

Quarkonium production in proton-nucleus collisions at collider energies

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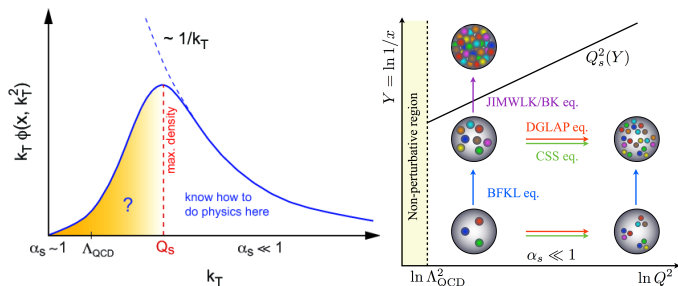
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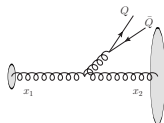


Gluon saturation in hadron/nucleus

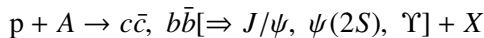


- At small value of Bjorken x , **Small- x /Color Glass Condensate (CGC) framework** is essential.
- Semihard Saturation scale : $Q_{sA}^2(x) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3} \gg \Lambda_{\text{QCD}}^2$
- $k_{\perp} < Q_s$: “**Saturation regime**” in Gluon TMD ϕ .
- Energy/rapidity dep. of $\phi \Rightarrow$ **NONLINEAR BK eq.** or **JIMWLK eq.**

Forward Quarkonium production in pA collisions



- Consider **Forward Quarkonium** productions in pA collisions;

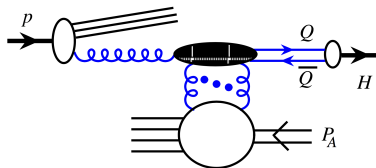


LO kinematics: $x_{1,2} = \frac{m_{\perp}}{\sqrt{s}} e^{\pm y}$ ($p_{\perp} = 1, y = +2$)

\sqrt{s}	200 GeV	5.02 TeV	8 TeV
$J/\psi : x_2$	2.20×10^{-3}	8.78×10^{-5}	5.51×10^{-5}
$\psi(2S) : x_2$	2.58×10^{-3}	1.03×10^{-4}	6.46×10^{-5}
$\Upsilon : x_2$	6.44×10^{-3}	2.56×10^{-4}	1.61×10^{-4}

- Forward Quarkoniums production in pA collisions can probe Small- x gluons! $\Rightarrow P_{\perp}$ distribution should reflect the saturation effect.

Effective Factorization : Short dist. and Long dist.



- Target nucleus \Rightarrow **Multiple scattering effect.**
- If $Q_{sA} \sim mv \sim Mv/2$ in $Q\bar{Q}$ rest frame \rightarrow **v -expansion is not ensured.**
- In the very forward rapidity region, owing to Lorentz time dilation,

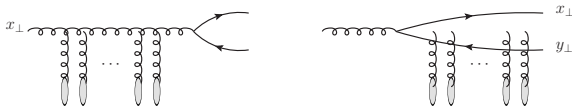
$$\frac{1}{mv} \frac{M \cosh y}{M} \gg \frac{1}{p_{\perp}} \sim \frac{1}{Q_{sA}}$$

The hadronization is effectively frozen when the $Q\bar{Q}$ passes through the nucleus. [Qiu,Sun,Xiao,Yuan (2013)]

- Yes, in principle. \checkmark CEM, \checkmark NRQCD

$Q\bar{Q}$ production in CGC framework

[Blaizot, Gelis, Venugopalan (2004)][Kovchegov, Tuchin (2006).] ...



- The scattering contributions are coherent as a whole.

$$\begin{aligned}
 M_{s_1 s_2; ij}(q, p) = & \\
 & \frac{g^2}{(2\pi)^4} \int d^2 k_{\perp} d^2 k_{1\perp} \frac{\rho_p(k_{1\perp})}{k_{1\perp}^2} \int d^2 x_{\perp} d^2 y_{\perp} e^{ik_{\perp} \cdot x_{\perp}} e^{i(P_{\perp} - k_{\perp} - k_{1\perp}) \cdot y_{\perp}} \\
 & \times \bar{u}_{s_1, i}(q) \left[T_g(k_{1\perp}) t^b W^{ba}(x_{\perp}) + T_{q\bar{q}}(k_{1\perp}, k_{\perp}) U(x_{\perp}) t^a U^{\dagger}(y_{\perp}) \right] v_{s_2, j}(p)
 \end{aligned}$$

- Wilson lines \Rightarrow **Multiple gluon scatterings** in Eikonal approximation
- In xsection, the nuclear wave function can be expressed by

$$S_{x_g}(r_{\perp}) = \frac{1}{N_c} \left\langle \text{Tr} \left[U(x_{\perp}) U^{\dagger}(y_{\perp}) \right] \right\rangle_{x_g}$$

