Phenomenological constraints on TSSAs from Lorentz invariance relations

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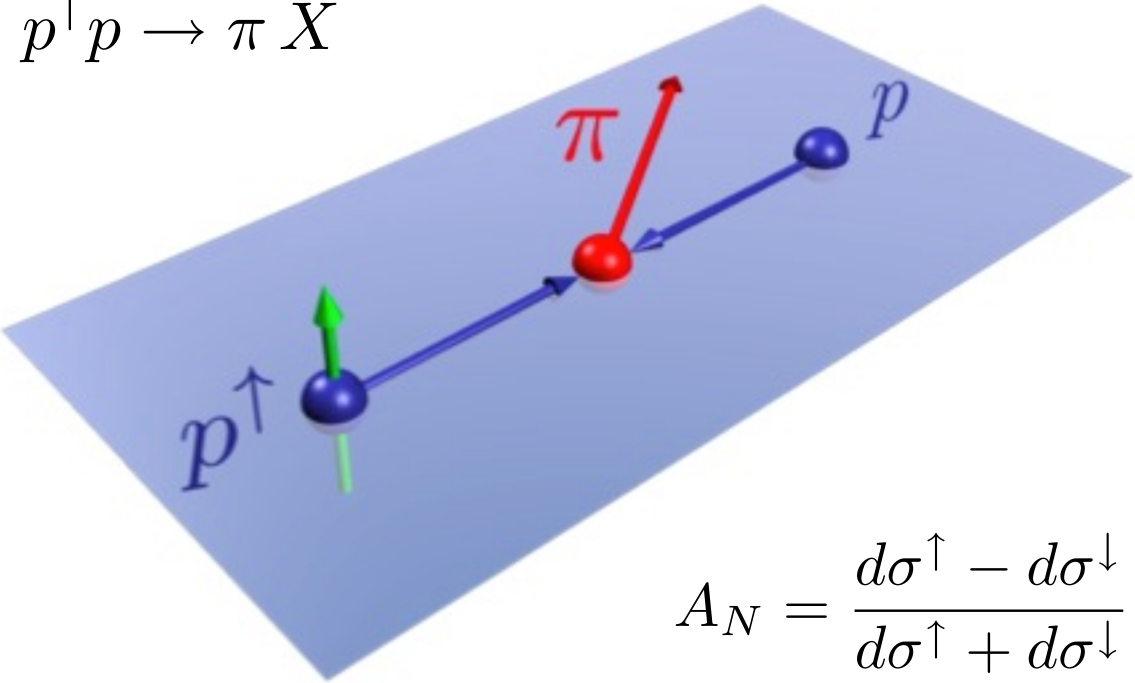
Outline

- Background
  - Transverse single-spin asymmetries (TSSAs): a 40+ year-old puzzle
  - Collinear twist-3 factorization
- Status as of 2014
  - Qiu-Sterman contribution and the “sign-mismatch” issue
  - Fragmentation contribution
- Recent developments
  - Lorentz invariance relations (LIRs) for twist-3 FFs
  - Revisiting the fragmentation contribution
  - TSSAs in *pA* collisions
- Future possibilities
  - Lepton-nucleon TSSAs
  - SIDIS and electron-positron annihilation
- Summary and outlook

Background

- TSSAs: a 40+ year-old puzzle

$$p^{\uparrow} p \to \pi \, X$$



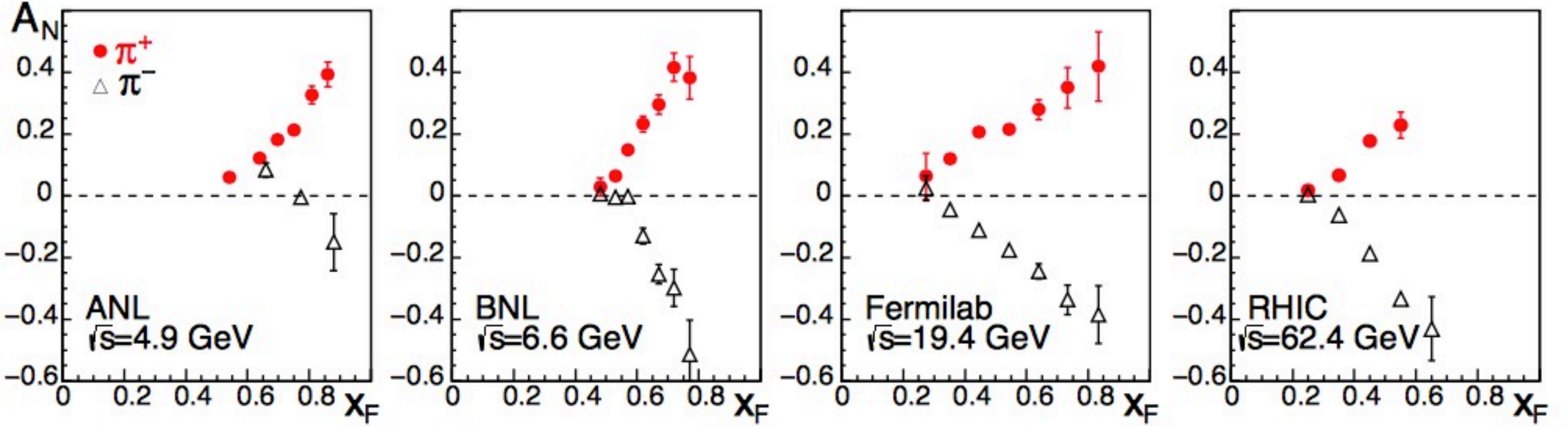
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

Data available from RHIC (BRAHMS, PHENIX, STAR), FNAL (E704, E581), AGS, and ANL

Background

- TSSAs: a 40+ year-old puzzle

Plot from Aidala, Bass, Hasch, Mallot (2013)



1976 ⟶

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$$

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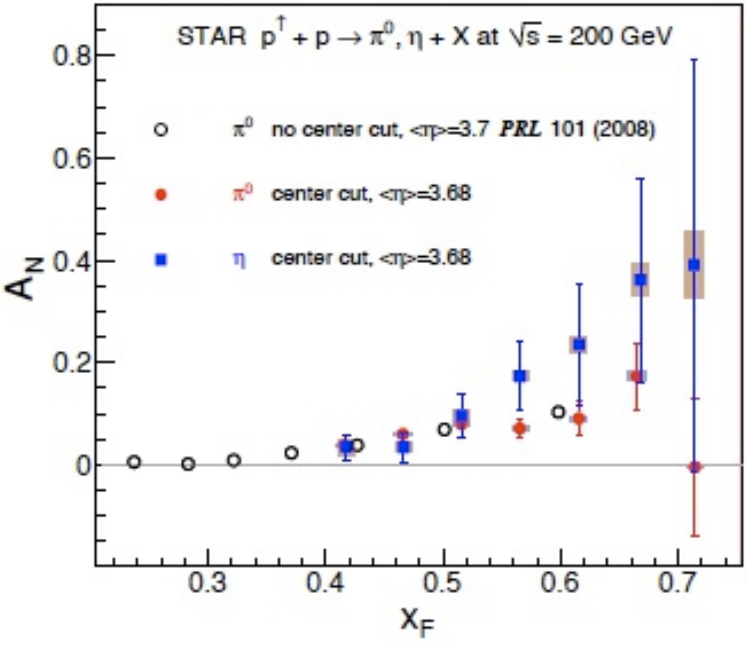
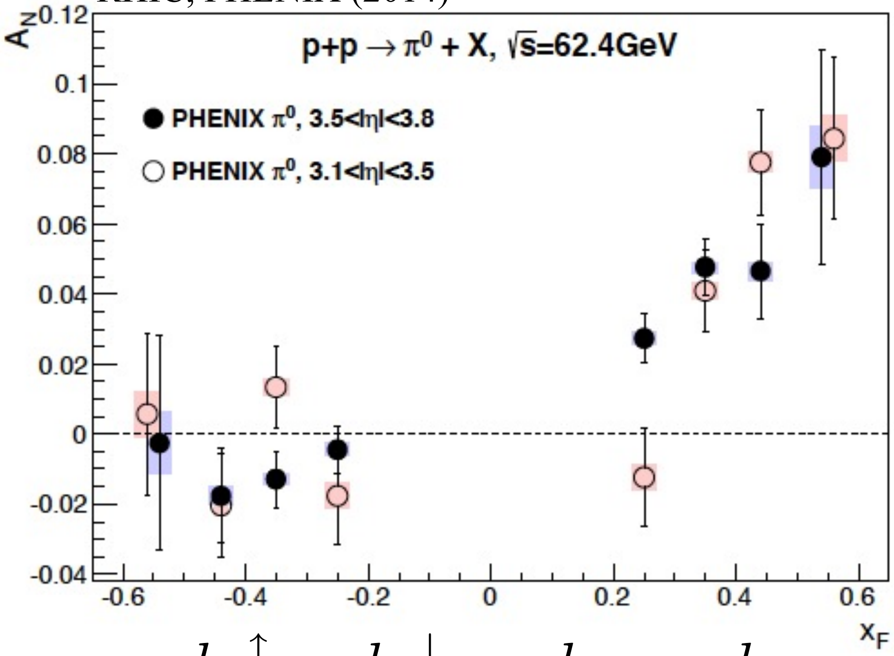
RHIC, STAR (2012)



RHIC, PHENIX (2014)



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- Collinear twist-3 factorization

$$
\begin{aligned}
d\sigma = \; & H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
& + H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
& + H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

A + B -> C + X

-One hadron T polarized (others U or L)

-One large scale ($P_{C,T}$)

