

Spin Dependent Quark GPDs in a Flexible Spectator Model and Deeply Virtual Lepton Scattering



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Tufts University
GHP 2017, Feb.1-3
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Abstract

Chiral Even and Odd Generalized Parton Distributions (GPDs) for Valence Quarks are obtained in a **"flexible" spectator model**. The parametrization is constrained by nucleon form factors, PDFs and some earlier Deeply Virtual Compton Scattering data. The model has been extended to cover the full range of experimentally attainable kinematics. A broad range of measured and measurable deeply virtual processes - cross sections and polarization asymmetries - is compared with existing data and predicted for future data. **Neutrino induced π** production is included and compared with electroproduction.



GPDs, Electroproduction & Extension to Neutrinos

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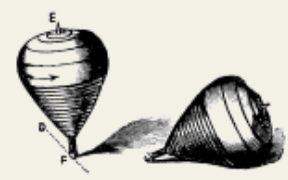
- GRG, O. Gonzalez-Hernandez, S.Liuti, PRD(2015) arXiv:1311.0483
- GRG, O. Gonzalez-Hernandez, S.Liuti, arXiv:1401.0438
- Ahmad, GRG, Liuti, PRD79, 054014, (2009)
- Gonzalez, GRG, Liuti PRD84, 034007 (2011)
- GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- Gonzalez Hernandez, Liuti, GRG, Kathuria, PRC88, 065206 (2013)
- GRG, Liuti, PoS DIS2016 (2016) 238
- GRG, Liuti, EPJ Web Conf. 112 (2016) 01009
- T. McAskill, Tufts U. dissertation (2014)
- GRG, Gonzalez Hernandez, Liuti, McAskill, Proc. NuFAct09, 0911.0495
- J.Poage, Tufts U. dissertation (2016)



OUTLINE

Collaborators: S. Liuti, O. Gonzalez Hernandez, T. McAskill, J. Poage

- **GPDs, Model– Reggeized spectator “flexible parameterization”**
- **Recall Valence quarks: Chiral Even & Odd**
- **Neutrinos & π production - extend “flexible”**
- **Gluon & Sea GPDs**
- **Preliminary results**
- **Some Observable quantities**



GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\boxed{H^q} \gamma^+ + \boxed{E^q} \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} \right] u(p, \lambda),$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^+ \gamma_5 \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\boxed{\tilde{H}^q} \gamma^+ \gamma_5 + \boxed{\tilde{E}^q} \frac{\gamma_5 \Delta^+}{2m} \right] u(p, \lambda),$$

Chiral even GPDs
-> Ji sum rule

$$\langle J_q^x \rangle = \frac{1}{2} \int dx [H(x, 0, 0) + E(x, 0, 0)] x$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i} \psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, z_T=0}$$

$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \left[\boxed{H_T^q} i\sigma^{+i} + \boxed{\tilde{H}_T^q} \frac{P^+ \Delta^i - \Delta^+ P^i}{m^2} \right.$$

$$\left. + \boxed{E_T^q} \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m} + \boxed{\tilde{E}_T^q} \frac{\gamma^+ P^i - P^+ \gamma^i}{m} \right] u(p, \lambda)$$

Chiral odd GPDs
-> transversity
How to measure
and/or
parameterize them?



Normalizing & constraining quark GPDs – Chiral even

Form factor,

Forward limit

$$\int_0^1 H_q(x, \xi, t) dx = F_1^q(t), \quad H_q(x, 0, 0) = q(x) \quad \text{Integrates to charge}$$

$$\int_0^1 E_q(x, \xi, t) dx = F_2^q(t) \quad \rightarrow \text{Anomalous magnetic moments}$$

$$\int_0^1 \tilde{H}_q(x, \xi, t) dx = g_A^q(t), \quad \tilde{H}_q(x, 0, 0) = \Delta q(x) = q_{\Rightarrow}^{\vec{}}(x) - q_{\Rightarrow}^{\leftarrow}(x)$$

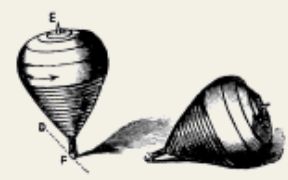
Integrates to axial charge

$$\int_0^1 \tilde{E}_q(x, \xi, t) dx = g_P^q(t)$$



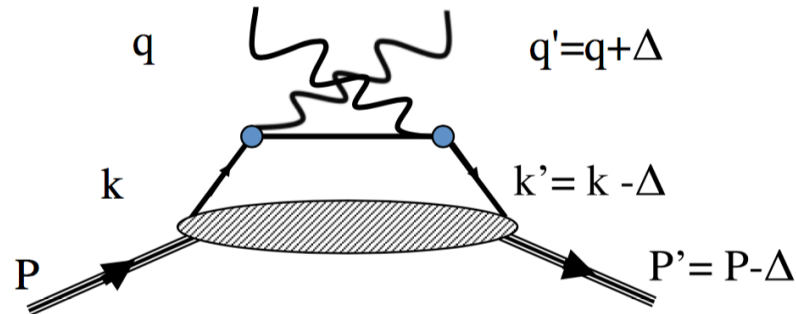
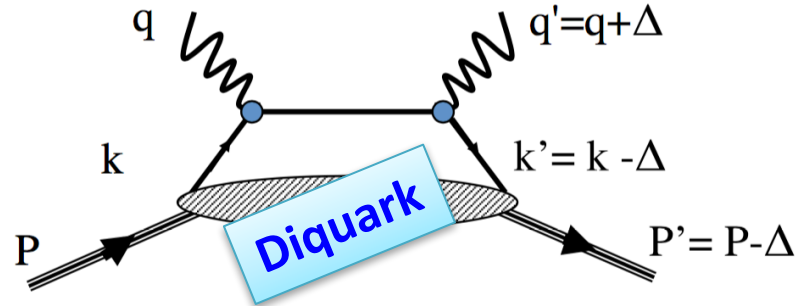
The Model for valence quarks– Reggeized Diquarks

The Model – first for Chiral Even –
Reggeized Diquark Spectator
Diquark: Color anti-3, scalar & axial vector



“flexible” covariant model

Gonzalez, GG, Liuti PRD84, 034007 (2011)



$$\begin{aligned}
 H = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \\
 & \times \int d^2 k_{\perp} \frac{[(m + MX)(m + M \frac{X-\zeta}{1-\zeta}) + \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2} \\
 & + \frac{\zeta^2}{4(1 - \zeta)} E,
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 E = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_{\perp} \\
 & \times \frac{-2M(1 - \zeta)[(m + MX) \frac{\tilde{\mathbf{k}} \cdot \Delta}{\Delta_{\perp}^2} - (m + M \frac{X-\zeta}{1-\zeta}) \frac{k_{\perp} \cdot \Delta}{\Delta_{\perp}^2}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2},
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 \tilde{H} = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \\
 & \times \int d^2 k_{\perp} \frac{[(m + MX)(m + M \frac{X-\zeta}{1-\zeta}) - \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2} \\
 & + \frac{\zeta^2}{4(1 - \zeta)} \tilde{E},
 \end{aligned} \tag{29}$$

$$\begin{aligned}
 \tilde{E} = & \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_{\perp} \\
 & \times \frac{-\frac{4M(1-\zeta)}{\zeta} [(m + MX) \frac{\tilde{\mathbf{k}} \cdot \Delta}{\Delta_{\perp}^2} + (m + M \frac{X-\zeta}{1-\zeta}) \frac{k_{\perp} \cdot \Delta}{\Delta_{\perp}^2}]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2},
 \end{aligned} \tag{30}$$



Reggeizing diquark model

Diquark+quark & fixed masses (e.g. at $\xi=0$)

$$H_{M_X^q, m_q}^{M_\Lambda^q} = \mathcal{N}_q \int \frac{d^2 k_\perp}{1-x} \frac{[(m_q + Mx)(m_q + Mx) + \mathbf{k}_\perp \cdot \tilde{\mathbf{k}}_\perp]}{[\mathcal{M}_q^2(x) - k_\perp^2/(1-x)]^2 [\mathcal{M}_q^2(x) - \tilde{k}_\perp^2/(1-x)]^2},$$

$$E_{M_X^q, m_q}^{M_\Lambda^q} = \mathcal{N}_q \int \frac{d^2 k_\perp}{1-x} \frac{-2M/\Delta_\perp^2 [(m_q + Mx)\tilde{\mathbf{k}}_\perp \cdot \mathbf{\Delta}_\perp - (m_q + Mx)\mathbf{k}_\perp \cdot \mathbf{\Delta}_\perp]}{[\mathcal{M}_q^2(x) - k_\perp^2/(1-x)]^2 [\mathcal{M}_q^2(x) - \tilde{k}_\perp^2/(1-x)]^2},$$

Diquark mass “spectrum”
as in Brodsky, Close & Gunion
Phys. Rev. D8, 3678 (1973)

$$F_T^q(X, \zeta, t) = \mathcal{N}_q \int_0^\infty dM_X^2 \rho(M_X^2) F_T^{(m_q, M_\Lambda^q)}(X, \zeta, t; M_X).$$

$$\rho(M_X^2) \approx \begin{cases} (M_X^2)^\alpha & M_X^2 \rightarrow \infty \\ \delta(M_X^2 - \bar{M}_X^2) & M_X^2 \text{ few GeV}^2 \end{cases}$$

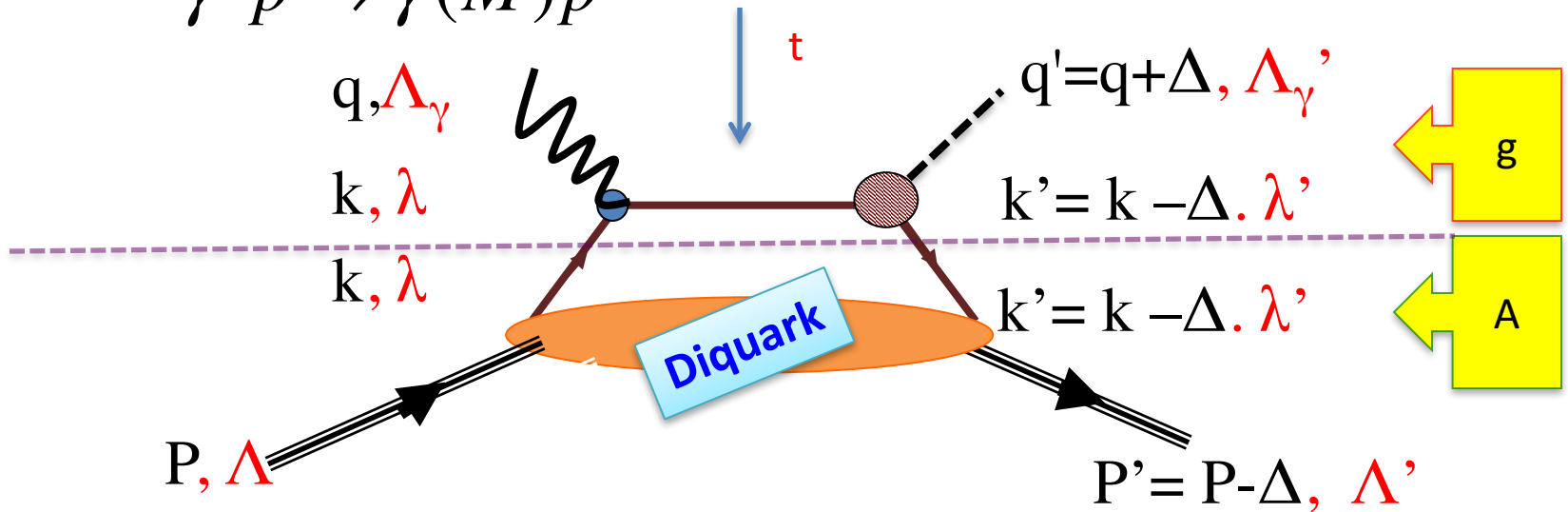
$$F_T^q(X, \zeta, t) \approx \mathcal{N}_q X^{-\alpha_q + \alpha'_q(X)t} F_T^{(m_q, M_\Lambda^q)}(X, \zeta, t; \bar{M}_X) = R_{p_q}^{\alpha_q, \alpha'_q}(X, \zeta, t) G_{M_X, m}^{M_\Lambda}(X, \zeta, t)$$

R × Dq



Connecting to exclusive processes (DVCS, DVMP...)

$$\gamma^* p \rightarrow \gamma(M) p'$$



Convolution of "hard part" with quark-proton **Helicity** amplitudes

$$f_{\Lambda_\gamma, \Lambda; \Lambda'_\gamma, \Lambda'} = \sum_{\lambda, \lambda'} g_{\lambda, \lambda'}^{\Lambda_\gamma, \Lambda'_\gamma(M)}(x, k_T, \zeta, t; Q^2) \otimes A_{\Lambda', \lambda'; \Lambda, \lambda}(x, k_T, \zeta, t)$$

$\lambda = +(-)$ λ' **chiral even** (odd)

See Ahmad, et al. PRD75, 0904003 (2007);
ibid, EPJC63, 407 (2009).

see Ahmad, GG, Liuti, PRD79, 054014, (2009)