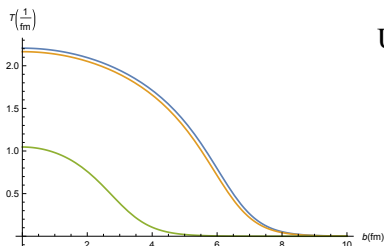
Energy and nuclear dependence

- Energy/Rapidity dependence \Rightarrow rc-BK eq. [Balitsky (2006)]

$$S_{Y=0}(r_{\perp}) = \exp \left[-\frac{(r_{\perp}^2 Q_{s,0}^2)^{\gamma}}{4} \ln \left(\frac{1}{|r_{\perp}| \Lambda} + e_c \cdot e \right) \right]$$

- Its IC is fixed by HERA-DIS global data analysis. [AAMQS][Lappi,Mantysaari (2013)]
- For Nucleus, change the saturation scale as

$$Q_{sA,0}^2(b) = A \hat{T}_A(b) S_{\perp} Q_{sp,0}^2 \equiv N Q_{sp,0}^2$$



Using Glauber Model with

- $S_{\perp} = \sigma_{inel}^{NN} \Rightarrow N_{MB} \approx 4 - 8.$
- $S_{\perp} = \int_p d^2b \Rightarrow N_{MB} \approx 1.6 - 1.9$
[Lappi, Mantysaari (2013)]

Note : N_{MB} is model dependent.

Hadronization from $Q\bar{Q}$ to ψ

- ✓ **Color Evaporation Model (CEM)** [Fujii, Gelis, Venugopalan (2006)]

$$\frac{d\sigma_\psi}{d^2P_\perp dy} = F_\psi \int_{2m_c}^{2M_D} dM \frac{d\sigma_{Q\bar{Q}}}{dM d^2P_\perp dy}$$

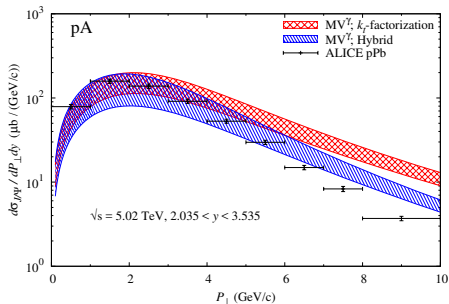
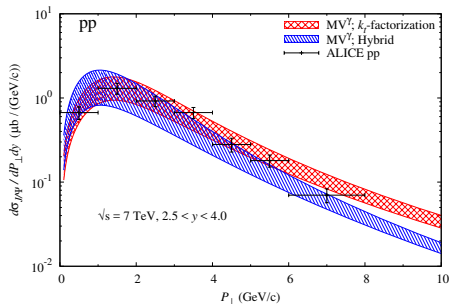
- Sum over all possible quantum states of $Q\bar{Q}$.
- $Q\bar{Q}$ converts into ψ with the probability $F_{Q\bar{Q} \rightarrow \psi}$.
- ✓ **Non-Relativistic QCD (NRQCD)** [Kang, Ma, Venugopalan (2013)]

$$\frac{d\sigma_\psi}{d^2P_\perp dy} = \sum_\kappa \frac{d\sigma_{Q\bar{Q}}^\kappa}{d^2P_\perp dy} \times \underbrace{\langle O_\kappa^\psi \rangle}_{\text{LDMEs}}$$

- Important LDMEs : $^3S_1^{[1]}$, $^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_0^{[8]}$
 $\underbrace{\hspace{10em}}_{\mathcal{O}(v^4)}$

J/ψ production in CGC + CEM

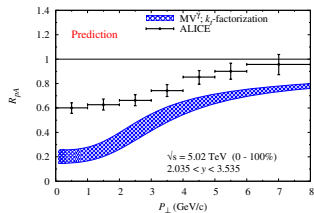
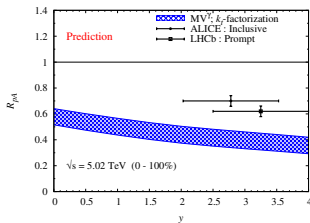
[Fujii and KW (2013)]



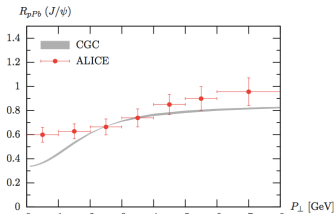
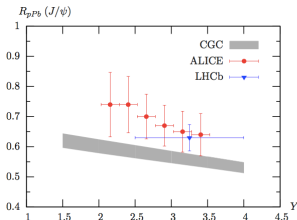
- The P_{\perp} broadening at low P_{\perp} can be described in CGC+CEM.

