Hadron structure from lattice QCD

Jeremy Green

NIC, DESY, Zeuthen

7th workshop of the
APS Topical Group on Hadronic Physics
February 1–3, 2017

Outline

1. Neutron beta decay couplings g_A, g_T, g_S
2. Sigma terms $\sigma_{\pi N}$, σ_s
3. Electromagnetic form factors
4. Proton charge radius
5. Strange form factors
6. PDFs and TMDs

Lattice QCD

…is a regularization of Euclidean-space QCD such that the path integral can be done fully non-perturbatively.

- Euclidean spacetime becomes a periodic hypercubic lattice, with spacing a and box size $L_s^3 \times L_t$.
- Path integral over fermion degrees of freedom is done analytically, for each gauge configuration. Solving the Dirac equation with a fixed source yields a source-to-all quark propagator.
- Path integral over gauge degrees of freedom is done numerically using Monte Carlo methods to generate an *ensemble* of *gauge configurations*.

The $a \to 0$ and $L_s, L_t \to \infty$ extrapolations need to be taken by using multiple ensembles.

Flavour Lattice Averaging Group (FLAG) reviews:

http://itpwiki.unibe.ch/flag

G. Colangelo *et al.*, Eur. Phys. J. C **71**, 1695 (2011)
S. Aoki *et al.*, Eur. Phys. J. C **74**, 2890 (2014)
S. Aoki *et al.*, 1607.00299

e.g., quark masses

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_{ud}	m_s
RBC/UKQCD 14B$^{\ominus}$	[10]	P	★	★	★	★	d	3.31(4)(4)	90.3(0.9)(1.0)
RBC/UKQCD 12$^{\ominus}$	[31]	A	★	○	★	★	d	3.37(9)(7)(1)(2)	92.3(1.9)(0.9)(0.4)(0.8)
PACS-CS 12*	[143]	A	★	■	■	★	b	3.12(24)(8)	83.60(0.58)(2.23)
Laiho 11	[44]	C	○	★	★	○	–	3.31(7)(20)(17)	94.2(1.4)(3.2)(4.7)
BMW 10A, 10B^{+}	[7, 8]	A	★	★	★	★	c	3.469(47)(48)	95.5(1.1)(1.5)
PACS-CS 10	[95]	A	★	■	■	★	b	2.78(27)	86.7(2.3)
MILC 10A	[13]	C	○	★	★	○	–	3.19(4)(5)(16)	–
HPQCD 10*	[9]	A	○	★	★	–	–	3.39(6)	92.2(1.3)
RBC/UKQCD 10A	[144]	A	○	○	★	★	a	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum 10†	[103]	A	○	■	○	★	–	3.44(12)(22)	97.6(2.9)(5.5)
PACS-CS 09	[94]	A	★	■	■	★	b	2.97(28)(3)	92.75(58)(95)
HPQCD 09A$^{\oplus}$	[18]	A	○	★	★	–	–	3.40(7)	92.4(1.5)
MILC 09A	[6]	C	○	★	★	○	–	3.25 (1)(7)(16)(0)	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	[89]	A	○	★	★	○	–	3.2(0)(1)(2)(0)	88(0)(3)(4)(0)
PACS-CS 08	[93]	A	★	■	■	■	–	2.527(47)	72.72(78)
RBC/UKQCD 08	[145]	A	○	■	★	★	–	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	[146]	A	■	★	★	■	–	$3.55(19)(^{+56}_{-20})$	$90.1(4.3)(^{+16.7}_{-4.3})$
HPQCD 05	[147]	A	○	○	○	○	–	$3.2(0)(2)(2)(0)^{\ddagger}$	$87(0)(4)(4)(0)^{\ddagger}$
MILC 04, HPQCD/ MILC/UKQCD 04	[107, 148]	A	○	○	○	■	–	2.8(0)(1)(3)(0)	76(0)(3)(7)(0)

Precision lattice QCD

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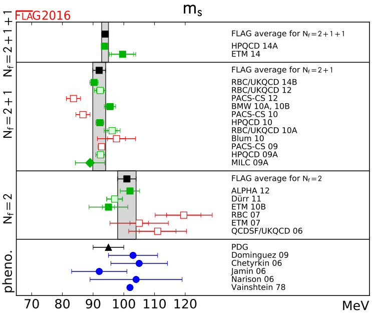
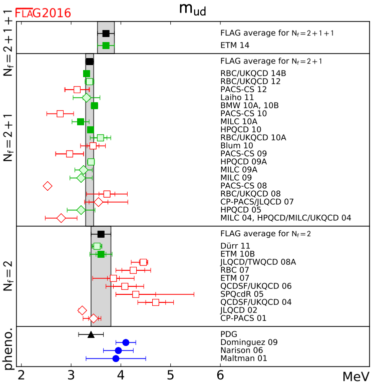
G. Colangelo *et al.*, Eur. Phys. J. C **71**, 1695 (2011)

