Initial conditions for hydrodynamics from pre-equilibrium evolution

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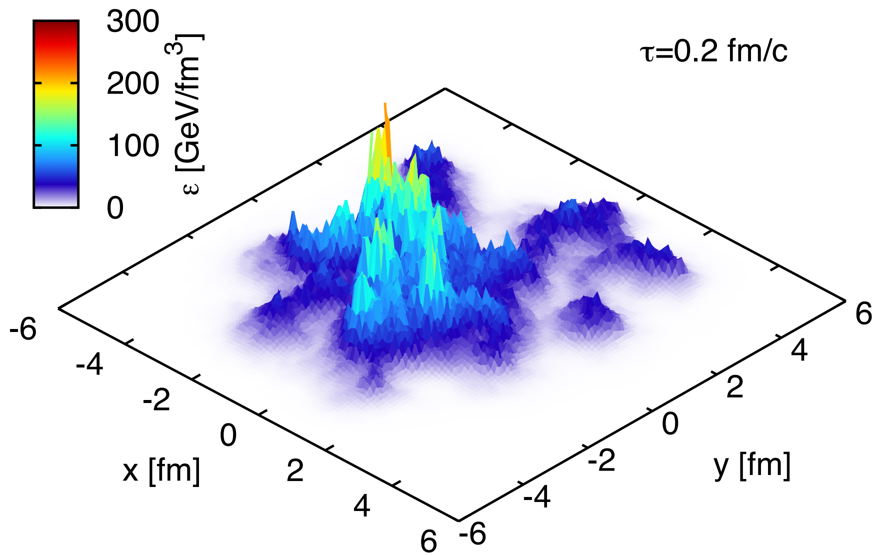
- Liam Keegan, Aleski Kurkela, Aleksas Mazeliauskas, DT, JHEP arXiv:1605.04287
- A. Kurkela, A. Mazeliauskas, JF Paquet, S. Schlichting, DT, in progress

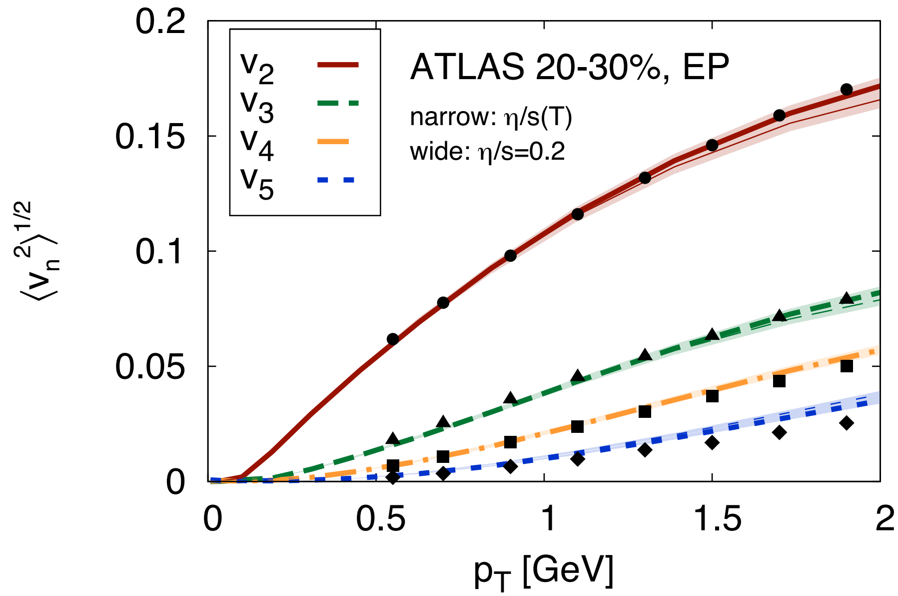
Aleksas Mazeliauskas, PhD 2017 !

Initial conditions and hydrodynamics at high energies: (Tribedy,Schenke,Venugopalan)

CGC Initial Conditions

Hydro Result

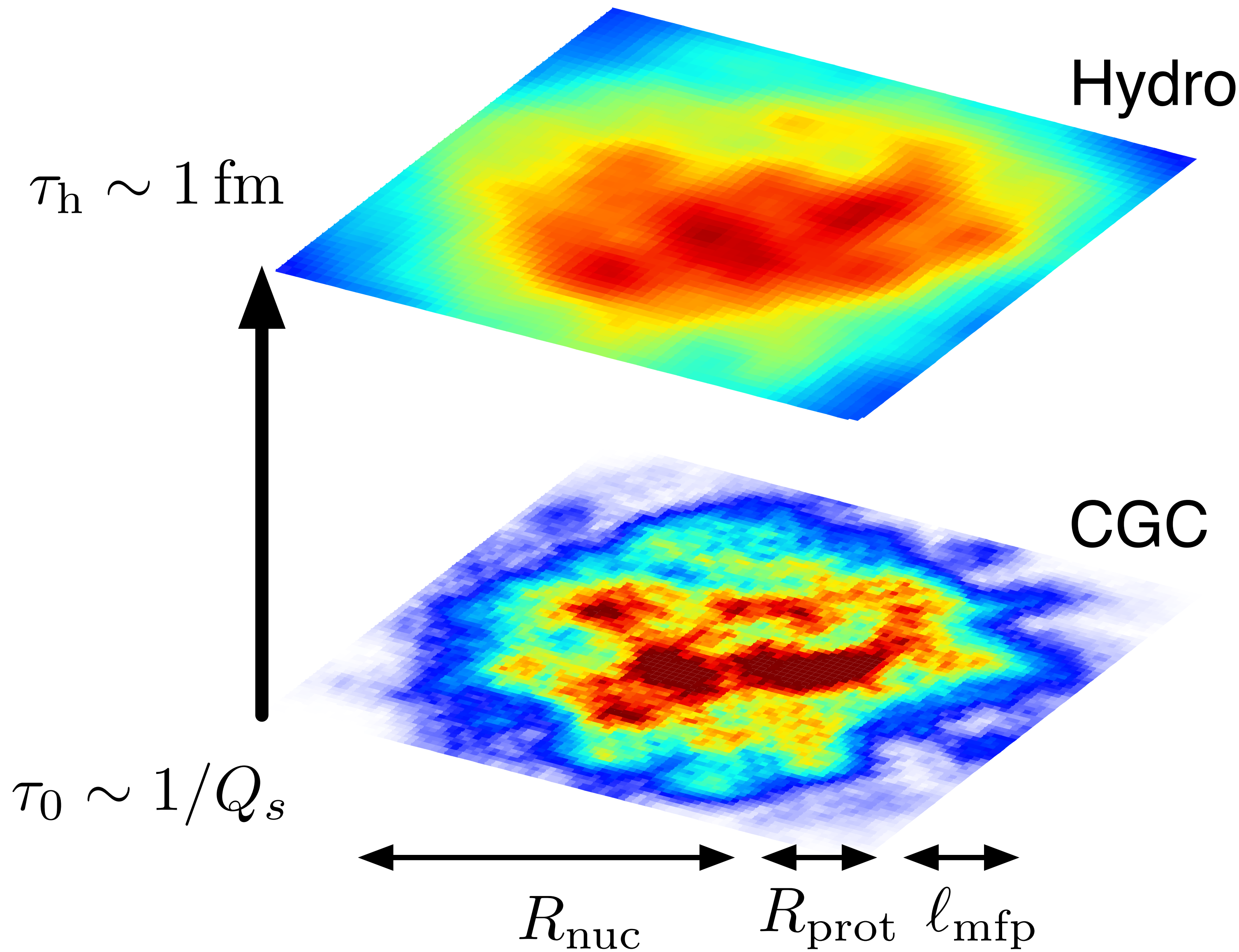




1. Large gradients
2. Gradients comparable to the mean free path

We will consistently map the IP-Glasma (CGC) initial conditions to hydrodynamics using QCD kinetic theory

