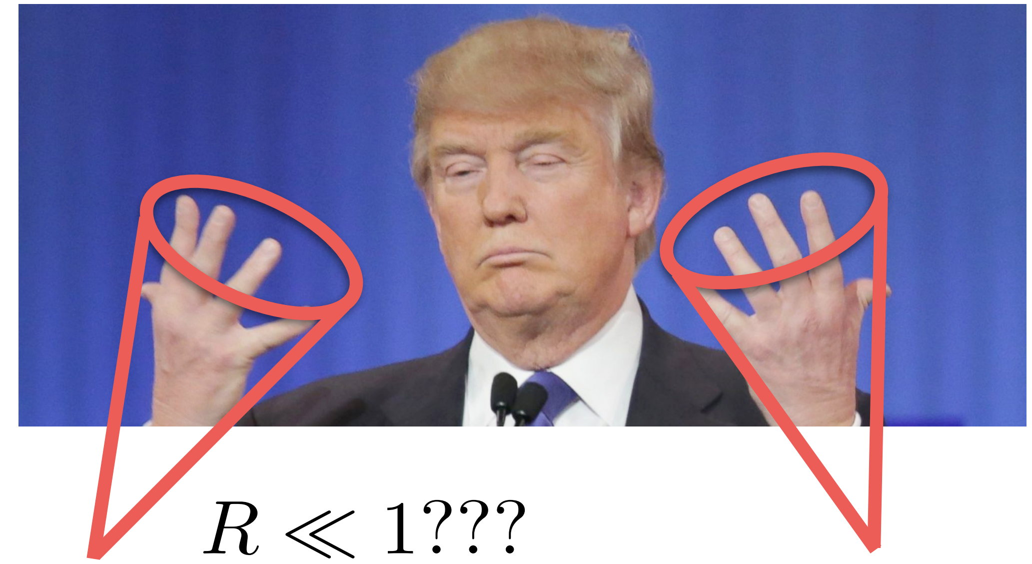
An EFT for small-*R* jets

Christopher Lee
Los Alamos National Laboratory
Theoretical Division, T-2 (Nuclear & Particle Physics, Astrophysics & Cosmology) Group

February 3, 2017

GHP Meeting
Washington, DC



An EFT for small-*R* jets

Christopher Lee
Los Alamos National Laboratory
Theoretical Division, T-2 (Nuclear & Particle Physics, Astrophysics & Cosmology) Group

February 3, 2017

GHP Meeting
Washington, DC

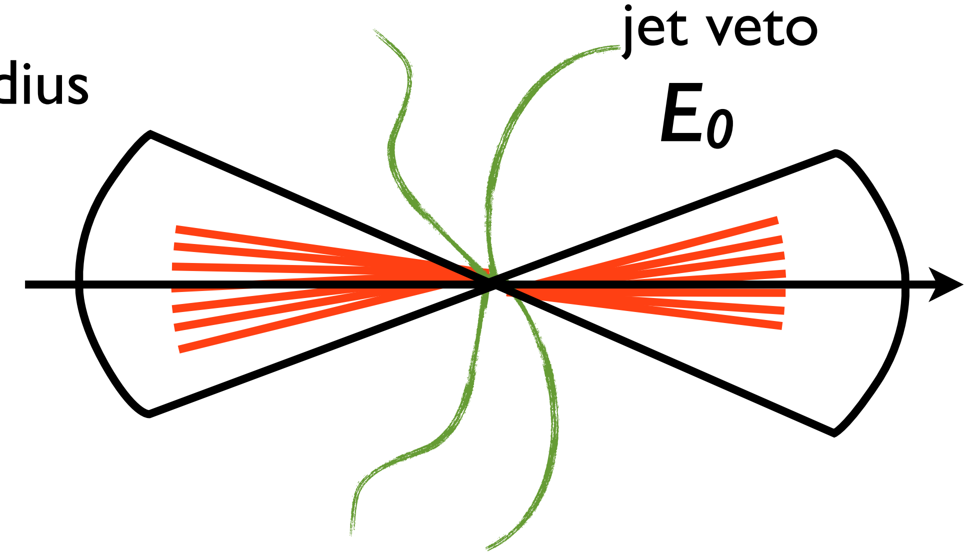
Los Alamos
NATIONAL LABORATORY
EST. 1943

Plan of Talk

- Jets and SCET (Soft Collinear Effective Theory)
- Jet Algorithms, Radii, and the soft-collinear scale
- $SCET_{+}$, $SCET_{++}$, and subjets

Exclusive Jet Cross Sections in QCD

- Example: e^+e^- to **two** jet cross section:



- One-loop cross section in QCD:
  - in a cone algorithm:

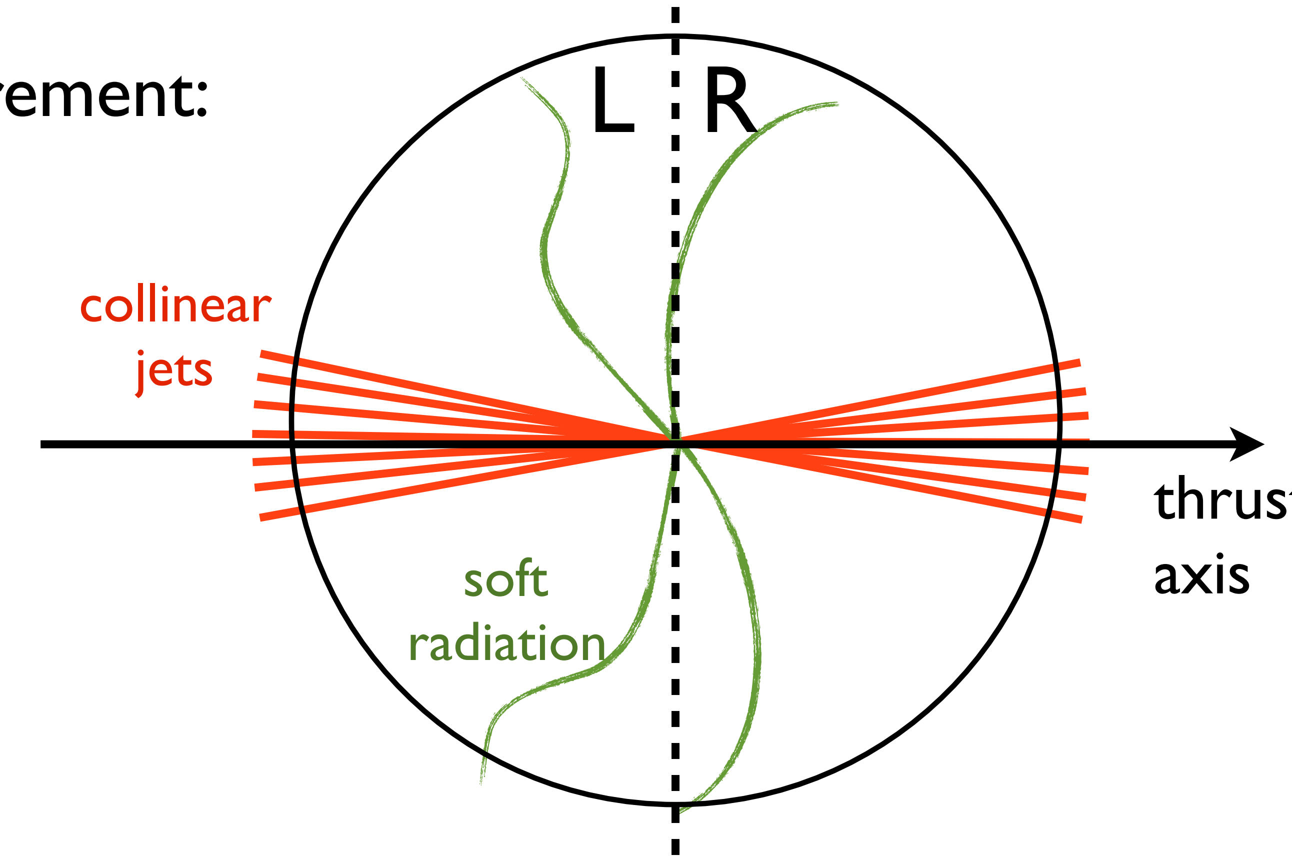
$$\frac{\sigma_{2\text{-jet}}}{\sigma_0} = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln\frac{2E_0}{Q}\ln R - 3\ln R - \frac{1}{2} + 3\ln 2\right)$$

  - in a kT-type recombination (or Sterman-Weinberg) algorithm:

$$\frac{\sigma_{2\text{-jet}}}{\sigma_0} = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln\frac{2E_0}{Q}\ln R - 3\ln R - \frac{\pi^2}{3} + \frac{5}{2}\right)$$

- Natural to use **SCET** to factorize and resume, but structure of logs is surprisingly subtle. Let's first review an application of standard SCET…

Global measurements



- Contrast a *global* event shape measurement:

  e.g. Thrust: $\tau = 1 - \frac{|\mathbf{p}_L| + |\mathbf{p}_R|}{Q}$

  Jet Mass: $m^2 = m_L^2 + m_R^2$

- One-loop cross section in QCD:

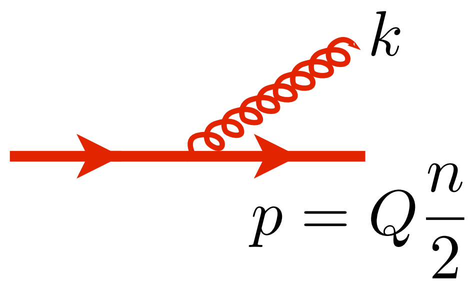
$$\sigma(\tau) = \frac{1}{\sigma_0}\int_0^\tau d\tau' \frac{d\sigma}{d\tau'}$$
$$= 1 - \frac{\alpha_s C_F}{2\pi}\left(4\ln^2\tau + 3\ln\tau + 1 - \frac{2\pi^2}{3}\right) + \mathcal{O}(\tau)$$

- Soft and collinear divergences controlled by *same* measurement: small thrust constrains all energetic radiation to be collimated along jet axis

- All soft radiation in the event captured in the single measurement and probed at a single scale

EFT for Jets

- EFT must match IR behavior of QCD for jet kinematics:

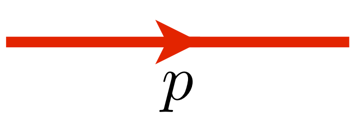


$$\not p\,\gamma^\mu \frac{i(\not p + \not k)}{(p+k)^2} \longrightarrow Qn\cdot k = QE_k(1-\cos\theta_k)$$

singular when $E_k \to 0$ (**soft**) or $\theta_k \to 0$ (**collinear**)

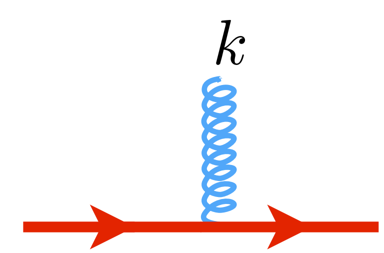
- EFT thus needs modes with soft and collinear momentum scalings
- EFT should reproduce QCD Feynman rules order-by-order in power expansion around soft and collinear limits:

e.g. collinear propagator



$$i\frac{\not p}{p^2} = i\frac{\bar n\cdot p}{\bar n\cdot p\, n\cdot p + p_\perp^2}\frac{\not n}{2}$$

soft gluon vertex



$$\not p\,\gamma^\mu \frac{i(\not p + \not k)}{(p+k)^2} \longrightarrow Q\frac{\not n}{2}\gamma^\mu\frac{iQ}{Qn\cdot k}\frac{\not n}{2} = Q\boxed{\frac{i}{n\cdot k}n^\mu\frac{\not{\bar n}}{2}}$$

eikonal propagator and vertex

Separation of scales

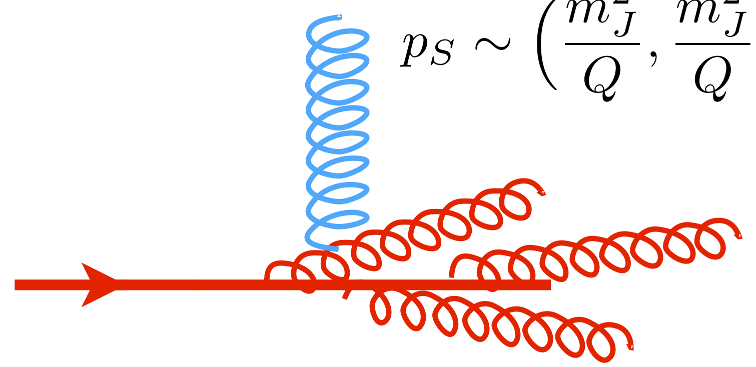
- Large logs in QCD arise from large ratios of physical scales defining the measurement or degree of exclusivity of a jet cross section.
- For jet cross sections, these are precisely ratios of hard to soft scales and ratios of collinear momentum components.
  - e.g. measurement of jet mass

$$p_J^2 = (p_c + p_s)^2 = m_J^2$$

$$p = (\bar{n}\cdot p, n\cdot p, p_\perp)$$

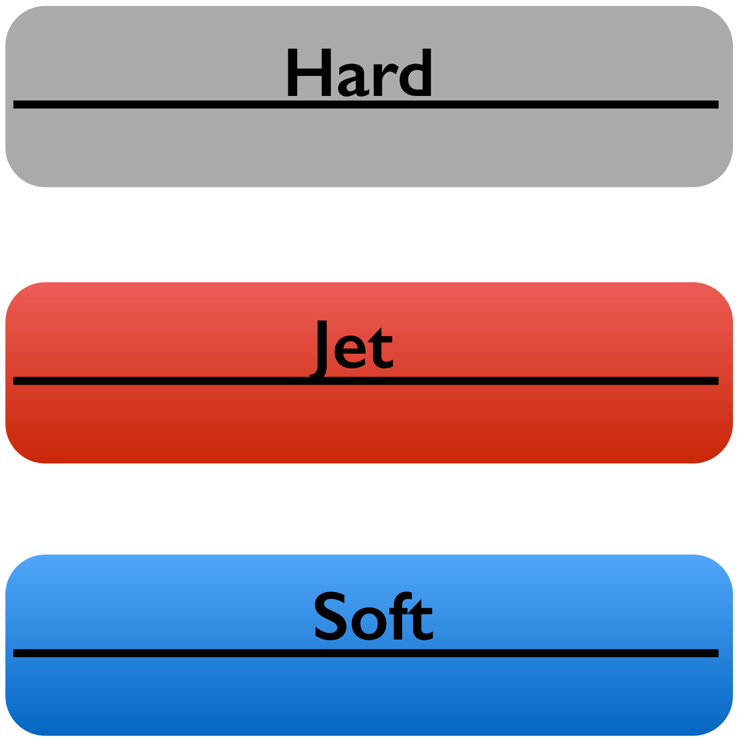
Hierarchy of scales

Factorize cross section into pieces depending on only one of these scales at a time.