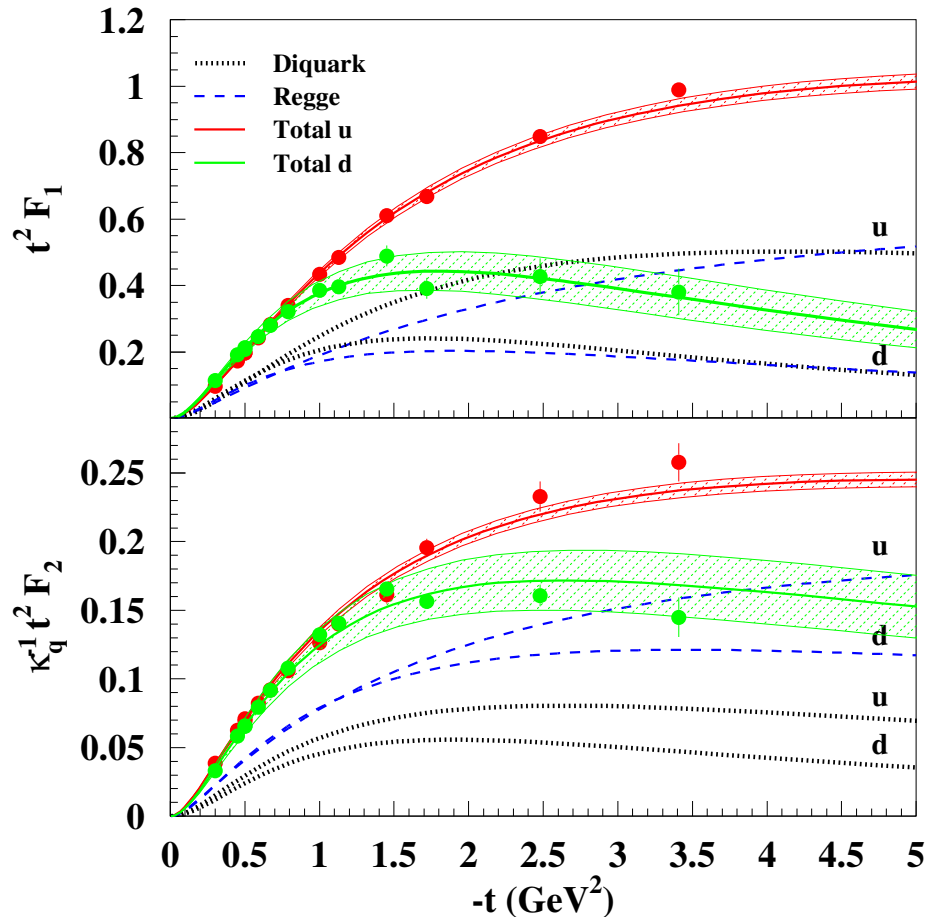
for first chiral odd GPD parameterization

Gonzalez, GG, Liuti PRD84, 034007 (2011) chiral even GPD



EM Form Factors (t dependence)

precision measurements \rightarrow tighter parameters

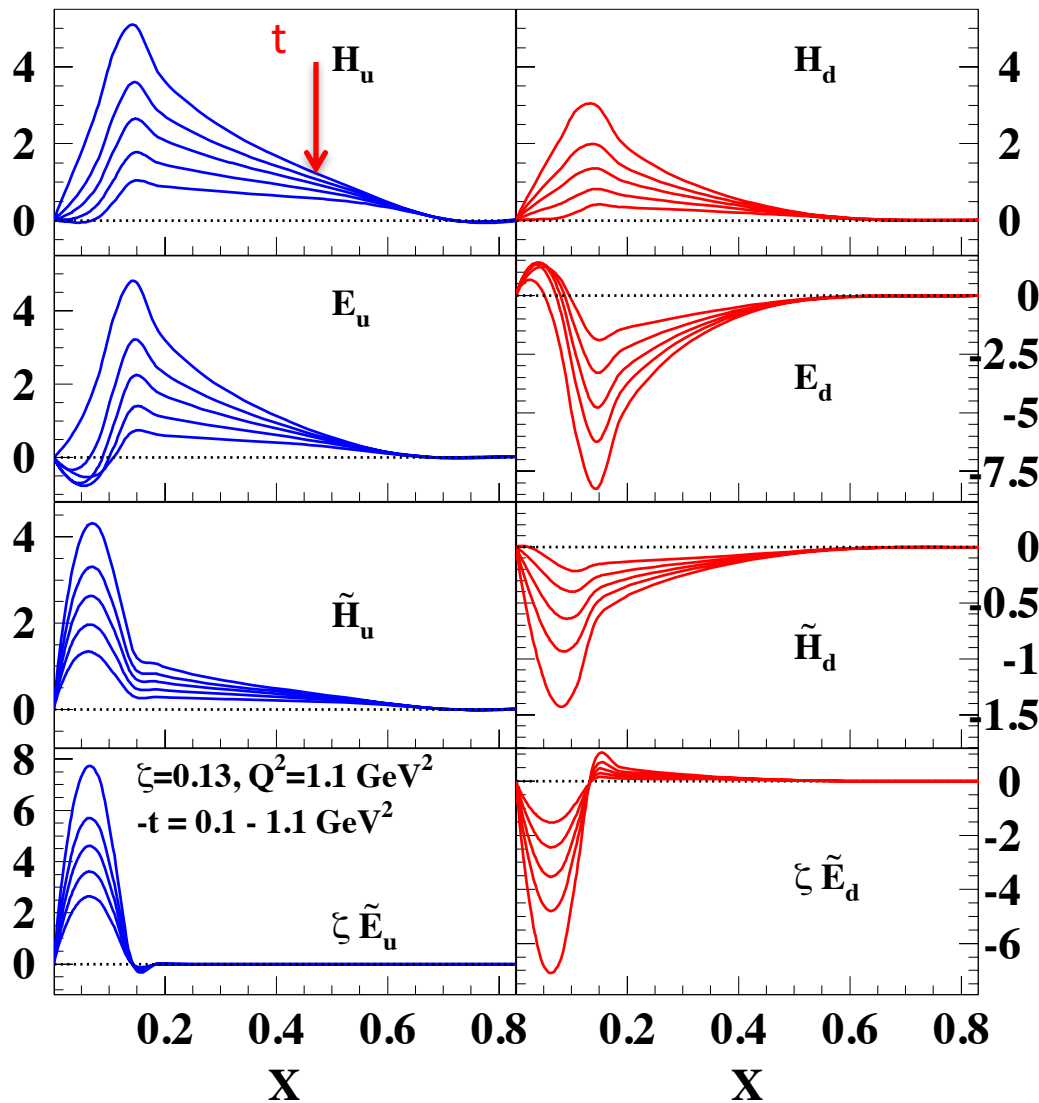


O.Gonzalez, GRG, S.Liuti, K.Kathuria PRC88, 065206(2013)

data: G.D. Cates, et al. PRL106,252003 (2011).



Chiral even GPDs



From GPDs
with evolution
to Compton
Form Factors

CFFs to helicity
amps

helicity amps to
observables

\leftrightarrow parameters



Some results – GG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011)

GOLDSTEIN, HERNANDEZ, AND LIUTI

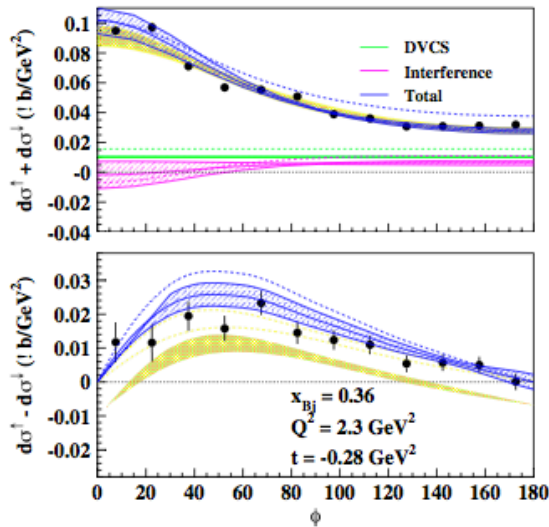


FIG. 16 (color online). Hall A data [49] for the “sum” (upper panel) and “difference” (lower panel) of the two electron beam polarizations. Shown are curves, including the contribution of the ζ -dependent factor from Eq. (34) (solid lines) and neglecting it (dashed lines). All terms (DVCS, Interference, and Total) are shown for the sum graph. The wide yellow bands in both panels represent the error of the data fit. The green band in the asymmetry graph is the theoretical error from our parametrization.

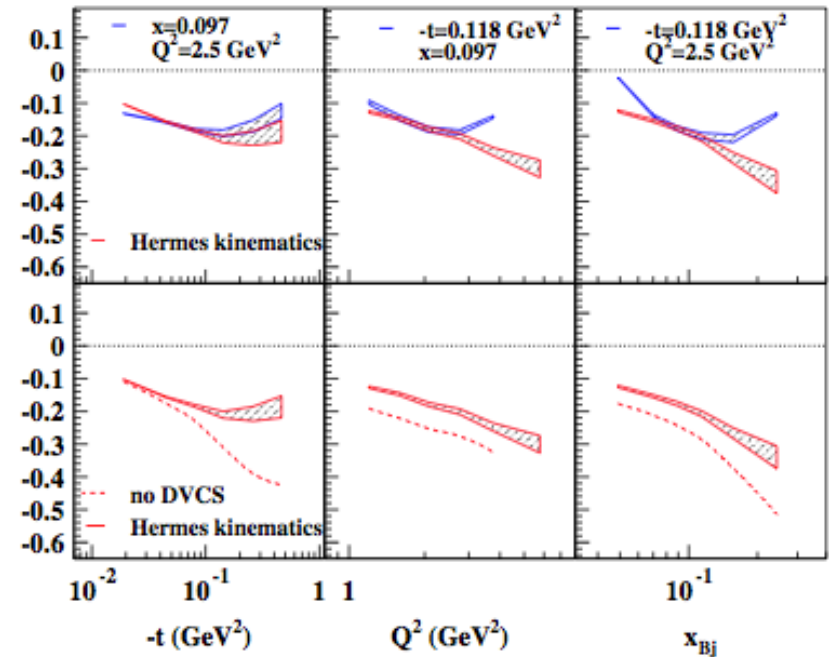


FIG. 18 (color online). Calculations at Hermes kinematics [52,53,56]. Shown is $A_{LL}(90^\circ)$ vs $-t$, Q^2 , and x_{Bj} , respectively, calculated at each kinematical bin provided by Hermes [56] (curve denoted as “Hermes kinematics”) and at the nominal average values presented in each panel. It is interesting to notice that, due to the correlation between x_{Bj} and Q^2 in the data, different features arise when using the average bin values. In the lower panels, we also show the effect of disregarding the DVCS term in the denominator (dashed curves).



Some more results – GG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011)

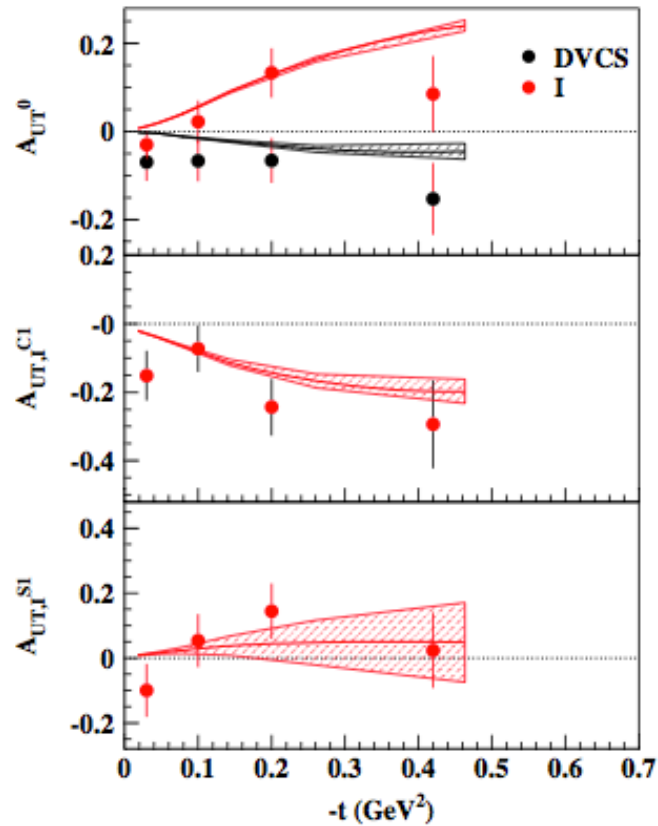
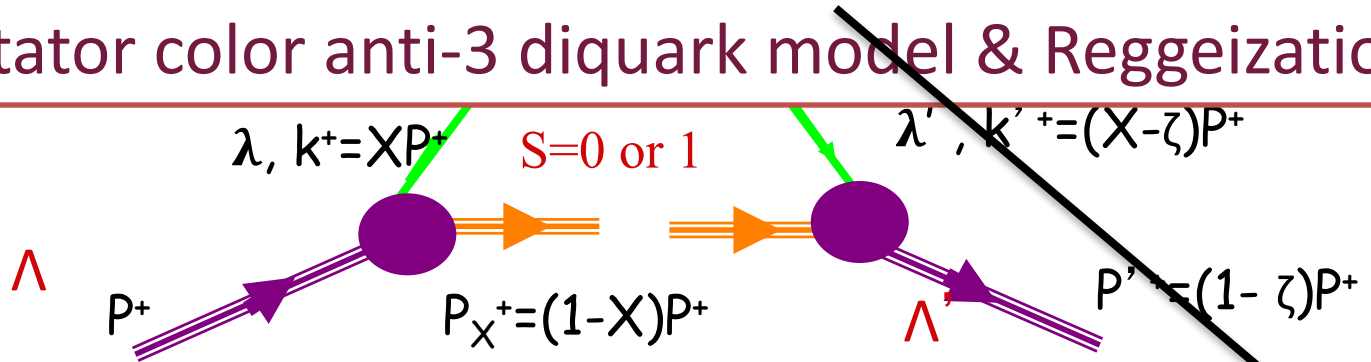


FIG. 21 (color online). Coefficients of the beam charge asymmetry, A_{UT} , extracted from experiment [52,53]. The upper panel shows the terms E and F from Eqs. (83) and (84), respectively; the middle panel shows G , and the lower panel H , both in Eq. (84). The curves are predictions obtained extending our quantitative fit of Jefferson Lab data to the Hermes set of observables.