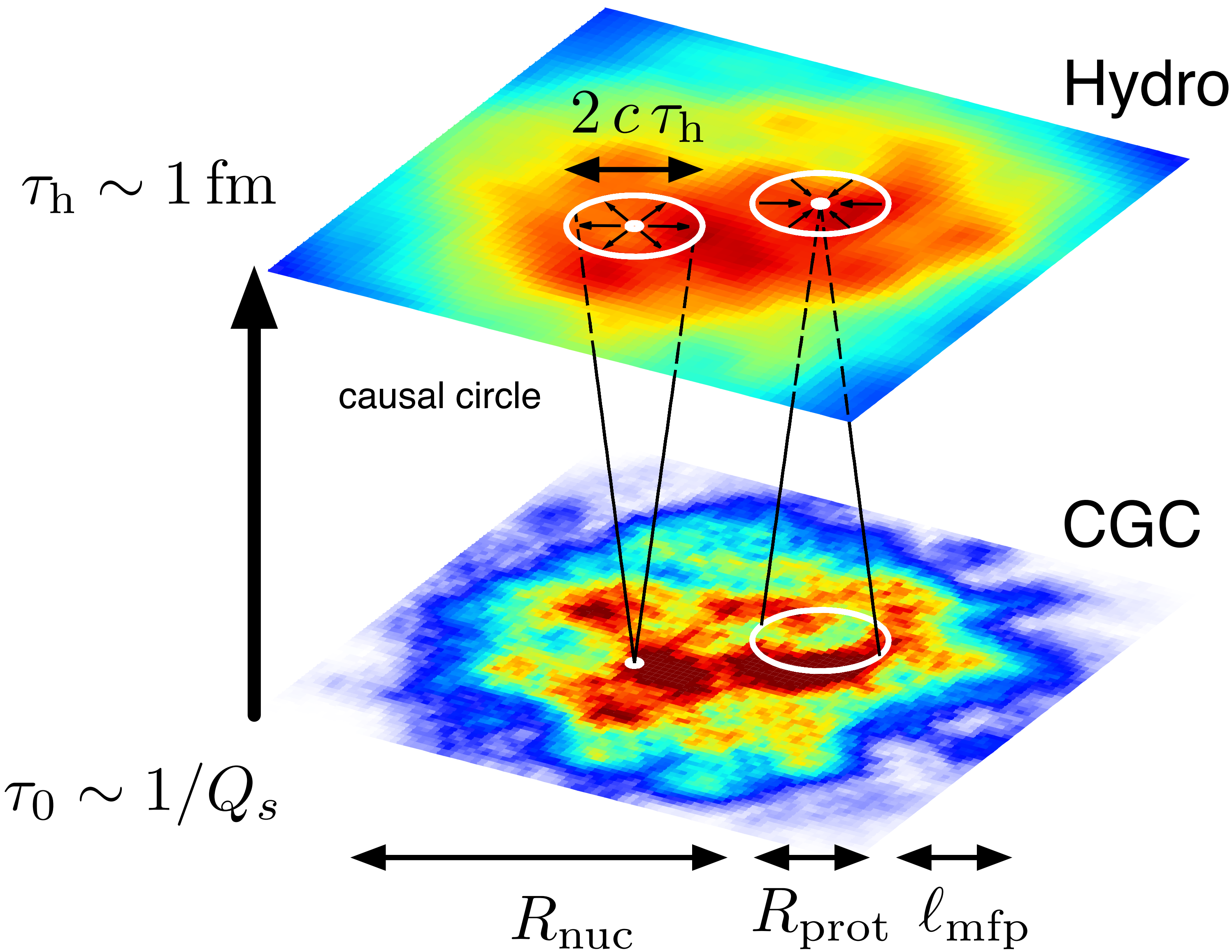
Mapping the CGC fluctuating initial conditions to hydro



Use QCD kinetic theory to map the CGC initial state to hydrodynamics with approximations:

$$R_{\mathrm{nuc}} \gg R_{\mathrm{prot}} \sim \ell_{\mathrm{mfp}}$$

Mapping the CGC fluctuating initial conditions to hydro

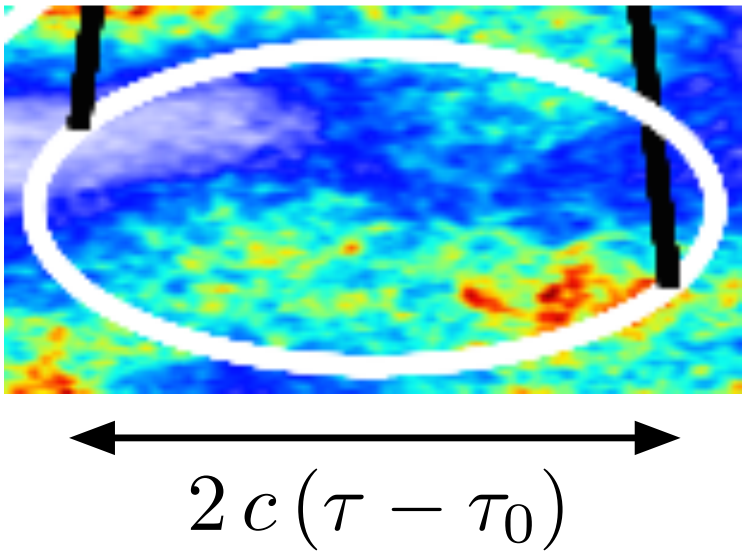


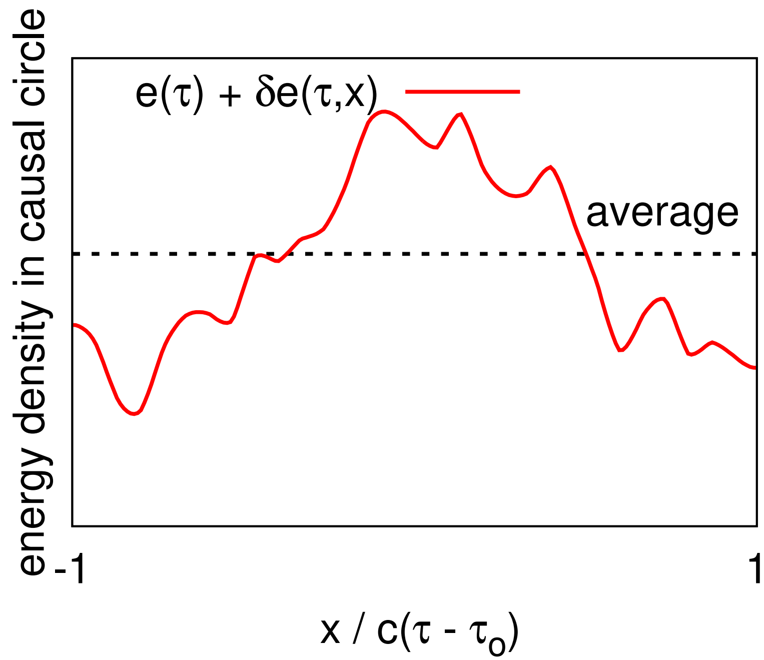
Causality limits the equilibration dynamics within a causal circle

$$R_{\mathrm{nuc}} \gg R_{\mathrm{prot}} \sim \ell_{\mathrm{mfp}} \sim c\tau_{\mathrm{h}}$$

An approximation scheme for the equilibration dynamics:

look in causal circle





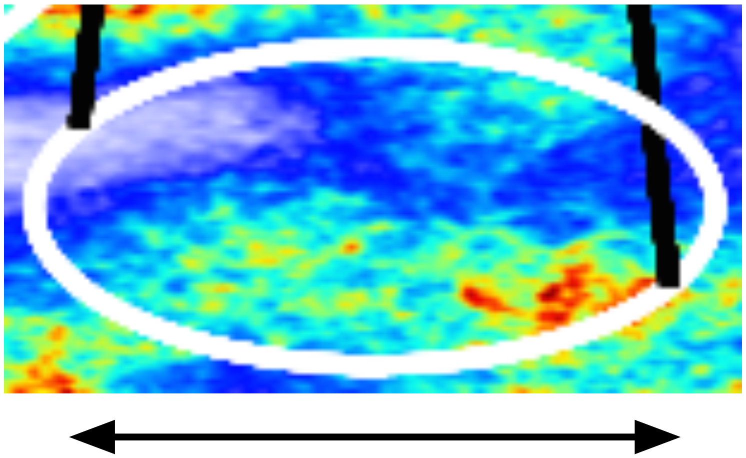
1. Determine the evolution of the average (homogeneous) background
2. Construct a Green function to propagate the linearized fluctuations.

$$\underbrace{\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)}}_{\text{final energy perturb}} = \int d^2\boldsymbol{x}' E(|\boldsymbol{x} - \boldsymbol{x}'|; \tau, \tau_o) \underbrace{\frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}}_{\text{initial energy perturb}}$$

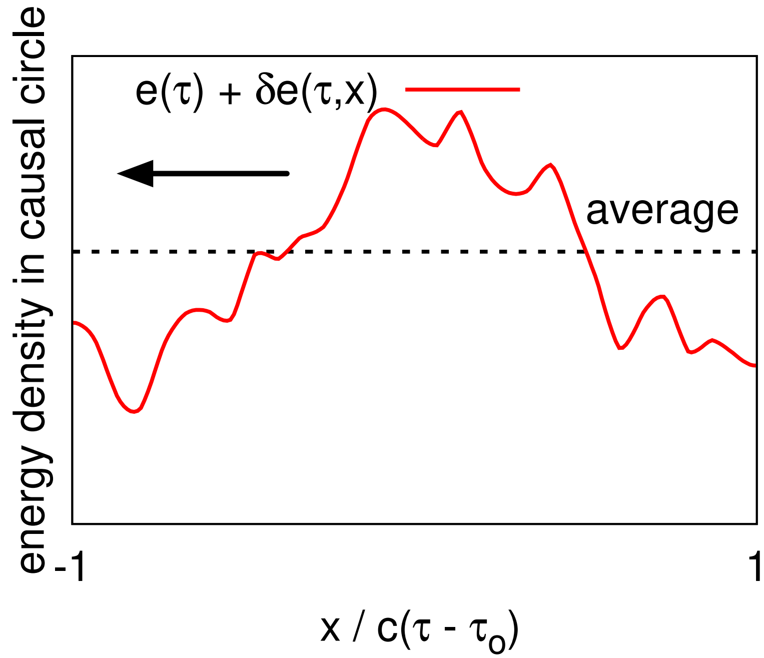
Determines the energy density $e(\tau) + \delta e(\tau, \boldsymbol{x})$ for hydrodynamics

