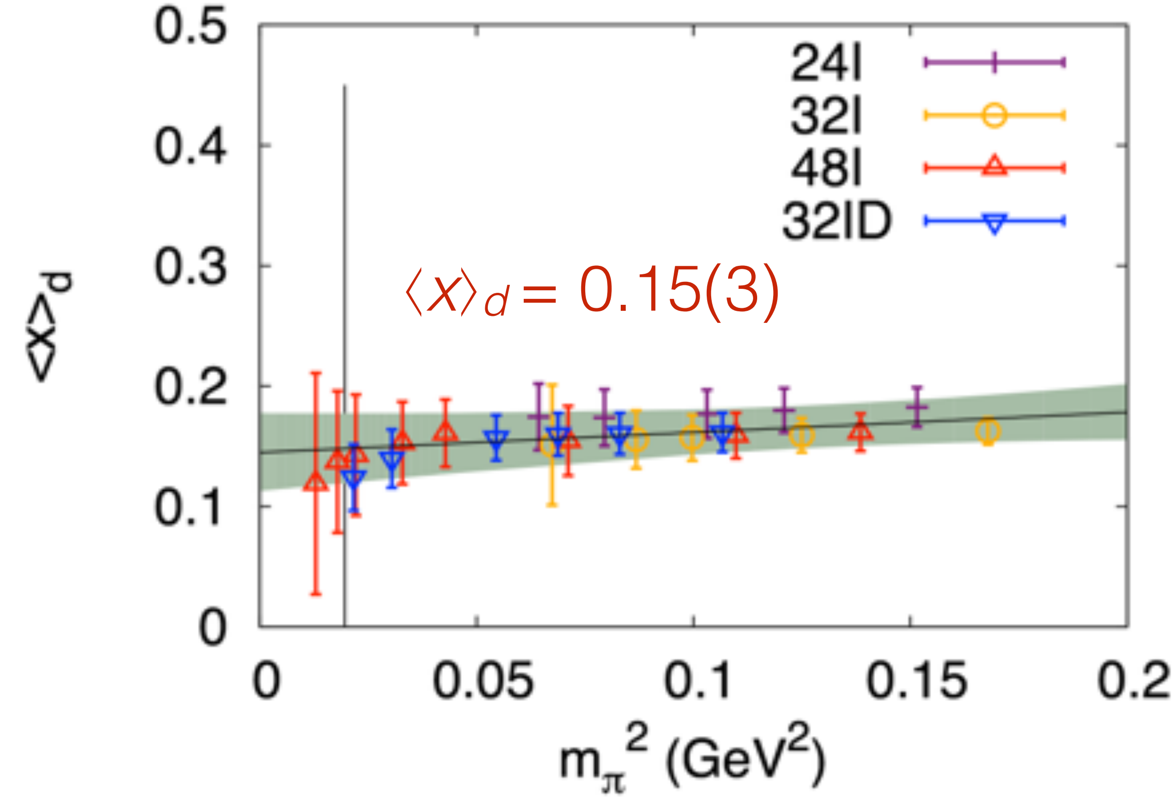
The glue field energy

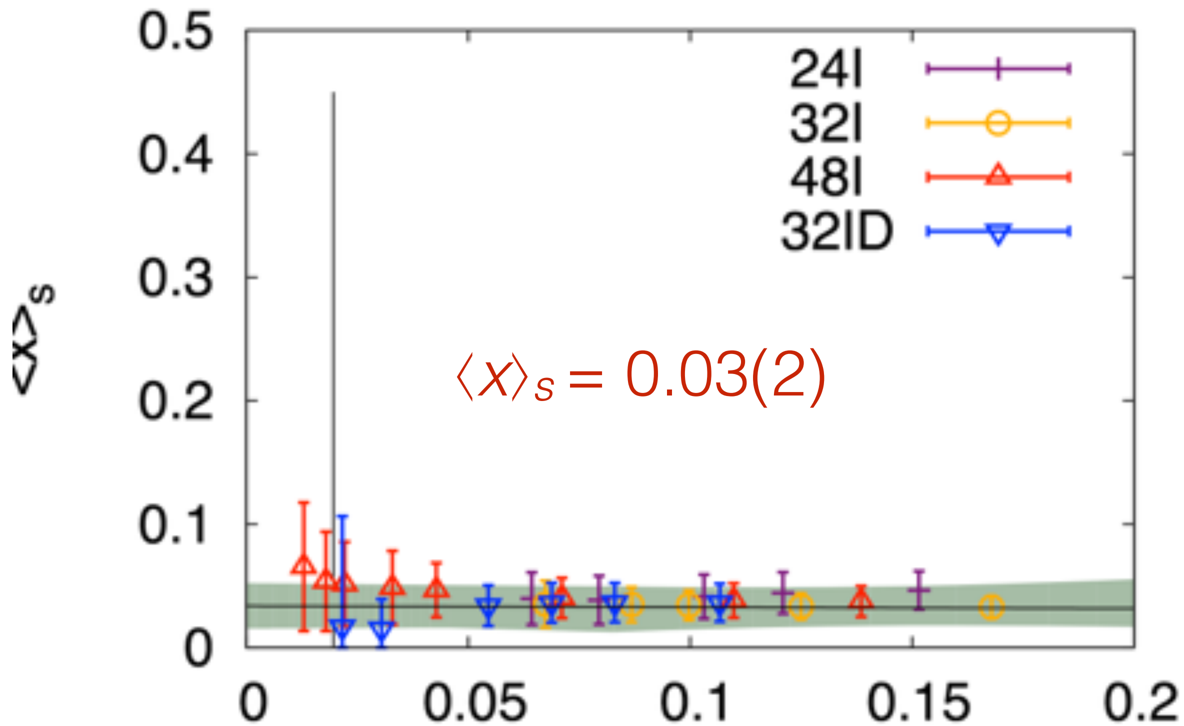
Quark part

Preliminary

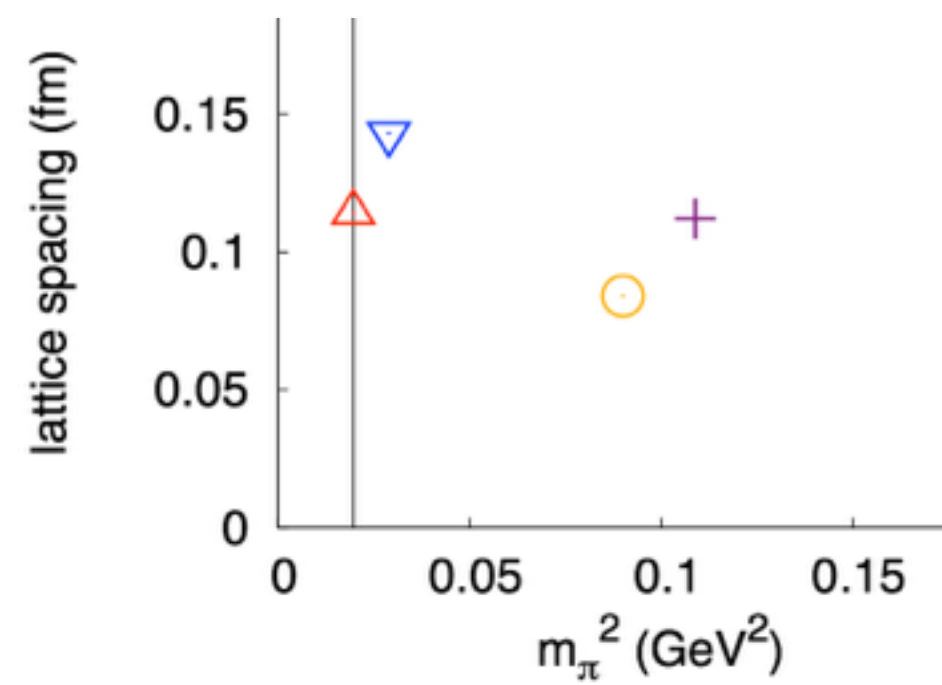
different flavors





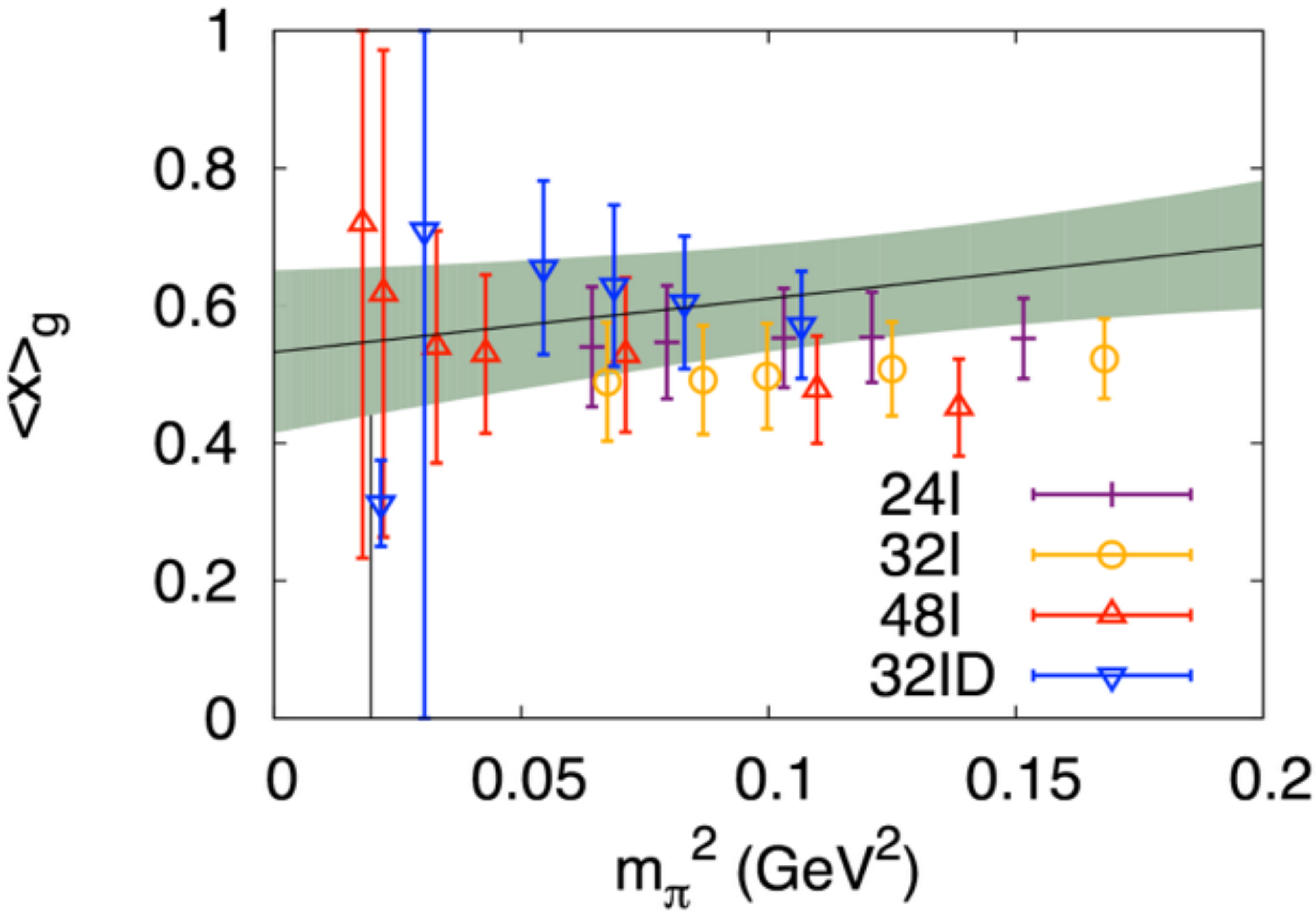


Lattice bare results evaluated to 2GeV



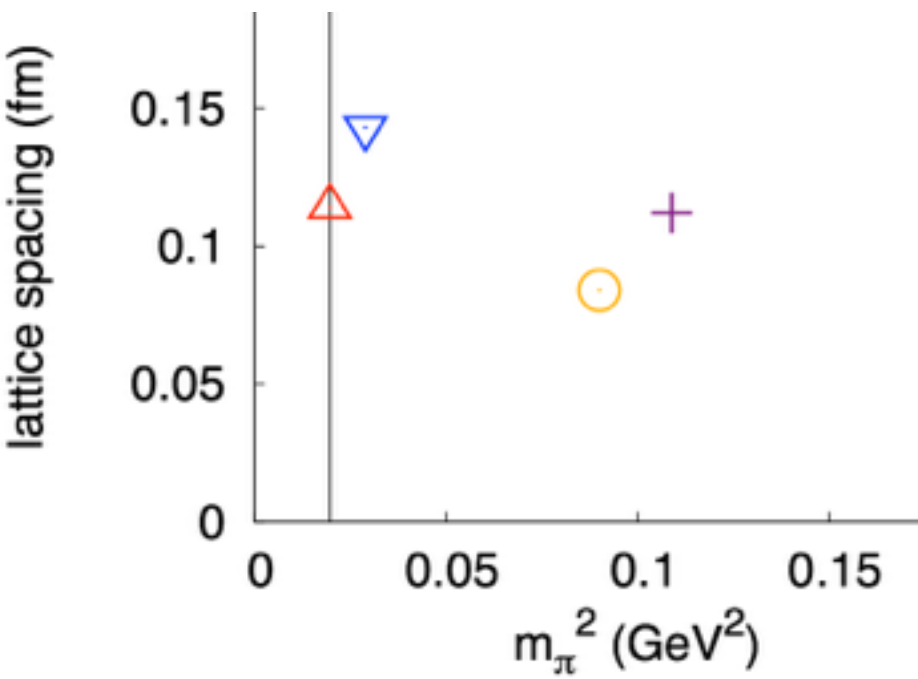
