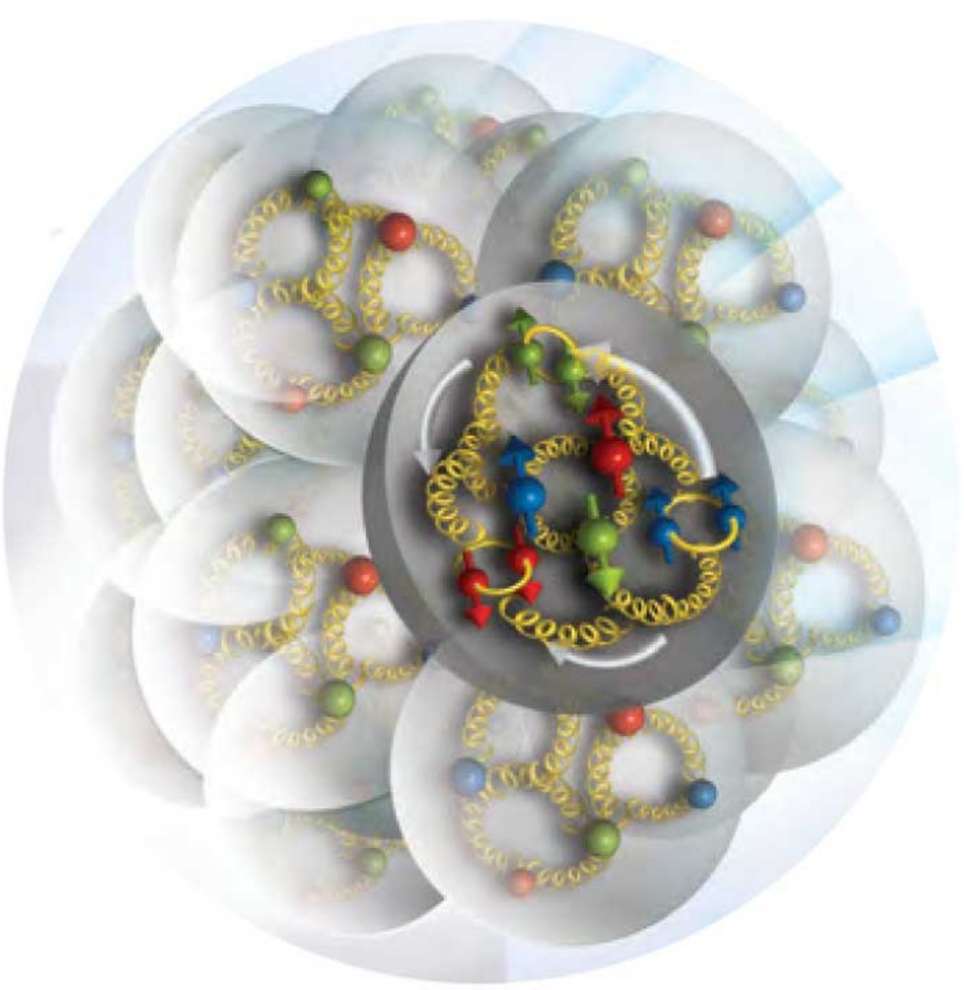
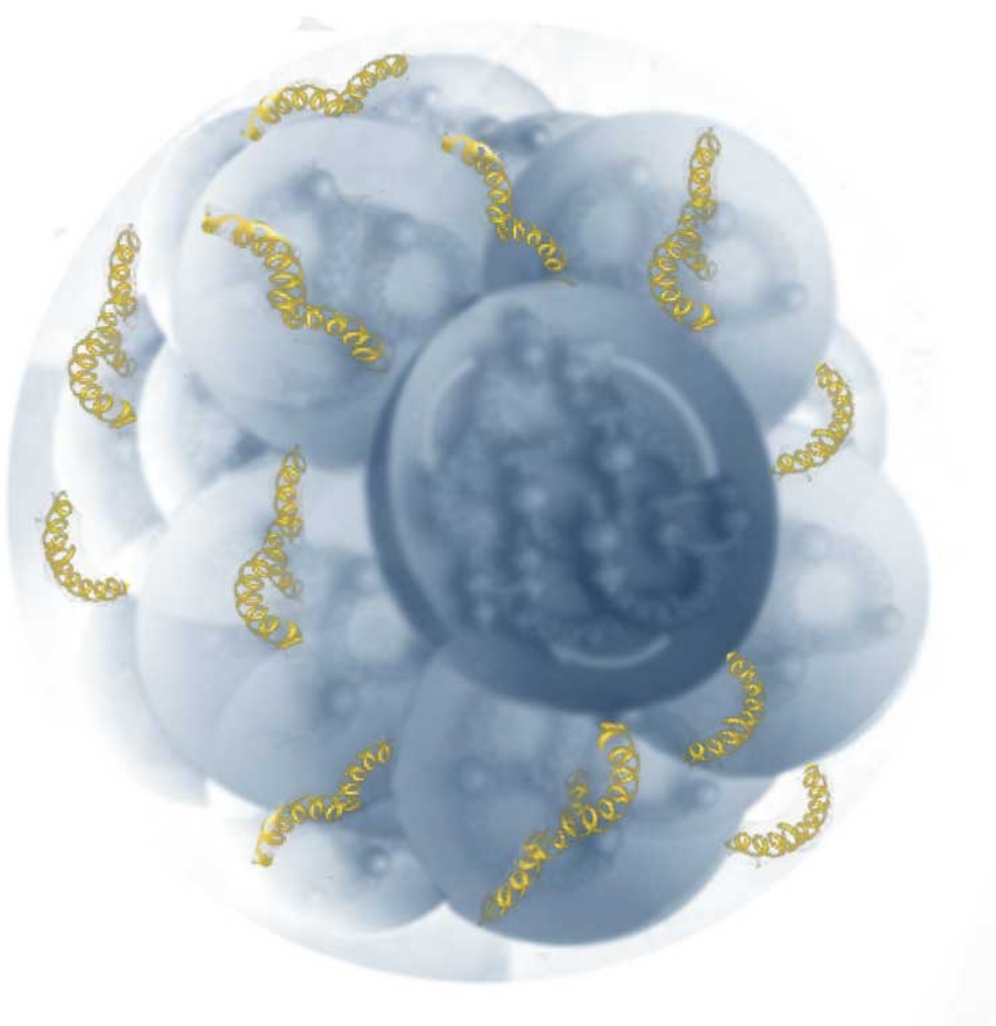
Gluonic Transversity from lattice QCD



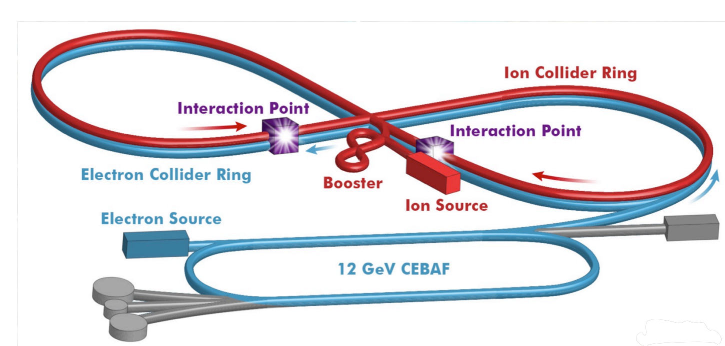


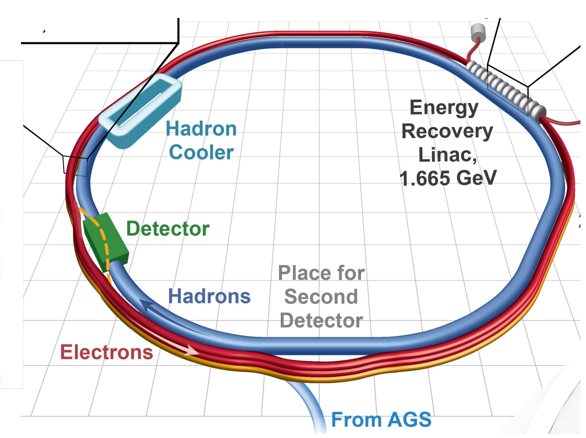
Phiala Shanahan, MIT

Gluonic Structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluonic structure (beyond gluon density) relatively unexplored
- Electron-Ion Collider
  - Priority in 2015 long range plan
  - "Understanding the glue that binds us all"
- What can LQCD do to help?

Cover image from EIC whitepaper arXiv::1212.1701

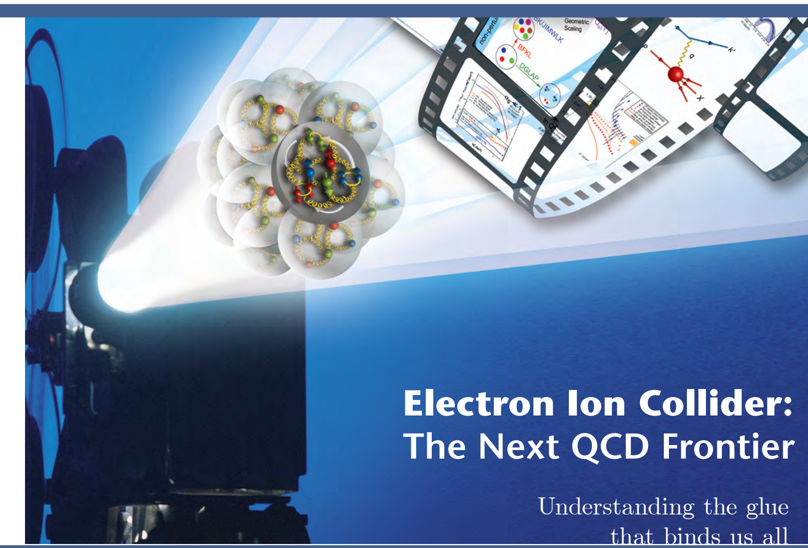




Lattice QCD for the EIC

Electron-Ion Collider

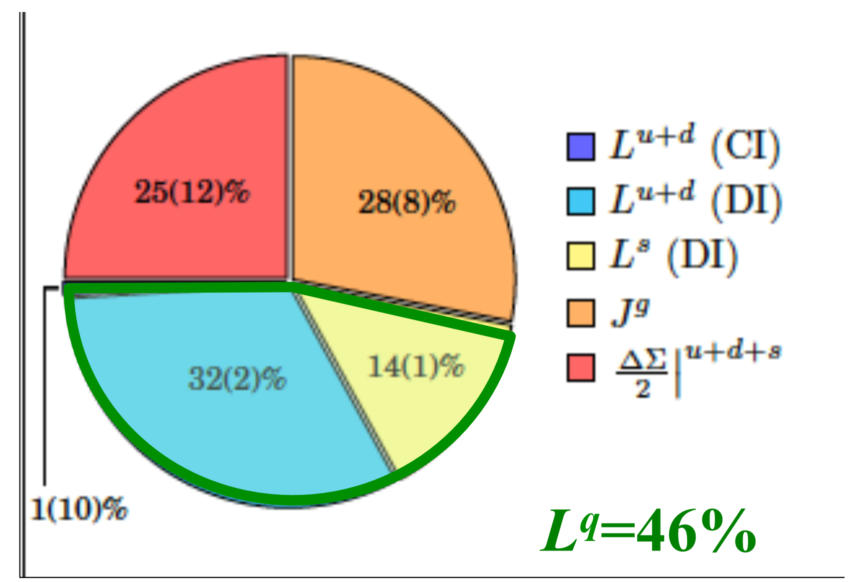
Precision gluon structure
Timescale~ 2025



Lattice QCD

Gluonic observables challenging!
Few results so far:

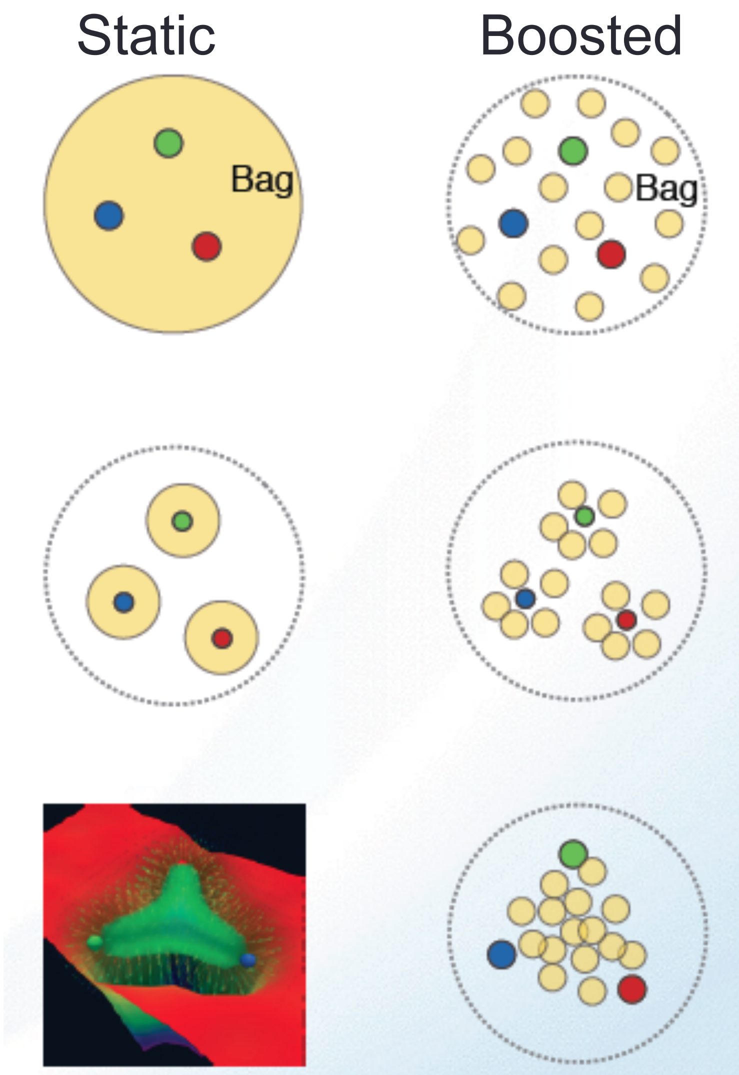
- Gluon momentum fraction
  [Meyer&Negele; Gockeler et al.;Yang et al.]
- Gluon contribution to helicity
  [Liu et al, Alexandru et al.]



K.F. Liu, C. Lorce, arXiv:1508.00911

Gluonic Structure

What does a proton look like?



Bag Model: Gluon field distribution is wider than the fast moving quarks.
Gluon radius > Charge Radius

Constituent Quark Model: Gluons and sea quarks hide inside massive quarks.
Gluon radius ~ Charge Radius

Lattice Gauge theory (with slow moving quarks), gluons more concentrated inside the quarks:
Gluon radius < Charge Radius

Gluonic Structure

Studying gluonic structure of hadrons/nuclei is hard

- Gluon probed only indirectly in electron scattering from hadrons/nuclei (does not couple to photon)

- Other processes less clean: heavy flavour production

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- Other processes less clean: heavy flavour production
- Quarks and gluons mix via evolution
  - **Uniquely quarky:** nonsinglet quantities
  - **Uniquely gluonic:** double helicity flip/ gluonic transversity

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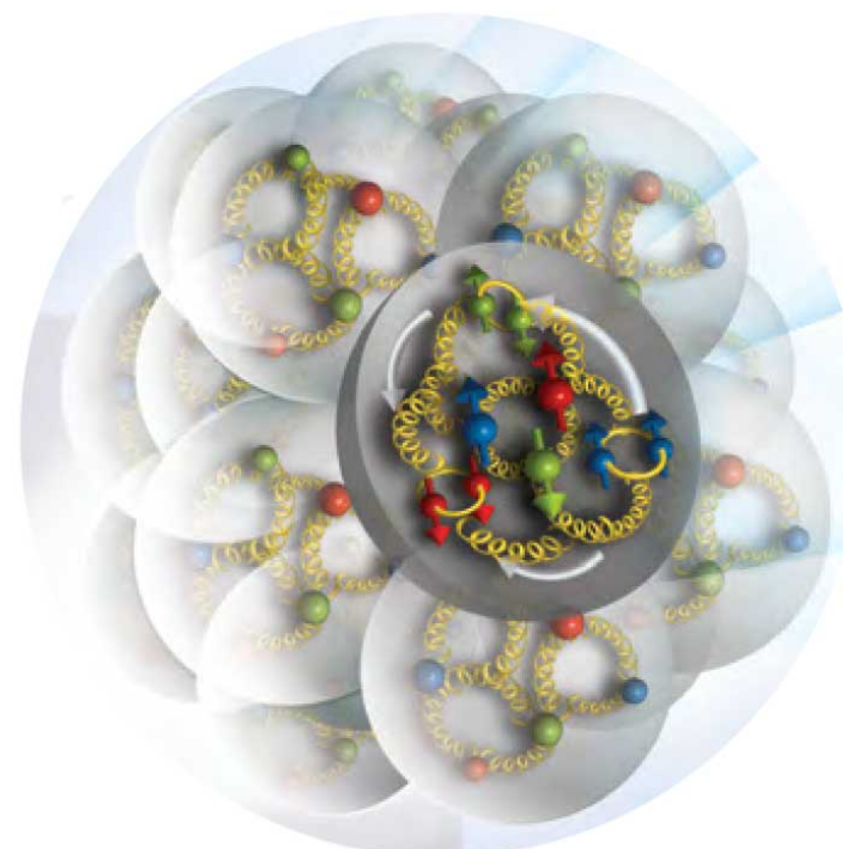
Gluonic Transversity

Double helicity flip structure function $\Delta(x,Q^2)$

- Purely gluonic observable
- Well-defined
- Hadrons: Gluonic Transversity
- Nuclei: Exotic Glue
    - gluons not associated with individual nucleons in nucleus
    - operator in nucleon = 0
      operator in nucleus ≠ 0



$$\langle p|\mathcal{O}|p\rangle = 0$$
$$\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$$

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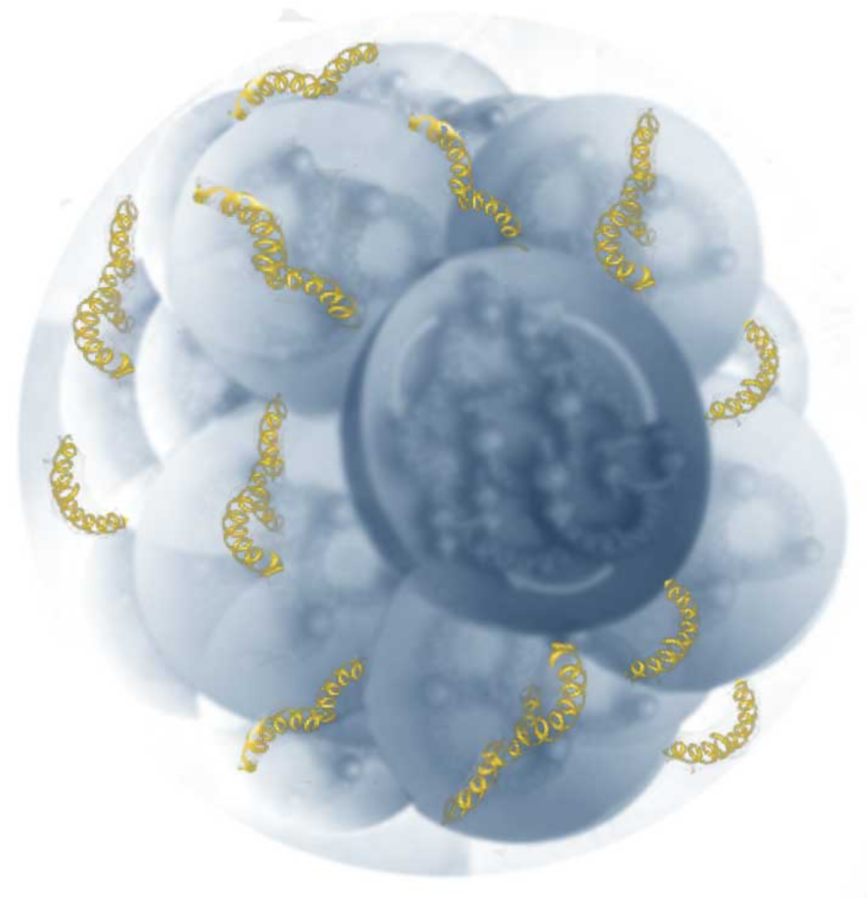


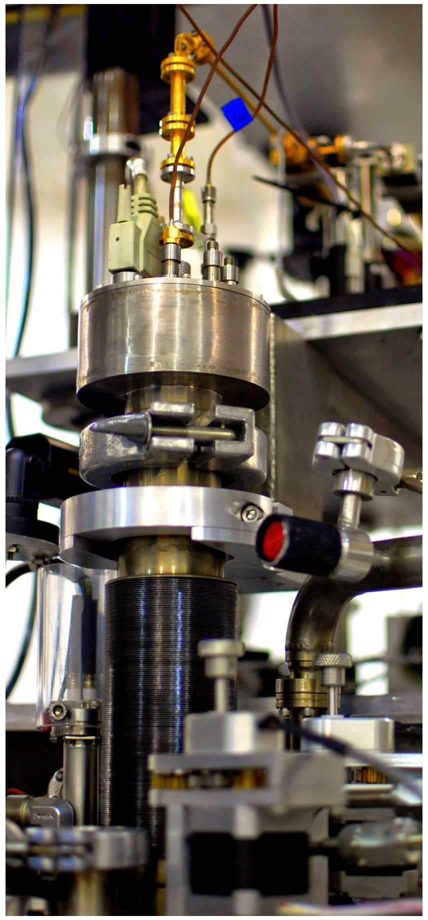
$$\langle p|\mathcal{O}|p\rangle = 0$$

$$\langle N, Z|\mathcal{O}|N, Z\rangle \neq 0$$

Double Helicity Flip Gluon Structure Function

Targets with J≥1 have leading twist gluon parton distribution $\Delta(x,Q^2)$: double helicity flip [Jaffe & Manohar 1989]

- Unambiguously gluonic: no analogous quark PDF at twist-2
- Vanishes in nucleon: measure of exotic glue in nuclei
- Experimentally measurable
  - Nitrogen target: JLab LoI 2015 [J. Maxwell]
  - Polarised nuclei at EIC [R. Milner]
- Moments calculable in LQCD



Double Helicity Flip Gluon Structure Function

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

$$W_{\mu\nu}^{\Delta=2} = \frac{1}{2}\left\{ \left[\left(E'^*_\mu - \frac{q\cdot E'^*}{\kappa\nu}\left(p_\mu - \frac{M^2}{\nu}q_\mu\right)\right)\left(E_\nu - \frac{q\cdot E}{\kappa\nu}\left(p_\nu - \frac{M^2}{\nu}q_\nu\right)\right) + (\mu \leftrightarrow \nu)\right] - \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{\kappa\nu^2}\left(p_\mu - \frac{\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{\nu}{q^2}q_\nu\right)\right]\left[E'^*\cdot E + \frac{M^2}{\kappa\nu^2}q\cdot E'^*\, q\cdot E\right]\right\}\boxed{\Delta(x,Q^2)}$$

structure function

Double Helicity Flip Gluon Structure Function

Hadronic tensor for inelastic lepton scattering from a polarized spin-one target:

structure function

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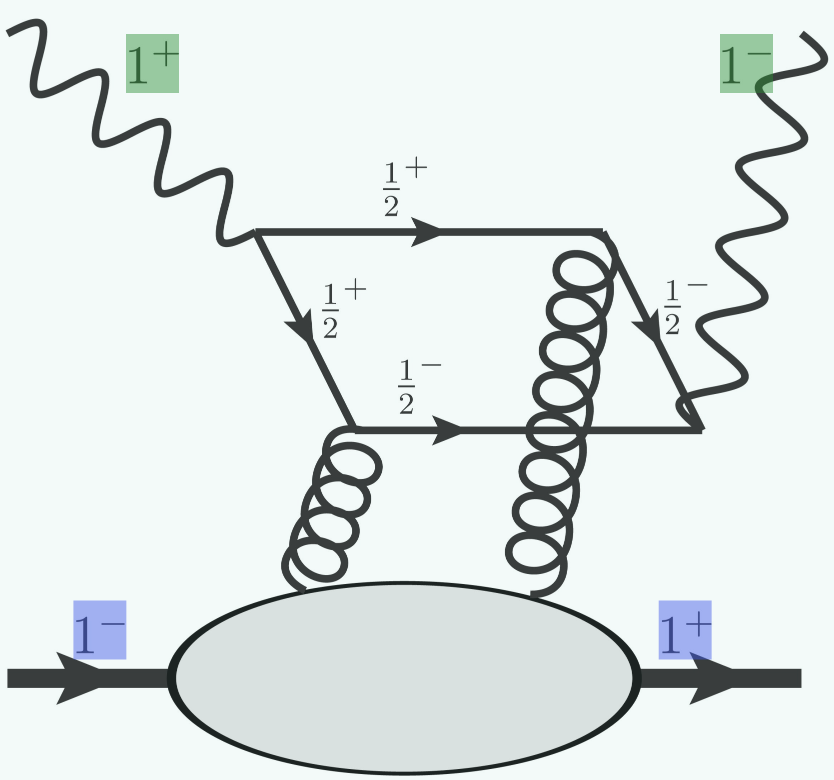
Helicity amplitude basis:

$$W_{\mu\nu}(p,q,E,E') = E'^{*\alpha}E^{\beta}W_{\mu\nu,\alpha\beta}(p,q)$$

$$W_{\mu\nu,\alpha\beta}(p,q) = \sum_{hH,h'H'} P(hH,h'H')_{\mu\nu,\alpha\beta}A_{hH,h'H'}(p,q)$$

