

# The Strange Proton

Strange quarks in nucleon structure  
from lattice QCD



THE UNIVERSITY  
*of* ADELAIDE



**COEPP**

ARC Centre of Excellence for  
Particle Physics at the Terascale

SPECIAL RESEARCH  
CENTRE FOR THE

**SUBATOMIC**



**STRUCTURE OF MATTER**



**Massachusetts  
Institute of  
Technology**

Phiala Shanahan

Massachusetts Institute of Technology

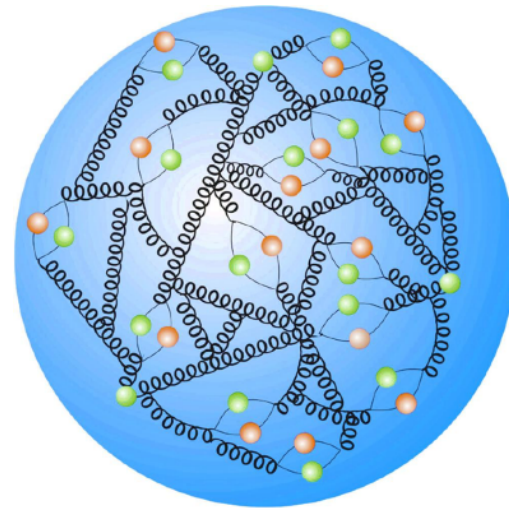
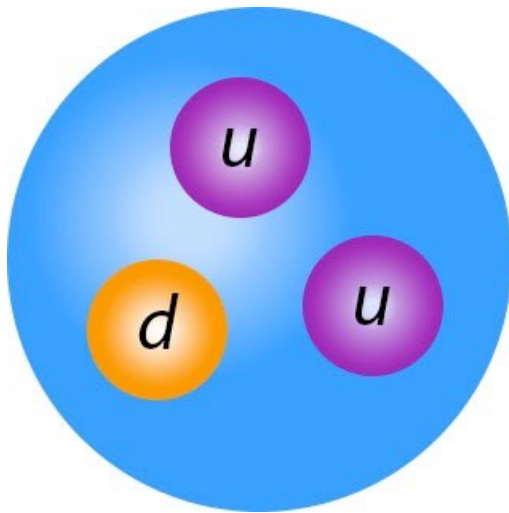
# Strangeness in the Nucleon

- **Strange nucleon form factors**
  - 'Hidden flavour' contributions to nucleon observables
  - Test for nonperturbative QCD

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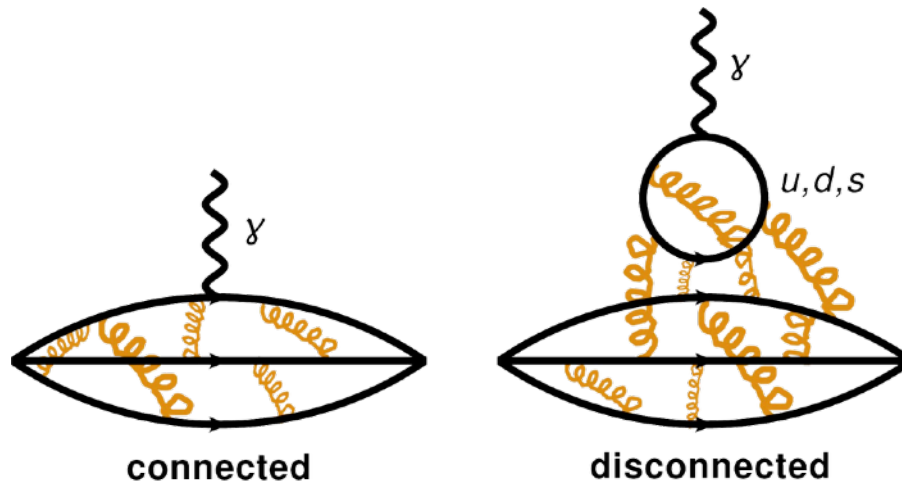


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Generated entirely by interactions with the vacuum



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### Strange quarks

Lightest of the 'sea only' quarks

→ play the largest role

1. **Strange quark electromagnetic form factors**
2. **Strange sigma terms** - relevant to dark matter direct detection experiments

# Strangeness in the Nucleon

- **Strange nucleon form factors**

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Experimental measurements limited by lack of **theory** knowledge of

- **Charge symmetry violating (CSV) form factors**

# Strangeness in the Nucleon

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Approach **both** problems through lattice QCD simulations of

- **Nucleon and hyperon EM form factors**

# Quantum Chromodynamics

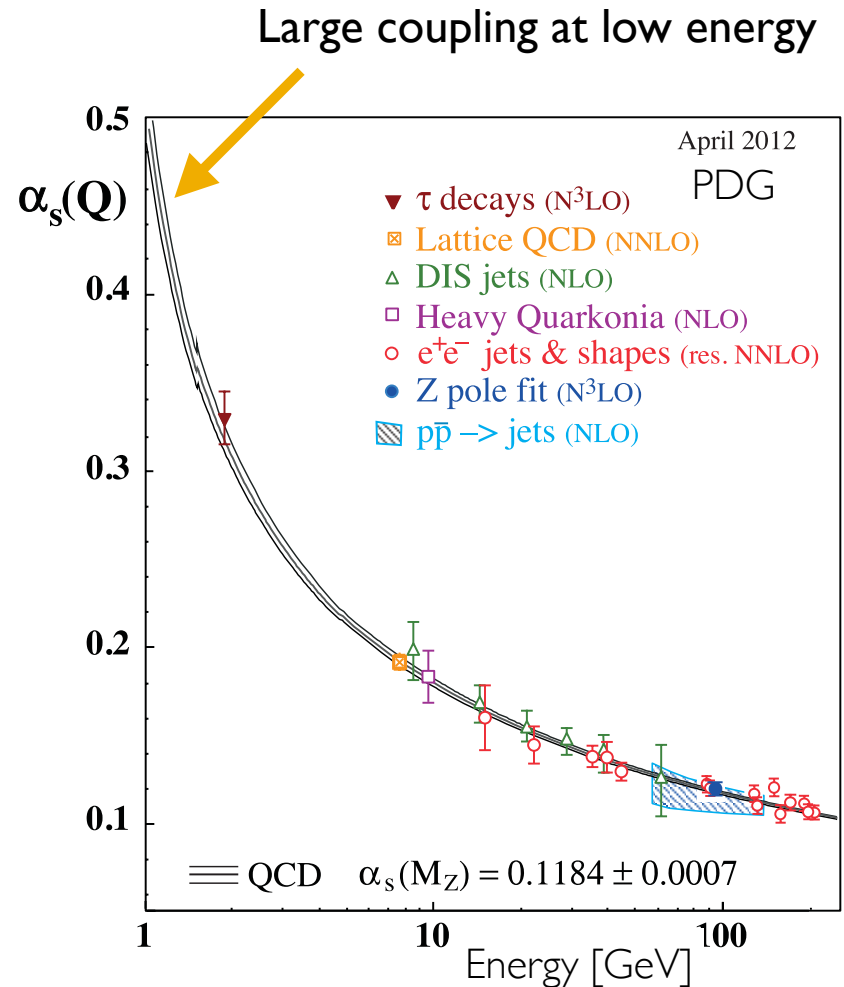
- Interaction strength depends on energy [Gross, Politzer, Wilczek, Nobel 2004]

- High energies: can use perturbation theory

$$\mathcal{O}_{\text{exact}} = \mathcal{O}_0 + \mathcal{O}_1\alpha_s + \mathcal{O}_2\alpha_s^2 + \dots$$

works for QED

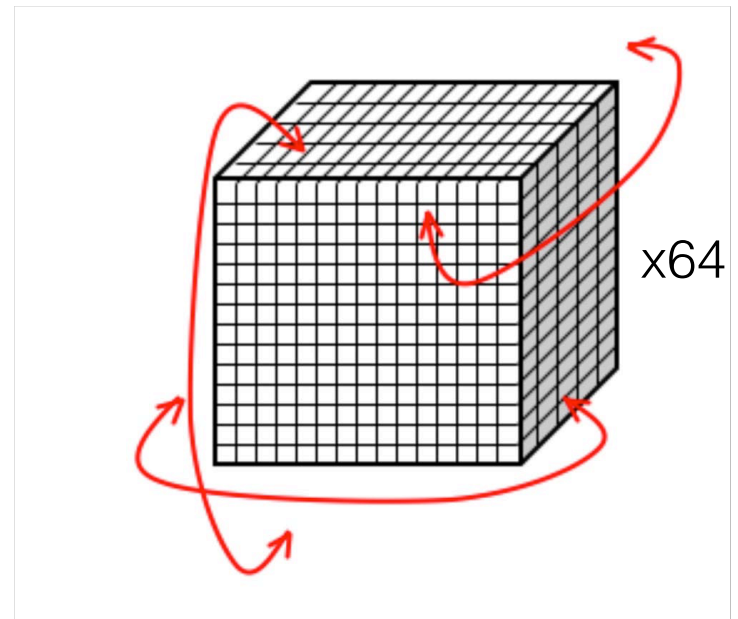
- Low energies: need an approach that does not require  $\alpha_s \ll 1$





# Lattice QCD

- Numerical first-principles approach
- Euclidean space-time  $t \rightarrow i\tau$ 
  - Finite lattice spacing  $a$
  - Volume  $L^3 \times T \approx 32^3 \times 64$
  - Boundary conditions
- Finite but large number of d.o.f ( $10^{12}$ )



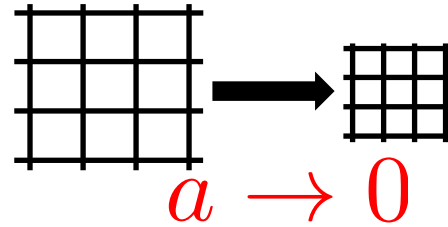
Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{O}[A, \bar{\psi}\psi] e^{-S[A, \bar{\psi}\psi]} \rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\text{conf}}} \sum_i^{N_{\text{conf}}} \mathcal{O}([U^i])$$

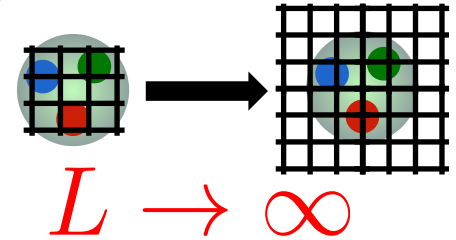
with field configurations  $U^i$  distributed according to  $e^{-S[U]}$

# Lattice QCD Systematics

- Finite lattice spacing  $a$   
Discretisation artifacts  
Continuum extrapolation