- Collinear twist-3 factorization

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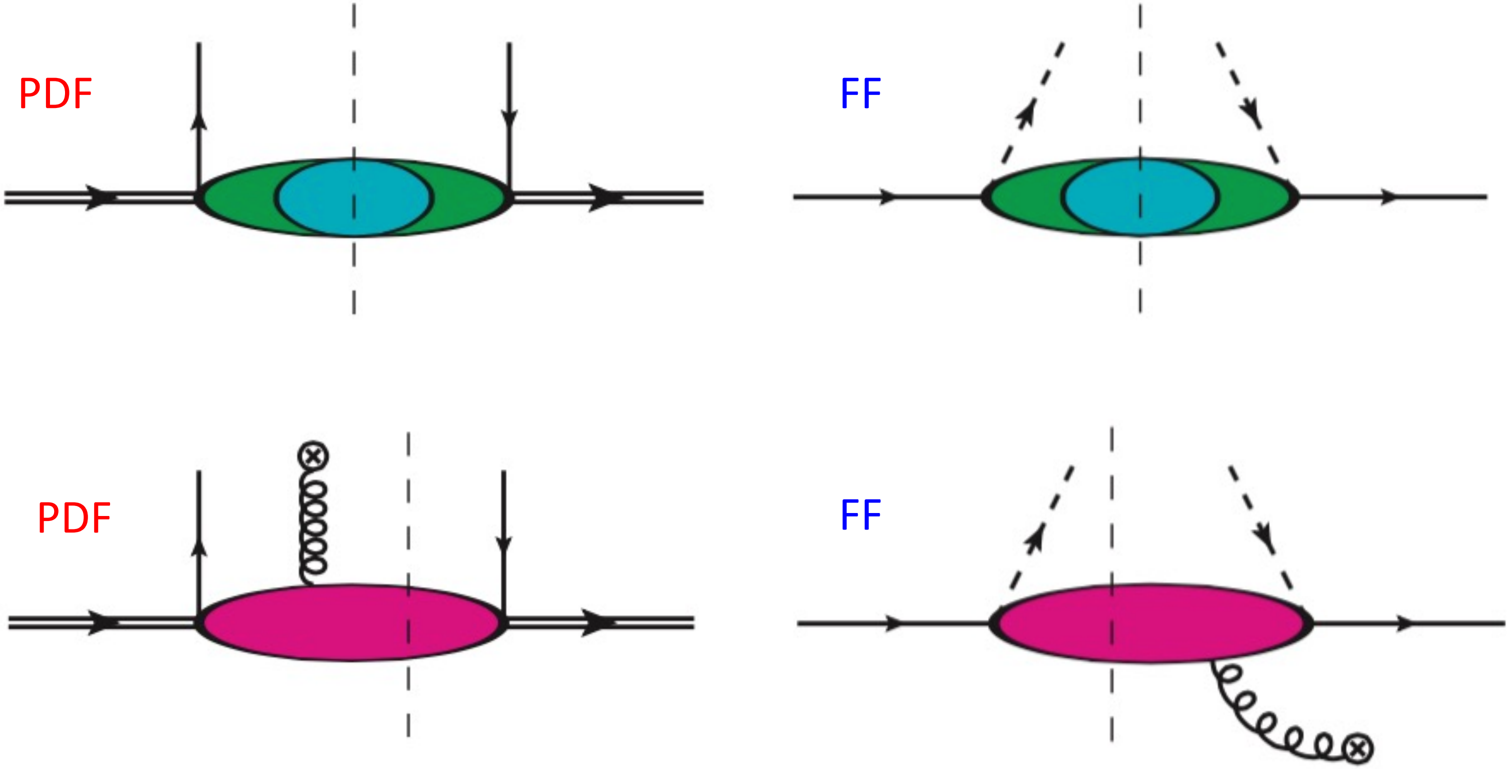

➢ Collinear twist-3 factorization

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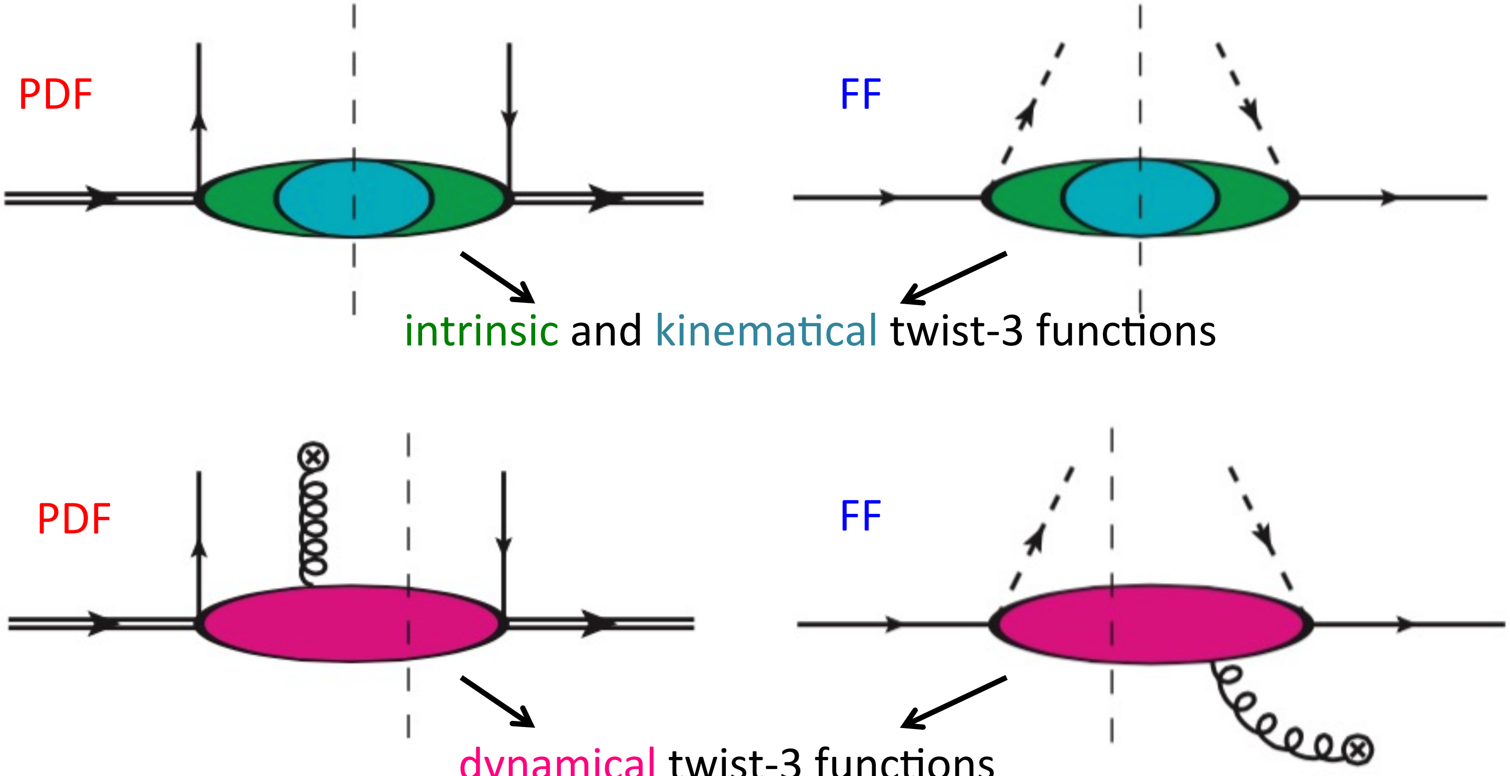
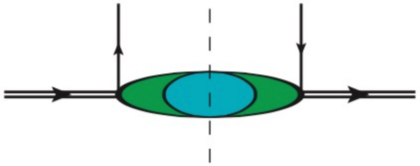
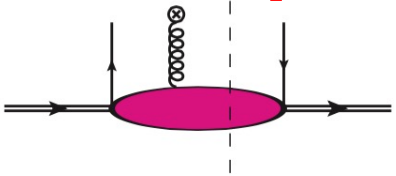
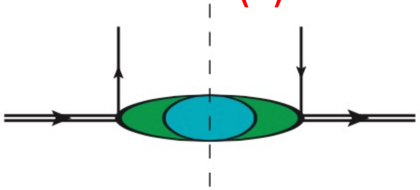
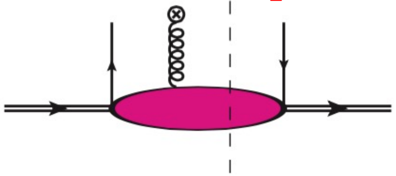
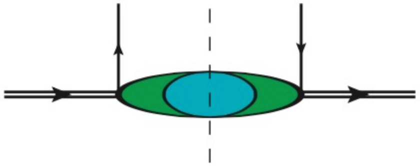
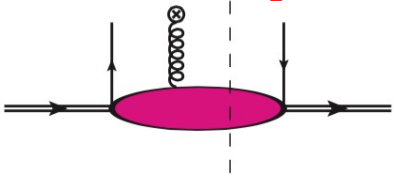


intrinsic and kinematical twist-3 functions

dynamical twist-3 functions

	PDF (x)		PDF (x, x_1)
Hadron Pol.			
	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}

Hadron Pol.	PDF (x) 		PDF (x, x_1) 
	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU} $h_1^{\perp(1)}(x) \sim H_{FU}(x,x)$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT} $f_{1T}^{\perp(1)}(x) \sim F_{FT}(x,x)$

Hadron Pol.	PDF (x) 		PDF (x, x_1) 
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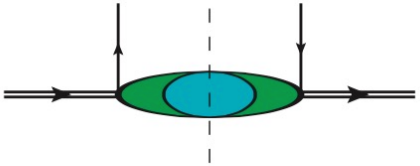
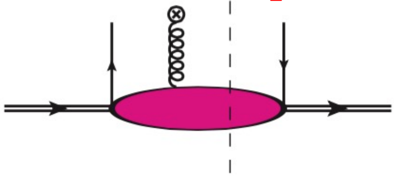
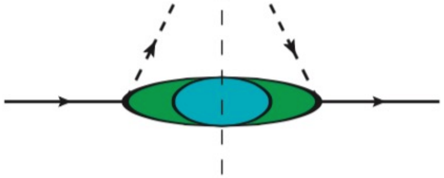
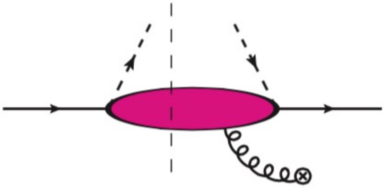
→ Qiu-Sterman function

Hadron Pol.	PDF (x) 		PDF (x, x_1) 	FF (z) 		FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