J/ψ suppression in CGC + CEM

- $Q_{sA}^2 = (4 - 6)Q_{sP}^2$ [Fujii, KW (2013)]

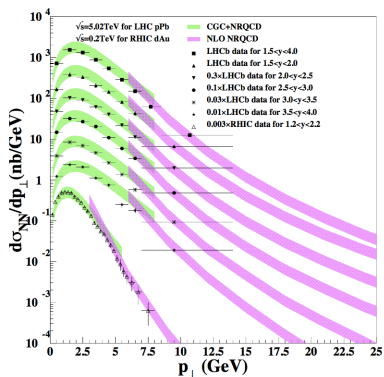
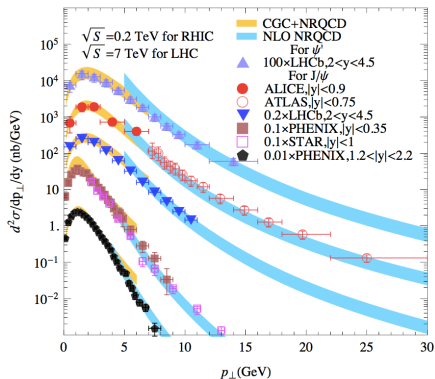


- Optical Glauber approach [Ducloue, Lappi, Mantysaari (2015)]



J/ψ production in CGC + NRQCD

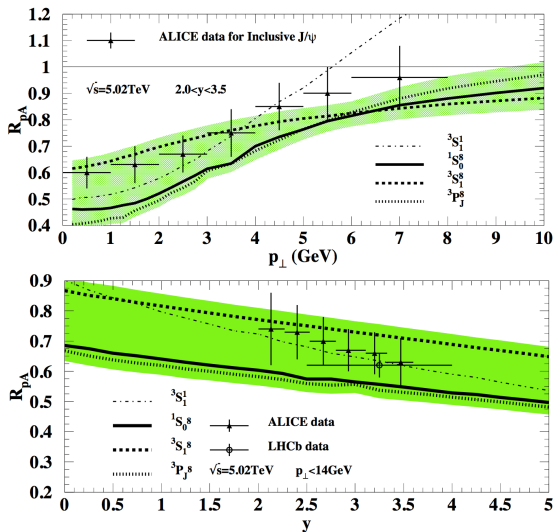
[Ma, Venugopalan (2014)] [Ma, Venugopalan, Zhang (2015)]



- Factorization assumed.
- The contribution of CS channel is **enhanced in pA** but relatively **small contribution**. (10% in pp, 15% – 20% in pA at small- p_{\perp})

J/ψ suppression in CGC + NRQCD

[Ma, Venugopalan, Zhang (2015)]

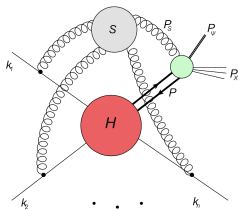


$\psi(2S)$ production

$\psi(2S)$ suppression in pA collisions is an interesting issue. Initial state interaction (saturation effect) is the same for J/ψ and $\psi(2S)$ due to similar kinematics and physical parameters. \Rightarrow **Final state interaction is required.**

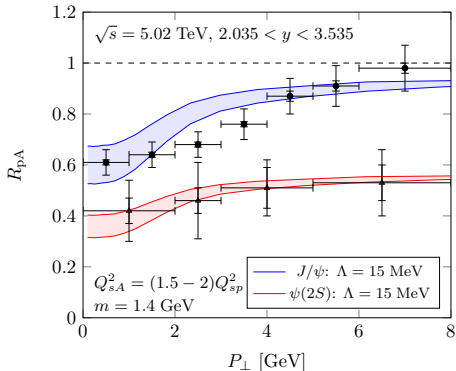
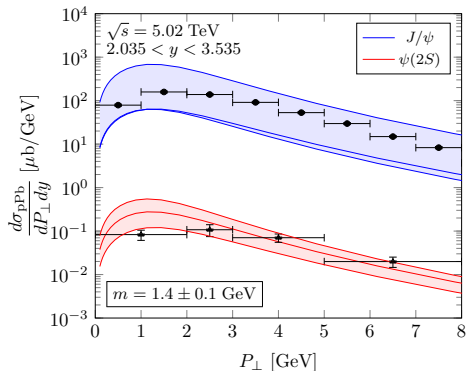
- ✓ **Comovers** produced in pA collisions [Ferreiro (2015)]
- ✓ **Soft color clouds** induced by $c\bar{c}$ traversing the nucleus \Leftarrow Improved CEM. [Ma, Vogt (2016)]

$$\frac{d\sigma_\psi}{d^2P_\perp dy} = F_\psi \int_{m_\psi}^{2m_D - \Lambda} dM \left(\frac{M}{m_\psi} \right)^2 \left. \frac{d\sigma_{c\bar{c}}}{dM d^2P'_\perp dy} \right|_{P'_\perp = \frac{M}{m_\psi} P_\perp}$$



$\psi(2S)$ production

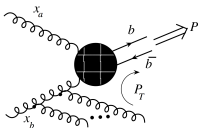
[Ma, Venugopalan, KW, Zhang in preparation]



- $Q_{sA}^2 = 2Q_{sp}^2$ fixed for MB events.
- Local "temperature" in $Q\bar{Q}$ rest frame : $\Lambda \sim O(\Delta E_{\psi(2S)})$

Υ production

Large mass scale $M \Rightarrow$ Large phase space $s \gg M^2 \gg p_{\perp}^2 \Rightarrow$ Parton shower (Sudakov resummation) effect is important. see [Berger, Qiu, Wang (2004)][[Sun, Yuan, Yuan (2012)]]



