Lattice QCD determination of quark masses and α_s

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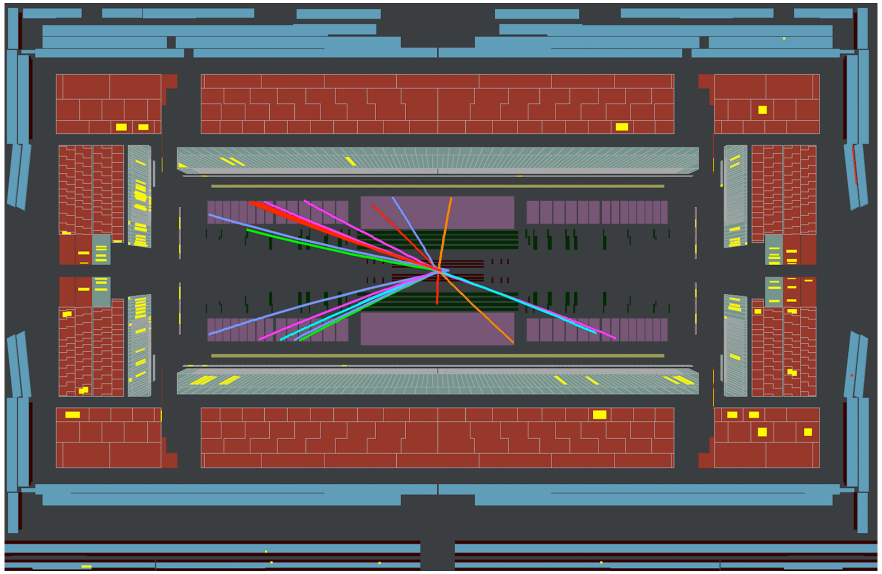
Quark masses and strong coupling are fundamental parameters of the SM but cannot be directly determined from experiment because we do not have direct access to quarks

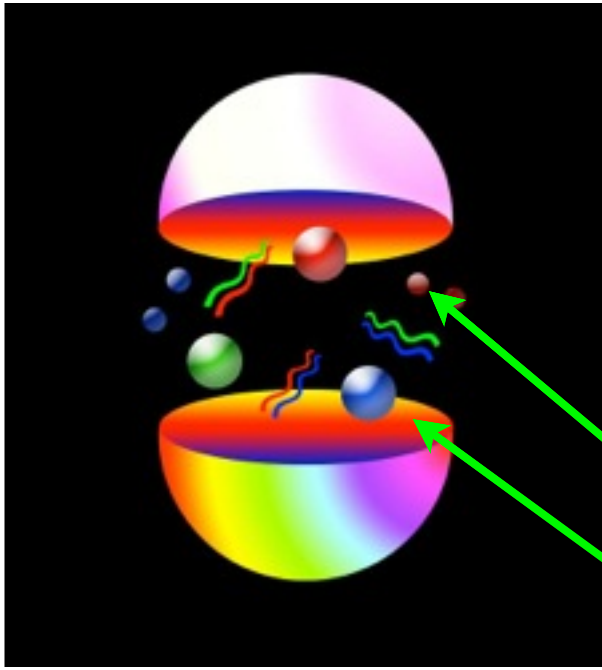


ATLAS@LHC

Well-defined m_q and α_s are scheme and scale-dependent. Convention is to use $\overline{MS}$.

Compare results from multiple approaches for strong test of QCD. Lattice QCD methods are particularly accurate.

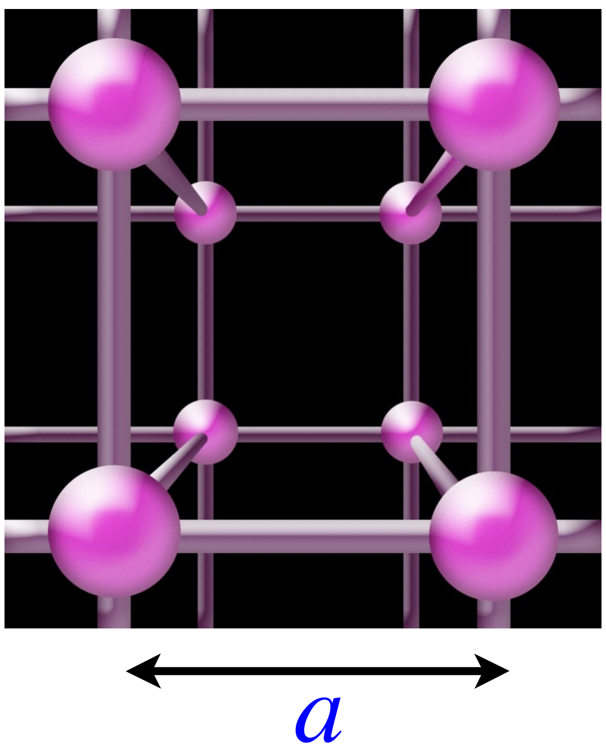
Masses/α_s are input to theoretical expressions for SM cross-sections e.g. $H \to c\overline{c}$



Lattice QCD: fields defined on 4-d discrete space-(Euclidean) time.
Lagrangian parameters: $\alpha_s, m_q a$

1) Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s, (c) sea quarks)

2) Calculate valence quark propagators and combine for "hadron correlators" . Fit for hadron masses and amplitudes

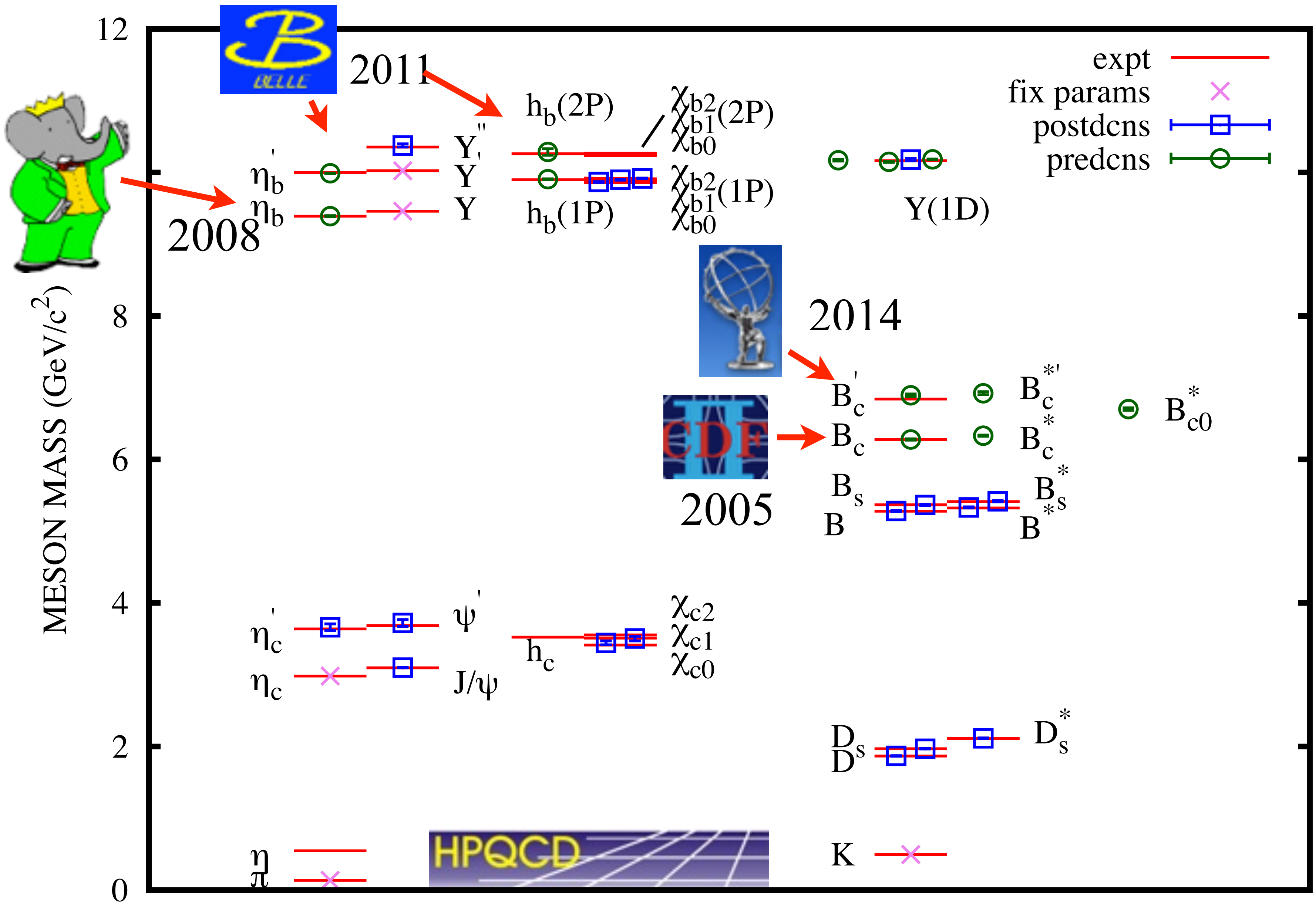


• Determine a to convert results in physical units. Fix m_q from hadron mass

numerically extremely challenging

• cost increases as $a \rightarrow 0, m_{u/d} \rightarrow \text{phys}$ and with statistics, volume.

Can tune bare lattice QCD mass parameters very accurately using experimentally very well-determined ground-state meson masses.



few MeV accuracy requires em effects to be considered

Mass parameters in Lattice QCD Lagrangian can be tuned very accurately against experimental hadron masses

Issue is:
Conversion of lattice quark masses to $\overline{MS}$ scheme

$$m_{\overline{MS}}(\mu) = Z_m(\mu a) m_{latt}$$

Options to calculate Z:

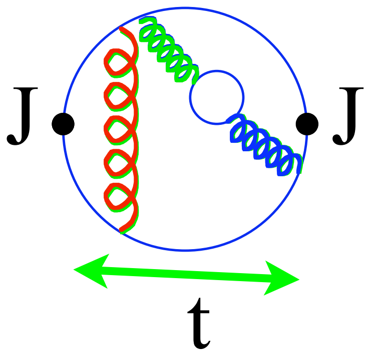
1) lattice QCD pert. th. - hard to do beyond NLO
2) Nonperturbative calculation of a quantity that can be matched to $\overline{MS}$ using continuum QCD pert. theory

* Error dominated by that of Z

Note: Z cancels in mass ratios, which are *completely nonperturbative* in lattice QCD in a given quark formalism. Provides critical test of procedure above.