Status as of 2014

- Qiu-Sterman contribution

$$
\begin{aligned}
d\sigma = {} & \boxed{H \otimes f_{a/A^{\uparrow}(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)}} \\
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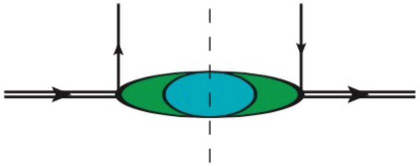
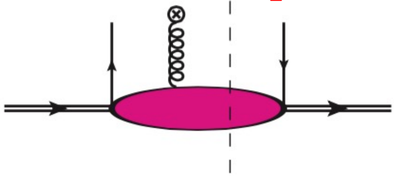
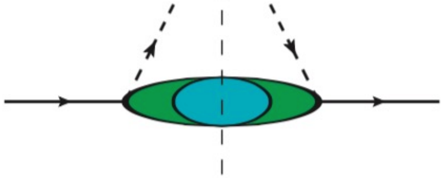
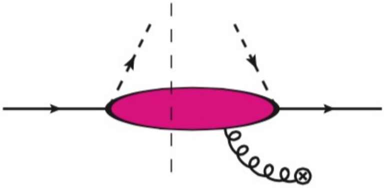
Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
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L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\mathfrak{R},\mathfrak{I}}$
T	g_T	$f_{1T}^{\perp(1)},$ $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT} SGP term (QS function)	D_T, G_T	$D_{1T}^{\perp(1)},$ $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\mathfrak{R},\mathfrak{I}}, \hat{G}_{FT}^{\mathfrak{R},\mathfrak{I}}$

Status as of 2014

- Qiu-Sterman contribution

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$$\begin{aligned} E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \; & \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^{1} \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^{1} \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x') \\ & \times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}} \right) \frac{1}{x} \left[T_{a,F}(x,x) - x \left(\frac{d}{dx} T_{a,F}(x,x) \right) \right] H_{ab\to c}(\hat{s},\hat{t},\hat{u}) \end{aligned}$$

$$\boxed{F_{FT} \sim T_F}$$

(Qiu and Sterman (1999), Kouvaris, et al. (2006))

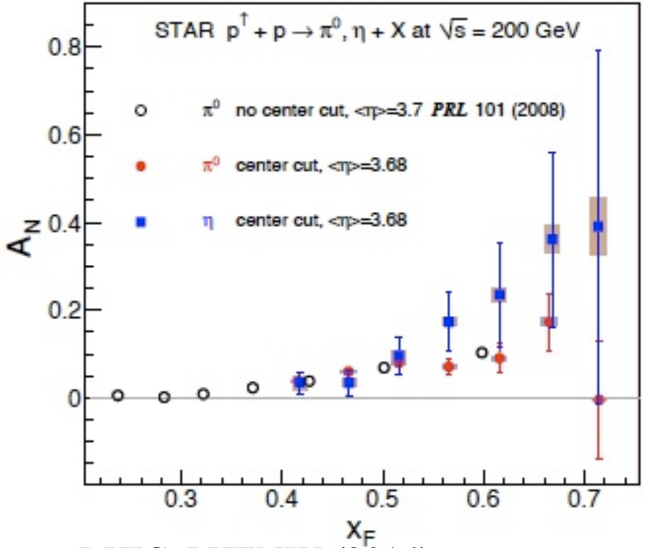
For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $p^\uparrow p \longrightarrow \pi \; X$

- The “sign mismatch” issue

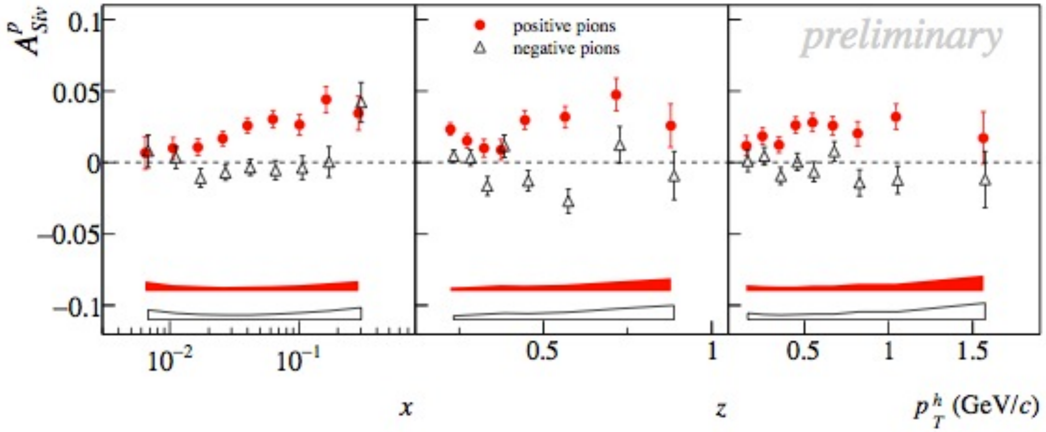
$$\boxed{p^{\uparrow}p \to h\,X}$$

$$\boxed{\ell N^{\uparrow} \to \ell'\,h\,X}$$

RHIC, STAR (2012)

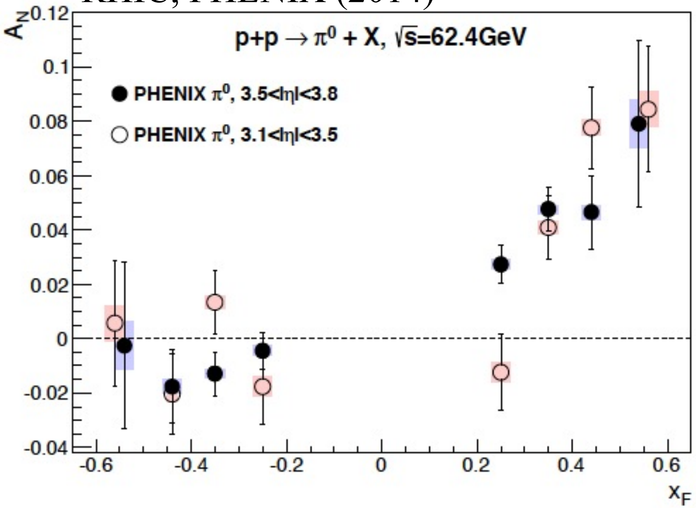


CERN, COMPASS (2013)



$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

RHIC, PHENIX (2014)



- The “sign mismatch” issue

$p^{\uparrow}p \to h\,X$

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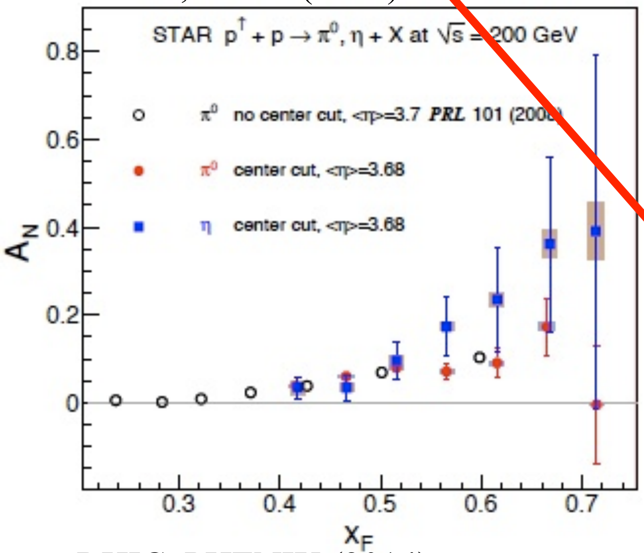

$\ell N^{\uparrow} \to \ell'\,h\,X$

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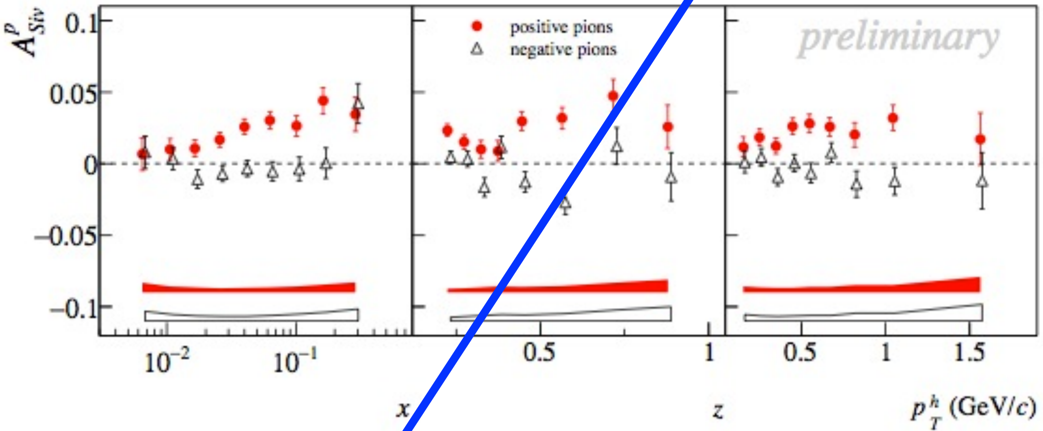

$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

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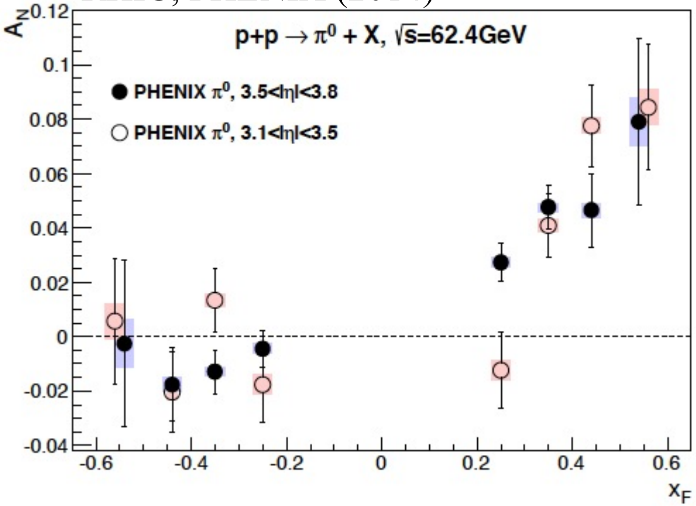