S. Aoki *et al.*, Eur. Phys. J. C **74**, 2890 (2014)

S. Aoki *et al.*, 1607.00299

e.g., quark masses





Nucleon matrix elements using lattice QCD

To find matrix elements, compute using an interpolating operator χ:

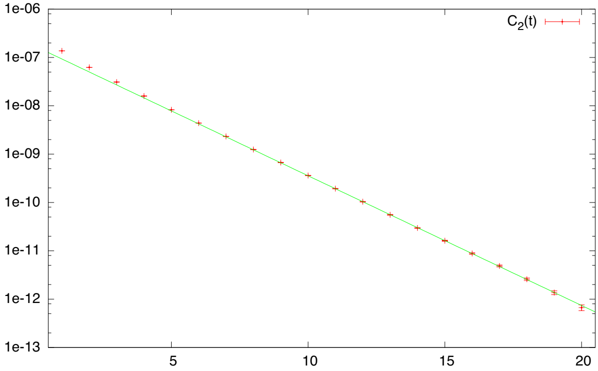
$$C_{2\mathrm{pt}}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \chi(\vec{x},t)\bar{\chi}(\vec{0},0)\rangle$$
$$\xrightarrow{t\to\infty} e^{-E(\vec{p})t} |\langle p|\bar{\chi}|\Omega\rangle|^2$$
$$C_{3\mathrm{pt}}(T,\tau;\vec{p},\vec{p}') = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}'\cdot\vec{x}} e^{i(\vec{p}'-\vec{p})\cdot\vec{y}} \langle \chi(\vec{x},T)\mathcal{O}(\vec{y},\tau)\bar{\chi}(\vec{0},0)\rangle$$
$$\xrightarrow[T-\tau\to\infty]{\tau\to\infty} e^{-E(\vec{p}')(T-\tau)} e^{-E(\vec{p})\tau} \langle\Omega|\chi|p'\rangle\langle p'|\mathcal{O}|p\rangle\langle p|\bar{\chi}|\Omega\rangle$$

Then form ratios to isolate $\langle p'|\mathcal{O}|p\rangle$.

Systematic error: excited states

With interpolating operator χ, compute, e.g.,

$$C_{2\text{pt}}(t) = \langle \chi(t)\chi^\dagger(0)\rangle = \sum_n e^{-E_n t}\left|\langle n|\chi^\dagger|0\rangle\right|^2$$



For a nucleon, the signal-to-noise asymptotically decays as $e^{-(m_N-\frac{3}{2}m_\pi)t}$.

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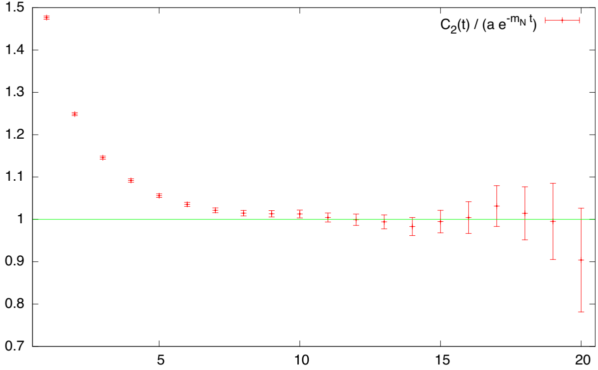
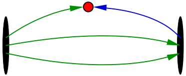


For a nucleon, the signal-to-noise asymptotically decays as $e^{-(m_N - \frac{3}{2}m_\pi)t}$.

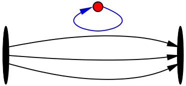
Connected and disconnected diagrams

Two kinds of quark contractions required for $C_{3\text{pt}}(\tau,T) = \langle \chi(T)\bar{q}\Gamma q(\tau)\bar{\chi}(0)\rangle$:

- Connected, which we usually evaluate with sequential propagators through the sink.



- Disconnected, which requires stochastic estimation to evaluate the *disconnected loop*,



$$T(\vec{q},t,\Gamma) = -\sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}}\,\mathrm{Tr}[\Gamma D^{-1}(x,x)].$$

Introduce noise sources η that satisfy $E(\eta\eta^\dagger) = I$. By solving $\psi = D^{-1}\eta$, we get

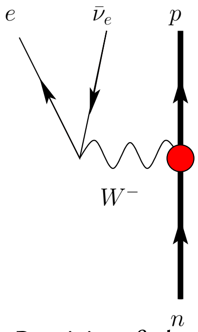
$$D^{-1}(x,y) = E(\psi(x)\eta^\dagger(y)),$$

so that

$$T(\vec{q},t,\Gamma) = E\left(-\sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}}\eta^\dagger(x)\Gamma\psi(x)\right).$$

We then need to compute the *correlation* between this loop and a two-point correlator.