An approximation scheme for the equilibration dynamics:

look in causal circle



$$2\,c\,(\tau - \tau_0)$$



1. Determine the evolution of the average (homogeneous) background

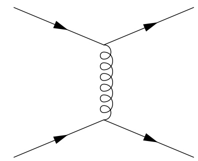
2. Construct a green functions to propagate the linearized fluctuations

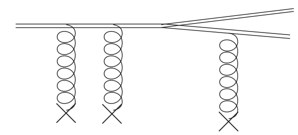
$$\underbrace{\frac{\delta g^i(\tau,\boldsymbol{x})}{e(\tau)}}_{\text{final momentum perturb}} = \int d^2\boldsymbol{x}' n^i G(|\boldsymbol{x}-\boldsymbol{x}'|;\tau,\tau_o) \underbrace{\frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}}_{\text{initial energy perturb}}$$

Determines the energy and momentum ($g^i \equiv T^{0i}$) densities for hydrodynamics

Pre-equilibrium evolution of transverse perturbations:

$$\partial_\tau f + \frac{\mathbf{p}}{|p|}\cdot\nabla f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{Bjorken expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]} \;,$$





Gluon distribution function for background and perturbations

$$f = \underbrace{\bar{f}_{\mathbf{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp,\mathbf{p}}\, e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}}_{\text{transverse perturbations}} \;.$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\mathbf{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

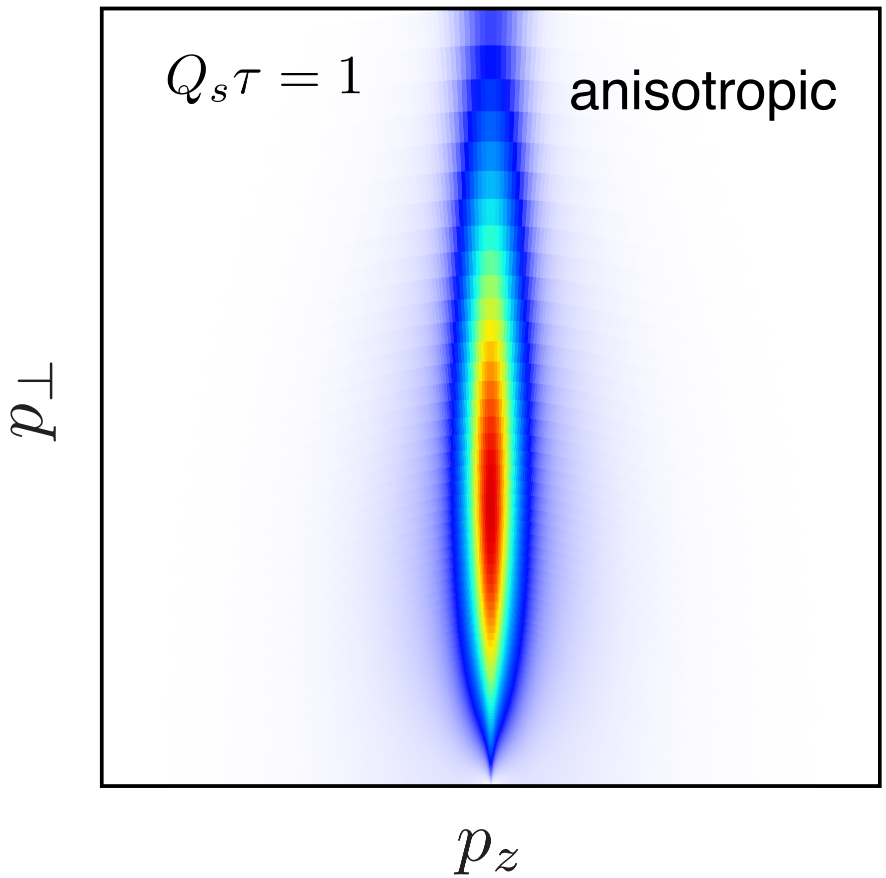
$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\mathbf{p}_\perp\cdot\boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp,\mathbf{p}} = -\delta\mathcal{C}[\bar{f},\delta f] \qquad \text{perturbation}$$

Compute energy $\delta e \equiv \delta T^{00}$ and momentum $g^x \equiv T^{0x}$ perturbations versus time.

Evolution of the background

- Follows the setup of bottom-up thermalization — Baier, Mueller, Schiff, Son (2001); Berges, Boguslavski, Schlichting, Venugopalan (2014)
- Builds upon the first numerical realization — Kurkela, Zhu (2015)

$p^2 f(p_\perp, p_z)$



Initialization:

1. Partons are initialized with:

$$\langle p_\perp^2 \rangle \sim Q_s^2 \qquad \langle p_z^2 \rangle \simeq 0$$

2. Take a coupling constant of $\alpha_s = 0.3$

$$\underbrace{\lambda \equiv 4\pi\alpha_s N_c = 10}_{\text{theorists variable}}$$

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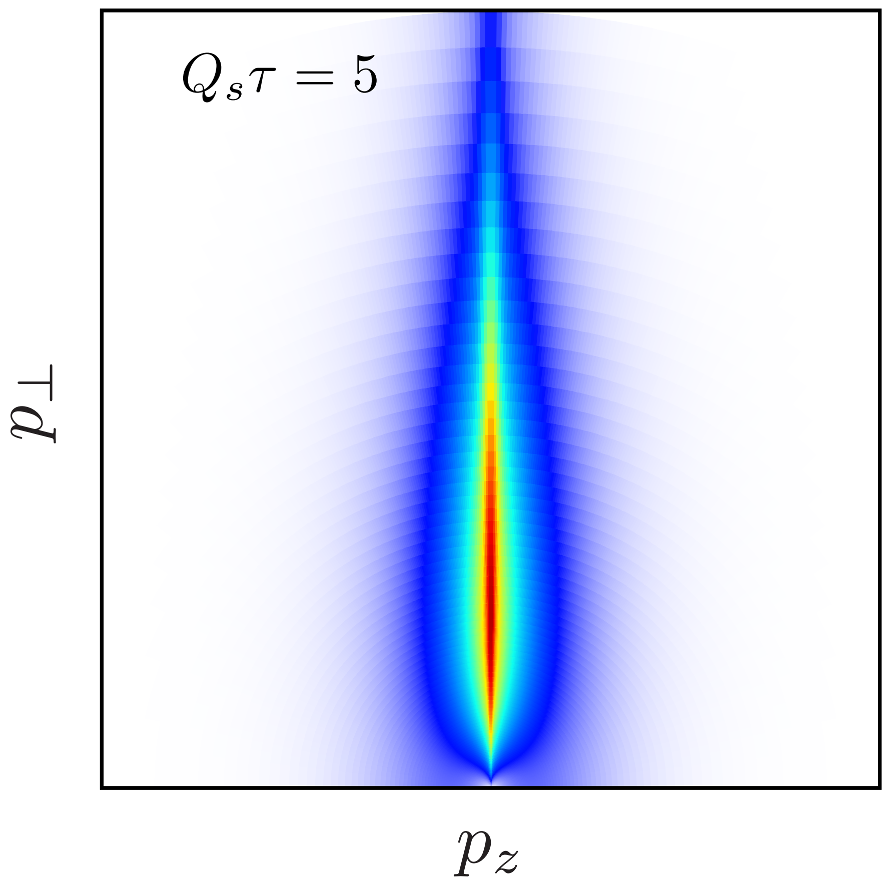

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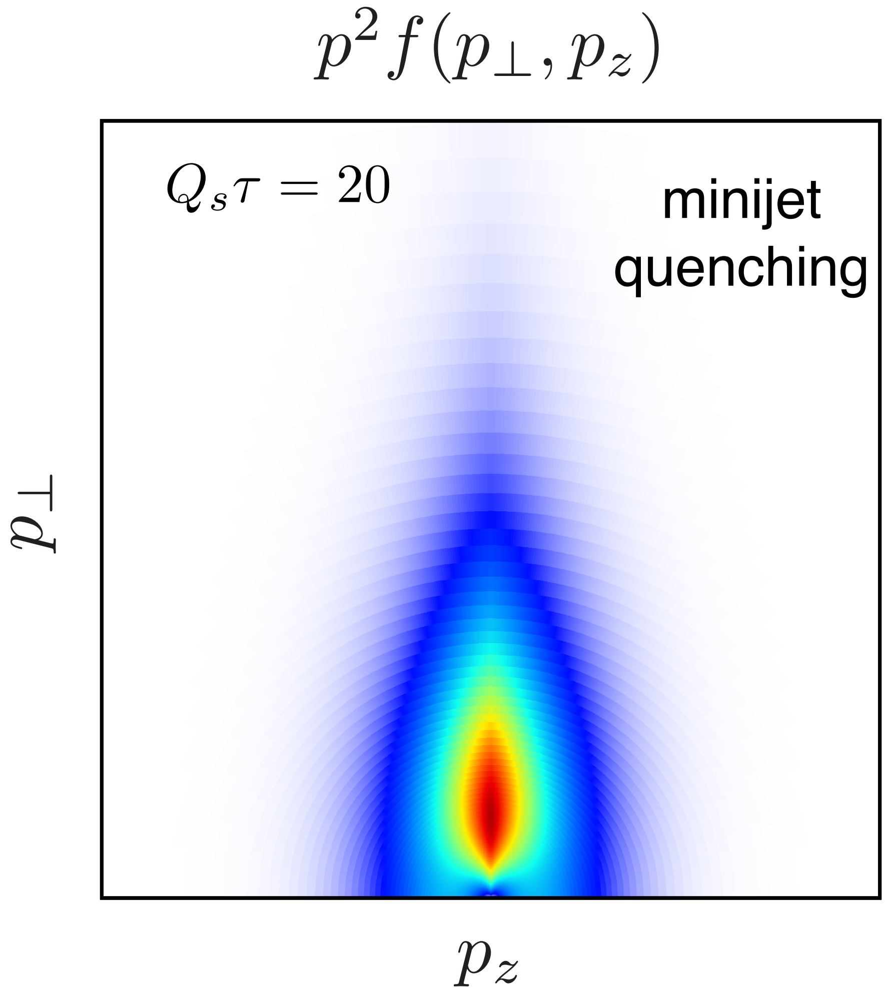