Small- x vs Sudakov

- $s \gg p_{\perp}^2$: Small- x resummation

$$\frac{\alpha_s N_c}{2\pi^2} \ln \frac{1}{x_g} \sim O(1) \Rightarrow \text{BFKL or BK/JIMWLK evolution}$$

- $M^2 \gg p_{\perp}^2$: Sudakov resummation

$$\frac{\alpha_s N_c}{2\pi} \ln^2 \frac{M^2}{p_{\perp}^2} \sim O(1) \Rightarrow \text{CSS evolution}$$

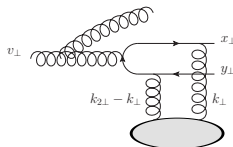
Sudakov implementation in CGC + CEM

Soft gluons emission on top of the BK resummation modifies p_\perp -dependence.

$$\frac{d\sigma_{Q\bar{Q}}}{d^2q_{Q\perp}d^2q_{\bar{Q}\perp}dy_qdy_{\bar{Q}}} = \frac{\alpha_s^2 \bar{S}_\perp}{16\pi^2 C_F} \int d^2l_\perp d^2k_\perp \frac{\Xi_{\text{coll}}(k_{2\perp}, k_\perp - z l_\perp)}{k_{2\perp}^2} \\ \times F_{\text{TMD}}(l_\perp) F_{Y_g}(k_\perp) F_{Y_g}(k_{2\perp} - k_\perp + l_\perp)$$

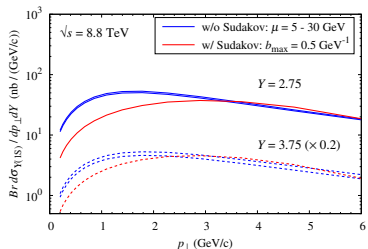
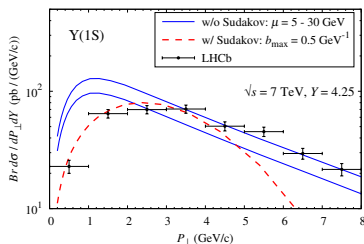
where the TMD gluon distribution is given by

$$F_{\text{TMD}}(M, l_\perp) = \int \frac{d^2b_\perp}{(2\pi)^2} e^{-ib_\perp \cdot l_\perp} e^{-S_{\text{Sud}}(M, b_\perp)} x_1 G\left(x_1, \mu = \frac{c_0}{b_\perp}\right).$$

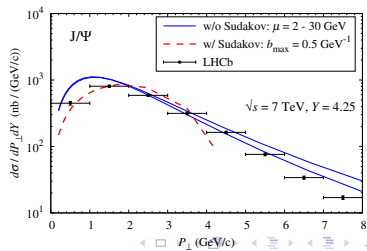


Υ production with Parton shower effect

[KW and Xiao (2015)]



- Parton shower (Sudakov) effect is dominant for low- P_{\perp} Υ production in pp collisions, while it is less pronounced in pA collisions.



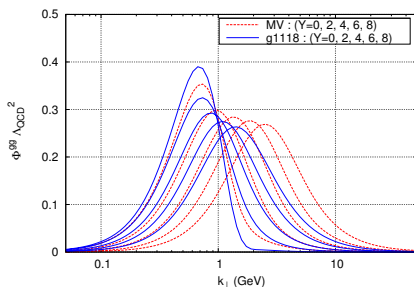
Summary

- J/ψ production can be described by CGC + CEM /NRQCD although the model dependence is a little bit messy.
- $\psi(2S)$ production in pA should be very sensitive to the color medium surrounding $Q\bar{Q}$. This color interaction enables us to interpret $\psi(2S)$ suppression easily in CGC + ICEM.
- Parton shower effect (Sudakov double logs) is essential to interpret data of Υ production.
- More importantly, full NLO calculations for quarkonium production are desired to complete.

Talk Plan

4 Appendix

Energy dependence of UGDF



- Energy dependence of the dipole amplitude ($S = 1 - T$) \Rightarrow
Balitsky-Kovchegov equation [Balitsky (1995)], [Kovchegov (1996)]

$$\frac{dT_Y(r)}{dY} = \mathcal{K} \otimes \left[\underbrace{T_Y(r_1) + T_Y(r - r_1) - T_Y(r)}_{\text{BFKL}} \underbrace{- T_Y(r_1)T_Y(r - r_1)}_{\text{Recombination}} \right]$$

Initial condition of Dipole amplitude

- GBW model [Golec-Biernat, Wusthoff (1998)]

$$S_Y(r_\perp) = 1 - T_Y(r_\perp) = \exp\left[-\frac{Q_s^2 r_\perp^2}{4}\right]$$

Not realistic model: high k_\perp contribution is strongly suppressed.