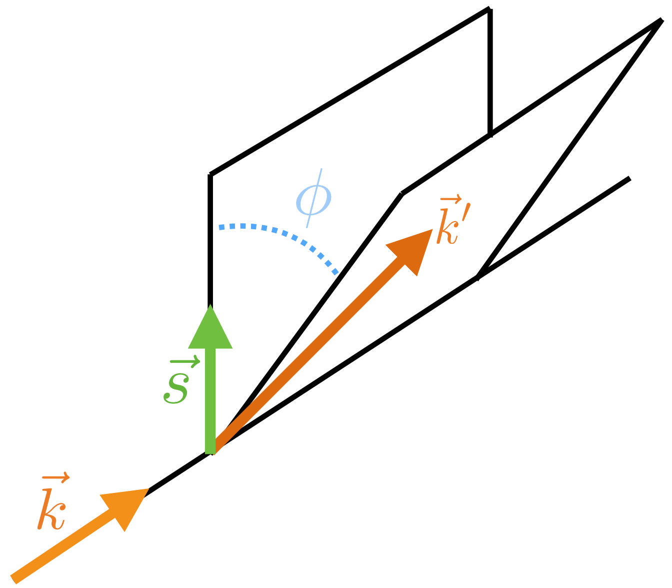
Changes both photon and target helicity by 2 units: $\Delta(x,Q^2) = A_{+-,-+}$



Double Helicity Flip Gluon Structure Function

Measurable in unpolarised electron DIS on transversely polarised target as azimuthal variation

- Nitrogen target: JLab LoI 2015 [J. Maxwell]
- Polarised nuclei at EIC [R. Milner]



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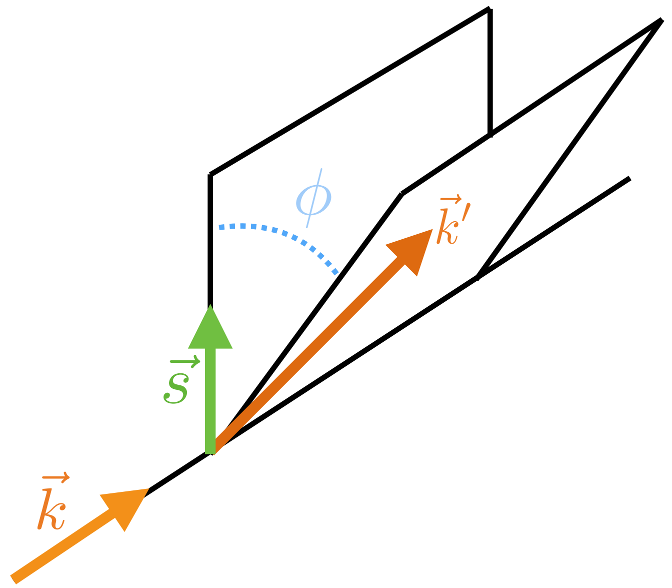


Parton model interpretation

$$\Delta(x, Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} \mathrm{Tr} \mathcal{Q}^2 \, x^2 \int_x^1 \frac{dy}{y^3} \left[g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(x, Q^2) \right]$$

$g_{\hat{x},\hat{y}}(y, Q^2)$: probability of finding a gluon with momentum fraction y linearly polarised in $\hat{x}$, $\hat{y}$ direction

"How much more momentum of transversely polarized particle carried by gluons aligned rather than perpendicular to it in the transverse plane"

Double Helicity Flip Gluon Structure Function

Moments of $\Delta(x,Q^2)$ are calculable in LQCD

Moment of Structure Function

Reduced Matrix Element

$$\int_0^1 dx x^{n-1} \Delta(x,Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \qquad n = 2,4,6\ldots,$$

Determined by matrix elements of local gluonic operators

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$\langle pE' | \underline{S} \left[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2} \right] | pE \rangle$$

Symmetrize and trace subtract in $\mu_1, \ldots, \mu_n$

$$= (-2i)^{n-2} \underline{S} \left[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} A_n(Q^2) \ldots,$$

Reduced Matrix Element

LQCD Calculation

First LQCD calculation [W Detmold & PES PRD 94 (2016), 014507]

- First moment in φ meson (simplest spin-1 system, eventually → nuclei)
- Lattice details: clover fermions, Lüscher-Weisz gauge action

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- Many systematics not addressed (yet)!
  - Quark mass effects
  - Volume effects
  - Discretisation
  - Renormalisation

LQCD Calculation

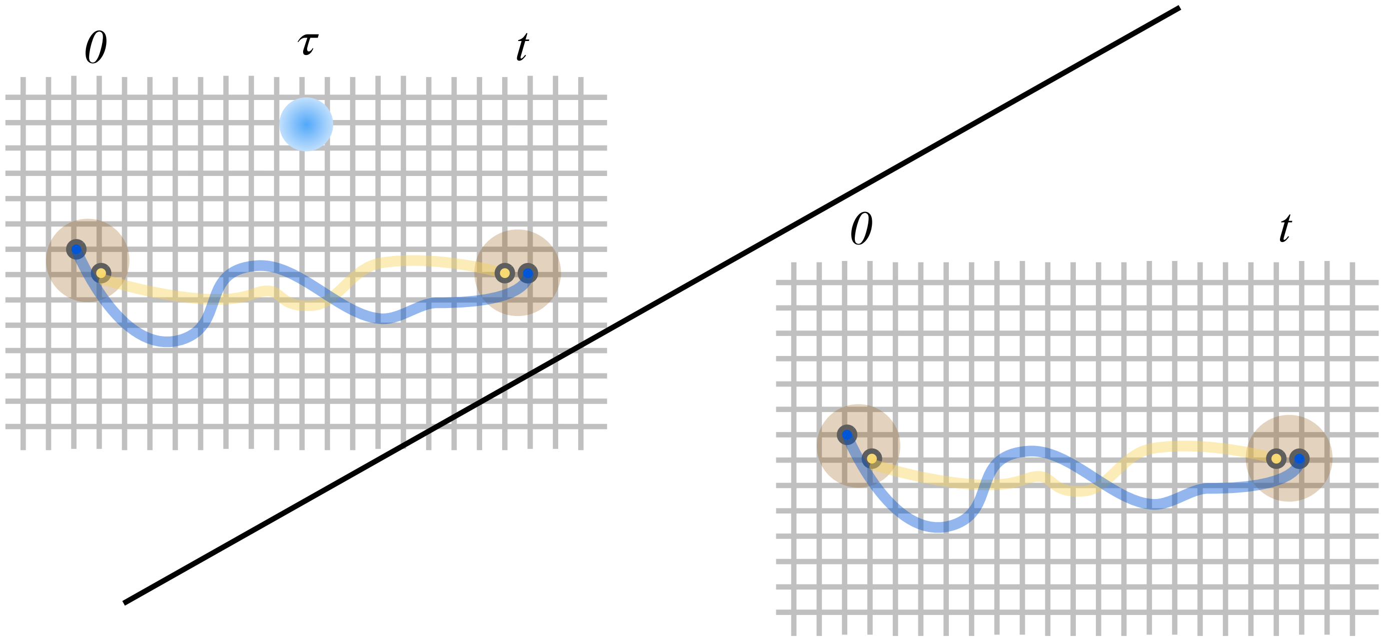
Calculate lowest moment of $\Delta(x,Q^2)$:

Moment of Structure Function

Reduced Matrix Element

$$\int_0^1 dx x^{n-1} \Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \qquad n = 2, 4, 6 \ldots,$$

Ratio of LQCD correlators $R_{jk}(t, \tau, \vec{p})$:



$$\propto A_2, \qquad 0 \ll \tau \ll t$$

LQCD Calculation

Calculate lowest moment of Δ(x,Q²):

Moment of Structure Function — Reduced Matrix Element

$$\boxed{\int_0^1 dx x^{n-1} \Delta(x, Q^2)} = \frac{\alpha_s(Q^2)}{3\pi} \frac{\boxed{A_n(Q^2)}}{n+2}, \qquad n = 2, 4, 6 \ldots,$$

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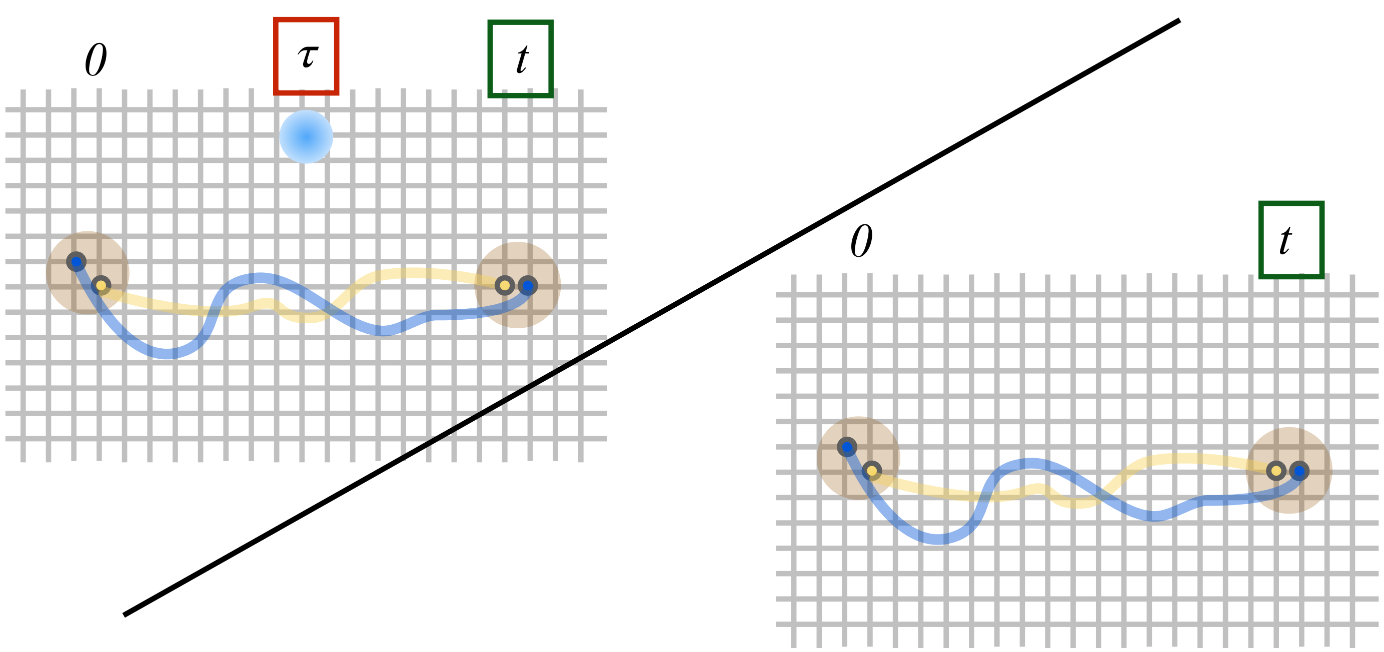