$$p_S \sim \left(\frac{m_J^2}{Q}, \frac{m_J^2}{Q}, \frac{m_J^2}{Q}\right)$$

$$p_c \sim \left(Q, \frac{m_J^2}{Q}, m_J\right)$$



$$\mu_H = Q$$

$$\mu_J = m_J$$

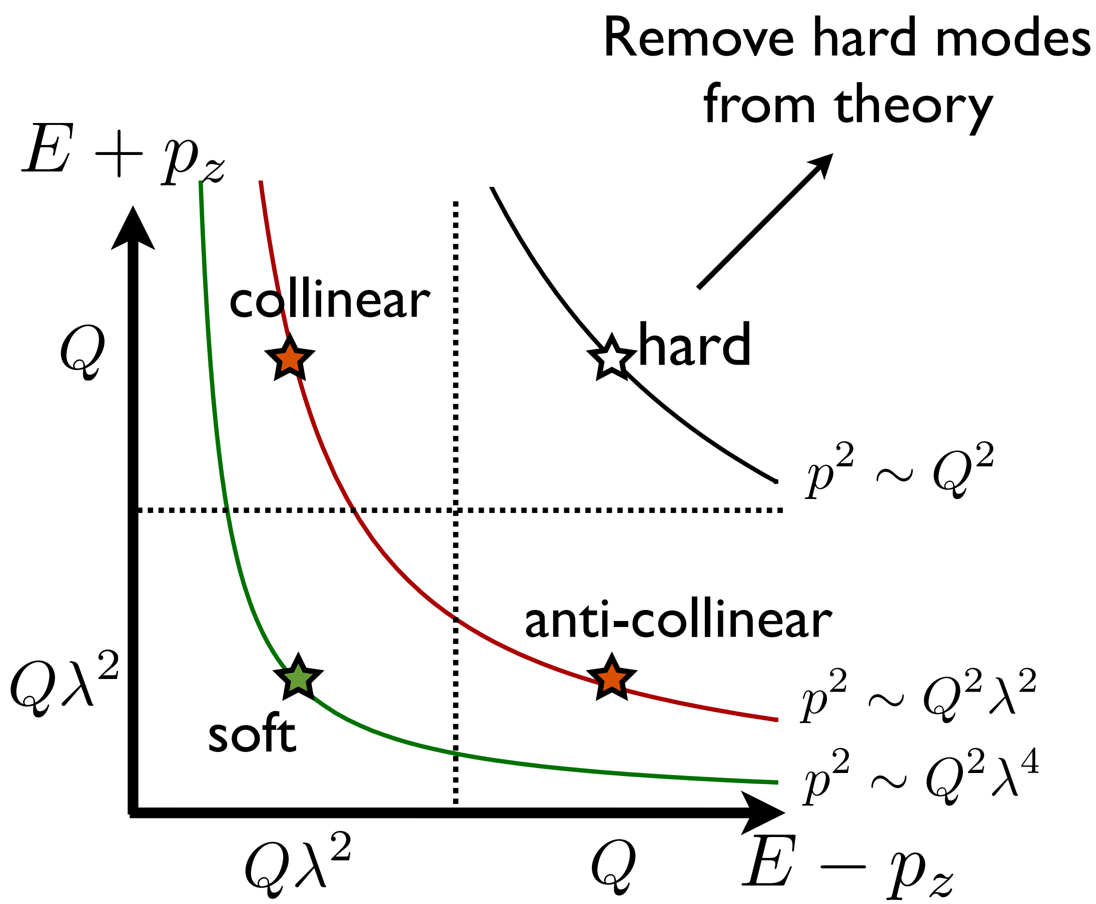
$$\mu_S = \frac{m_J^2}{Q}$$

Bauer, Fleming, Luke (2000)
Bauer, Fleming, Pirjol, Stewart (2001)
Bauer, Fleming, Pirjol, Rothstein, Stewart (2002)

Soft Collinear Effective Theory

- $\mathrm{SCET_I}$

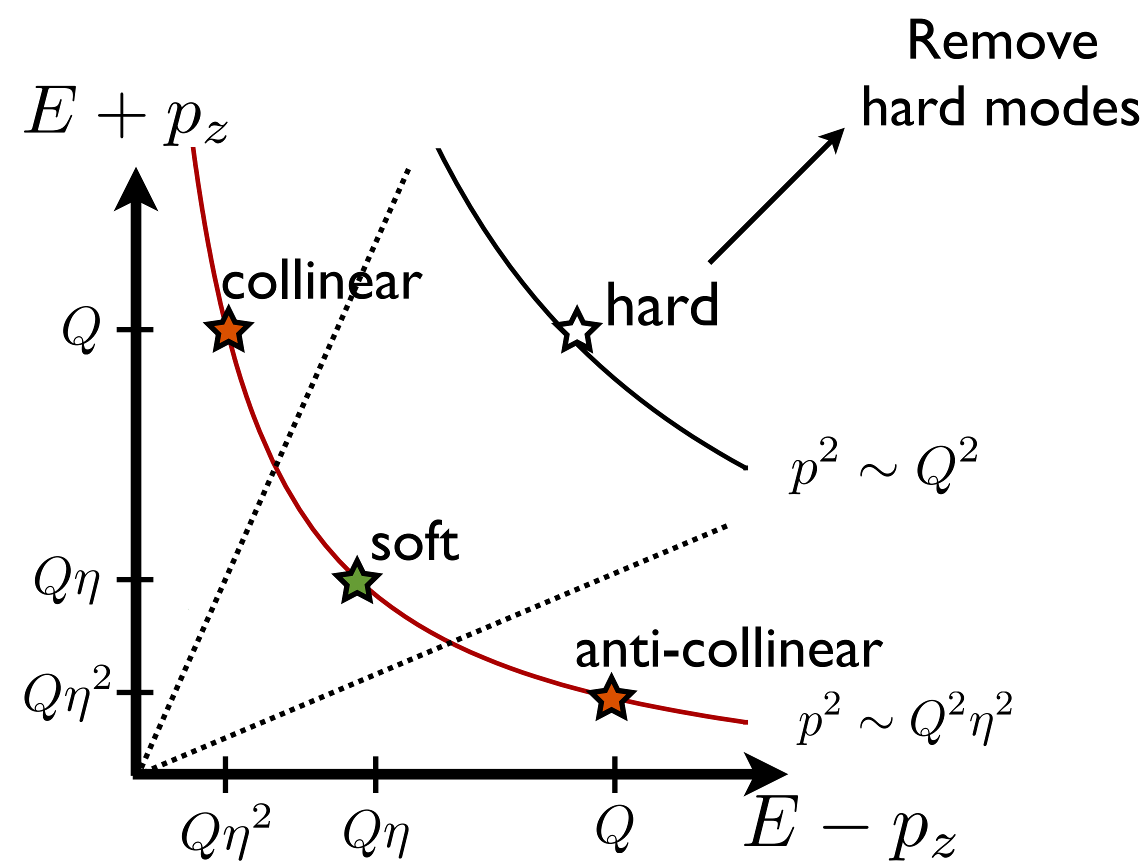
Theory for jets constrained by mass



- Hard, collinear, soft all separated by **virtuality**
- Collinear/soft decoupling and factorization
- Dim. Reg. regulates all divergences

- $\mathrm{SCET_{II}}$

Theory for jets constrained by transverse momentum or for exclusive collinear hadrons

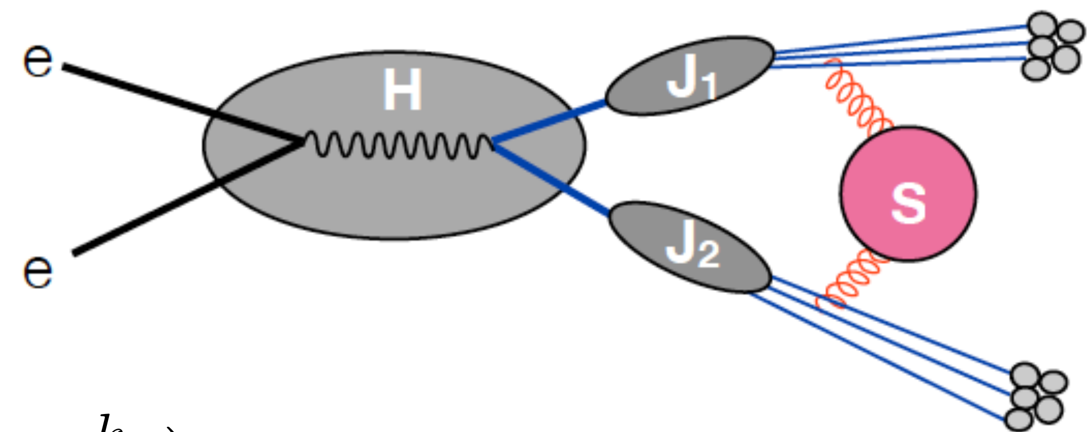


- Hard separated from coll. and soft by virtuality, collinear & soft separated by **rapidity**
- Inherits $\mathrm{SCET_I}$ collinear-soft decoupling
- Dim. Reg. regulates virtuality divergences but not rapidity divergences ➡ need additional regulator

Chiu, Jain, Neill, Rothstein (2011, 2012)

Factorization

Collins, Soper, Sterman

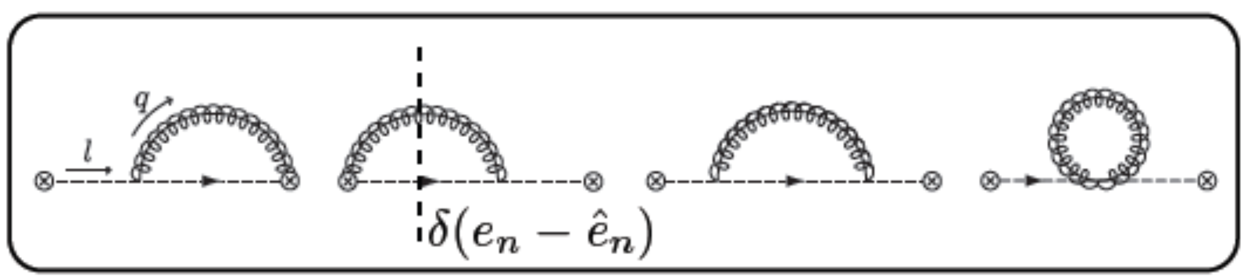


- Consider hemisphere jet mass (thrust) distribution: $Q\tau = m_J^2$
- Factorized cross section takes the form:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H_2(Q^2,\mu)\int dt_n dt_{\bar n} dk_S\,\delta\Big(\tau - \frac{t_n + t_{\bar n}}{Q^2} - \frac{k_S}{Q}\Big) J_n(t_n,\mu) J_{\bar n}(t_{\bar n},\mu) S_2(k_S,\mu)$$

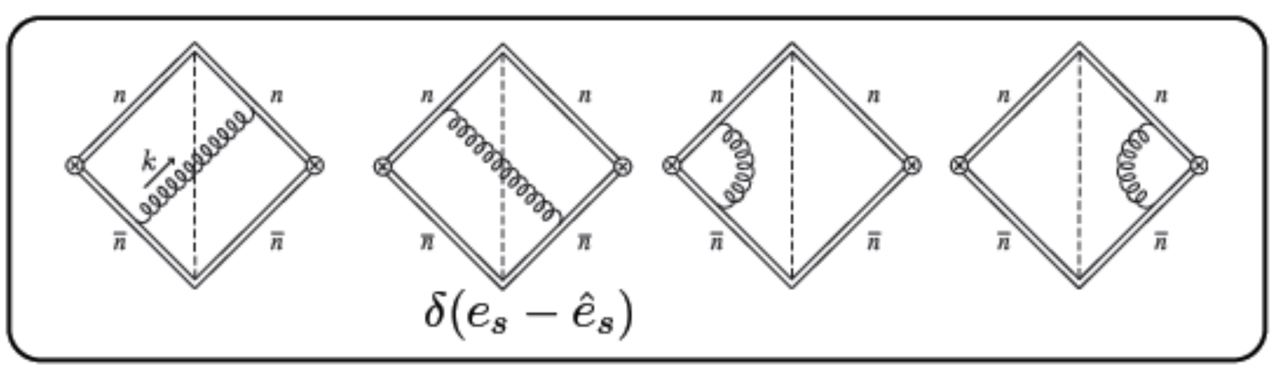
Fleming, Hoang, Mantry, Stewart (2007)
Bauer, Fleming, CL, Sterman (2008)

- Jet Function



Collinear matrix elements with measurement of jet mass $p_c^2 = t_n$

- Soft Function



Soft Wilson line matrix elements with measurement of small light-cone contribution to jet mass $k_S = n \cdot k_s$

Glaubers and factorization violation in SCET:
see Rothstein, Stewart (2016) and others

Hard, Jet, and Soft Functions

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H_2(Q^2,\mu)\int dt_n dt_{\bar{n}} dk_S\, \delta\Big(\tau - \frac{t_n + t_{\bar{n}}}{Q^2} - \frac{k_S}{Q}\Big) J_n(t_n,\mu) J_{\bar{n}}(t_{\bar{n}},\mu) S_2(k_S,\mu)$$

$$H = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left[-8 + \frac{7\pi^2}{6} + \ln^2\frac{\mu^2}{Q^2} + 3\ln\frac{\mu^2}{Q^2}\right]$$ (from matching calculation)

$$\int J(t) = 1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left[2\ln^2\frac{t}{\mu^2} - 3\ln\frac{t}{\mu^2} + 7 - \pi^2\right]$$

$$\int S(k) = 1 + \frac{\alpha_s(\mu)C_F}{4\pi}\left[8\ln^2\frac{k}{\mu} + \frac{\pi^2}{3}\right]$$

natural scales:

$$\mu_H \sim Q$$

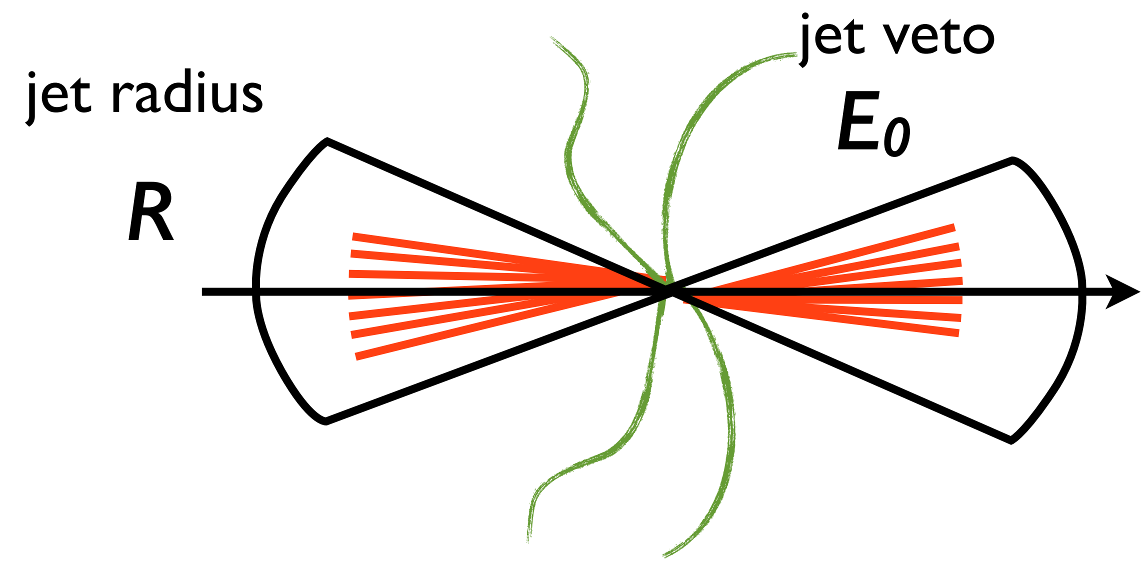
$$\mu_J \sim \sqrt{t} \to Q\sqrt{\tau}$$

$$\mu_S \sim k \to Q\tau$$

- Each function contains logs only of its *single* relevant physical scale, over the arbitrary factorization scale in DR μ
- If each function could be evaluated at its natural scale, the logs would be zero. But we can only pick one μ.
- RG Evolution tells us how to evolve or run each function to another scale.
- Full cross section in QCD is independent of this μ

Attempt: SCET for jet rates?

