

Towards combined QCD global analysis of polarized and unpolarized PDFs and fragmentation functions

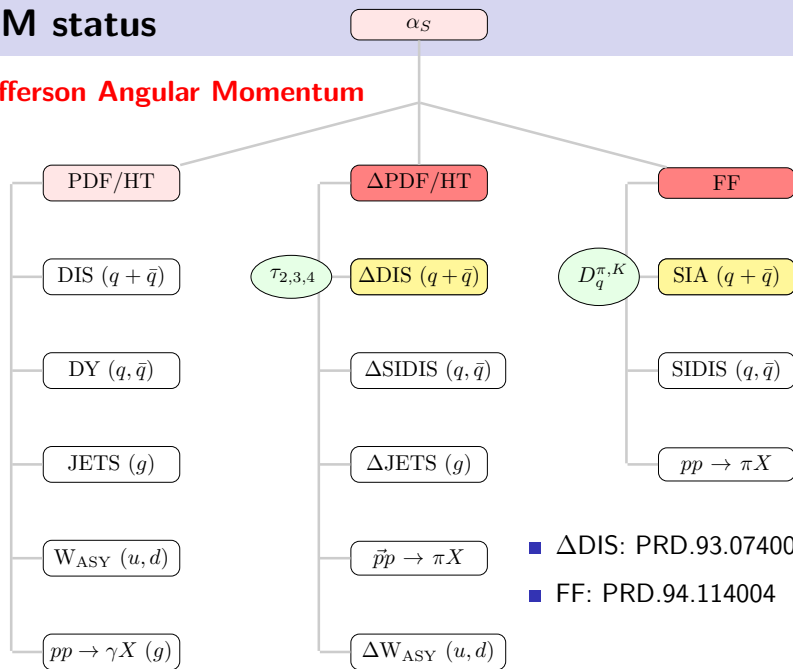
Nobuo Sato



**7th Workshop of the APS Topical Group on Hadronic Physics
Feb, 2017**

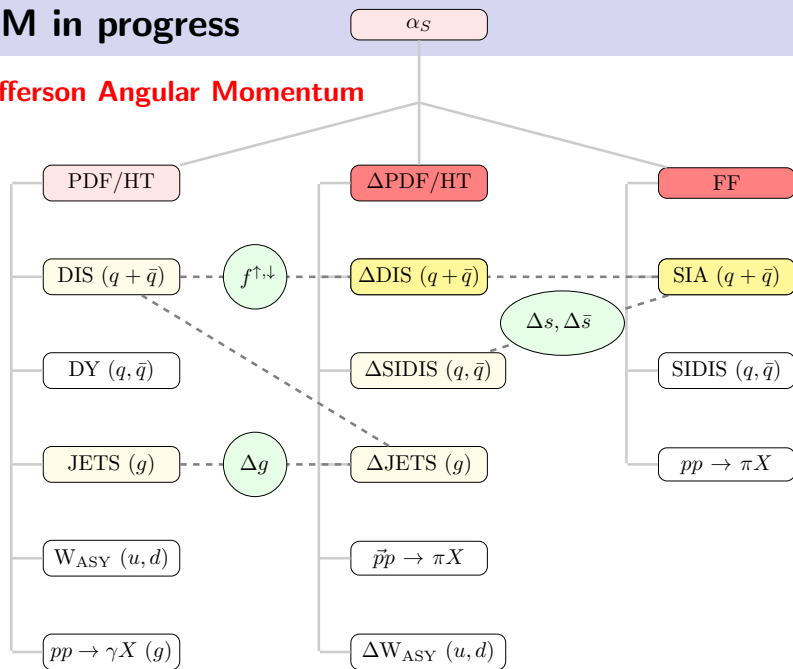
JAM status

Jefferson Angular Momentum



JAM in progress

Jefferson Angular Momentum



Theory of fitting

The goal is to estimate:

$$E[\mathcal{O}] = \int d^n a \mathcal{P}(\mathbf{a}|data) \mathcal{O}(\mathbf{a})$$

$$V[\mathcal{O}] = \int d^n a \mathcal{P}(\mathbf{a}|data) [\mathcal{O}(\mathbf{a}) - E[\mathcal{O}]]^2$$