Glue part

Preliminary



Lattice bare results evaluated to 2GeV,

$\langle x \rangle_g = 0.54(11)$



Renormalization

Y. Yang, et.al. [χQCD], arXiv: 1612.02855

of the momentum fractions

From the lattice bare quantities with the chiral fermion and HYP smeared Iwasaki gluon to that under the MS-bar scheme, at a scale μ=1/a,

$$\begin{pmatrix} \mathcal{T}_Q^{\overline{\rm MS}} \\ \mathcal{T}_G^{\overline{\rm MS}} \end{pmatrix} = \begin{pmatrix} 1.0202 & 0.0123N_f \\ 0.1565 & 2.08(25) - 0.0239N_f \end{pmatrix} \begin{pmatrix} \mathcal{T}_Q^{lat} \\ \mathcal{T}_G^{lat} \end{pmatrix} + O(g^4),$$

the off-diagonal part of $\mathcal{T}^{\mu\nu}$

$$\begin{pmatrix} \mathcal{T}_Q^{\overline{\rm MS}} \\ \mathcal{T}_G^{\overline{\rm MS}} \end{pmatrix} = \begin{pmatrix} 1.0175 & -0.0069N_f \\ 0.1528 & 1.84(18) - 0.0239N_f \end{pmatrix} \begin{pmatrix} \mathcal{T}_Q^{lat} \\ \mathcal{T}_G^{lat} \end{pmatrix} + O(g^4),$$