Neutron beta decay



Coupling to W boson via axial current depends on nucleon axial charge,

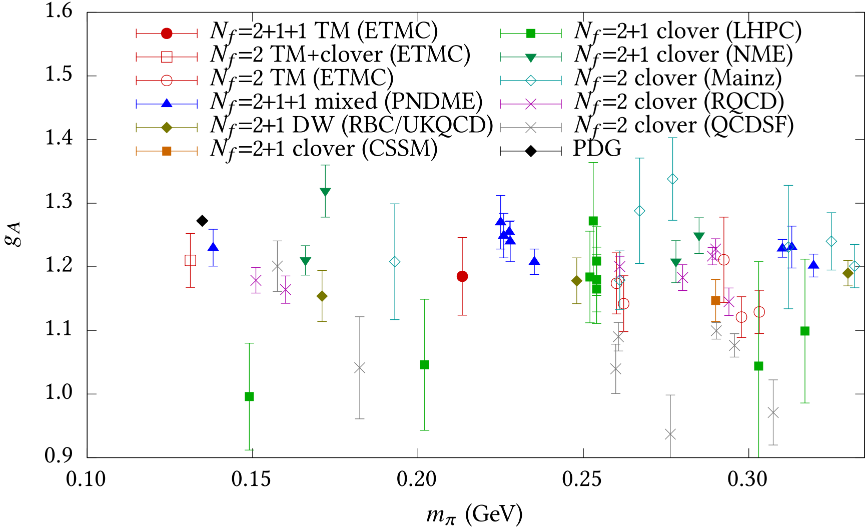
$$\langle p(P,s')|\bar{u}\gamma^\mu\gamma_5 d|n(P,s)\rangle = g_A \bar{u}_p(P,s')\gamma^\mu\gamma_5 u_n(P,s).$$

Well-measured experimentally: $g_A = 1.2723(23)$.

Precision β-decay experiments may be sensitive to BSM physics; leading contributions are controlled by the (not measured experimentally) scalar and tensor charges: T. Bhattacharya *et al.*, Phys. Rev. D **85** (2012) 054512 [1110.6448]

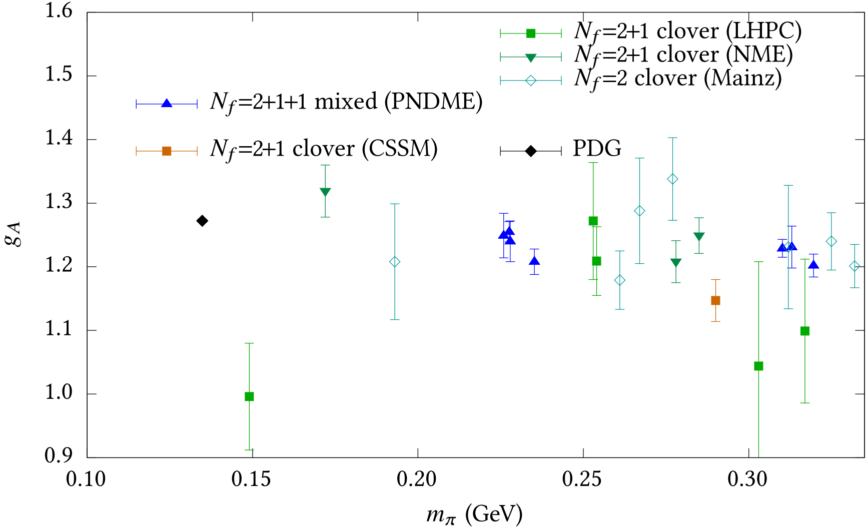
$$\langle p(P,s')|\bar{u}d|n(P,s)\rangle = g_S \bar{u}_p(P,s')u_n(P,s),$$
$$\langle p(P,s')|\bar{u}\sigma^{\mu\nu}d|n(P,s)\rangle = g_T \bar{u}_p(P,s')\sigma^{\mu\nu}u_n(P,s).$$

Axial charge g_A: “benchmark” observable



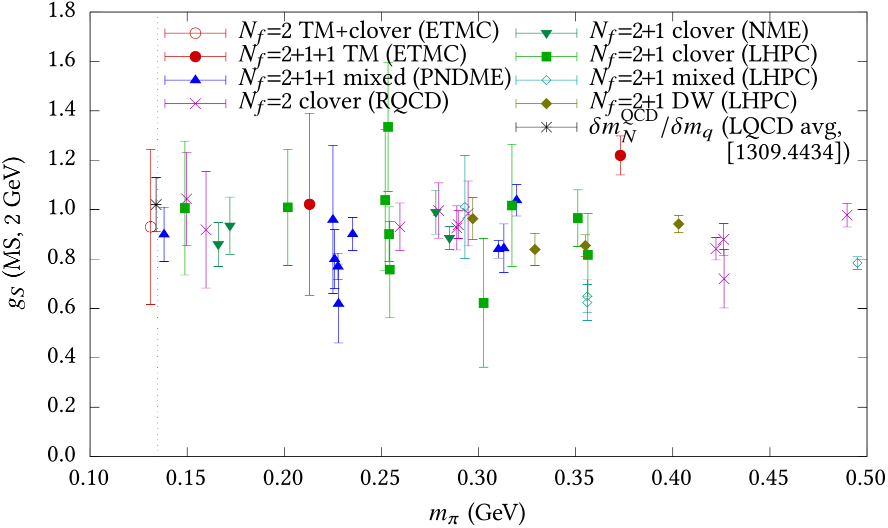
Works that extend below 300 MeV.