- Naive construction of SCET for an exclusive jet rate:



Hard $\mu_H = Q$

Jet $\mu_J = QR$ $\quad p_c = Q(1, R^2, R)$

Soft $\mu_S = E_0$ $\quad p_s = (E_0, E_0, E_0)$

- Naive factorization:

Ellis, Hornig, CL, Vermilion, Walsh (2010)

$$\sigma^{\text{alg}}_{2\text{-jet}} = H_2(Q^2, \mu) J^{\text{alg}}_{\text{un}}(QR, \mu)^2 S_{\text{veto}}(E_0, R, \mu)$$

  - No convolution between jet and soft since they do not “talk” or contribute to the same measurement

Ellis, Hornig, CL, Vermilion, Walsh (2010)

Attempt: SCET for jet rates?

- "Unmeasured" jet function:

$$J_{\rm un}^{\rm alg}(QR,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(4\ln^2\frac{\mu}{QR} + 6\ln\frac{\mu}{QR} + c_J^{1,\rm alg}\right)$$

$$c_J^{1,\rm cone} = 7 + 6\ln 2 - \frac{5\pi^2}{6} \qquad c_J^{1,\rm kT} = 13 - \frac{3\pi^2}{2}$$

  - note: different double log than "measured" jet function. puzzle: **why?**

$$\int J(t) = 1 + \frac{\alpha_s C_F}{4\pi}\left(8\ln^2\frac{\mu}{\sqrt{t}} + 6\ln\frac{\mu}{\sqrt{t}} + 7 - \pi^2\right)$$

- Jet veto soft function:

$$S_{\rm veto}(E_0,R,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(8\ln R\ln\frac{\mu^2}{4E_0^2 R} - \frac{2\pi^2}{3}\right)$$

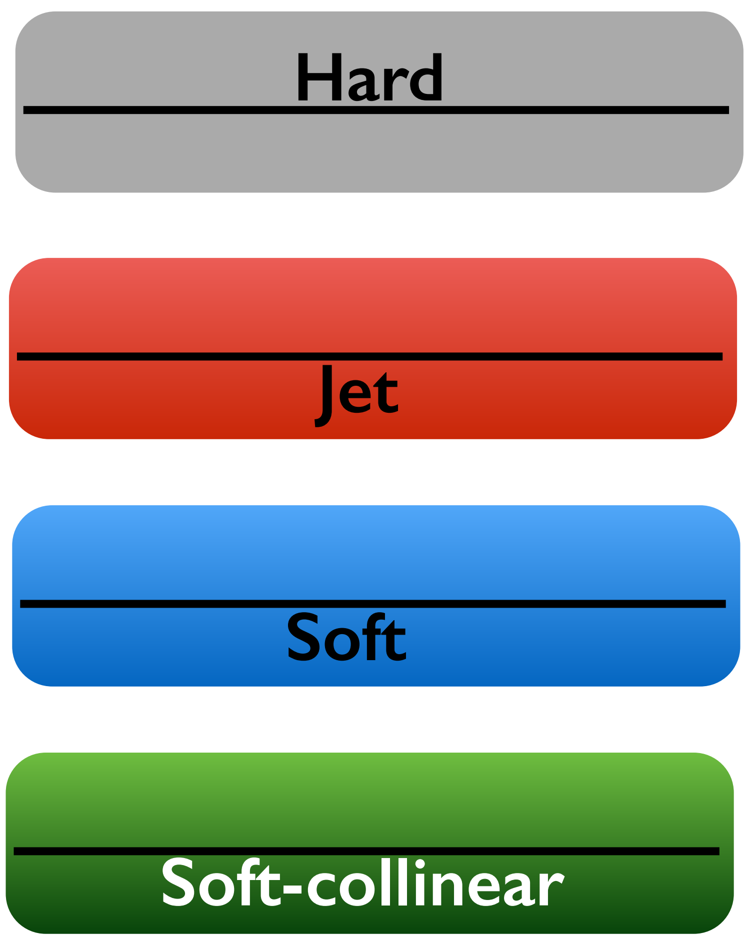
  - puzzle: what is the correct soft scale? $2E_0$? $2E_0\sqrt{R}$?

Clue to the soft-collinear mode

- Jet veto soft function can be rewritten:

$$S_{\rm veto}(E_0, R, \mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(8\ln^2\frac{\mu}{2E_0} - 8\ln^2\frac{\mu}{2E_0 R} - \frac{2\pi^2}{3}\right)$$

- Sensitivity to a new scale: $2E_0R$



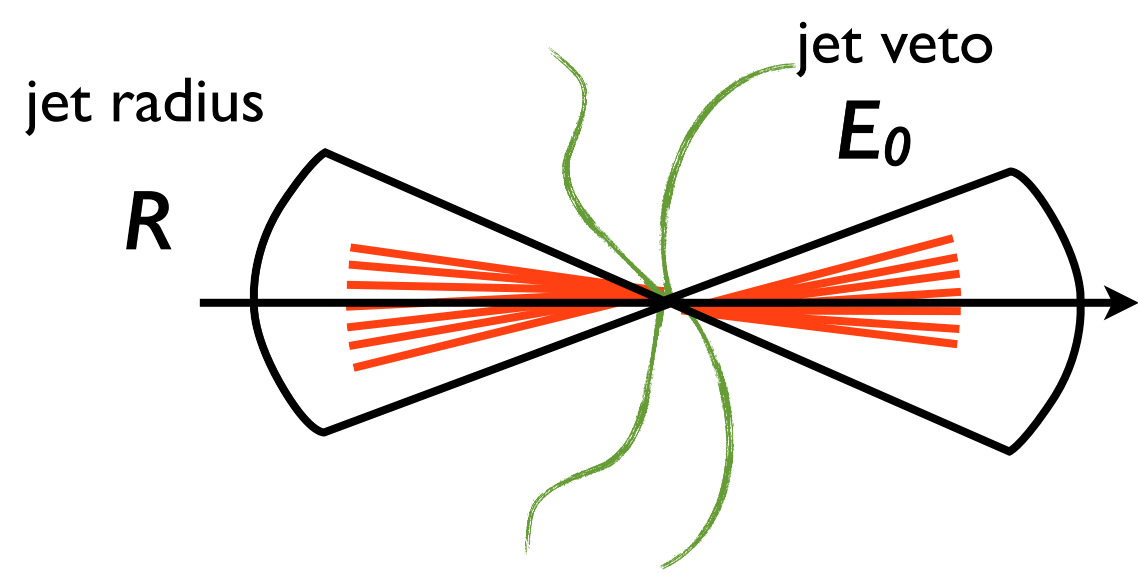
$\mu_H = Q$

$\mu_J = QR$

$\mu_S = 2E_0$

$\mu_{sc} = 2E_0R$

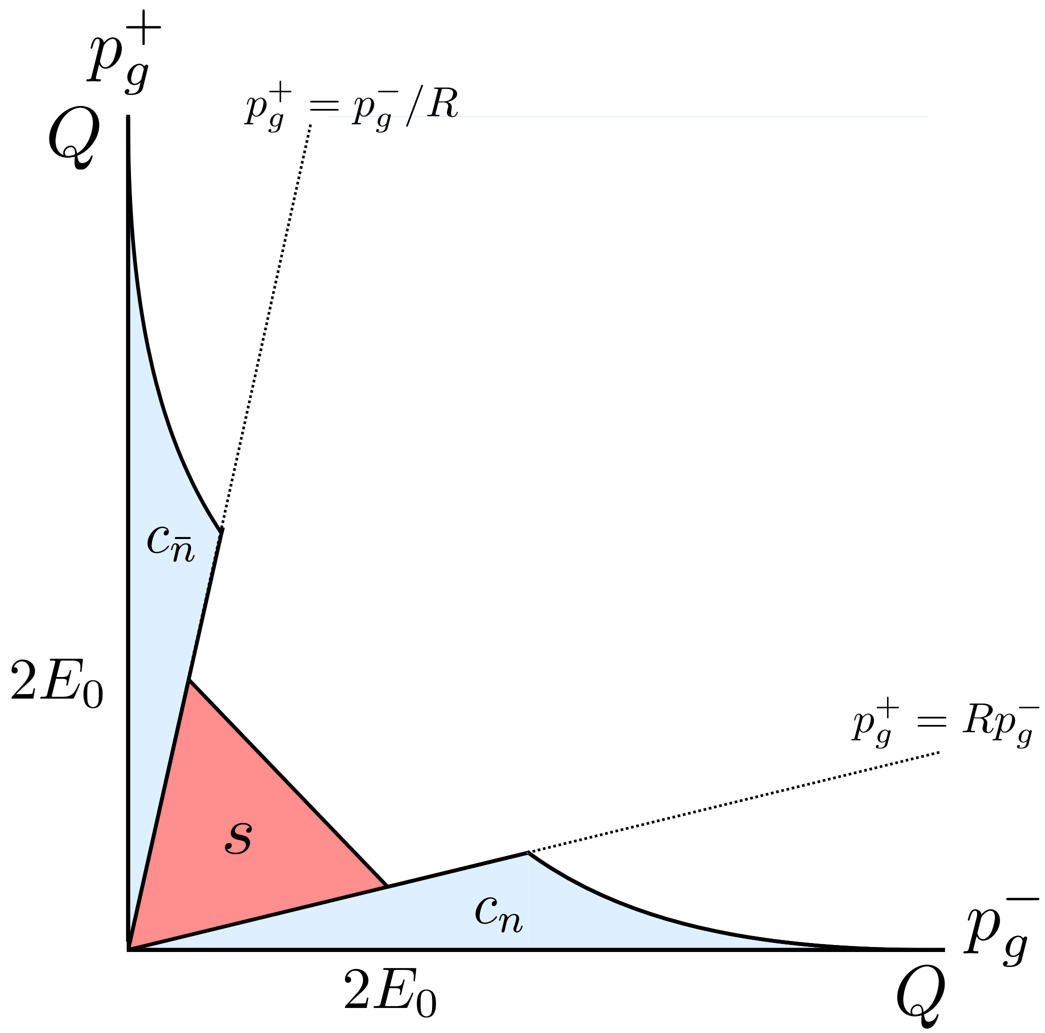
- The soft radiation at *E₀* is sensitive to the cone angle by being forced ***outside*** of it:

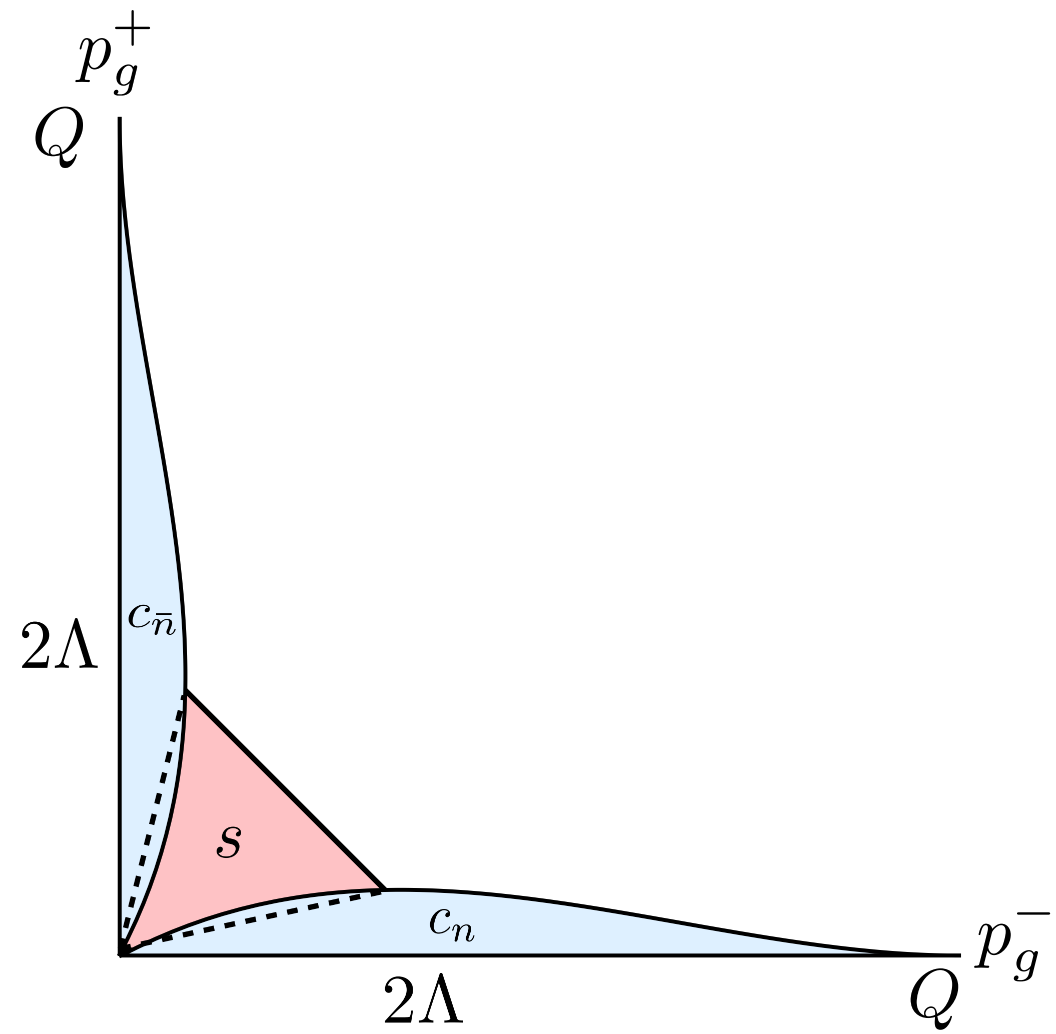


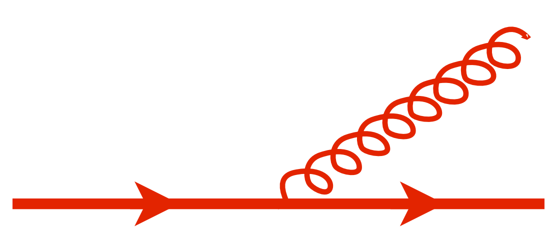
Becher, Neubert, Rothen, Shao (2015)
Chien, Hornig, CL (2015)