$\propto A_2, \qquad 0 \ll \boxed{\tau} \ll \boxed{t}$

operator insertion time

sink time

LQCD Calculation

More specifically,

$$C^{3\text{pt}}_{jk}(t,\tau,\vec{p}) = \sum_{\vec{x}}\sum_{\vec{y}} e^{i\vec{p}\cdot\vec{x}}\langle \eta_j(t,\vec{p})\ \mathcal{O}(\tau,\vec{y})\ \eta_k^\dagger(0,\vec{0})\rangle$$
$$= Z_\phi e^{-Et}\sum_{\lambda\lambda'}\epsilon_j^{(E)}(\vec{p},\lambda)\epsilon_k^{(E)*}(\vec{p},\lambda')\langle \vec{p},\lambda|\mathcal{O}|\vec{p},\lambda'\rangle$$

$$C^{2\text{pt}}_{jk}(t,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\langle \eta_j(t,\vec{x})\eta_k^\dagger(0,\vec{0})\rangle$$
$$= Z_\phi\left(e^{-Et}+e^{-E(T-t)}\right)\sum_{\lambda\lambda'}\epsilon_j^{(E)}(\vec{p},\lambda)\epsilon_k^{(E)*}(\vec{p},\lambda')$$

ratio depends on polarisations, momentum

$$R_{jk}(t,\tau,\vec{p}) = \frac{C^{3\text{pt}}_{jk}(t,\tau,\vec{p}) + C^{3\text{pt}}_{jk}(T-t,T-\tau,\vec{p})}{C^{2\text{pt}}_{jk}(t,\vec{p})}$$

$$\epsilon^\mu(\vec{p},\lambda) = \left(\frac{\vec{p}\cdot\vec{e}_\lambda}{m}, \vec{e}_\lambda + \frac{\vec{p}\cdot\vec{e}_\lambda}{m(m+E)}\vec{p}\right)$$
$$\vec{e}_\pm = \mp\frac{m}{\sqrt{2}}(0,1,\pm i),$$
$$\vec{e}_0 = m(1,0,0).$$

LQCD Calculation

More specifically,

$$C_{jk}^{3\text{pt}}(t,\tau,\vec{p}) = \sum_{\vec{x}}\sum_{\vec{y}} e^{i\vec{p}\cdot\vec{x}}\langle \eta_j(t,\vec{p})\ \mathcal{O}(\tau,\vec{y})\ \eta_k^\dagger(0,\vec{0})\rangle$$

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- All polarisation combinations (j,k)
- Boost momenta up to (1,1,1)
- Examine all elements of each hypercubic irrep.

$$\epsilon^\mu(\vec{p},\lambda) = \left(\frac{\vec{p}\cdot\vec{e}_\lambda}{m}, \vec{e}_\lambda + \frac{\vec{p}\cdot\vec{e}_\lambda}{m(m+E)}\vec{p}\right)$$

$$\vec{e}_\pm = \mp\frac{m}{\sqrt{2}}(0,1,\pm i),$$

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LQCD Calculation

Ratio of correlation functions determines reduced matrix elt.

- p=(0,0,0)

$$\begin{array}{c c} & \begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\ \begin{array}{c} \rho_0 \\ \rho_+ \\ \rho_- \end{array} & \begin{pmatrix} \frac{2m^2 A_2}{\sqrt{3}} & 0 & 0 \\ 0 & -\frac{m^2 A_2}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{m^2 A_2}{\sqrt{3}} \end{pmatrix} \end{array}$$

LQCD Calculation

Ratio of correlation functions determines reduced matrix elt.

- p=(0,0,0)

$$
\begin{array}{c}
 \\ \rho_0 \\ \rho_+ \\ \rho_-
\end{array}
\begin{array}{c}
\begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\
\begin{pmatrix}
\frac{2m^2 A_2}{\sqrt{3}} & 0 & 0 \\
0 & -\frac{m^2 A_2}{\sqrt{3}} & 0 \\
0 & 0 & -\frac{m^2 A_2}{\sqrt{3}}
\end{pmatrix}
\end{array}
$$

- p=(1,1,1) (lattice units)

$$
\begin{array}{c}
 \\ \rho_0 \\ \rho_+ \\ \rho_-
\end{array}
\begin{array}{c}
\begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\
\begin{pmatrix}
\frac{2\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} \\
\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} \\
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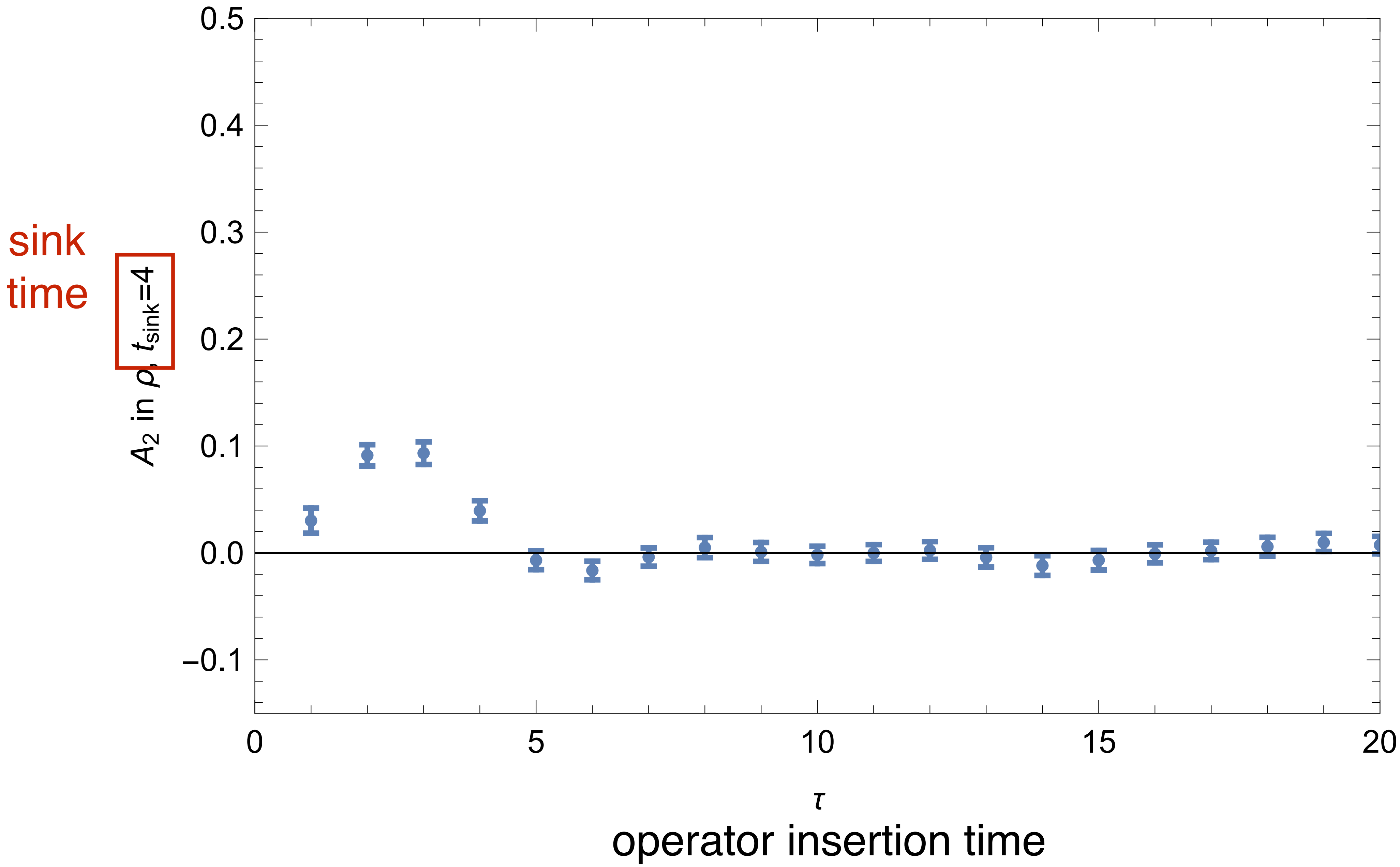
\end{pmatrix}
\end{array}
$$

LQCD Calculation

Lattice QCD quantity $\propto A_2, \qquad 0 \ll \tau \ll t$



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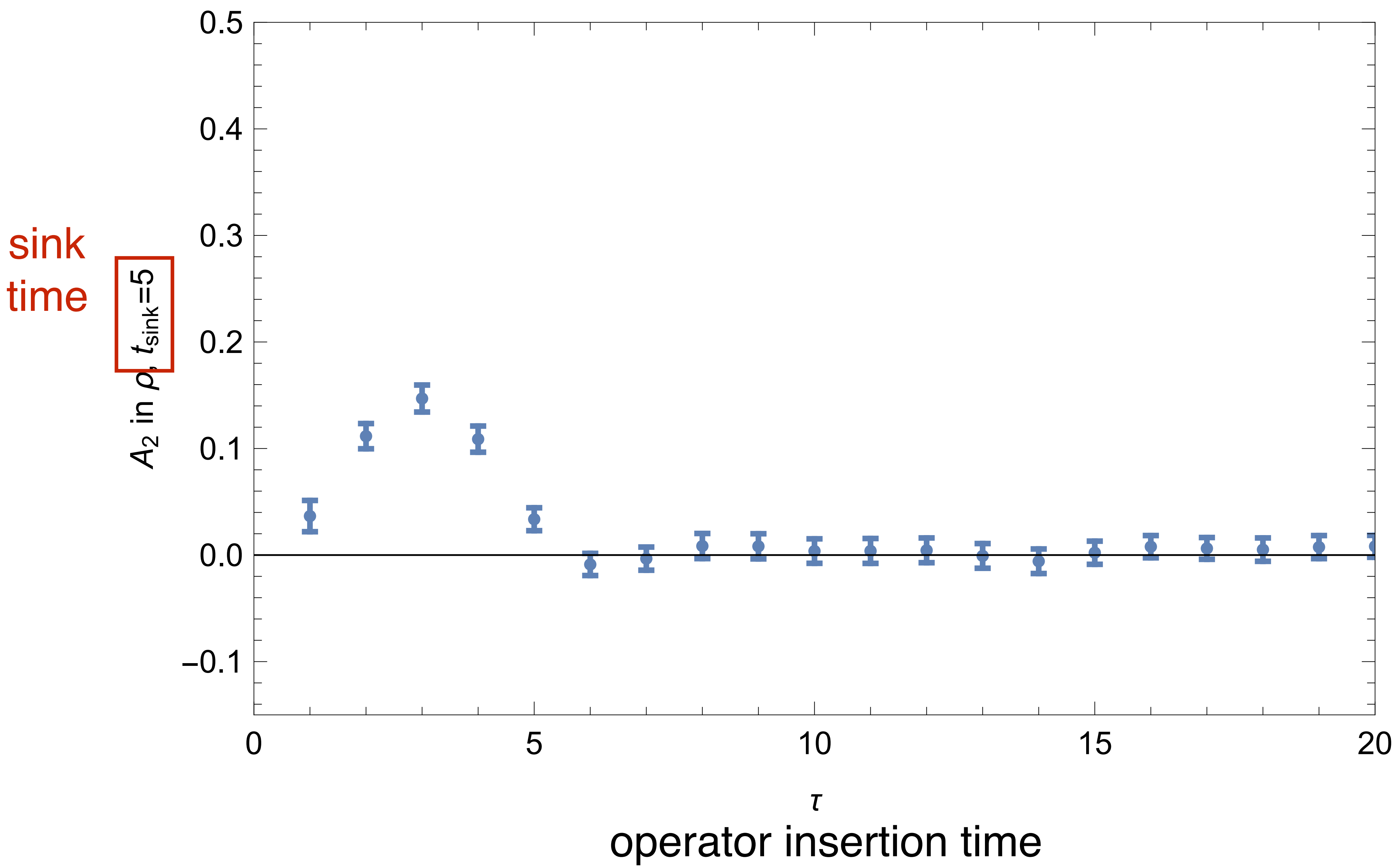