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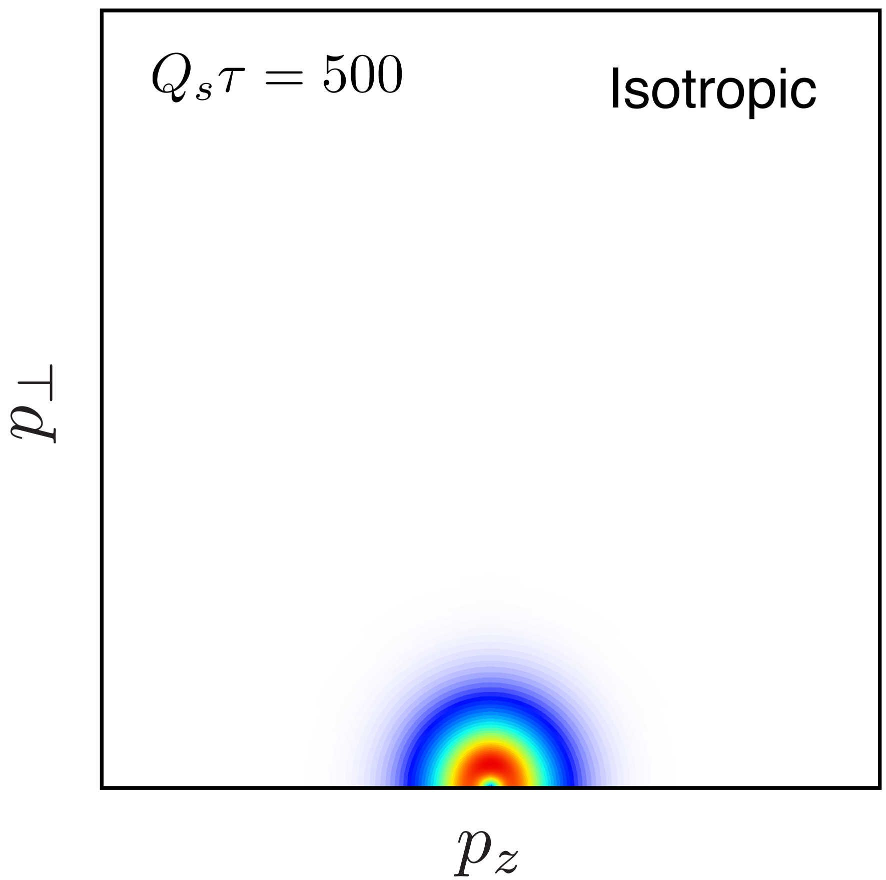

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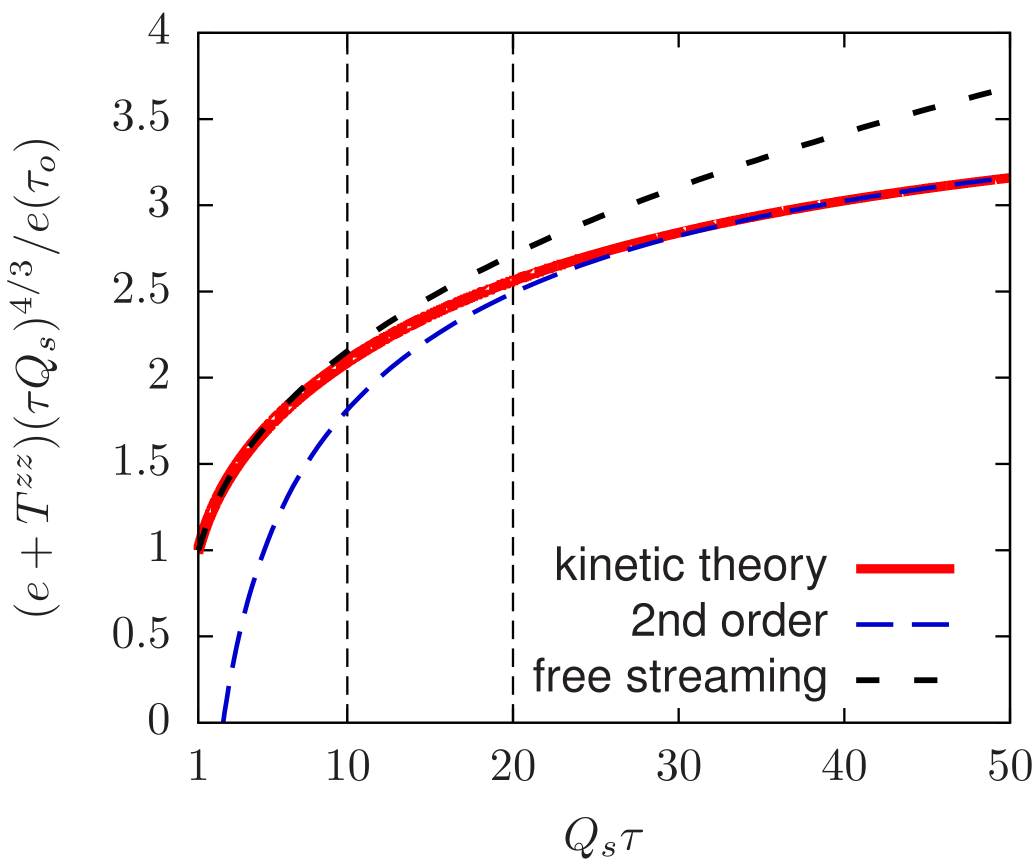
Approach to 2nd order hydro:

- We will check the constitutive relation:

$c_s^2 = \frac{1}{3}$, $\eta/s = 0.63$ for $\lambda = 10$

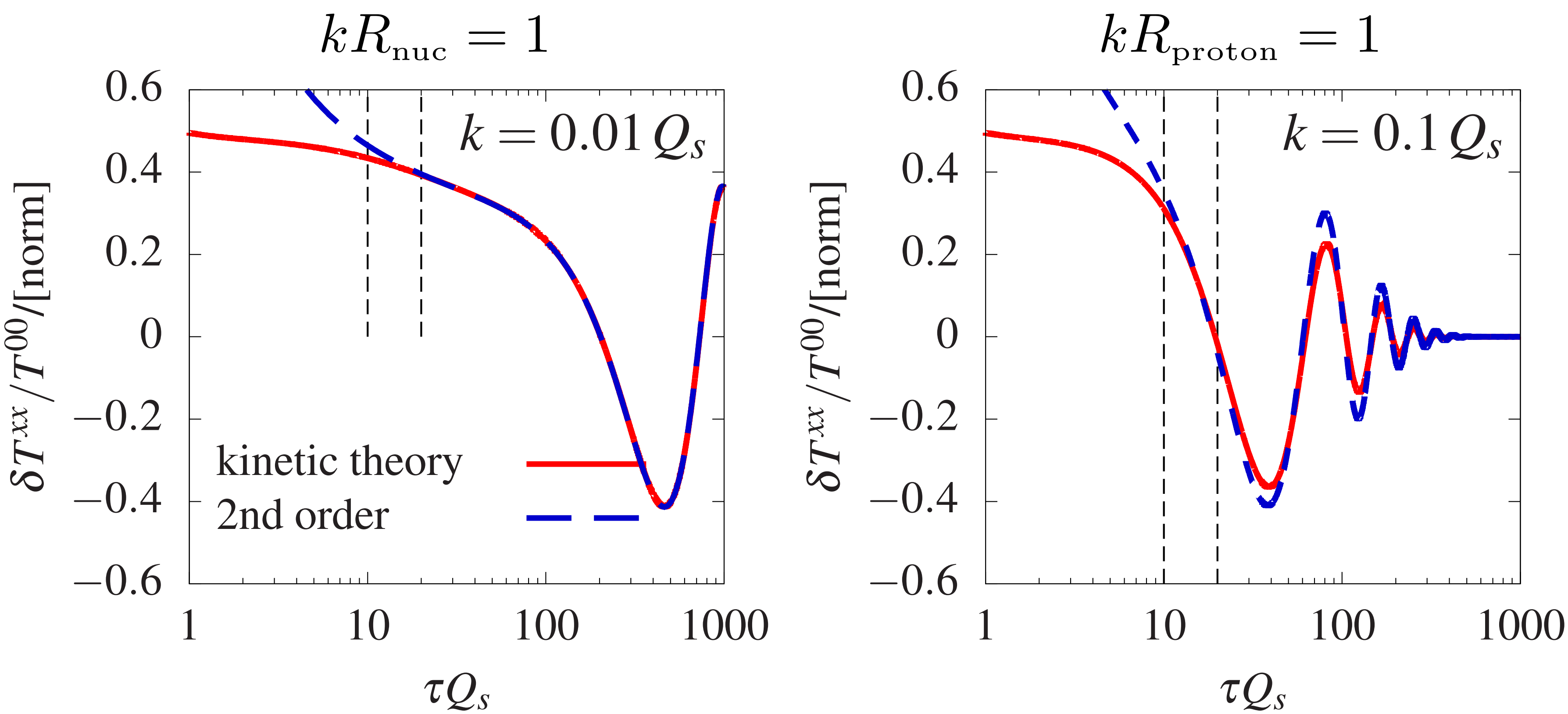
$$T^{zz}(e) = \underbrace{\frac{1}{3}e - \frac{4}{3}\frac{\eta}{\tau} - \frac{8}{9}\frac{\eta\tau_\pi - \lambda_1}{\tau^2}}_{\text{hydro prediction}}$$