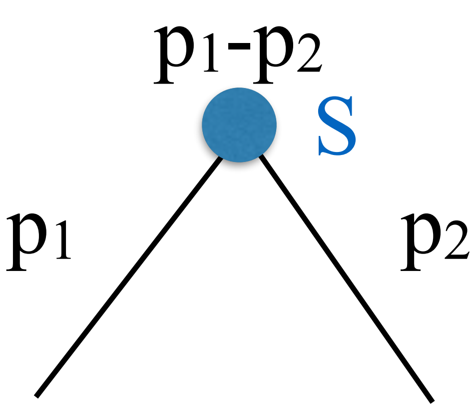
various of these

Lattice QCD: determining quark masses and providing non-perturbative tests of the determination



m_c, m_b

current-current correlators



$m_s, m_u/m_d$

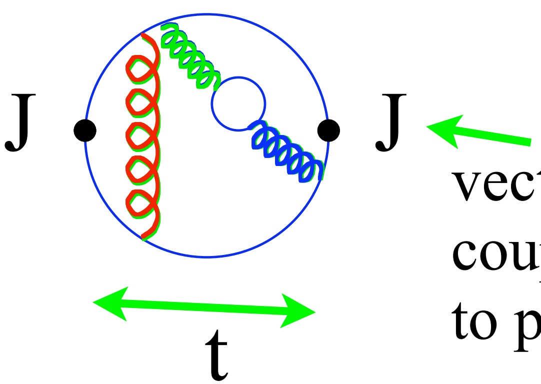
Green's function in RI-SMOM scheme

$$\frac{m_c}{m_s}$$

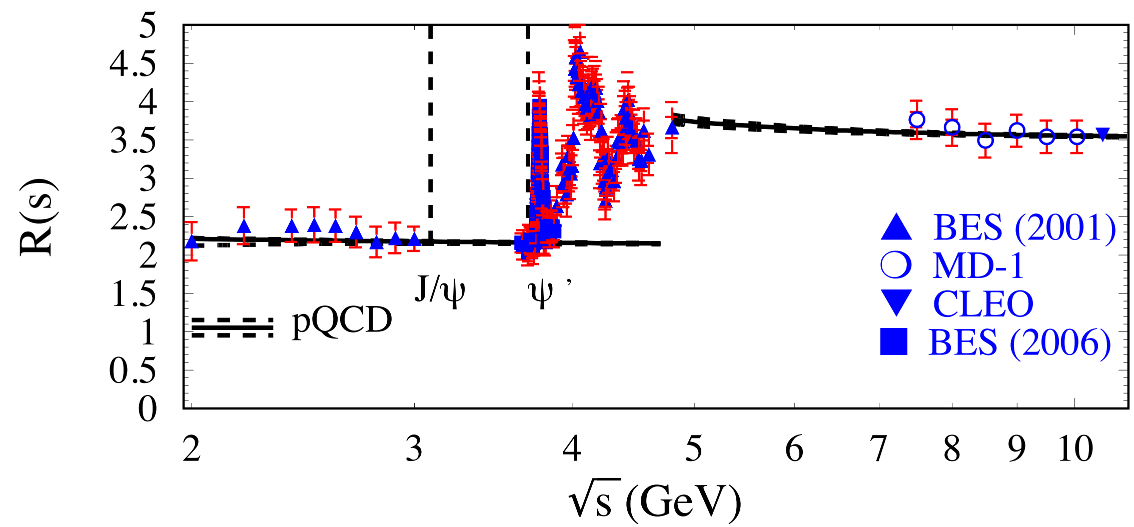
determined directly from lattice QCD

Current-current correlator method for m_c (and m_b)

Time-moments of lattice QCD correlators extrapolated to the continuum limit can be related to s^{-1}-moments of $R_{e^+e^-}$ and to continuum QCD perturbation theory known through α_s^3 (NNNLO) `e.g. Kuhn et al, hep-ph/0702103`



vector coupling to photon



$$G_n = \sum_t (t/a)^n G(t)$$

$$R_{e^+e^-}(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha^2/(3s)}$$

$$n = 4, 6, 8, 10 \ldots$$

$$n = 2k + 2$$

$$\mathcal{M}_k \equiv \int \frac{ds}{s^{k+1}} R_c(s)$$

c quark contribn

Continuum QCD perturbation theory for the moments is a function of quark mass and known through α_s^3

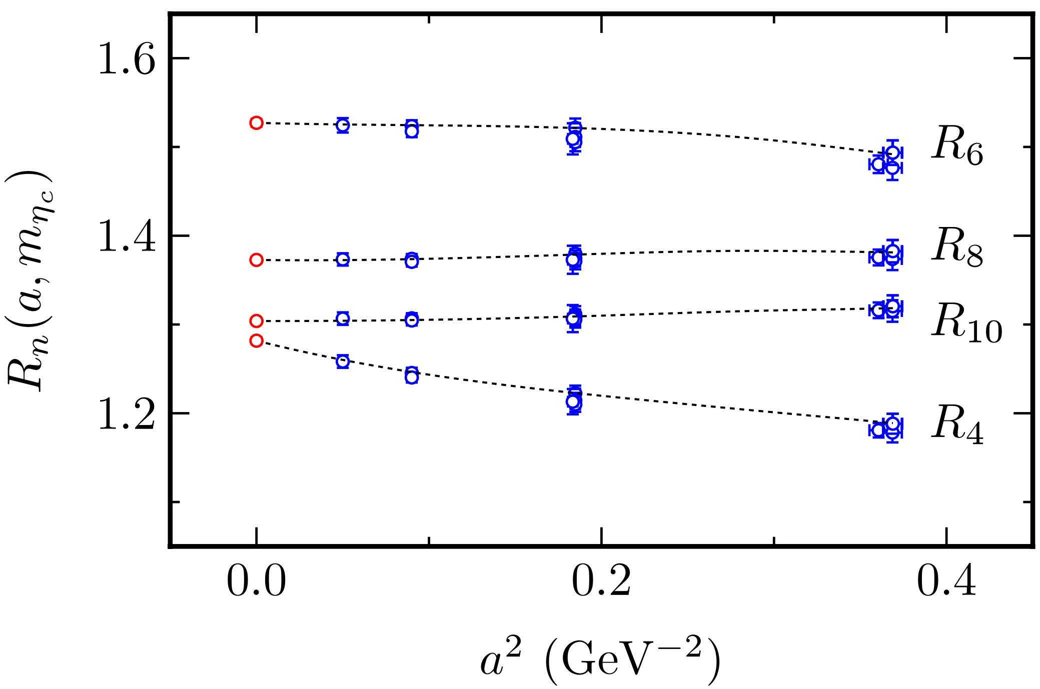
In lattice QCD can calculate moments not available to expt.

e.g. for pseudoscalar density correlator for c quarks:

$$R_{n,latt} = G_4/G_4^{(0)} \quad n=4$$

← ratio to results with no gluon field improves disc. errors

$$= \frac{am_{\eta_c}}{2am_c}(G_n/G_n^{(0)})^{1/(n-4)} \quad n = 6, 8, 10\ldots$$



$$R_{n,cont} = \frac{m_{\eta_c}}{2m_c(\mu)}\frac{C_k^P}{C_k^{P,0}}$$

$$\frac{C_k^P}{C_k^{P,0}} = 1 + \sum c_i\alpha_s^i(\mu)$$

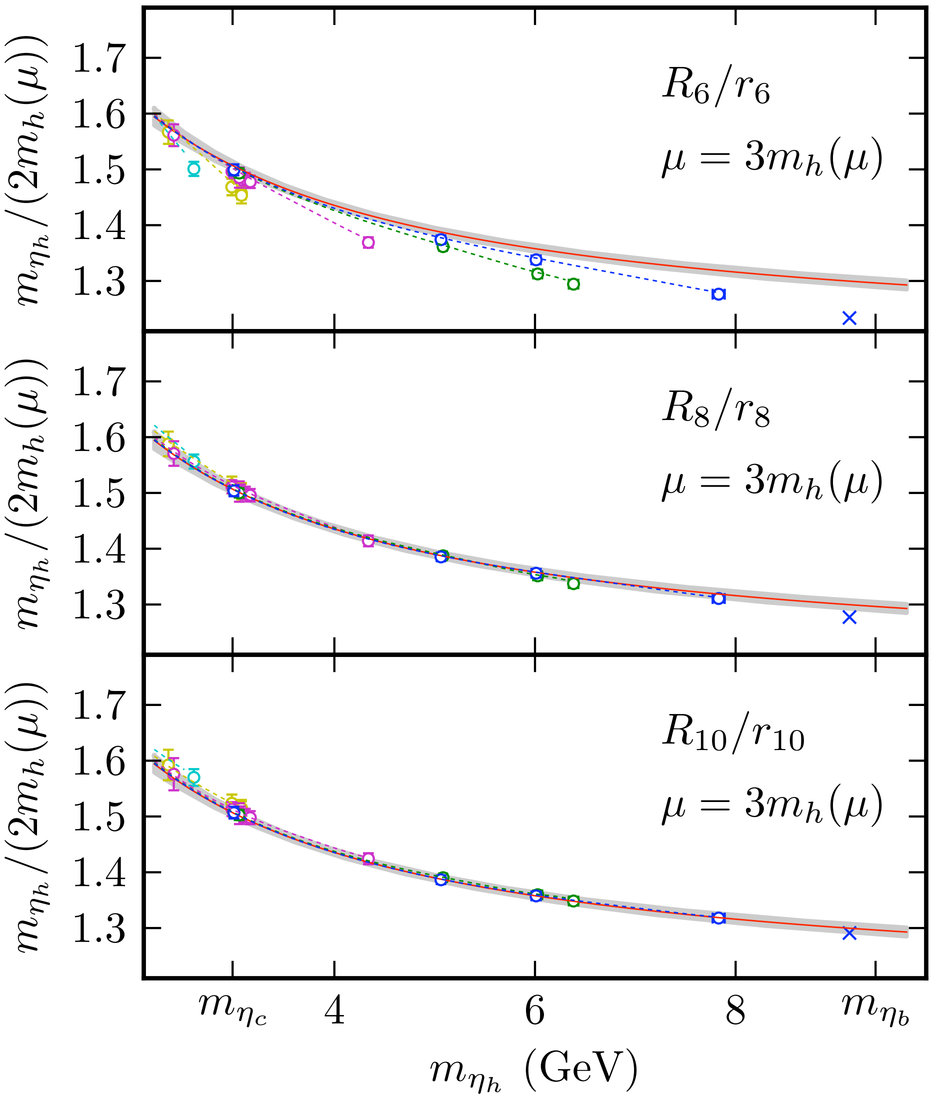
simultaneous fit to multiple moments - gives α_s, m_c

