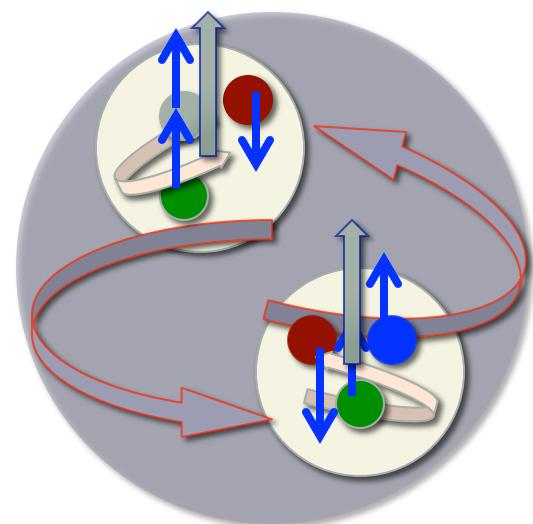


# PARTON OAM: EXPERIMENTAL LEADS

7<sup>TH</sup> GHP WORKSHOP  
FEBRUARY 1-3, 2017  
WASHINGTON, DC

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Simonetta Liuti  
University of Virginia



# Based on

## Parton transverse momentum and orbital angular momentum distributions

PHYSICAL REVIEW D 94, 034041 (2016)

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The quark orbital angular momentum component of proton spin,  $L_q$ , can be defined in QCD as the integral of a Wigner phase space distribution weighting the cross product of the quark's transverse position and momentum. It can also be independently defined from the operator product expansion for the off-forward Compton amplitude in terms of a twist-three generalized parton distribution. We provide an explicit link between the two definitions, connecting them through their dependence on partonic intrinsic transverse momentum. Connecting the definitions provides the key for correlating direct experimental determinations of  $L_q$  and evaluations through lattice QCD calculations. The direct observation of quark orbital angular momentum does not require transverse spin polarization but can occur using longitudinally polarized targets.

DOI: 10.1103/PhysRevD.94.034041

...and A. Rajan et al., soon to be posted

# Outline

1. Definitions
2. Lorentz Invariant Relations → OAM is given by a twist three distribution
3. Equations of Motion Relations
4. A probe of QCD at the amplitude level
5. Process dependence of OAM distributions/universality
6. Conclusions

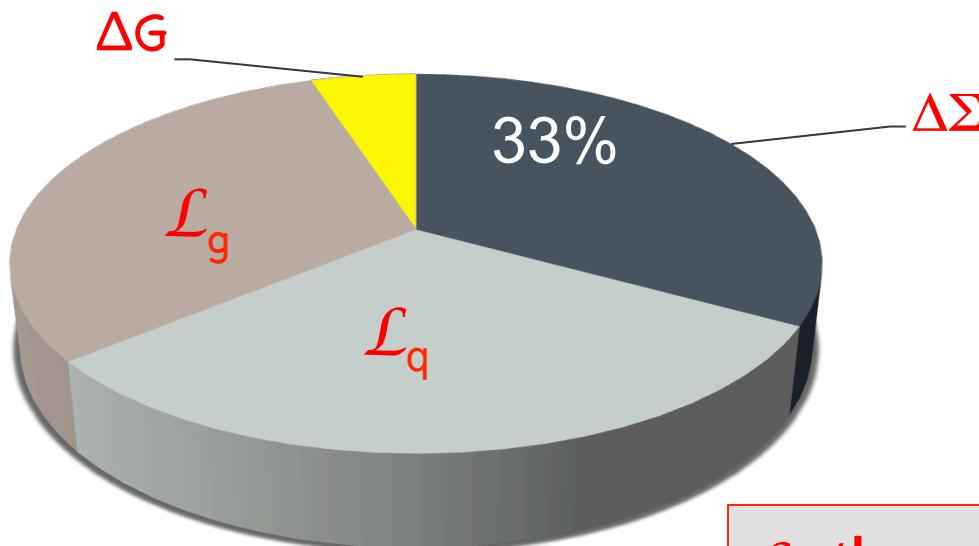
# 1. DEFINITIONS

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# The spin crisis in a cartoon

Jaffe Manohar

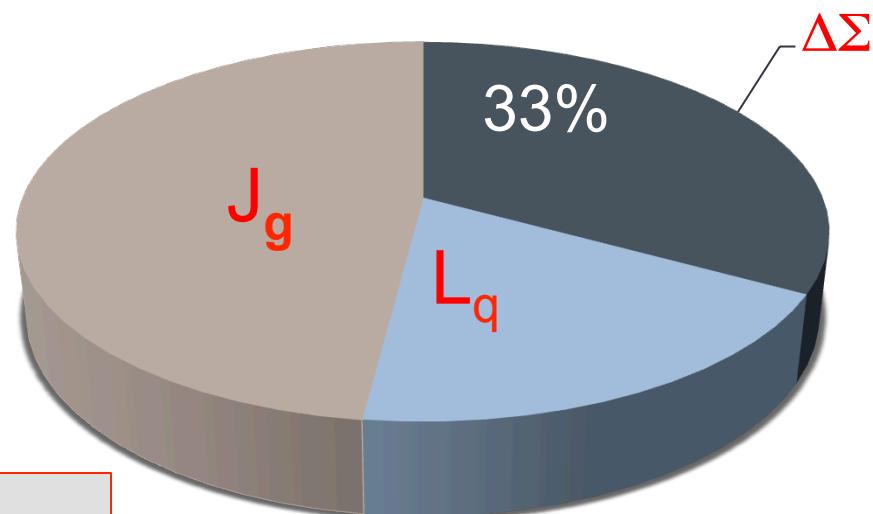
$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$



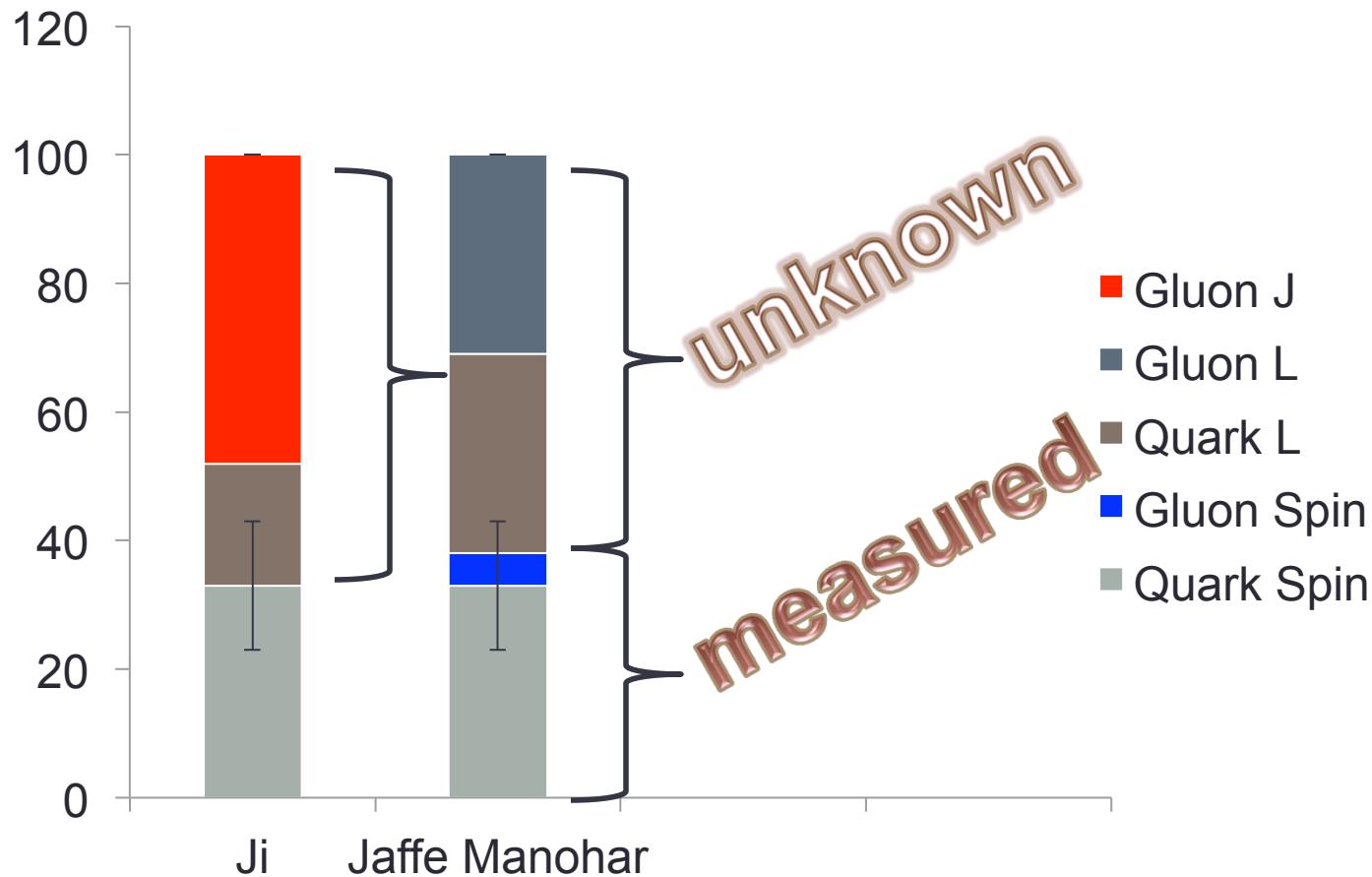
$$\mathcal{L}_q \neq \mathcal{L}_q$$
$$J_g \neq \mathcal{L}_g + \Delta G$$

Ji

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + L_q + J_g$$



# Angular Momentum Budget

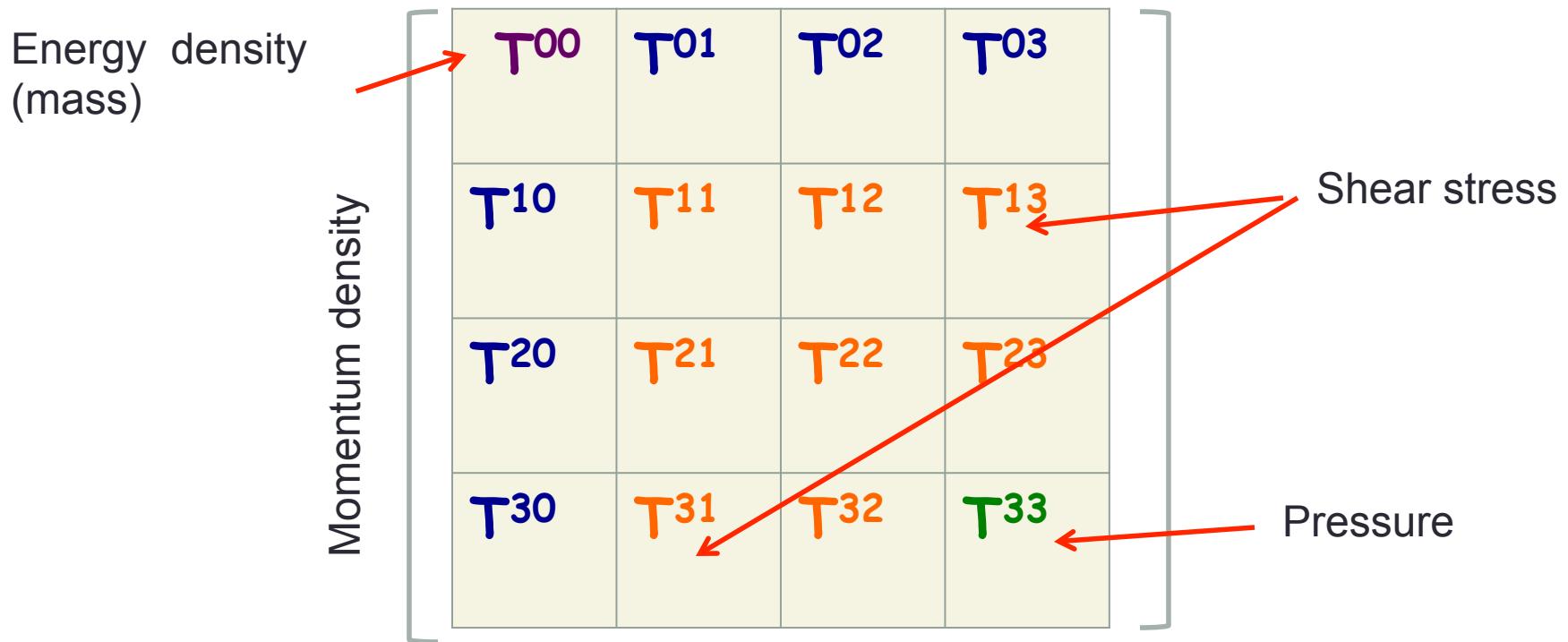


## The Starting Point

Jaffe and Manohar's field theoretical description of the quark and gluon orbital angular momentum through its relation to the QCD Energy Momentum Tensor.