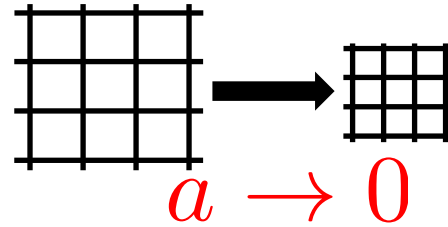


- Finite box size  $L$   
Momentum quantised, finite-volume effects  
Finite-volume corrections

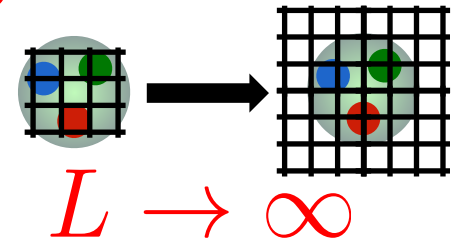


# Lattice QCD Systematics

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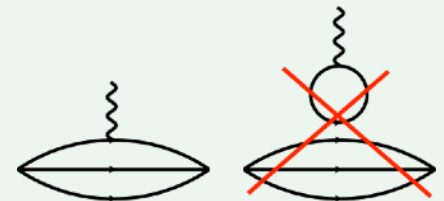
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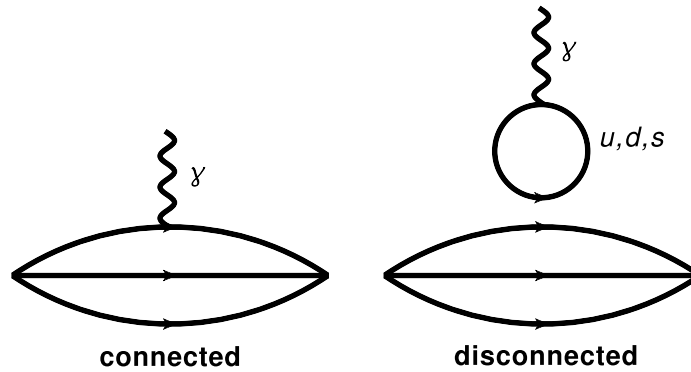
- Large pion mass  $m_\pi$   
Chiral extrapolation  
BUT: Can map out pion mass dependence of observables

$$m_\pi \rightarrow 140\text{MeV}$$

- Omitted disconnected loops  
BUT: can separate 'connected' and 'disconnected' contributions

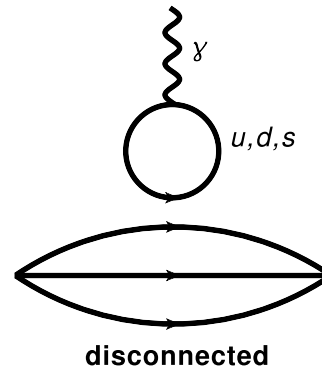
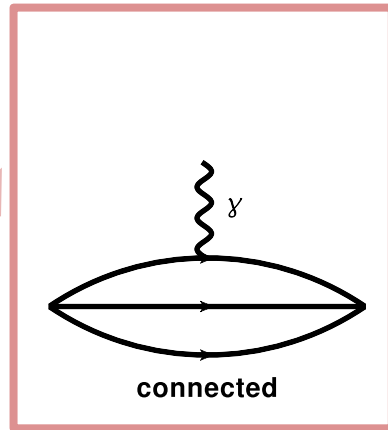


# Strange Proton Observables



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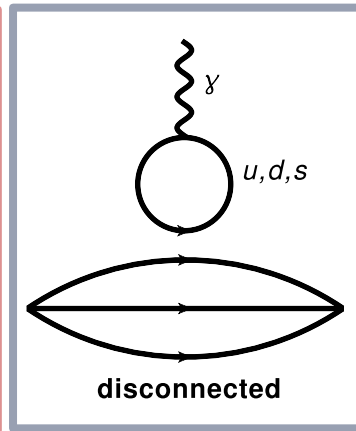
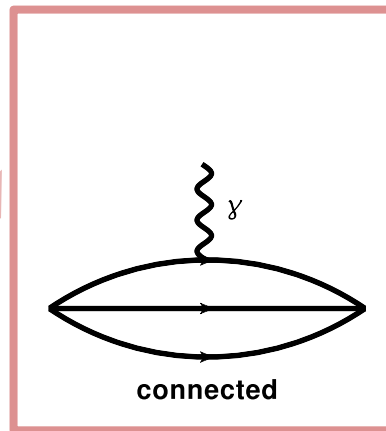
Good control  
over lattice  
systematics



# Strange Proton Observables

Direct Calculation of Strange Proton Observables is Expensive

Good control  
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systematics

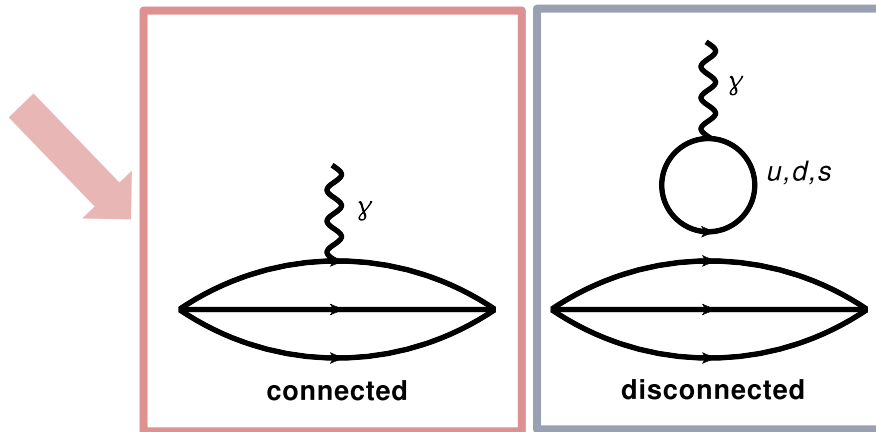


Naively, factor of the  
lattice volume  
(~millions)  
more expensive

# Strange Proton Observables

Direct Calculation of Strange Proton Observables is Expensive

Good control over lattice systematics



Naively, factor of the lattice volume (~millions) more expensive

A place for indirect lattice methods

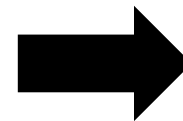
# Indirect Lattice Methods

## A place for indirect approaches

**PRECISION** lattice results  
in unphysical parameter space



Clever ways to deal with  
systematic effects

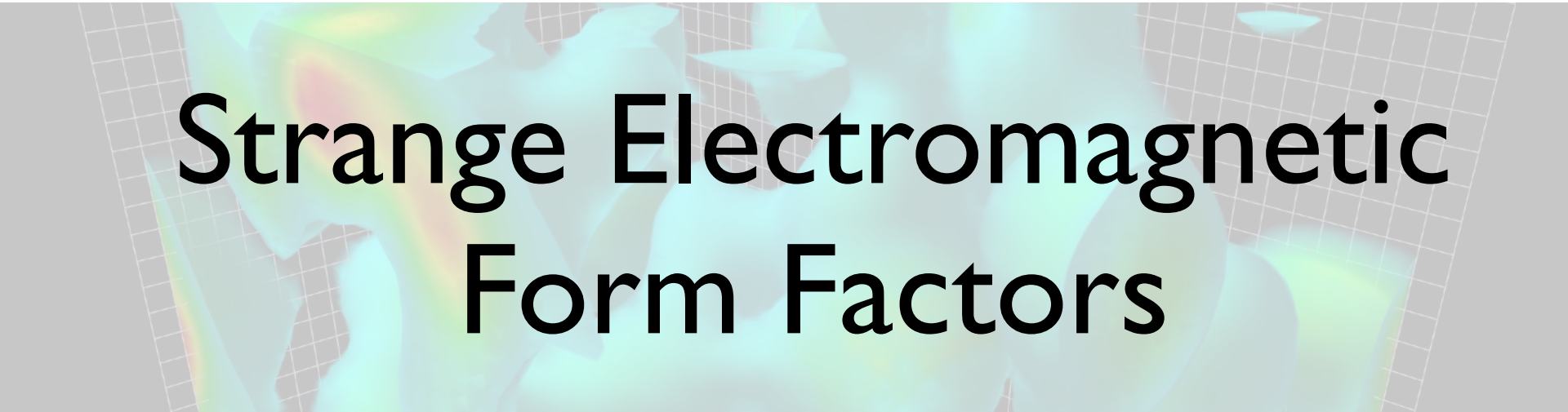


Physical results



Experiment

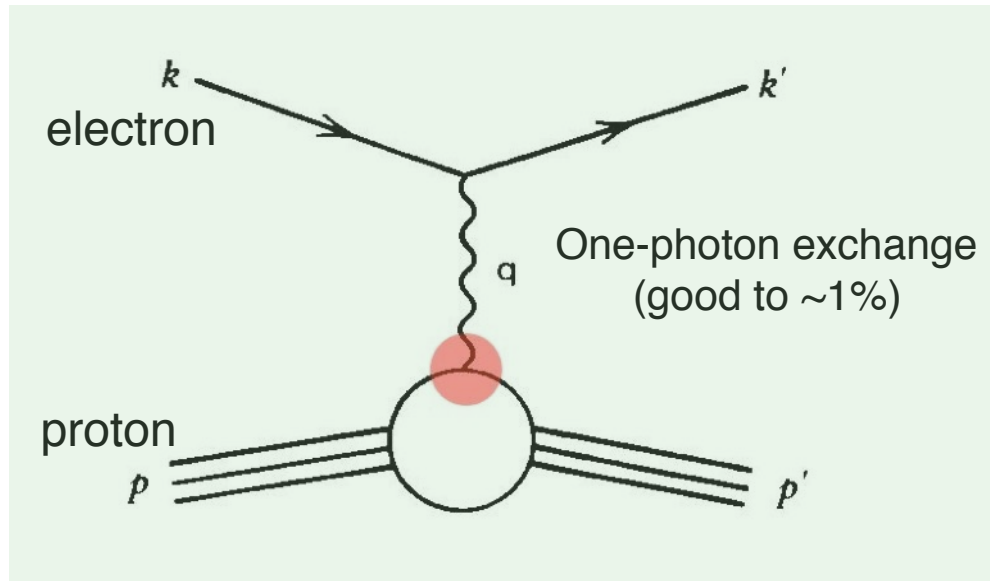


The background of the slide features a light gray grid pattern overlaid on a series of overlapping, semi-transparent shapes in shades of cyan, green, and yellow. The shapes are somewhat irregular and resemble stylized, overlapping planes or surfaces.