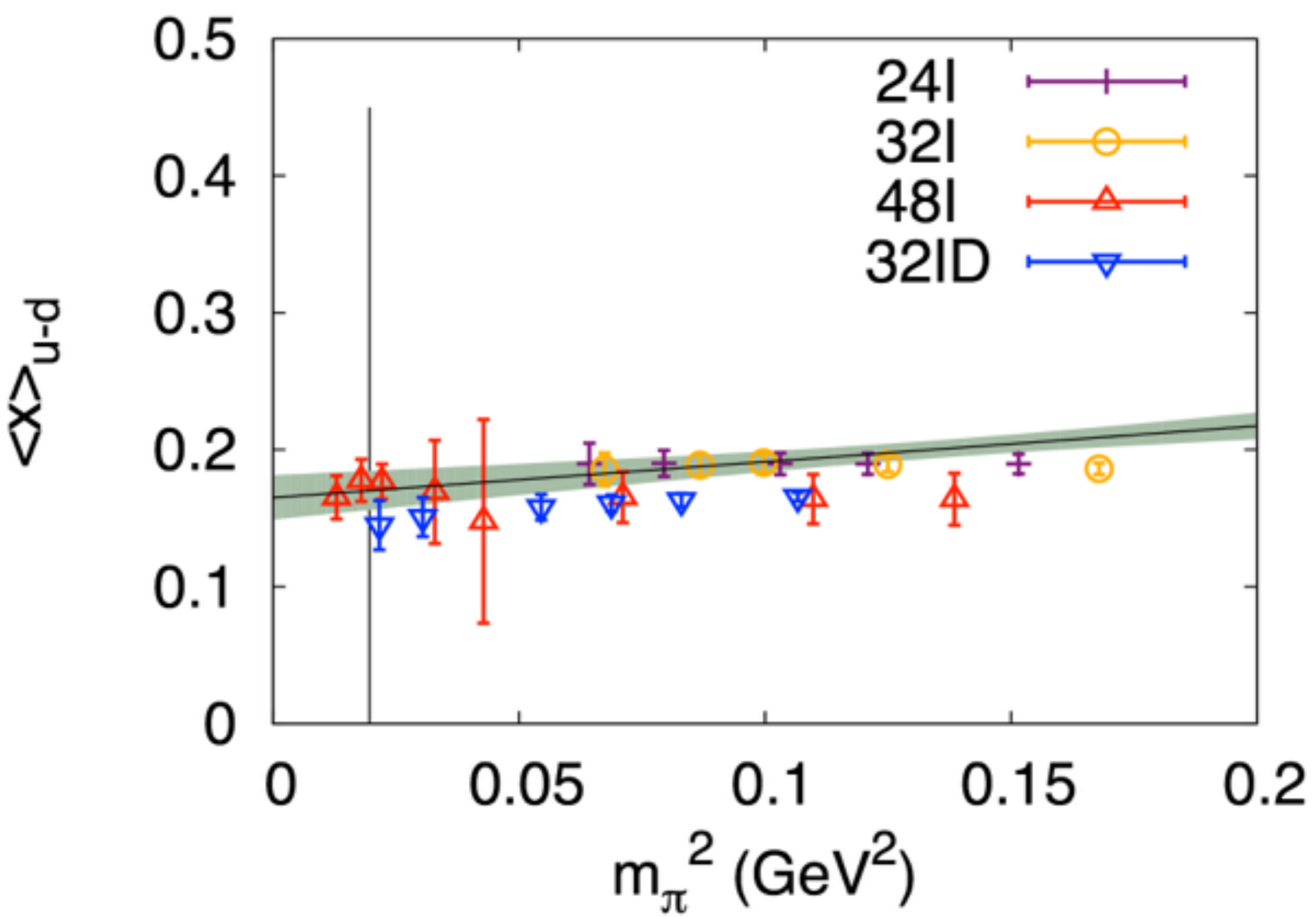
the traceless diagonal part

- With the global fit, $\langle x \rangle_q$ **= 50(7)%** at MS-bar 2GeV.
- For the gluon operator renormalization at 1-loop level, the value and the uncertainty (from the estimate of the 4-gluon vertex tadpole contribution) are large and then indicate the convergence problem.
- The bare value of $\langle x \rangle_g$ is **54(11)%** and that deduced from the momentum fraction sum rule is $\langle x \rangle_g$ **= 50(7)%**.