$$T^{\mu\nu} \rightarrow M^{\mu\nu\lambda} = x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}$$

Angular Momentum density



$$T^{\mu\nu} = \frac{1}{4}iq\bar{\psi}\left(\gamma^\mu\vec{D}^\nu + \gamma^\nu\vec{D}^\mu\right)\psi + Tr\left\{F^{\mu\alpha}F_\alpha^\nu - \frac{1}{2}g^{\mu\nu}F^2\right\}$$

Jaffe Manohar:

\*

$$M^{+12} = \psi^\dagger \sigma^{12} \psi + \psi^\dagger [\vec{x} \times (-i\partial)]^3 \psi + Tr(\epsilon^{+-ij} F^{+j} A^j) + 2i Tr F^{+j} (\vec{x} \times \partial) A^j$$

$$\Delta\Sigma$$

$$\mathcal{L}_q$$

$$\Delta G$$

$$\mathcal{L}_g$$

Ji:

$$M^{+12} = \psi^\dagger \sigma^{12} \psi + \psi^\dagger [\vec{x} \times (-i\vec{D})]^3 \psi + [\vec{x} \times (\vec{E} \times \vec{B})]^3$$

$$\Delta\Sigma$$

$$\mathcal{L}_q$$

$$J_g$$



$$J_q$$

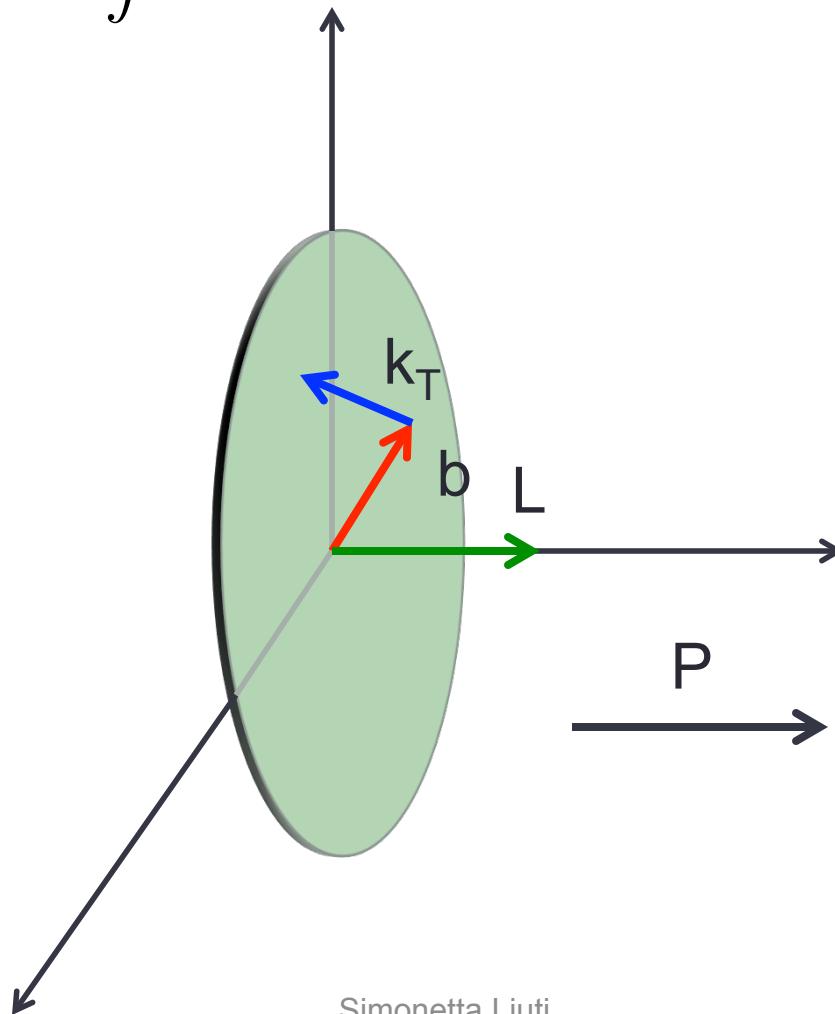
\*Chen, Goldman et al., are consistent with this definition (see K.F.Liu et al.)

The first step towards an observable effect...

# Partonic OAM: Wigner Distributions

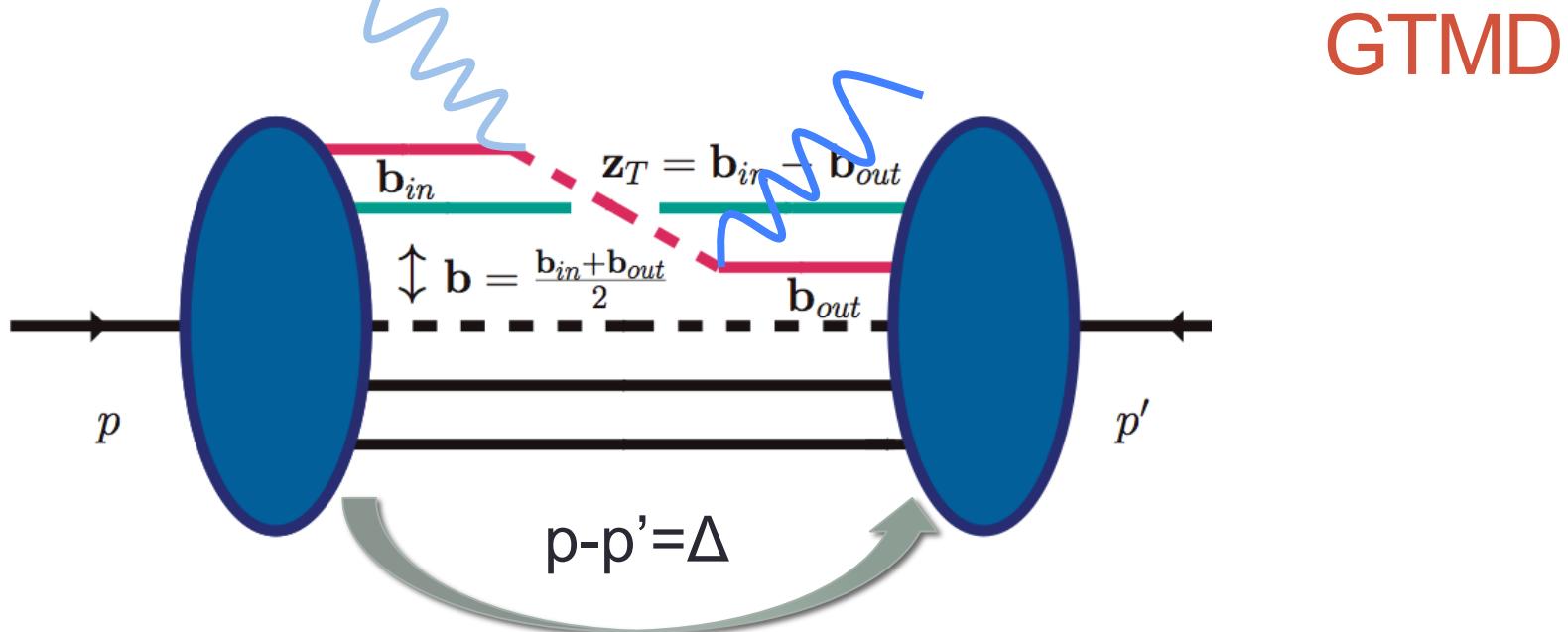
$$L_q^{\mathcal{U}} = \int dx \int d^2\mathbf{k}_T \int d^2\mathbf{b} (\mathbf{b} \times \mathbf{k}_T)_z \mathcal{W}^{\mathcal{U}}(x, \mathbf{k}_T, \mathbf{b})$$

Hatta  
Lorce, Pasquini,  
Xiong, Yuan



# Wigner Distribution

$$\mathcal{W}^U = \frac{1}{2} \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{i \Delta_T \cdot b} \boxed{\int dz^- d^2 \mathbf{z}_T e^{ikz} \langle P - \Delta, \Lambda' | \bar{q}(0) \gamma^+ \mathcal{U}(0, z) q(z) | P, \Lambda \rangle |_{z^+=0}}$$



- $\Delta_T$  Fourier conjugate:  $\mathbf{b}$  = transverse position of the quark inside the proton
- $k_T$  Fourier conjugate:  $\mathbf{z}_T$  = transverse distance traveled by the struck quark between the initial and final scattering

# Which GTMD?

The quark-quark correlator for a spin  $\frac{1}{2}$  hadron has been parametrized up to **twist four** in terms of **GTMDs**, **TMDs** and **GPDs**, in a complete way in:

Generalized parton correlation functions for a spin- $1/2$   
hadron

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Stephan Meißner,<sup>a</sup> Andreas Metz<sup>b</sup> and Marc Schlegel<sup>c</sup>

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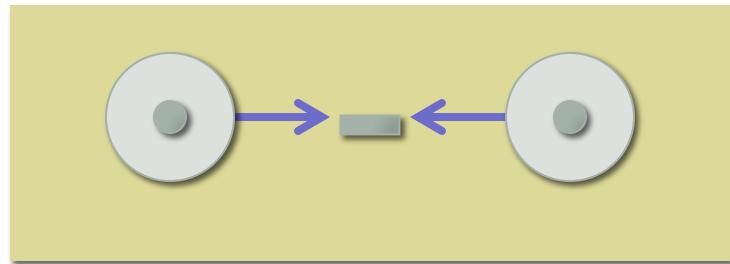
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12000 Jefferson Avenue, Newport News, VA 23606, U.S.A.  
E-mail: [stephan.meissner@tp2.rub.de](mailto:stephan.meissner@tp2.rub.de), [metza@temple.edu](mailto:metza@temple.edu),  
<http://tp2.rub.de/~schlegel/>

JHEP08(2009)