- MV model [McLerran, Venugopalan (1994)]

$$S_Y(r_\perp) = \exp\left[-\frac{r_\perp^2 Q_{s,0}^2}{4} \ln\left(\frac{1}{|r_\perp| \Lambda} + e\right)\right]$$

- MV^γ and MV^e models [AAMQS(2010)][Lappi-Mäntysaari(2013)]

$$S_Y(r_\perp) = \exp\left[-\frac{(x_\perp^2 Q_{s,0}^2)^\gamma}{4} \ln\left(\frac{1}{|r_\perp| \Lambda} + e_c \cdot e\right)\right]$$

rcBK

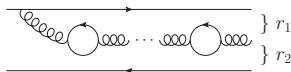
- Balitsky-Kovchegov equation

$$\frac{dT_Y(r)}{dY} = \mathcal{K} \otimes \left[\underbrace{T_Y(r_1) + T_Y(r - r_1) - T_Y(r)}_{\text{BFKL}} - T_Y(r_1)T_Y(r - r_1) \right]$$

- The running coupling kernel in Balitsky's prescription:

$$\mathcal{K}(r_\perp, r_{1\perp}) = \frac{\alpha_s(r^2)N}{2\pi^2} \left[\frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

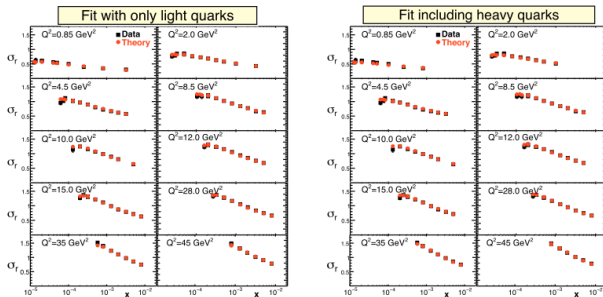
$$\text{with } \alpha_s(r^2) = \left[\frac{9}{4\pi} \ln \left(\frac{4C^2}{r^2 \Lambda^2} + a_{\text{cutoff}} \right) \right]^{-1}$$



- NLO BK equation can be used. See [Lappi, Mäntysaari (2015,2016)]

Constraint from DIS at HERA

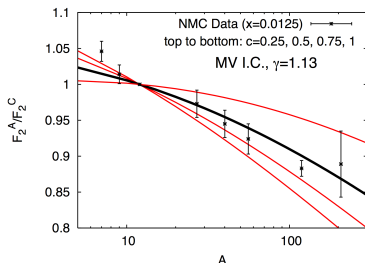
[AAMQS(2010)][Lappi-Mäntysaari(2013)]



set	$Q_{s0,p}^2/\text{GeV}^2$	γ	$\alpha_s(r \rightarrow \infty)$	e_c	$\chi^2/\text{d.o.f.}$
MV	0.2	1	0.5	1	—
MV $^\gamma$	0.1597	1.118	1.0	1	1.12
MV e	0.06	1	0.7	18.9	1.15

IC for the rcBK for nuclei

The initial condition of the rcBK equation for heavy nuclei is poorly understood due to lack of numerous data of eA scattering. A fit to the available NMC (New Muon Collab.) data (fixed target eA) on the nuclear structure functions $F_{2,A}(x, Q^2)$ gives valuable information in this respect.



- Set the initial saturation at $x = 0.01$ as $Q_{s,A}^2(x_2) = cA^{1/3}Q_{s0}^2$
- For MV^γ model, $c = 0.5$ is favored from the fit of data. [Dusling, Gelis, Lappi, Venugopalan (2009)]

Extrapolation at $x \geq 0.01$

- From QCD scaling law, [Fujii, Gelis, Venugopalan][Fujii, KW]

$$\phi_{p,Y}(k_{\perp}) = \phi_{p,Y_0}(k_{\perp}) \left(\frac{1-x}{1-x_0} \right)^4 \left(\frac{x_0}{x} \right)^{0.15}$$

- Fitting from collinear PDF [Ma, Venugopalan (2014)]

$$F_Y(k_{\perp}) \stackrel{x \geq x_0}{=} a(x) F_{Y_0}(k_{\perp})$$

where (e.g.)

$$a(x) = \frac{x G_{\text{CTEQ}}(x, Q_0^2)}{x G^{\text{Dipole}}(x_0, Q_0^2)}$$

with the identity

$$x G^{\text{Dipole}}(x, Q_0^2) = \frac{1}{4\pi^3} \int_0^{Q_0^2} dk_{\perp}^2 \phi_{Y,p}^{g,g}(k_{\perp})$$

R_p is determined by fitting of collinear gluon PDF.

These extrapolations cause systematic uncertainty.

$Q\bar{Q}$ production in the CGC framework

The $q\bar{q}$ production amplitude from the background gauge fields of $\mathcal{O}(\rho_p^1 \rho_A^\infty)$ can be described as

$$M_{s_1 s_2; ij}(q, p) = \frac{g^2}{(2\pi)^4} \int d^2 k_\perp d^2 k_{1\perp} \frac{\rho_p(k_{1\perp})}{k_{1\perp}^2} \int d^2 x_\perp d^2 y_\perp e^{ik_\perp \cdot x_\perp} e^{i(P_\perp - k_\perp - k_{1\perp}) \cdot y_\perp} \times \bar{u}_{s_1, i}(q) \left[T_g(k_{1\perp}) t^b W^{ba}(x_\perp) + T_{q\bar{q}}(k_{1\perp}, k_\perp) U(x_\perp) t^a U^\dagger(y_\perp) \right] v_{s_2, j}(p)$$