Soft and Soft-Collinear phase space

- collinear and soft phase space for cone and kT algorithms:



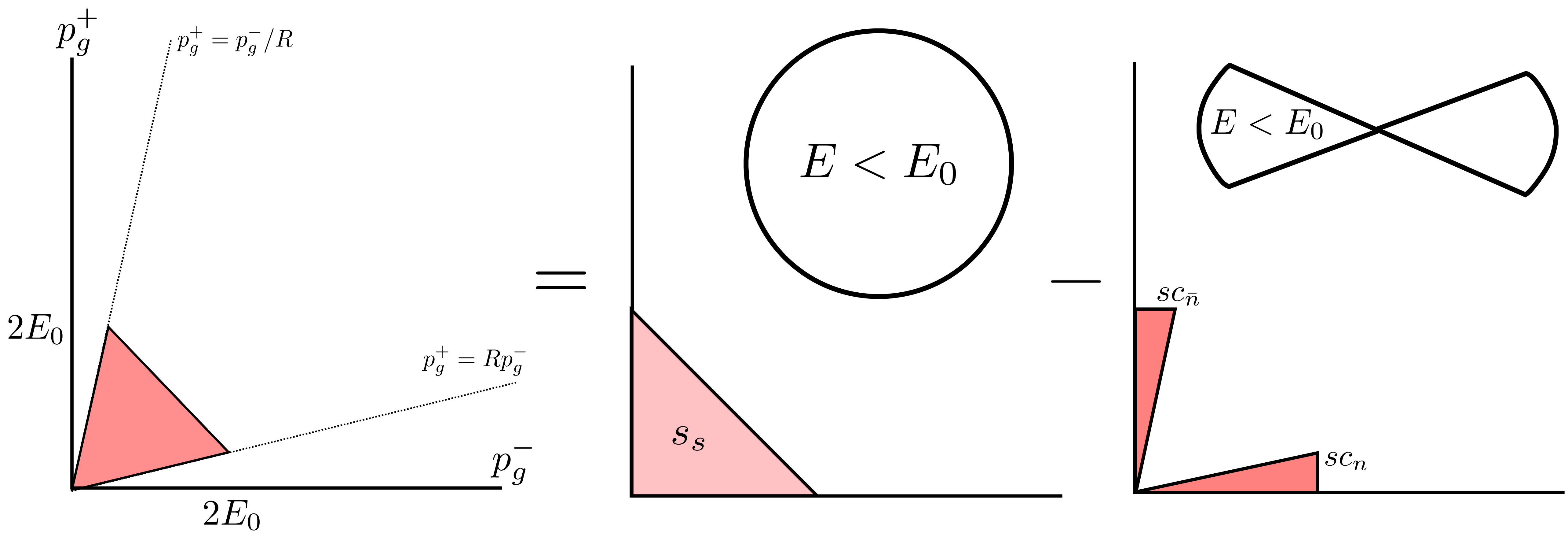




Ellis, Hornig, CL, Vermilion, Walsh (2010)
Chien, Hornig, CL (2015)

Soft and Soft-Collinear phase space

- Soft phase space splits into two, single-scale-sensitive regions:

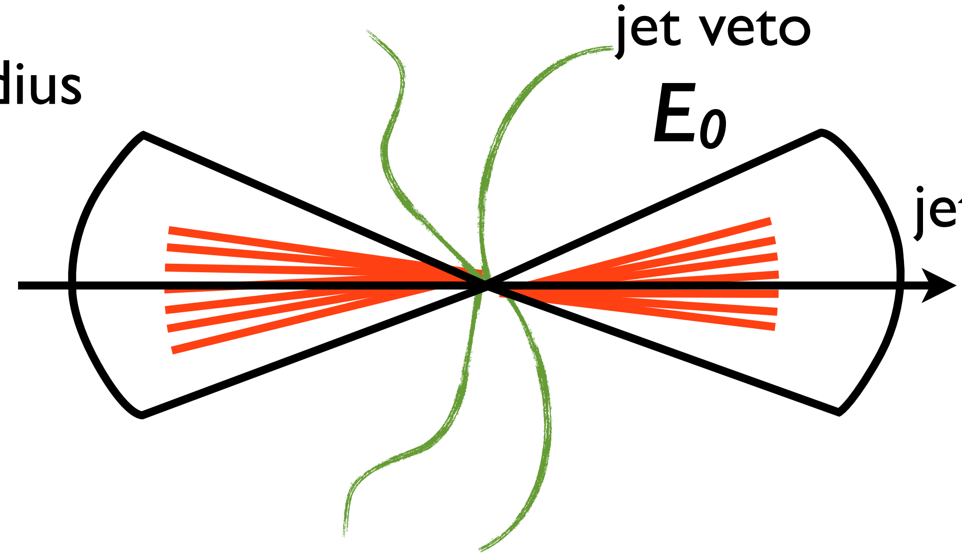


$S_{\rm veto}(E_0, R, \mu)$ $\qquad$ $S_s(E_0, \mu)$ $\qquad$ $-2S_{sc}(E_0 R, \mu)$

$$\frac{\alpha_s C_F}{4\pi}\left(8\ln R\ln\frac{\mu^2}{4E_0^2 R} - \frac{2\pi^2}{3}\right) \qquad \frac{\alpha_s C_F}{4\pi}\left(8\ln^2\frac{\mu}{2E_0} - \pi^2\right) \qquad \frac{\alpha_s C_F}{4\pi}\left(-8\ln^2\frac{\mu}{2E_0 R} + \frac{\pi^2}{3}\right)$$

Finer probe: Jet thrust

- We can learn much about the factorization structure of the jet cross section by measuring a more differential probe like jet thrust:



$$\tau = \frac{1}{2E_J} \sum_{i \in J} n \cdot p_i$$

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{alg}}(E_0,R)}{d\tau} = H_2(Q^2,\mu) \int dt_n dt_{\bar{n}} dk_s J_n^{\text{alg}}(t_n,\mu) J_{\bar{n}}^{\text{alg}}(t_{\bar{n}},\mu) S(k_s,E_0,R,\mu)\delta\left(\tau - \frac{t_n + t_{\bar{n}}}{Q^2} - \frac{k_s}{Q}\right)$$

- "Measured" jet function same as global thrust jet function, plus algorithm-dependent power correction:

Jouttenus (2009)
Ellis, Hornig, CL, Vermilion, Walsh (2010)

$$\int J_n^{\text{cone}}(t) = 1 + \frac{\alpha_s C_F}{4\pi}\left(2\ln^2\frac{t}{\mu^2} - 3\ln\frac{t}{\mu^2} + 7 - \pi^2\right) + \Delta J^{\text{cone}}(t)$$

$$\Delta J^{\text{cone}}(t) = \theta(Q^2R^2 - t)\frac{\alpha_s C_F}{4\pi} 6\ln\frac{t + Q^2R^2}{Q^2R^2}$$

Jet thrust+veto soft function

- One-loop soft function:

$$S(k, E_0, R, \mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(4\ln R \ln\frac{\mu^2}{4E_0^2 R} - \frac{\pi^2}{3} - 4\ln^2\frac{k}{\mu R}\right)$$

- Suggests further separation of scales:

$$S(k, E_0, R, \mu) = S_{\text{in}}\left(\frac{k}{R}, \mu\right)^2 S_s(E_0, \mu) S_{sc}^2(E_0 R, \mu)$$

$$S_{\text{in}} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{\pi^2}{6} - 4\ln^2\frac{k}{\mu R}\right)$$

larger in-jet soft scale, by 1/R!

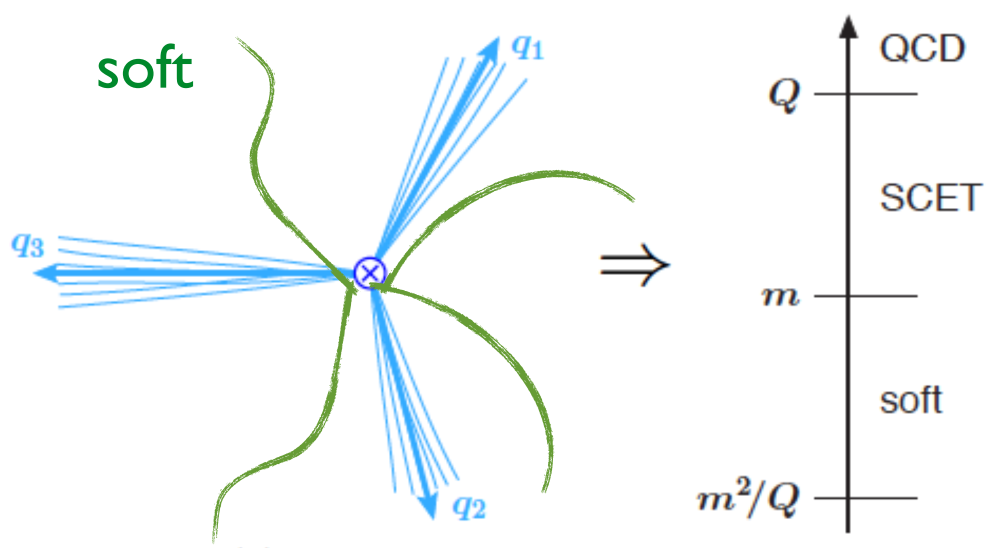
$$S_s = 1 + \frac{\alpha_s C_F}{4\pi}\left(8\ln^2\frac{\mu}{2E_0} - \pi^2\right)$$

$$S_{sc} = 1 + \frac{\alpha_s C_F}{4\pi}\left(-4\ln^2\frac{\mu}{2E_0 R} + \frac{\pi^2}{6}\right)$$

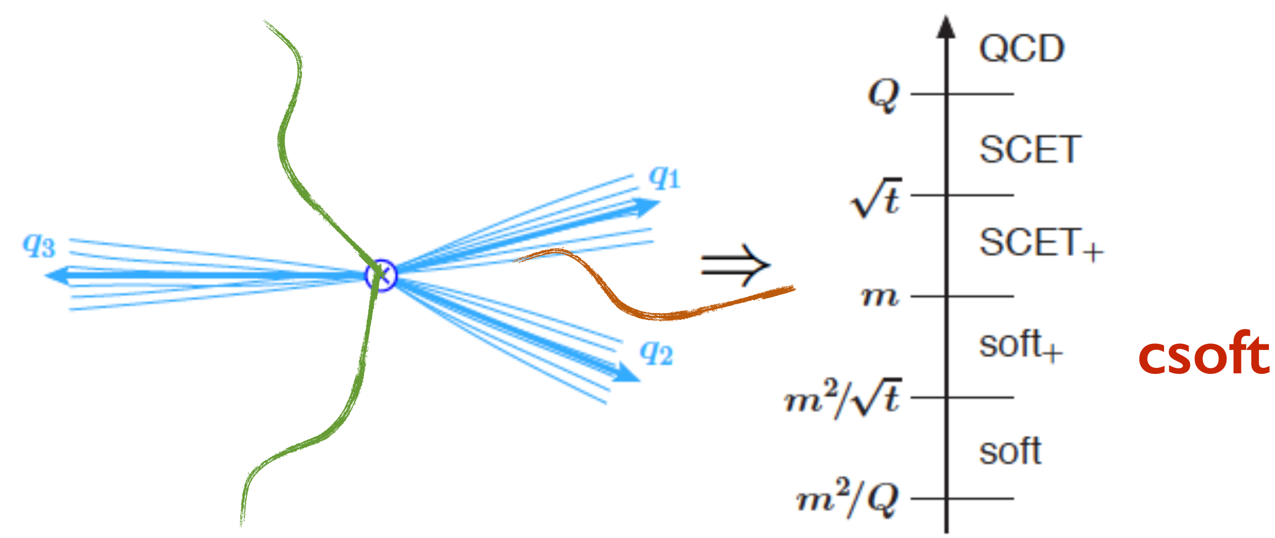
SCET$_+$ and Subjets

- First introduced for (sub)jets that get close together:

Bauer, Tackmann, Walsh, Zuberi (2011)



(a) All jets equally separated.