# F<sub>14</sub>

$$\begin{aligned}
 W_{\Lambda\Lambda'}^{\gamma^+} &= \frac{1}{2P^+} \bar{U}(p', \Lambda') \left[ \gamma^+ F_{11} + \frac{i\sigma^{i+}\Delta_T^i}{2M} (2F_{13} - F_{11}) + \frac{i\sigma^{i+}\bar{k}_T^i}{2M} (2F_{12}) + \frac{i\sigma^{ij}\bar{k}_T^i\Delta_T^j}{M^2} F_{14} \right] U(p, \Lambda) \\
 &= \delta_{\Lambda,\Lambda'} F_{11} + \delta_{\Lambda,-\Lambda'} \frac{-\Lambda\Delta_1 - i\Delta_2}{2M} (2F_{13} - F_{11}) + \delta_{\Lambda,-\Lambda'} \frac{-\Lambda\bar{k}_1 - i\bar{k}_2}{2M} (2F_{12}) + \delta_{\Lambda,\Lambda'} i\Lambda \frac{\bar{k}_1\Delta_2 - \bar{k}_2\Delta_1}{M^2} F_{14}
 \end{aligned}$$

helicity non-flip



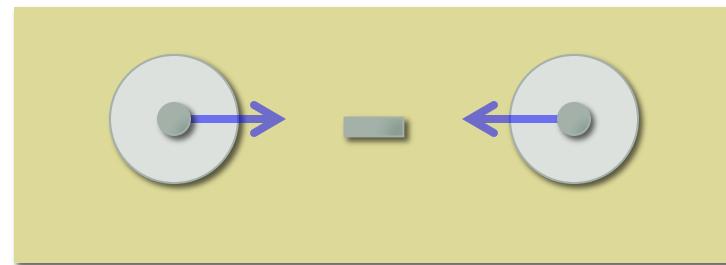
UL correlation: unpolarized quark density in a longitudinally polarized proton

$$\mathbf{G}_{11} \quad (L \cdot S) \rightarrow \frac{1}{2} \int d^2 \mathbf{b} (\mathbf{b} \times \mathbf{k}_T)_z \langle \bar{q}(0) \gamma^+ \gamma_5 q(z) \rangle$$

$$W_{\Lambda\Lambda'}^{\gamma^+\gamma_5} = \frac{1}{2M} \overline{U}(p', \Lambda') \left[ -\frac{i\epsilon_T^{ij} k_T^i \Delta_T^j}{M^2} G_{11} + \frac{i\sigma^{i+}\gamma^5 k_T^i}{P^+} G_{12} + \frac{i\sigma^{i+\gamma^5} \Delta_T^i}{P^+} G_{13} + i\sigma^{+-}\gamma^5 G_{14} \right] U(p, \Lambda)$$

$$= \left[ -\frac{i(k_1\Delta_2 - k_2\Delta_1)}{M^2} G_{11} + \Lambda G_{14} \right] \delta_{\Lambda\Lambda'} + \left[ \frac{\Delta_1 + i\Lambda\Delta_2}{M} \left( G_{13} + \frac{i\Lambda(k_1\Delta_2 - k_2\Delta_1)}{2M^2} G_{11} \right) + \frac{k_1 + i\Lambda k_2}{M} G_{12} \right] \delta_{-\Lambda, \Lambda'}$$

helicity non-flip



UL correlation: longitudinally polarized quark density in an unpolarized proton

## Integral relations

$$L_q = - \int_0^1 dx \int d^2 k_T \frac{k_T^2}{M^2} F_{14} = - \int_0^1 dx F_{14}^{(1)}$$

$$L_q \cdot S_q = - \int_0^1 dx \int d^2 k_T \frac{k_T^2}{M^2} G_{11} = - \int_0^1 dx G_{11}^{(1)}$$

Lorce, Pasquini, Xiong, Yuan  
Hatta, Yoshida  
Ji, Xiong, Yuan

## 2. LIR

---

# Lorentz Invariance Relations (LIR)

(see D. Pitonyak and A. Rajan's talks)

- LIR in the off-forward sector: relations between **twist-3 GPDs** ( $\rightarrow$  PDFs) and  $k_T$  moments of GTMDs ( $\rightarrow$  TMDs)
- Based on the most general Lorentz invariant decomposition of the fully unintegrated quark-quark correlator
- LIRs are a consequence of there being a smaller number of independent unintegrated terms in the decomposition than the number of GTMDs

$$\Phi^U = \int \frac{d^4 z}{(2\pi)^4} e^{i(k \cdot z)} \langle P', \Lambda' | \bar{\psi}(0) \gamma^+ \mathcal{U}(0, \infty | n) \psi(z) | P, \Lambda \rangle$$

→ parametrized in terms of invariant functions  $A_1, A_2, \dots$

(Meissner, Metz and Schlegel (2009), Mulders, Tangerman, Pijlman, Bacchetta....)

$$\tilde{\Phi}^U = \int dk^- \Phi^U$$

→ parametrized in terms of invariant functions  $F_{11}, F_{12}, \dots, F_{21}, F_{22} \dots$

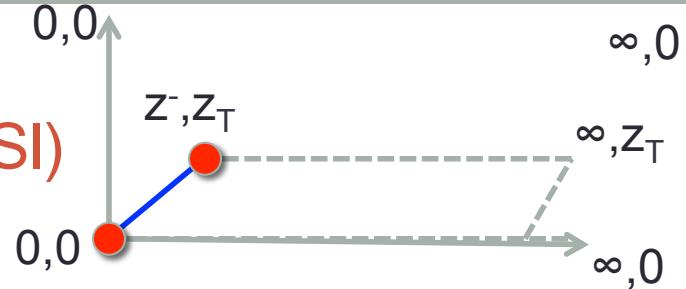
Specifically, one finds the following relations (A. Rajan's talk)

**tw 2**

$$\left\{ \begin{array}{l} \frac{k_T \cdot \Delta_T}{\Delta_T^2} F_{12} + F_{13} = 2P^+ \int dk^- \left( \frac{k_T \cdot \Delta_T}{\Delta_T^2} A_5 + A_6 - \frac{xP^2 - k \cdot P}{M^2} (A_8 + xA_9) \right) \\ F_{14} = 2P^+ \int dk^- (A_8 + xA_9) \end{array} \right.$$

**tw 3**

$$\frac{k_T \cdot \Delta_T}{\Delta_T^2} F_{27} + F_{28} = 2P^+ \int dk^- \left( \frac{k_T \cdot \Delta_T}{\Delta_T^2} A_5 + A_6 + \frac{1}{M^2} \left( \frac{(k_T \cdot \Delta_T)^2}{\Delta_T^2} - k_T^2 \right) A_9 \right)$$



## Generalized LIR for straight gauge link (no FSI)

Obtained by studying in detail the  $k_T$  structure of GTMDs and twist 3 GPDs for a straight gauge link (Ji's definition)

$L_q(x)$

OAM is given by a twist 3 GPD

$$\frac{d}{dx} \int d^2 k_T \frac{k_T^2}{M^2} F_{14} = \tilde{E}_{2T} + H + E$$

$k_T$  moment of a GTMD

$$\frac{d}{dx} \int d^2 k_T \frac{k_T^2}{M^2} G_{11} = - \left( 2\tilde{H}'_{2T} + E'_{2T} + \tilde{H} \right)$$

twist 3 GPD

Integrating in  $dx$  one finds the OAM distribution function

$$F_{14}^{(1)} = - \int_x^1 dy (\tilde{E}_{2T} + H + E) \Rightarrow -L_q = \int_0^1 dx F_{14}^{(1)} = \int_0^1 dx x G_2$$

$L_q(x)$

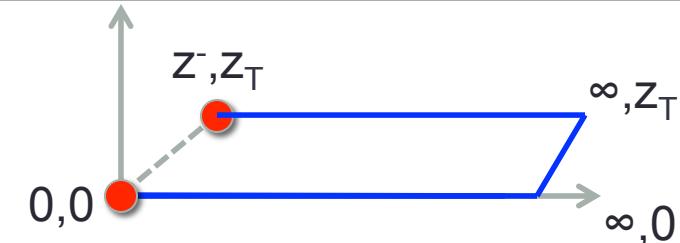
- $F_{14}$  and  $\tilde{E}_{2T}$  give us similar information on the distribution in  $x$  of OAM! new result
- In addition: we confirm and corroborate the global/integrated OAM result deducible from Ji et al

Different notation!

$$G_2 \rightarrow \tilde{E}_{2T} + H + E$$

Polyakov et al. Meissner, Metz and Schlegel, JHEP(2009)

## Generalized LIR for a staple link



$$\frac{d}{dx} \int d^2 k_T \frac{k_T^2}{M^2} F_{14} = \tilde{E}_{2T} + H + E + \mathcal{A}$$

LIR violating term

# 3. EQUATIONS OF MOTION

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# Equations of Motion (EoM) relation

Now insert the EoM in the correlator for a longitudinally polarized proton

$$\int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T \cdot z_T} \langle p', \Lambda' | \bar{\psi}(-z/2) (\Gamma \mathcal{U} i \vec{\not{D}} + i \vec{\not{D}} \Gamma \mathcal{U}) \psi(z/2) | p, \Lambda \rangle_{z^+=0} = 0$$

We find

$$\tilde{E}_{2T} = - \int_x^1 \frac{dy}{y} (H + E) + \left[ \frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2} \tilde{H} \right] + \left[ \frac{1}{x} \mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2} \mathcal{M}_{F_{14}} \right]$$

$\int_0^1 dx \dots$  relates to

L	=	J	- S	+	0
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## Consistent with OPE based relation

Polyakov et al.(2000), Hatta(2012)

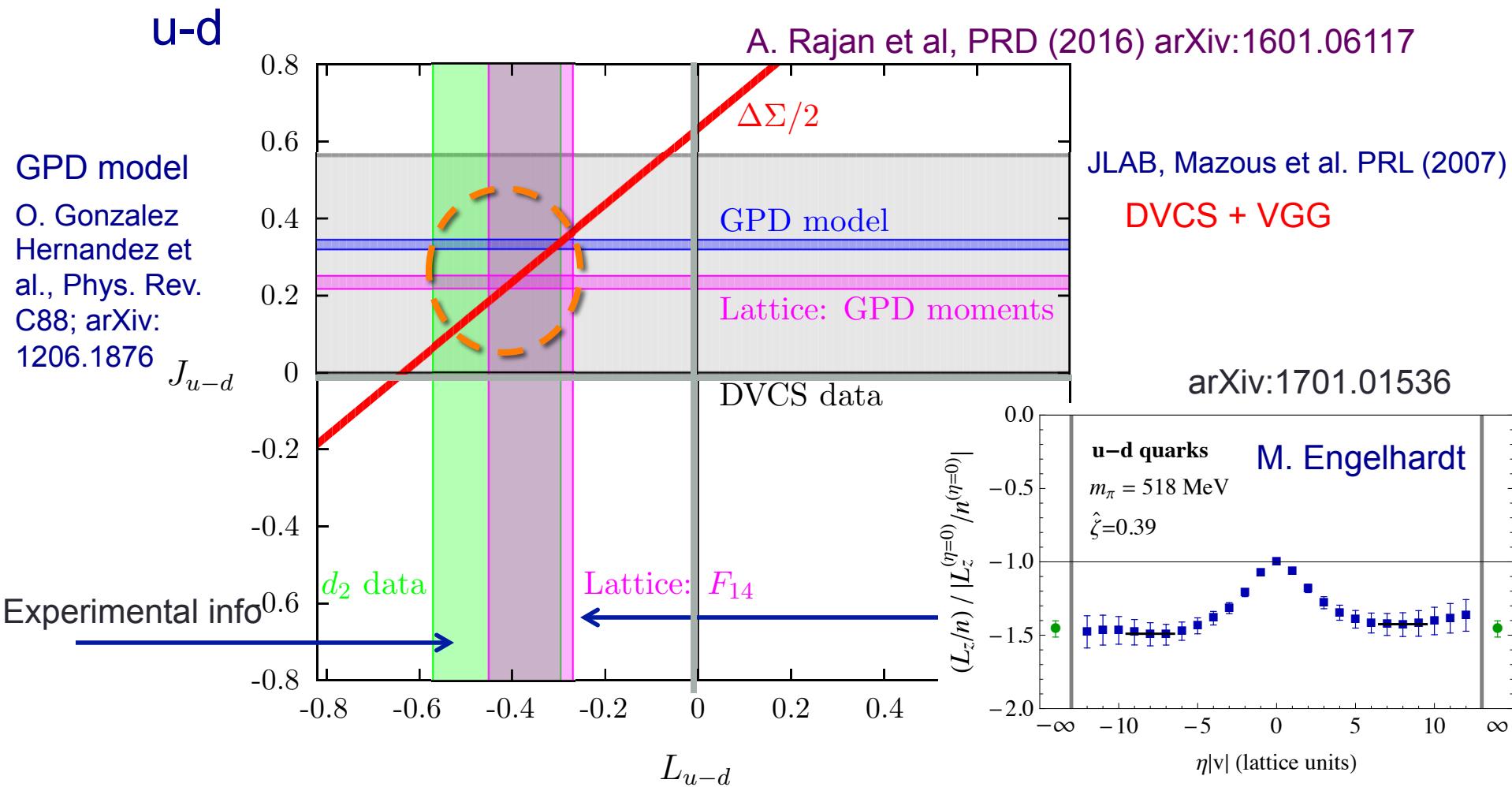
$$\int_0^1 dx x G_2 = -\frac{1}{2} \int_0^1 dx x(H + E) + \frac{1}{2} \int_0^1 dx \tilde{H}$$