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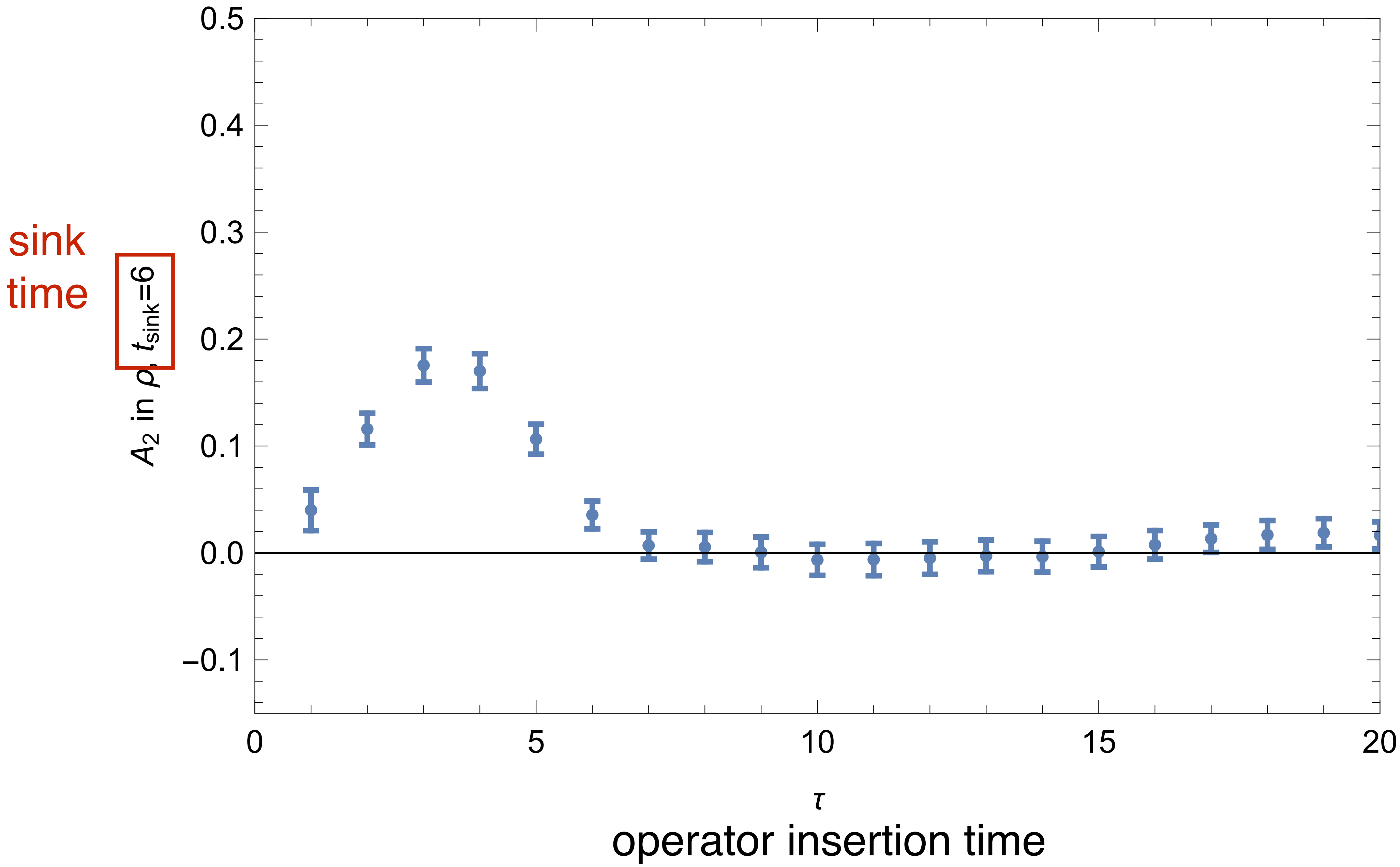

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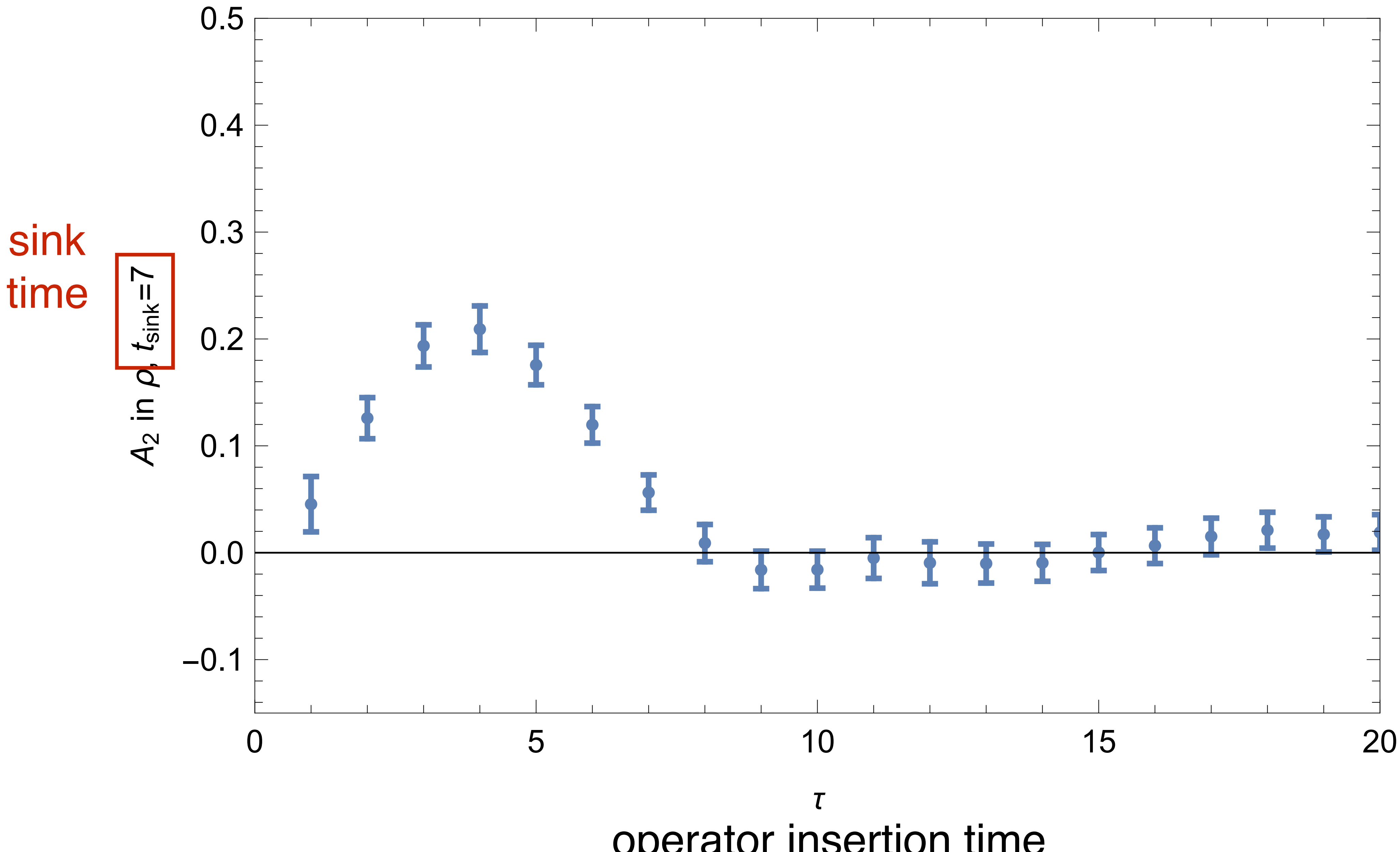

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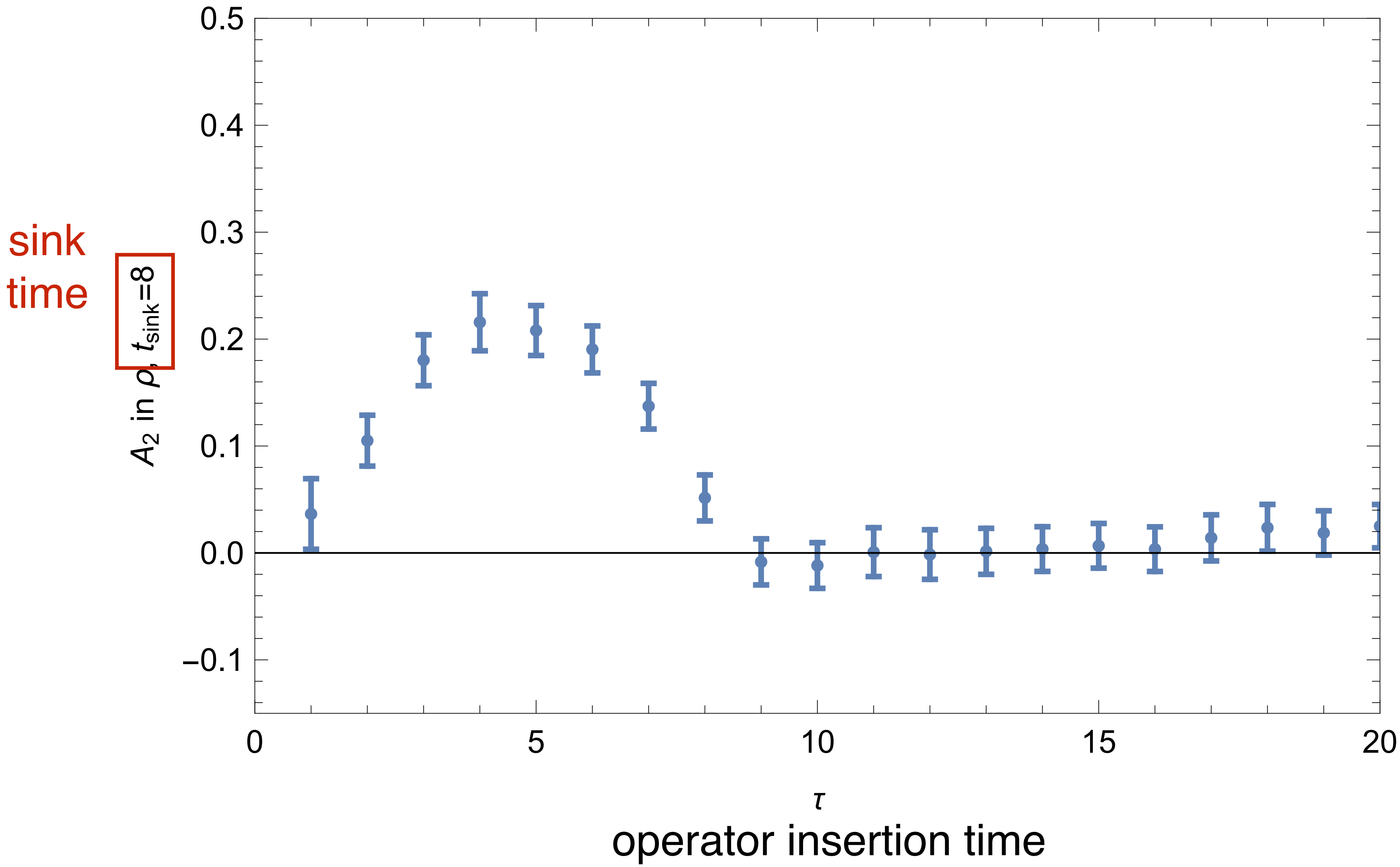