HPQCD + Chetyrkin et al, 0805.2999, C. Mcneile et al, HPQCD,1004.4285

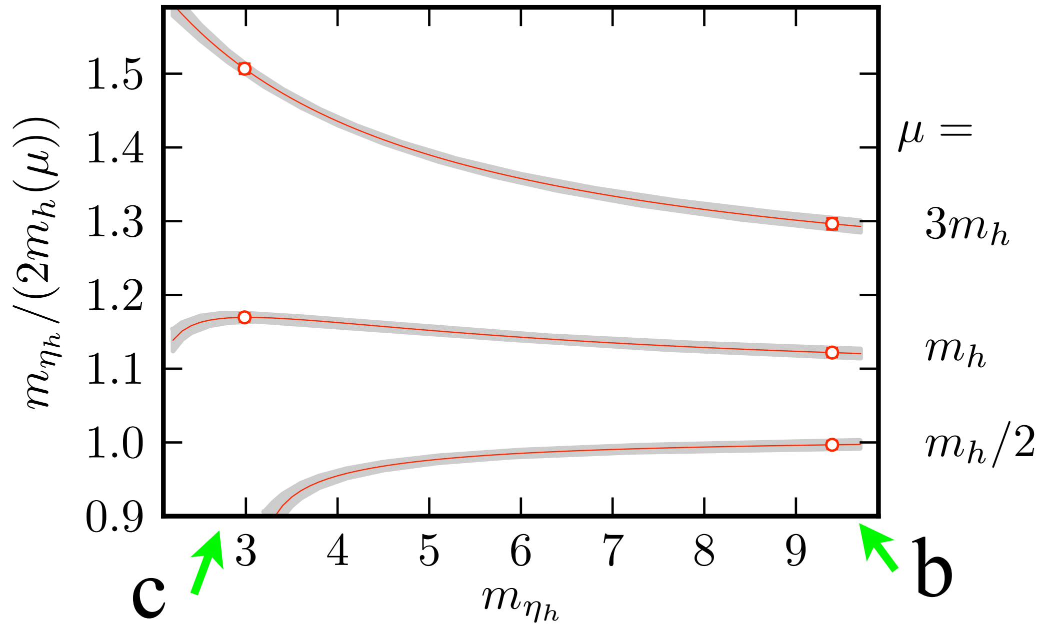
Current-current correlator method -HISQ

HPQCD, 1004.4285

- Repeat calcln for $m_q \geq m_c$ inc. ultrafine lattices



Agrees well with contnm results using $R_{e^+e^-}$



Can determine m_h/m_{η_h} for heavy quarks - extrapolate (slightly) to b.

$$\overline{m}_b^{n_f=5}(\overline{m}_b) = 4.164(23)\text{GeV}$$

key error is now extrapoln in a

Example error budget for HISQ current-current method

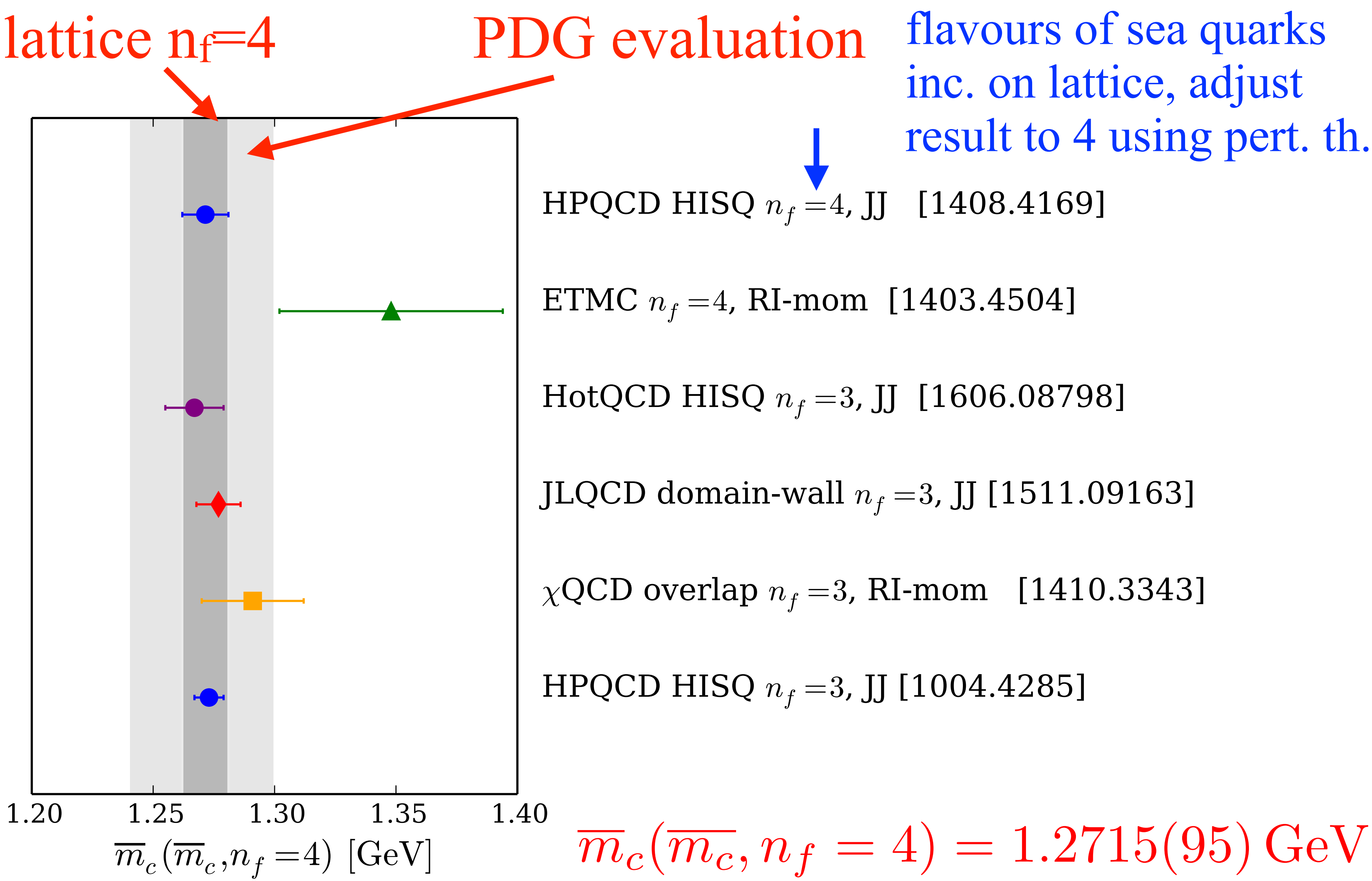
TABLE IV. Error budget [31] for the c mass, QCD coupling, and the ratios of quark masses m_c/m_s and m_b/m_c from the $n_f = 4$ simulations described in this paper. Each uncertainty is given as a percentage of the final value. The different uncertainties are added in quadrature to give the total uncertainty. Only sources of uncertainty larger than 0.05% have been listed.

HPQCD, 1408.4169

	$m_c(3)$	$\alpha_{\overline{\rm MS}}(M_Z)$	m_c/m_s	m_b/m_c
Perturbation theory	0.3	0.5	0.0	0.0
Statistical errors	0.2	0.2	0.3	0.3
$a^2 \to 0$	0.3	0.3	0.0	1.0
$\delta m_{uds}^{\rm sea} \to 0$	0.2	0.1	0.0	0.0
$\delta m_c^{\rm sea} \to 0$	0.3	0.1	0.0	0.0
$m_h \neq m_c$ (Eq. (15))	0.1	0.1	0.0	0.0
Uncertainty in w_0, w_0/a	0.2	0.0	0.1	0.4
α_0 prior	0.0	0.1	0.0	0.0
Uncertainty in m_{η_s}	0.0	0.0	0.4	0.0
$m_h/m_c \to m_b/m_c$	0.0	0.0	0.0	0.4
δm_{η_c}: electromag., annih.	0.1	0.0	0.1	0.1
δm_{η_b}: electromag., annih.	0.0	0.0	0.0	0.1
Total:	0.64%	0.63%	0.55%	1.20%