The diagram illustrates the derivation of the generalized Wandzura Wilczeck relation. Three terms are shown at the top, each with a grey arrow pointing downwards to a central box. The first term is  $\int_0^1 dx x G_2$ , the second is  $-\frac{1}{2} \int_0^1 dx x(H + E)$ , and the third is  $\frac{1}{2} \int_0^1 dx \tilde{H}$ . The central box contains the equation  $J_q = L_q + \frac{1}{2} \Delta \Sigma_q$ , which is highlighted with a red border.

$$J_q = L_q + \frac{1}{2} \Delta \Sigma_q$$

A generalized Wandzura Wilczeck relation obtained using OPE for twist 2 and twist 3 operators for the off-forward matrix elements

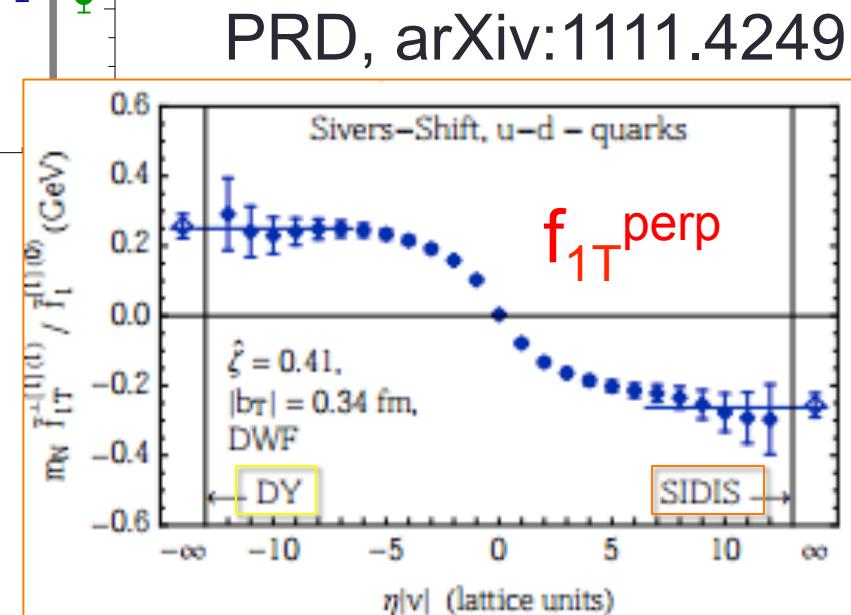
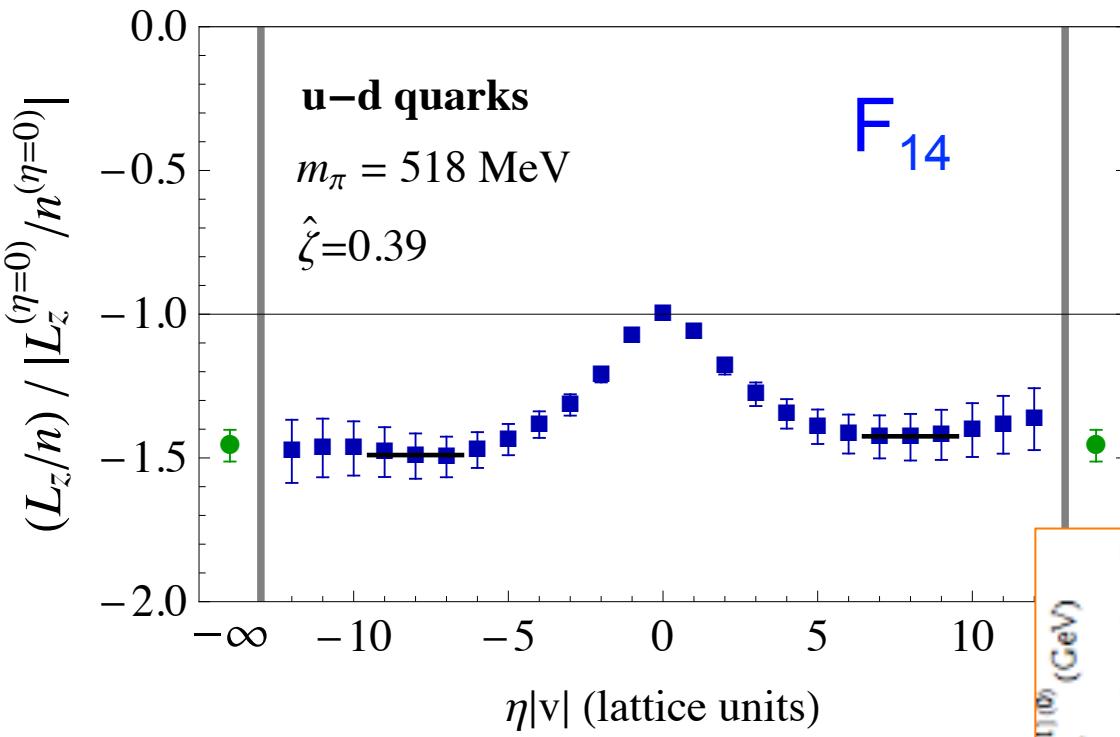
# Validation of Ji's Sum Rule: $J_q = L_q + \frac{1}{2}\Delta\Sigma_q$ through three independently measured quantities



# 4. A PROBE OF QCD AT THE AMPLITUDE LEVEL

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large effect from lattice (M. Engelhardt, arXiv:1701.01536)



insight into non-perturbative aspects of QCD associated with **dynamical chiral symmetry breaking**

# PT transformation

Forward case: Sivers function (J. Collins, 2002)

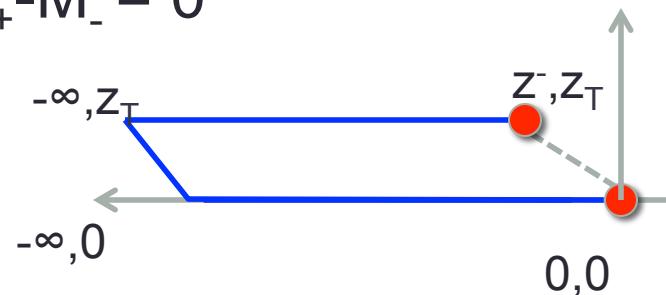
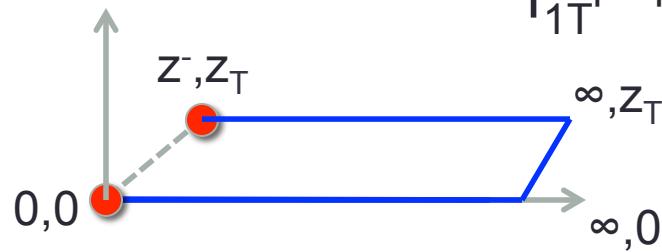
PT:

$$\langle \mathcal{P}, \mathcal{S} | \bar{\psi}(0)\gamma^+ \psi(z) | \mathcal{P}, \mathcal{S} \rangle = \langle \mathcal{P}, -\mathcal{S} | \bar{\psi}(0)\gamma^+ \psi(z) | \mathcal{P}, -\mathcal{S} \rangle$$

$M_+$

$$f_{1T}^{\text{perp}} = M_+ - M_- = 0$$

$M_-$



PT:

$$\langle \mathcal{P}, \mathcal{S} | \bar{\psi}(0)\gamma^+ U(v, z) \psi(z) | \mathcal{P}, \mathcal{S} \rangle = \langle \mathcal{P}, -\mathcal{S} | \bar{\psi}(0)\gamma^+ U(-v, z) \psi(z) | \mathcal{P}, -\mathcal{S} \rangle$$

$$M_+^\nu - M_-^{-\nu} = 0$$



$$f_{1T}^{\text{perp, SIDIS}} = M_+^\nu - M_-^{-\nu} = -f_{1T}^{\text{perp, DY}} = M_+^{-\nu} - M_-^{-\nu}$$

Off forward case:  $F_{14}$

PT:

$$\langle P - \Delta, S | \bar{\psi}(0)\gamma^+ U(v, z)\psi(z) | P, S \rangle = \langle P, -S | \bar{\psi}(0)\gamma^+ U(-v, z)\psi(z) | P - \Delta, -S \rangle$$

$\underbrace{\hspace{10em}}_{L_+^{v, \Delta}}$        $\underbrace{\hspace{10em}}_{L_-^{-v, -\Delta}}$

$$L_+^{v, \Delta} - L_-^{-v, -\Delta} = 0$$


$$(k_T \times \Delta_T) F_{14} \text{"SIDIS"} = L_+^{v, \Delta} - L_-^{-v, -\Delta} = (k_T \times \Delta_T) F_{14} \text{"DY"} = L_+^{-v, -\Delta} - L_-^{v, \Delta}$$

## Genuine/intrinsic twist three term in Equation of Motion relation

$$\mathcal{M}_{\Lambda\Lambda'}^i = \frac{1}{4} \int \frac{dz^- d^2 z_T}{(2\pi)^3} e^{ixP^+ z^- - ik_T \cdot z_T}$$

$$\langle p', \Lambda' | \bar{\psi}(-z/2) \left[ (\vec{\partial} - ig\mathcal{A})\mathcal{U}\Gamma \Big|_{-z/2} + \Gamma\mathcal{U}(\vec{\partial} + ig\mathcal{A}) \Big|_{z/2} \right] \psi(z/2) | p, \Lambda \rangle_{z^+=0}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{staple}} + \mathcal{M}_{F_{14}}^{\text{staple}}$$

$$-x\tilde{E}_{2T} = \tilde{H} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{\text{straight}} + \mathcal{M}_{F_{14}}^{\text{straight}}$$

$$\mathcal{A} = \frac{d}{dx} (\mathcal{M}^{\text{staple}} - \mathcal{M}^{\text{straight}})$$