See approach 2nd order hydro by $Q_s\tau \simeq 20$ for $\lambda = 10$

Do the perturbations obey 2nd order hydro ?

$$\delta T^{xx}(k,e) = \underbrace{\frac{\delta e}{e}\left[c_s^2 + \ldots\right] + \frac{g^x}{e}\left[-ik\eta + \ldots\right]}_{\text{Hydro prediction (2nd order constitutive relation)}}$$

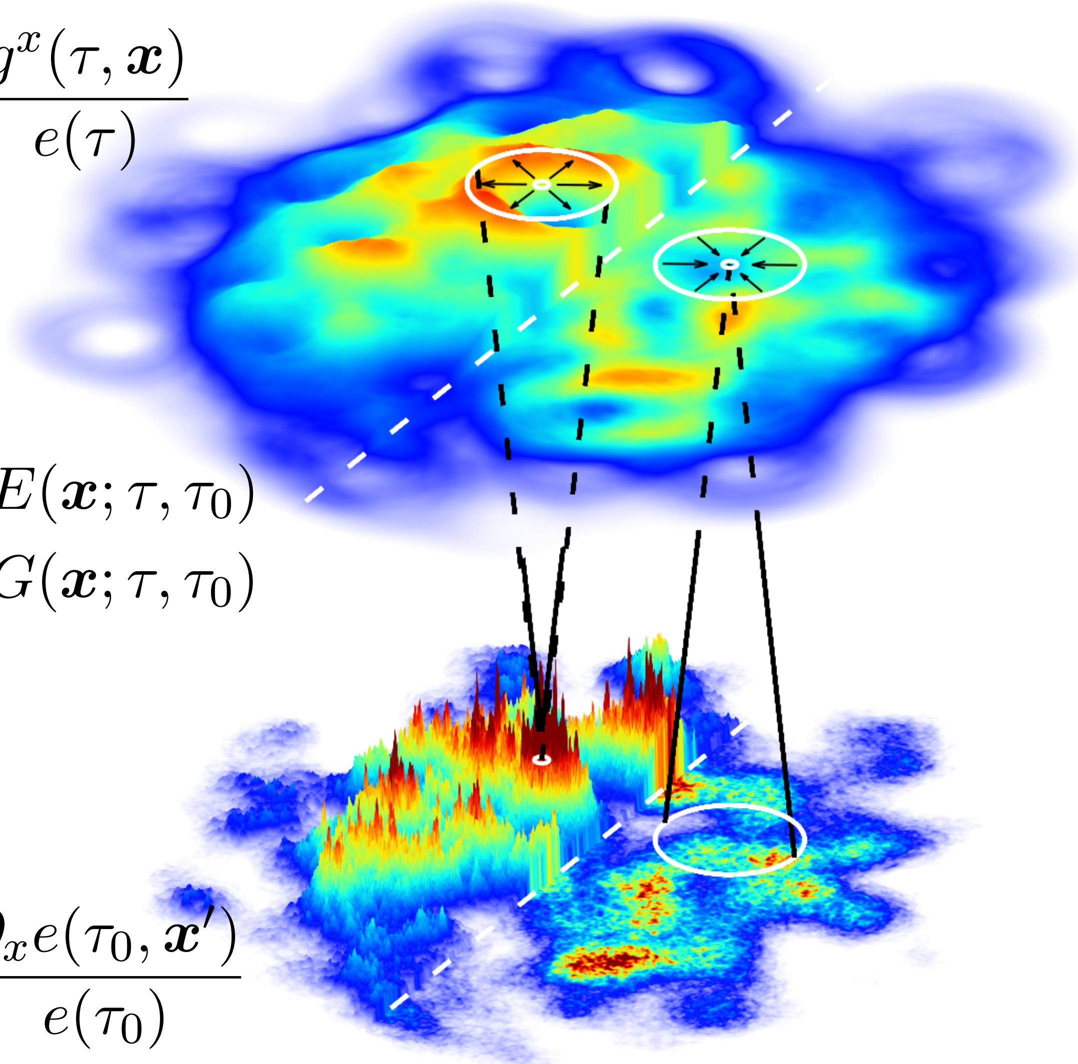


The perturbations are still approximately hydro-like for $kR_{\rm proton} \gtrsim 1$

Constructing the Green functions:

$$\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)}, \frac{g^x(\tau, \boldsymbol{x})}{e(\tau)}$$

Green functions $E(\boldsymbol{x}; \tau, \tau_0)$ $G(\boldsymbol{x}; \tau, \tau_0)$



$$\frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}, \frac{\partial_x e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}$$