Axial charge g_A: "benchmark" observable

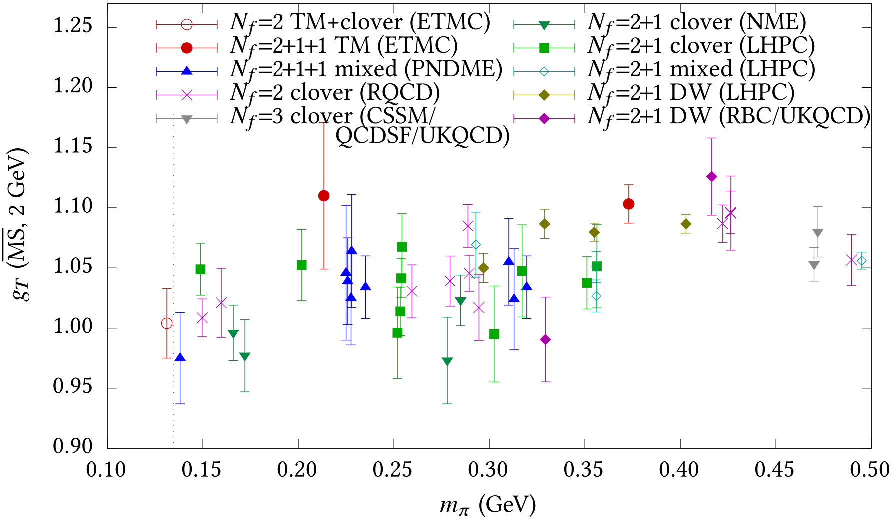


Works that extend below 300 MeV, have $m_\pi L \geq 4$, and control exc. states.

Scalar charge g_S



Tensor charge g_T

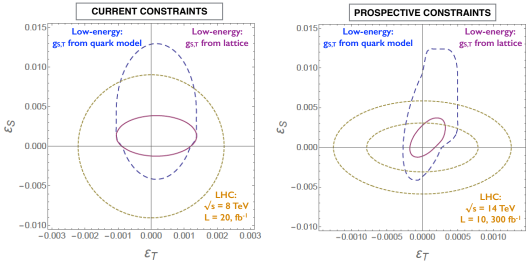


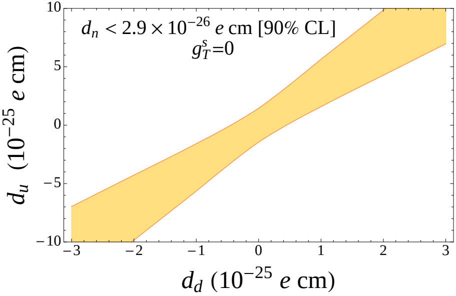
Scalar and tensor charges: applications

T. Bhattacharya *et al.* (PNDME),

Phys. Rev. D **94**, 054508 [1606.07049]

Using g_S, g_T: constraints on beta decay parameters imply constraints on non-standard 4-fermion couplings ε_S, ε_T.





T. Bhattacharya *et al.* (PNDME),

Phys. Rev. Lett. **115**, 212002 [1506.04196]

Using g_T^u, g_T^d: constraint on neutron electric dipole moment implies constraints on quark EDMs d_u, d_d.

Sigma terms

The contributions from quark masses to the nucleon mass.

$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle \qquad \sigma_q = m_q\langle N|\bar{q}q|N\rangle$$

Control the sensitivity of direct-detection dark matter searches to WIMPs that interact via Higgs exchange.

Two main lattice approaches:

1. Direct calculation (requires disconnected diagrams).
2. Feynman-Hellmann theorem:

$$\sigma_q = m_q \frac{\partial m_N}{\partial m_q}$$

   (requires multiple ensembles with different m_q).

Recent results using (near-)physical m_π:

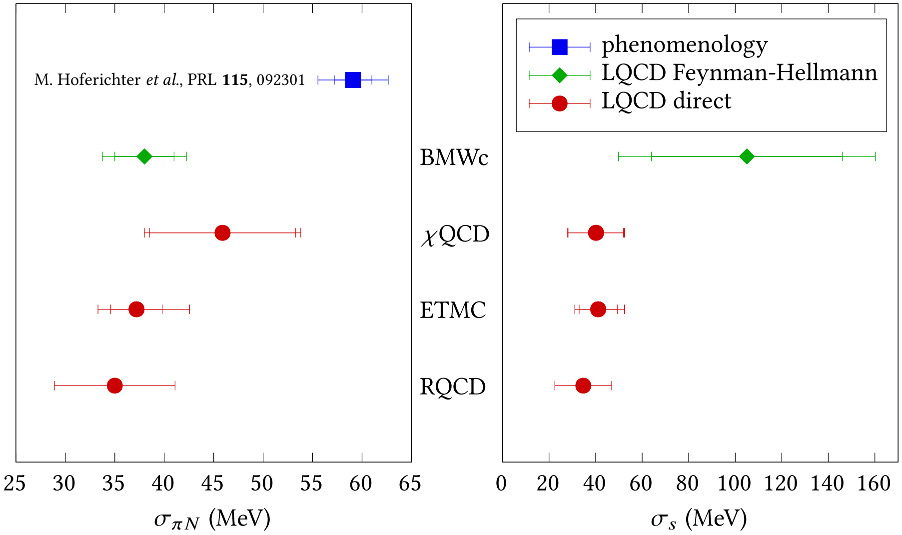
Direct: Y.-B. Yang *et al.* (χQCD), Phys. Rev. D **94**, 054503 [1511.09089] → parallel talk today, 16:00