Procedure to construct **Chiral Even GPDs** & observables Spectator color anti-3 diquark model & Reggeization



Product of diquark l.c.w.f.'s $\rightarrow A_{\Lambda\lambda; \Lambda'\lambda'=\lambda}$

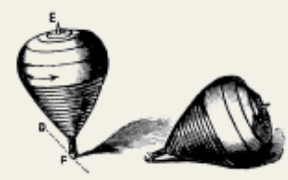
Odd

$A_{\Lambda\lambda; \Lambda'\lambda}$ \rightarrow chiral even GPDs + Evolution

$g \otimes A \rightarrow$ exclusive process helicity amps

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

$A_{\Lambda\lambda; \Lambda'\lambda} \rightarrow$ all chiral even GPDs \rightarrow DVCS, DVMP



Chiral odd quark GPDs

One question is: how do we **normalize** chiral-odd GPDs?

The only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_T(x, 0, 0) = q_{\uparrow\uparrow}^{\uparrow}(x) - q_{\uparrow\uparrow}^{\downarrow}(x) = h_1(x)$$

Transversity

Form Factors

Integrates to tensor charge δ_q

$$\int H_T^q(x, \xi, t) dx = \delta q(t)$$

$$\int \bar{E}_T^q(x, \xi, t) dx = \int \left(2\tilde{H}_T^q + E_T^q \right) dx = \kappa_T^q(t)$$

"transverse moment" κ_T^q

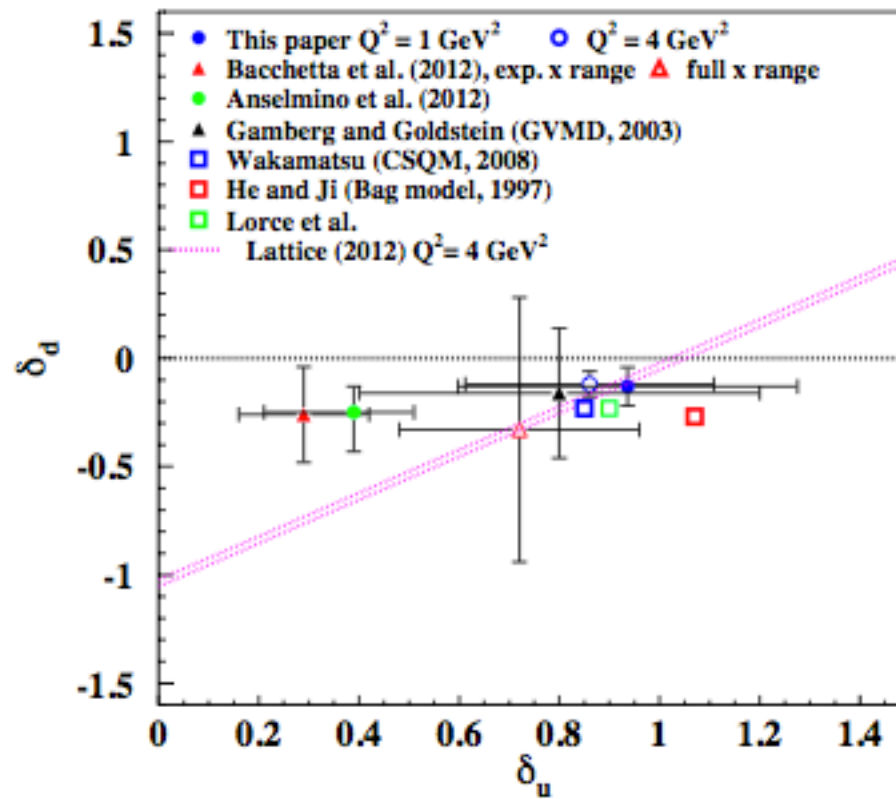
$$\int \tilde{E}_T(x, \xi, t) dx = 0$$

No direct interpretation of E_T .



Extraction of tensor charge–

GRG, O. Gonzalez-Hernandez, S.Liuti, PRD91, 114013 (2015)

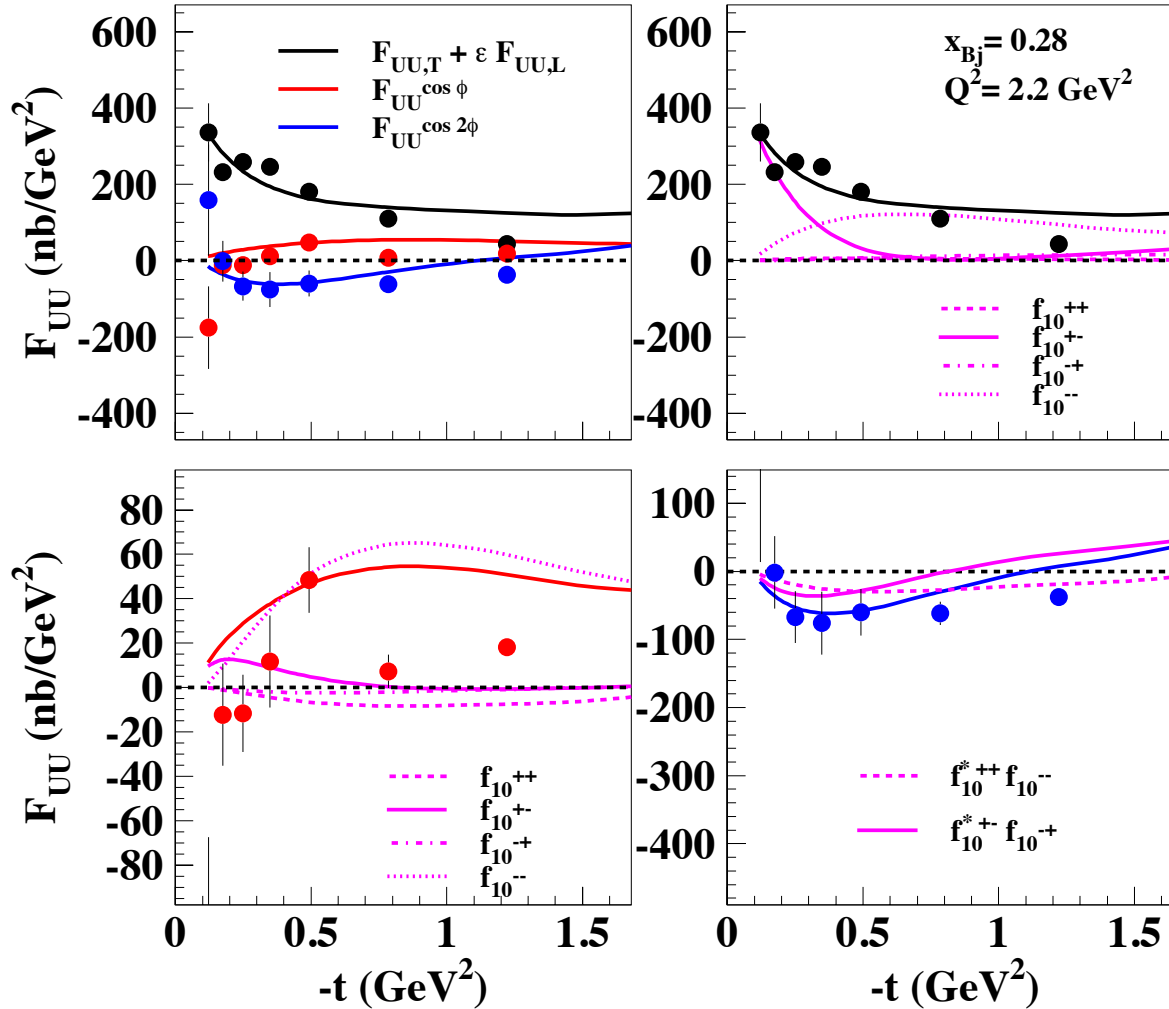




How well do the parameters fixed with DVCS data reproduce π^0 electroproduction? **Transverse $d\sigma/dt$ dominates \Rightarrow Chiral Odd**

Hall B data, Bedlinskii, et al. PRL 109, 112001 (2012)

$e+p \rightarrow e'+\pi^0+p'$



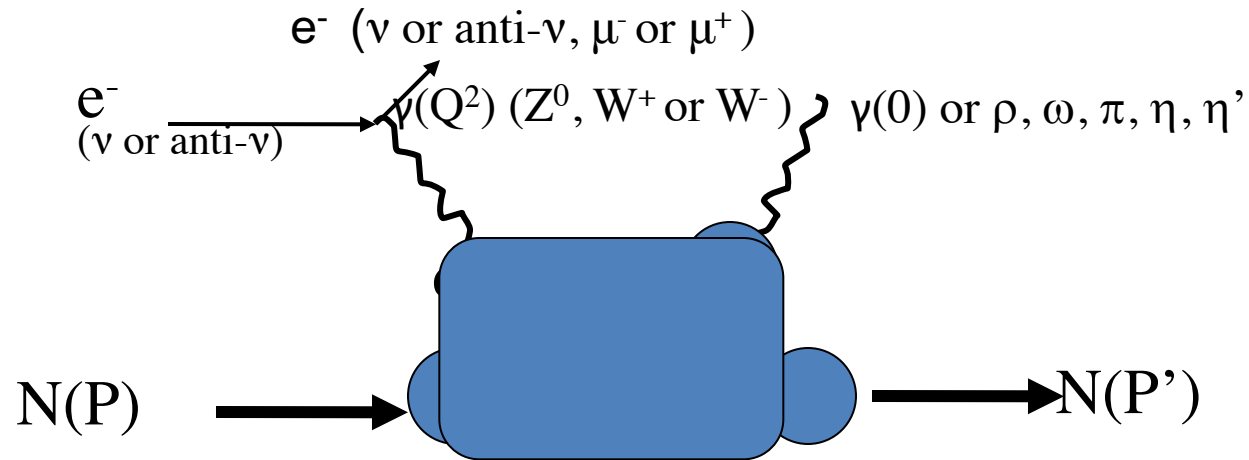


Extending Phenomenology of Flexible Model

Neutrino Production - What Processes?
How to Measure?



From exclusive electroproduction to neutrino-production



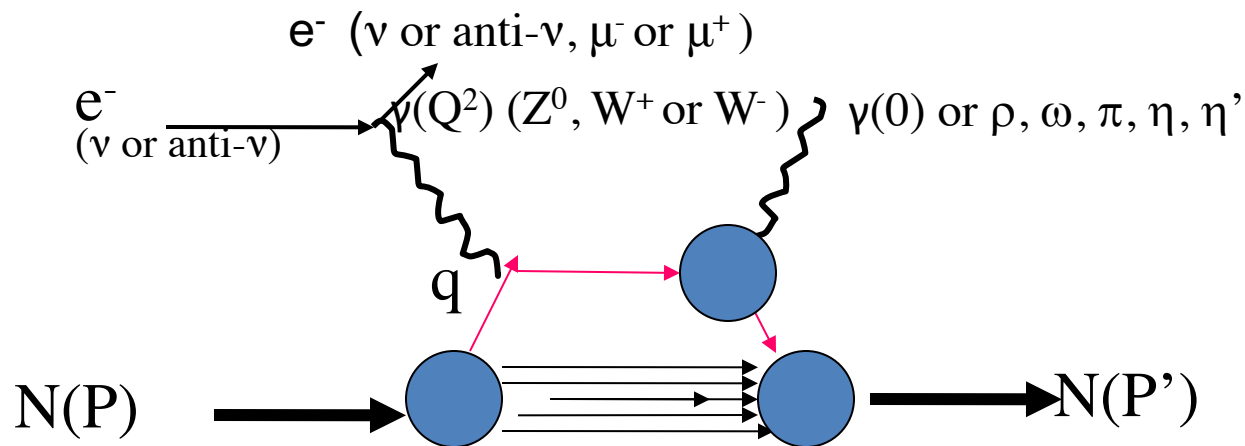
Replace EM current entering with Weak current.
Replace outgoing photon with meson e.g. $\pi^{0,+,-}$.