Monte Carlo methods

- $\mathcal{P}(\mathbf{a}|data) \rightarrow \{\mathbf{a}_k\}$
- $E[\mathcal{O}] \approx \frac{1}{N} \sum_k \mathcal{O}(\mathbf{a}_k)$
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Maximum Likelihood

- Maximize $\mathcal{P}(\mathbf{a}|data) \rightarrow \mathbf{a}_0$
- $E[\mathcal{O}] \approx \mathcal{O}(\mathbf{a}_0)$
- $V[\mathcal{O}] \approx \text{hessian}, \Delta\chi^2 \text{ envelope}, \dots$

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The $\Delta\chi^2$ paradox

- there is a controversy about how to treat errors in global fits
- A particular example occurred in the IC context
 - Jimenez-Delgado et al PRL114 (2015) $\Delta\chi^2 = 1$
 - Brodsky, Gardner PRL116 (2016) $\Delta\chi^2 \neq 1$
- similar issues in other analysis (TMD, FF)
- I will attempt clarify this!!!
- to understand this lets consider an example ...

The $\Delta\chi^2$ paradox: Example

- consider two parameters:

$$\theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2}$$

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- change of variables

$$\theta_{1,2} \rightarrow t_{1,2} = \frac{\theta_{1,2} - \mu_{1,2}}{\sigma_{1,2}} \rightarrow r, \phi$$

$$\begin{aligned} d\theta_1 d\theta_2 p(\theta_1, \theta_2) &= dt_1 dt_2 \frac{1}{2\pi} \exp\left[-\frac{1}{2}(t_1^2 + t_2^2)\right] \\ &= d\phi r dr p(t_1, t_2) = \frac{d\phi}{2\pi} r dr \exp\left[-\frac{1}{2}r^2\right] \end{aligned}$$

The $\Delta\chi^2$ paradox: Example

- confidence volume:

$$\text{CV} = \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^R dr r \exp\left[-\frac{1}{2}r^2\right] = \int_0^R dr r \exp\left[-\frac{1}{2}r^2\right]$$

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- notice that

$$R^2 = t_1^2 + t_2^2 = \left(\frac{\theta_1 - \mu_1}{\sigma_1}\right)^2 + \left(\frac{\theta_2 - \mu_2}{\sigma_2}\right)^2 \equiv \chi^2$$

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- the *confidence region* for the parameters is

$$\max[t_{1,2}] = R \quad \rightarrow \quad \theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2} R$$

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- the *confidence region* for the parameters is

$$\max[t_{1,2}] = R \rightarrow \theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2} R$$

- ... but we know that

$$\theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2}$$

- this is the origin of the paradox.

The $\Delta\chi^2$ paradox: Example

- **The resolution** → **Use calculus!!**

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- The expectation value:

$$\begin{aligned} E[\theta_{1,2}] &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^\infty dr p(r, \phi) \theta_{1,2} \\ &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^\infty dr r \exp\left[-\frac{1}{2}r^2\right] [\mu_{1,2} + t_{1,2}\sigma_{1,2}] \\ &= \mu_{1,2} \end{aligned}$$

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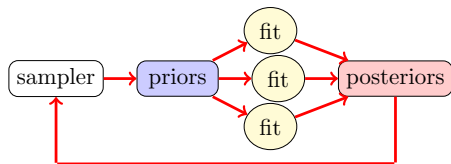
- The variance value:

$$\begin{aligned} V[\theta_{1,2}] &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^\infty dr p(r, \phi) [\theta_{1,2} - \mu_{1,2}]^2 \\ &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^\infty dr r \exp\left[-\frac{1}{2}r^2\right] [t_{1,2}\sigma_{1,2}]^2 \\ &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^\infty dr r \exp\left[-\frac{1}{2}r^2\right] [r\{\cos, \sin\}(\phi)\sigma_{1,2}]^2 \\ &= \sigma_{1,2}^2 \end{aligned}$$