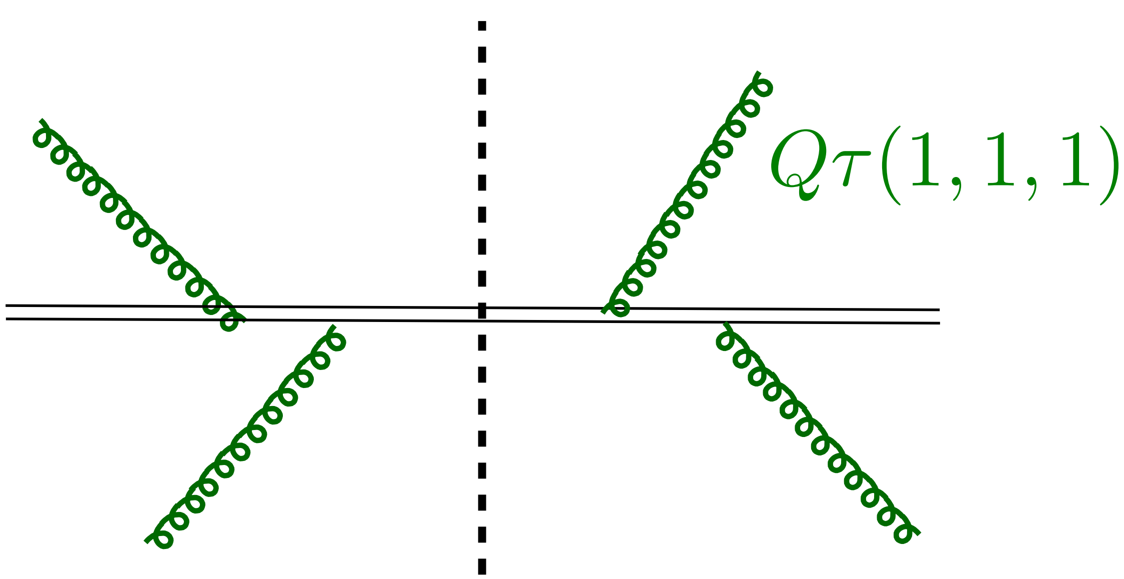


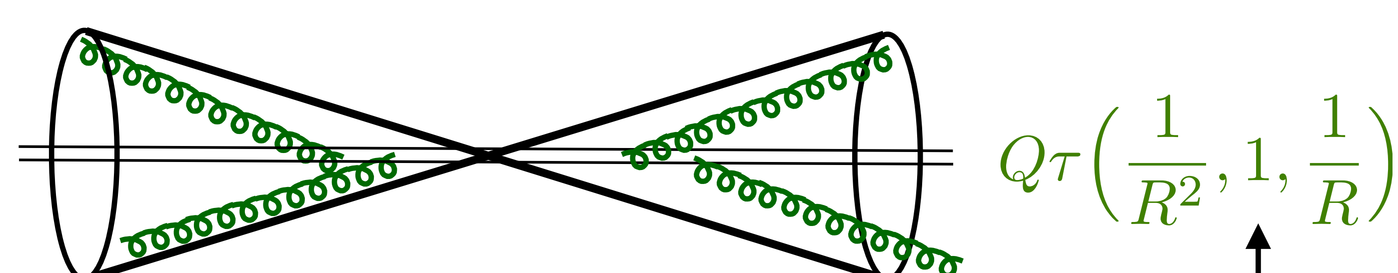
(b) Two jets close to each other.

- "Collinear-soft" scale appears here because of squeezing by cone:

Ellis, Hornig, CL, Vermilion, Walsh (2010)

Chien, Hornig, CL (2015)

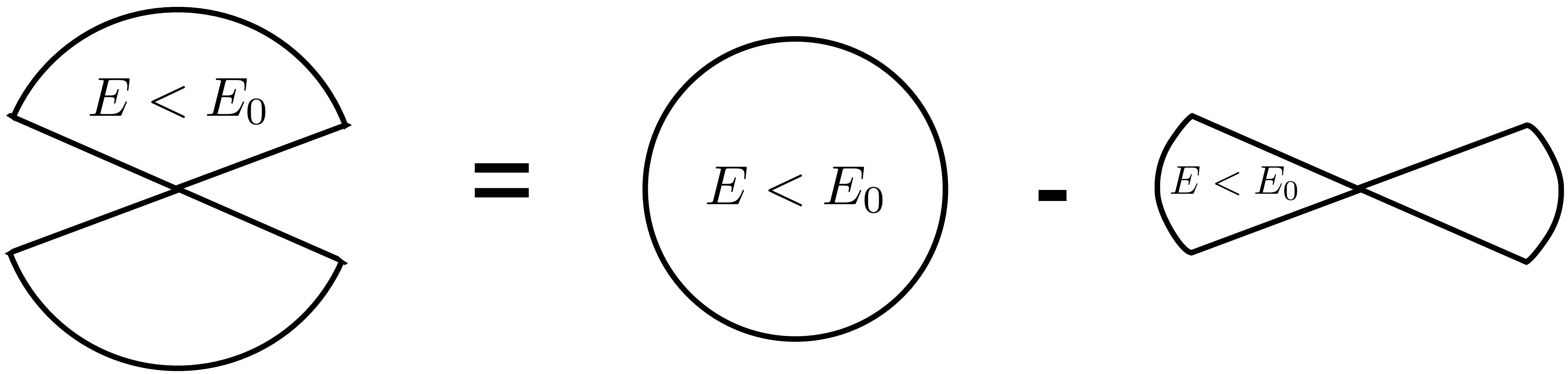




smallest component is measured, others *increase* by collinearity *R*

Soft-collinear mode

- Soft-c mode, in contrast, appears due to cone size sensitivity ***outside***:



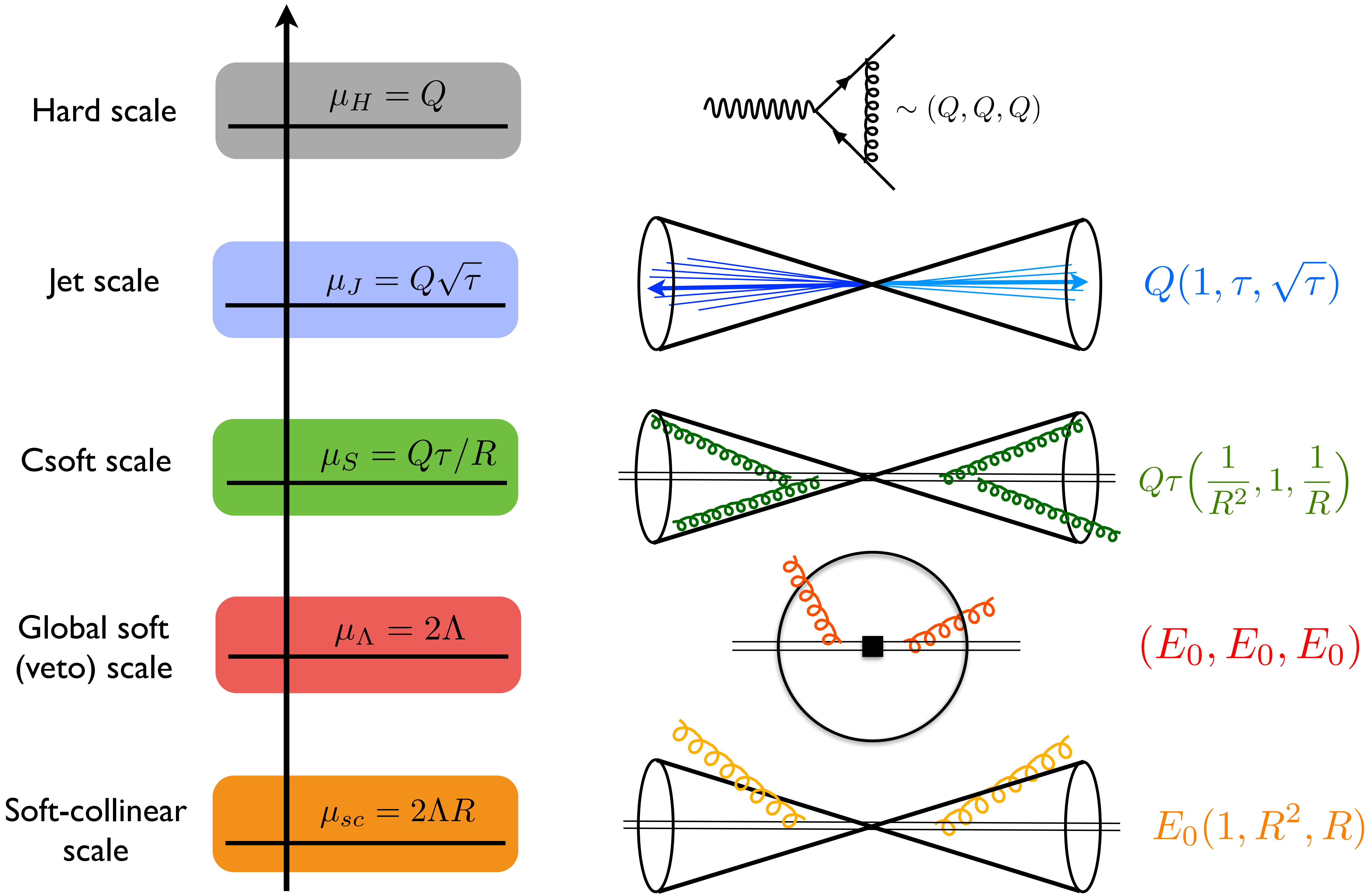
- Effect of cone restriction is **opposite** to csoft mode, **reducing** scale:

$$E_0(1,1,1) \longrightarrow E_0(1,R^2,R)$$

measuring energy constrains **largest** component of momentum, others **decrease** by collinearity *R*

SCET++

Chien, Hornig, CL (2015)



Checks and Consequences

- 1-loop factorization is easy to check: see above
- First nontrivial check is at two loops
  - Luckily, the full (but unrefactorized) two-loop jet thrust + veto soft function $S(k, E_0, R, \mu)$ was computed von Manteuffel, Schabinger, Zhu (2013)
  - Disentangling this result to achieve factorization and resummation of logs *R* required identifying all the scales in SCET++. Chien, Hornig, CL (2015)
    - Showed it takes exactly the form predicted by the SCET++ factorization
    - Extracted evolution $\gamma_{\mathrm{in},ss,sc}$ to 2 loops from vMSZ result, and in fact proved an all-orders relation to global thrust soft anom. dim. γ_{hemi} which is known to three loops
- Leads to confidence in all orders factorization, but some extensions and formal proofs are still needed.

work in progress:
Chien, Neill, CL, Ringer (2017)

Jet thrust+veto soft function

- Two-loop soft function: $S^c(k,\Lambda,R,\mu) = S_{C_F}(k,\Lambda,R,\mu) + \left(\frac{\alpha_s}{4\pi}\right)^2 S^{(2)}_{nA}(k,\Lambda,R,\mu)\,,$

$$S_{C_F}(k,\Lambda,R,\mu) = 1 + \frac{\alpha_s}{4\pi}\Big[2\Gamma_0\Big(-\ln^2\frac{\mu R}{k} + \ln R\,\ln\frac{\mu^2}{4\Lambda^2 R}\Big) - \frac{\pi^2}{3}C_F\Big] + \left(\frac{\alpha_s}{4\pi}\right)^2\Big\{2(\Gamma_0)^2\Big(-\ln^2\frac{\mu R}{k} + \ln R\,\ln\frac{\mu^2}{4\Lambda^2 R}\Big)^2 + 2\Gamma_0\frac{\pi^2}{3}C_F\Big(\ln^2\frac{\mu R}{k} - \ln R\,\ln\frac{\mu^2}{4\Lambda^2 R}\Big) - \frac{4\pi^2}{3}(\Gamma_0)^2\Big(\ln^2\frac{\mu R}{k} + \ln^2 R\Big) - 16\zeta_3\Gamma_0^2\ln\frac{\mu R}{k} + c^{(2)}_{C_F}\Big\}, \quad (65)$$

$$\begin{aligned} S^{(2)}_{nA[38]}(k,\Lambda,R,\mu) = C_A C_F\Big[&-\frac{176}{9}\ln^3\frac{\mu}{k} + \Big(-\frac{176\ln R}{3} + \frac{8\pi^2}{3} - \frac{536}{9}\Big)\ln^2\frac{\mu}{k} \\ &+ \Big(-\frac{176}{3}\ln^2 R + \frac{16}{3}\pi^2\ln R - \frac{1072}{9}\ln R + 56\zeta_3 + \frac{44\pi^2}{9} - \frac{1616}{27}\Big)\ln\frac{\mu}{k} \\ &+ \Big(-\frac{176}{3}\ln^2 R - \frac{16}{3}\pi^2\ln R + \frac{1072}{9}\ln R - \frac{44\pi^2}{9}\Big)\ln\frac{\mu}{2\Lambda} + \frac{176}{3}\ln R\ln^2\frac{\mu}{2\Lambda} - \frac{8}{3}\pi^2\ln^2\frac{k}{2\Lambda R^2} \\ &+ \Big(-16\zeta_3 - \frac{8}{3} + \frac{88\pi^2}{9}\Big)\ln\frac{k}{2\Lambda R^2} - \frac{682\zeta_3}{9} + \frac{109\pi^4}{45} - \frac{1139\pi^2}{54} - \frac{1636}{81}\Big] \\ + C_F T_F n_f\Big[&\frac{64}{9}\ln^3\frac{\mu}{k} + \Big(\frac{64\ln R}{3} + \frac{160}{9}\Big)\ln^2\frac{\mu}{k} + \Big(\frac{64}{3}\ln^2 R + \frac{320}{9}\ln R - \frac{16\pi^2}{9} + \frac{448}{27}\Big)\ln\frac{\mu}{k} \\ &+ \Big(\frac{64}{3}\ln^2 R - \frac{320}{9}\ln R + \frac{16\pi^2}{9}\Big)\ln\frac{\mu}{2\Lambda} - \frac{64}{3}\ln R\ln^2\frac{\mu}{2\Lambda} + \Big(\frac{16}{3} - \frac{32\pi^2}{9}\Big)\ln\frac{k}{2\Lambda R^2} \\ &+ \frac{248\zeta_3}{9} + \frac{218\pi^2}{27} - \frac{928}{81}\Big] - 4\Gamma_1\ln^2 R\,. \end{aligned} \quad \text{(C1)}$$

von Manteuffel, Schabinger, Zhu (2013)

Factored 2-loop soft function

- Recognizing presence of csoft, soft, and soft-collinear scales:

$$
\begin{aligned}
S^{(2)}_{nA[38]} &= \frac{4}{3}\Gamma_0\beta_0\Big(-\ln^3\frac{\mu R}{k}+\ln^3\frac{\mu}{2\Lambda}-\ln^3\frac{\mu}{2\Lambda R}\Big)+2\Gamma_1\Big(-\ln^2\frac{\mu R}{k}+\ln R\,\ln\frac{\mu^2}{4\Lambda^2 R}\Big)\\
&+\Big[\Big(56\zeta_3-\frac{1616}{27}\Big)C_FC_A+\frac{448}{27}C_FT_Fn_f+\frac{4\pi^2}{3}\beta_0C_F\Big]\ln\frac{\mu R}{k}\\
&+\Big[\Big(\frac{1616}{27}-56\zeta_3\Big)C_FC_A-\frac{448}{27}C_FT_Fn_f-\frac{8\pi^2}{3}\beta_0C_F\Big]\ln\frac{\mu}{2\Lambda}\\
&+\Big[\Big(56\zeta_3-\frac{1616}{27}\Big)C_FC_A+\frac{448}{27}C_FT_Fn_f+\frac{4\pi^2}{3}\beta_0C_F\Big]\ln\frac{\mu}{2\Lambda R}+c^{(2)}_{nA}\\
&-\frac{8}{3}\pi^2C_FC_A\ln^2\frac{k}{2\Lambda R^2}+\Big[\Big(-16\zeta_3-\frac{8}{3}\Big)C_FC_A+\frac{16}{3}C_FT_Fn_f+\frac{8\pi^2}{3}\beta_0C_F\Big]\ln\frac{k}{2\Lambda R^2}\,.
\end{aligned}
$$