# Strange Electromagnetic Form Factors

# Electromagnetic Form Factors

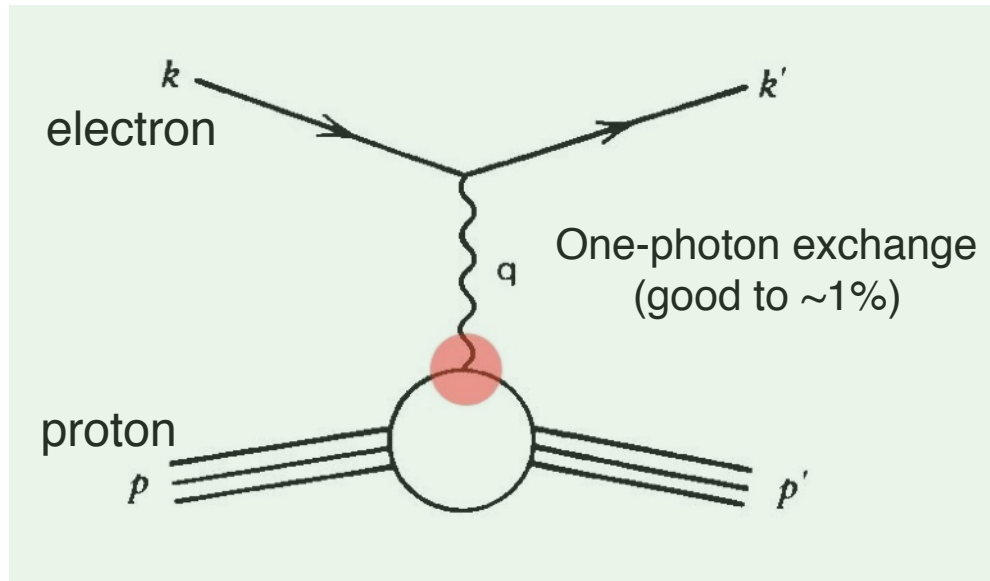
Form factors characterise the extended nature of composite particles



$N$  current matrix element  $G_E(Q^2), G_M(Q^2)$   
Form factors

# Electromagnetic Form Factors

Form factors characterise the extended nature of composite particles



**Static limit:**

charge and magnetic moment

$$G_E(Q^2 = 0) = Q_e$$

$$G_M(Q^2 = 0) = \mu$$

*N* current matrix element  $G_E(Q^2), G_M(Q^2)$

Form factors

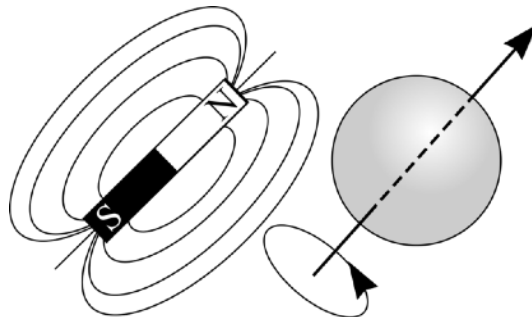
# Strange Magnetic Moment

## Strange Magnetic Moment

Magnetic moment quantifies how much torque a proton experiences in a magnetic field

Predictions of theory calculations vary widely!

- Magnitude?
- Sign?



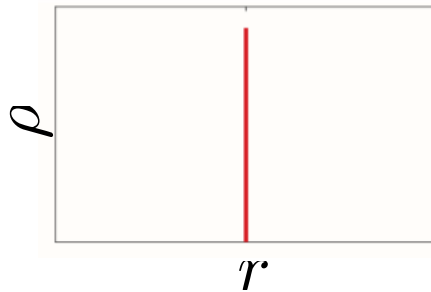
Type of calculation	$\mu_s$ (n.m.)
Poles	$-0.31 \pm 0.09$
Kaon loops	$-0.31 \rightarrow -0.40$
Kaon loops	$-0.026$
Kaon loops	$ \mu_s  = 0.8$
$SU(3)$ Skyrme (broken)	$-0.13$
$SU(3)$ Skyrme (symmetric)	$-0.33$
$SU(3)$ chiral hyperbag	$+0.42$
$SU(3)$ chiral color dielectric	$-0.20 \rightarrow -0.026$
$SU(3)$ chiral soliton	$-0.45$
Poles	$-0.24 \pm 0.03$
Kaon loops	$-0.125 \rightarrow -0.146$
NJL soliton	$-0.05 \rightarrow +0.25$
QCD equalities	$-0.75 \pm 0.30$
Loops	$+0.035$
Dispersion	$-0.10 \rightarrow -0.14$
Chiral models	$-0.25, -0.09$
Poles	$0.003$
$SU(3)$ Skyrme (broken)	$+0.36$

# Electromagnetic Form Factors

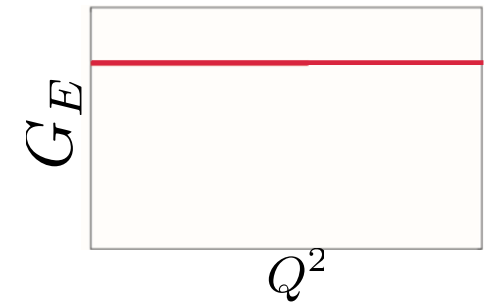
Interpretation (non-relativistic) of  $G_E$  as the Fourier transform of the charge distribution

$$G_E(Q^2) = \int e^{i\vec{q}\cdot\vec{x}} \rho(r) d^3r$$

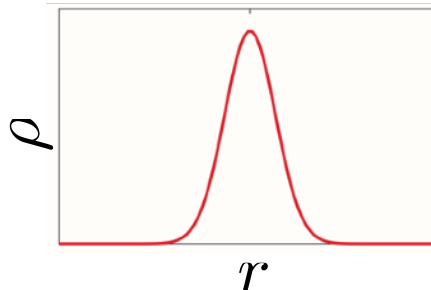
Point-like  
object



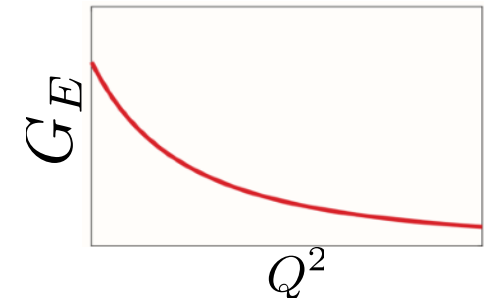
Constant FF



Extended  
object

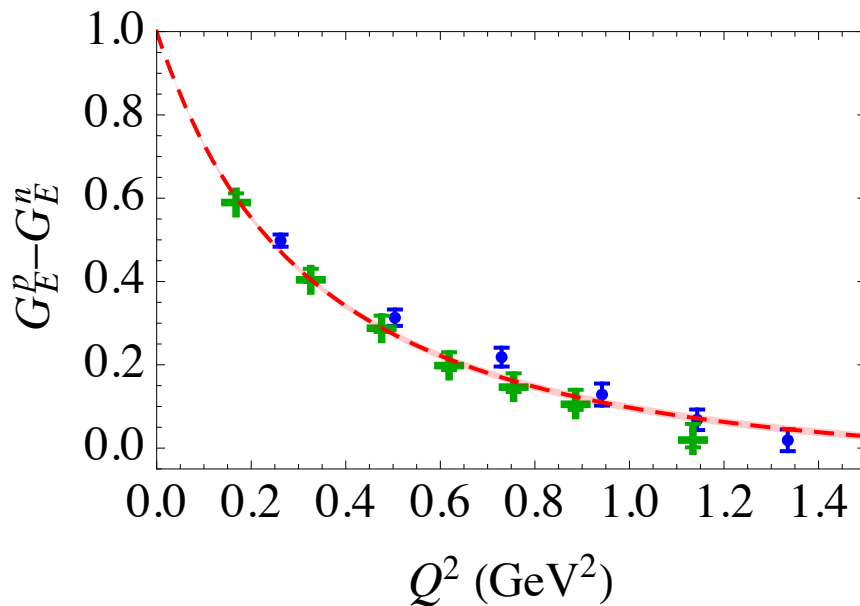


Deviation from  
constant

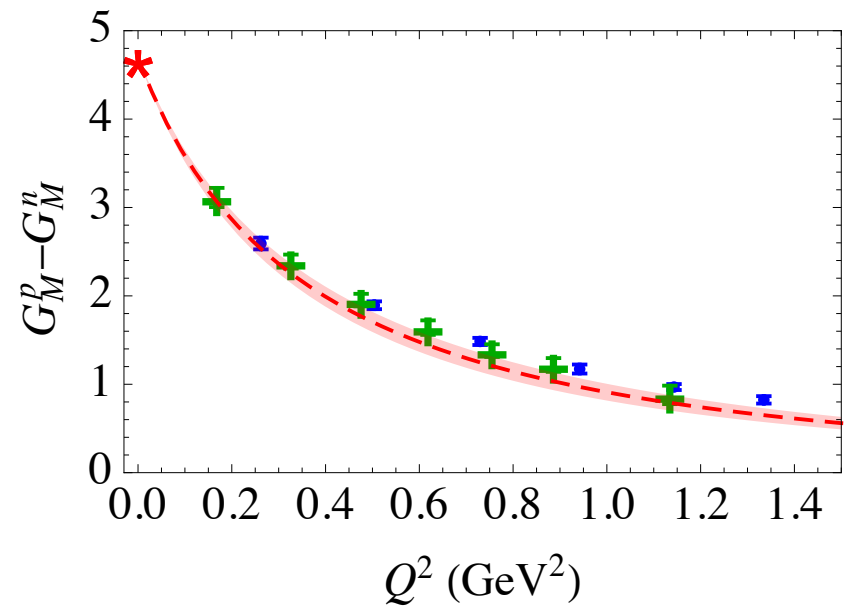


# Electromagnetic Form Factors

## Electric



## Magnetic



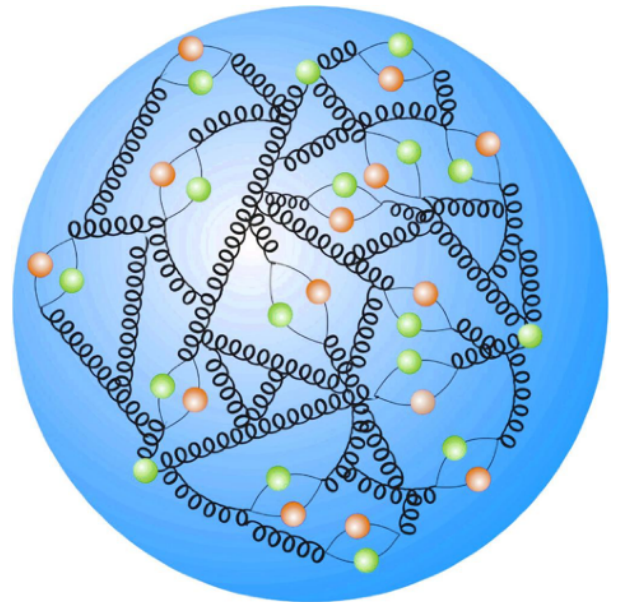
**Red:** Parameterization of experimental results