Neutrino Deeply Virtual Compton Scattering or
 Neutrino Deeply Virtual Meson Production

$$T_W^\nu = i \int d^4x \int d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle N'(p_2, s_2) | T \{ J_{\pi^0}(y) J_W^\nu(x) \} | N(p_1, s_1) \rangle$$



Connecting to GPDs



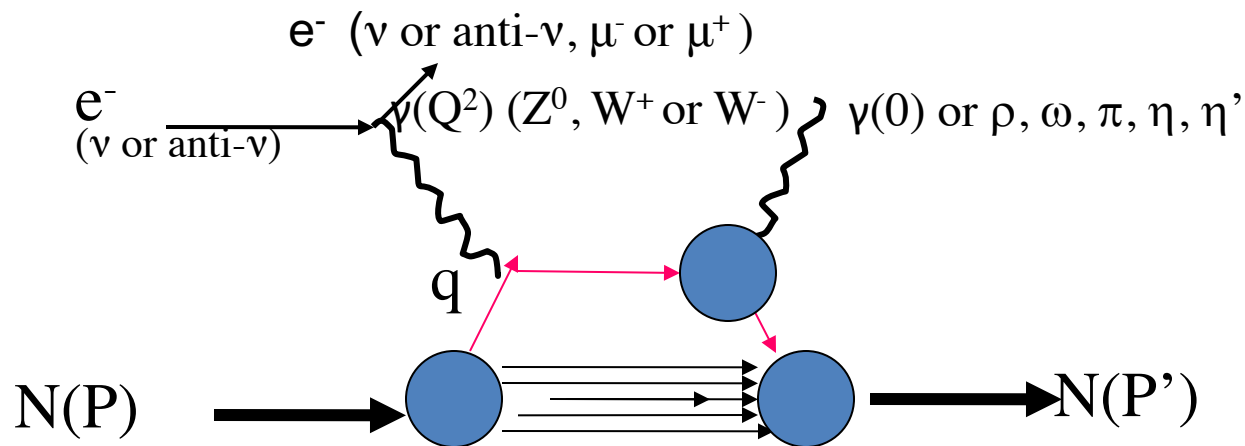
$$f_{\Lambda_\gamma, \Lambda; 0, \Lambda'} = \sum_{\lambda, \lambda'} g_{\Lambda_\gamma, \lambda; 0, \lambda'}(X, \zeta, t, Q^2) \otimes A_{\Lambda', \lambda'; \Lambda, \lambda}(X, \zeta, t).$$

$g_{\Lambda_\gamma, \lambda; 0, \lambda'}$ = same helicity structure as $W+p \rightarrow \pi+p'$
 with $W^\mu \sim c_V \gamma^\mu - c_A \gamma^\mu \gamma^5$ & $\pi \sim \varphi_A q'_\nu \gamma^\nu \gamma^5 + \varphi_P \gamma^5$

$A_{\Lambda', \lambda'; \Lambda, \lambda} \rightarrow 2 \times 6$ helicity amps \supset GPDs



Hadrons to quarks



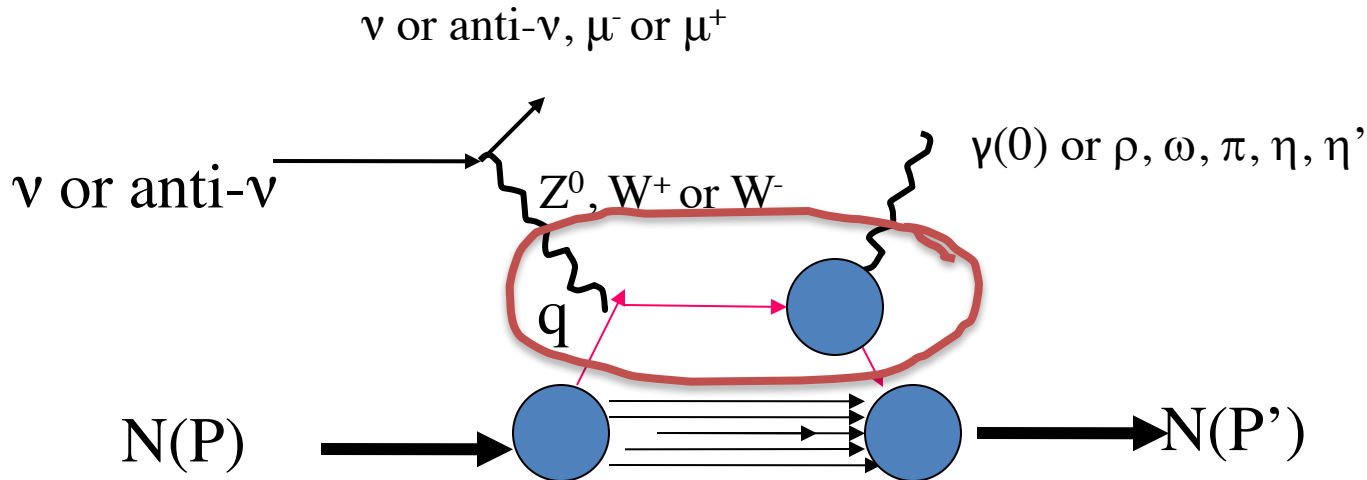
$g_{\Lambda^W, \lambda; 0, \lambda'} = \text{same helicity structure as } W+p \rightarrow \pi+p'$
 with $W^\mu \sim c_V \gamma^\mu - c_A \gamma^\mu \gamma^5$ & $\pi \sim \varphi_A q'_\nu \gamma^\nu \gamma^5 + \varphi_P \gamma^5$
 $A_{\Lambda'; \lambda'; \Lambda, \lambda} \supset GPDs$

One of 4 terms:
$$\frac{-i}{2} \bar{u}(k') i\gamma_5 \left[\frac{i((k+q)^\mu \gamma_\mu + m_3)}{(k+q)^2 - m_3^2} \varepsilon^\nu \gamma_\nu (c_V - c_A \gamma_5) \right] u(k)$$

+ u-channel



Hard subprocess & Q



Consider π production $V^* + q \rightarrow \pi + q'$ with DA for $\pi \supset q'_\mu \gamma^\mu \gamma^5$ twist2 & γ^5 twist 3
 Helicity amps:

Longitudinal with non-flip quark – leading $\sim Q/M$: $q'_\mu \gamma^\mu \gamma^5$

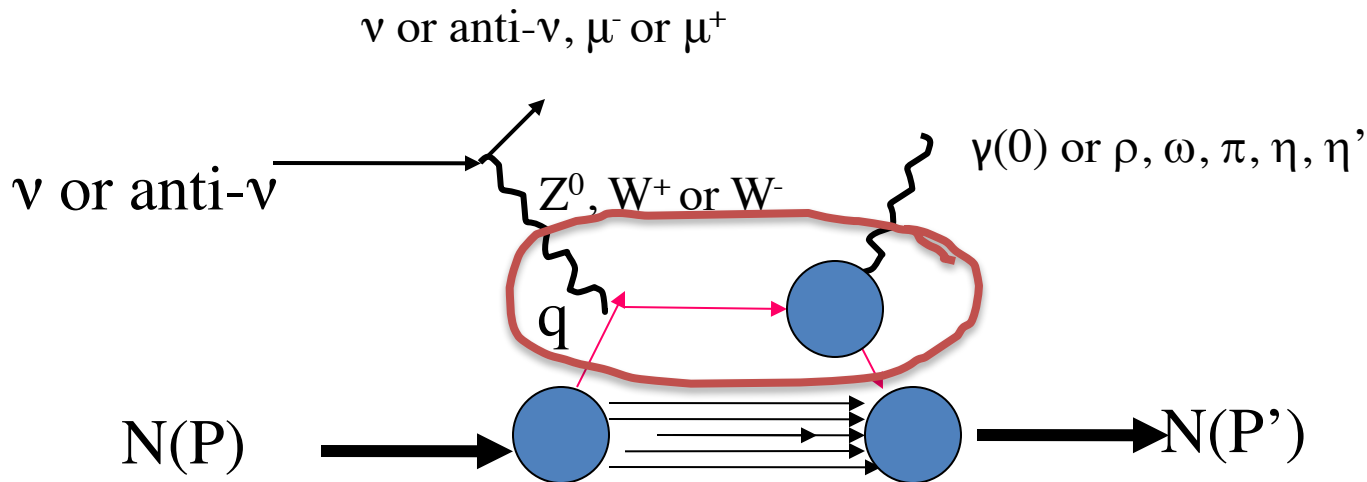
Transverse with flip quark \sim constant (or Δ^2/Q^2 cross channel) : γ^5

Transverse non-flip $\sim \Delta/M$: $q'_\mu \gamma^\mu \gamma^5$ (Δ by angular momentum conservation)

Longitudinal with flip $\sim \Delta/Q$ (Δ by angular momentum conservation)



Hard subprocesses



Consider π production $V^* + q \rightarrow \pi + q'$ with DA for $\pi \supset q'_\mu \gamma^\mu \gamma^5$ twist2 & γ^5 twist 3

Helicity amps: **Longitudinal** with non-flip quark – leading $\sim Q/M$: $q'_\mu \gamma^\mu \gamma^5$

Transverse with flip quark \sim constant (or Δ^2/Q^2 cross channel) : γ^5

Transverse non-flip $\sim \Delta/M$: $q'_\mu \gamma^\mu \gamma^5$ (Δ by angular momentum conservation)

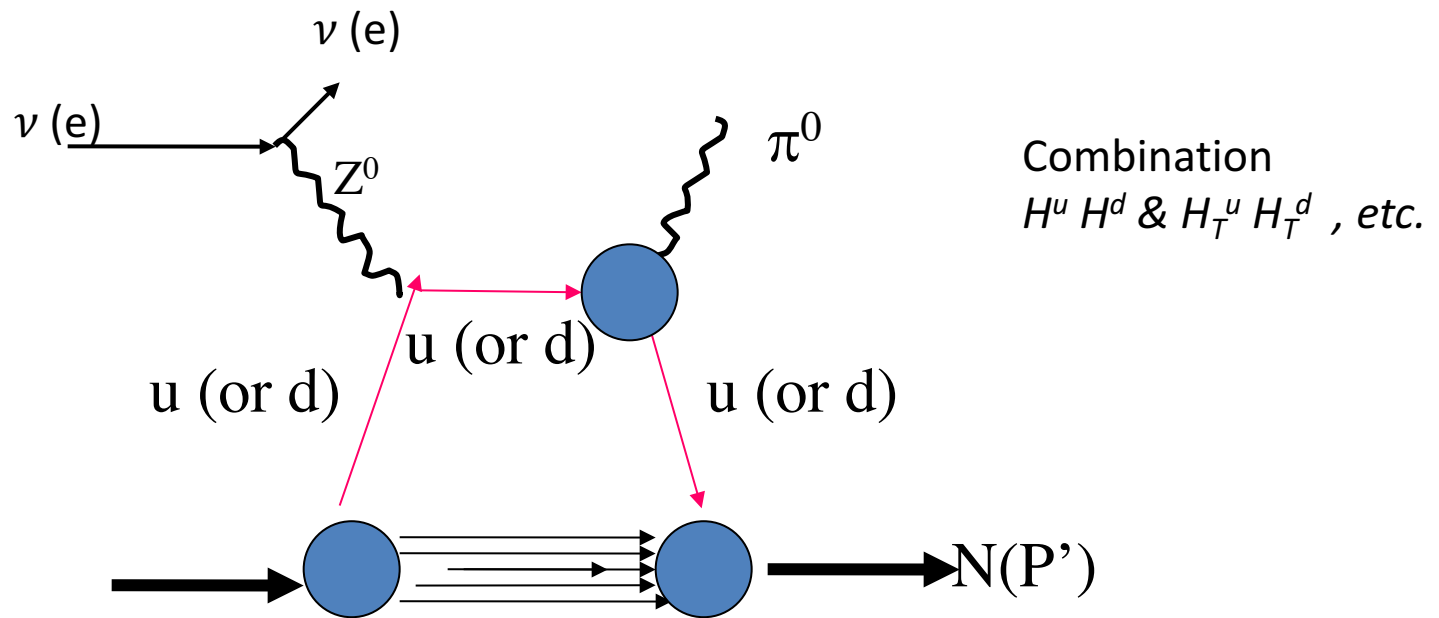
Longitudinal with flip $\sim \Delta/Q$ (Δ by angular momentum conservation)

- Question:** Does Twist 3 dominate ν production of π as in electroproduction?
 i.e. will measurements be sensitive to this twist 3 DA π mechanism?
 Will Transversity be important?
 See Kopeliovich, Schmidt, Siddikov: Twist 2 & longitudinal dominate.
 See Pire & Szymanowski: Twist 3 dominates **for heavy quarks**.



Neutral Current interaction

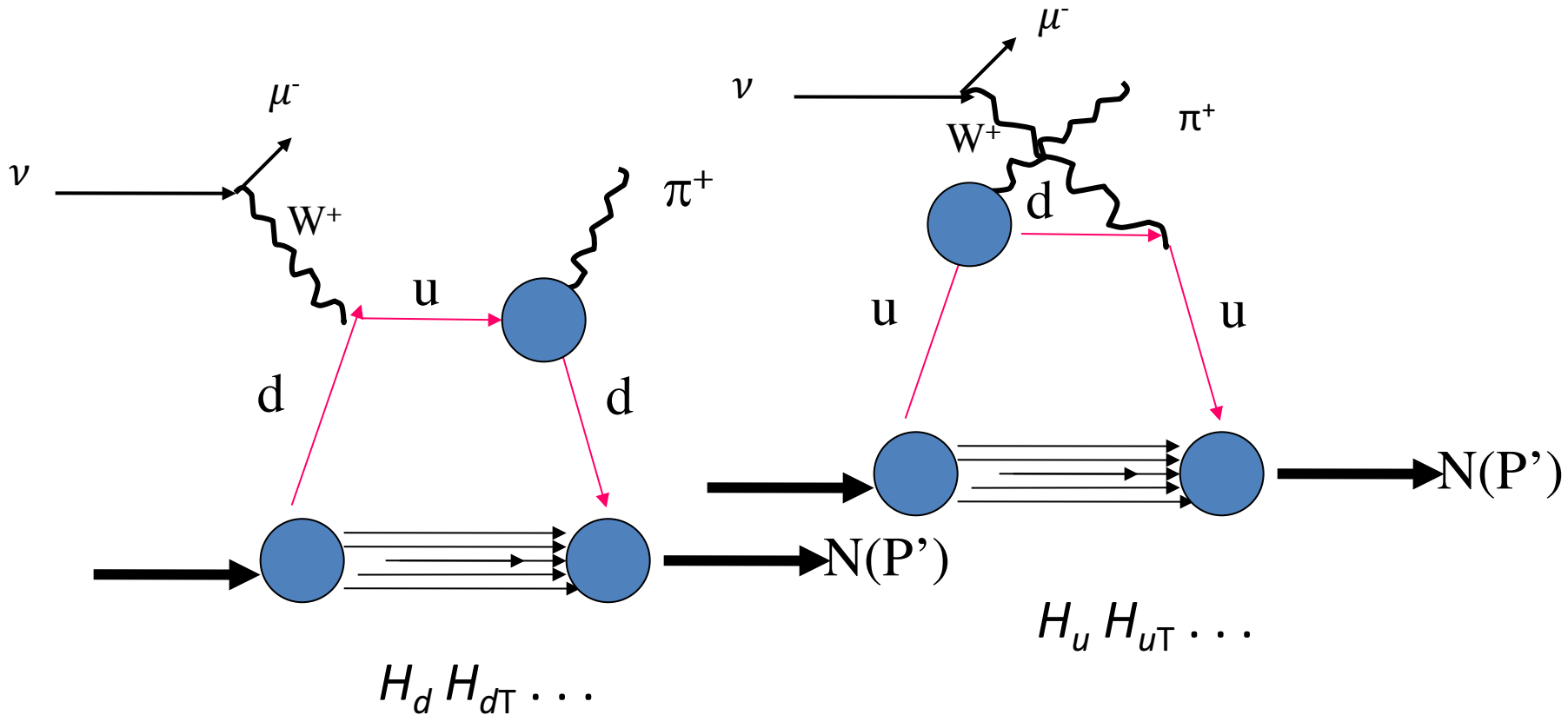
- NC & parity violating electroproduction





Neutrino & flavor separation

- Charge currents separate u from d





Differential cross sections

Neutral Current:

$$\frac{d^4\sigma}{dx_{Bj}dQ^2d|t|d\phi} = \frac{G_F^2 x_{Bj}^2}{32\pi^3 \cos^4(\theta_W) Q^2 (1 + Q^2/M_Z^2)^2 (1 + \gamma^2)^{3/2}} |T|^2$$

Charge Current: replace M_Z with M_W and set $\theta_W=0$.