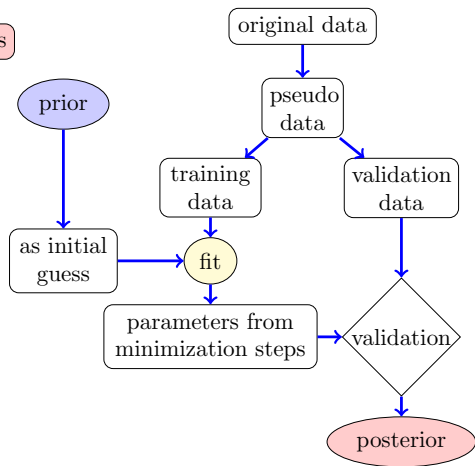
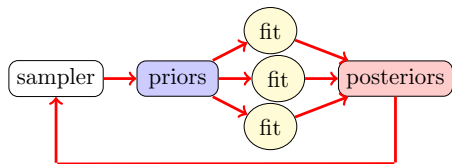
The $\Delta\chi^2$ paradox: The resolution

- If:
 - the likelihood function is approximately gaussian in the parameters
 - If the model is well described by a linear approximation
- Then,
 - 1σ CL is characterized by the hessian criterion with $\Delta\chi^2 = 1$ for any arbitrary number of parameters
- Else,
 - Use MC methods

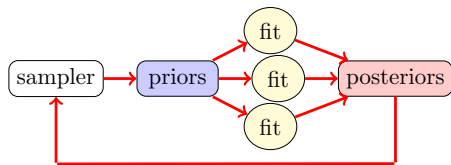
Iterative Monte Carlo analysis (IMC)



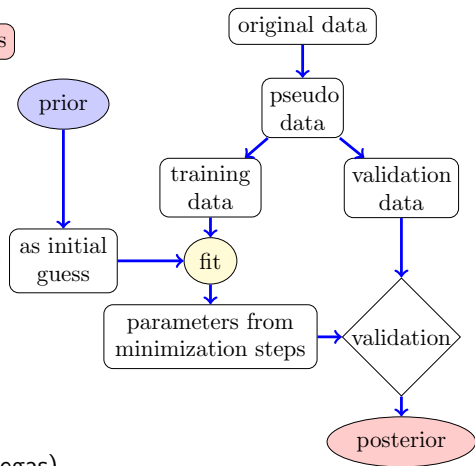
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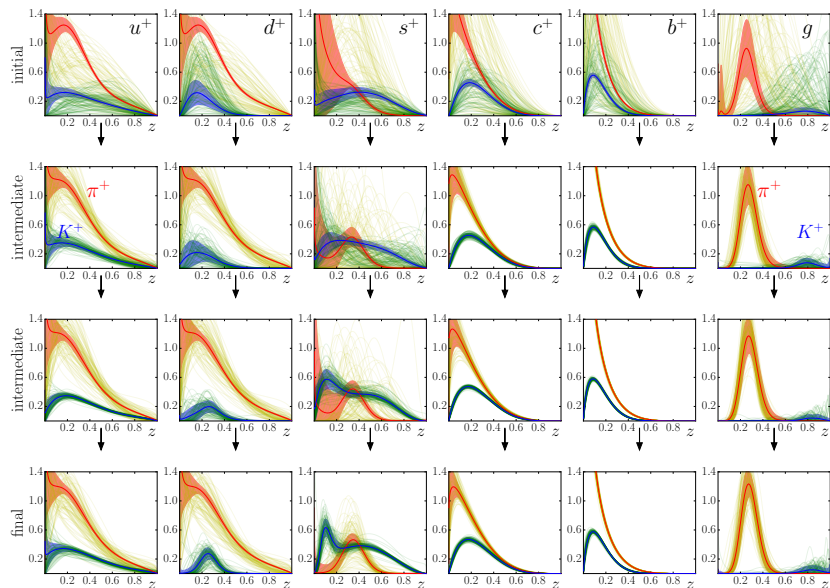
Iterative Monte Carlo analysis (IMC)



- Use traditional ansatz
$$xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$$
- Keep all the parameters free.
No assumptions on the exponents
- Avoid over-fitting by Cross-Validation
- **Iterative procedure**
→ Adaptive MC integration (like in Vegas)
- Robust estimation of uncertainties



IMC in action (example in FF case)