Direct: A. Abdel-Rehim *et al.* (ETMC), Phys. Rev. Lett. **116**, 252001 [1601.01624]

Direct: G. S. Bali *et al.* (RQCD), Phys. Rev. D **93**, 094504 [1603.00827]

Feynman-Hellmann: S. Durr *et al.* (BMWc), Phys. Rev. Lett. **116**, 172001 [1510.08013]

Sigma terms: recent results



There is a disagreement for $\sigma_{\pi N}$ between LQCD and phenomenology.

Electromagnetic form factors

Proton matrix elements of vector current parameterized by Dirac and Pauli form factors:

$$\langle p',s'|J^\mu_{\text{em}}|p,s\rangle = \bar{u}(p',s')\left(\gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m_p}F_2(Q^2)\right)u(p,s),$$

where $q = p' - p$, $Q^2 = -q^2$. Or alternatively, by the electric and magnetic Sachs form factors,

$$G_E(Q^2) = F_1(Q^2) - \tfrac{Q^2}{(2m_p)^2}F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

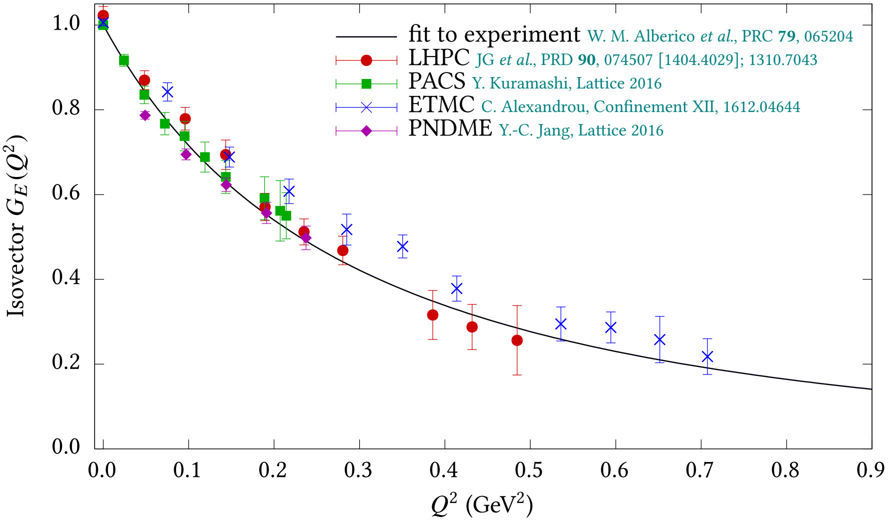
At $Q^2 = 0$, these give the charge and magnetic moment of the proton, and their derivatives define the mean-squared electric and magnetic radii:

$$G_E^p(Q^2) = 1 - \tfrac{1}{6}(r_E^2)^p Q^2 + O(Q^4), \qquad G_M^p(Q^2) = \mu^p\left(1 - \tfrac{1}{6}(r_M^2)^p Q^2 + O(Q^4)\right).$$

To eliminate disconnected diagrams, we take the isovector combination,

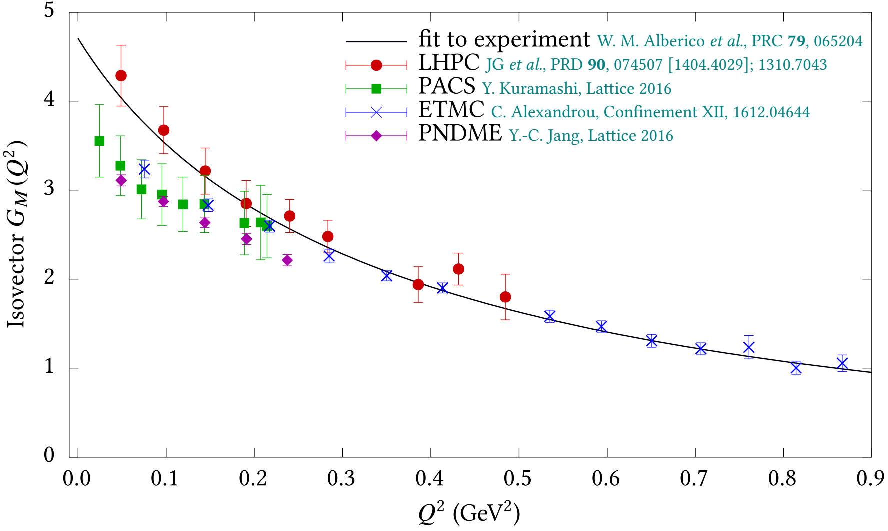
$$G^v_{E,M} = G^p_{E,M} - G^n_{E,M}, \qquad F^v_{1,2} = F^p_{1,2} - F^n_{1,2}.$$

Isovector $G_E(Q^2)$



Results for near-physical m_π.