**Blue/Green:** Numerical simulations of QCD (different lattice sizes/spacings)

# Strange EM Form Factors

## Recall: **Hidden flavor**

- Fundamental challenge for hadronic physics
- Contributions entirely through interactions with QCD vacuum



# Strange EM Form Factors

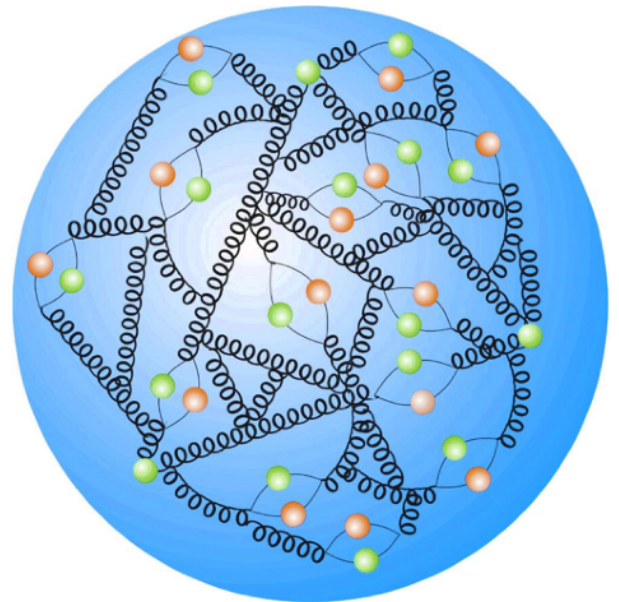
## Recall: **Hidden flavor**

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- Contributions entirely through interactions with QCD vacuum

## Extensive **experimental** efforts

- JLAB (G0, HAPPEX)
- MIT-Bates (SAMPLE)
- Mainz (A4)

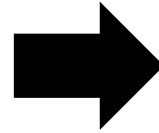
## Compare to theory: **Lattice QCD**





# Strange EM Form Factors

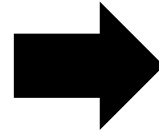
Direct lattice calculations:  
Expensive, Large systematics



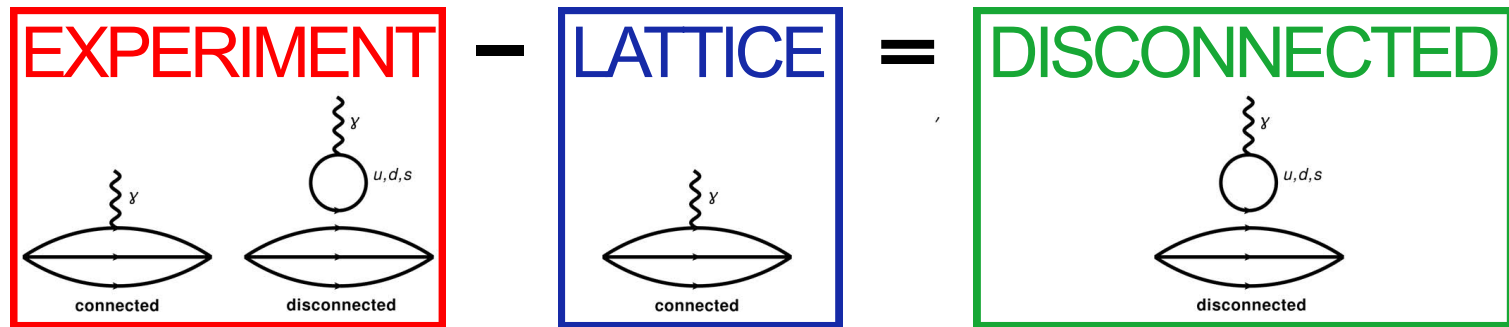
**INDIRECT  
APPROACH**

# Strange EM Form Factors

Direct lattice calculations:  
Expensive, Large systematics



INDIRECT  
APPROACH



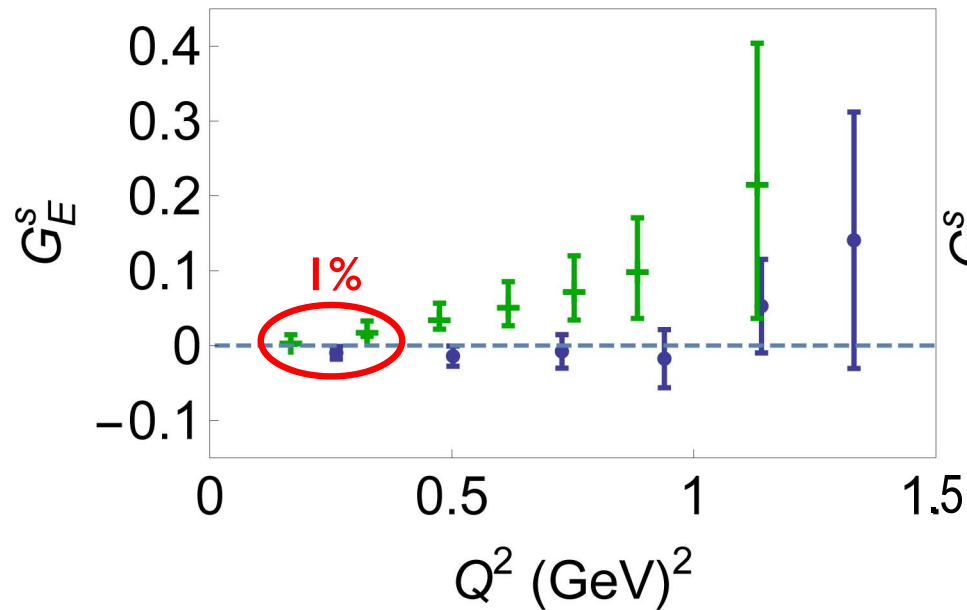
Take advantage of precise results for connected diagrams on the lattice!

**ONLY WORKS IF SYSTEMATICS ARE UNDER CONTROL**

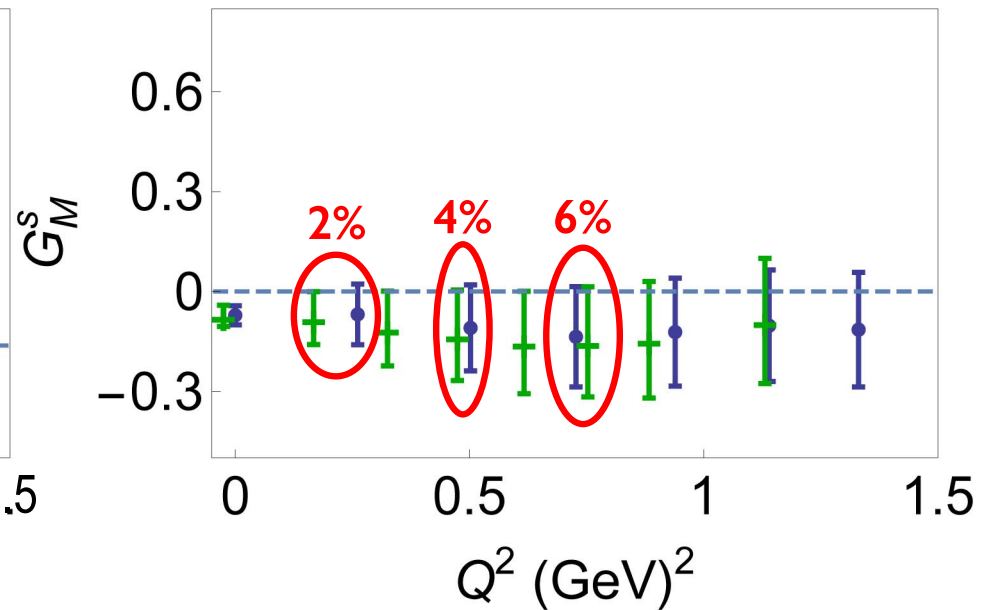
**Simple idea but many technical challenges!**

# Strange EM Form Factors

## Electric



## Magnetic

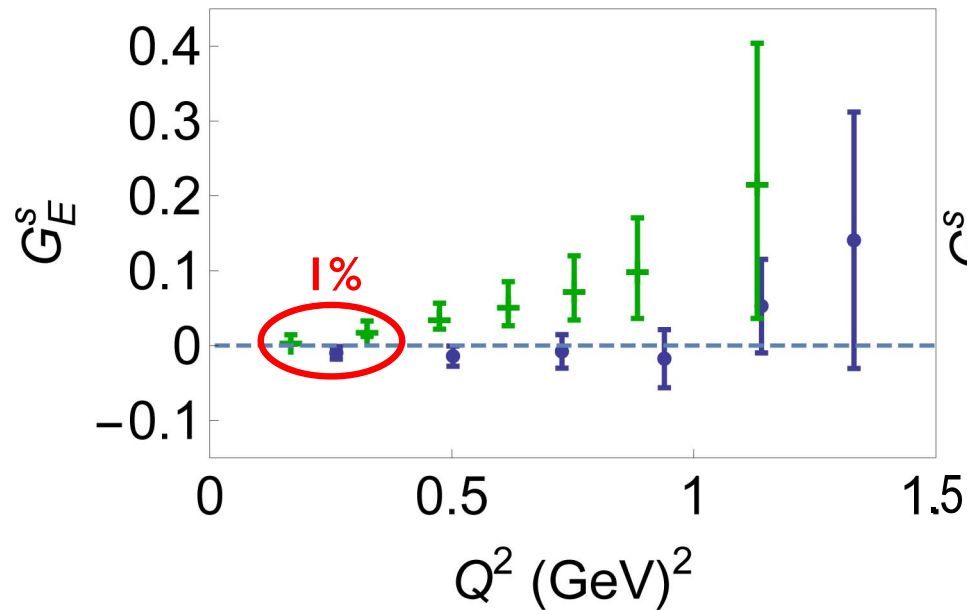


**Red:** Parity-violating electron scattering experiments, JLAB, MIT-BATES, MAINZ

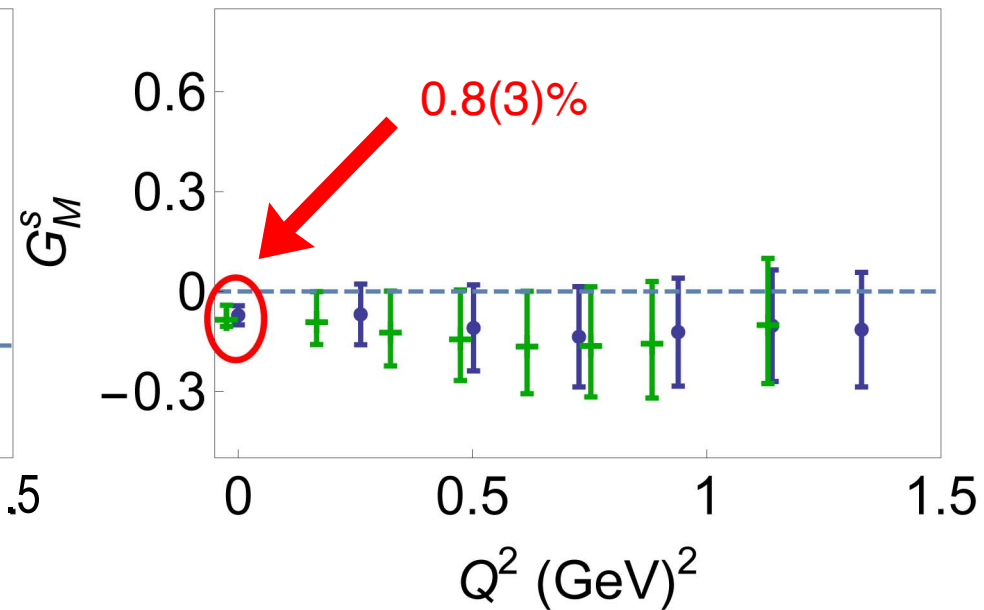
**Blue/Green:** Lattice (different lattice sizes/spacings)