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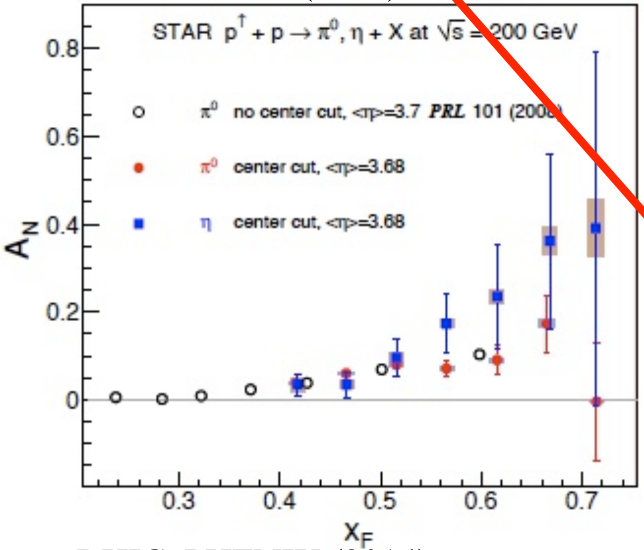
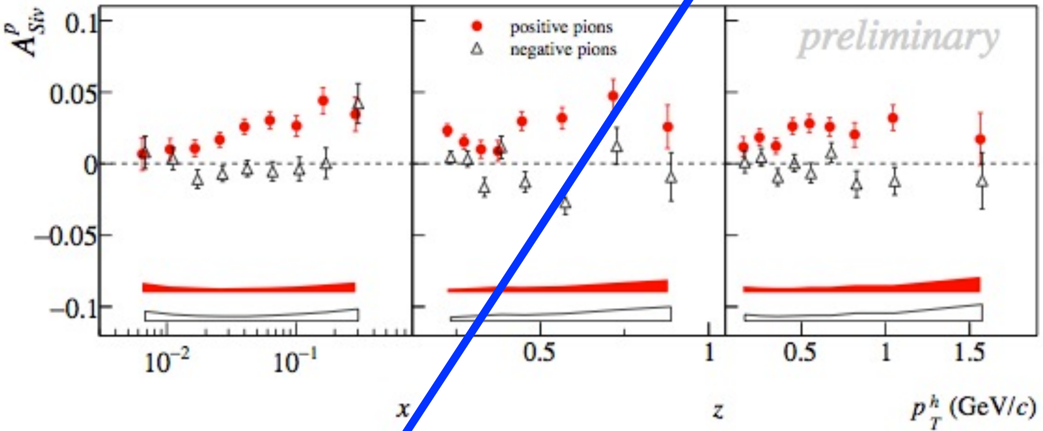


$\ell N^\uparrow \to \ell'\, h\, X$

CERN, COMPASS (2013)



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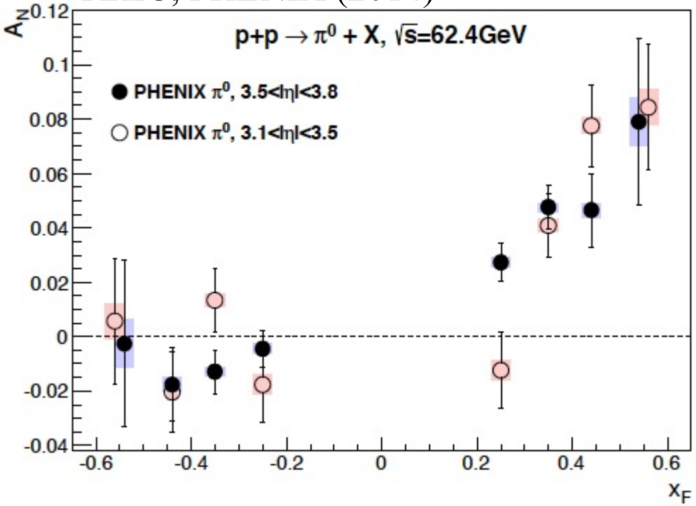
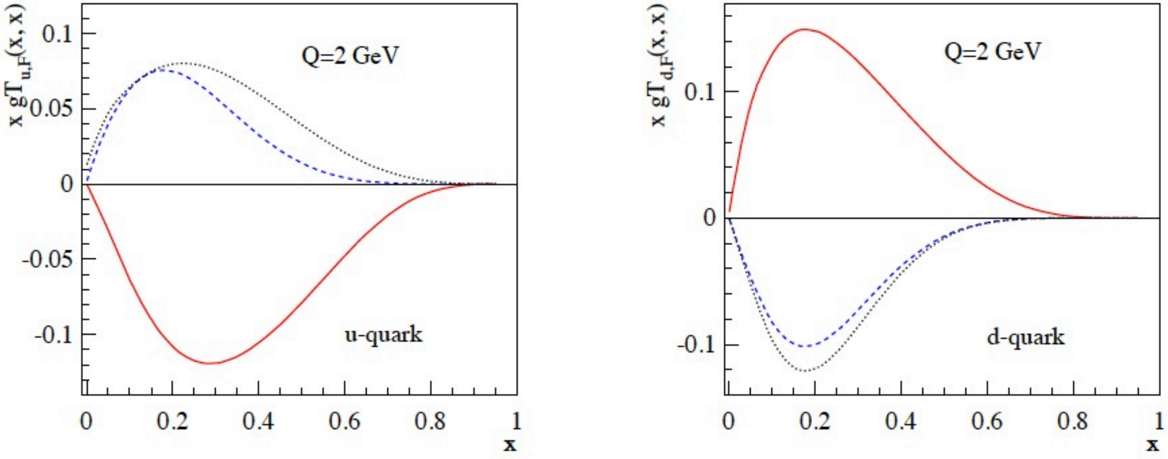
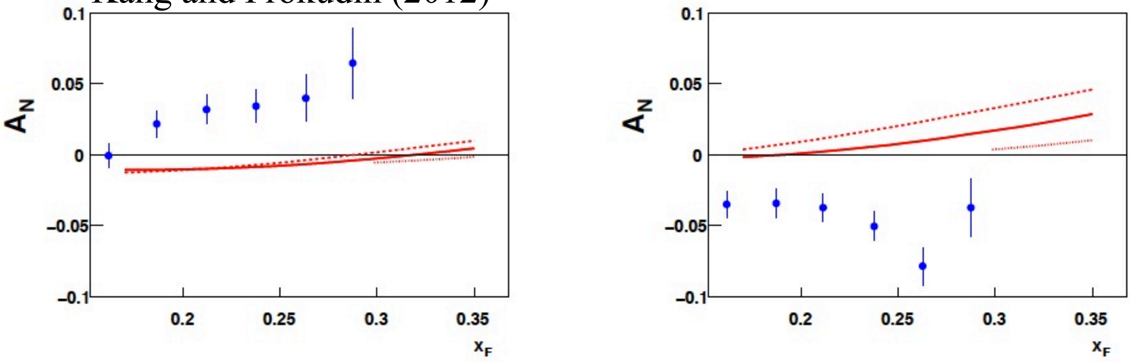


“sign mismatch” (Kang, Qiu, Vogelsang, Yuan (2011))

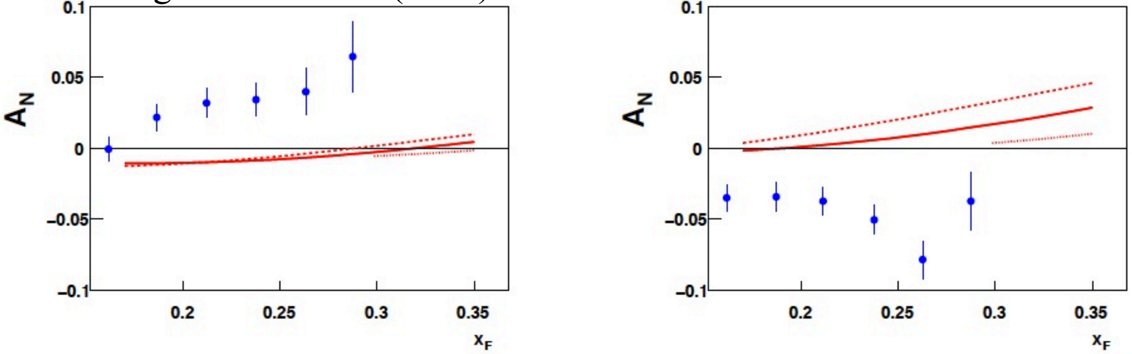




Kang and Prokudin (2012)

Proton-proton data from BRAHMS for π^+ (left) and π^- (right)

Nodes in Sivers cannot resolve issue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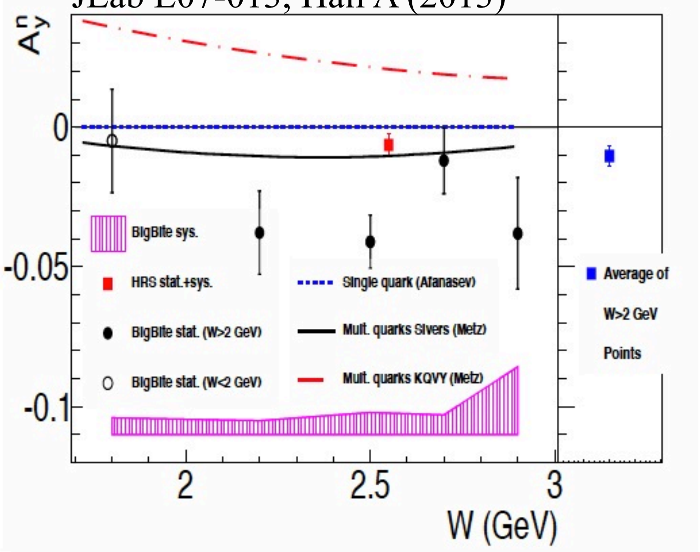
JLab E07-013, Hall A (2013)

Neutron TSSA in inclusive DIS

Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou - PRD **86** (2012)

Sivers input agrees reasonably well with the JLab data ➡ FIRST INDICATION on the PROCESS DEPENDENENCE of the Sivers function (see also Gamberg, Kang, Prokudin (2013))

KQVY input gives the wrong sign ➡ Qiu-Sterman function cannot be the main cause of the large TSSAs seen in pion production from *pp* collisions

$$
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d\sigma = \; & H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
+ \; & H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)} \\
+ \; & H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
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$$

$$
\begin{aligned}
d\sigma = \; & H \otimes f_{a/A^\uparrow(3)} \otimes f_{b/B(2)} \otimes D_{c/C(2)} \\
& + \boxed{H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}} \Longrightarrow \\
& + H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}
\end{aligned}
$$

Negligible
(Kanazawa and
Koike (2000))

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& + \boxed{H' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(3)} \otimes D_{c/C(2)}} \Rightarrow \\
& + \boxed{H'' \otimes f_{a/A^\uparrow(2)} \otimes f_{b/B(2)} \otimes D_{c/C(3)}}
\end{aligned}
$$