At high scattering energies, Eikonal approximation is valid. The interaction between incident quark (gluon) and the dense gluons in the target nucleus can be expressed as the Wilson line in the fundamental (adjoint) representation:

$$U(x_\perp) = \mathcal{P}_+ \exp \left[ig \int_{-\infty}^{+\infty} dz^+ A_A^-(z^+, x_\perp) \cdot t \right],$$

$$W(x_\perp) = \mathcal{P}_+ \exp \left[ig \int_{-\infty}^{+\infty} dz^+ A_A^-(z^+, x_\perp) \cdot T \right]$$

$Q\bar{Q}$ production cross section from the CGC

At classical level, it is required to average the squared amplitude over the distributions of the classical color sources ρ_P and ρ_A

$$\frac{d\hat{\sigma}_{q\bar{q}}}{d^2q_{\perp}d^2p_{\perp}dy_qdy_p} = \int \frac{d^2b_{\perp}}{[2(2\pi)^3]^2} \int \mathcal{D}\rho_P \mathcal{D}\rho_A W_P[\rho_P] W_A[\rho_A] \times |M_{s_1 s_2; ij}(q, p)|^2$$

with W_P and W_A are the weight functionals of ρ_P and ρ_A .

- Rapidity dependence of operator $\langle O \rangle = \int \mathcal{D}\rho_P O W_P[\rho_P]$ is embodied in the weight functionals which obey the JIMWLK equation

$Q\bar{Q}$ production in CGC framework at LO

[Blaizot, Gelis and Venugopalan (2004)][Fujii, Gelis and Venugopalan (2006)]

$$\begin{aligned}
 \phi_A^{g,g}(\mathbf{l}_\perp) &= \text{Diagram 1: A horizontal bar representing a nucleus with a vertical dashed line. A red wavy line labeled } \mathbf{l}_\perp \text{ enters from the left, and another red wavy line labeled } \mathbf{l}_\perp \text{ exits to the right. A green circle with a dot in the center is connected to the two wavy lines by two horizontal lines.} \\
 \phi_A^{q\bar{q},g}(\mathbf{l}_\perp; \mathbf{k}_\perp) &= \text{Diagram 2: Similar to Diagram 1, but the green circle contains a dot and is labeled } \mathbf{k}_\perp \text{ with a vertical arrow. The wavy lines are labeled } \mathbf{l}_\perp - \mathbf{k}_\perp \text{ and } \mathbf{l}_\perp. \\
 \phi_A^{q\bar{q},q\bar{q}}(\mathbf{l}_\perp; \mathbf{k}_\perp, \mathbf{k}'_\perp) &= \text{Diagram 3: Similar to Diagram 2, but with two green circles. The first contains } \mathbf{k}_\perp \text{ and the second contains } \mathbf{k}'_\perp. \text{ The wavy lines are labeled } \mathbf{l}_\perp - \mathbf{k}_\perp, \mathbf{l}_\perp - \mathbf{k}'_\perp, \text{ and } \mathbf{l}_\perp.
 \end{aligned}$$

$$\begin{aligned}
 \frac{d\sigma_{Q\bar{Q}}}{d^2q_\perp d^2p_\perp dy_q dy_p} &= \frac{\alpha_s}{(2\pi)^6 C_F} \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{\varphi_{p,Y_p}(k_{1\perp})}{k_{1\perp}^2 k_{2\perp}^2} \\
 &\times \left[\Xi^{g,g} \phi_{A,Y_A}^{g,g} + \int \frac{d^2k_\perp}{(2\pi)^2} \Xi^{q\bar{q},g} \phi_{A,Y_A}^{q\bar{q},g} + \int \frac{d^2k_\perp d^2k'_\perp}{(2\pi)^4} \Xi^{q\bar{q},q\bar{q}} \phi_{A,Y_A}^{q\bar{q},q\bar{q}} \right] \\
 &\Rightarrow \int \frac{d^2k_\perp}{(2\pi)^2} \Xi(k_{1\perp}, k_{2\perp}, k_\perp) \phi_{A,Y_A}^{q\bar{q},g}(k_{2\perp}, k_\perp)
 \end{aligned}$$

Multipoint function

The unintegrated gluon distribution function

$$\varphi_{p,Y_p}(k_\perp) = \frac{(2\pi)^2 \alpha_s S_\perp}{k_\perp^2} \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \langle \rho_p(0) \rho_p(x_\perp) \rangle_{Y_p}.$$

and the higher multi-point Wilson line correlator in large- N_c limit

$$\phi_{A,Y_A}^{q\bar{q},g}(k_{2\perp}, k_\perp) \approx S_\perp \frac{N_c k_{2\perp}^2}{4} F_{Y_A}(k_{2\perp} - k_\perp) F_{Y_A}(k_\perp)$$

Fourier transform of the fundamental dipole amplitude is defined as

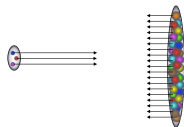
$$F_Y(k_\perp) = \int d^2 x_\perp e^{-ik_\perp \cdot x_\perp} \underbrace{\frac{1}{N_c} \langle \text{Tr} [U(x_\perp) U^\dagger(0_\perp)] \rangle_Y}_{\text{Dipole amplitude}}.$$