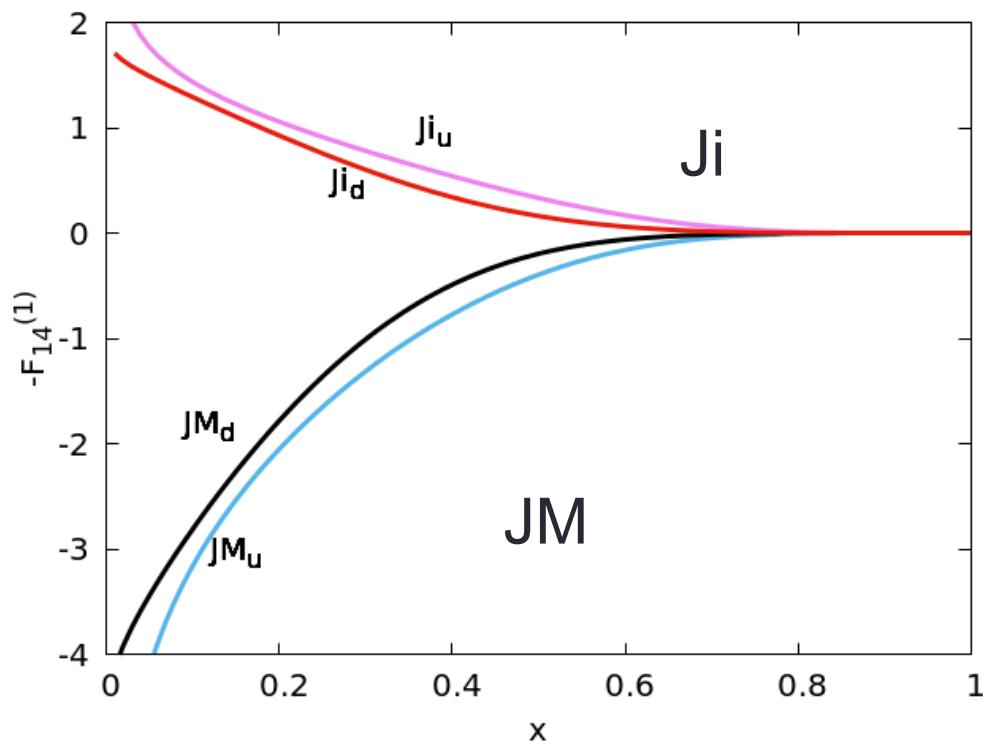
LIR violating term

## Generalized Qiu Sterman term

$$\int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{JM} - \int d^2 k_T \frac{k_T^2}{M^2} F_{14}^{Ji} = T_F(x, x, \Delta)$$

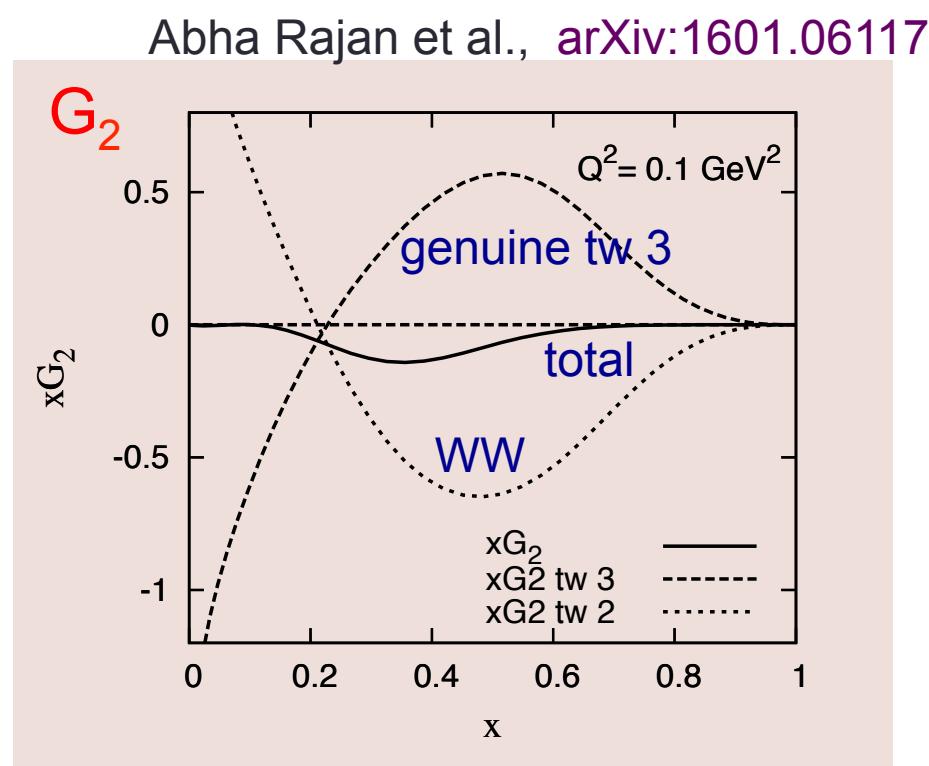
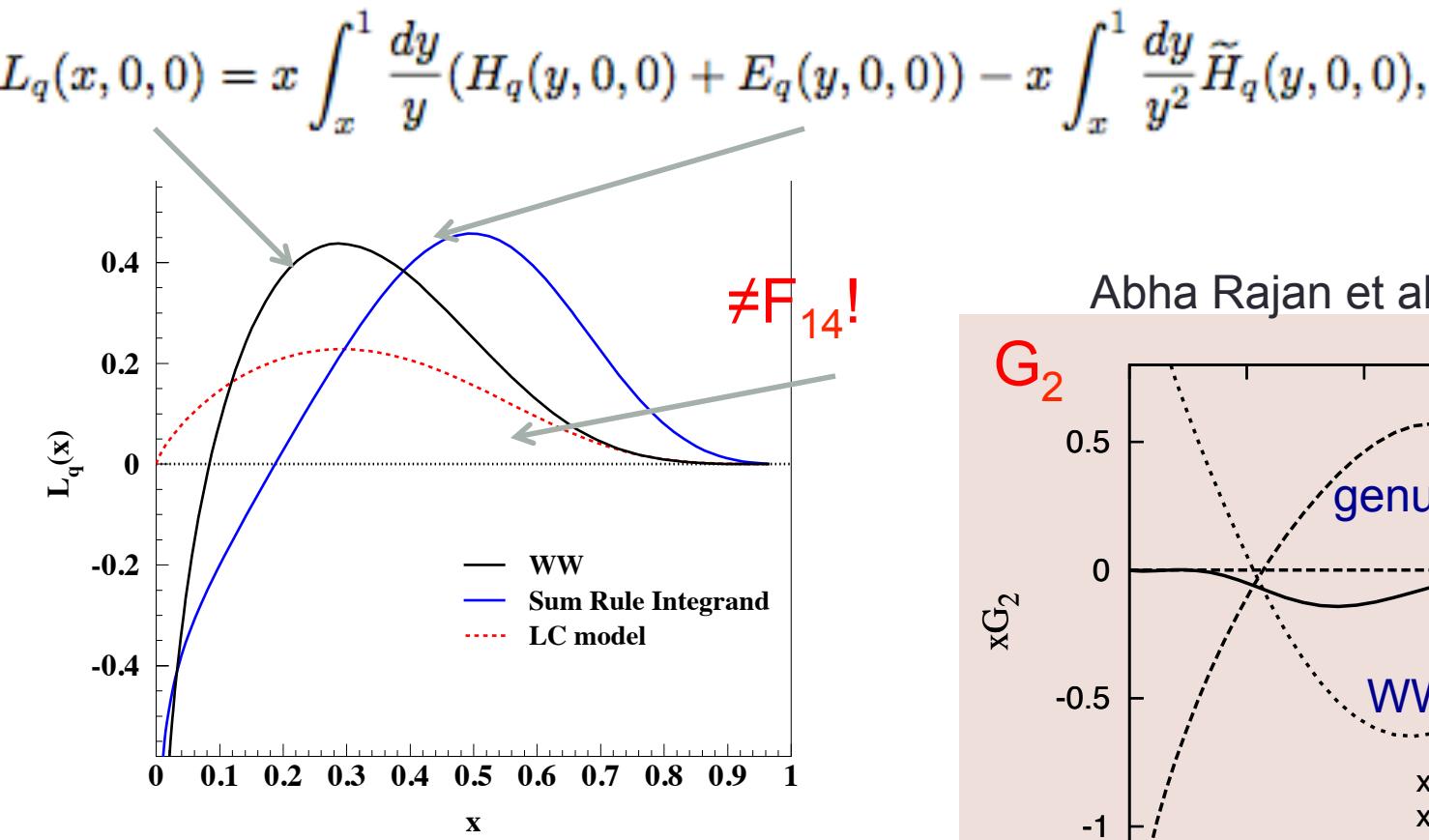
$F_{14}^{(1)}$ 