Separating the Vector boson + nucleon scattering:

$$\frac{d\sigma_Z}{dt} = \frac{\sqrt{2}G_F M_Z^2 x_{Bj}^2 |T|^2}{16\pi Q^4 (1 + \gamma^2)}$$

$$\frac{d\sigma_W}{dt} = \frac{G_F M_Z^2 x_{Bj}^2 |T|^2}{16\pi \sqrt{2} Q^4 (1 + \gamma^2)}$$



Azimuthal correlations to separate P violating parts

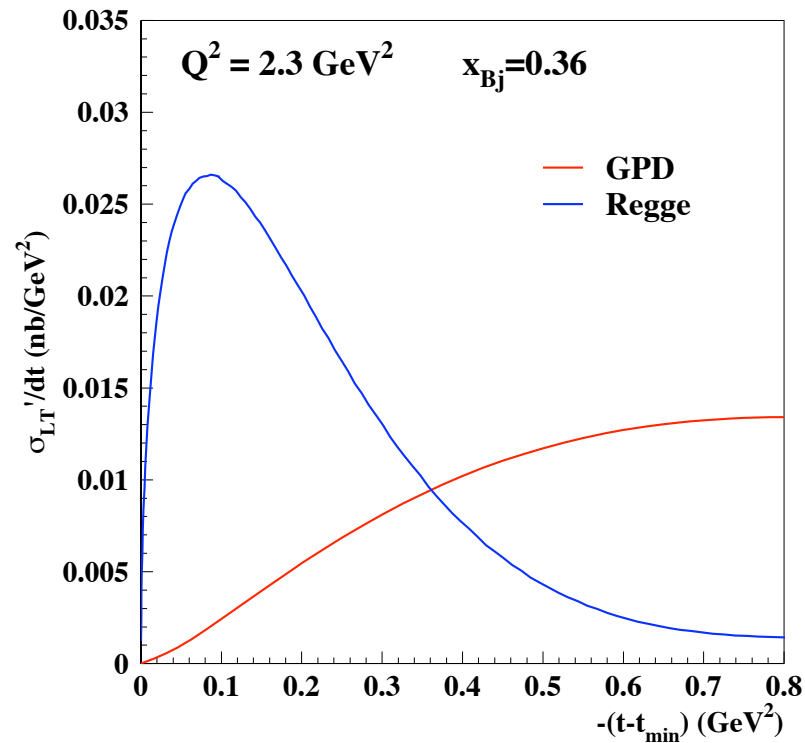
$$\begin{aligned}
 \frac{d^4 \sigma}{d\Omega d\varepsilon_2 d\phi dt} &= \Gamma \left[\frac{d\sigma_T}{dt} + \varepsilon_L \frac{d\sigma_L}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} \right. \\
 &+ \sqrt{2\varepsilon_L(\varepsilon + 1)} \cos \phi \frac{d\sigma_{LT}}{dt} \\
 &+ \varepsilon \sin 2\phi \frac{d\sigma_{T'T}}{dt} \\
 &\left. \pm \sqrt{2\varepsilon_L(\varepsilon + 1)} \sin \phi \frac{d\sigma_{L'T}}{dt} \right] \quad (
 \end{aligned}$$



To Measure Chiral Even-Odd Interference

Contribution to π^0 exclusive ν production cross section,
 $d\sigma_{L,T}/dt$ from model GPDs (c.f. Regge description).

Few GeV region accessible in current & future neutrino experiments.



GRG, Gonzalez-Hernandez, Liuti, McSkill, Proc. NuFact09, 0911.0495; in preparation 2016.



Gluon GPDs



Gluon GPDs

$$\frac{1}{\bar{P}^+} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P', \Lambda' | G^{+i}(-\frac{1}{2}z) G^{+i}(\frac{1}{2}z) | P, \Lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+} \bar{U}(P', \Lambda') [H^g(x, \xi, t) \gamma^+ + E^g(x, \xi, t) \frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}] U(P, \Lambda)$$

Even parity & Gluon helicity conserving

$$\frac{-i}{\bar{P}^+} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P', \Lambda' | G^{+i}(-\frac{1}{2}z) \tilde{G}^{+i}(\frac{1}{2}z) | P, \Lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} =$$

$$\frac{1}{2\bar{P}^+} \bar{U}(P', \Lambda') [\tilde{H}^g(x, \xi, t) \gamma^+ \gamma_5 + \tilde{E}^g(x, \xi, t) \frac{\gamma_5(-\Delta^+)}{2M}] U(P, \Lambda)$$

Odd parity & Gluon helicity conserving



• Gluon & Sea quark distributions Spectator Model

– generalize Regge-diquark spectator model

- $N \rightarrow g +$ “color octet N” spectator ($8 \otimes 8 \supset 1$)
(could be spin $\frac{1}{2}$ or $\frac{3}{2}$)

- ($N \rightarrow$ *anti-u* + color 3 “tetraquark” uuud)

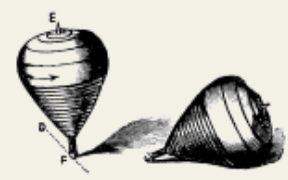
- How to normalize?

$$H_g(x, \xi, t)_Q^2 \rightarrow H_g(x, 0, 0)_Q^2 = xG(x)_Q^2$$

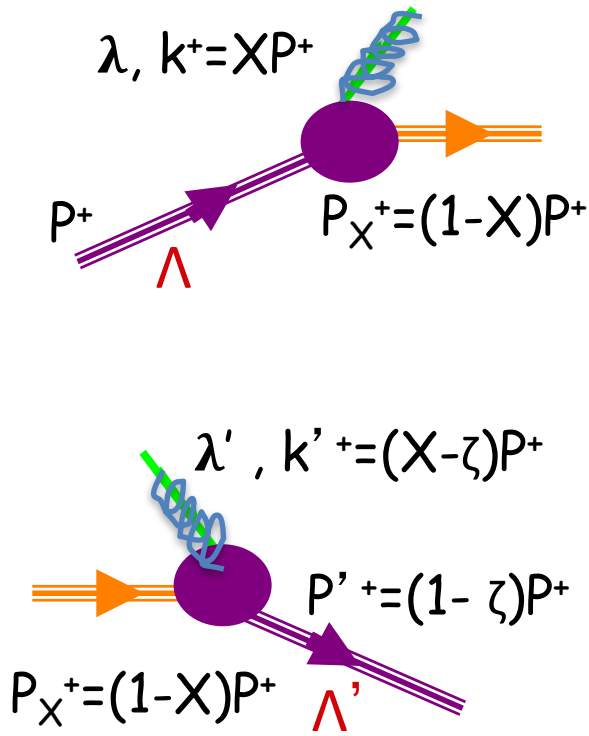
- Evolution & small x phenomenology
- Sea quark distributions $H_{\text{anti-u}}(x, 0, 0) \dots$

- Use pdf’s to fix x dependence

- Small x \sim Pomeron



Gluon 'vertex functions' $\mathcal{G}_{\Lambda X}; \Lambda g, \Lambda$



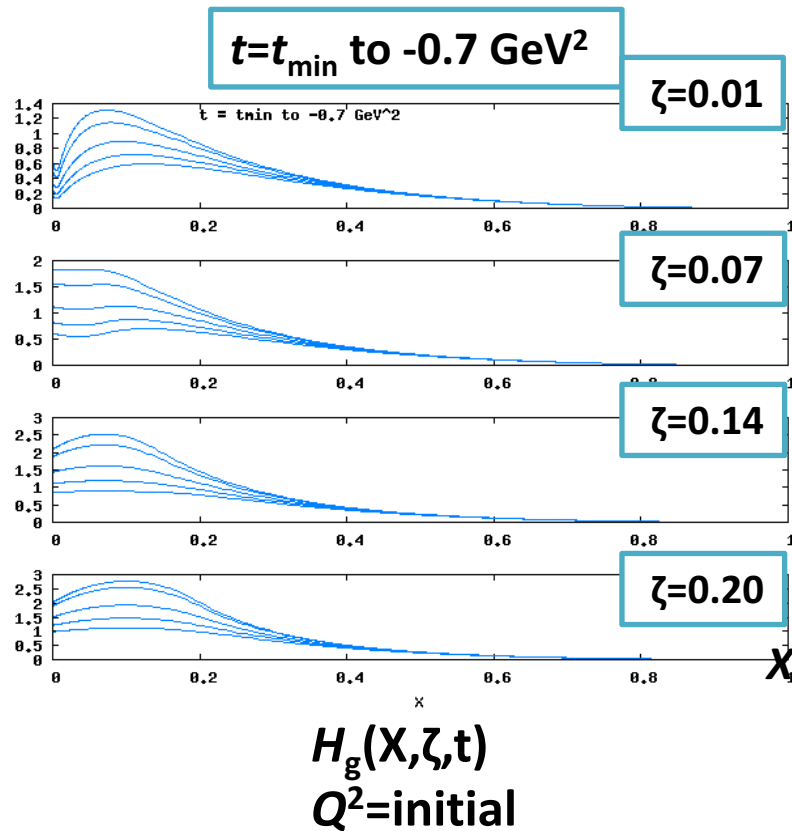
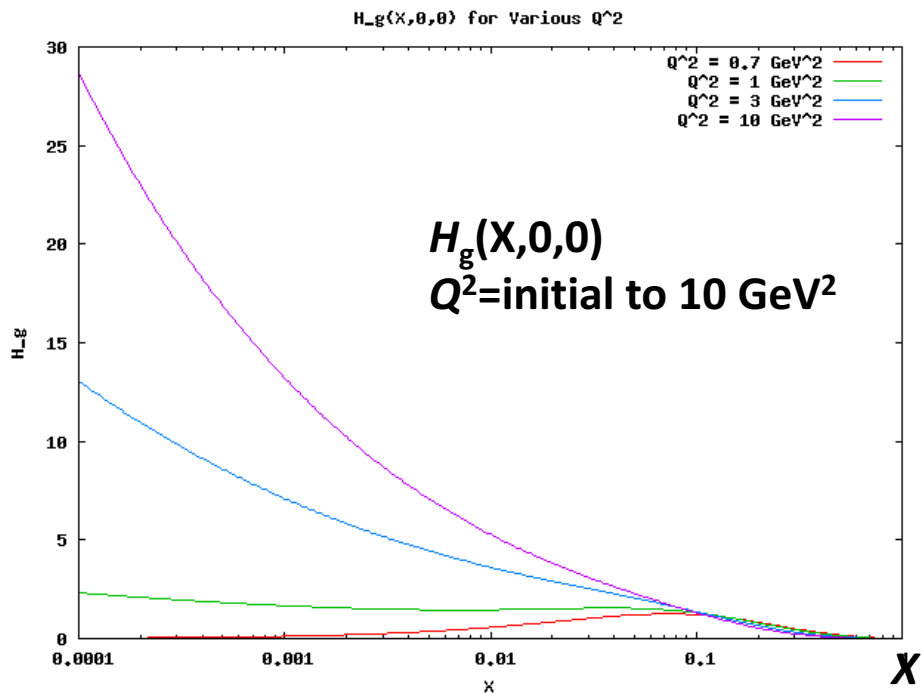
$\mathcal{G}_{+++}(x, \vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}} \frac{(k_x - ik_y)}{X}$
$\mathcal{G}_{-++}(x, \vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}} (M(1-X) - M_x)$
$\mathcal{G}_{++-}(x, \vec{k}_T^2)$	0
$\mathcal{G}_{-+-}(x, \vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}} (1-X) \frac{(k_x - ik_y)}{X}$
$\mathcal{G}_{+++}^*(x, \vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}} \frac{(\tilde{k}_x + i\tilde{k}_y)}{X'}$
$\mathcal{G}_{-++}^*(x, \vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}} (M(1-X') - M_x)$
$\mathcal{G}_{++-}^*(x, \vec{k}_T'^2)$	0
$\mathcal{G}_{-+-}^*(x, \vec{k}_T'^2)$	$-\frac{2}{\sqrt{2(1-X')}} (1-X') \frac{(\tilde{k}_x + i\tilde{k}_y)}{X'}$

$$X' = \frac{X-\zeta}{1-\zeta}, \quad \tilde{k}_{i=1,2} = k_i - \frac{1-X}{1-\zeta} \Delta_i$$

GG, Gonzalez Hernandez, Liuti, Poage, in progress



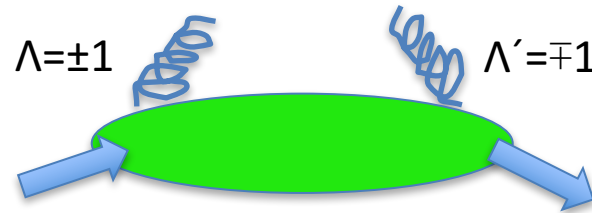
After pdf's vs. $Q^2 \rightarrow$ fix x dependence
 Regge behavior determines t dependence
 Spectator determines ζ dependence



from J. Poage



Extension to Gluon “Transversity”



$$\begin{aligned}
 & -\frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \mathbf{S} F^{+i}(-\frac{1}{2}z) F^{+j}(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^+=0, \mathbf{z}_T=0} \\
 &= \mathbf{S} \frac{1}{2P^+} \frac{P^+ \Delta^j - \Delta^+ P^j}{2mP^+} \\
 &\times \bar{u}(p', \lambda') \left[H_T^g i\sigma^{+i} + \tilde{H}_T^g \frac{P^+ \Delta^i - \Delta^+ P^i}{m^2} \right. \\
 &\quad \left. + E_T^g \frac{\gamma^+ \Delta^i - \Delta^+ \gamma^i}{2m} + \tilde{E}_T^g \frac{\gamma^+ P^i - P^+ \gamma^i}{m} \right] u(p, \lambda).
 \end{aligned}$$

4 GPDs: M.Diehl, EPJC19, 485 (2001)



Using the Reggeized Spectators Model

How to Measure? What Processes?