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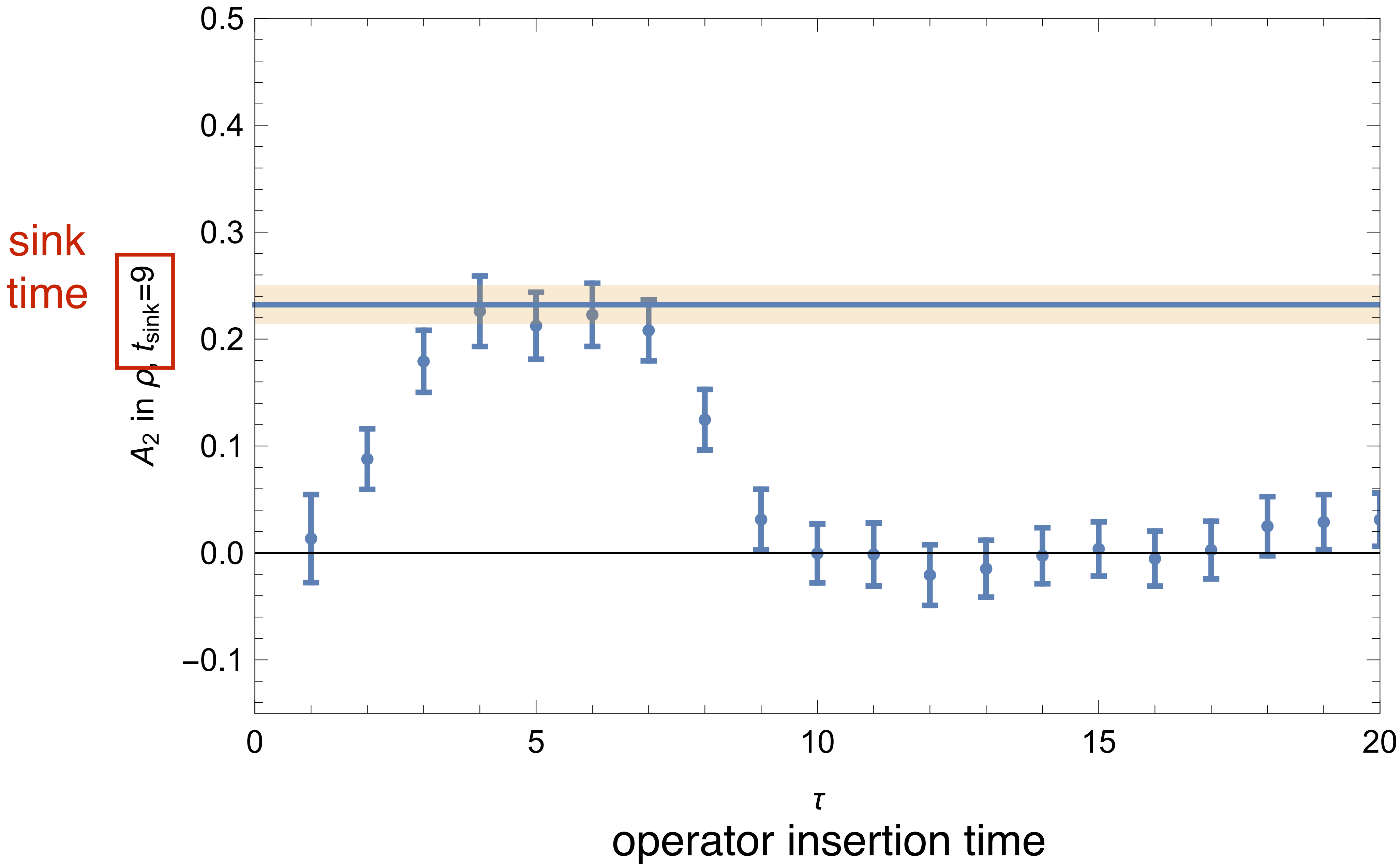
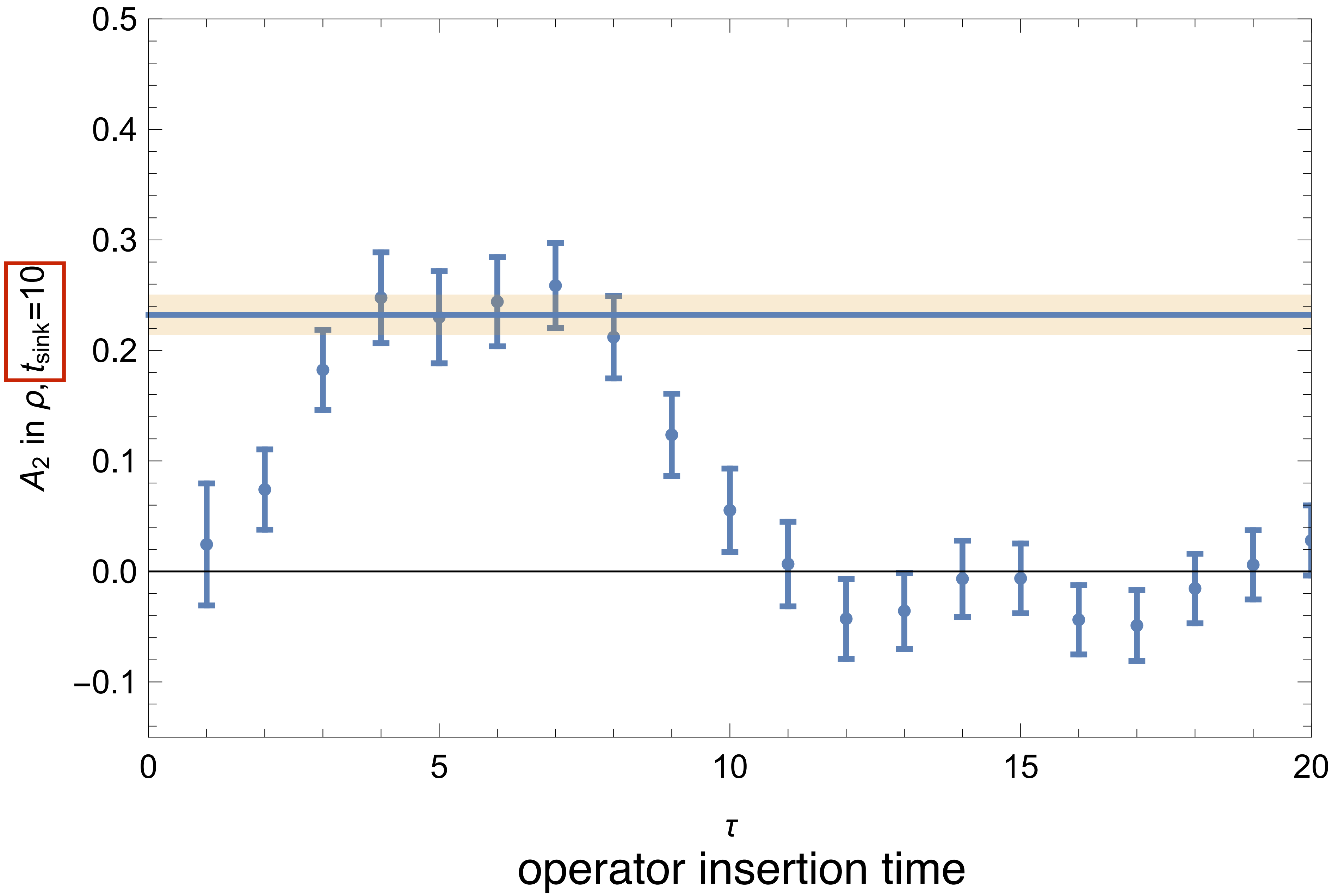


LQCD Calculation

Lattice QCD quantity $\propto A_2, \qquad 0 \ll \tau \ll t$

sink time



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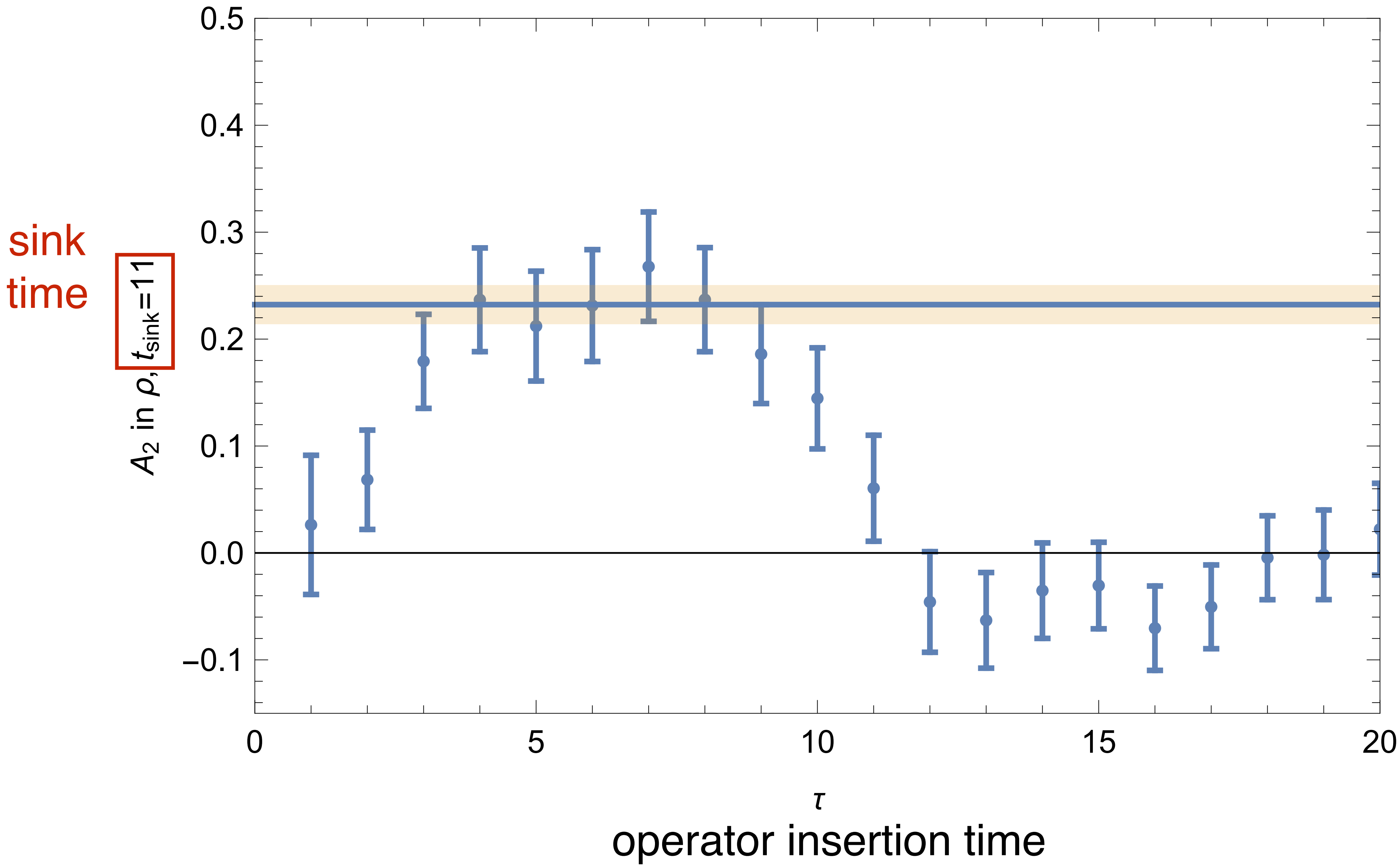