- Structure only apparent once relevant scales are all identified:

$$
\begin{aligned}
S(k_n,k_{\bar n},\Lambda,R,\mu) &= \int_0^\Lambda dE\, S_{\rm in}(k_n/R,\mu)S_{\rm in}(k_{\bar n}/R,\mu)\\
&\times S_s(E,\mu)\otimes S^2_{sc}(ER,\mu)\otimes S_{ng}(k_{n,\bar n},E,R)\,, \quad (50)
\end{aligned}
$$

Chien, Hornig, CL (2015)

Factored 2-loop soft function

- Collinear-soft function:

$$S_{\rm in}^{\rm c}(k/R,\mu) = 1 + \frac{\alpha_s}{4\pi}\Big(-\Gamma_0 \ln^2\frac{\mu R}{k} + c_{\rm in}^1\Big) + \Big(\frac{\alpha_s}{4\pi}\Big)^2\bigg[\frac{1}{2}\Gamma_0^2 \ln^4\frac{\mu R}{k} - \frac{2}{3}\Gamma_0\beta_0\ln^3\frac{\mu R}{k} + \Big(-\Gamma_1 - c_{\rm in}^1\Gamma_0 - \frac{\pi^2}{3}(\Gamma_0)^2\Big)\ln^2\frac{\mu R}{k} + \big(\gamma_{\rm in}^1 + 2c_{\rm in}^1\beta_0 - 4\zeta_3(\Gamma_0)^2\big)\ln\frac{\mu R}{k} + c_{\rm in}^2\bigg], \tag{119}$$

- Global (veto) soft function:

$$S_s^{\rm c}(\Lambda,\mu) = 1 + \frac{\alpha_s}{4\pi}\Big(2\Gamma_0 \ln^2\frac{\mu}{2\Lambda} + c_{ss}^1\Big) + \Big(\frac{\alpha_s}{4\pi}\Big)^2\bigg[2\Gamma_0^2 \ln^4\frac{\mu}{2\Lambda} + \frac{4}{3}\Gamma_0\beta_0\ln^3\frac{\mu}{2\Lambda} + \Big(2\Gamma_1 + 2c_{ss}^1\Gamma_0 - \frac{4\pi^2}{3}(\Gamma_0)^2\Big)\ln^2\frac{\mu}{2\Lambda} + \big(\gamma_{ss}^1 + 2c_{ss}^1\beta_0 - 16\zeta_3(\Gamma_0)^2\big)\ln\frac{\mu}{2\Lambda} + c_{ss}^2\bigg], \tag{120}$$

- Soft-collinear function:

$$S_{sc}^{\rm c}(\Lambda R,\mu) = 1 + \frac{\alpha_s}{4\pi}\Big(-\Gamma_0 \ln^2\frac{\mu}{2\Lambda R} + c_{sc}^1\Big) + \Big(\frac{\alpha_s}{4\pi}\Big)^2\bigg[\frac{1}{2}\Gamma_0^2 \ln^4\frac{\mu}{2\Lambda R} - \frac{2}{3}\Gamma_0\beta_0\ln^3\frac{\mu}{2\Lambda R} + \Big(-\Gamma_1 - c_{sc}^1\Gamma_0 - \frac{\pi^2}{3}(\Gamma_0)^2\Big)\ln^2\frac{\mu}{2\Lambda R} + \big(\gamma_{sc}^1 + 2c_{sc}^1\beta_0 - 4\zeta_3(\Gamma_0)^2\big)\ln\frac{\mu}{2\Lambda R} + c_{sc}^2\bigg], \tag{121}$$

Chien, Hornig, CL (2015)

Resumed jet thrust cross section

- Thanks to this organized structure, we were able to deduce the cusp anomalous dimensions to three loops and non-cusp anomalous dimensions of each piece to two loops

$$\gamma_{ss}^{1} = -2\gamma_{\rm in}^{1} = -2\gamma_{sc}^{1} = C_F\left[\left(\frac{1616}{27} - 56\zeta_3\right)C_A - \frac{448}{27}T_F n_f - \frac{2\pi^2}{3}\beta_0\right]$$

- and an all-orders relation:

$$\gamma_{\rm hemi} = \gamma_{\rm in} = \gamma_{sc} = -\frac{\gamma_{ss}}{2}$$

$\gamma_{\rm hemi}$ → known to 3 loops!

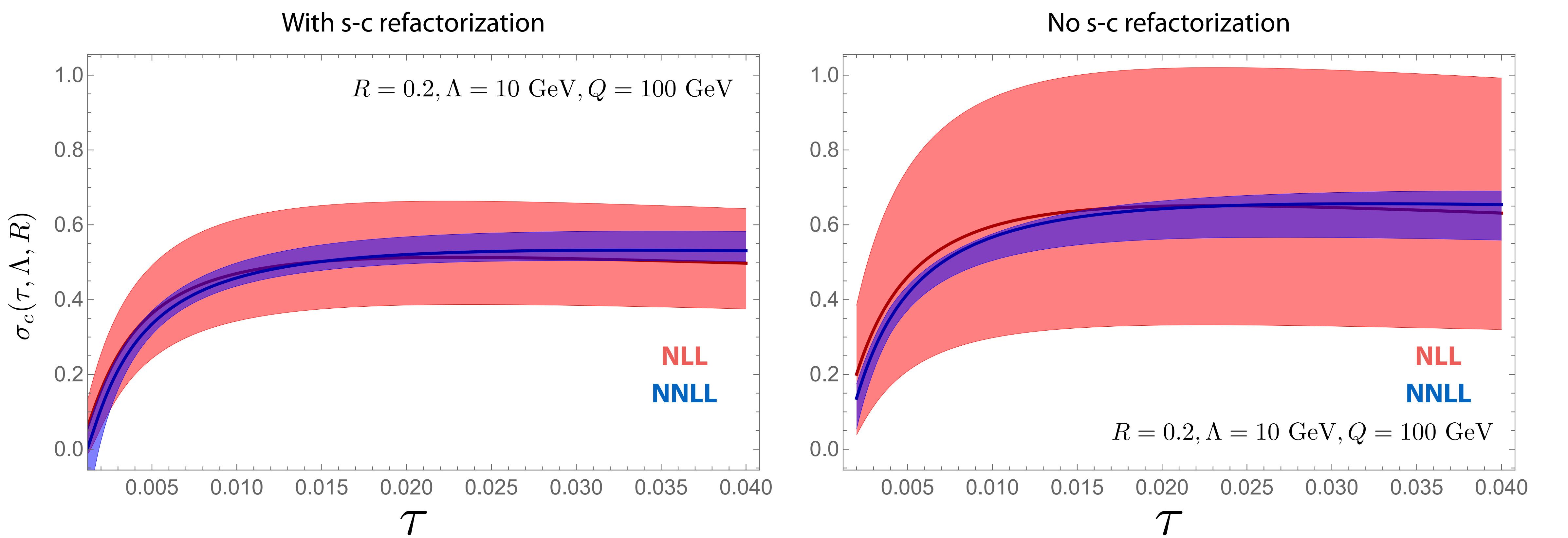
- Resummed cross section from RG evolution in SCET++

$$\sigma_c(\tau, E_0, R) = e^{\mathcal{K}(\mu_H,\mu_J,\mu_{\rm in},\mu_{ss},\mu_{sc},\mu)}\left(\frac{\mu_H}{Q}\right)^{\omega_H(\mu_H,\mu)}\left(\frac{\mu_J^2}{Q^2\tau}\right)^{2\omega_J(\mu_J,\mu)}\left(\frac{\mu_{\rm in}R}{Q\tau}\right)^{2\omega_{\rm in}(\mu_{\rm in},\mu)}$$
$$\times\left(\frac{\mu_{ss}}{2E_0}\right)^{\omega_{ss}(\mu_{ss},\mu)}\left(\frac{\mu_{sc}}{2E_0R}\right)^{2\omega_{sc}(\mu_{sc},\mu)} H(Q^2,\mu_H)\theta(\tau)\theta(E_0)$$
$$\times\widetilde{J}\left(\partial_\Omega + \ln\frac{\mu_J^2}{Q^2\tau},\mu_J\right)^2\widetilde{S}_{\rm in}\left(\partial_\Omega + \ln\frac{\mu_{\rm in}}{Q\tau},\mu_{\rm in}\right)^2\frac{e^{\gamma_E\Omega}}{\Gamma(1-\Omega)}$$
$$\times\widetilde{S}_s\left(\partial_\Upsilon + \ln\frac{\mu_{ss}}{2E_0},\mu_{ss}\right)\widetilde{S}_{sc}\left(\partial_\Upsilon + \ln\frac{\mu_{sc}}{2E_0R},\mu_{sc}\right)^2\frac{e^{\gamma_E\Upsilon}}{\Gamma(1-\Upsilon)}\otimes S_{ng}(Q\tau/(2E_0R^2))$$

Chien, Hornig, CL (2015)

Resummed jet thrust cross section

- Integrated jet thrust in e^+e^-:

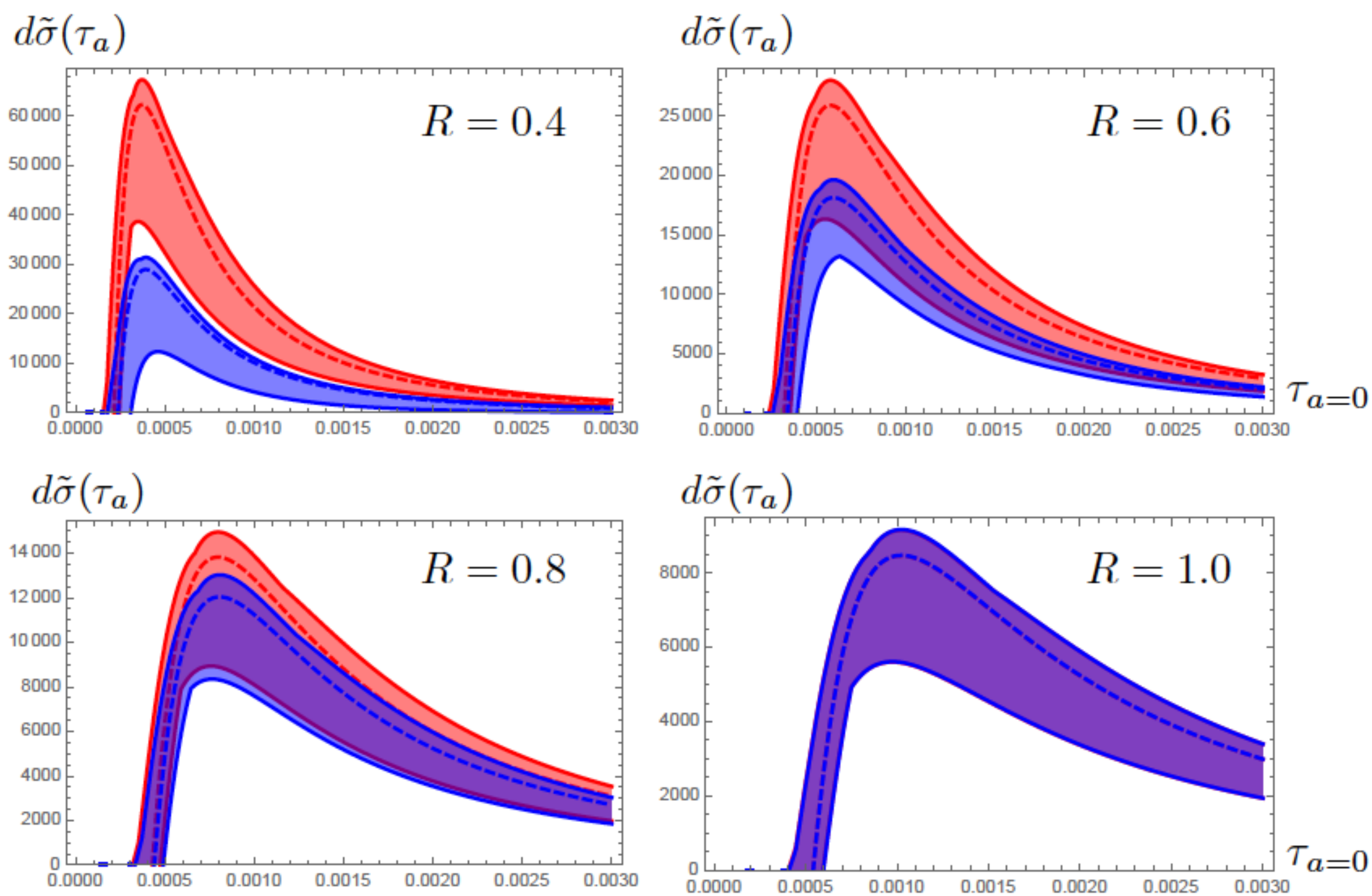


- Improved perturbative convergence thanks to additional logs resummed after soft-collinear refactorization

Resummed jet thrust cross section

- *pp* jet angularity differential distribution:

A. Hornig, Y. Makris, T. Mehen (2016)



without soft-collinear refactorization

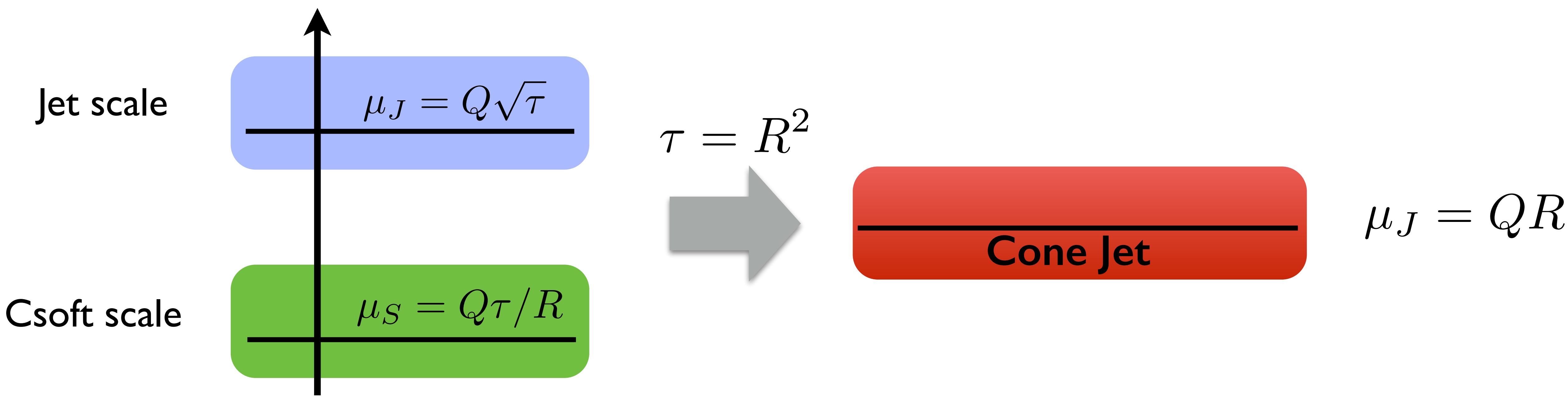
with soft-collinear refactorization

- Larger impact on a differential shape

Chien, Hornig, CL (2015)