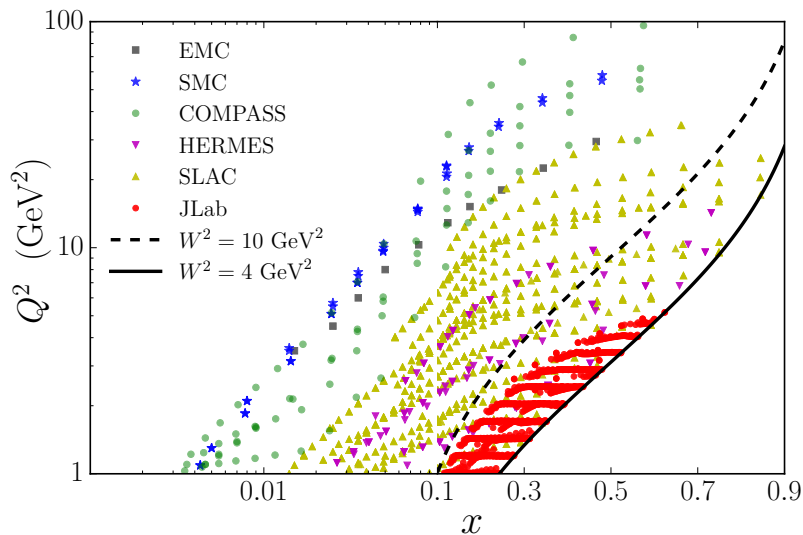


Spin PDFs from polarized DIS

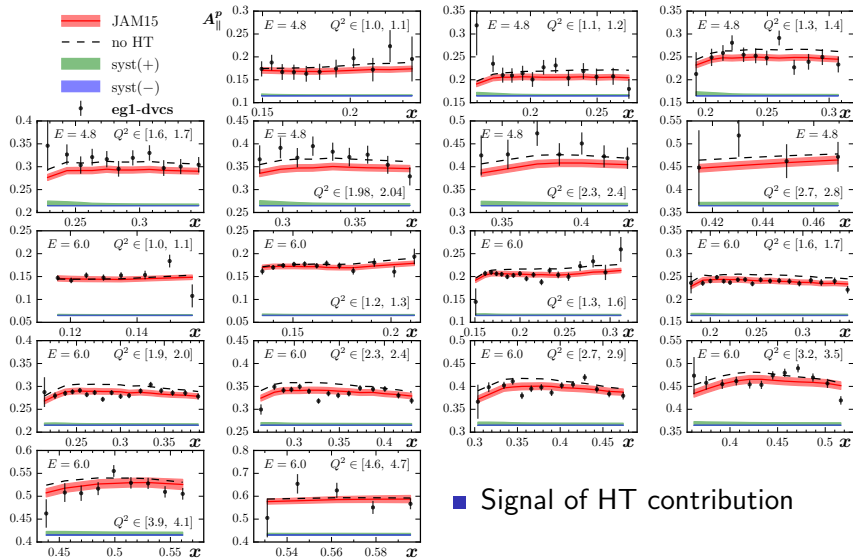
In Collaboration with:

- J. J. Ethier (College of William and Mary)
- W. Melnitchouk (Jefferson Lab)
- A. Accardi (Hampton U. and Jefferson Lab)
- S. E. Kuhn (Old Dominion U.)

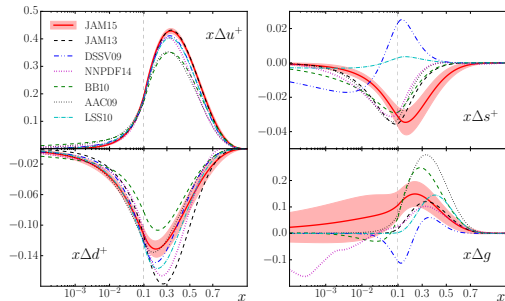
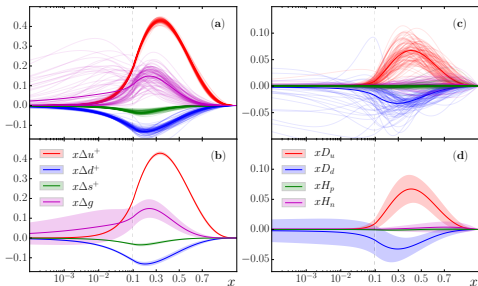
Global polarized DIS data