Forward production : Hybrid formalism

At forward rapidity where $x_1 \sim 1$, the phase space of the gluon distribution in projectile proton shrinks. By taking the limit $k_{1\perp} \rightarrow 0$ in the hard scattering parts and replacing the unintegrated gluon distribution function of proton with the collinear gluon distribution function (PDF), Hybrid-formula ($x_1 \sim 1$ and $x_2 \ll 1$) can be written as

$$\frac{d\sigma_{q\bar{q}}}{d^2p_{q\perp}d^2p_{\bar{q}\perp}dy_qdy_{\bar{q}}} = \frac{N_c\alpha_s S_{\perp}^A}{64\pi^2 C_F} \int_{k_{\perp}} \Xi_{\text{coll}}(k_{2\perp}, k_{\perp}) x_1 G(x_1, \mu) \\ \times F_{x_2}(k_{2\perp} - k_{\perp}) F_{x_2}(k_{\perp})$$

with $p_{q\perp} + p_{\bar{q}\perp} = k_{2\perp}$ and $x_1 G(x_1, \mu) \equiv C \int^{\mu^2} dk_{\perp}^2 \varphi_{p, Y_p}(k_{\perp})$



Quarkonium Production Models

- Color Evaporation Model **Fritzsch (1977)** ...
 - Constant transition rate : $F_{J/\psi}$
 - Color Singlet Model **Chang (1980)** ...
 - α_s expansion
 - Gluon Jet fragmentation **Braaten and Yuan (1993)** ...
 - High P_\perp production
 - NRQCD **Braaten and Fleming (1995)** ...
 - α_s and $v \sim \alpha_s(mv)$ expansion : $mv^2 \ll mv \ll m$
 - Power expansion **Nayak, Qiu, and Sterman (2005)** ...
 - M/P_\perp and α_s expansion : High P_\perp production
- ... and More!

Quarkonium production at low $P_\perp < M$

Interesting but challenging! \Rightarrow Small- x /CGC framework provides important information of quarkonium production mechanism.

CGC framework + NRQCD

[Kang, Ma, Venugopalan (2013)]

Color singlet channel

$$\begin{aligned}
 & \frac{d\sigma_{q\bar{q}}^{\text{CS}}}{d^2P_{\perp} dy} \propto \\
 & \int \frac{d^2k_{1\perp} d^2k_{\perp} d^2k'_{\perp}}{(2\pi)^6} \frac{\varphi_{p,Y_p}(k_{1\perp})}{k_{1\perp}^2} \frac{1}{2J+1} \sum_{J_z} \mathcal{F}_{q\bar{q}}^{J_z}(P, k_{1\perp}, k_{\perp}) \mathcal{F}_{q\bar{q}}^{J_z\dagger}(P, k_{1\perp}, k'_{\perp}) \\
 & \times \int d^2x_{\perp} d^2x'_{\perp} d^2y_{\perp} d^2y'_{\perp} e^{i(k_{\perp}\cdot x_{\perp} - k'_{\perp}\cdot x'_{\perp})} e^{i(k_{2\perp} - k_{\perp})\cdot y_{\perp}} e^{-i(k_{2\perp} - k'_{\perp})\cdot y'_{\perp}} \\
 & \times \frac{1}{N_c} \langle \text{tr}[U(x_{\perp}) t^a U^{\dagger}(y_{\perp})] \text{tr}[U(y'_{\perp}) t^a U^{\dagger}(x'_{\perp})] \rangle_{Y_A} . \\
 & \underbrace{\hspace{15em}} \\
 & \approx \frac{1}{2} [Q_{Y_A}(x_{\perp}, y_{\perp}; y'_{\perp}, x'_{\perp}) - S_{Y_A}(x_{\perp}, y_{\perp}) S_{Y_A}(y'_{\perp}, x'_{\perp})]
 \end{aligned}$$

The quadrupole amplitude

$$Q_{Y_A}(x_{\perp}, y_{\perp}; y'_{\perp}, x'_{\perp}) \equiv \frac{1}{N_c} \text{tr} \langle U(x_{\perp}) U^{\dagger}(x'_{\perp}) U(y'_{\perp}) U^{\dagger}(y_{\perp}) \rangle_{Y_A} .$$

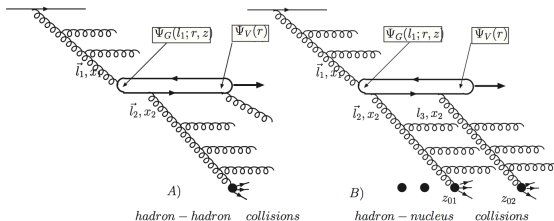
Enhancement mechanism

[D. Kharzeev, E. Levin, M. Nardi and K. Tuchin]

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- J/ψ has $J^{PC} = 1^{--}$: odd number of gluons are needed.
- pp collisions : $g + g \rightarrow J/\psi + \text{soft-g} \rightarrow O(\alpha_s^5)$
- pA collisions : $g + g + g \rightarrow J/\psi \rightarrow O(\alpha_s^6 A^{2/3}) \sim O(\alpha_s^2)$ when $\alpha_s^2 A^{1/3} \sim 1$.