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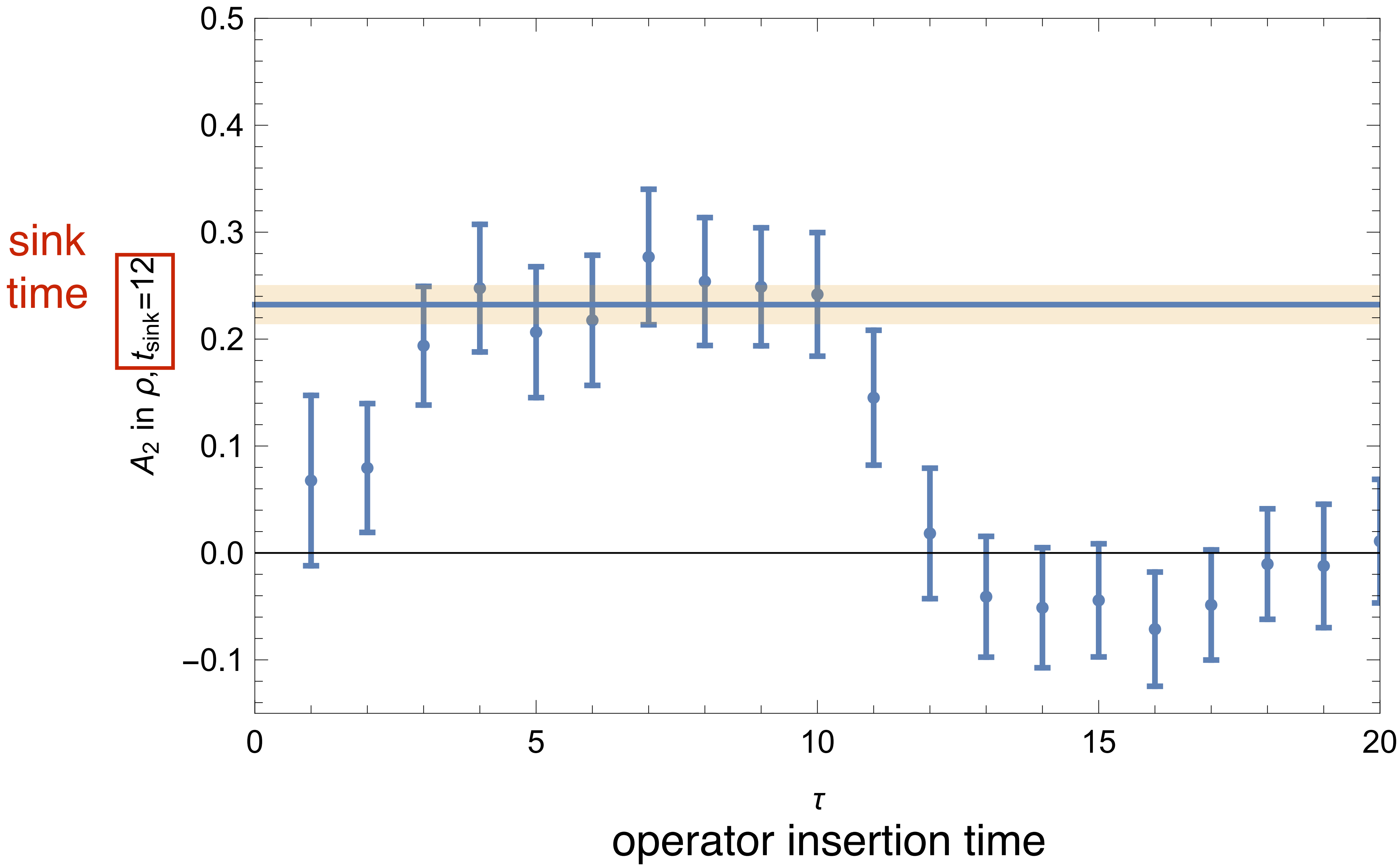

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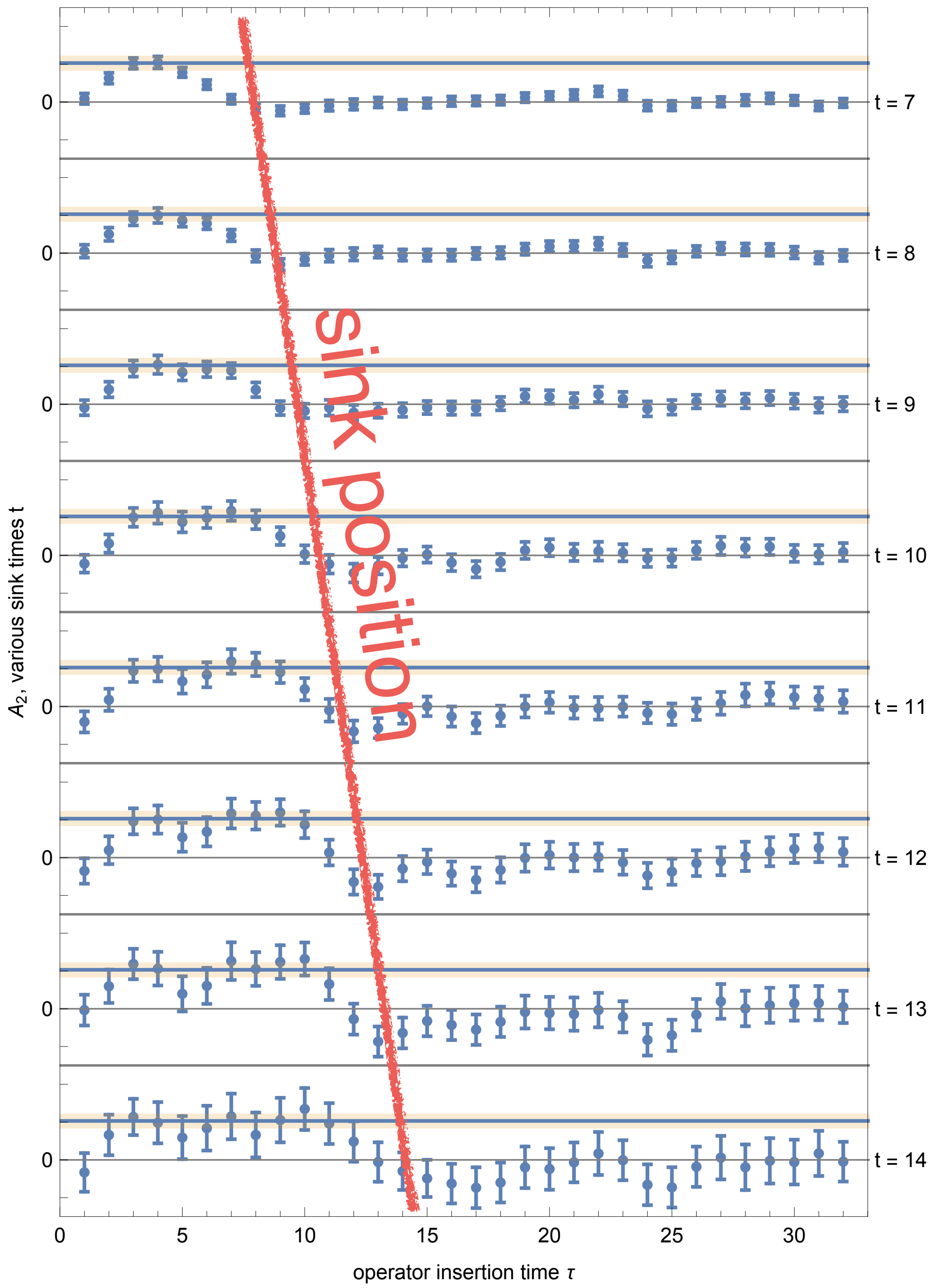