Integrate to get the jet rate

- We can integrate jet thrust up to $\tau = R^2$ to get the total (cone) 2-jet rate
- Note the merging of hard collinear and collinear-soft scales:



- Leads to the identification:

$$J_{\rm un}^{\rm cone}(QR,\mu) = \int_0^{R^2} d\tau \int dt\, dk\, \delta\Big(\tau - \frac{t}{Q^2} - \frac{k}{Q}\Big) J_n^{\rm cone}(t,R,\mu) S_{\rm in}\Big(\frac{k}{R},\mu\Big)$$

csoft function

- Requires keeping "power-suppressed" algorithm-dependent part of measured jet function
- Explains factor two in double log vs. measured jet function: unmeasured jet function contains extra contribution from csoft radiation

Unmeasured jet function

- 1-loop unmeasured jet function:

$$J_n^{\rm cone}\otimes S_{\rm in} = 1+\frac{\alpha_s}{4\pi}\Big[\Gamma_0\ln^2\frac{\mu}{QR}+\gamma_J^0\ln\frac{\mu}{QR} + \Big(7-\frac{5\pi^2}{6}+6\ln 2\Big)C_F\Big]$$

$$J_n^{{\rm k}_T}\otimes S_{\rm in} = 1+\frac{\alpha_s}{4\pi}\Big[\Gamma_0\ln^2\frac{\mu}{QR}+\gamma_J^0\ln\frac{\mu}{QR} + \Big(13-\frac{3\pi^2}{2}\Big)C_F\Big],$$

- 2-loop unmeasured jet function:

Chien, Hornig, CL (2015)

$$J_{\rm un}^{\rm cone} = 1+\frac{\alpha_s}{4\pi}\Big[\Gamma_0\ln^2\frac{\mu}{QR}+\gamma_J^0\ln\frac{\mu}{QR}+\Big(7-\frac{5\pi^2}{6}+6\ln 2\Big)C_F\Big] \tag{98}$$
$$+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Big\{\frac{(\Gamma_0)^2}{2}\ln^4\frac{\mu}{QR}+\Gamma_0\Big(\gamma_J^0+\frac{2}{3}\beta_0\Big)\ln^3\frac{\mu}{QR}+\Big[\Gamma_1+\Gamma_0\Big(7-\frac{5\pi^2}{6}+6\ln 2\Big)C_F+\frac{(\gamma_J^0)^2}{2}+\gamma_J^0\beta_0\Big]\ln^2\frac{\mu}{QR}$$
$$+\Big[\gamma_J^1+\gamma_{\rm in}^1+(\gamma_J^0+2\beta_0)\Big(7-\frac{5\pi^2}{6}+6\ln 2\Big)C_F\Big]\ln\frac{\mu}{QR}+\text{consts.}\Big\}.$$

- consistent with the form of a multiplicatively renormalized function, with

$$\gamma_{J_{\rm un}} = \gamma_J+\gamma_{\rm in} = -\frac{\gamma_H}{2}$$

Check to two loops

- Leads to the prediction for the rate:

Chien, Hornig, CL (2015)

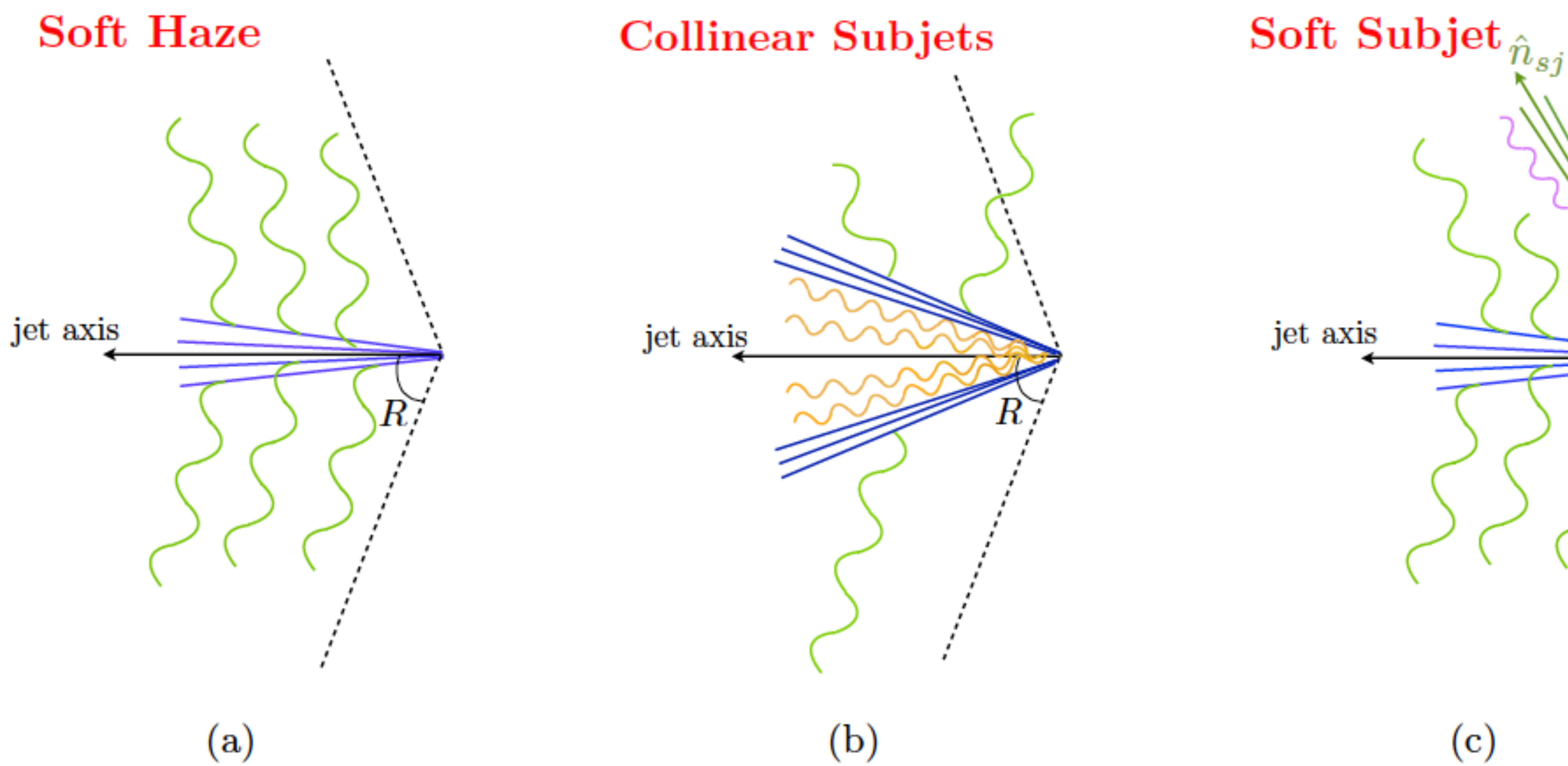
$$
\begin{aligned}
\sigma^{(2)}_{\text{cone}} = 4C_F^2\Big\{ &\Big(32\ln^2\frac{2\Lambda}{Q} + 48\ln\frac{2\Lambda}{Q} + 18 - \frac{16\pi^2}{3}\Big)\ln^2 R + \Big[(8-48\ln 2)\ln\frac{2\Lambda}{Q} + \frac{9}{2} + 2\pi^2 - 24\zeta_3 - 36\ln 2\Big]\ln R\Big\} \\
+ 4C_F C_A\Big\{ &\Big(\frac{44}{3}\ln\frac{2\Lambda}{Q} + 11\Big)\ln^2 R + \Big[\frac{44}{3}\ln^2\frac{2\Lambda}{Q} + \Big(\frac{4\pi^2}{3} - \frac{268}{9}\Big)\ln\frac{2\Lambda}{Q} - \frac{57}{2} + 12\zeta_3 - 22\ln 2\Big]\ln R \\
&- \frac{2\pi^2}{3}\ln^2\frac{2\Lambda}{Q} + \Big(\frac{2}{3} + 4\zeta_3 - \frac{11\pi^2}{9}\Big)\ln\frac{2\Lambda}{Q}\Big\} \\
+ 4C_F T_F n_f\Big\{ &\Big(-\frac{16}{3}\ln\frac{2\Lambda}{Q} - 4\Big)\ln^2 R + \Big(-\frac{16}{3}\ln^2\frac{2\Lambda}{Q} + \frac{80}{9}\ln\frac{2\Lambda}{Q} + 10 + 8\ln 2\Big)\ln R - \Big(\frac{4}{3} - \frac{4\pi^2}{9}\Big)\ln\frac{2\Lambda}{Q}\Big\} \\
+ \text{consts.}
\end{aligned} \tag{107}
$$

- Agrees with full QCD in all *ln R* terms and all n_f terms: these come from emissions from one collinear subjet in the cone

- To capture additional soft logs, we need to include additional subjets

work in progress:
Chien, Neill, CL, Ringer (2017)

Additional sources of soft logs



- one subjet: included in our factorization theorem so far
- two collinear subjets: not yet included, but allowed by a restriction only cone angle (as opposed to invariant mass). Will generate single soft logs at 2 loops.
- a soft subjet: emissions from it going out of the jet region responsible for the leading non-global log at 2 loops

Larkoski, Moult, Neill (2015)

Conclusions

- **$SCET_{++}$ provides first separation of all scale ratios of *R***
  - hard ratio: Q to QR for energetic collinear modes confined inside jets
  - soft ratio: E_0 to $E_0 R$ for soft radiation outside cones, sensitive to cone boundary
  - resolves several mysteries about structure of factorization and evolution of exclusive jet cross sections, and the relation with integrated jet thrust
- **Resummation of logs of *R* to all orders**
  - showed resummation to NNLL
- **Provides framework to go forward to include effects to capture additional soft logs of jet veto**
  - soft subjets and collinear subjets

Backup

SCET Operators

- Match QCD currents onto SCET operators
- e.g. for e^+e^- to 2 jets, DIS, or Drell-Yan processes:

Manohar (2003)
Bauer, CL, Manohar, Wise (2003)

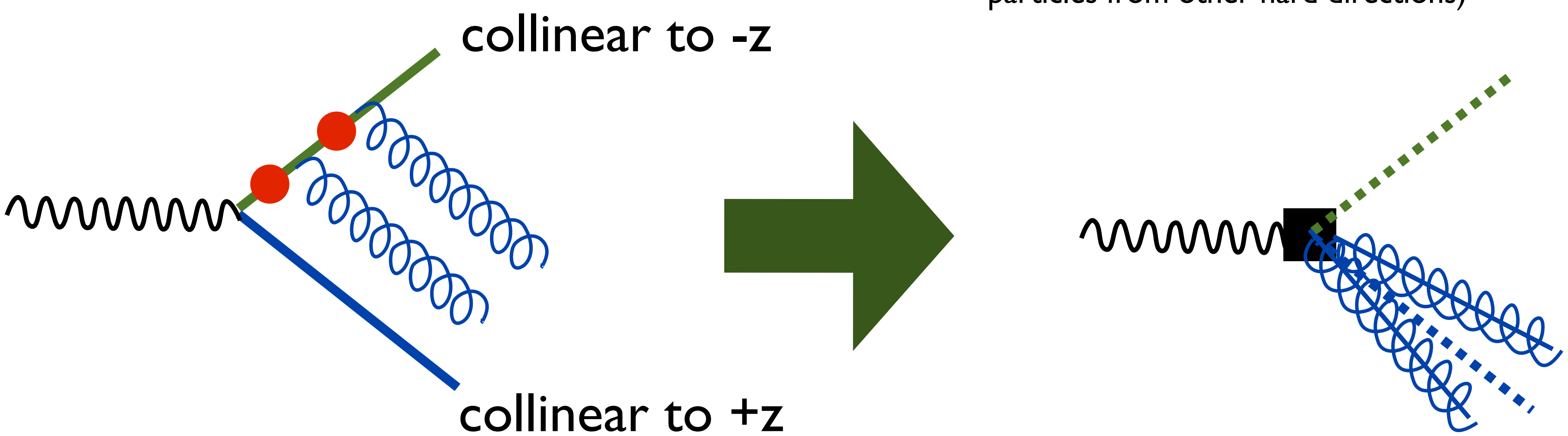
$$j^\mu = \bar{\psi}\gamma^\mu\psi \longrightarrow C_2(\tilde{p}_1\cdot\tilde{p}_2,\mu)[\bar{\xi}_{n_1}W_{n_1}]_{p_1}\gamma^\mu[W^\dagger_{n_2}\xi_{n_2}]_{p_2} = C_2\mathcal{O}_2$$

EM current

matching coefficient
(UV regulator dependent)

collinear Wilson lines
(required by gauge invariance)
(also, arise from emission of collinear particles from other hard directions)

collinear quark fields
(in separate directions)



decouples different collinear directions

- Determine matching coefficient C_2 by equating matrix elements of both sides. Must agree in IR. Mismatch in UV compensated by matching coeff.