# Strange EM Form Factors

## Electric



## Magnetic

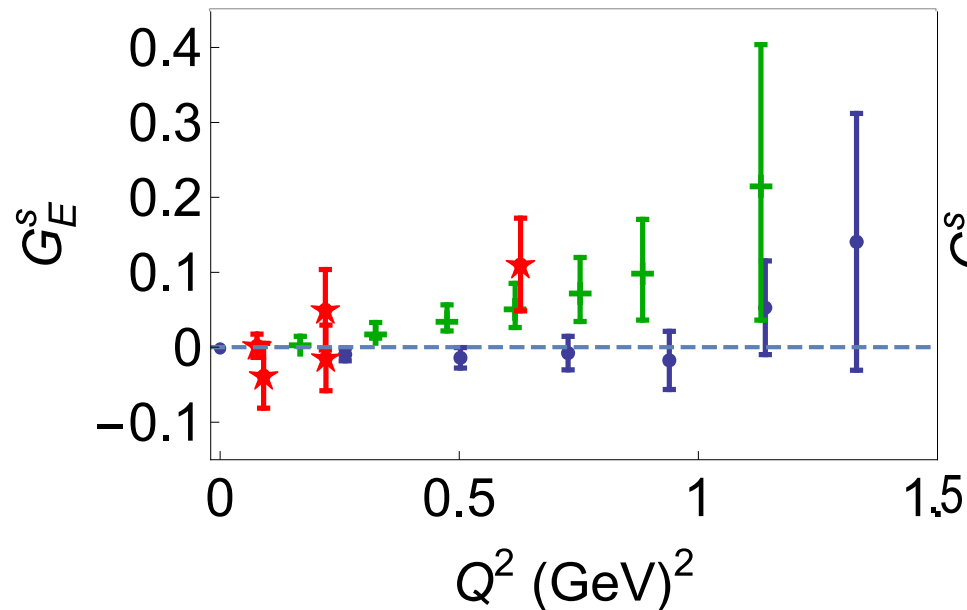


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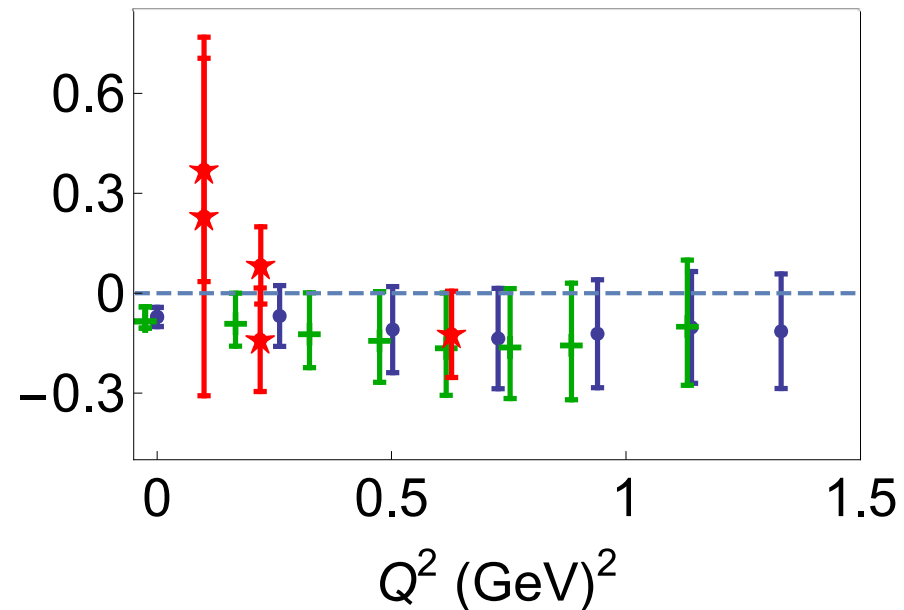
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# Strange EM Form Factors

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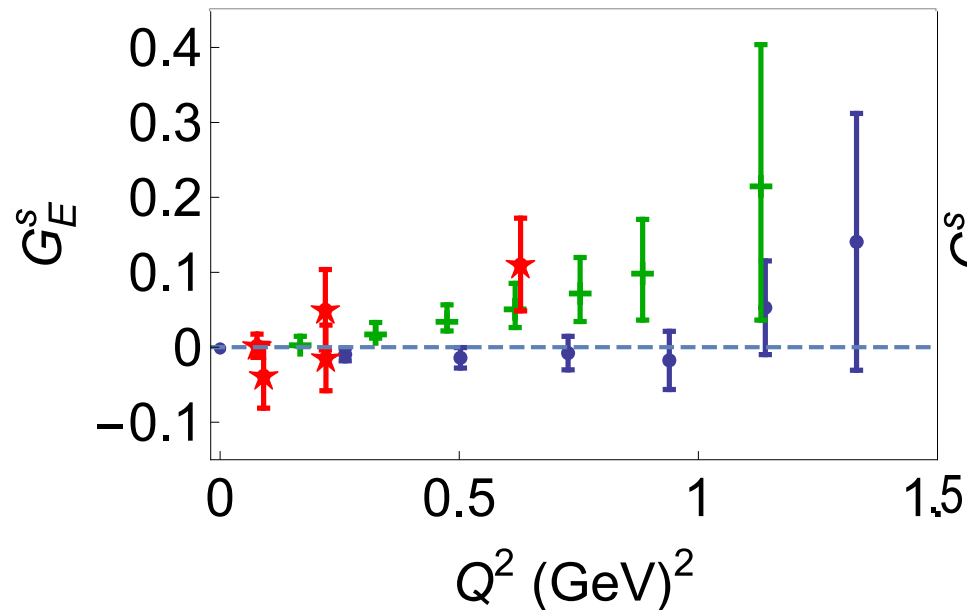
## Magnetic



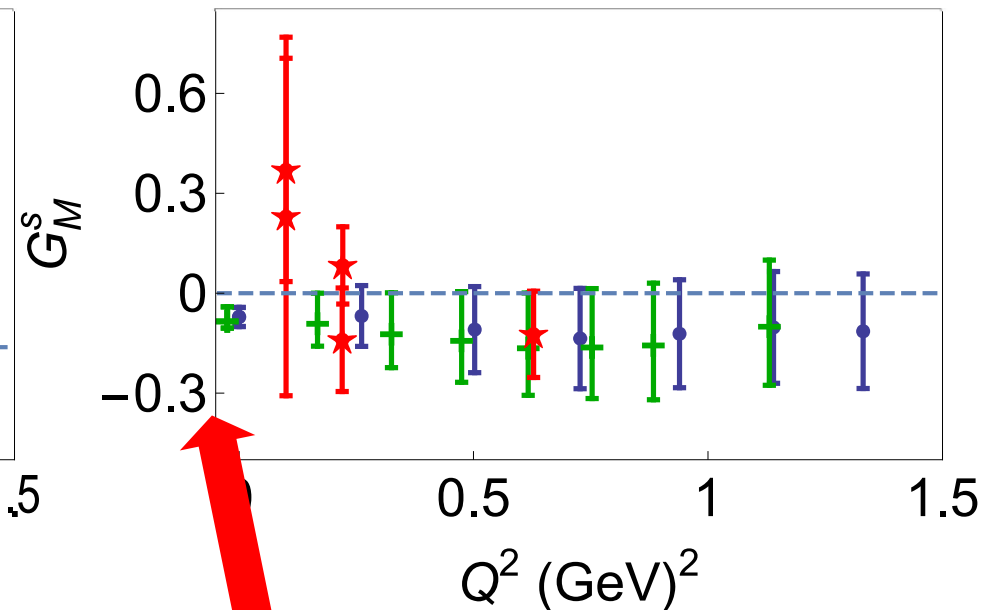
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# Strange EM Form Factors

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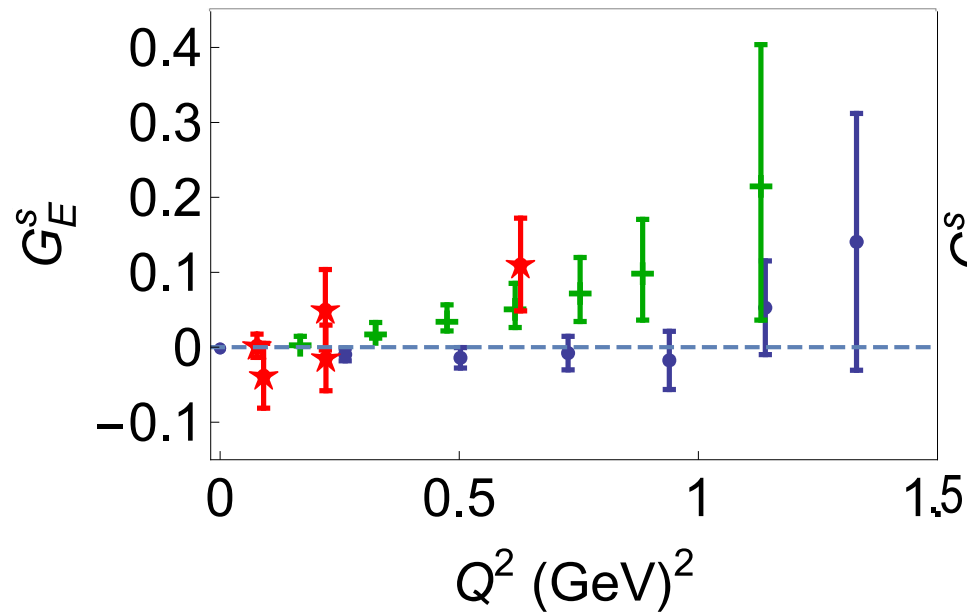