m_c summary

Good consistency between lattice actions using JJ method



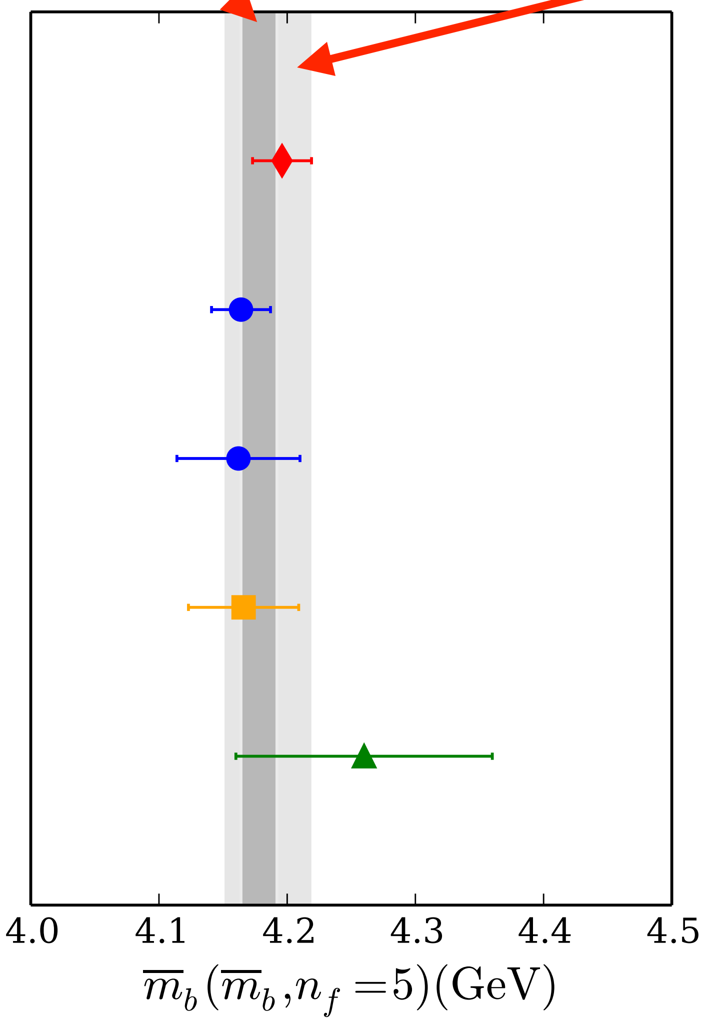
$$\overline{m}_c(\overline{m_c}, n_f = 4) = 1.2715(95)\ \text{GeV}$$

m_b summary

Several different methods here. Good consistency between different methods and b-quark formalisms

lattice av. PDG evaluation

flavours of sea quarks inc. on lattice. Use pert. th. to inc. to 5



HPQCD NRQCD JJ $n_f = 4$ [1408.5768]

HPQCD HISQ JJ $n_f = 3$ [1004.4285]

HPQCD HISQ ratio $n_f = 4$ [1408.4169]

HPQCD NRQCD E_0 $n_f = 3$ [1302.3739]

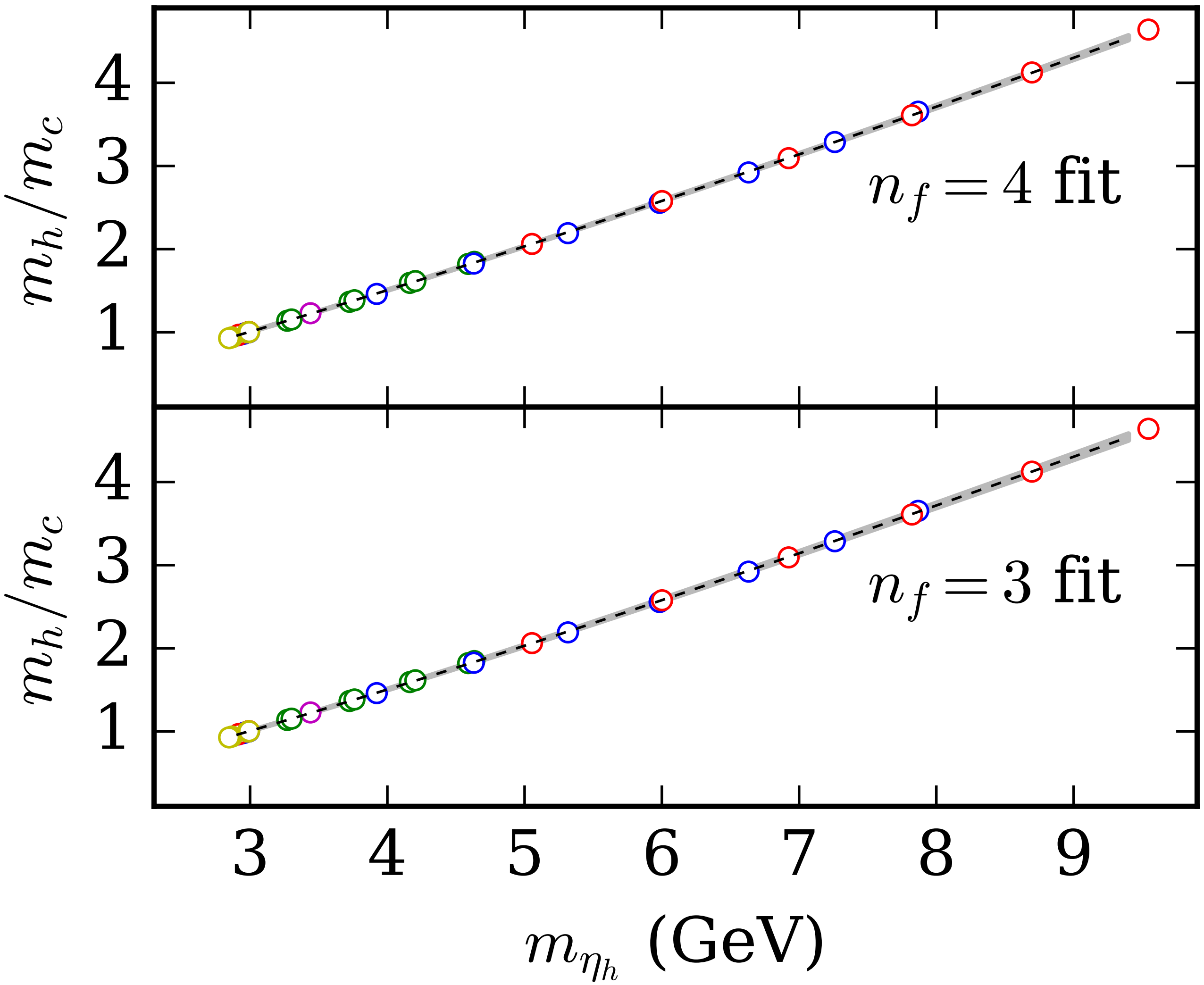
ETMC ratio method $n_f = 4$ [1603.04306]

Lattice average:

4.178(14) GeV

m_b/m_c from lattice QCD

update to HPQCD, 1408.4169



$$\left(\frac{m_{q1,latt}}{m_{q2,latt}}\right)_{a=0} \text{ in QCD}$$

$$= \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

completely nonperturbative determination of ratio gives:

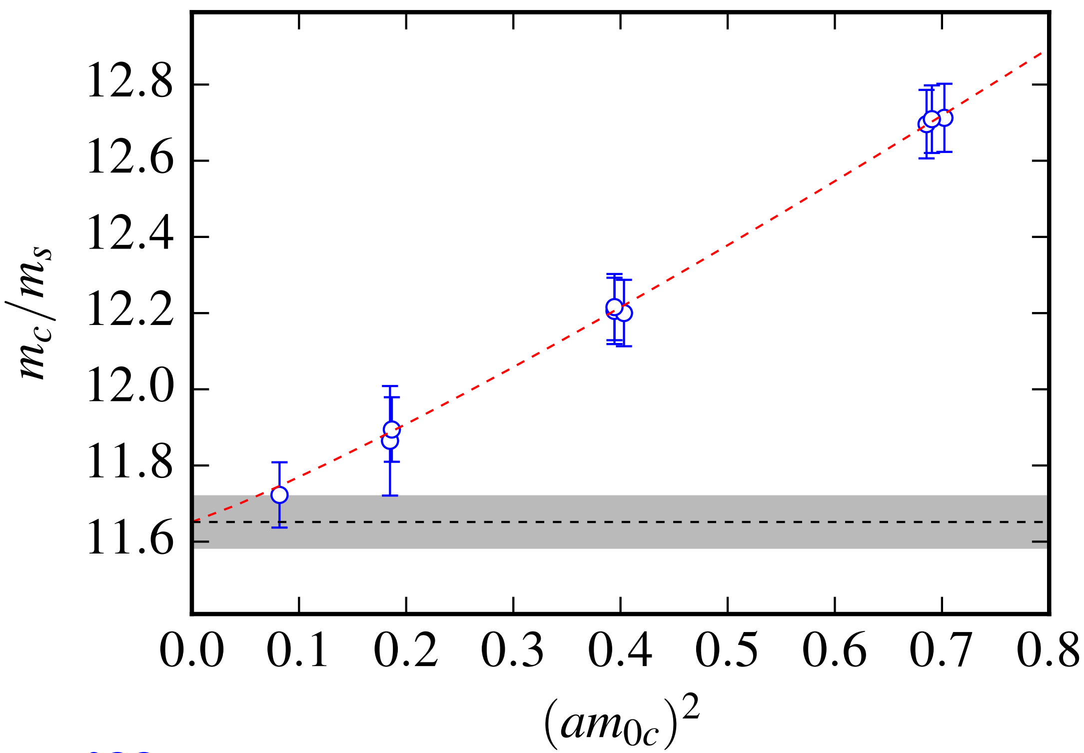
$$\frac{m_b}{m_c} = 4.541(26)$$

Agrees with that from current-current correlator method - test of pert. th.