Soft-Collinear Decoupling

- At leading power, soft-collinear interactions are eikonal:

$$\bar{\xi}_n(in\cdot D)\xi_n = \bar{\xi}_n(in\cdot\partial + gn\cdot A_s)\xi_n$$

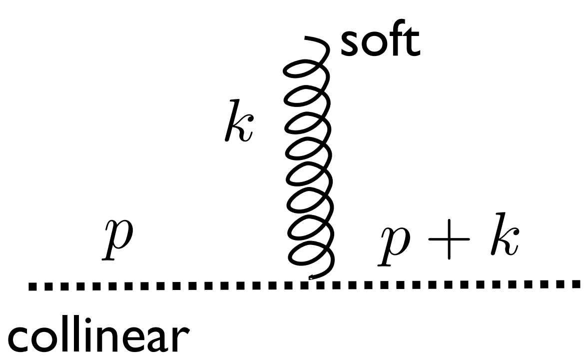
- They can be summed up into soft Wilson lines:

$$Y_n(x) = P\exp\Big[ig\int_{-\infty}^{0} ds\, n\cdot A_s(ns+x)\Big]$$

- Perform a field redefinition: use: $in\cdot D_s Y_n = 0$ $\quad D_s^\mu = \partial^\mu - igA_s^\mu$

$$\xi_n(x) = Y_n(x)\xi_n^{(0)}(x) \qquad \bar{\xi}_n(in\cdot D_s)\xi_n \longrightarrow \bar{\xi}_n^{(0)}(in\cdot\partial)\xi_n^{(0)}$$

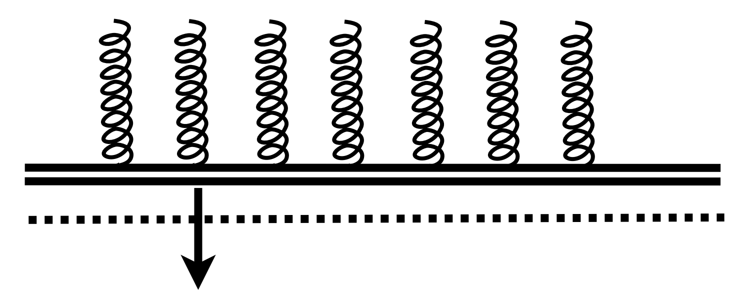
no soft gluons



$$\sim \not{p}\,\frac{ig\gamma^\mu t^a}{(p+k)^2}\not{p}$$

keep leading order part of diagram

soft gluons "see" only direction and color of jet



$$\frac{n^\mu t^a}{n\cdot k}\frac{\not{n}}{2}$$

independent of p and collinear splittings

- Soft Wilson lines then reappear in operators: $\mathcal{O}_2 = \bar{\chi}_n Y_n \gamma^\mu Y_{\bar{n}}^\dagger \chi_{\bar{n}}$

Soft-Collinear Decoupling

- At leading power, soft-collinear interactions are eikonal:

$$\bar{\xi}_n (in \cdot D) \xi_n = \bar{\xi}_n (in \cdot \partial + g n \cdot A_s) \xi_n$$

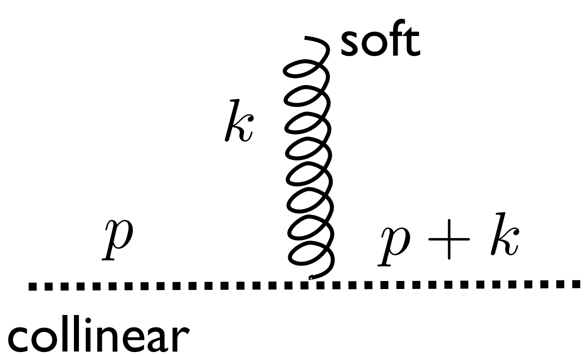
- They can be summed up into soft Wilson lines:

$$Y_n(x) = P \exp\Big[ig \int_{-\infty}^{0} ds\, n \cdot A_s(ns + x) \Big]$$

- Perform a field redefinition: use: $in \cdot D_s Y_n = 0$ $\qquad D_s^\mu = \partial^\mu - ig A_s^\mu$

$$\xi_n(x) = Y_n(x) \xi_n^{(0)}(x) \qquad \bar{\xi}_n (in \cdot D_s) \xi_n \longrightarrow \bar{\xi}_n^{(0)} (in \cdot \partial) \xi_n^{(0)}$$

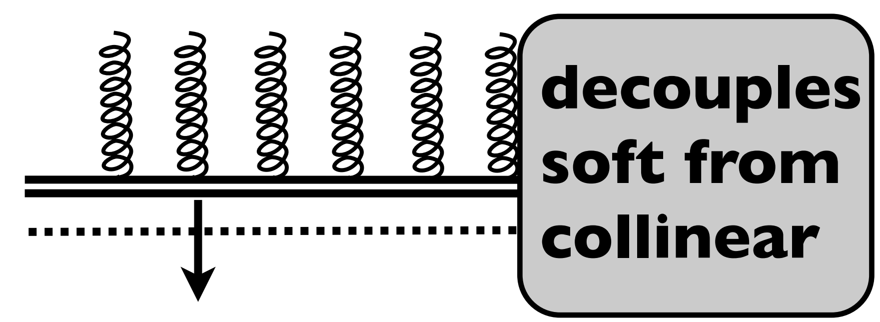
no soft gluons



$\sim \not{p} \dfrac{ig\gamma^\mu t^a}{(p+k)^2} \not{p}$

soft gluons "see" only direction and color of jet

keep leading order part of diagram



$\dfrac{n^\mu t^a}{n \cdot k} \dfrac{\not{n}}{2}$

independent of p and collinear splittings

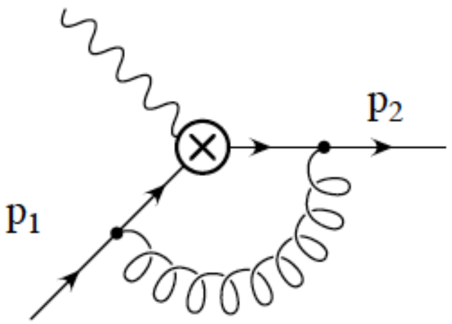
- Soft Wilson lines then reappear in operators: $\mathcal{O}_2 = \bar{\chi}_n Y_n \gamma^\mu Y_{\bar{n}}^\dagger \chi_{\bar{n}}$

Bauer, Fleming, Pirjol, Rothstein, Stewart (2002);
Bauer, **CL**, Manohar, Wise (2003)

Matching Computation

- Match matrix elements in QCD and SCET, e.g. quark-(anti)quark external states:

QCD:

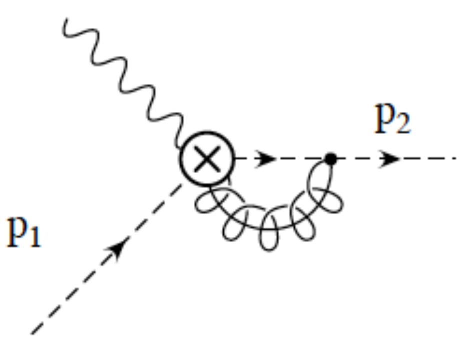


regulating UV and IR divergences in pure dim reg:

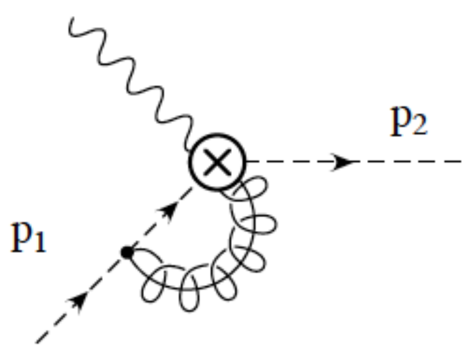
$$\frac{\alpha_s(\mu)C_F}{4\pi}\gamma^\mu\Big[-\frac{2}{\epsilon_{\rm IR}^2}-\frac{2}{\epsilon_{\rm IR}}\ln\frac{\mu^2}{Q^2}-\frac{3}{\epsilon_{\rm IR}}-\ln^2\frac{\mu^2}{-2p_1\cdot p_2}-3\ln\frac{\mu^2}{-2p_1\cdot p_2}+\frac{\pi^2}{6}-8\Big]$$

(no UV divergence as this current is conserved/not renormalized in QCD)

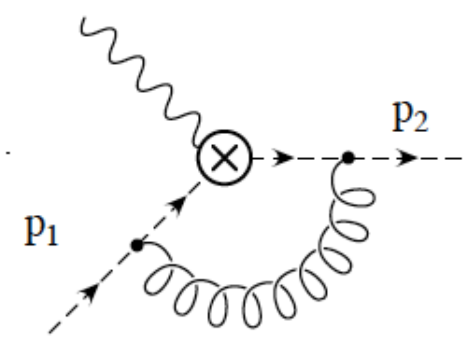
SCET:



n_2-collinear



n_1-collinear



soft

These diagrams are actually scaleless in pure dim reg and thus zero (the scale $p_1.p_2$ does not flow through the loops)

Add up to:

$$\frac{\alpha_s(\mu)C_F}{4\pi}\gamma^\mu\Big[\frac{2}{\epsilon_{\rm UV}^2}-\frac{2}{\epsilon_{\rm IR}^2}+\frac{2}{\epsilon_{\rm UV}}\ln\frac{\mu^2}{-2p_1\cdot p_2}-\frac{2}{\epsilon_{\rm IR}}\ln\frac{\mu^2}{-2p_1\cdot p_2}+\frac{3}{\epsilon_{\rm UV}}-\frac{3}{\epsilon_{\rm IR}}\Big]$$

(can also identify coefficients of IR poles by using explicit IR regulator like quark off-shellness)

Matching coefficient:

$$C_2(p_1\cdot p_2,\mu)=1+\frac{\alpha_s(\mu)C_F}{4\pi}\Big[-8+\frac{\pi^2}{6}-\ln^2\frac{\mu^2}{-2p_1\cdot p_2}-3\ln\frac{\mu^2}{-2p_1\cdot p_2}\Big]$$

Operator renormalization:

$$Z_2=1+\frac{\alpha_s(\mu)C_F}{4\pi}\Big[-\frac{2}{\epsilon_{\rm UV}^2}-\frac{2}{\epsilon_{\rm UV}}\ln\frac{\mu^2}{-2p_1\cdot p_2}-\frac{3}{\epsilon_{\rm UV}}\Big]$$

Counting of Logs

- QCD perturbative expansion takes the form:

$$\sigma(\tau) \equiv \int_0^\tau d\tau' \frac{1}{\sigma_0}\frac{d\sigma}{d\tau'} = 1 + \frac{\alpha_s}{4\pi}\Big(F_{12}\ln^2\tau + F_{11}\ln\tau + F_{10}\Big)$$

$$+ \left(\frac{\alpha_s}{2\pi}\right)^2\Big(F_{24}\ln^4\tau + F_{23}\ln^2\tau + F_{22}\ln^2\tau + F_{21}\ln\tau + F_{20}\Big)$$

- Reorganize the series:

$$\begin{aligned}\ln\sigma(\tau) \sim\ & \alpha_s(\ln^2\tau + \ln\tau) \\ & + \alpha_s^2(\ln^3\tau + \ln^2\tau + \ln\tau) \\ & + \alpha_s^3(\ln^4\tau + \ln^3\tau + \ln^2\tau + \ln\tau) \\ & + \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots\end{aligned}$$

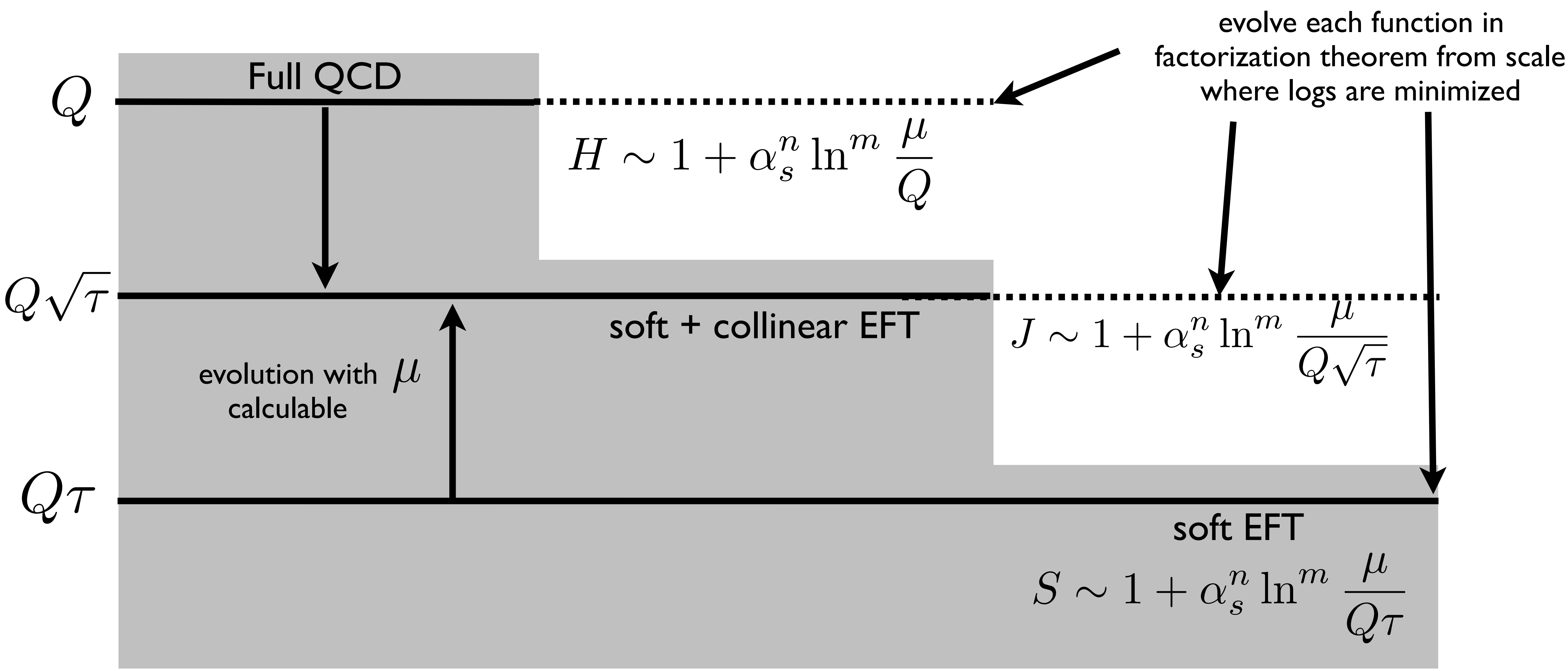
Leading Log (LL)	Next-to-Leading Log (NLL)	NNLL
$\frac{1}{\alpha_s}$	1	α_s

Heuristic power counting:

$$\ln\tau \sim \frac{1}{\alpha_s}$$

Resummation from Evolution

- Effective theory gives equations for evolution of hard, jet, and soft functions in factorization theorem with energy scale μ.
  (essentially, RGEs describe variation with respect to arbitrary boundaries between hard, jet, and soft regions)
- Solutions of these equations sum logs to all orders in α_s



- **Solutions of evolution equations contain logs resummed to all orders in α_s**