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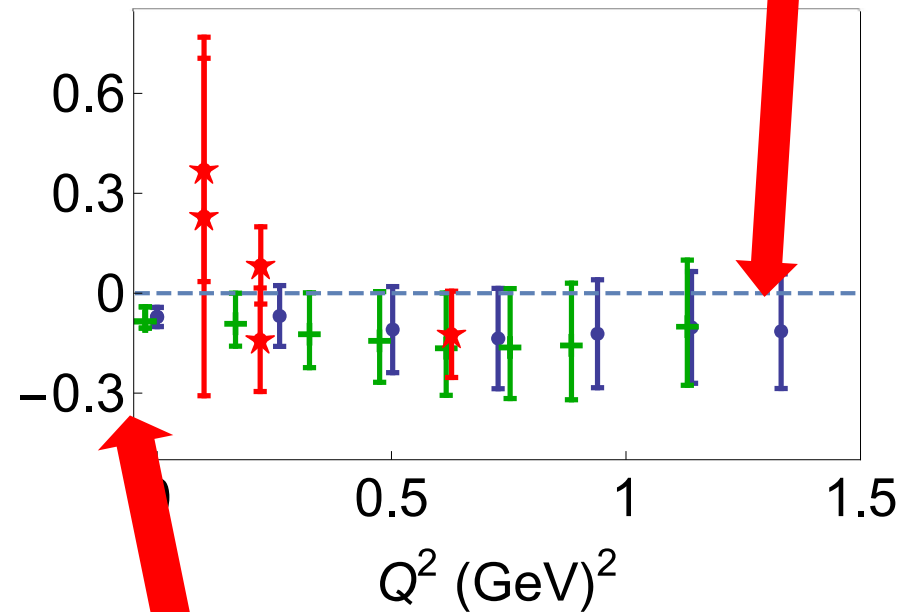
Theory uncertainties leading experiment by an order of magnitude : **PREDICTION**

# Strange EM Form Factors

## Electric



## Magn



First determination  
(theory or expt.)  
at these momenta

Red: Experiment, JLAB, MIT-BATES, MAINZ  
Blue/Green: Lattice (different lattice sizes/spacings)

Theory uncertainties leading  
experiment by an order of  
magnitude : **PREDICTION**

# Strange EM Form Factors

## Experimental determinations of $G^s$ :

EM and weak vector currents give access to different combinations of  $G^{(u/d/s)}$

$$G^{p,\gamma} = \frac{2}{3}G^{p,u} - \frac{1}{3}(G^{p,d} + G^{p,s})$$

$$G^{p,Z} = \left(1 - \frac{8}{3}\sin^2\theta_W\right)G^{p,u} - \left(1 - \frac{4}{3}\sin^2\theta_W\right)(G^{p,d} + G^{p,s})$$

$$G_{E/M}^s = \underbrace{\left(1 - 4\sin^2\theta_W\right)}_{\text{well determined}} \underbrace{G_{E/M}^{p,\gamma}}_{\text{well determined}} - \underbrace{G_{E/M}^{n,\gamma}}_{\text{well determined}} - \underbrace{G_{E/M}^{p,Z}}_{\text{PVES}} + \underbrace{G_{\text{CSV}}}_{\text{??}}$$

### Parity-violating electron scattering

JLab (G0, HAPPEX), MIT-Bates (SAMPLE), Mainz (A4)

Models and/or lattice QCD



# Charge Symmetry Violation

Breaking of symmetry between  
 $u$  quark in proton and  $d$  quark in neutron

$$G_{\text{CSV}} = \frac{2}{3}(G_{E/M}^{p,u} - G_{E/M}^{n,d}) - \frac{1}{3}(G_{E/M}^{p,d} - G_{E/M}^{n,u})$$

# Charge Symmetry Violation

Breaking of symmetry between  
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$$G_{\text{CSV}} = \frac{2}{3}(G_{E/M}^{p,u} - G_{E/M}^{n,d}) - \frac{1}{3}(G_{E/M}^{p,d} - G_{E/M}^{n,u})$$

## CSV indirectly from the lattice

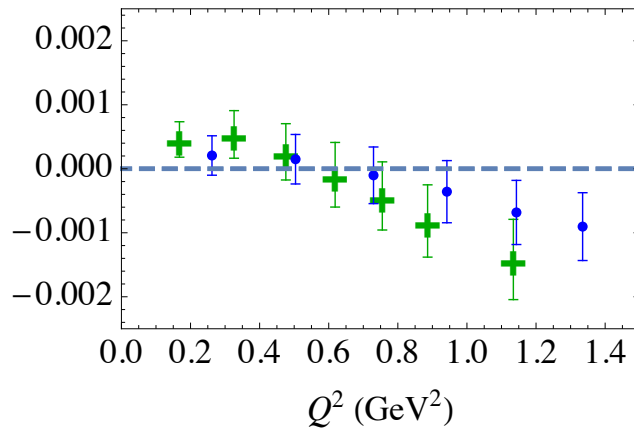
- lattice simulations have  $m_u = m_d$

Chiral perturbation theory expressions for  $m_u \neq m_d$   
have the **same free parameters** as isospin-averaged case

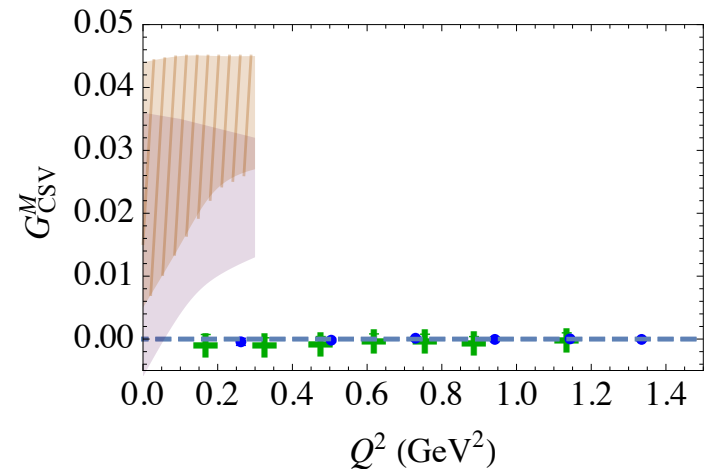
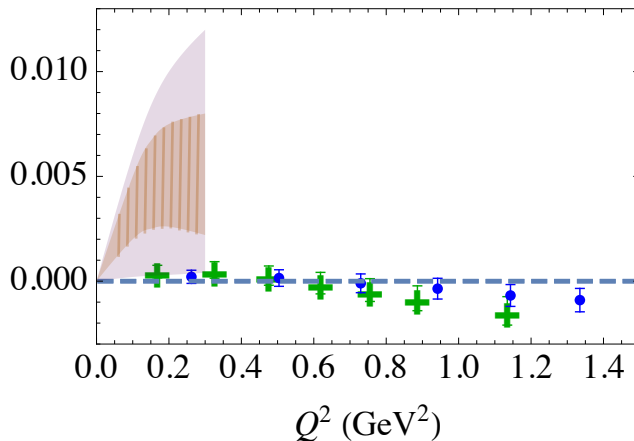
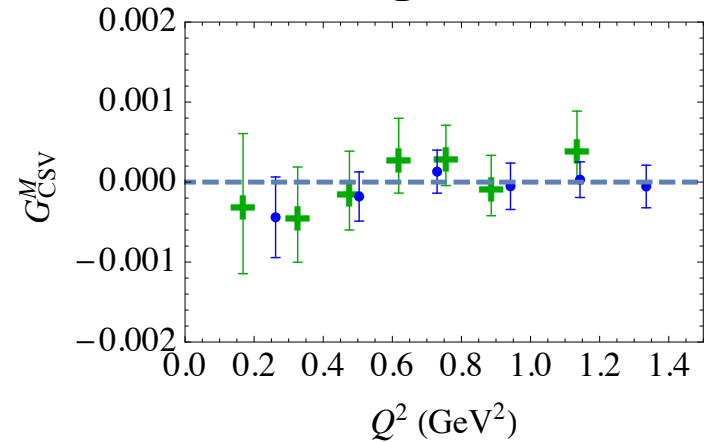
- Determine free parameters from isospin-averaged fits
- Input  $m_u/m_d$  from experiment or lattice