Data vs theory: proton JLab eg1-dvcs



Results



moment	truncated	full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta \Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.011 ± 0.004	0.011 ± 0.004
d_2^n	-0.002 ± 0.002	-0.002 ± 0.002
h_p	-0.000 ± 0.001	0.000 ± 0.001
h_n	0.001 ± 0.002	0.001 ± 0.003

- $\chi^2/N_{npts} = 1.07$
- Sign of τ_3 distributions is the same as τ_2
- **Negative Δs^+**
- Δg compatible with the most recent DSSV fits
- moments of Δg not constrained.

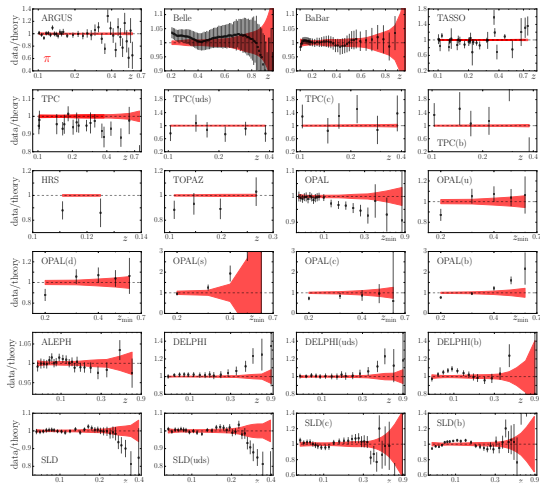
Fragmentation Functions from SIA data

In Collaboration with:

- J. J. Ethier (College of William and Mary)
- W. Melnitchouk (Jefferson Lab)
- A. Accardi (Hampton U. and Jefferson Lab)
- S.Kumano (KEK, J-PARC)
- M.Hirai (Nippon Institute of Technology)

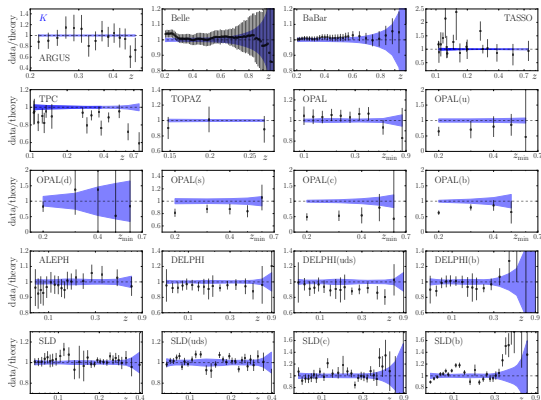
π analysis

$$(\chi^2/N_{\text{npts}} = 1.31)$$



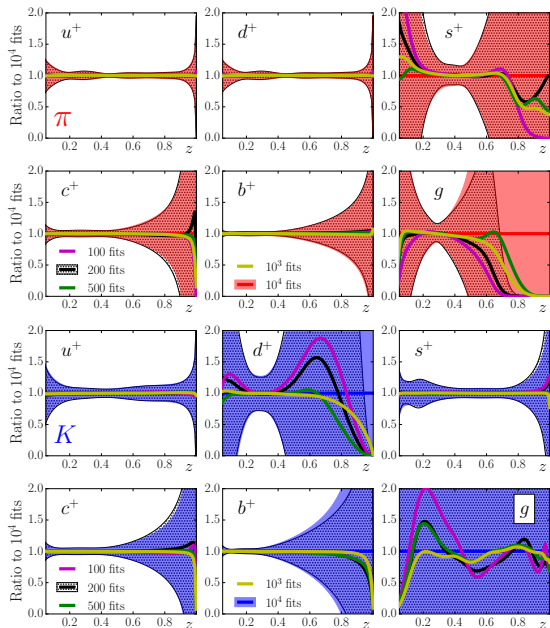
- $z_{\text{cut}} > 0.1$ for low energies
- $z_{\text{cut}} > 0.05$ for high energies
- We use BaBar prompt data set
- Belle data set needs 10% normalization
- Good agreement at low- z for inclusive data sets
- Data inconsistencies at large- z for $Q^2 = M_z^2$