Negligible
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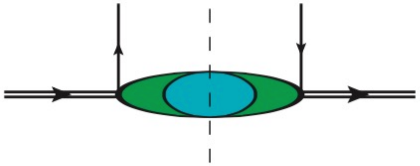
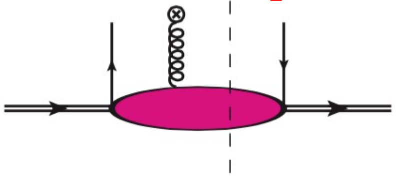
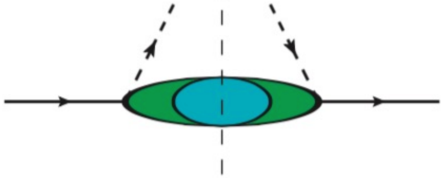
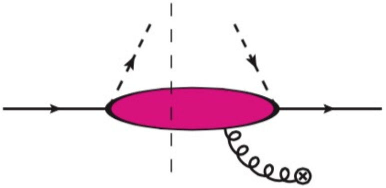
Hadron Pol.	PDF (x) 		PDF (x, x_1) 	FF (z) 		FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	$E, \boxed{H}$	$\boxed{H_1^{\perp(1)}}$	$\boxed{\hat{H}_{FU}^{\Re,\boxed{\Im}}}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)},$ $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)},$ $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

- Fragmentation contribution

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$
$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$
$$\left. + \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

(Metz and DP - PLB **723** (2013))

- Fragmentation contribution

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

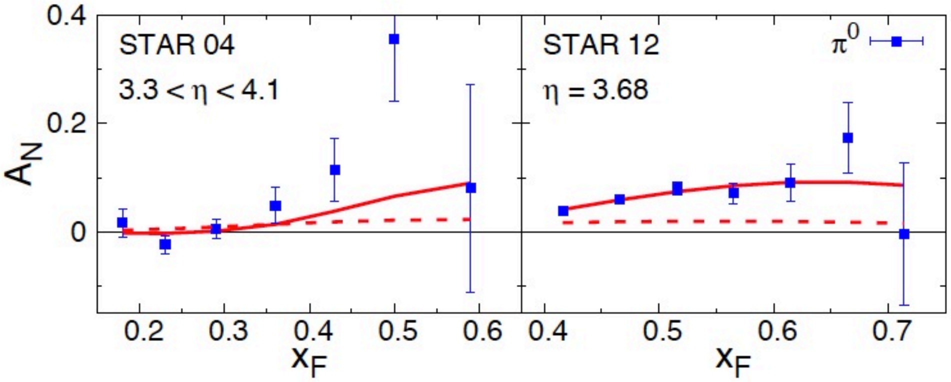
$$\left. + \frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

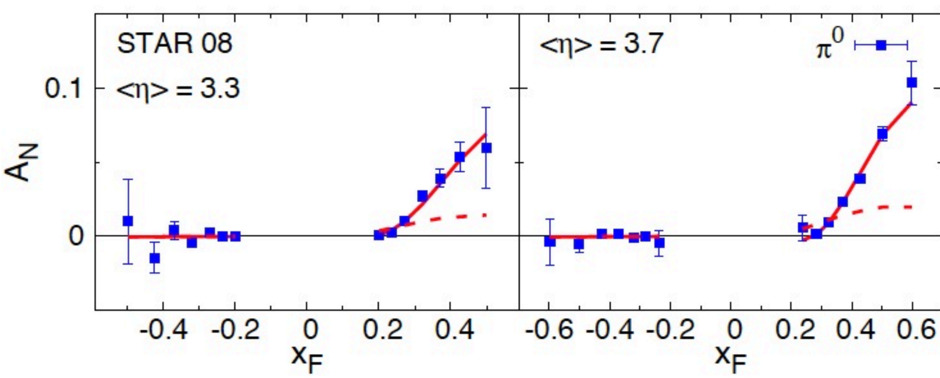
(Metz and DP - PLB **723** (2013))

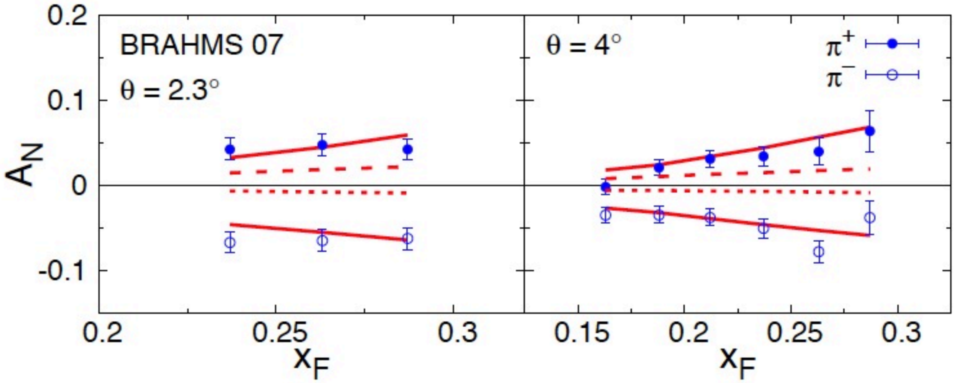
$$H^q(z) \;=\; -2z\,H_1^{\perp(1),q}(z) + \boxed{2z\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{q,\Im}(z,z_1)}$$

QCD e.o.m. relation (EOMR)

$$\longrightarrow\;\equiv\tilde{H}^q(z)$$







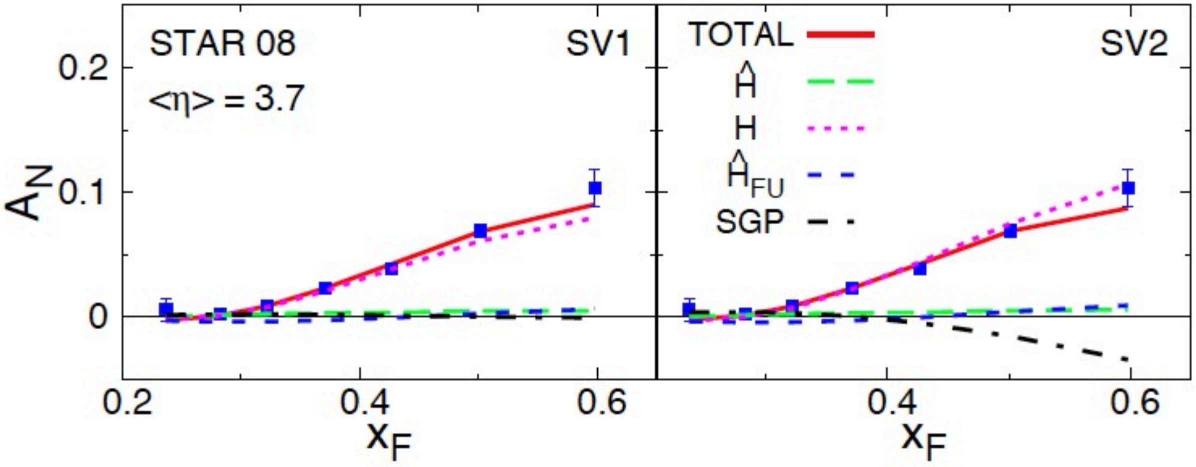
-Used Sivers function from SIDIS as input for Qiu-Sterman function $f_{1T}^{\perp(1)}(x) \propto F_{FT}(x,x)$

-Used Collins function and transversity TMD extracted by Anselmino, et. al (2013) from the Collins asymmetry in SIDIS/e^+e^- using DGLAP-type evolution

-Used EOM relation for H

-Extracted $\hat{H}_{FU}^{\Im}(z,z_1)$

(Kanazawa, Koike, Metz, DP (2014))



Fragmentation term is the dominant source of A_N

First consistent description of TSSAs in SIDIS, e^+e^-, and *pp*

(Kanazawa, Koike, Metz, DP (2014))

Recent Developments

- Lorentz invariance relations (LIRs) for twist-3 FFs

$$H^q(z) \;=\; -2z\, H_1^{\perp(1),q}(z) + \tilde{H}^q(z)$$

EOMR

Recent Developments

- Lorentz invariance relations (LIRs) for twist-3 FFs

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + \tilde{H}^q(z)$$

EOMR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\, \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

Derived for all twist-3 FFs for the first time

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

➢ Revisiting the fragmentation contribution

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]\tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z)+\frac{1}{z}\tilde{H}^c(z)\right]\tilde{S}^i_H\right\}$$

where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, arXiv:1701.09170)

- Revisiting the fragmentation contribution

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}}$$

$$\times\; h_1^a(x)\, f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]\tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z)\right]\tilde{S}^i_H\right\}$$

where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, arXiv:1701.09170)

-Use Qiu-Sterman function extracted by Echevarria, Idilbi, Kang, Vitev (2014) from the Sivers asymmetry in SIDIS using full TMD evolution ➔ negligible (and opposite sign to the data)

$$\boxed{f_{1T,\mathrm{SIDIS}}^{\perp q(\alpha)}(x,b;Q)} = \left(\frac{ib^\alpha}{2}\right)\boxed{T_{q,F}(x,x,c/b_*)}\exp\left\{-\int_{c/b_*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}\exp\left\{-b^2\left(g_1^{\mathrm{sivers}}+\frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

(*b*-space) Sivers function

Qiu-Sterman function

-Use first k_T-moment of Collins function and **transversity PDF** extracted by Kang, Prokudin, Sun, Yuan (2016) from the Collins asymmetry in SIDIS/e^+e^- using full TMD evolution