# Charge Symmetry Violation

## Electric

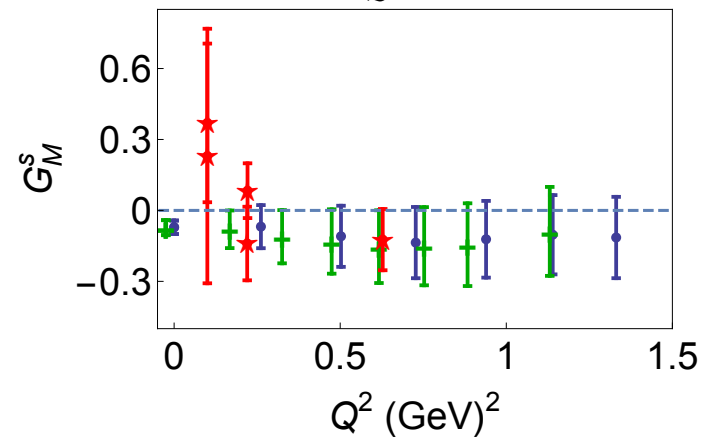
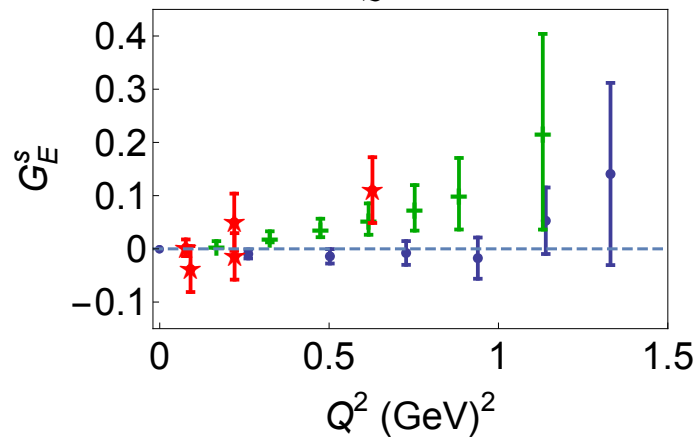
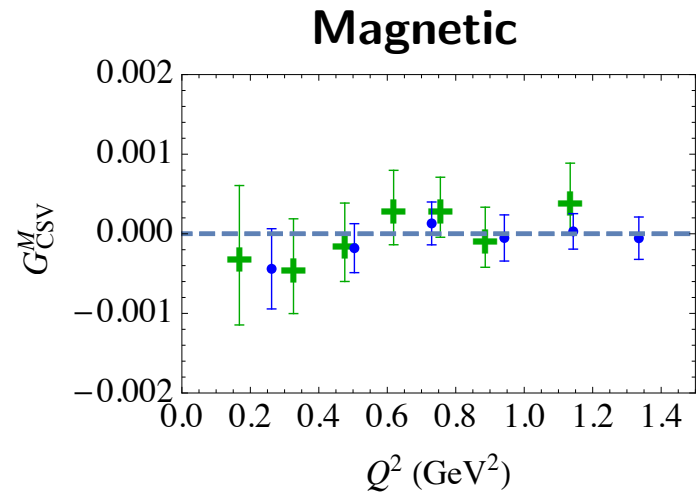
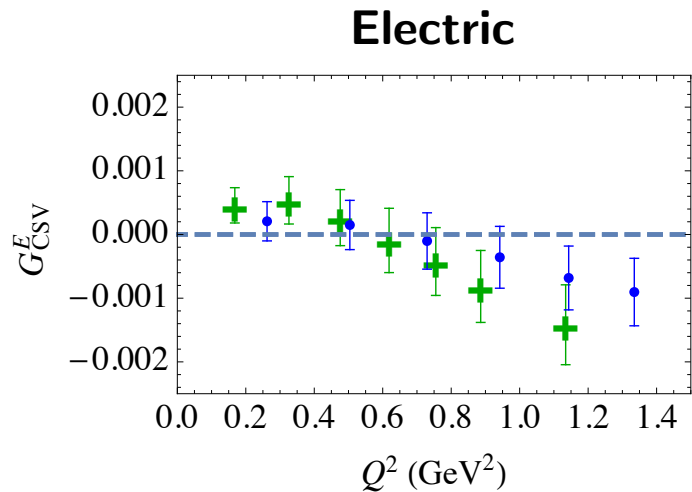


## Magnetic



Kubis & Lewis, PRC (2006), Wagman & Miller, PRC (2014)

# Charge Symmetry Violation



**Red stars:** G0, SAMPLE, HAPPEX, A4  
**Green/Blue points:** Lattice QCD (PES *et al.* PRL (2015))

# Charge Symmetry Violation

CSV in the nucleon EMFFs < 1%

i.e., order of magnitude smaller than the precision of existing PVES studies measuring the strange nucleon form factors

**HAPPEX** ( $Q^2 = 0.109\text{GeV}^2$ ) [PRL 98 (2007)]

$$G_E^s + 0.09G_M^s$$

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Experimental	0.007(14)
Previous CSV uncertainty (theory):	0.009
New CSV uncertainty (this work):	0.0009

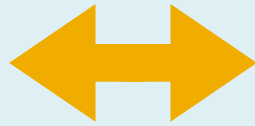


Strange Sigma Terms

➔ Dark Matter Cross Sections

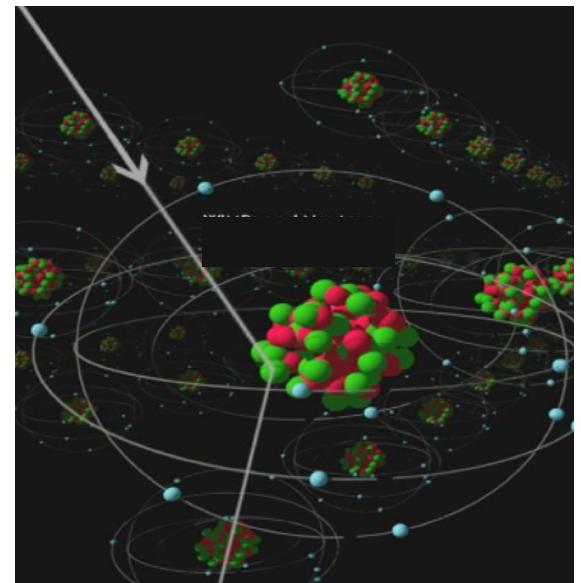
# Sigma Terms

$q$ -quark  
sigma term



contribution of  $q$ -quarks  
to the proton mass

- **Strange quark sigma term:**  
Measure of vacuum-quark contributions to proton mass
- Dominant uncertainty in amplitude for spin-independent scattering of **weakly-interacting dark matter** from nucleons



# Strange Sigma Term

- Not directly accessible to experiment

 **LATTICE QCD**



# Strange Sigma Term

- Not directly accessible to experiment

 **LATTICE QCD**

- **RECALL:** Disconnected terms expensive + noisy
- **BUT:** Feynman-Hellmann Theorem relates sigma terms to mass-dependence

$$\sigma_s = \langle N | m_s \bar{s} s | N \rangle = m_s \frac{\partial M_N}{\partial m_s}$$

# Strange Sigma Term

- Not directly accessible to experiment

 **LATTICE QCD**

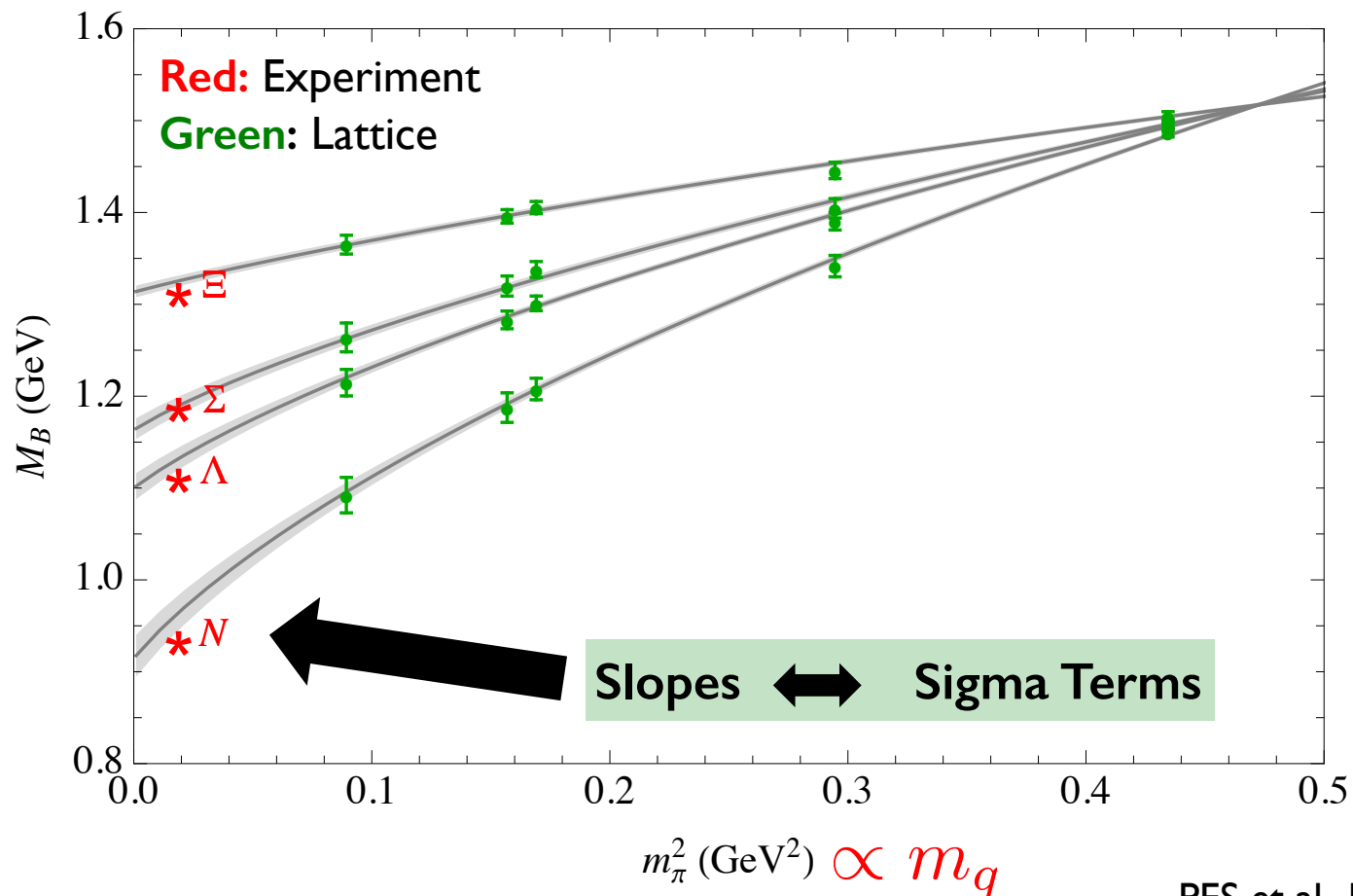
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$$\sigma_s = \langle N | m_s \bar{s} s | N \rangle = m_s \frac{\partial M_N}{\partial m_s}$$

Can vary quark masses on the lattice!

# Strange Sigma Term

Baryon masses as a function of quark mass!