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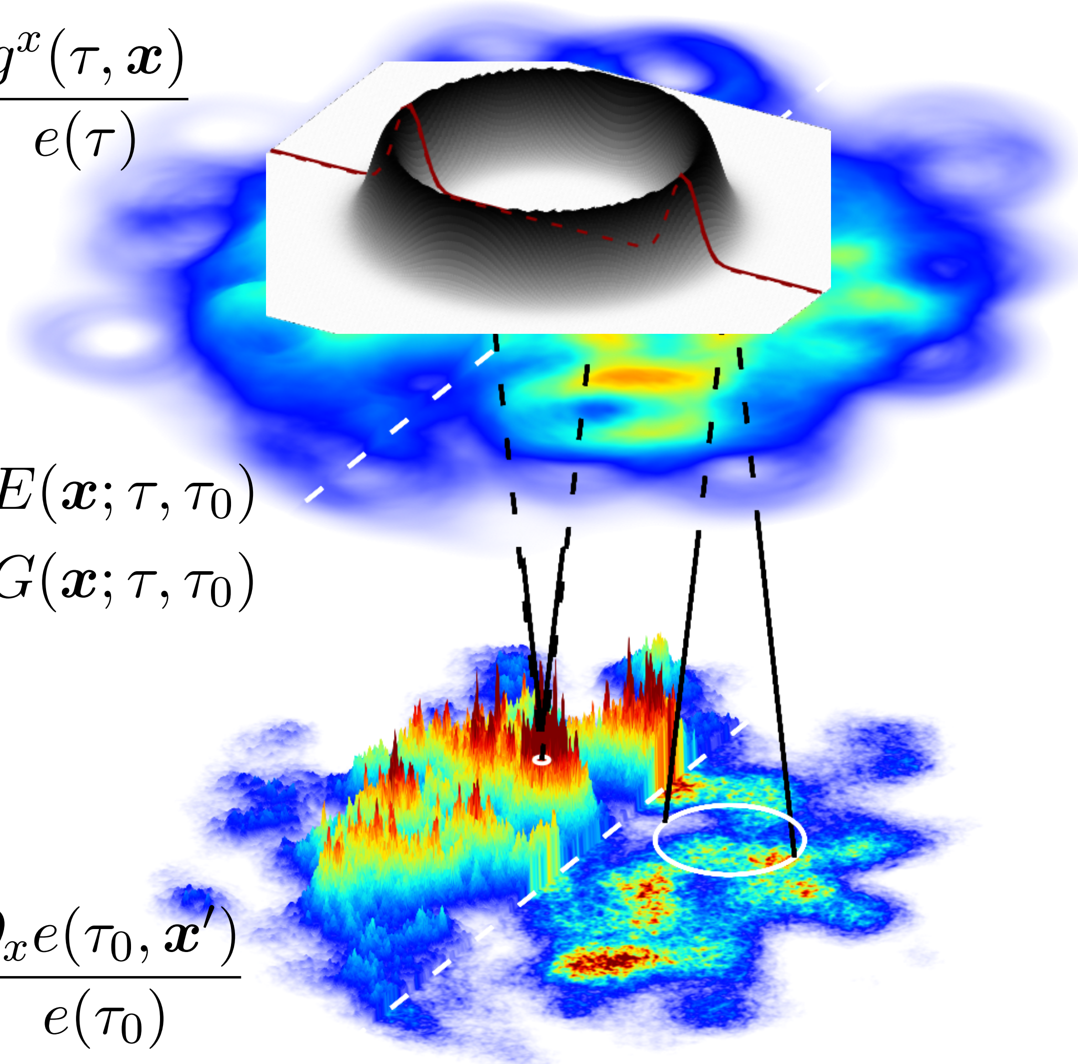
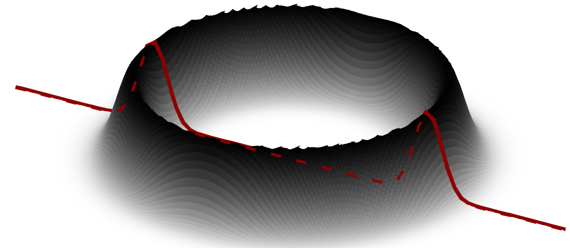
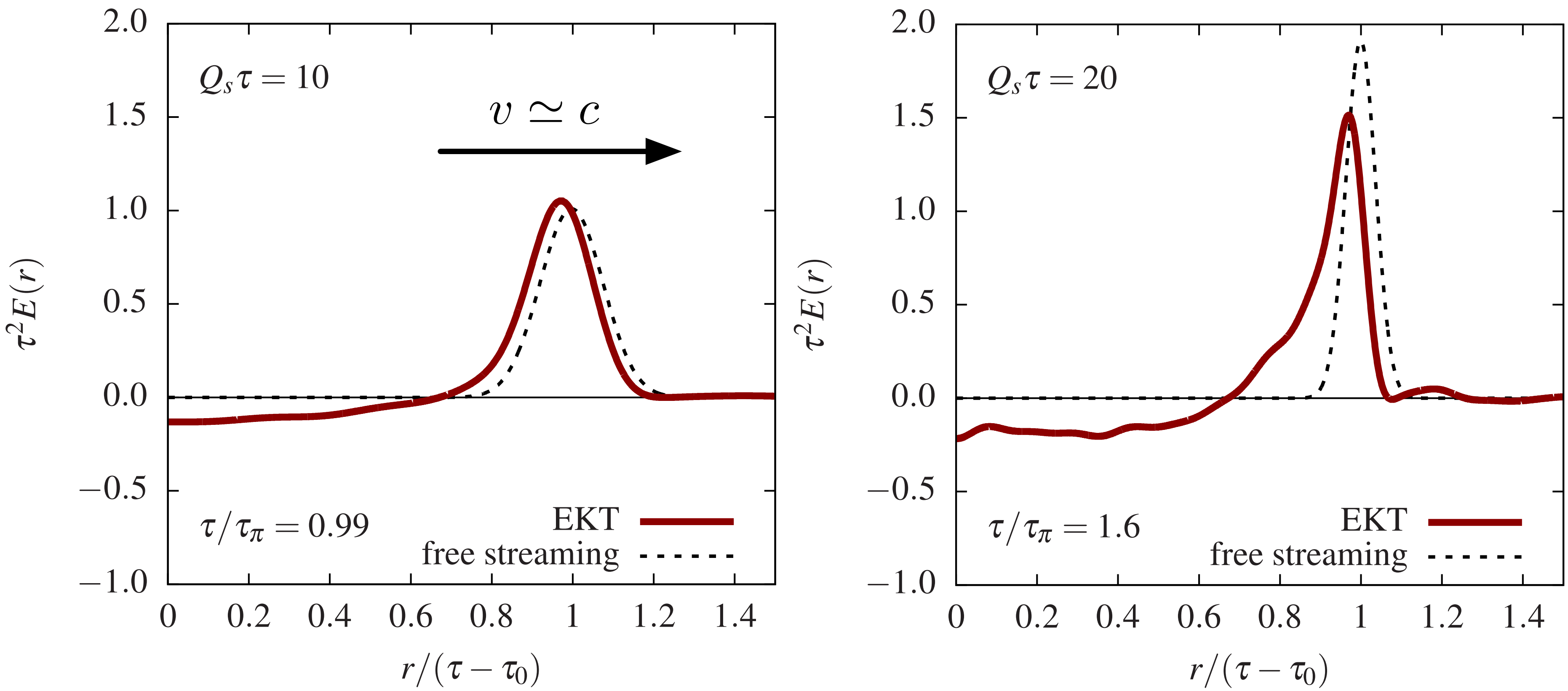


Green functions in coordinate space

$$\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)} = \int d^2\boldsymbol{x}' \underbrace{E(|\boldsymbol{x} - \boldsymbol{x}'|; \tau, \tau_o)}_{\text{Green function}} \frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}$$



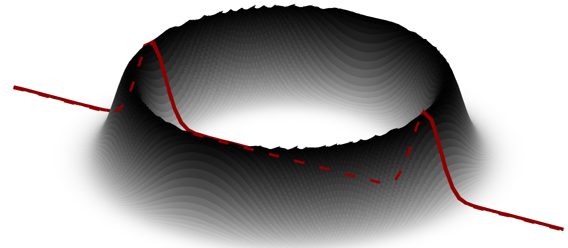


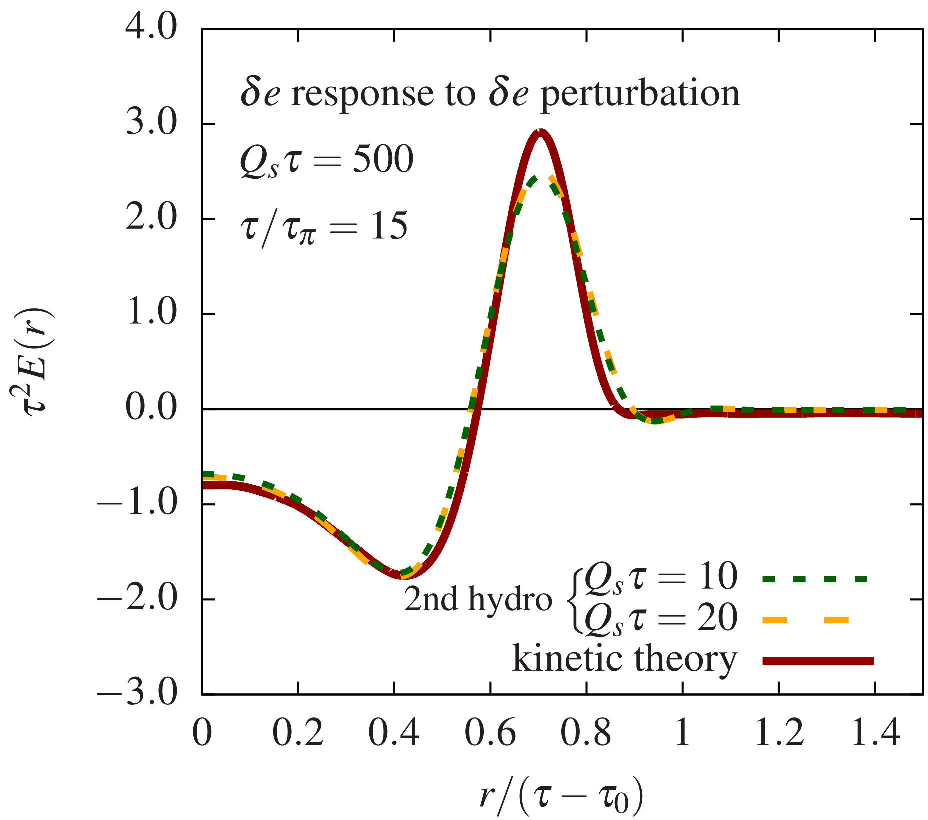
Start to see significant deviations from free streaming by $Q_s\tau = 20$

(when you should start using hydro)

Very late times the response approaches hydrodynamics

$$\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)} = \int d^2\boldsymbol{x}' \underbrace{E(|\boldsymbol{x} - \boldsymbol{x}'|; \tau, \tau_o)}_{\text{Green function}} \frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}$$





At late times the Green function is an outgoing sound pulse as expected.