see also: HPQCD, 1004.4285; ETM,1603.04306; HotQCD, 1606.08798

m_c/m_s

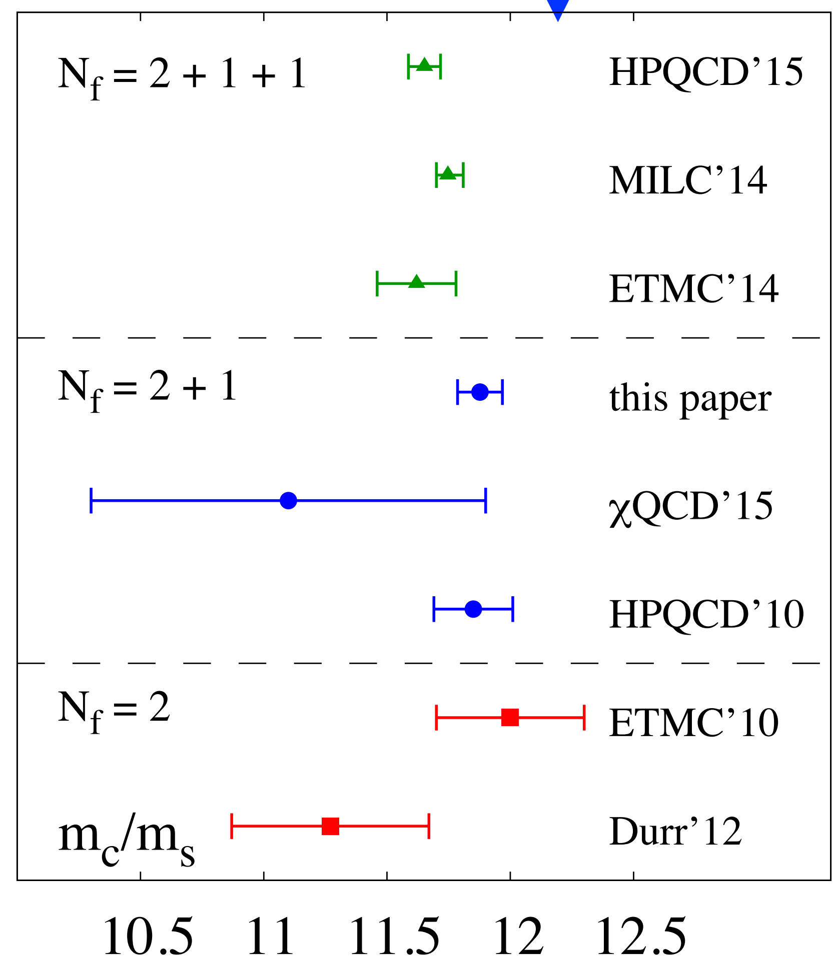
Mass ratio can be obtained directly from lattice QCD if same quark formalism is used for both quarks.
Not possible with any other method ...



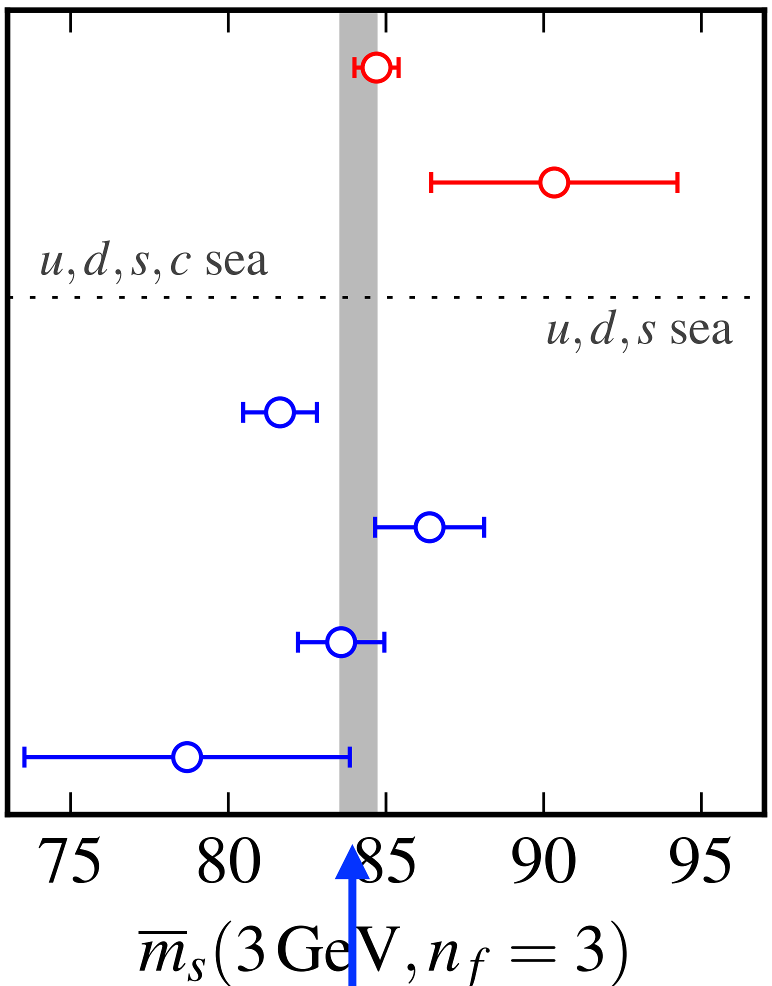
$$\frac{m_c}{m_s} = 11.652(65)$$

HPQCD, 1408.4169

summary from hotQCD, 1606.08798



Combining m_c and m_c/m_s leads to 1% accuracy in m_s - also compare to RI-SMOM scheme determination



HPQCD 1408.4169 mc/ms+mc

ETMC 1403.4504 RI-MOM

RBC/UKQCD 1411.7017 RI-SMOM

Durr et al 1011.2403 RI-MOM

HPQCD 0910.3102 mc/ms+mc

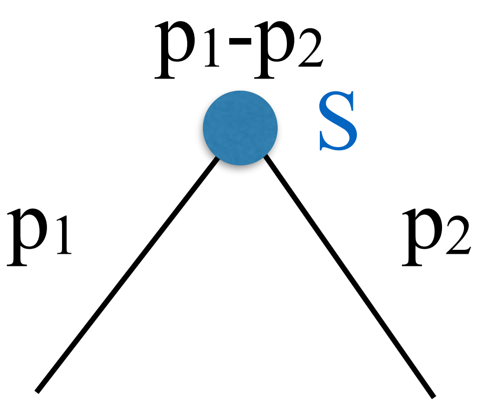
HPQCD (pert) 0511160 lattice pert

Also hotQCD:1606.08798
83.6(1.5) MeV

$$\overline{m}_s(3\mathrm{GeV}, n_f = 3) = 84.1(5)\mathrm{MeV}$$

Alternative method to determine (light) quark masses: RI-SMOM scheme

RBC/UKQCD, 0712.1061



$$\frac{Z_O}{Z_q}\Lambda_O(p_1,p_2) = O^{tree}$$

$$Z_m = Z_S^{-1}$$

Impose a 'MOM' renormalisation scheme directly on the lattice, i.e. fix an amputated vertex function to its tree-level value (in Landau gauge).

Match to $\overline{MS}$ perturbatively - α_s^2 = NNLO

Important improvement:

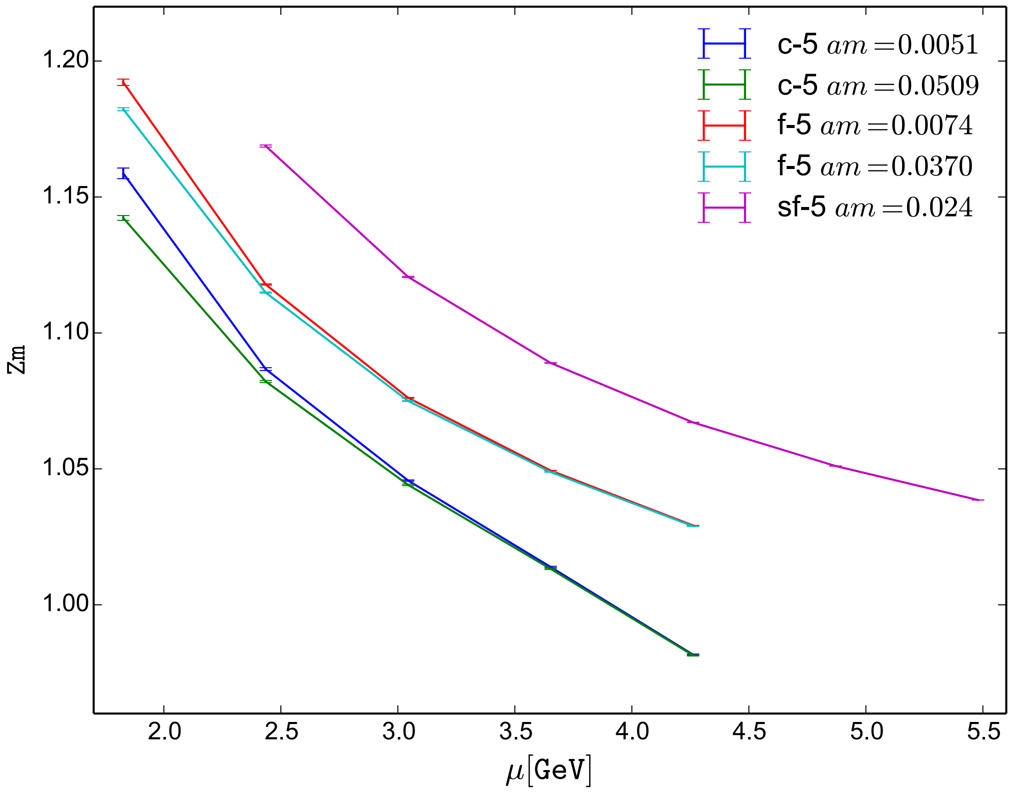
'Non-exceptional' kinematics:

$$p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$$

much smaller systematic errors from non-pert effects AND pert. matching

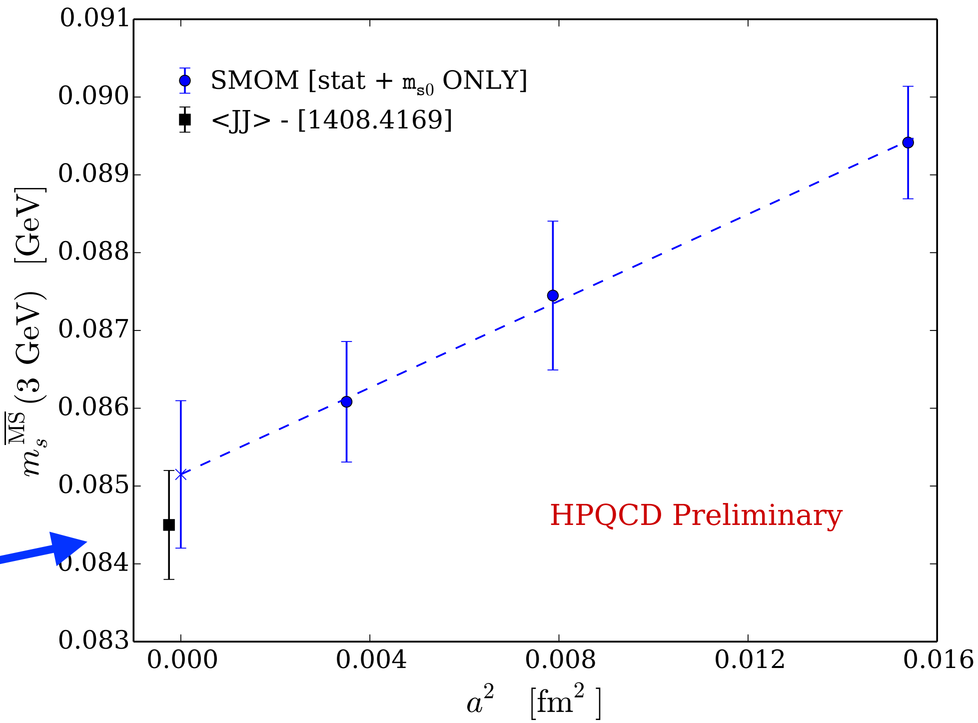
Sturm et al, 0901.2599; Gorbahn, Jäger, 1004.3997

Calculate lattice Z_m - multiply by tuned lattice bare mass and pert. matching to $\overline{MS}$

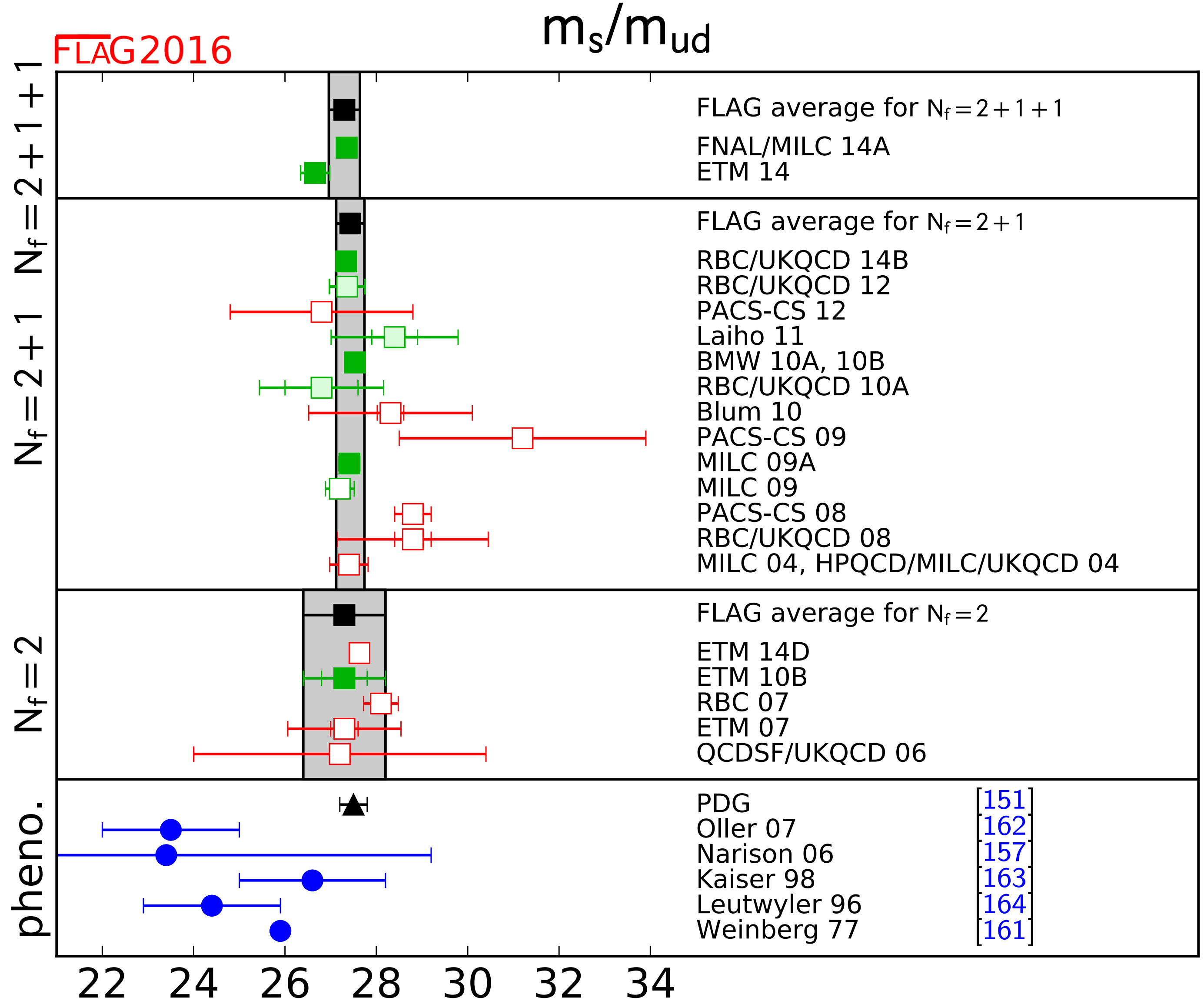


HPQCD, Lytle et al, preliminary, 1511.06547

Agrees well with result expected from JJ method for m_c and m_c/m_s



Lattice QCD determination of m_s/m_{ud} requires consideration of em effects via charged/neutral π/K



$$m_{ud} = \frac{m_u + m_d}{2}$$

$$\frac{m_s}{m_{ud}} \overset{\text{LO chi-PT}}{=} \frac{M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2}{M_{\pi^+}^2} = 25.9$$

summary from 1607.00299

PDG average dominated by lattice = 27.3(7)

Lattice QCD determination of α_s

Lattice QCD Lagrangian has parameter g^2 = lattice scheme coupling at scale π/a - determination of a fixes the coupling.

However, again it is conversion to $\overline{MS}$ which is issue.

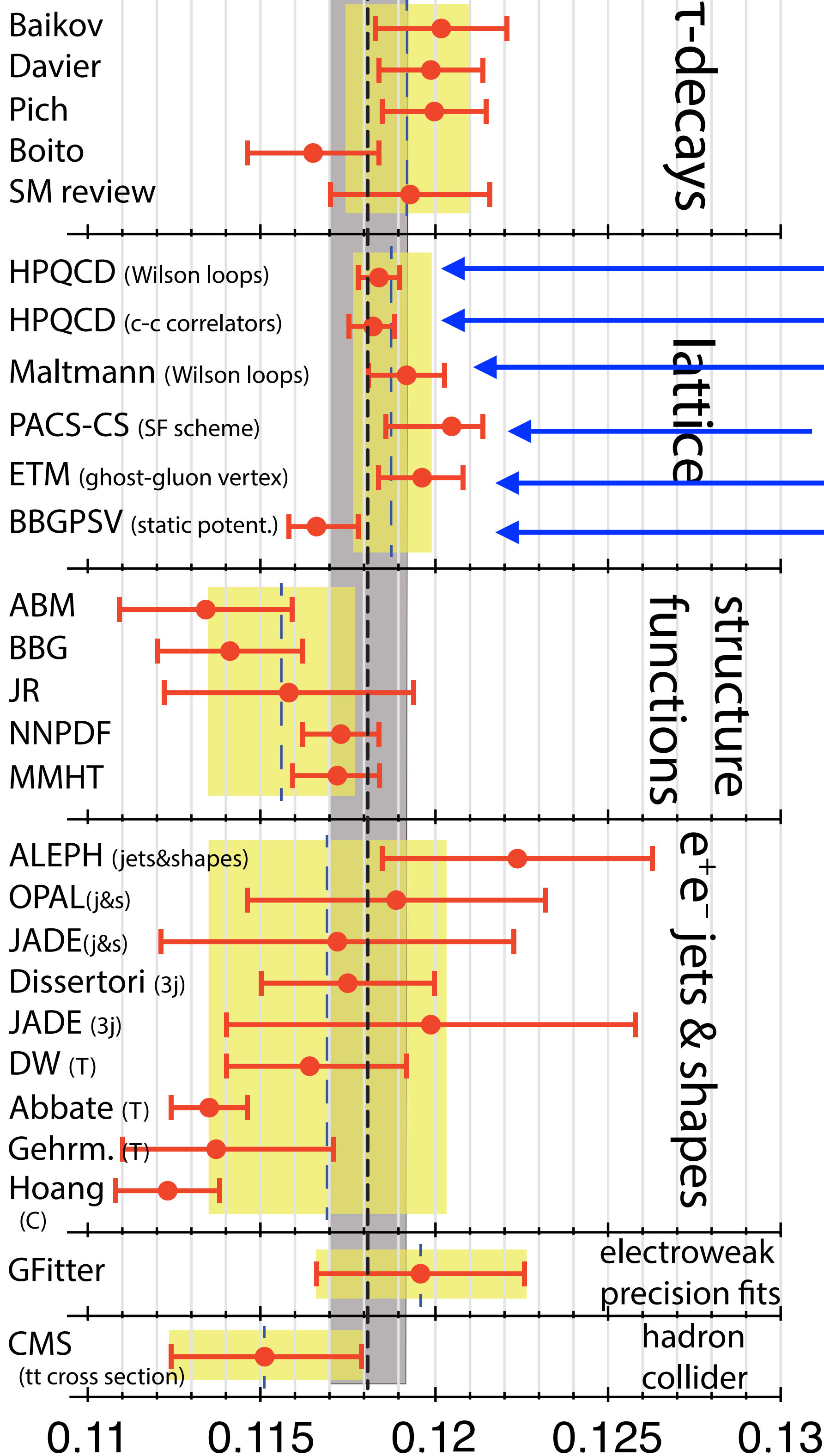
$$Q = a_0 + a_1\alpha_s(\mu) + a_2\alpha_s(\mu)^2 + \ldots$$

Calculate in lattice QCD. Could be e.g. continuum limit of 4th moment of JJ correlator. Minimal exptl uncty

Choice of μ depends on Q e.g. $\sim 3m_h$ in JJ . Fixing it requires determination of a using e.g. a hadron mass.