# Strange Sigma Term

Strange Sigma Term (MeV)

Phenomenology (1990s)

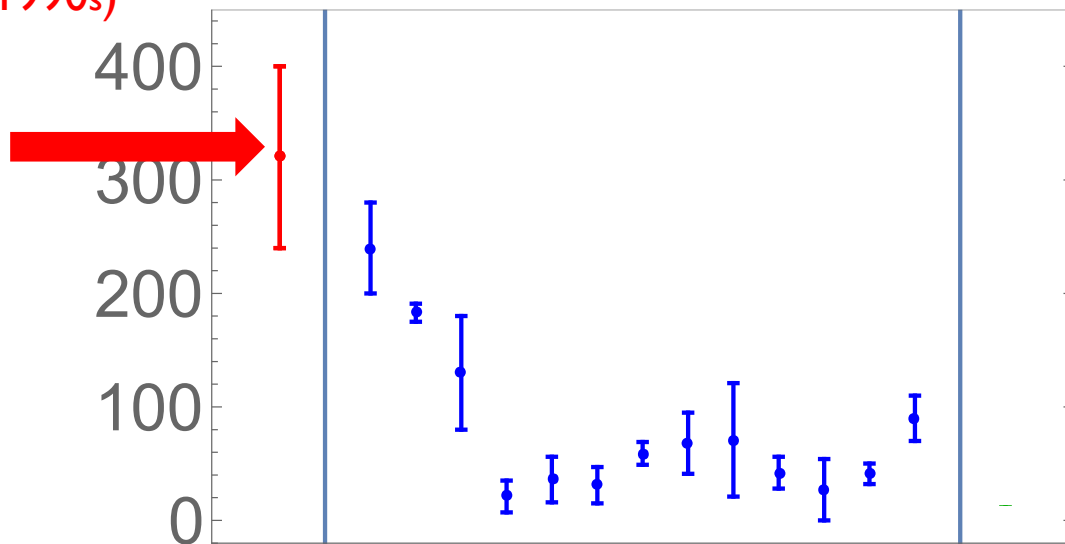
Implies up to 1/3 nucleon mass from strange quarks



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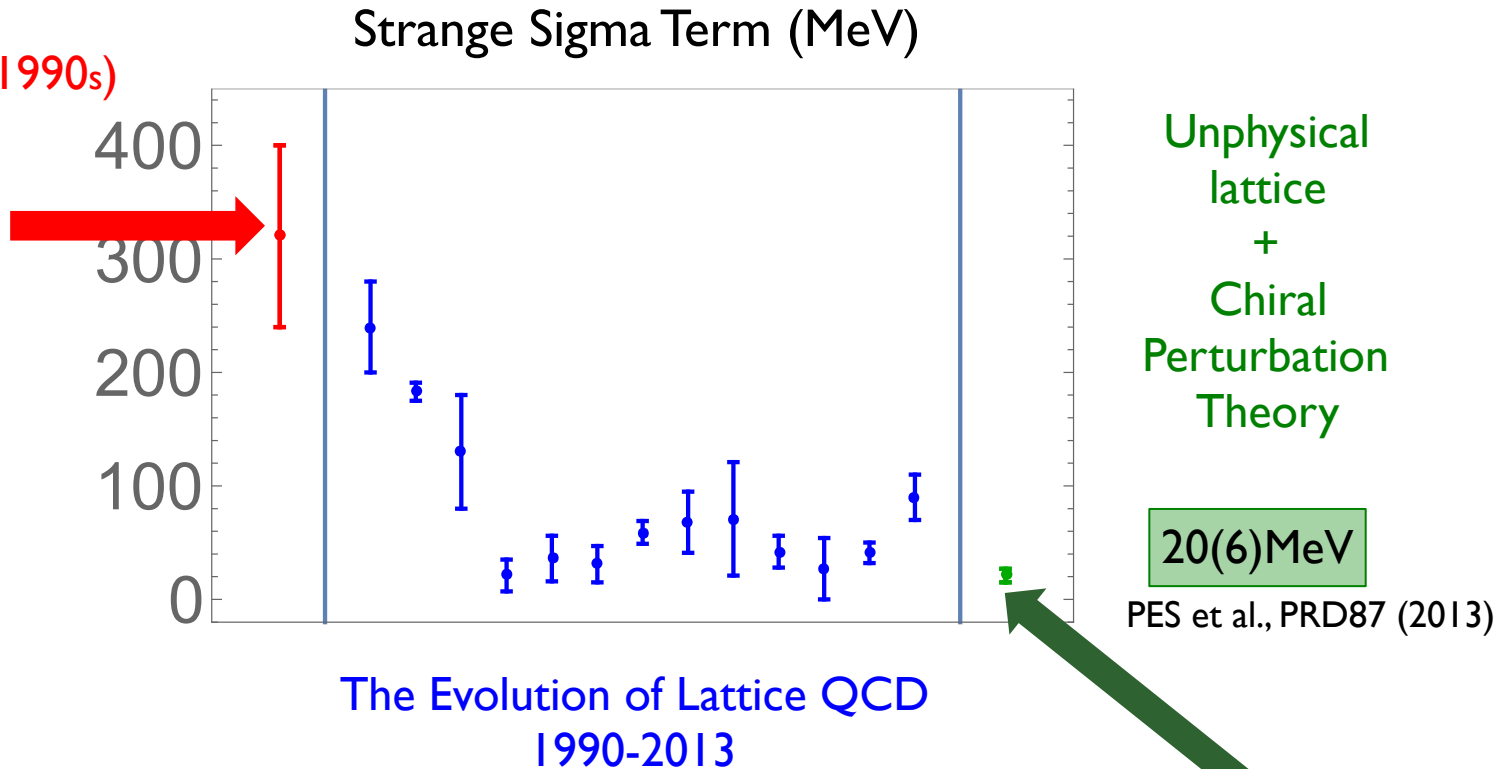
The Evolution of Lattice QCD  
1990-2013

Fukugita '95, Dong '96  
SESAM '98, Toussaint, Freeman '09,  
JLQCD '11, Durr '11, Alexandrou '13,  
Ren '14, Abdel-Rehim '16

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## Phenomenology (1990s)

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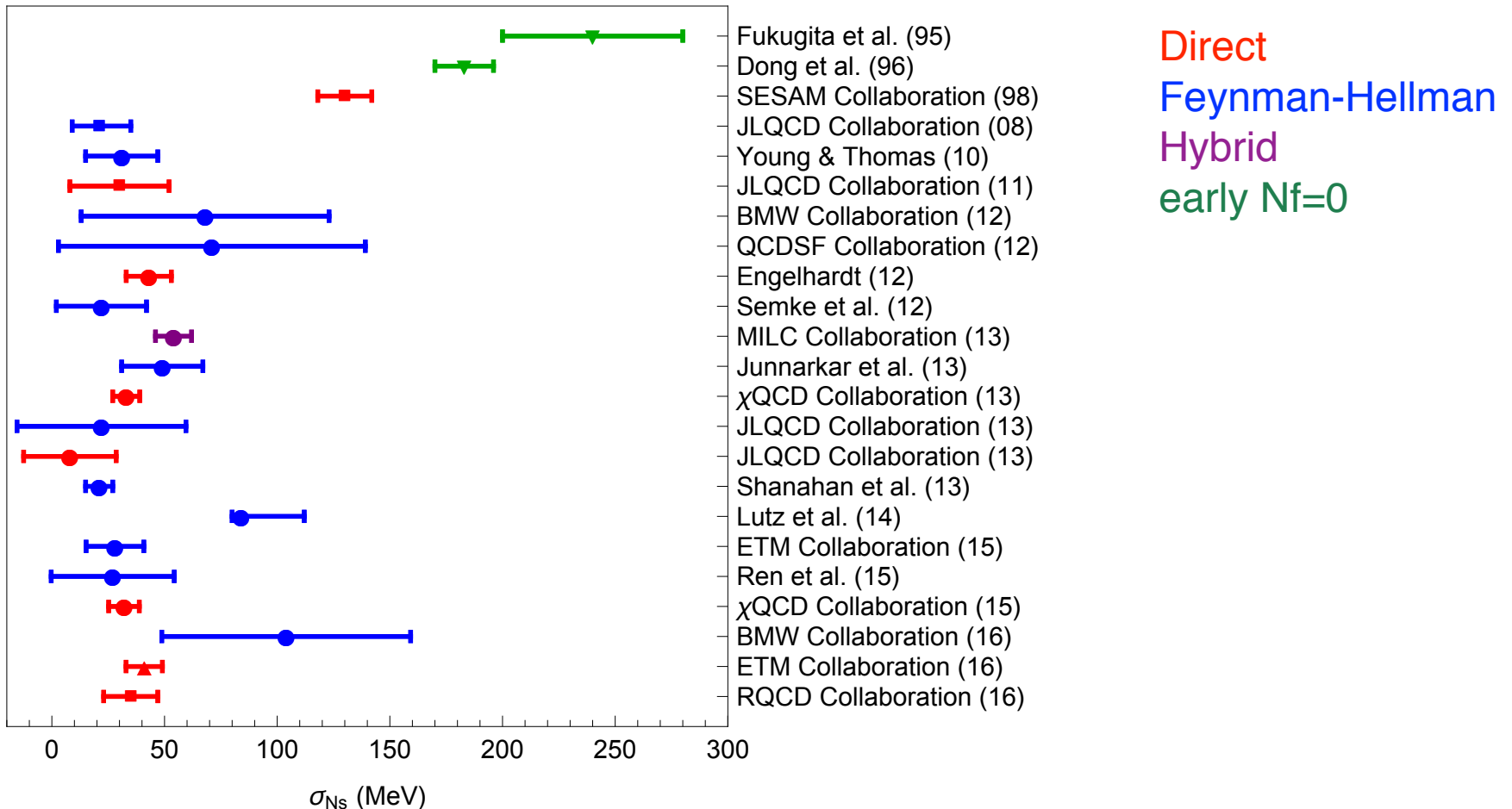


Predicted dark matter cross-sections reduced by an order of magnitude

Fukugita '95, Dong '96  
SESAM '98, Toussaint, Freeman '09,  
JLQCD '11, Durr '11, Alexandrou '13,  
Ren '14, Abdel-Rehim '16

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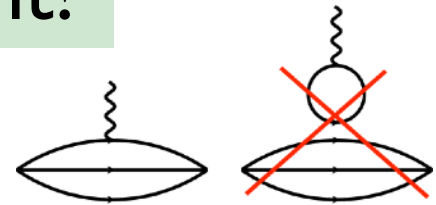
● Sigma term ~ 2016



# Summary

Quantitatively understanding  
proton strangeness is important!

- Direct calculations from QCD are **hard**.

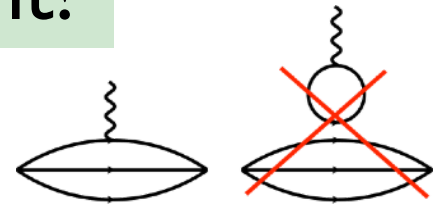




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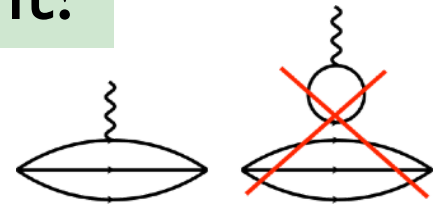
- Direct calculations from QCD are **hard**.
- Precise physics results available **NOW** by combining information from numerical simulations with experiment and models
- Set benchmarks for experimental tests of **nonperturbative QCD**
- Strange quark effects  $\sim$  a few percent for proton properties
  - **Strange sigma terms:** new level of precision for direct dark matter searches
  - **Strange magnetic moment:** new benchmark for experiment



# Summary

## Quantitatively understanding proton strangeness is important!

- Direct calculations from QCD are **hard**.
- Precise physics results available **NOW** by combining information from numerical simulations with experiment and models
- Set benchmarks for experimental tests of **nonperturbative QCD**
- Strange quark effects ~ a few percent for proton properties
  - **Strange sigma terms**: new level of precision for direct dark matter searches
  - **Strange magnetic moment**: new benchmark for experiment
- What next? **Gluon distributions**: predictions for studies at a proposed electron-ion collider



# References

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