with B. Kriesten and A. Rajan,  
using diquark model



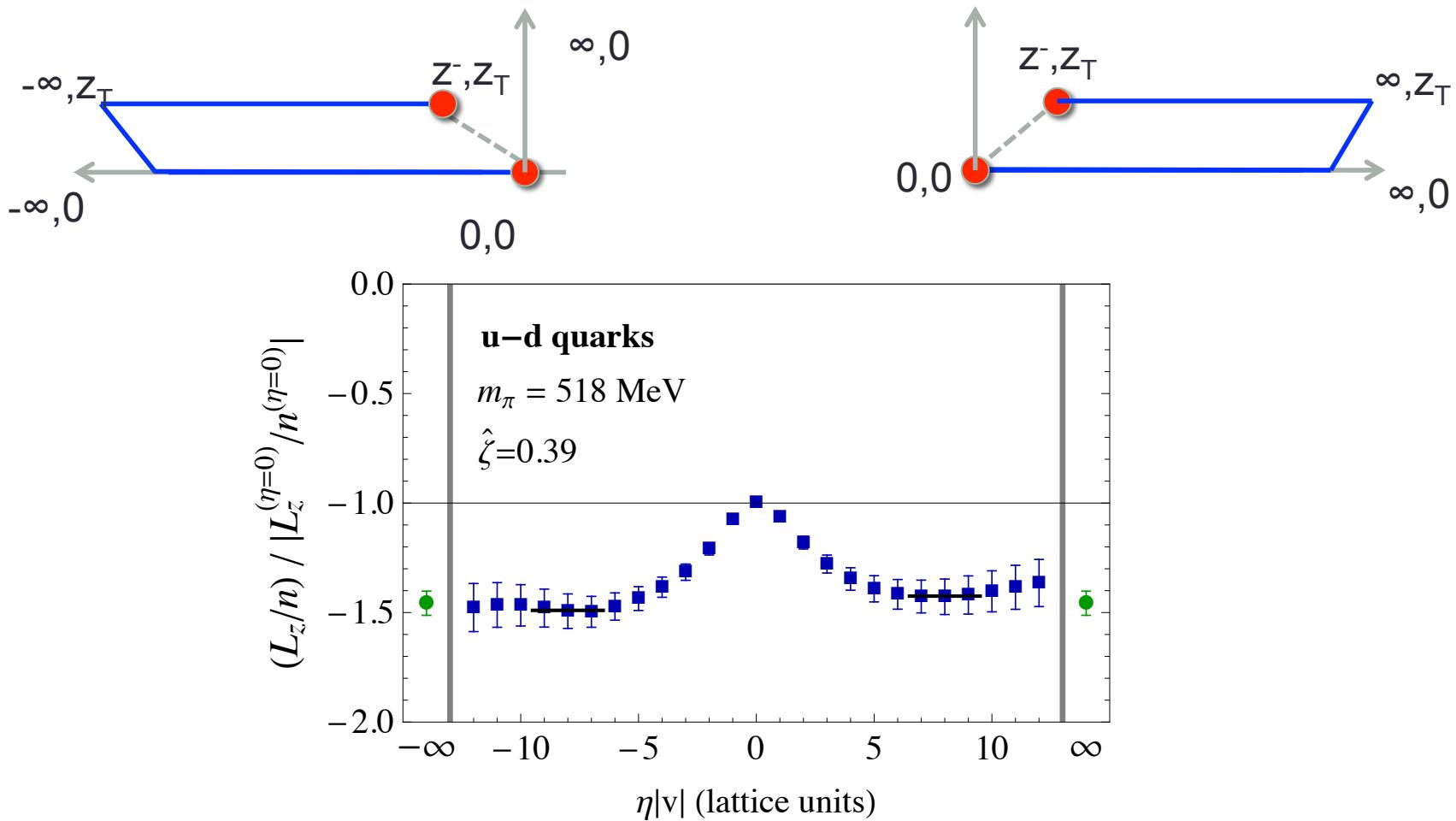
Ratio  $L_{Ji}/L_{JM}=0.72$

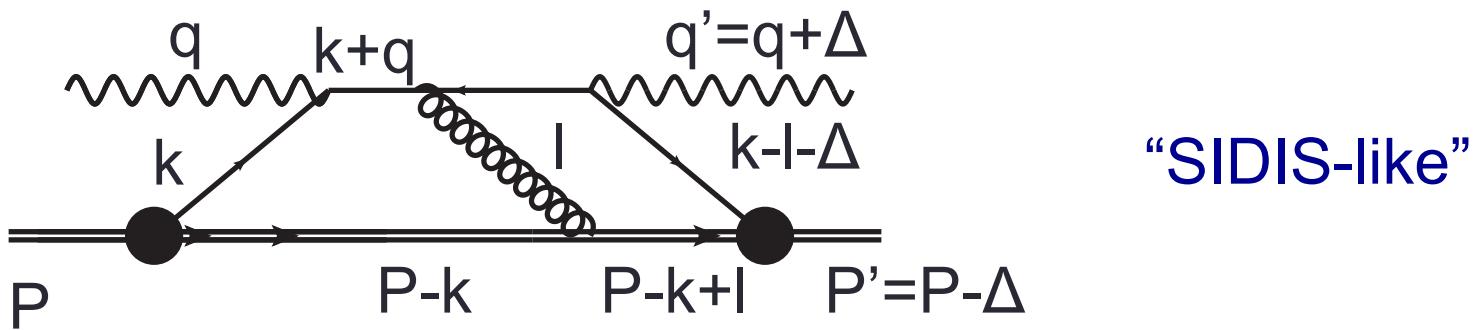
Lattice Ratio  $L_{Ji}/L_{JM}=0.62\pm0.16(\text{stat})$   
(extrapolated at  $\zeta=\infty$ )



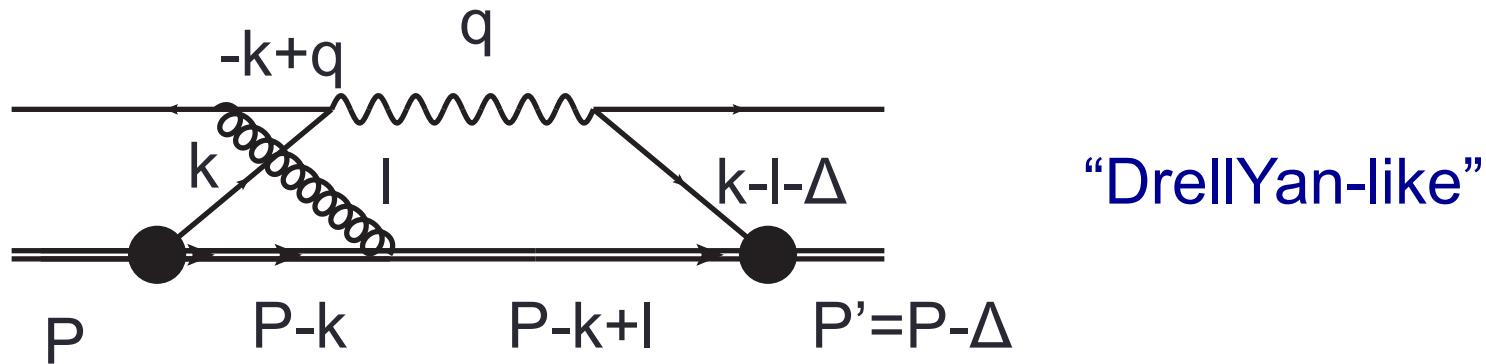
# 5. PROCESS DEPENDENCE

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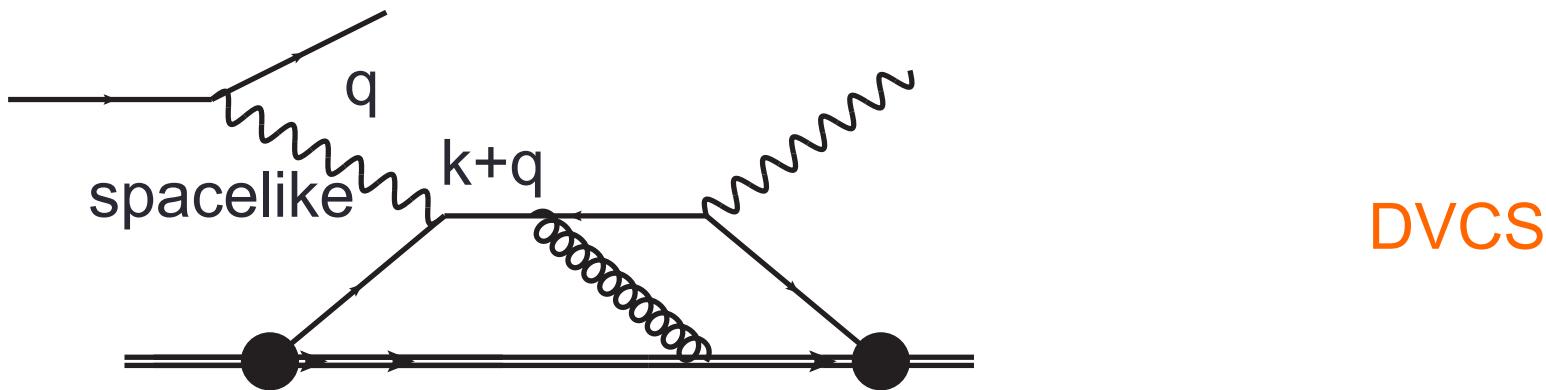




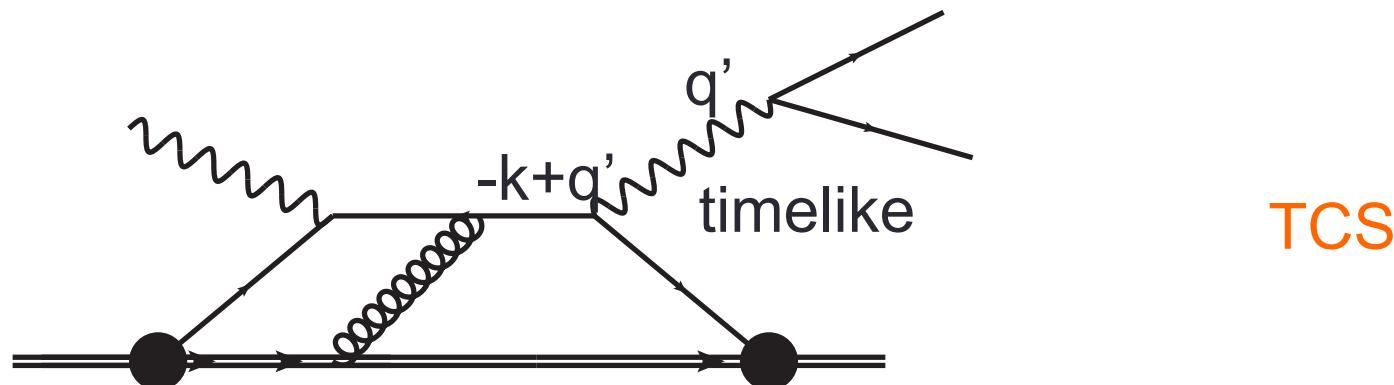
$$F_{1,4} = \int \frac{d^2 l}{(2\pi)^2} \frac{e_c^2 g_s^2 M^2 2P^+ (1-x)^2 \left(1 + \frac{l_T}{k_T} \cos \phi_l\right)}{2x(l_T^2 + m_g^2)((k-l)^2 - M_\Lambda^2)^2 ((k-\Delta)^2 - M_\Lambda^2)^2}$$