Perturbative expansion - using continuum QCD pert. th. in $\overline{MS}$ if Q is cont. quantity. Needs to be high-order.

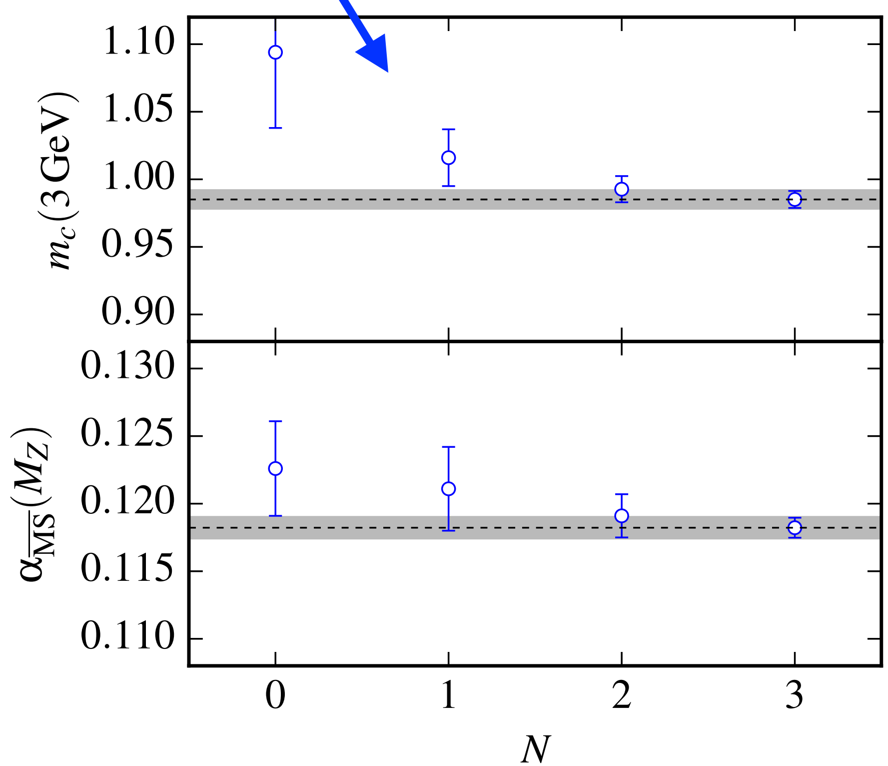
$\alpha_s(\overline{MS}, n_f = 5, M_Z)$, PDG $= 0.1181(11)$



small Wilson loops
JJ
small Wilson loops
Schro. functional
ghost-gluon vertex
static quark potential

Lattice results most precise; several different methods

Effect of inc. pert. theory order in JJ method



Conclusions

Lattice QCD results available from multiple quark formalisms and methods now. - good consistency

$m_c(m_c)$ is determined to 1% and
$m_b(m_b)$ to 0.5% from continuum and lattice methods.

$\alpha_s(M_Z)$ to 1% from lattice - multiple methods

1% accurate m_c/m_s ratio allows 1% in m_s also, along with RI-SMOM methods

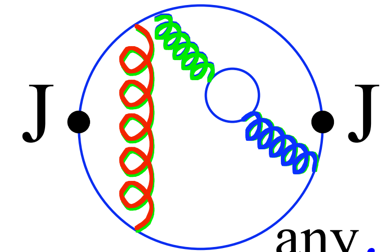
Tests of perturbation theory from completely non-perturbative mass ratios and JJ/RI-SMOM comparison

Future improvements from higher order pert th. (?possible) and finer lattices to push up μ values.

Backup slides

Current-current correlator method for lattice m_c

HPQCD + Chetyrkin et al, 0805.2999, C. Mcneile et al, HPQCD,1004.4285



any J^{PC} current now

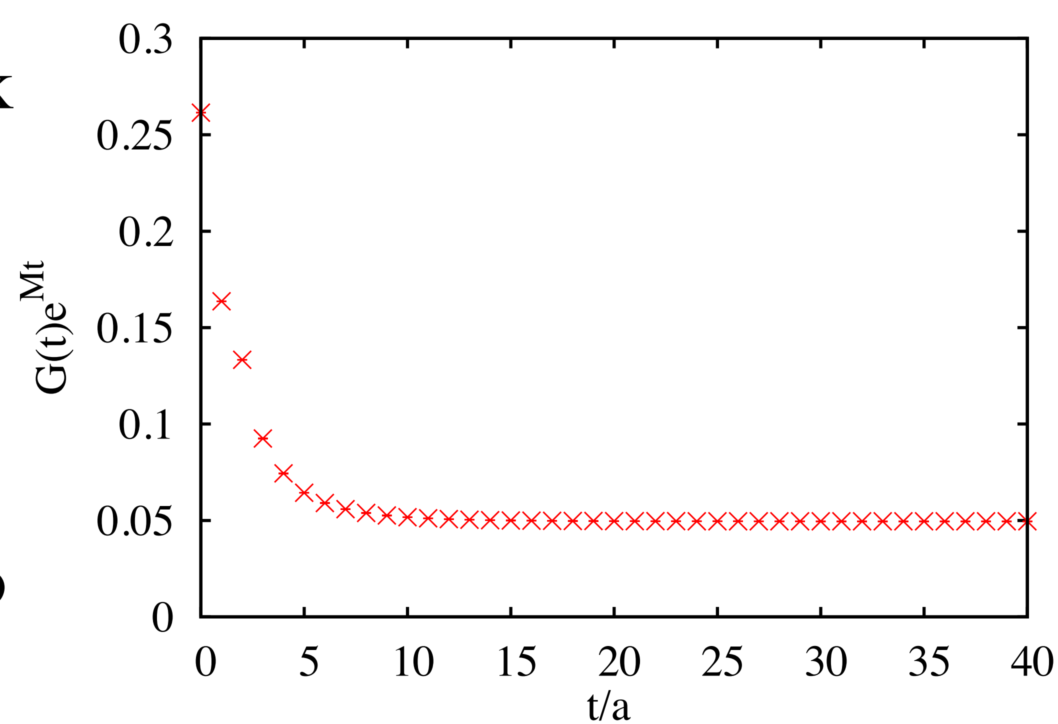
- Substitute time-moment of lattice charmonium correlator for experiment. In principle can use any current J now.
- For HISQ quarks pseudoscalar η_c correlator is most accurate. J is absolutely normalised.

step 1: calculate η_c correlators by combining lattice charm quark propagators

step 2: large time - fit to exponential, gives η_c mass

step 3: tune lattice quark mass so η_c mass correct.

step 4: calculate time moments to compare to QCD pert. theory. Emphasises short-time contribns.



Further check of JJ method:
compare vector moments (after normalising current) to those extracted from

$R_{e^+e^-}$

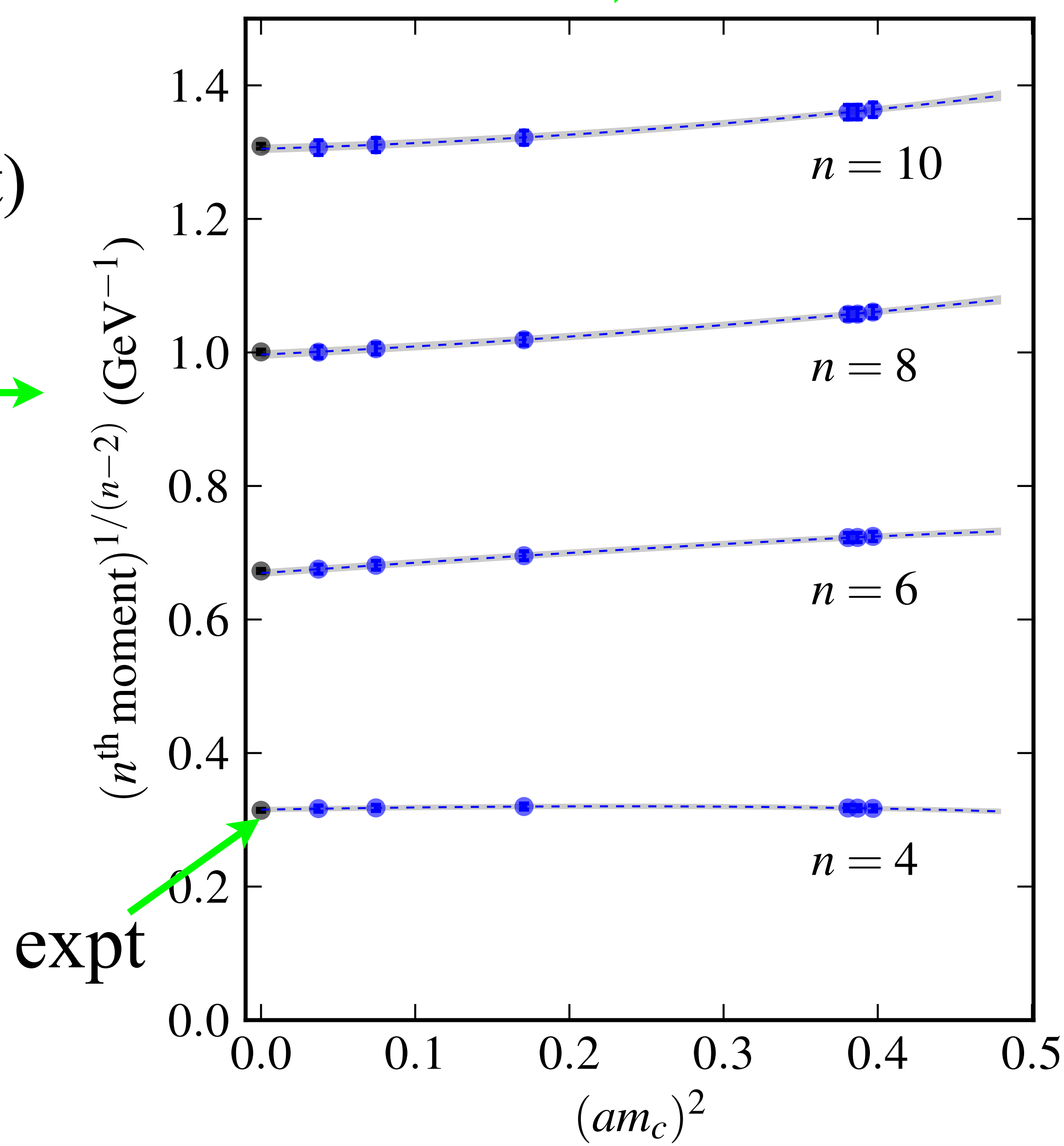
Agreement is a 1% test of (lattice) QCD.