Isovector $G_M(Q^2)$



Results for near-physical m_π.

Proton radius problem

Two disagreeing values ($> 5\sigma$ discrepancy) for the proton charge radius r_E^p:

1. "high" value: 0.8751(61) fm
   CODATA 2014
2. "low" value: 0.84087(39) fm
   A. Antognini *et al.*, Science **339**, 417 (2013)

	e	μ
spectroscopy	high	low
scattering	high	?
QCD theory	?	

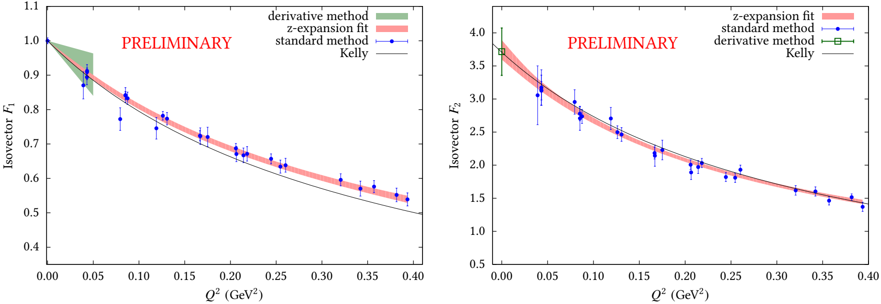
Traditional lattice strategy to determine r_E: fitting to $G_E(Q^2)$.

Problem: $Q^2_{\text{min}} \approx (2\pi/L)^2 \gtrsim 0.05\ \text{GeV}^2$, an order of magnitude higher than for data from experiment.

New methods for directly computing radius (and magnetic moment):

- Correlators in a background field
  Z. Davoudi and W. Detmold, Phys. Rev. D **92**, 074506 [1507.01908]
- $\frac{\partial}{\partial q^2}$ using position-space moments
  C. C. Chang *et al.*, Lattice 2016, 1610.02354 → D. Richards parallel talk today, 17:15
- $\frac{\partial}{\partial q_\mu}$ using semi-position-space moments
  C. Alexandrou *et al.*, Phys. Rev. D **94**, 074508 [1605.07327]
- $\frac{\partial}{\partial q_\mu}$ via derivative of twisted boundary conditions
  N. Hasan *et al.*, Lattice 2016, 1611.06805

Isovector Dirac and Pauli form factors



N. Hasan, M. Engelhardt, JG, S. Krieg, S. Meinel, J. Negele, A. Pochinsky, S. Syritsyn, 1611.01383

$m_\pi \approx 135$ MeV, $m_\pi L \approx 4$

Momentum derivative method yields $F_2(0)$ and $F_1'(0)$.

Strange form factors

Define strange electromagnetic form factors using strange vector current:

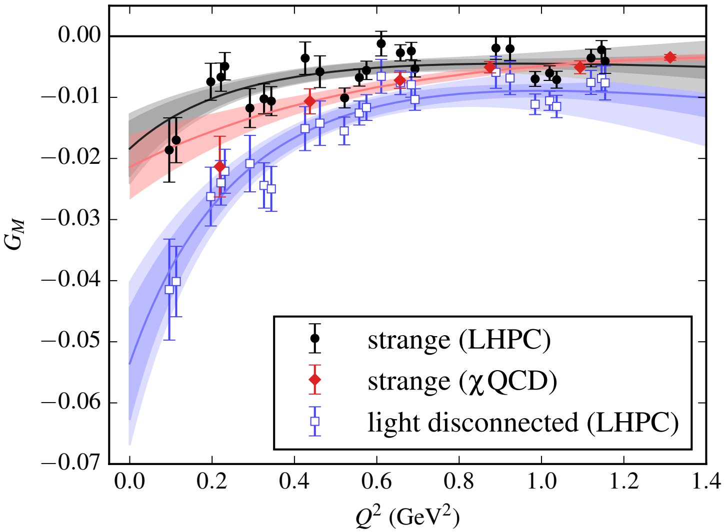
$$J^\mu_{\text{em}} \to J^\mu_s \equiv \bar{s}\gamma^\mu s, \qquad G_{E,M}(Q^2) \to G^s_{E,M}(Q^2).$$

Determined by measuring the parity-violating asymmetry in elastic $\vec{e}p$ scattering, which is sensitive to Z exchange.
In lattice calculations, these arise purely from disconnected diagrams.
Two recent calculations with a clear signal:

1. LHPC: JG, S. Meinel *et al.*, Phys. Rev. D **92**, 031501(R) [1505.01803]
   - One ensemble, $m_\pi = 317$ MeV, Wilson-clover fermions.
   - Strange and light disconnected loops.
2. χQCD: R. S. Sufian, Y.-B. Yang *et al.*, Phys. Rev. Lett. **118**, 042001 [1606.07075]
   - Four ensembles including $m_\pi = 139$ MeV, overlap on domain wall.
   - Also varied valence $m_{ud} \to 24$ different m_π (partially quenched).
   - Strange disconnected loops.