Quark part

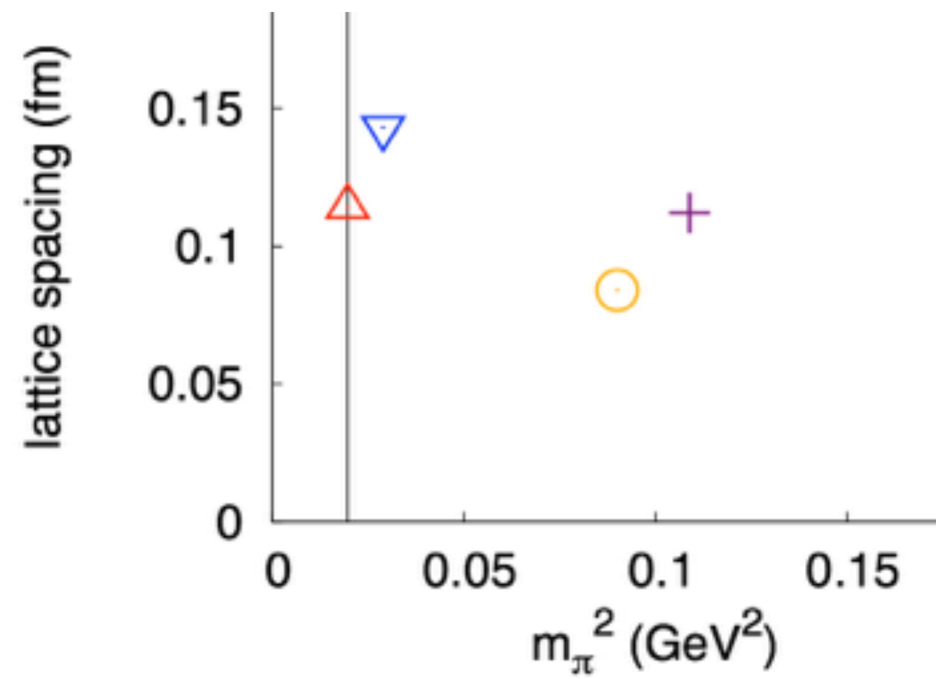
Preliminary

Renormalized u-d



Renormalized value at MS-bar 2GeV,

$\langle x \rangle_{u\text{-}d} = 0.170(14)$



The quark/gluon energy

Preliminary

from the momentum fractions

$$\langle H_E \rangle = \frac{3}{4}\langle x \rangle_q M - \frac{3}{4}\langle H_m \rangle,$$

$$\langle H_q \rangle = \frac{3}{4}\langle x \rangle_q M + \frac{1}{4}\langle H_m \rangle.$$

$$\langle H_g \rangle = \frac{3}{4}\langle x \rangle_g M.$$

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

From the last section,

$\langle H_m \rangle / M_N = 9(2)\%$ $\langle x \rangle_q = 50(7)\%$ and $\langle x \rangle_g = 50(7)\%$