K analysis ($\chi^2/N_{\text{npts}} = 1.01$)



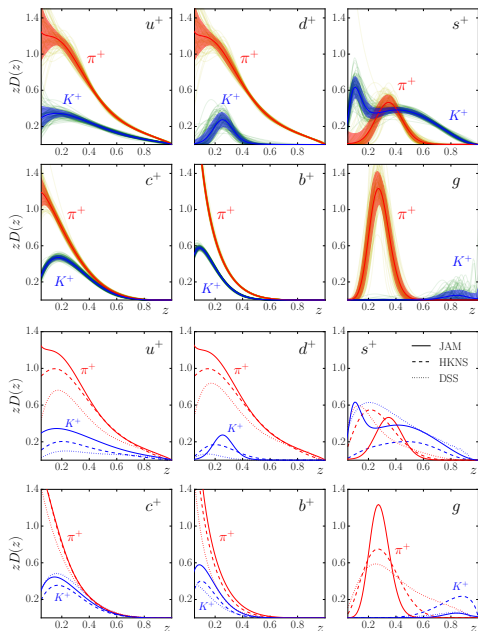
- $z_{\text{cut}} > 0.2$ for low energies to avoid hadron mass corrections
- $z_{\text{cut}} > 0.05$ for high energies
- Smaller χ^2 than π due to larger errors
- Consistent shapes across all z
- Inconsistencies mostly due to normalizations

Fragmentation functions



- Larger constraints on favored FFs than unfavored FFs
- More sensitivity on D_g^π than D_g^K
- D_g^K is unknown
- HQ tagged data provides similar constraints for π and K
- Convergence attained with ~ 200 posteriors

Fragmentation functions



- Similar behavior of unfavored $D_{s^+}^\pi$ and $D_{d^+}^K$
- In contrast the D_g^π and D_g^K behave differently
- The charm and bottom FFs become compatible at large- z
- Favored $D_{u^+}^\pi$ and $D_{s^+}^K$ have similar shape at large- z
- JAM $D_{s^+}^K$ is compatible with DSS. **Will it change the sign of Δ_{s^+} ?**

Summary and outlook

Polarized DIS data

- New JAM analysis to study impact of all JLab 6 GeV inclusive DIS data at low W and high x
- New extraction of LT & HT distributions
- Constraints on d_2

SIA data

- New study of SIA data including recent Belle and BaBar data
- New extraction of fragmentation functions
- The setup for combined pol DIS, SIA and pol SIDIS is ready!

JAMLIB

- JAM SPDFs and FFs are available at [github](#)
- Python, Fortran and LHAPDF interfaces provided

JAM for public

The screenshot shows the homepage of the Jefferson Lab Angular Momentum Collaboration. The header features the JAM logo and the text "Jefferson Lab Angular Momentum Collaboration". Below the header is a navigation menu with "LINKS" and "About". The "About" section contains two paragraphs: the first describes the collaboration as an enterprise involving theorists and experimentalists, and the second explains the unique capabilities of the Jefferson Lab's CEBAF accelerator. Below the "About" section is a "Library" section, followed by a "Theory Framework" section and a "Results" section. The "Results" section is titled "Pion and Kaon fragmentation from JAM16" and contains three plots showing the fragmentation function $D_{h/q}^A(z, Q^2)$ versus z for π^+ , K^+ , and π^0 mesons.

Jefferson Lab Angular Momentum Collaboration

LINKS

- HOME
- JAMLIB
- References
- Database
- Talks
- Collaboration
- LINKS

About

The JAM (Jefferson Lab Angular Momentum) Collaboration is an enterprise involving theorists and experimentalists from the Jefferson Lab community to study the quark and gluon spin structure of the nucleon by performing global fits of spin-dependent parton distribution functions (PDFs).

Because of the unique capabilities of Jefferson Lab's CEBAF accelerator in measuring small cross sections at extreme kinematics, the JAM spin PDFs are particularly tailored for studies of the large Bjorken- x region, as well as the resonance-deep inelastic scattering transition region at low and intermediate values of W^2 and Q^2 .