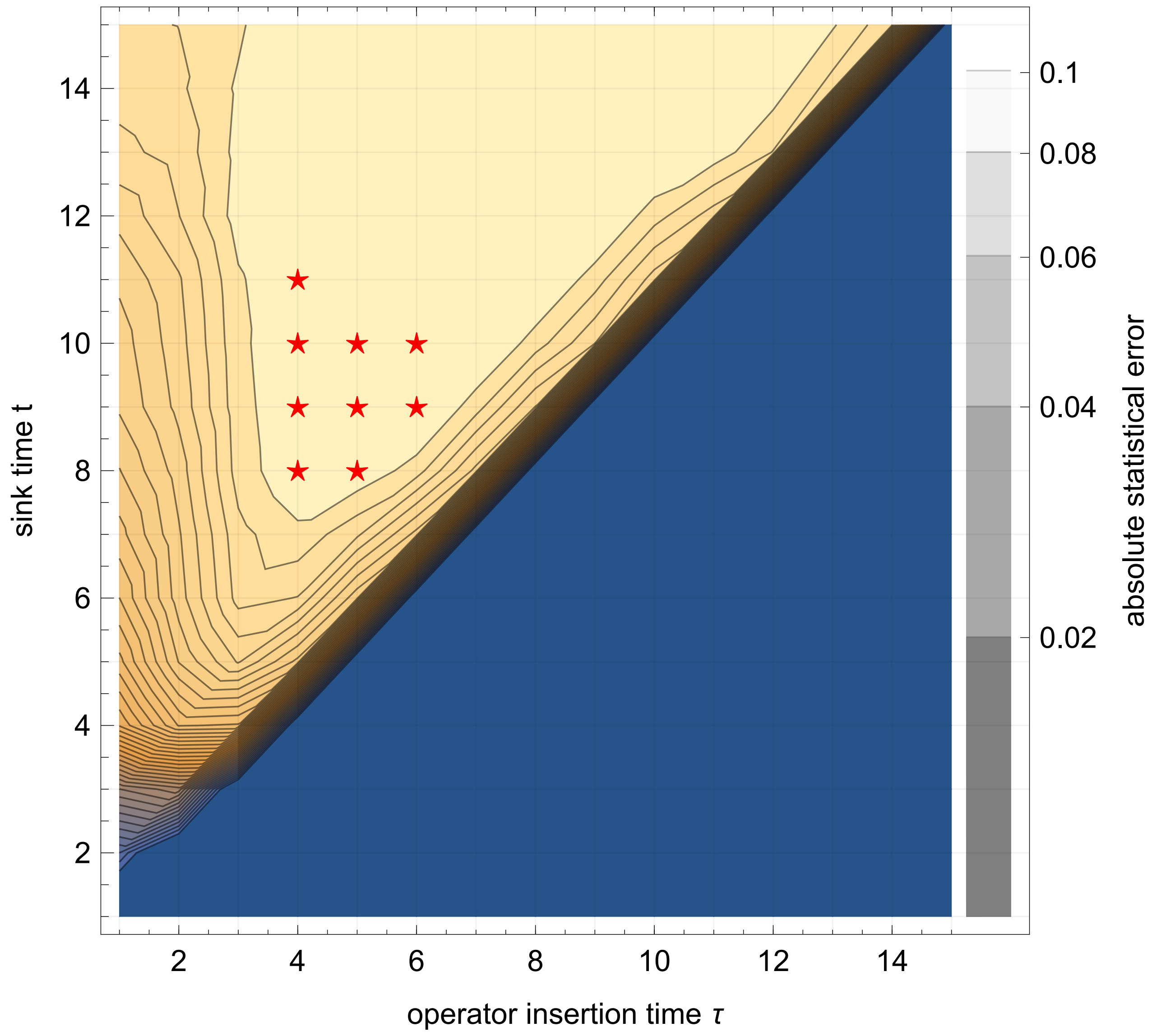

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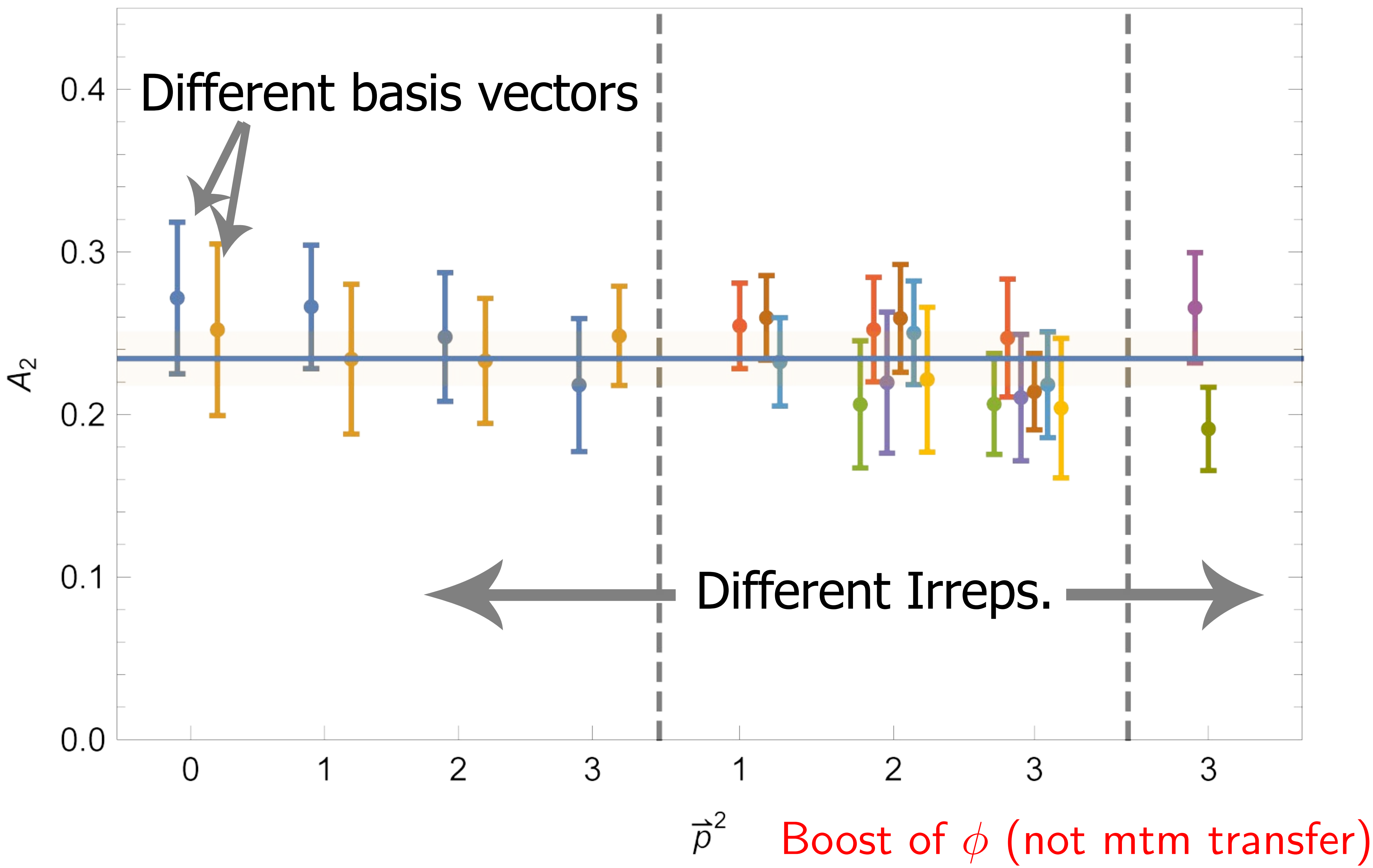


Gluonic Radii

Off-forward matrix elements are complicated

- Eg: moments of $\Delta(x,Q^2)$ related to many form factors

$$
\begin{aligned}
&\left\langle p'E' \left| S\left[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2} \right] \right| pE \right\rangle \\
&= \sum_{\substack{m \text{ odd} \\ m=3}}^{n} \Big\{ A^{(n)}_{1,m-3}(t,\mu^2)\, S\left[(P_\mu E_{\mu_1} - E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \right] \\
&\quad + A^{(n)}_{2,m-3}(t,\mu^2)\, S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2}) \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \right] \\
&\quad + A^{(n)}_{3,m-3}(t,\mu^2)\, S\Big[\big((\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) - (\Delta_\mu E'^*_{\mu_1} - E'^*_\mu \Delta_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2}) \big) \\
&\qquad\qquad \times \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \Big] \\
&\quad + A^{(n)}_{4,m-3}(t,\mu^2)\, S\left[(E_\mu E'^*_{\mu_1} - E_{\mu_1} E'^*_\mu)(P_\nu \Delta_{\mu_2} - P_{\mu_2} \Delta_\nu) \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \right] \\
&\quad + \frac{A^{(n)}_{5,m-3}(t,\mu^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2}) \\
&\qquad\qquad + (E'^*\cdot P)(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(\Delta_\nu E_{\mu_2} - E_\nu \Delta_{\mu_2}) \big) \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \Big] \\
&\quad + \frac{A^{(n)}_{6,m-3}(t,\mu^2)}{M^2} S\Big[\big((E\cdot P)(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) \\
&\qquad\qquad - (E'^*\cdot P)\, (P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2}) \big) \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \Big] \\
&\quad + \frac{A^{(n)}_{7,m-3}(t,\mu^2)}{M^2} (E'^*\cdot E) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu \Delta_{\mu_2} - \Delta_\nu P_{\mu_2}) \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \right] \\
&\quad + \frac{A^{(n)}_{8,m-3}(t,\mu^2)}{M^4} (E\cdot P)(E'^*\cdot P) S\left[(P_\mu \Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu \Delta_{\mu_2} - \Delta_\nu P_{\mu_2}) \Delta_{\mu_3} \ldots \Delta_{\mu_{m-1}} P_{\mu_m} \ldots P_{\mu_n} \right] \Big\}
\end{aligned}
$$

Gluonic Radii

Off-forward matrix elements are complicated

- Eg: moments of $\Delta(x,Q^2)$ related to many form factors

$$
\left\langle p'E' \left| S\left[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2} \right] \right| pE \right\rangle
$$

$$
= \sum_{\substack{m\ \mathrm{odd} \\ m=3}}^{n} \Big\{ A^{(n)}_{1,m-3}(t,\mu^2)\, S\left[(P_\mu E_{\mu_1} - E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}} P_{\mu_m}\ldots P_{\mu_n}\right]
$$

$$
+ A^{(n)}_{2,m-3}(t,\mu^2)\, S\left[(\Delta_\mu E_{\mu_1} - E_\mu \Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2} - E'^*_\nu \Delta_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}} P_{\mu_m}\ldots P_{\mu_n}\right]
$$

$$
+ A^{(n)}_{3,m-3}(t,\mu^2)\, S\left[\left((\Delta_\mu E_{\mu_1} - E_\mu\Delta_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2}) - (\Delta_\mu E'^*_{\mu_1} - E'^*_\mu \Delta_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\right) \right.
$$

$$
\times \Delta_{\mu_3} \ldots \Delta_{\mu_m} \cdots
$$

$$
+ A^{(n)}_{4,m-3}(t,\mu^2)\, S\left[(E_\mu E_{\mu_1} \cdots \right.
$$

$$
+ \frac{A^{(n)}_{5,m-3}(t,\mu^2)}{M^2} S\left[\left((E\cdot P)(P_\mu\Delta \cdots \right.\right.
$$

$$
+ (E'^* \cdot P)(P \cdots P_{\mu_n}\Big]
$$

$$
+ \frac{A^{(n)}_{6,m-3}(t,\mu^2)}{M^2} S\left[\left((E'\cdot P)(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2} - E'^*_\nu P_{\mu_2})\right.\right.
$$

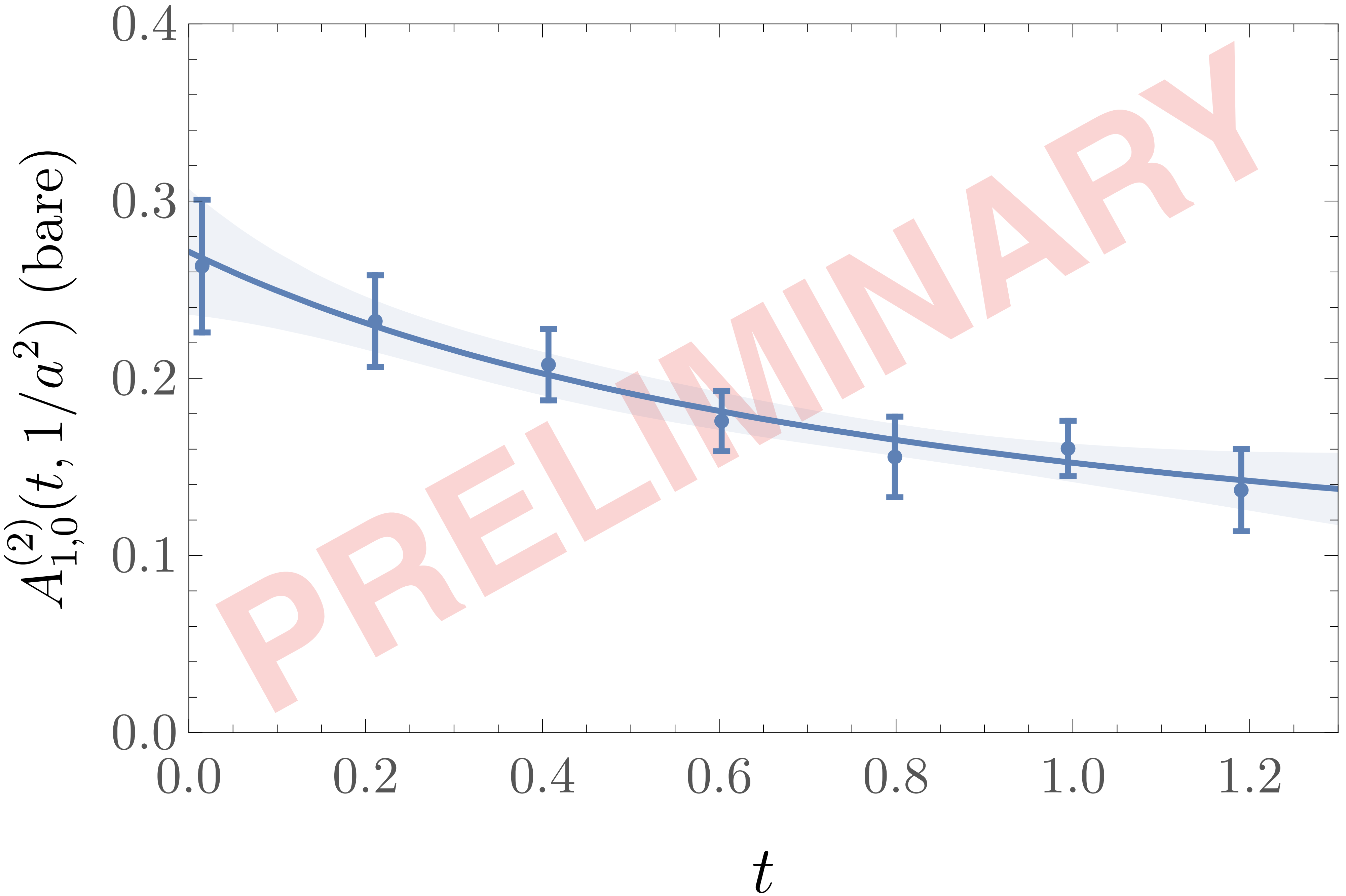
$$
\left.\left. - (E'^* \cdot P)\right)(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2} - E_\nu P_{\mu_2})\right)\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}} P_{\mu_m}\ldots P_{\mu_n}\Big]
$$

$$
+ \frac{A^{(n)}_{7,m-3}(t,\mu^2)}{M^2} (E'^* \cdot E) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}} P_{\mu_m}\ldots P_{\mu_n}\right]
$$

$$
+ \frac{A^{(n)}_{8,m-3}(t,\mu^2)}{M^4} (E\cdot P)(E'^*\cdot P) S\left[(P_\mu\Delta_{\mu_1} - \Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2} - \Delta_\nu P_{\mu_2})\Delta_{\mu_3}\ldots\Delta_{\mu_{m-1}} P_{\mu_m}\ldots P_{\mu_n}\right] \Big\}
$$

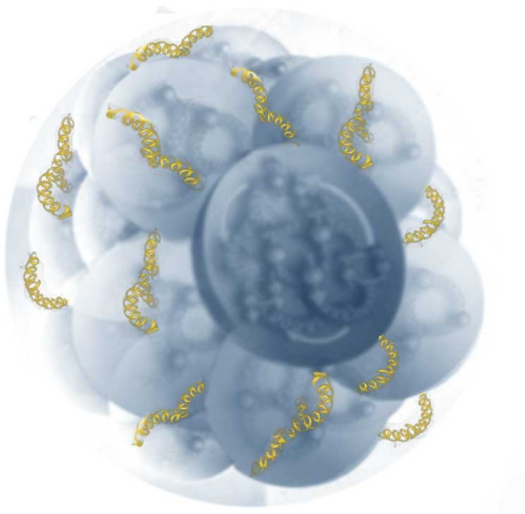
Many gluonic radii:
Defined by slope of each form factor at $Q^2=t=0$

Gluonic Radii



Gluonic Structure circa 2025

- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
  - Work towards a complete 3D picture of parton structure (PDFs, GPDs, TMDs)
  - $\Delta(x,Q^2)$ has an interesting role
    - Purely gluonic
    - Non-nucleonic: directly probe nuclear effects



  - Compare quark and gluon distributions in hadrons and nuclei
- Lattice QCD calculations in hadrons and light nuclei will be available