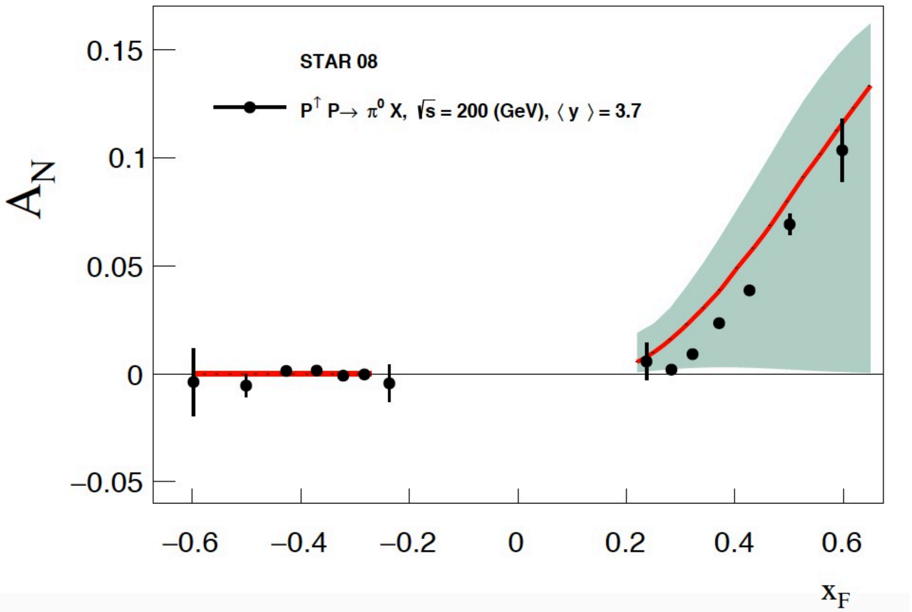
$$\boxed{\widetilde{H}_{1\,h/q}^{(sub)\perp\alpha}(z_h,b,\rho;Q^2,Q)} = \left(\frac{-ib^\alpha}{2z_h}\right)e^{-\frac{1}{2}S_{\mathrm{pert}}(Q,b_*)-S_{\mathrm{NP}}^{D_1}(Q,b)}\,\widetilde{\mathcal{H}}_c(\alpha_s(Q))\,\delta\hat{C}_{q'\leftarrow q}\otimes\boxed{\hat{H}_{h/q'}^{(3)}(z_h,\mu_b)}$$

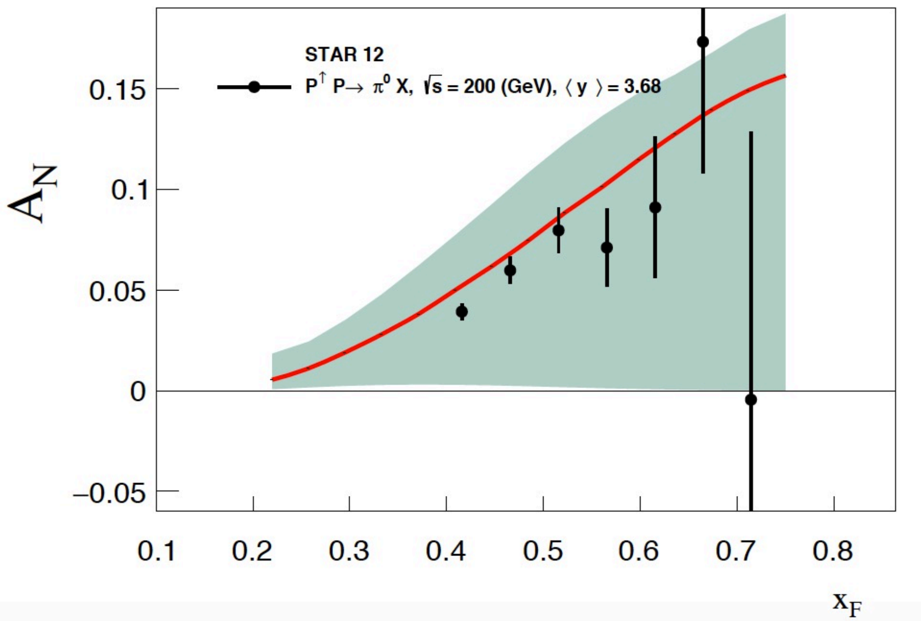
(*b*-space) Collins function

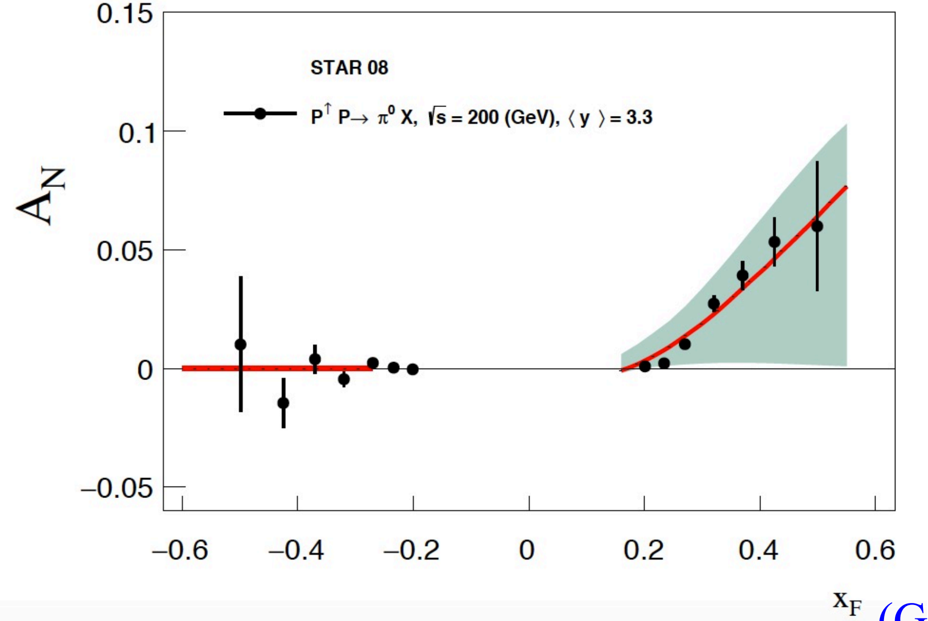
first k_T-moment of Collins function

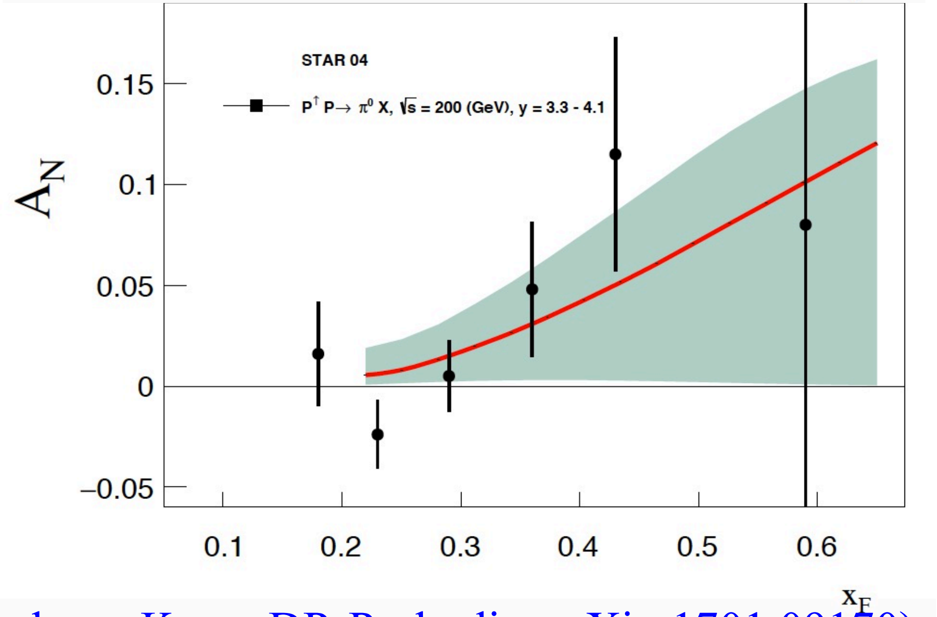
-Do not consider piece involving $\tilde{H}$ (BUT IT CANNOT BE ZERO)

(Gamberg, Kang, DP, Prokudin, arXiv:1701.09170)