Dependence on the shear viscosity (or coupling constant) and a scaling variable

1. Changing the coupling constant changes the relaxation time:

$$\tau_R \equiv \underbrace{\frac{\eta}{sT}}_{\text{kinetic estimate for relaxation time}}$$

2. Measure time in units of the relaxation time to compare couplings:

$$\underbrace{w(\tau)}_{\text{scaled time}} \propto \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_R(\tau')}$$

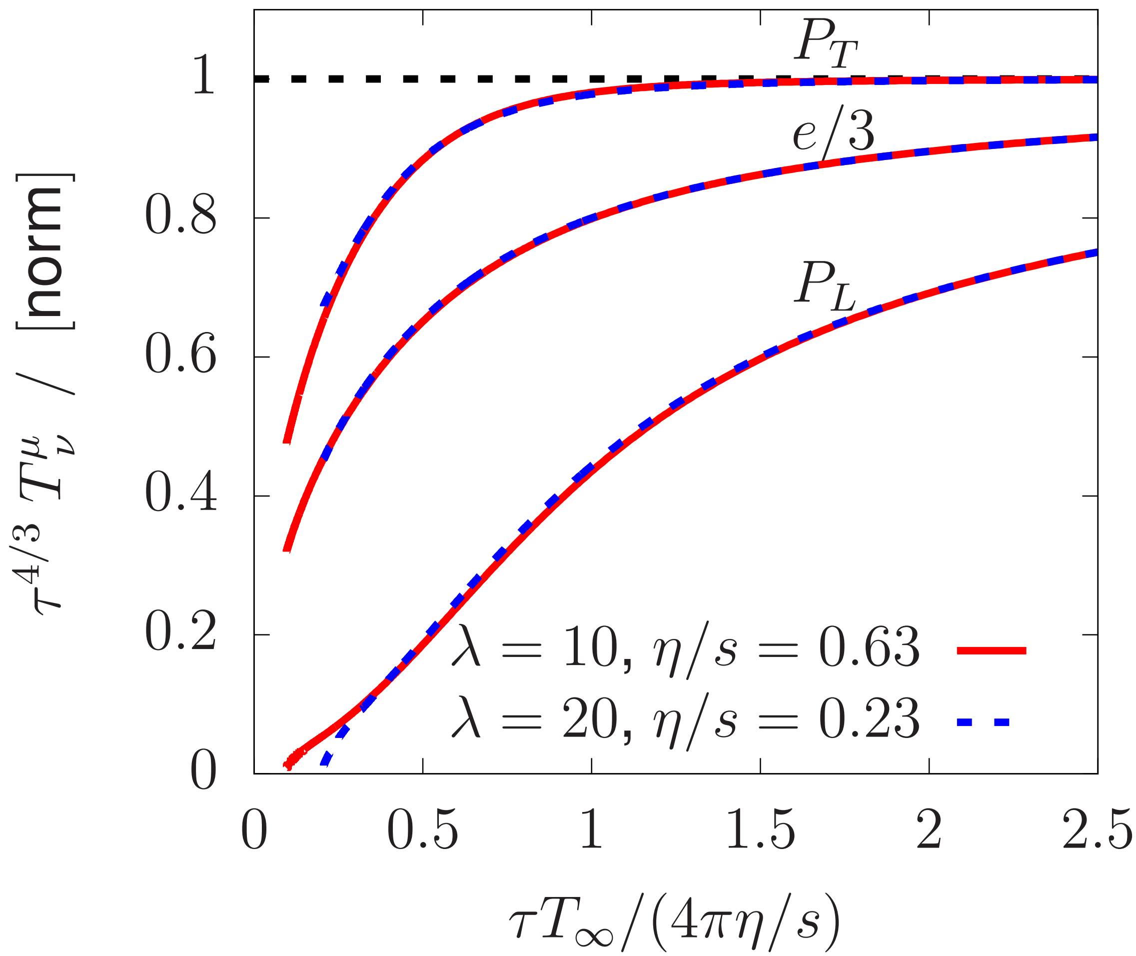
3. At late times the temperature is

$$T_\infty(\tau) = \frac{C}{\tau^{1/3}} \qquad \text{and} \qquad w \propto \frac{C\tau^{2/3}}{(\eta/s)} \propto \frac{\tau T_\infty(\tau)}{\eta/s}$$

4. And thus define the scaling variable:

$$w \equiv \frac{\tau T_\infty(\tau)}{4\pi(\eta/s)} \equiv \text{integrated number of relaxation times up to time } \tau$$

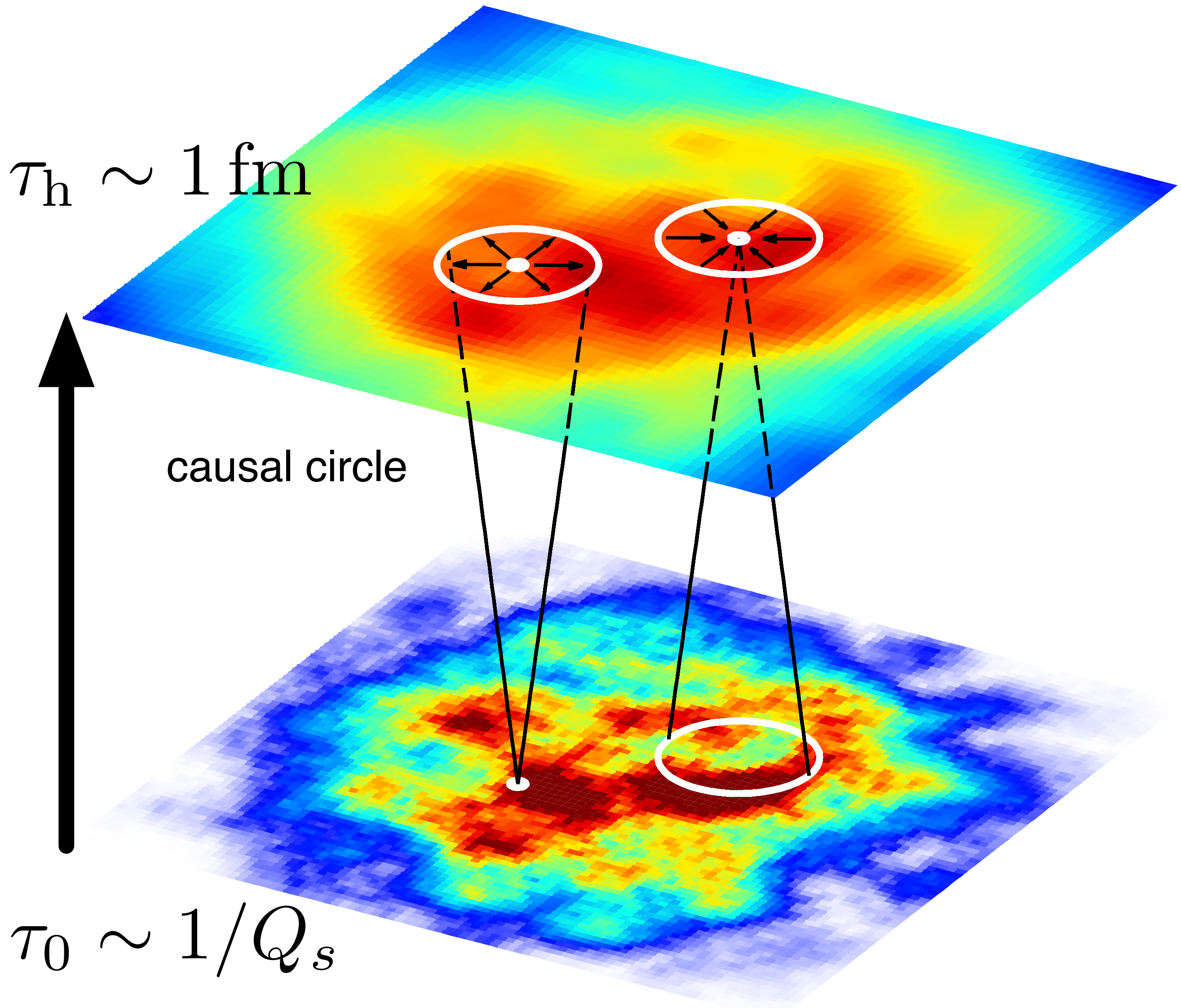
Scaling of background stress tensor (Heller,Kurkela,Romatschke)



The equilibration of the background and perturbations (not shown) lie on a universal curve.

All dependence on η/s is in the scaling variable w.