→ see also P. Shanahan plenary talk Friday, 11:10

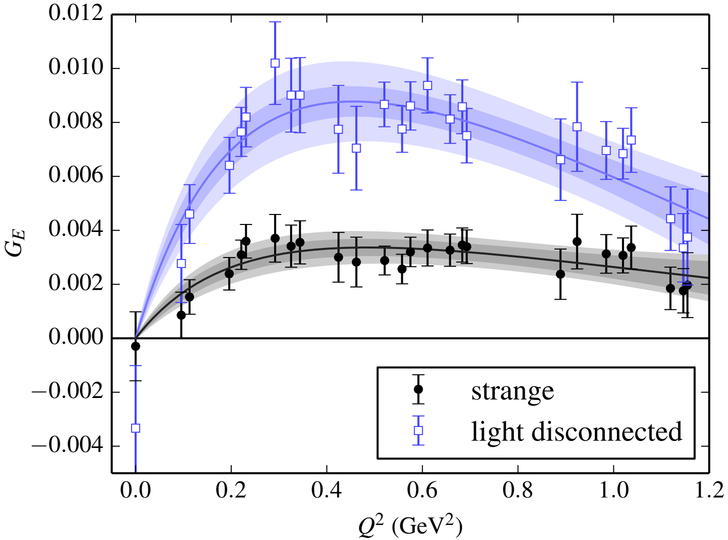
Strange magnetic form factor: comparison



Similar pion mass: 317 MeV (LHPC) and 330 MeV (χQCD).

At physical point, χQCD gets $\mu^s = -0.064(14)(9)\mu_N$.

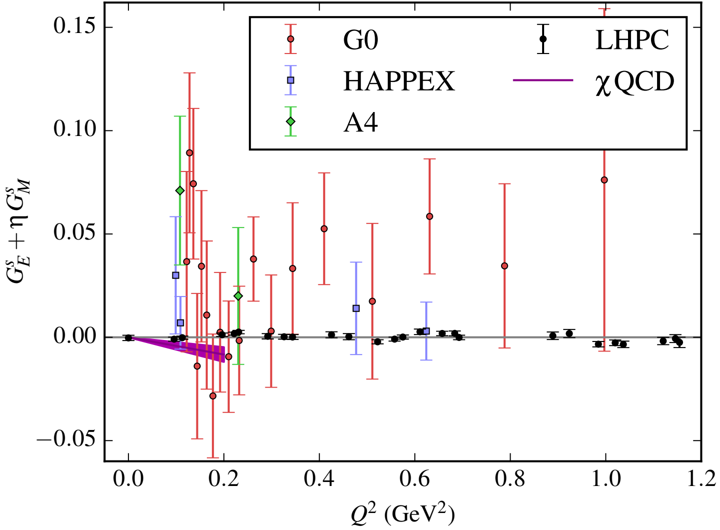
Strange electric form factor: $m_\pi = 317$ MeV



This corresponds to $(r_E^2)^s = -0.0054(9)(13)$ fm^2.

At physical point, χQCD gets $(r_E^2)^s = -0.0043(16)(11)$ fm^2.

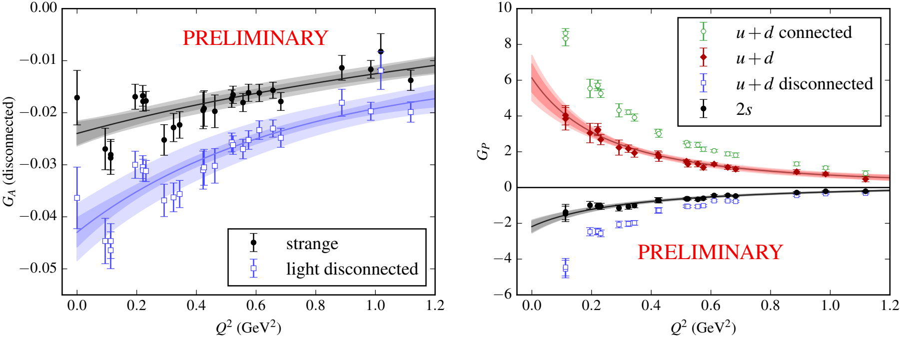
Forward-angle parity-violating elastic scattering experiments



$\eta \approx AQ^2$, $A = 0.94\ \text{GeV}^{-2}$

Disconnected axial form factors

$$\langle p'|\bar{q}\gamma_\mu\gamma_5 q|p\rangle = \bar{u}(p')\left[\gamma_\mu G_A^q(Q^2) + \frac{(p'-p)_\mu}{2m_p} G_P^q(Q^2)\right]\gamma_5 u(p)$$



JG, Lattice 2016

Small mixing between light and strange quarks accounted for.