The color singlet production in pA collisions is **enhanced** compared to pp collisions.



¹Phys. Rev. Lett. **102**, 152301 (2009)

CGC framework + NRQCD

Color Octet state

$$\frac{d\sigma_{q\bar{q}}^{\text{CO}}}{d^2P_{\perp} dy} = \frac{\alpha_s S_{\perp}}{(2\pi)^3 (N_c^2 - 1)} \int \frac{d^2k_{1\perp} d^2k_{\perp}}{(2\pi)^4} \frac{\varphi_{p, Y_p}(k_{1\perp})}{k_{1\perp}^2} \\ \times F_{Y_A}(k_{2\perp} - k_{\perp}) F_{Y_A}(k_{\perp}) \Xi^{\text{CO}}$$

- Of particular importance is that the nuclear effect is the same both in CEM and the color octet state in NRQCD with the large- N_c approximation.

Conventional TMD framework

One loop calculations in NRQCD factorization framework [Sun, Yuan, Yuan (2012)]

$$\left. \frac{d\sigma}{d^2P_\perp dy} \right|_{|P_\perp| \ll M} = \int \frac{d^2b_\perp}{(2\pi)^2} e^{iP_\perp \cdot b_\perp} W(M, b_\perp, x_1, x_2) + (\text{Y-term})$$

with

$$W(M, b_\perp, x_1, x_2) = e^{-S_{\text{sud}}(M, b_\perp)} W(M, b_\perp, C_1, C_2)$$

$$W(M, b_\perp, C_1, C_2) = \sigma_0 \frac{M^2}{s} \int \frac{dx}{x} \frac{dx'}{x'} C_{gg} \left(\frac{x_1}{x} \right) C_{gg} \left(\frac{x_2}{x'} \right) G(x_1, \mu) G(x_2, \mu)$$

- TMD factorization [Collins-Soper-Sterman (1985)] [Collins (2011)]
- Y-term is power suppressed by P_\perp/M at low P_\perp
- $\mu = \frac{c_0}{b_\perp}$ with $c_0 = 2e^{-\gamma_E} \simeq 1$

Collins-Soper-Sterman (CSS) formalism

Split the Sudakov factor into two parts

$$\boxed{S_{\text{Sud}}(M, b) = S_{\text{perp}}(M, b_\star) + S_{\text{NP}}(M, b)} \quad \text{with } b_\star = \frac{b}{\sqrt{1 + (b/b_{\text{max}})^2}}$$

- Perturbative : $b_\star \sim b \ll b_{\text{max}}$

$$S_{\text{perp}}(M, b) = \int_{c_0/b^2}^{M^2} \frac{d\mu^2}{\mu^2} \left[A \ln \left(\frac{M^2}{\mu^2} \right) + B \right]$$

$$A = \sum_{i=1} A^{(i)} \left(\frac{\alpha_s}{\pi} \right)^i, \quad B = \sum_{i=1} B^{(i)} \left(\frac{\alpha_s}{\pi} \right)^i. \quad A^{(1)} = C_A \text{ and} \\ B^{(1)} = -(b_0 + \delta_{8c}/2)N_c \text{ with } b_0 = \left(\frac{11}{6}N_c - \frac{n_f}{3} \right) \frac{1}{N_c}.$$

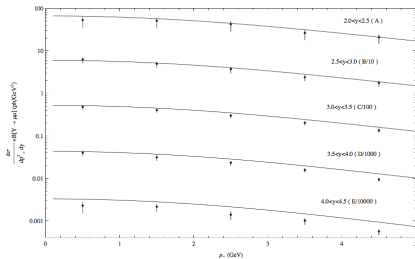
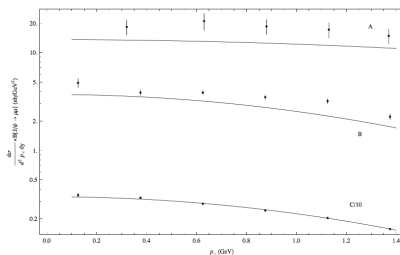
- Non-perturbative : $b > b_{\text{max}}$ [Sun, Yuan, Yuan (2012)]

$$S_{\text{NP}}(M, b) = \exp \left[b^2 \left(-g_1 - g_2 \ln \left(\frac{M}{2Q_0} \right) - g_1 g_3 \ln(100x_1 x_2) \right) \right]$$

$$g_1 = 0.03, \quad g_2 = 0.87, \quad g_1 g_3 = -0.17 \text{ with } Q_0 = 1.6 \text{ GeV}, \quad b_{\text{max}} = 0.5 \text{ GeV}.$$

Lesson from conventional TMD calculation

[Sun, Yuan, Yuan (2012)]



- Left fig: J/ψ at LHC forward, RHIC mid and forward. Right fig: Υ at LHC forward.
- The agreement between theoretical results and experimental data for J/ψ production is not as good as that for Υ production