Two additional processes: DVCS and TCS twist three contributions



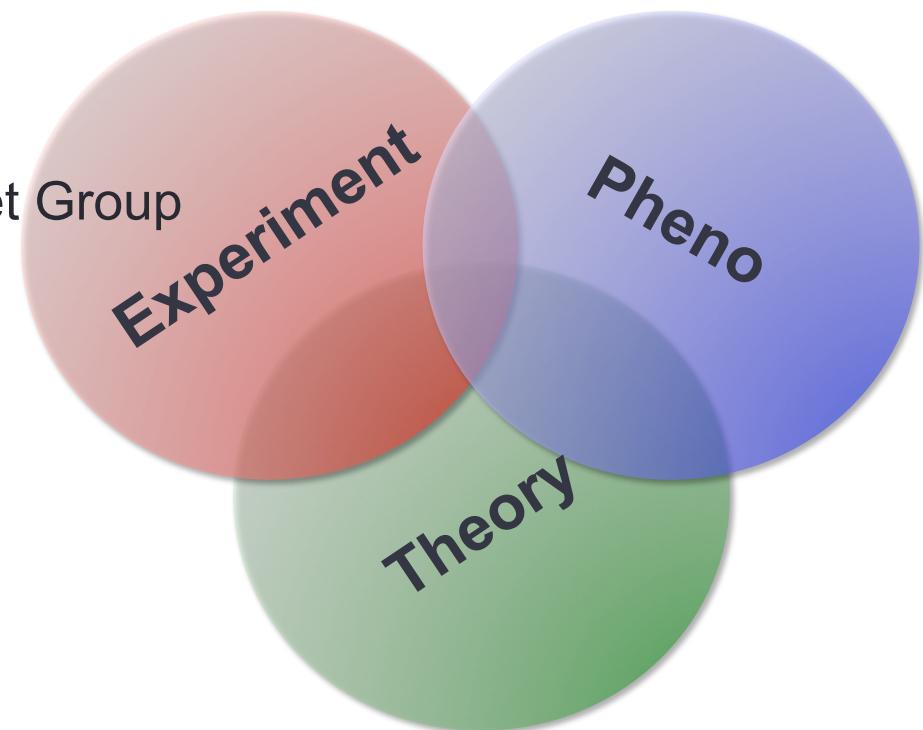
DVCS



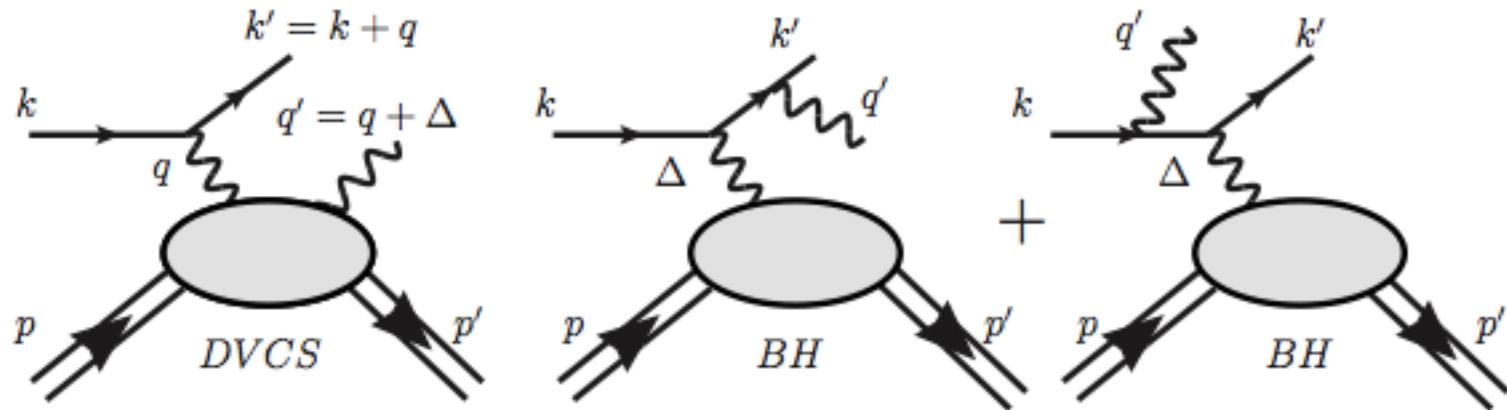
TCS

Extracting twist 3 GPDs from these processes will allow us to zoom into aspects of the “sign change”

Dustin Keller & U.Va. Polarized Target Group



# Deeply Virtual Exclusive Photoproduction



$$\frac{d^5\sigma}{dx_{Bj} dQ^2 d|t| d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s - M^2)^2 \sqrt{1 + \gamma^2}} |T|^2 ,$$

$$T(k, p, k', q', p') = T_{DVCS}(k, p, k', q', p') + T_{BH}(k, p, k', q', p'),$$

# BASIC MODULE (based on helicity amplitudes)

$$\sum_{(\Lambda'_\gamma, \Lambda')} \left( T_{DVCS, \Lambda \Lambda'}^{h \Lambda'_\gamma} \right)^* T_{DVCS, \Lambda \Lambda'}^{h \Lambda'_\gamma} =$$

$$\frac{1}{Q^2} \frac{1}{1-\epsilon} \left\{ (F_{\Lambda+}^{11} + F_{\Lambda-}^{11} + F_{\Lambda+}^{-1-1} + F_{\Lambda-}^{-1-1}) + \epsilon (F_{\Lambda+}^{00} + F_{\Lambda-}^{00}) \right.$$

$$+ 2\sqrt{\epsilon(1+\epsilon)} \operatorname{Re} (-F_{\Lambda+}^{01} - F_{\Lambda-}^{01} + F_{\Lambda+}^{0-1} + F_{\Lambda-}^{0-1}) + 2\epsilon \operatorname{Re} (F_{\Lambda+}^{1-1} + F_{\Lambda-}^{1-1})$$

$$+(2h) \left[ \sqrt{1-\epsilon^2} (F_{\Lambda+}^{11} + F_{\Lambda-}^{11} - F_{\Lambda+}^{-1-1} - F_{\Lambda-}^{-1-1}) \right]$$

polarized lepton

$$- 2\sqrt{\epsilon(1-\epsilon)} \operatorname{Re} (F_{\Lambda+}^{01} + F_{\Lambda-}^{01} + F_{\Lambda+}^{0-1} + F_{\Lambda-}^{0-1}) \Big] \Big\}$$

Helicity amplitudes

Virtual Photon helicities

$$F_{\Lambda\Lambda'}^{\Lambda^{(1)}\gamma^*\Lambda^{(2)}\gamma^*} = \sum_{\Lambda_{\gamma'}} \left( f_{\Lambda\Lambda'}^{\Lambda^{(1)}\gamma^*\Lambda\gamma'} \right)^* f_{\Lambda\Lambda'}^{\Lambda^{(2)}\gamma^*\Lambda\gamma'}$$

Initial and final proton helicities

$$F_{++}^{11} = (1-\xi^2) \mid \mathcal{H} + \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[ (\mathcal{H}^* + \widetilde{\mathcal{H}})^*(\mathcal{E} + \widetilde{\mathcal{E}}) + (\mathcal{H} + \widetilde{\mathcal{H}})(\mathcal{E}^* + \widetilde{\mathcal{E}}^*) \right]$$

$$F_{--}^{11} = (1-\xi^2) \mid \mathcal{H} - \widetilde{\mathcal{H}} \mid^2 - \xi^2 \left[ (\mathcal{H}^* - \widetilde{\mathcal{H}})^*(\mathcal{E} - \widetilde{\mathcal{E}}) + (\mathcal{H} - \widetilde{\mathcal{H}})(\mathcal{E}^* - \widetilde{\mathcal{E}}^*) \right]$$

$$F_{+-}^{11} = \frac{t_0-t}{4M^2} \mid \mathcal{E} + \xi \widetilde{\mathcal{E}} \mid^2$$

$$F_{-+}^{11} = \frac{t_0-t}{4M^2} \mid \mathcal{E} - \xi \widetilde{\mathcal{E}} \mid^2$$

# Phase dependence

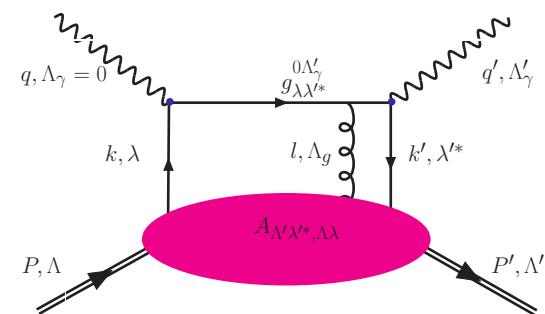
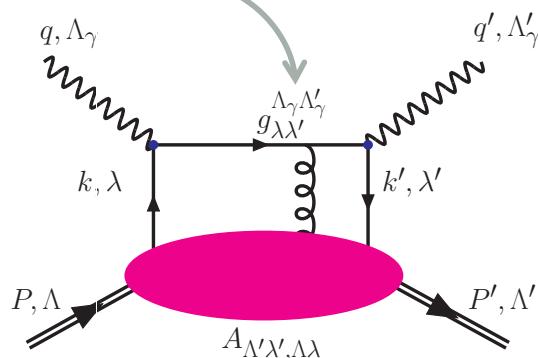
$$f \rightarrow e^{i[\Lambda_{\gamma^*} - \Lambda_{\gamma'} - (\Lambda - \Lambda')] \phi}$$

The phase is determined entirely by the virtual photon helicity which can be different for the amplitude and its conjugate

## Twist 3

$$f_{\Lambda\Lambda'}^{01} = g_{-\star+}^{01} \otimes A_{\Lambda'+,\Lambda-\star} + g_{-+\star}^{01} \otimes A_{\Lambda'+\star,\Lambda-} + g_{+\star-}^{01} \otimes A_{\Lambda'-,\Lambda+\star} + g_{+-\star}^{01} \otimes A_{\Lambda'-\star,\Lambda+}$$

“Bad” component (exchanged gluon flips the quark chirality)



## The unpolarized cross section: example

$$\sigma^{UU} = \frac{\Gamma}{Q^2(1-\epsilon)} \left[ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} \right]$$

$$F_{UU,T} = 2(F_{++}^{11} + F_{+-}^{11} + F_{-+}^{11} + F_{--}^{11}),$$

$$F_{UU,L} = 2F_{++}^{00}$$

$$F_{UU}^{\cos \phi} = \text{Re} [F_{++}^{01} + F_{--}^{01}]$$

$$F_{UU}^{\cos 2\phi} = \text{Re} [F_{++}^{1-1} + F_{+-}^{1-1} + F_{-+}^{1-1} + F_{--}^{1-1}]$$

**Twist 2**

**Twist 4**

**Twist 3**

**Photon helicity flip:  
transverse gluons**

We connect the two 3 amps DVCS formalism with the TMD, GPD, GTMD comprehensive parametrization in Meissner Metz and Schlegel, JHEP08 (2009)

### Example

$$A_{+-,++^*} = \frac{1}{2} \left( \tilde{E}_{2T} - \bar{E}_{2T} + \tilde{E}'_{2T} + \bar{E}'_{2T} \right)$$

$$A_{+-^*,++} = \frac{1}{2} \left( -\tilde{E}_{2T} + \bar{E}_{2T} + \tilde{E}'_{2T} + \bar{E}'_{2T} \right)$$

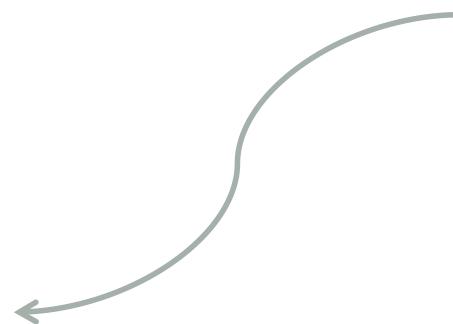
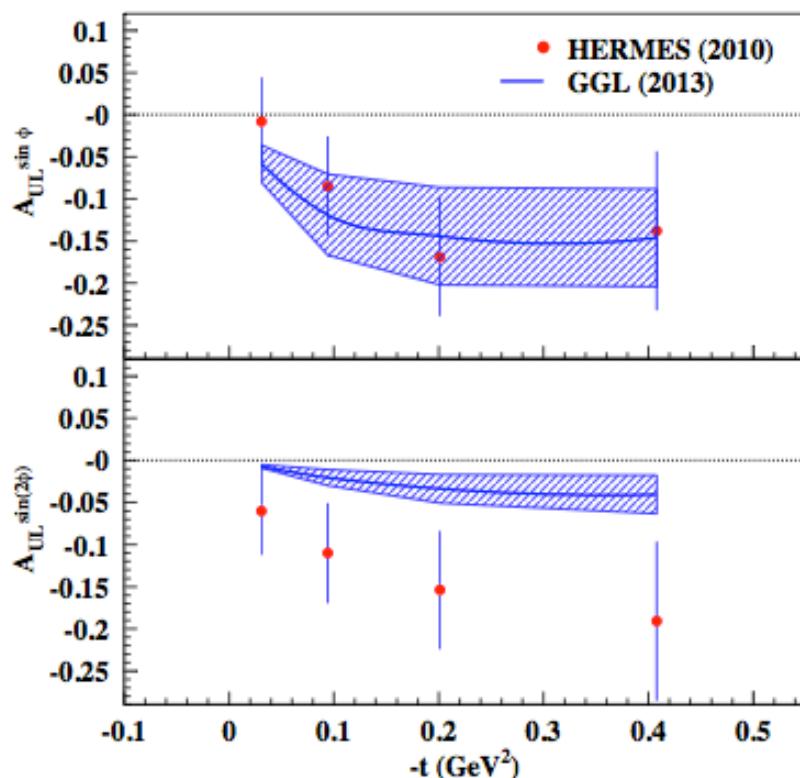
Orbital angular momentum