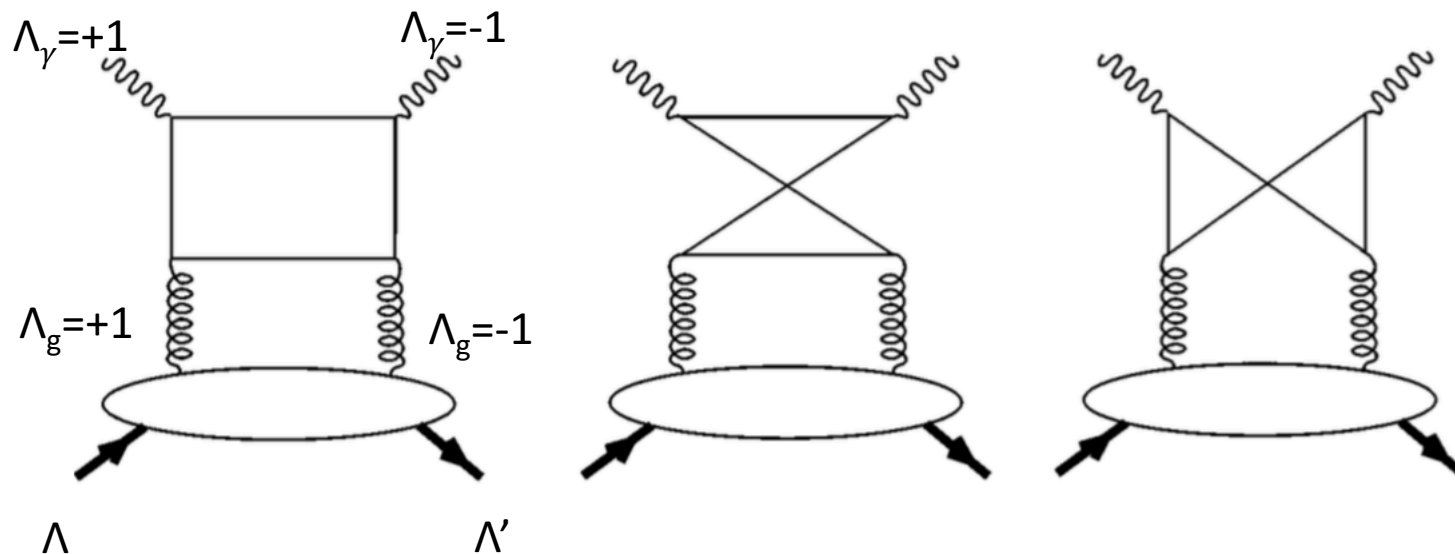


$A_{\Lambda', -1; \Lambda, +1}$ Gluon Transversity contributes to DVCS order α_s

$$M_{\Lambda', \Lambda' \gamma = -1; \Lambda, \Lambda \gamma = +1} = -\frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda', \Lambda' g = -1; \Lambda, \Lambda g = +1}(x, \xi, t)}{(\xi - x - i\epsilon)(\xi + x - i\epsilon)} C'(x, \xi, Q^2)$$

DVCS cross sections: $d\sigma/dt \propto \sum |M_{\Lambda' \dots}|^2$

*** **Interference** with Bethe-Heitler contains $\cos 3\varphi$ modulation to distinguish from (leading twist) quark contribution ***



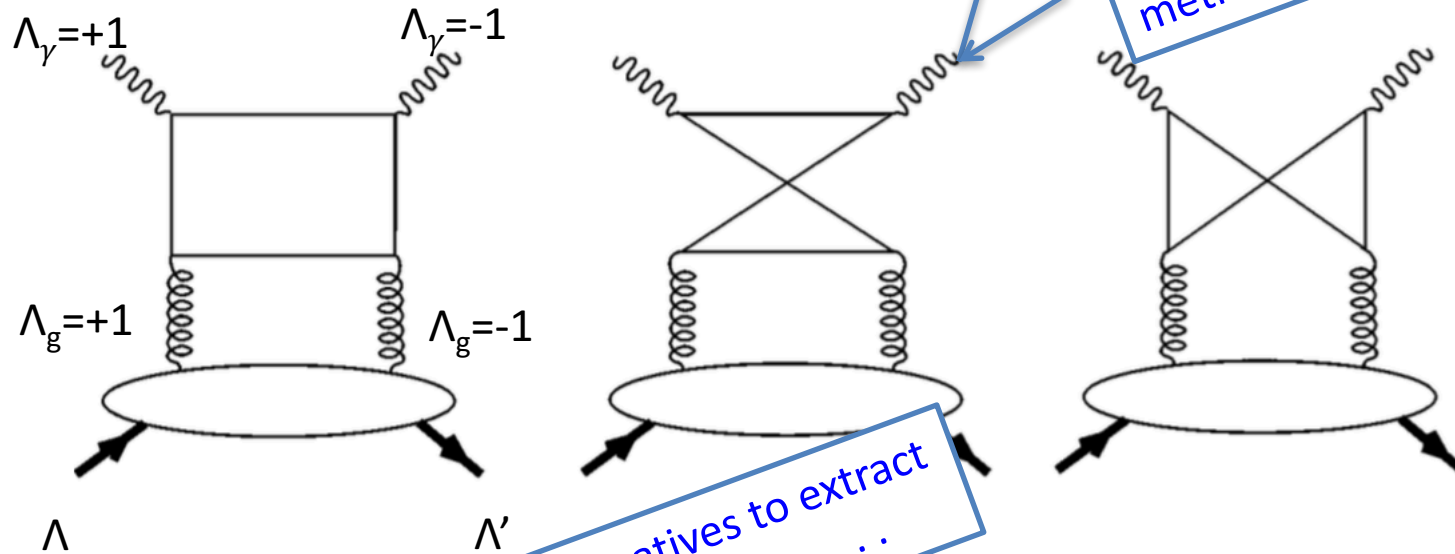
See Hoodbhoy & Ji, PRD58, 054006 (1998)



$A_{\Lambda', -1; \Lambda, +1}$ contributes to DVCS at order α_s

$$M_{\Lambda', \Lambda' \gamma = -1; \Lambda, \Lambda \gamma = +1} = -\frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda', \Lambda' g = -1; \Lambda, \Lambda g = +1}(x, \xi, t)}{(\xi - x - i\epsilon)(\xi + x - i\epsilon)} C'(x, \xi, Q^2)$$

Interference with Bethe-Heitler contains $\cos 3\phi$ modulation to distinguish from (leading twist) quark contribution



See Hoodbhoy & Ji, PRD58, 054006 (1998)



Measuring Gluons in Nucleons

DVCS

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2$$

$$T(k, p, k', q', p') = T_{DVCS}(k, p, k', q', p') + T_{BH}(k, p, k', q', p'),$$

$$|T|^2 = |T_{BH} + T_{DVCS}|^2 = |T_{BH}|^2 + |T_{DVCS}|^2 + \mathcal{I}.$$

$$\mathcal{I} = T_{BH}^* T_{DVCS} + T_{DVCS}^* T_{BH}.$$

For unpolarized $e+p \rightarrow e'+\gamma+p'$ cross section depends on azimuthal angle ϕ .
 $\cos 3\phi$ term in interference $d\sigma$ measures gluon transversity GPDs (CFF's)

$$\frac{\sqrt{t_0 - t}^3}{8M^3} \left[H_T^g F_2 - E_T^g F_1 - 2\tilde{H}_T^g \left(F_1 + \frac{t}{4M^2} F_2 \right) \right] \cos 3\phi$$

$$\mathcal{H}_T^g \sim \int dx H_T^g / (x-\xi)(x+\xi) \text{ CFF's}$$

See Diehl, *et al.* PLB411, 193 (1997);
 Diehl, EPJC25, 223 (2002);
 Belitsky, Mueller, PLB486, 369 (2000).



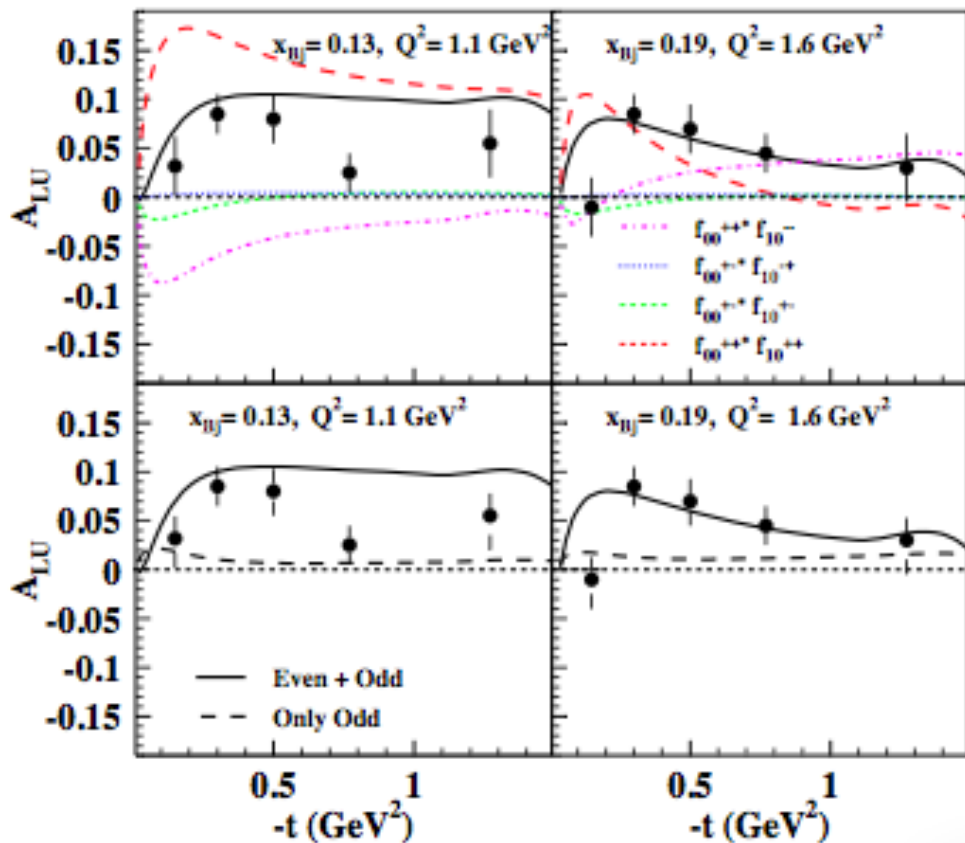
Summary

- Flexible parameterization for chiral even valence quarks from form factors, pdfs & DVCS $R \times Dq$
- Extended $R \times Dq$ to $R \times \text{Spectator}$
- Extend phenomenology to *v production*
- **New Extension to gluons & the sea**
- Considered Gluon sector
 - Helicity conserving & Helicity \rightarrow gluon *Transversity*
- Measurements?
- **More phenomenology to come**

Backup & extra slides



Beam spin asymmetry
 shows importance of H chiral even (CLAS data -DeMasi, et al.)