Then

$\langle H_E + H_g \rangle / M_N = 69(2)\%,$

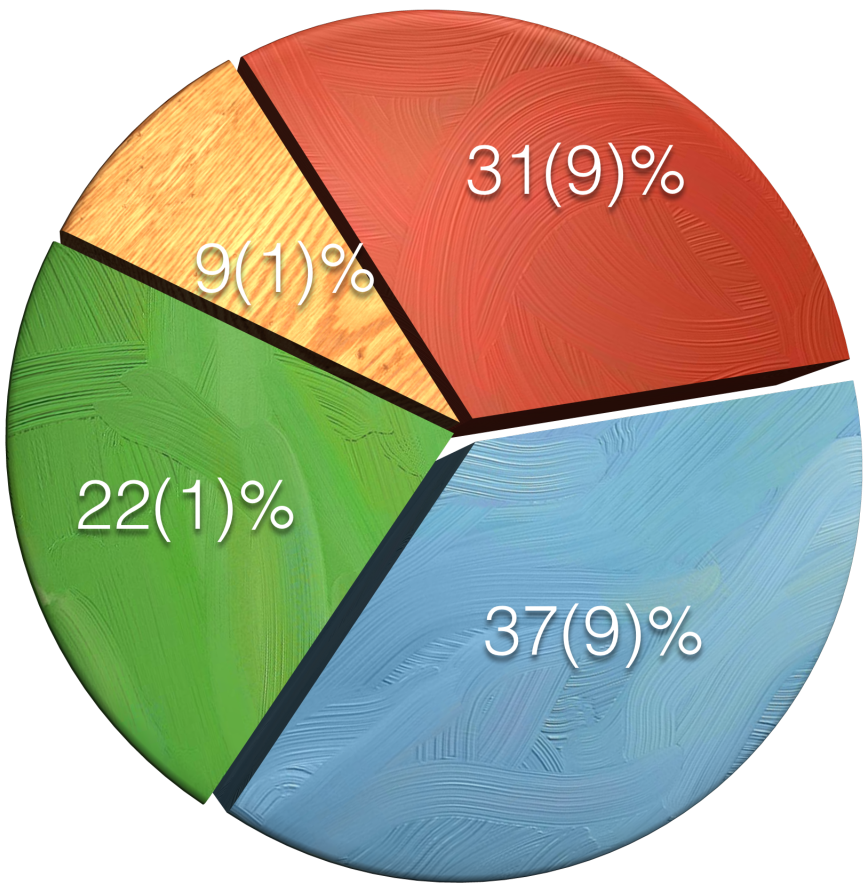
$\langle H_E \rangle / M_N = 31(5)\%,$

$\langle H_q \rangle / M_N = \langle H_E + H_m \rangle / M_N = 40(5)\%,$

$\langle H_g \rangle / M_N = 37(5)\%$

Proton Mass decomposition

by type



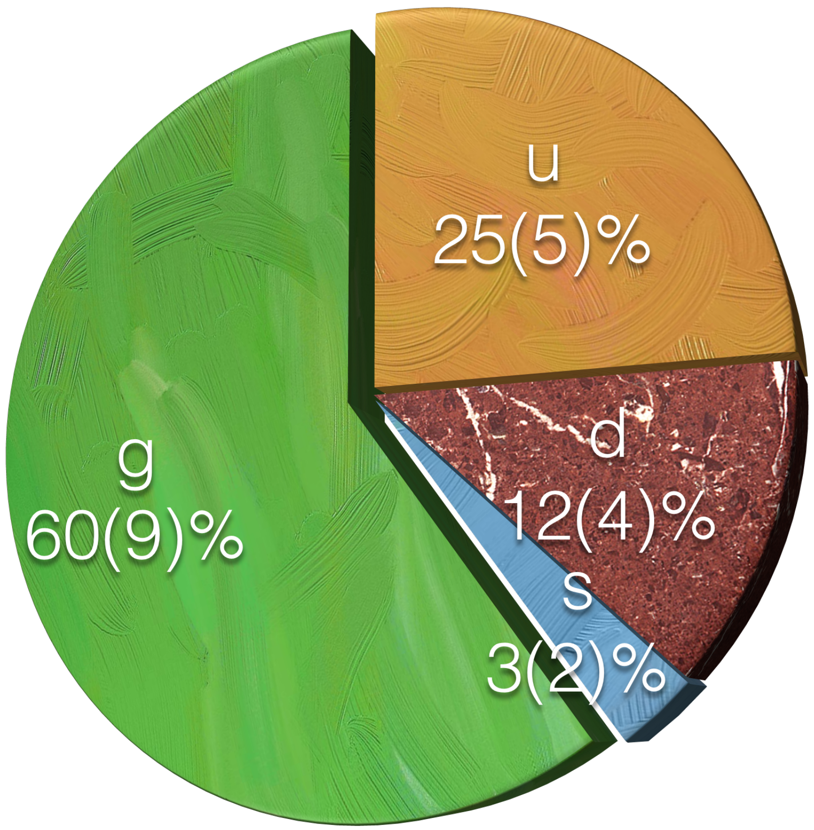
quark mass	quark energy
9(2) %	31(5) %
QCD anomaly	**glue energy**
23(1) %	37(5) %

- *Renormalized momentum fraction at MS-bar 2GeV.*
- *QCD anomaly and gluon energy are deduced by the sum rule.*
- *The contribution from heavy quarks ignored since the simulation is based on 2+1 flavor ensembles.*

Preliminary

Proton Mass decomposition

by u/d/s flavors+glue



- *Quark part: quark mass term+quark energy term.*
- *Glue part: glue energy + QCD anomaly.*

u	d
25(4) %	12(4) %
s	**g**
3(2) %	60(5) %

- *Renormalized momentum fraction at MS-bar 2GeV.*
- *QCD anomaly and gluon energy are deduced by the sum rule.*
- *The contribution from heavy quarks ignored since the simulation is based on 2+1 flavor ensembles.*

Preliminary

Summary

- **The mass decomposition based on the energy-momentum tensor is an old story, while Lattice QCD simulation gives it a second life.**

- **We decompose the proton mass into quark and gluon components in lattice simulation.**

1. **The joint u/d/s quark mass term contribute 9(2)%.**
2. **The glue momentum fraction is 50(7)%.**
3. **The joint quark/glue energy contributes 69(2)%.**
4. **The joint glue contributes half of the proton mass.**

- **Further analysis of the gluon contribution are in progress.**