Also gives charm quark contribution to anomalous magnetic moment of the muon

$a_\mu^c = 14.4(4) \times 10^{-10}$

HPQCD, 1208.2855, 1403.1778

lattice and expt errors similar size

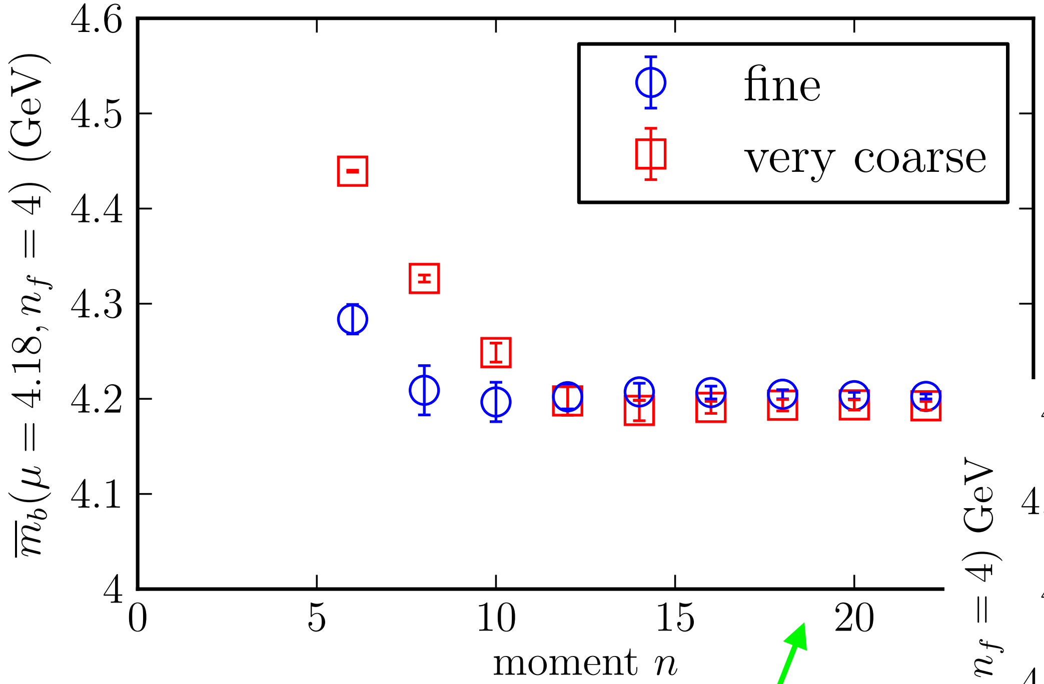


see also ETMC, 1111.5252

Alternative determinations of m_b

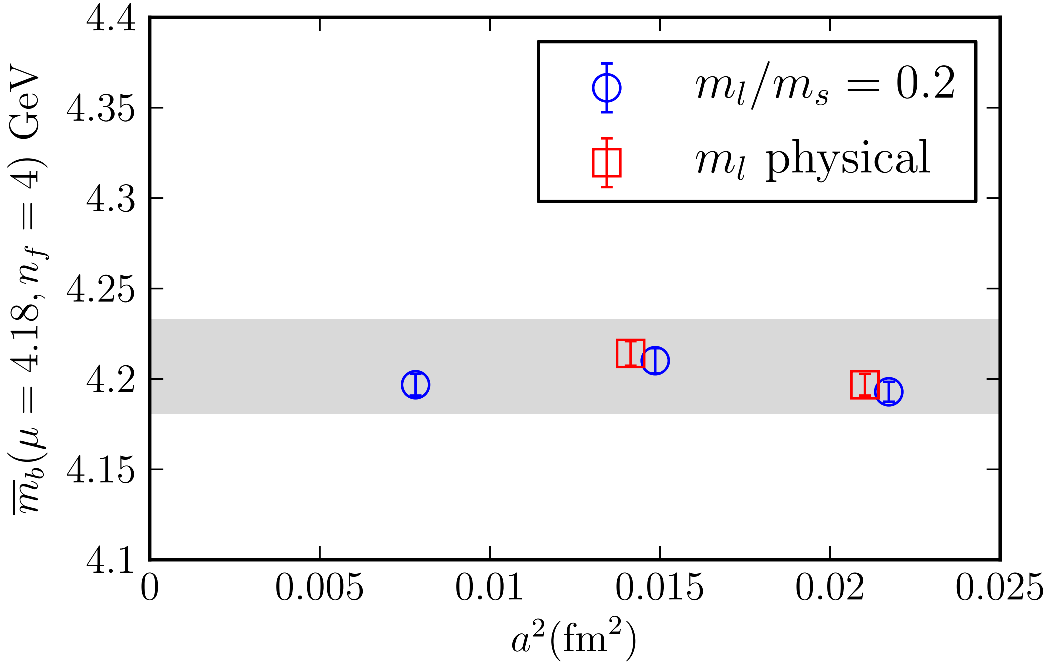
HPQCD, 1408.4768

Current-current correlator method using vector bottomonium correlators calculated with improved NRQCD b quarks



n_f = 2+1+1 HISQ sea quarks

$\overline{m}_b(\overline{m}_b, n_f = 5) = 4.196(25)\text{GeV}$



Larger moment numbers more nonrelativistic - use

Update and improved method

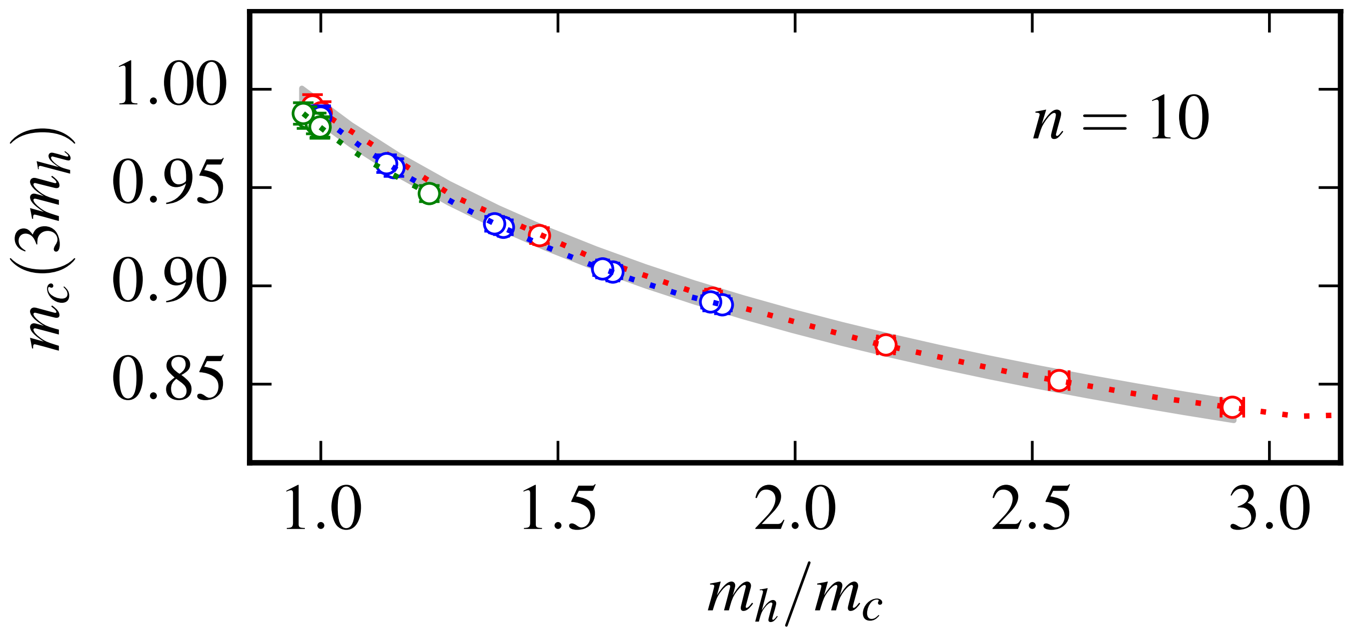
HPQCD, 1408.4169

Use improved $n_f = 2+1+1$ gluon field configs, more accurate lattice spacing determination etc etc.

Determine m_c at higher scales by using multiple m_h

$$\tilde{R}_n = \frac{1}{m_c}\left(G_n/G_n^{(0)}\right)^{1/(n-4)} \to \frac{1}{m_c(\mu)}\frac{C_k^P}{C_k^{P.0}}$$

tuned m_c — G_n at m_h — $\mu = 3m_h$



$$\frac{m_{0h}}{m_{0c}} = \frac{m_h(\mu)}{m_c(\mu)}$$

$$m_c(m_c) = 1.2715(95)\text{GeV}$$