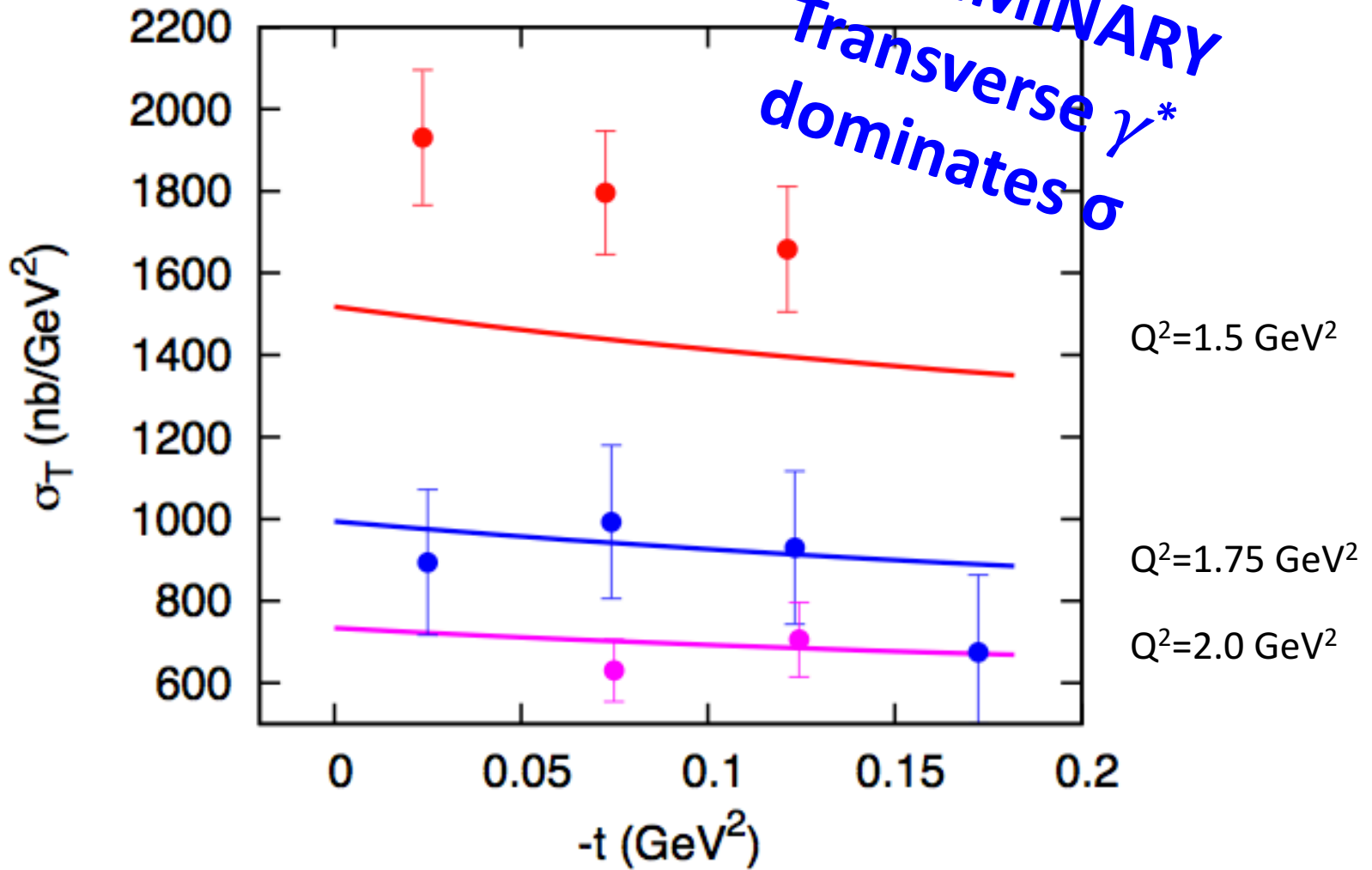




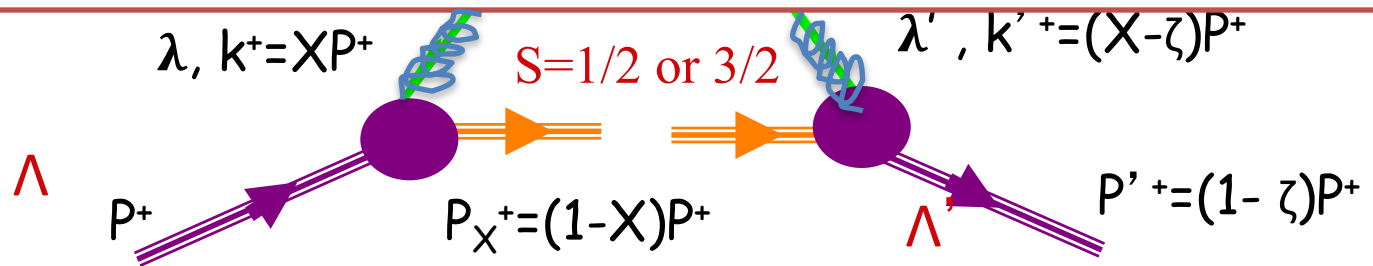
Hall A data $x_B=0.36$

courtesy F. Sabatie & M. Defurne

PRELIMINARY
Transverse γ^*
dominates σ



Procedure to construct **Gluon** GPDs & observables Spectator color octet “nucleon” model & Reggeization



Product of baryon l.c.w.f.'s $\rightarrow A_{\Lambda\lambda; \Lambda'\lambda' = \lambda}$

$A_{\Lambda\lambda; \Lambda'\lambda} \rightarrow$ gluon GPDs

$g \otimes A \rightarrow$ exclusive process helicity amps

pdf's, FF's, $d\sigma/d\Omega$ & Asymmetries: parameters & predictions

vertex parity? $\rightarrow A_{\Lambda\lambda; -\Lambda' -\lambda'} \rightarrow$ “transversity” gluon GPDs ?



What about Gluon “transversity”?

Transversity for **on-shell** gluons or photons : no $|0\rangle$ helicity

$$|+1\rangle_{trans} = \{|+1\rangle + |-1\rangle\} / 2 = |-1\rangle_{trans}$$

$$|0\rangle_{trans} = \{|+1\rangle - |-1\rangle\} / \sqrt{2}$$

$$\text{helicity } |\pm 1\rangle = \{- / + \hat{x} - i \hat{y}\} / \sqrt{2}$$

$$\hat{x} = -|0\rangle_{trans} = P_{parallel}$$

Linear polarization in the plane

$$\hat{y} = i\sqrt{2} |+1\rangle_{trans} = P_{normal}$$

Linear polarization normal to the plane

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).

pdf's fix x dependence



Gluon

anti-u

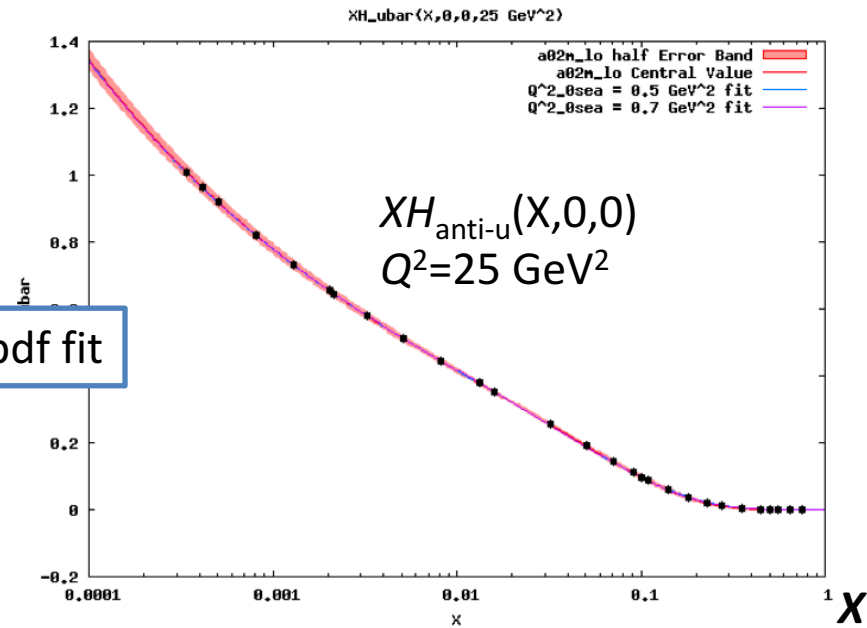
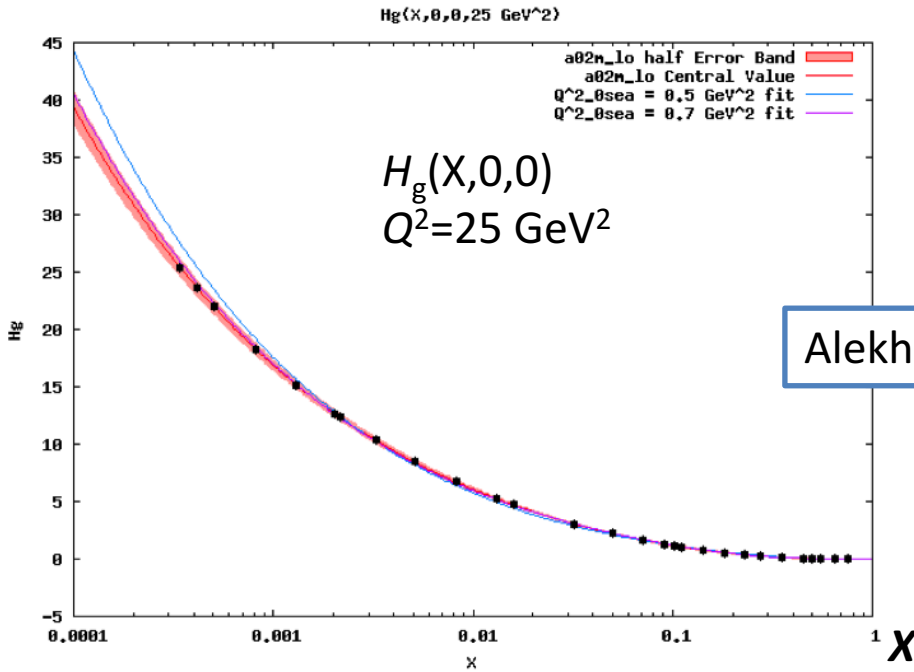


Figure 3: The plot above shows the distribution $XH_{\bar{u}}(X,0,0,25 \text{ GeV}^2)$ for the two fits. Alekhin's distribution $X\bar{u}(X)$ used in the fit procedure is included with an error band of one half of the error for the set a02m.lo. The X points used in the fit procedure are indicated by black dots.

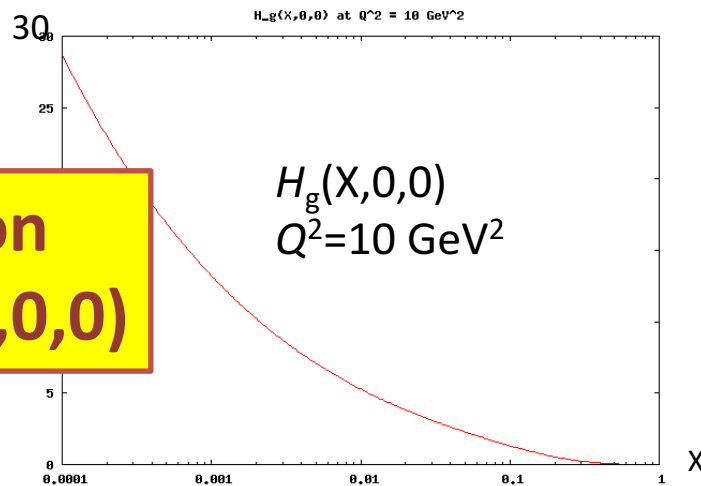
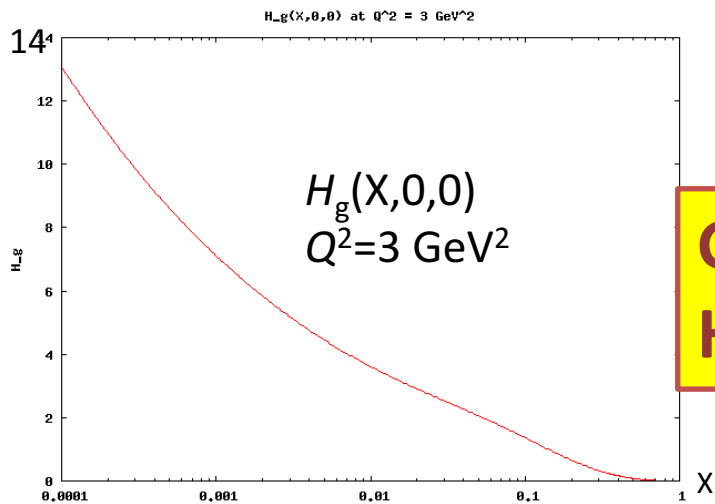
Single Q^2 value shown --- fit known pdf's all Q^2
from J. Poage

GG, Gonzalez Hernandez, Liuti, Poage, in progress

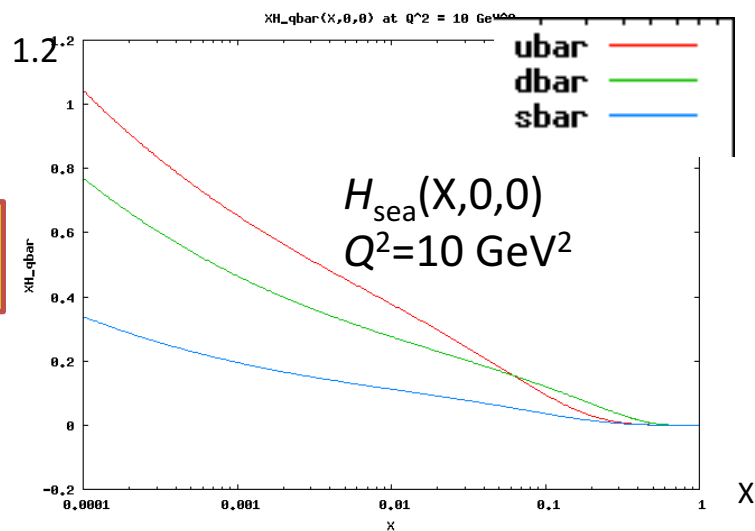
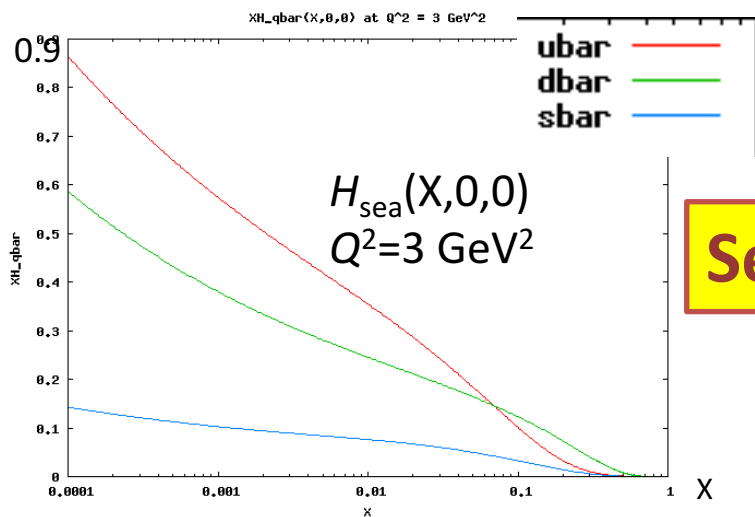