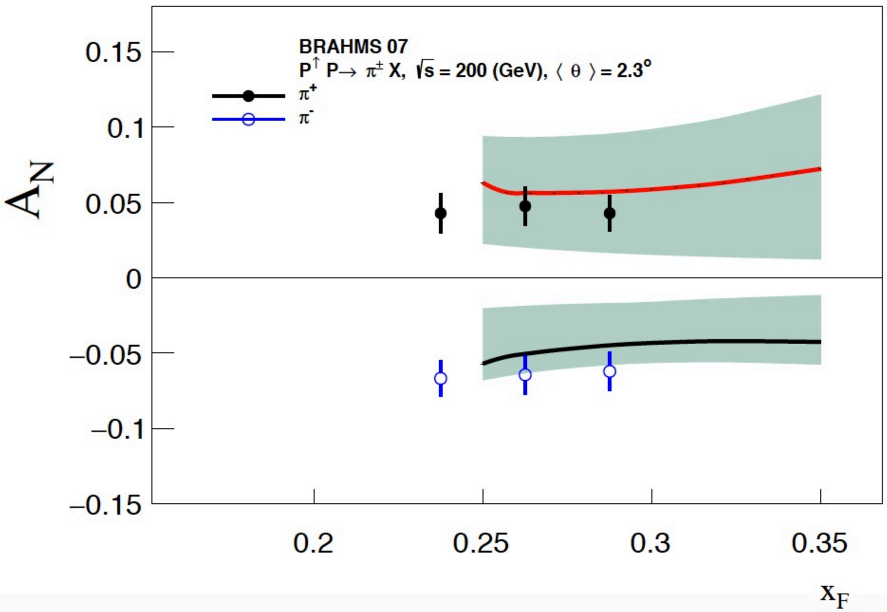


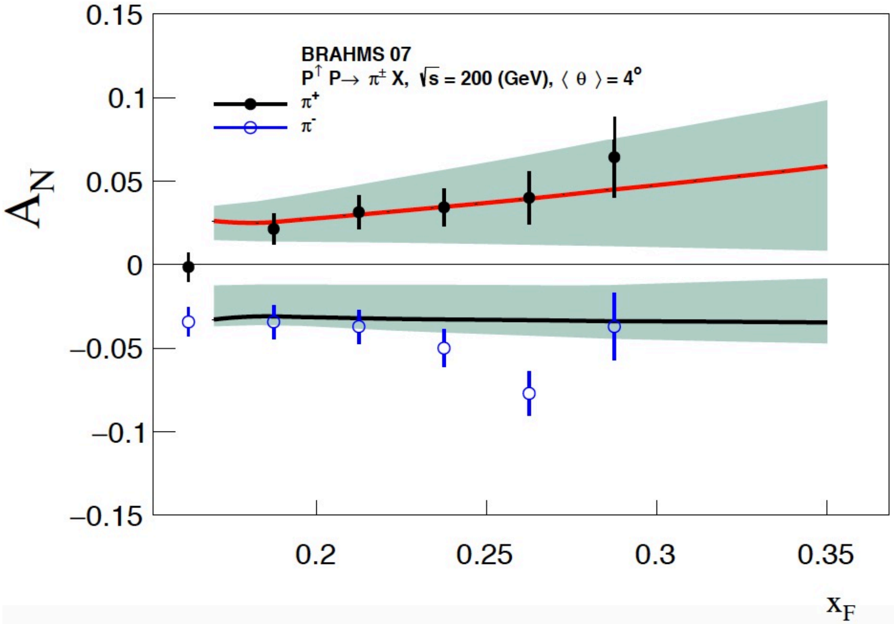


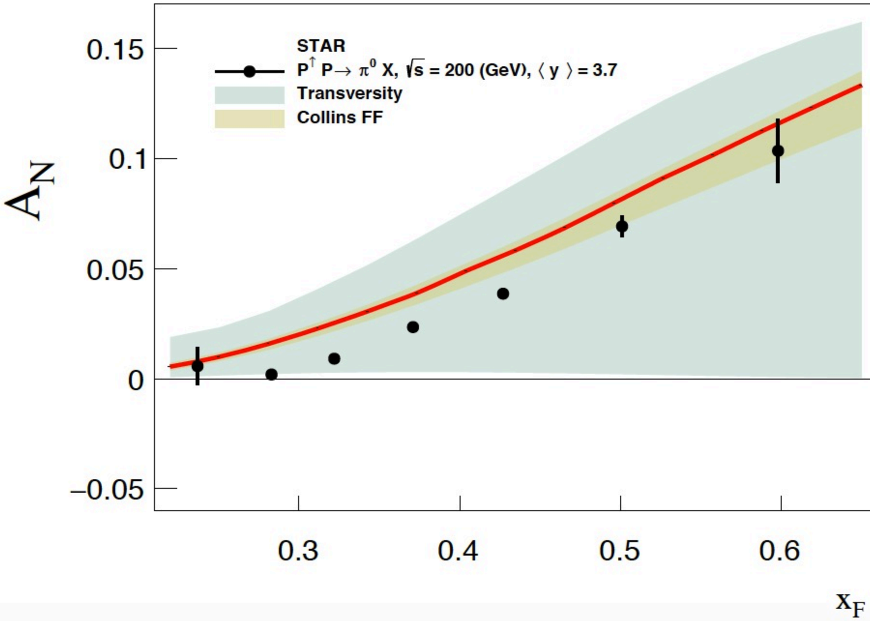




(Gamberg, Kang, DP, Prokudin, arXiv:1701.09170)







-Confirms the original work of Kanazawa, Koike, DP, Metz (2014) that the twist-3 fragmentation term can dominate A_N

-Encouraged that we will be able to fully describe A_N through this mechanism even with the additional constraint from the LIR – still need to fit $\tilde{H}(z)$

-Large error due to uncertainty in transversity at large-x

(Gamberg, Kang, DP, Prokudin, arXiv:1701.09170)

EOMR

$$H^q(z) \,=\, -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2}\, \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

LIR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2}\, \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

EOMR

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

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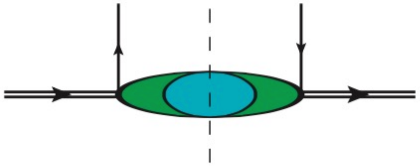
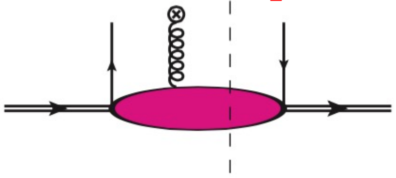
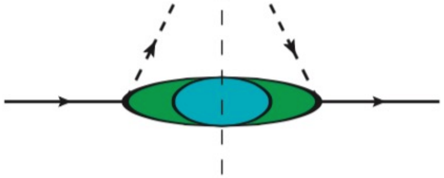
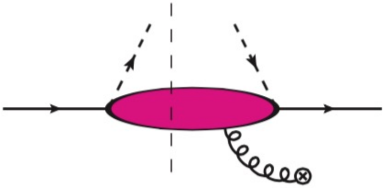
$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

⬇

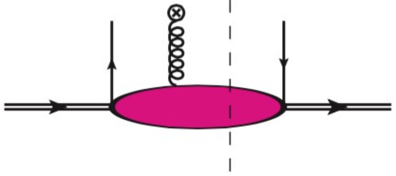
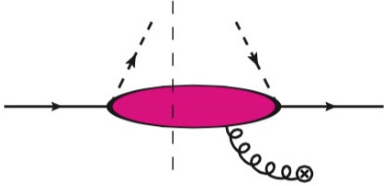
$$H(z) = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\, 2 \left[\frac{\left(2(\frac{2}{z_1} - \frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right) \delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2) \right]$$

$$H_1^{\perp(1)}(z) = -\frac{2}{z} \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} \frac{\left(\frac{2}{z_1} - \frac{1}{z_2}\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \hat{H}_{FU}^{\Im}(z_1, z_2)$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	$\cancel{e}$	$\cancel{h_1^{\perp(1)}}$	H_{FU}	$\cancel{E}, \cancel{H}$	$\cancel{H_1^{\perp(1)}}$	$\hat{H}_{FU}^{\Re,\Im}$
L	$\cancel{h_L}$	$\cancel{h_{1L}^{\perp(1)}}$	H_{FL}	$\cancel{E_L}, \cancel{H_L}$	$\cancel{H_{1L}^{\perp(1)}}$	$\hat{H}_{FL}^{\Re,\Im}$
T	$\cancel{g_T}$	$\cancel{f_{1T}^{\perp(1)}}$, $\cancel{g_{1T}^{\perp(1)}}$	F_{FT}, G_{FT}	$\cancel{D_T}, \cancel{G_T}$	$\cancel{D_{1T}^{\perp(1)}}$, $\cancel{G_{1T}^{\perp(1)}}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Hadron Pol.	PDF (x, x_1)	FF (z, z_1)
		
U	dynamical H_{FU}	dynamical $\hat{H}_{FU}^{\mathfrak{R},\mathfrak{I}}$
L	H_{FL}	$\hat{H}_{FL}^{\mathfrak{R},\mathfrak{I}}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\mathfrak{R},\mathfrak{I}}, \hat{G}_{FT}^{\mathfrak{R},\mathfrak{I}}$

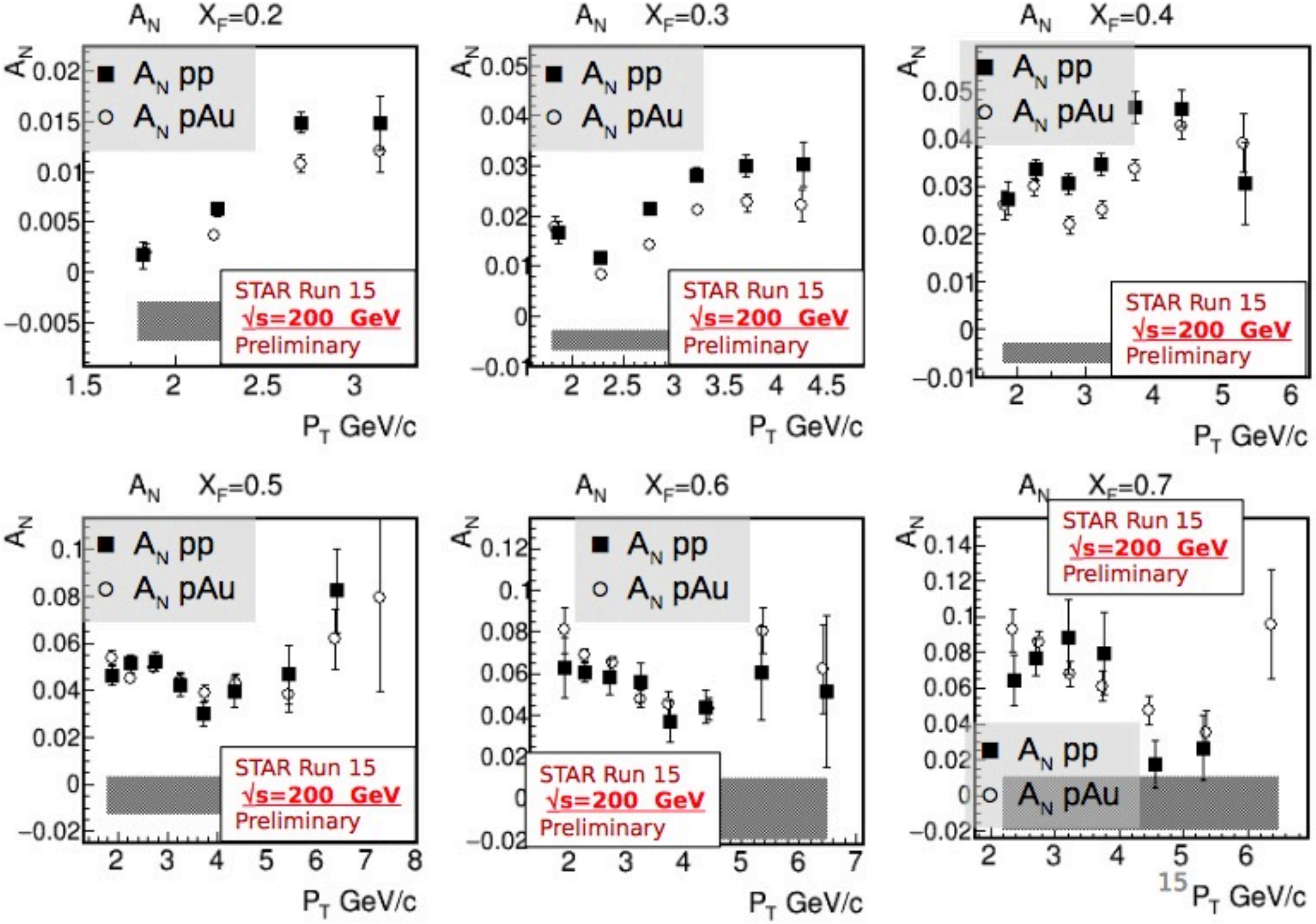
Transverse spin observables are driven by *multi-parton correlations*

In particular, the quark-gluon-quark FF $\hat{H}_{FU}^{\mathfrak{I}}(z, z_1)$ could be the main cause of A_N in *pp* collisions