→ Affects $G_A^s(Q^2)$ by up to 10% at $\mu = 2$ GeV.

Disconnected contribution to $G_P(Q^2)$ is significant.

Parton distribution functions

$q(x,\mu)$ is interpreted as the probability to find a quark q with fraction x of the proton's momentum.

PDFs can be defined using a light cone operator:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixz(n\cdot p)} \langle p|\bar{\psi}(0)W(0,zn)(n\cdot\gamma)\psi(zn)|p\rangle,$$

where $p = (p_0, 0, 0, p_z)$, $n = (1,0,0,-1)/\sqrt{2}$, and W is a straight gauge link. This can't be implemented on a Euclidean lattice.

Idea: take n spacelike, $n = (0,0,0,-1)$ and define a quasi-PDF $\tilde{q}(x,\mu,p_z)$. These are related at large p_z:

$$\tilde{q}(x,\mu,p_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{p_z}\right) q(x,\mu) + O\left(\frac{\Lambda_{\text{QCD}}^2}{p_z^2}, \frac{m^2}{p_z^2}\right)$$

X. Ji, Phys. Rev. Lett. **110**, 262002 [1305.1539]; many follow-up papers

PDFs from lattice QCD

Exploratory studies of PDFs $q(x)$, $\Delta q(x)$, and $\delta q(x)$ by two groups:

1. H.-W. Lin, J.-W. Chen, S. D. Cohen, X. Ji, Phys. Rev. D **91**, 054510 [1402.1462]
   J.-W. Chen, S. D. Cohen, X. Ji, H.-W. Lin, J.-H. Zhang, Nucl. Phys. B **911**, 246–273 [1603.06664]
2. C. Alexandrou, K. Cichy, V. Drach *et al.*, Phys. Rev. D **92**, 014502 [1504.07455]
   C. Alexandrou, K. Cichy, M. Constantinou *et al.*, 1610.03689

Also study of total gluon helicity using large-momentum methods:

Y.-B. Yang, R. S. Sufian *et al.* (χQCD) Phys. Rev. Lett. (to appear), 1609.05937

Additional challenges over standard observables:

- Need to take large-p_z limit. New "momentum smearing" yields improved signal at large p_z. G. S. Bali *et al.* (RQCD), PRD **93**, 094515 [1602.05525]
- Removal of power divergence and renormalization.

→ J. Zhang, M. Constantinou, K. Orginos in parallel session on lattice PDFs Thursday, 11:00

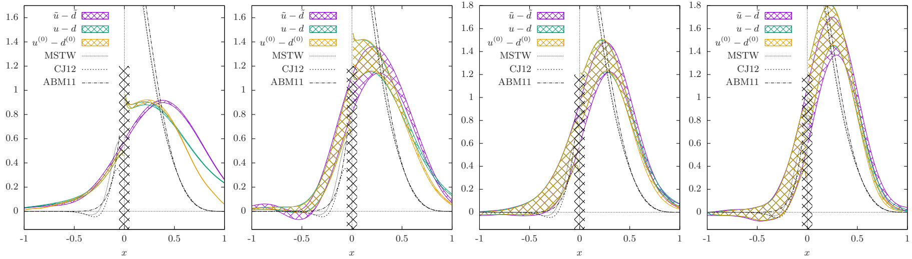
Similar program to study TMDs on the lattice using staple-shaped links:

B. U. Musch *et al.*: Europhys. Lett. **88**, 61001 [0908.1283]; Phys. Rev. D **83**, 094507 [1011.1213]
B. U. Musch, M. Engelhardt *et al.*: Phys. Rev. D **85**, 094510 [1111.4249]; PRD **93**, 054501 [1506.07826]
→ M. Engelhardt parallel talk on orbital angular momentum Thursday, 12:15

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese,

1610.03689

$p_z = 1.0$ GeV $p_z = 1.5$ GeV $p_z = 2.0$ GeV $p_z = 2.5$ GeV



$x < 0 \rightarrow$ antiquark PDFs $\bar{q}(x) = -q(-x)$

$m_\pi = 370$ MeV

Effect of matching $\tilde{q} \rightarrow q$ and target mass corrections reduced at large p_z.

Qualitative behaviour is similar to phenomenological curves.

Summary and outlook

Calculations of quark-connected nucleon matrix elements and form factors are approaching maturity:

- Multiple collaborations are working with near-physical pion masses.
- Excited states and exponential growth of noise remain problems.
- Better control over remaining systematics ($a \to 0$, $L \to \infty$) still needed.

There is a serious effort to calculate disconnected diagrams:

- Modern techniques seem effective at controlling the noise from stochastic estimation.
- Non-perturbative renormalization can also be done.

Exploratory calculations are ongoing for many more challenging observables:

- Direct computation of charge radii.
- Form factors at high Q^2.
- Parton distribution functions and transverse momentum-dependent PDFs.