⋮

Spin Orbit interaction

# DVCS: bilinears of tw 2 and tw 3 CFFs

$$F_{++}^{01} = \mathcal{P} \left[ \mathcal{H}^*(\tilde{\mathcal{E}}_{2T} - \bar{\mathcal{E}}_{2T} + \dots), \dots \right]$$

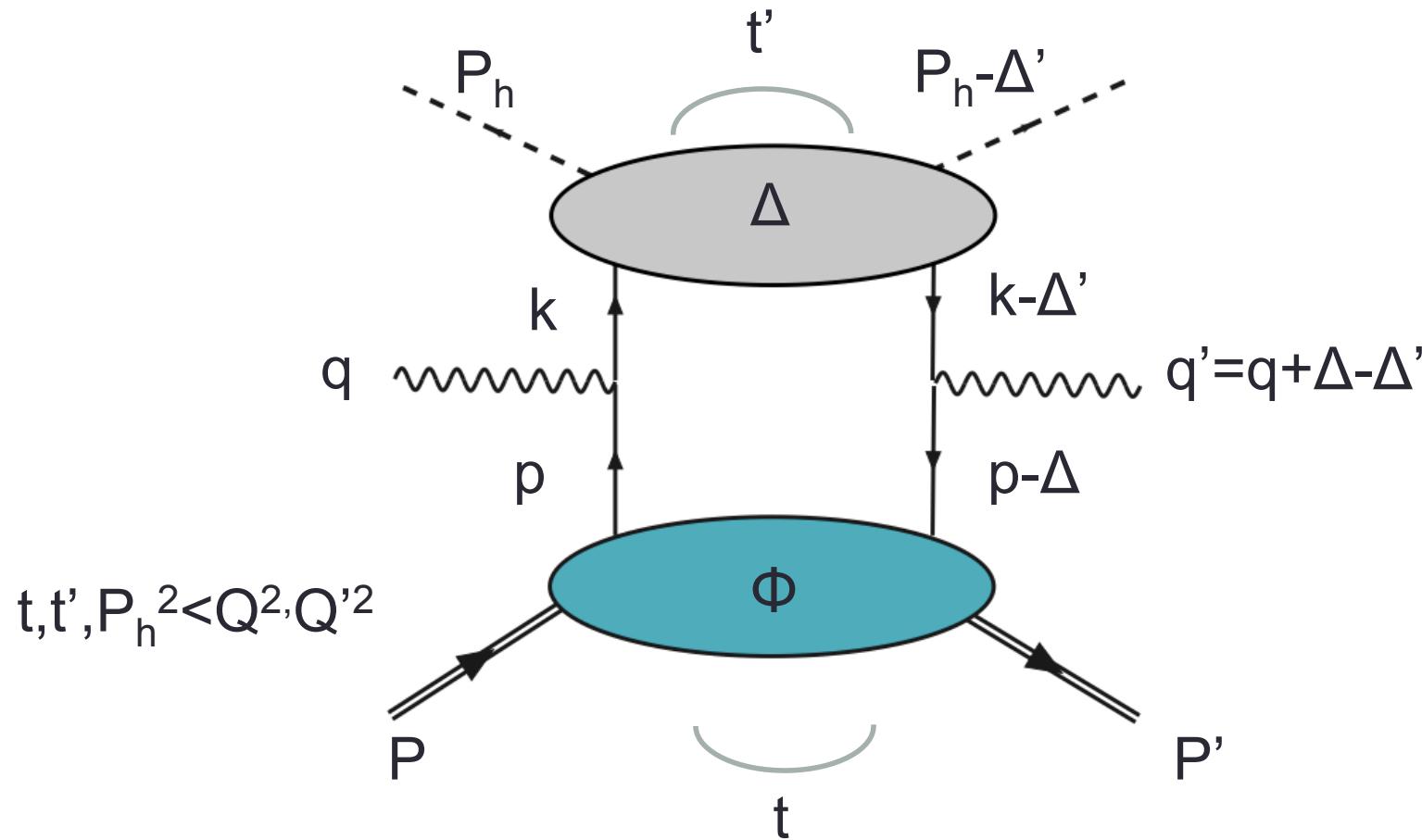


Extraction from experiment  
using Wandzura Wilczek approximation

A.Courtoy, G.Goldstein, O.Gonzalez  
Hernandez, S.L. and A.Rajan, PLB  
731(2014)

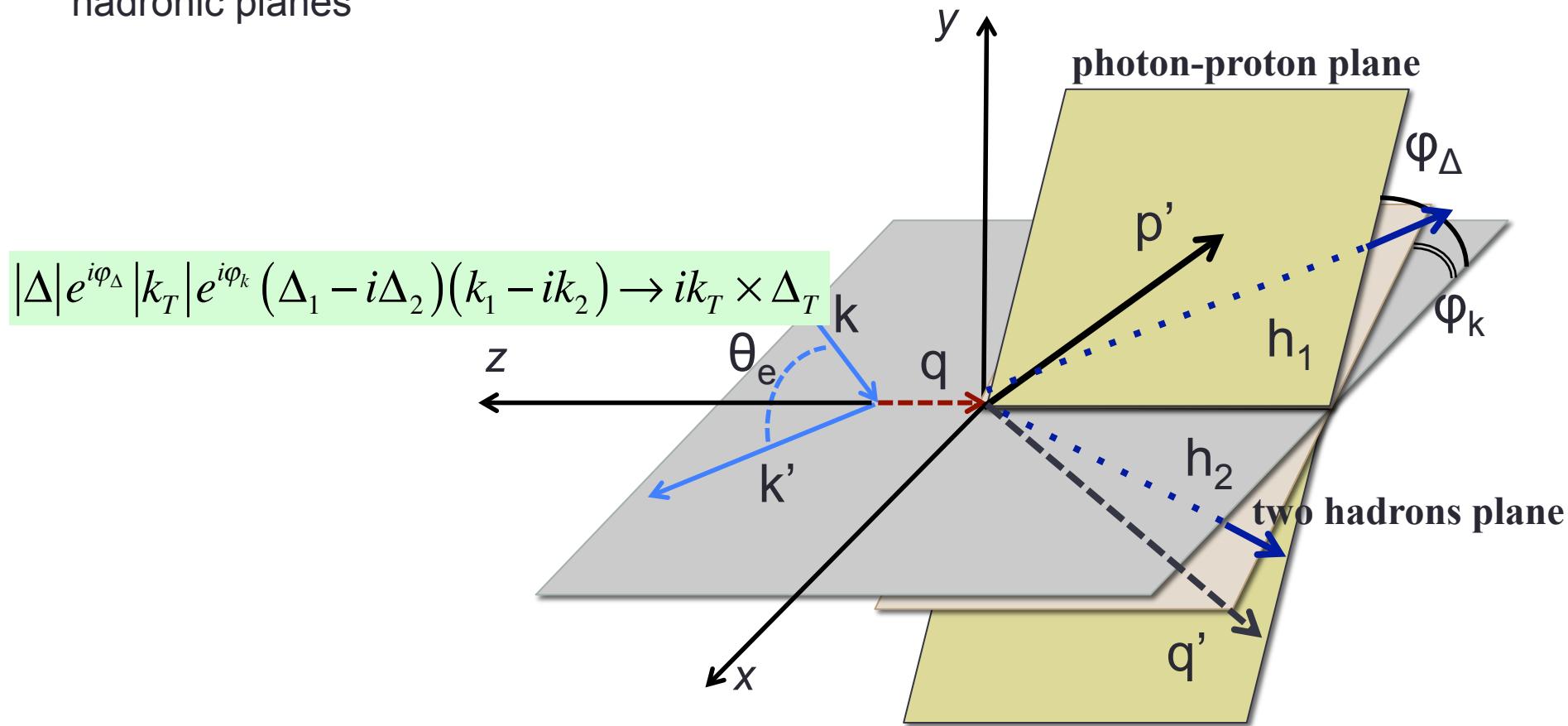
# Our proposal: GTMDs from Double DVCS hadron production (off-forward SIDIS)

$$ep \rightarrow e' \pi^+ \pi^- \mu^+ \mu^- p'$$



## Helicity amplitude formalism for DDVCS hadron production

- To measure  $F_{14}$  one has to be in a frame where the reaction cannot be viewed as a two-body quark-proton scattering
- In the CoM the amplitudes are imaginary → UL term goes to 0 unless one defines two hadronic planes



# Conclusions and Outlook

The connection we established through the new relations between (G)TMDs and Twist 3 GPDs, not only allows us to evaluate the angular momentum sum rule, it also opens many interesting avenues:

- It allows us to study in detail the role of quark-gluon correlations, in a framework where the role of  $k_T$  and off-shellness,  $k^2$ , is manifest.
- OAM was obtained so far by subtraction (also in lattice). We can now both calculate OAM on the lattice (GTMD) and validate this through measurements (twist 3 GPD)
- It provides an ideal setting to test renormalization issues, evolution etc...
- QCD studies at the amplitude level shed light on chiral symmetry breaking
- **TWIST THREE GPDs ARE CRUCIAL TO STUDY QCD AT THE AMPLITUDE LEVEL**

# Back up

# Helicity and Transverse Spin Structures of H+E

$$(A_{++,++} + A_{+-,+-} + A_{-+,--+} + A_{--,--}) + (A_{++,+-} + A_{+-,--} - A_{--,+-} - A_{-+,++})$$

$$(A_{++,++}^X + A_{+-,+-}^X + A_{-+,--+}^X + A_{--,--}^X) + (A_{++,+-}^X + A_{+-,--}^X - A_{--,+-}^X - A_{-+,++}^X)$$

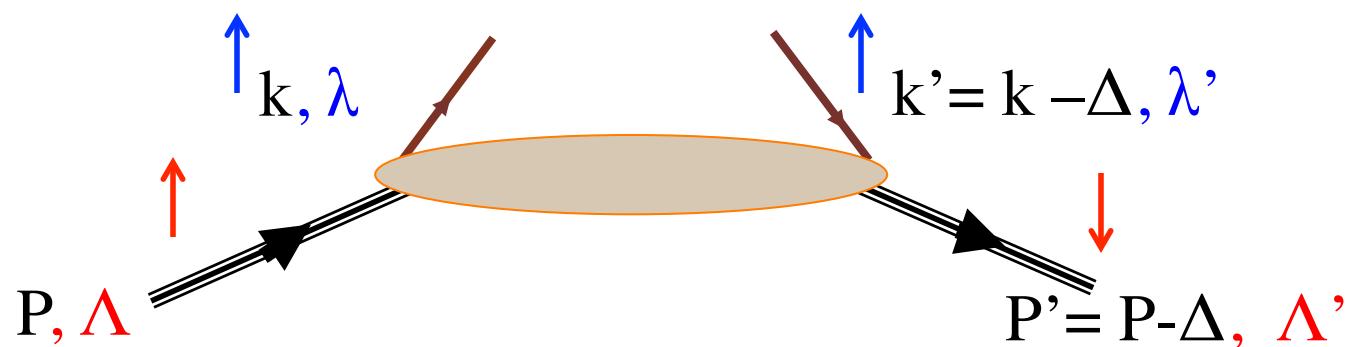
$$\approx H - i\Delta_2 E$$

Helicity flips

=  
Transv. spin  
conserved  
“non flip”

E measures J, not L, but a change of one unit of L (because of the helicity flip)

$$S_z = -1/2 \rightarrow 1/2 \Rightarrow \Delta L_z = 1 \quad \text{at fixed } J$$



Brodsky and Drell '80s, Belitsky, Ji and Yuan, '90's