Gluon & sea distributions

J. Poage



Gluon
 $H_g(x,0,0)$



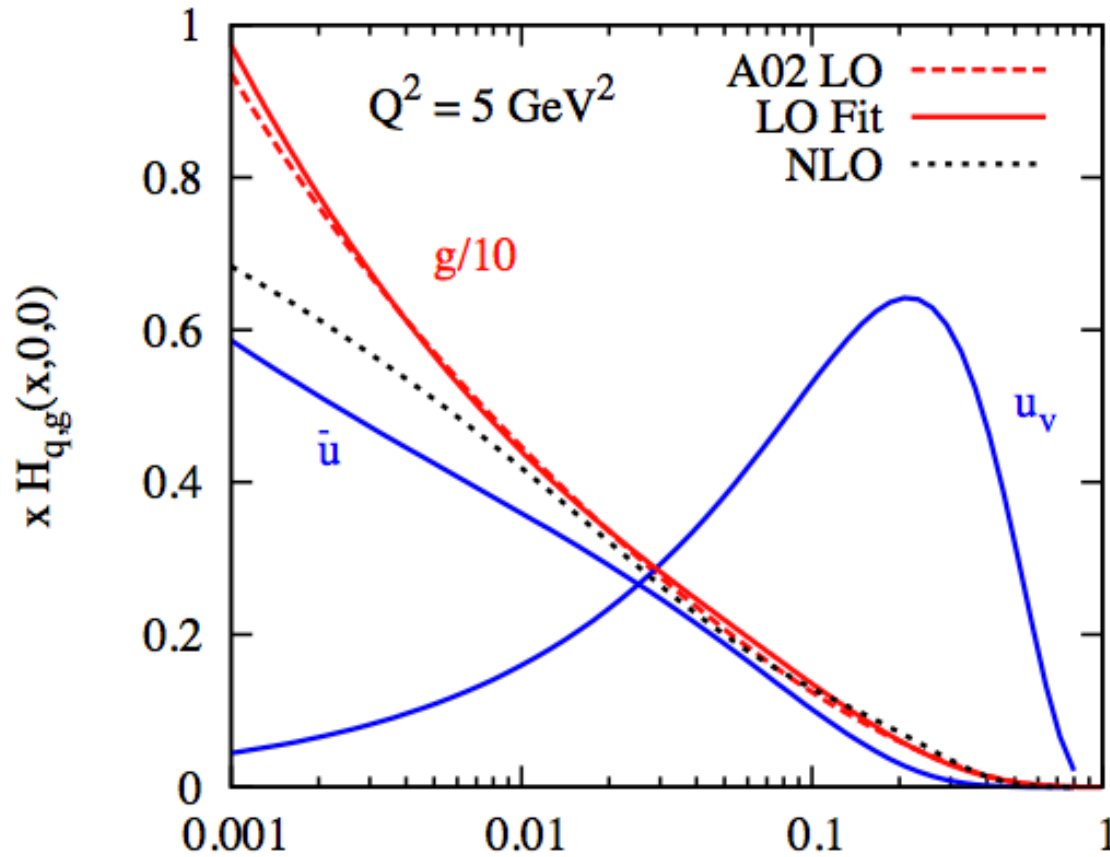
Sea

GG, Gonzalez Hernandez, Liuti, Poage, in progress



Fitting gluon pdf's

c.f. Alekhin, .. etc.



GG, Gonzalez Hernandez, Liuti, Poage, in progress



Preliminary: x and t dependence of $H_g(x, 0, t)$ for input scale

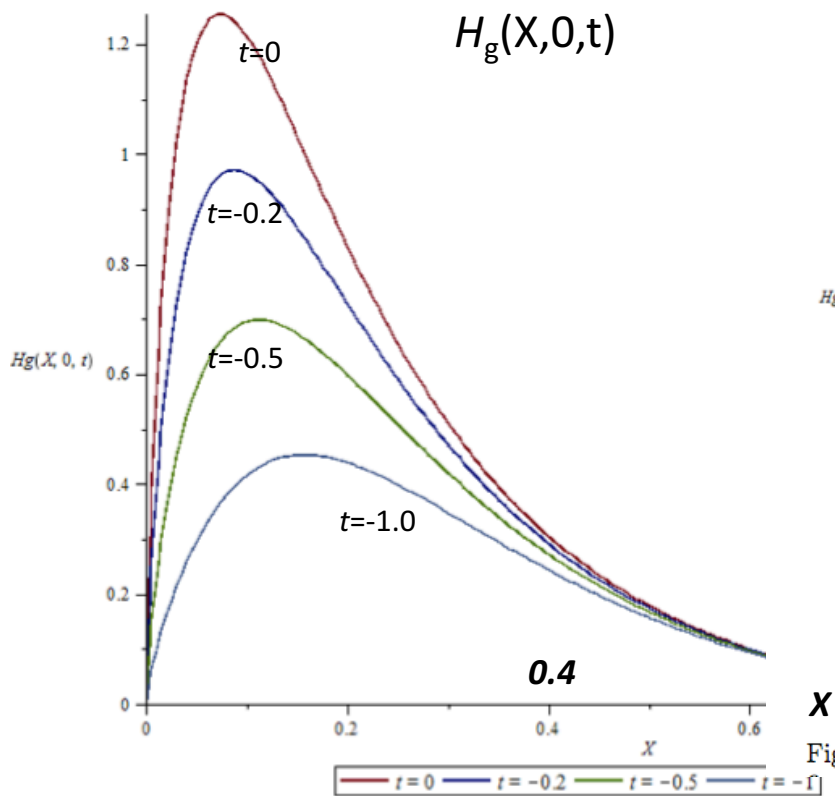


Figure 9: The plot above displays the distribution $H_g(X, 0, t)$ for a range of t values.

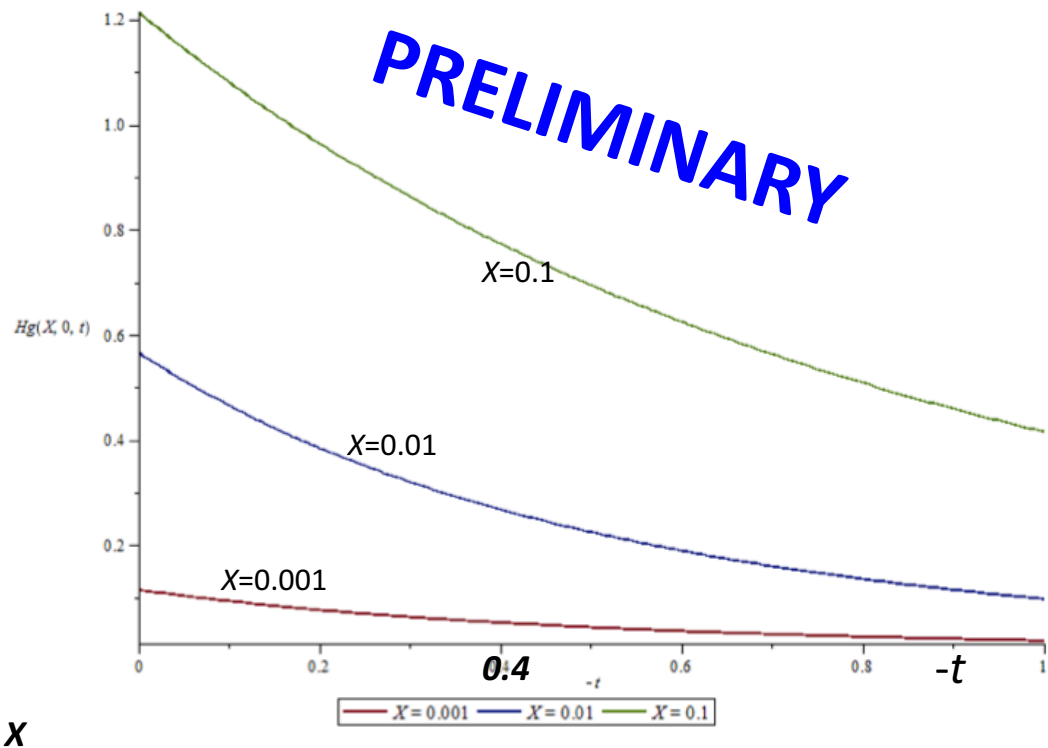


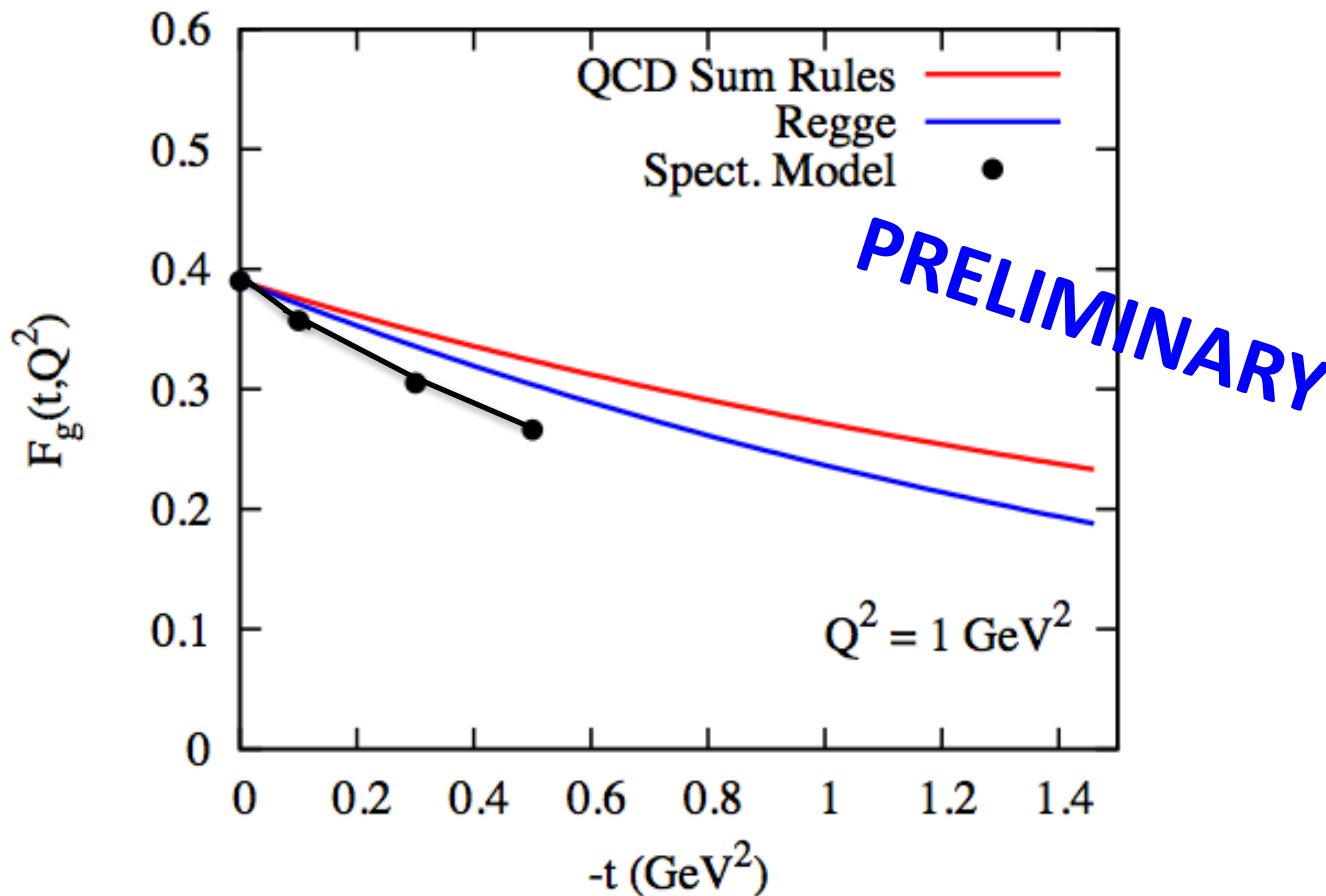
Figure 10: The plot above displays the distribution $H_g(X, 0, t)$ as a function of t , for several

GG, Gonzalez Hernandez, Liuti, Poage, in progress



$H_g(x,0,t)$ What constrains t-dependence?

Spectator t-dependence w/o Regge small x behavior:
 & hybrid Regge-Spectator model combines



Compare Gluon form factor via QCD sum rules

Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)

GG, Gonzalez Hernandez, Liuti, Poage, in progress