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

➢ TSSAs in *pA* collisions

STAR, preliminary



No *A* dependence observed up to x_F = 0.7

2013 expression from Metz and DP

$$
\begin{aligned}
E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = & -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})} \\
& \times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right. \\
& \left. \qquad + \frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}
\end{aligned}
$$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\,\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\underbrace{\boxed{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]}}_{\sim A^{-1/3}} S^i_{H_1^\perp} + \underbrace{\boxed{\frac{1}{z}H^c(z)\,S^i_H}}_{\sim A^{-1/3}} + \underbrace{\boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}}_{\sim A^{0}}\right\}$$

Include saturation corrections to calculate *pA* TSSA
(Hatta, Xiao, Yoshida, Yuan (2017))

Note: QS term in *pA* ~ A^0, in agreement with STAR data
(Hatta, Xiao, Yoshida, Yuan (2016))

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\, f_1^b(x')\left\{\boxed{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]} S^i_{H_1^\perp} + \boxed{\frac{1}{z}H^c(z)\,S^i_H}\right.$$

$$\left. + \boxed{\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

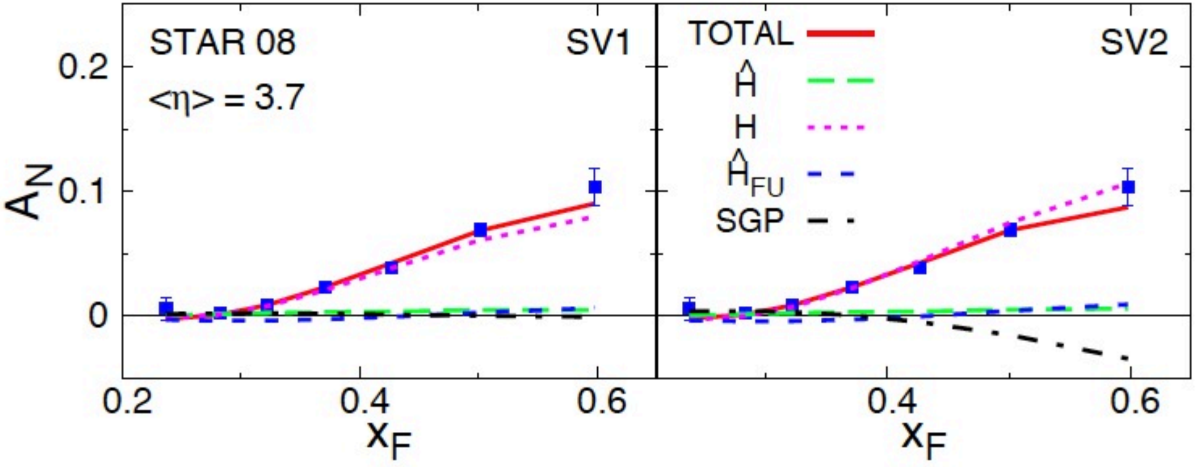
~ $A^{-1/3}$

~ $A^{-1/3}$

~ A^0

Include saturation corrections to calculate *pA* TSSA (Hatta, Xiao, Yoshida, Yuan (2017))

Kanazawa, Koike, Metz, DP (2014)



Term in blue is negligible
➔ fragmentation mechanism for *pA* TSSAs is suppressed by $A^{-1/3}$, contrary to the STAR data

2013 expression from Metz and DP

$$
\begin{aligned}
E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = &-\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1 \frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})} \\
&\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\,\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right. \\
&\left. + \boxed{\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}
\end{aligned}
$$

~ A^0

EOMR + LIR ➔

$$
\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\,\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z) + z\,\frac{dH_1^{\perp(1),c}(z)}{dz} - \frac{1}{z}\tilde{H}^c(z)
$$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

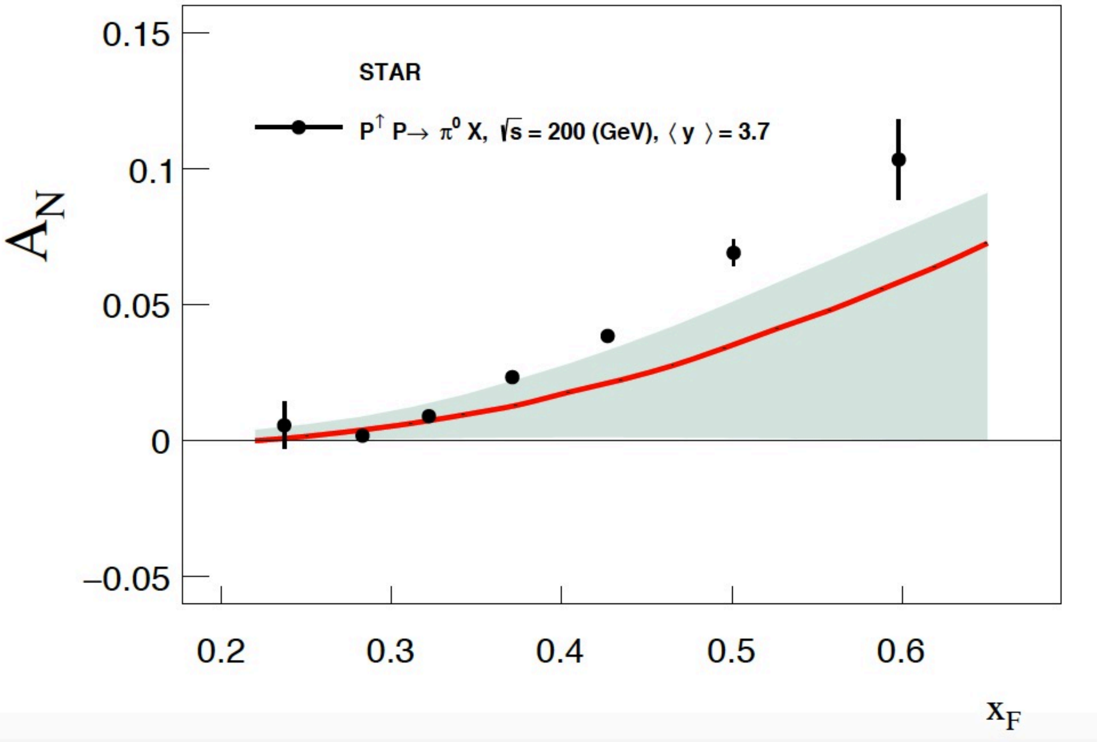
$$\left. + \boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

~ A^0

EOMR + LIR ➔

$$\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z) + z\frac{dH_1^{\perp(1),c}(z)}{dz} - \frac{1}{z}\tilde{H}^c(z)$$

Calculate pieces involving the (first k_T-moment of the) Collins function to get a new estimate for the term in blue



Fragmentation term is not inconsistent with the STAR *pA* TSSA data

(Gamberg, Kang, DP, Prokudin, arXiv:1701.09170)

Future Possibilities

$$E_h \frac{d\Delta\sigma^{QS}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_S^2 M}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{\pi}{\hat{s}\hat{u}}$$
$$\times\; f_1^b(x')\,D_1^c(z)\left[\boldsymbol{F_{FT}^a(x,x)} - x\frac{d\boldsymbol{F_{FT}^a(x,x)}}{dx}\right]S^i_{F_{FT}}$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}}$$
$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[\boldsymbol{H_1^{\perp(1),c}(z)} - z\frac{d\boldsymbol{H_1^{\perp(1),c}(z)}}{dz}\right]\tilde{S}^i_{H_1^\perp} + \left[-2\boldsymbol{H_1^{\perp(1),c}(z)} + \frac{1}{z}\boldsymbol{\tilde{H}^c(z)}\right]\tilde{S}^i_H\right\}$$

A_N in $\ell\, N^{\uparrow} \longrightarrow \pi\, X$

$$
\begin{aligned}
E_h \frac{d\sigma_{\rm LO}(S_N)}{d^3\vec{P}_h} &= \frac{8\alpha_{\rm em}^2}{S}\sum_q e_q^2 \int_0^1 dx \int_0^1 \frac{dz}{z^3}\,\delta(\hat{s}+\hat{t}+\hat{u}) \\
&\times\, \epsilon^{lPP_hS_N}\Bigg\{\pi M\left(1-x\frac{\mathrm{d}}{\mathrm{d}x}\right)\boldsymbol{F_{FT}^{q}(x,x)}\,D_1^q(z)\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^3\hat{u}}\right) \\
&+ M_h\, h_1^q(x)\Bigg[\left(-2\boldsymbol{H_1^{\perp(1),q}(z)}+\frac{1}{z}\boldsymbol{\tilde{H}^q(z)}\right)\left(\frac{2(\hat{u}-\hat{s})}{\hat{t}^3}\right)+\left(1-z\frac{\mathrm{d}}{\mathrm{d}z}\right)\boldsymbol{H_1^{\perp(1),q}(z)}\left(\frac{2\hat{u}}{\hat{t}^3}\right)\Bigg]\Bigg\}
\end{aligned}
$$

-Data from HERMES (2013) and JLab Hall A (2013) find sizable effects – most likely need NLO calculation (Hinderer, Schlegel, Vogelsang (2015); D'Alesio, Flore, Murgia (2017))

-Future data from an EIC – larger P_T and measure in the forward region of the transversely polarized nucleon (like at RHIC)

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T (Bacchetta, et al. (2007))

$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}^a(z)}{z}$$

$A_{UT}^{\sin\phi_S}$ in $e^+e^- \rightarrow h_1 h_2 X$ integrated over q_T (Boer, Jakob, Mulders (1997))

$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}(z_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

And also the TMD version of these (and other) observables (but with many more terms)

-Note: data from COMPASS, HERMES, and Belle show nonzero effects for the unintegrated version of the above asymmetries

Summary and Outlook

- TSSAs in proton-proton collisions have been around for over 40 years, but the underlying mechanism remained unclear
  - ➔ 2014: collinear twist-3 fragmentation could finally give us an explanation
  - ➔ Now: new constraints from LIRs and data on *pA*

- New constraint from LIR has been employed and an estimate of A_N confirms the fragmentation term dominates the asymmetry
  - ➔ This mechanism is not inconsistent with *pA*
  - ➔ Still need to fit the remaining (3-parton) FF $\tilde{H}(z)$

- $\tilde{H}$ is not a "new" function! – it shows up in (TMD and collinear) asymmetries in SIDIS and e^+e^-

- Transverse spin observables are driven by 3-parton (dynamical) functions