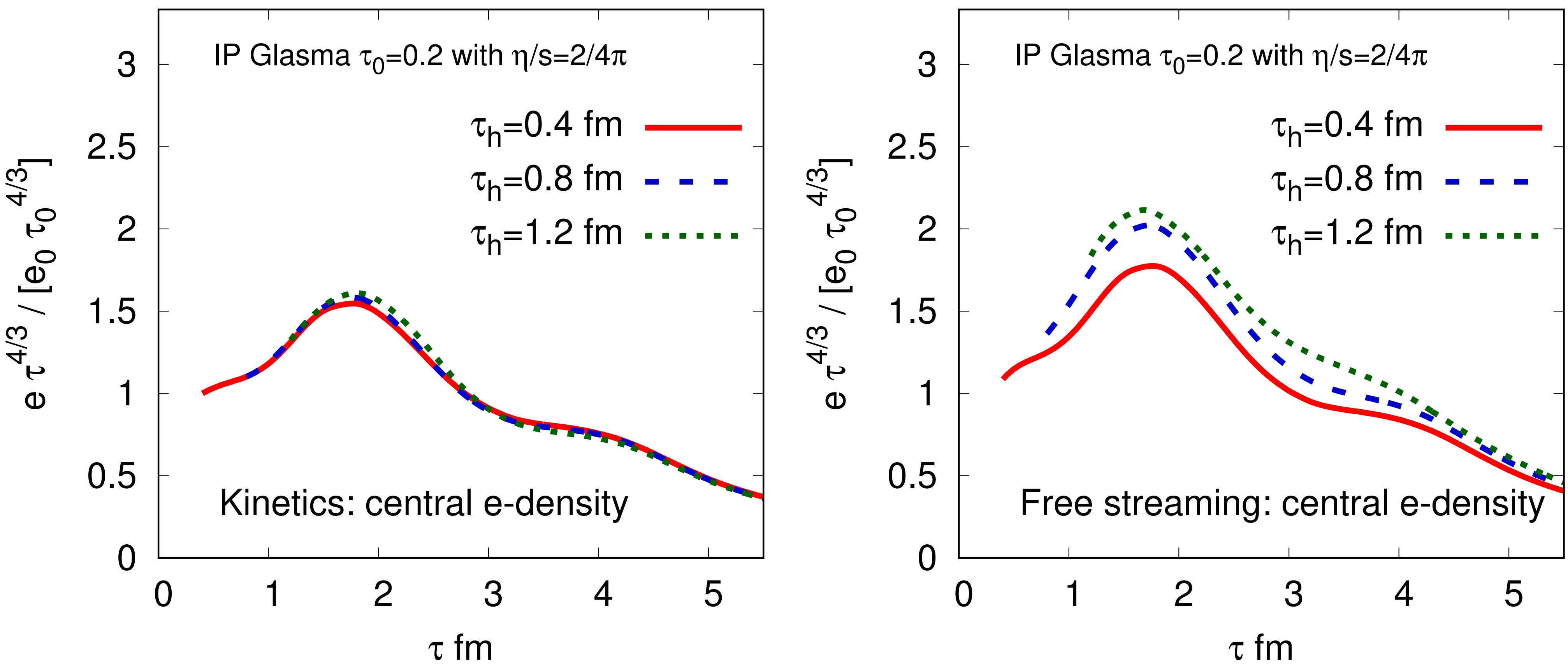
A practical algorithm



Find the number of relaxation times, w, between τ_0 and τ_h for given η/s

Putting it all together: the energy density at the center in a complete model

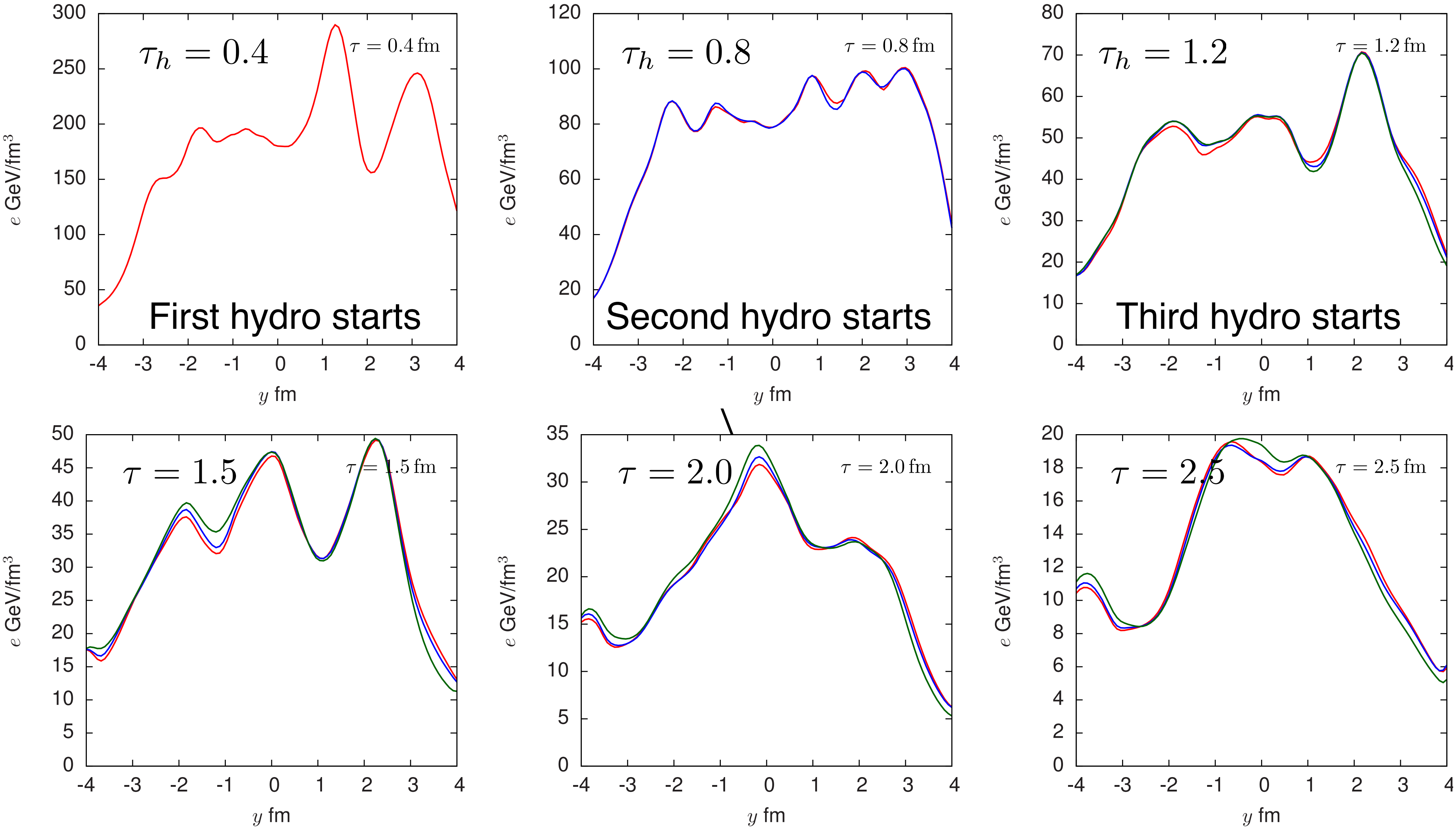
(i) IP Glasma (ii) Linearized Kinetic Theory (iii) Hydrodynamics



See a smooth transition to hydro independent of τ_{h}.

Many more plots to come – see A. Mazeliauskas at QM2017 next week!

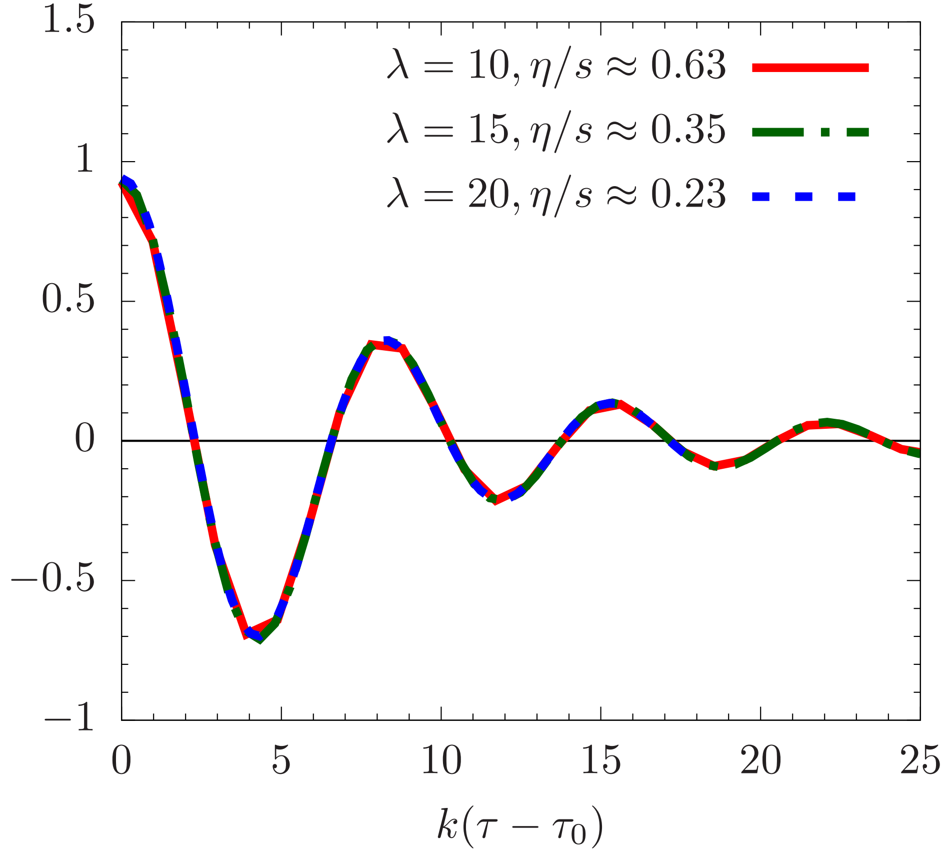
Summary – Use QCD kinetics to smoothly match to hydro at late times



More to come – Thank you!

Rescaling of response perturbations in k-space

$$\frac{\delta e(k,\tau)}{e(\tau)} = E(k,\tau,\tau_0)\frac{\delta e(k,\tau)}{e(\tau)}$$



$$k\tau = k$$