Library

We provide a collection of codes/scripts (in fortran, python, mathematica), along with interpolation tables for the collinear parton distributions in the nucleon, as well as the collinear parton to hadron fragmentation functions. The library is available at [JAMLIB](#).

Theory Framework

Our analysis uses collinear factorization at NLO in perturbative QCD with an emphasis on the large- x region which includes a treatment of higher twist as well as target mass corrections. A treatment of nuclear corrections for deuteron and ^3He targets in DIS data sets is included.

Results

Pion and Kaon fragmentation from JAM16

The three plots show the fragmentation function $D_{h/q}^A(z, Q^2)$ versus z for π^+ , K^+ , and π^0 mesons. The x-axis represents z from 0.2 to 0.8, and the y-axis represents $D_{h/q}^A(z, Q^2)$ from 0.2 to 1.4. The plots show the fragmentation function for the quark q and the antiquark \bar{q} for each meson. The π^+ plot shows the fragmentation function for u and \bar{u} quarks. The K^+ plot shows the fragmentation function for u and \bar{u} quarks. The π^0 plot shows the fragmentation function for u and \bar{u} quarks.

The screenshot shows the GitHub repository page for JeffersonLab/JAMLIB. The repository is titled "JeffersonLab / JAMLIB" and has 209 commits, 1 branch, 4 releases, and 44 contributors. The repository contains several files, including "README.md", "README.py", "README.f", "README.m", "README.c", "README.f90", "README.f95", "README.f03", "README.f08", "README.f15", "README.f18", "README.f22", "README.f27", "README.f32", "README.f37", "README.f42", "README.f47", "README.f52", "README.f57", "README.f62", "README.f67", "README.f72", "README.f77", "README.f82", "README.f87", "README.f92", "README.f97", "README.f102", "README.f107", "README.f112", "README.f117", "README.f122", "README.f127", "README.f132", "README.f137", "README.f142", "README.f147", "README.f152", "README.f157", "README.f162", "README.f167", "README.f172", "README.f177", "README.f182", "README.f187", "README.f192", "README.f197", "README.f202", "README.f207", "README.f212", "README.f217", "README.f222", "README.f227", "README.f232", "README.f237", "README.f242", "README.f247", "README.f252", "README.f257", "README.f262", "README.f267", "README.f272", "README.f277", "README.f282", "README.f287", "README.f292", "README.f297", "README.f302", "README.f307", "README.f312", "README.f317", "README.f322", "README.f327", "README.f332", "README.f337", "README.f342", "README.f347", "README.f352", "README.f357", "README.f362", "README.f367", "README.f372", "README.f377", "README.f382", "README.f387", "README.f392", "README.f397", "README.f402", "README.f407", "README.f412", "README.f417", "README.f422", "README.f427", "README.f432", "README.f437", "README.f442", "README.f447", "README.f452", "README.f457", "README.f462", "README.f467", "README.f472", "README.f477", "README.f482", "README.f487", "README.f492", "README.f497", "README.f502", "README.f507", "README.f512", "README.f517", "README.f522", "README.f527", "README.f532", "README.f537", "README.f542", "README.f547", "README.f552", "README.f557", "README.f562", "README.f567", "README.f572", "README.f577", "README.f582", "README.f587", "README.f592", "README.f597", "README.f602", "README.f607", "README.f612", "README.f617", "README.f622", "README.f627", "README.f632", "README.f637", "README.f642", "README.f647", "README.f652", "README.f657", "README.f662", "README.f667", "README.f672", "README.f677", "README.f682", "README.f687", "README.f692", "README.f697", "README.f702", "README.f707", "README.f712", "README.f717", "README.f722", "README.f727", "README.f732", "README.f737", "README.f742", "README.f747", "README.f752", "README.f757", "README.f762", "README.f767", "README.f772", "README.f777", "README.f782", "README.f787", "README.f792", "README.f797", "README.f802", "README.f807", "README.f812", "README.f817", "README.f822", "README.f827", "README.f832", "README.f837", "README.f842", "README.f847", "README.f852", "README.f857", "README.f862", "README.f867", "README.f872", "README.f877", "README.f882", "README.f887", "README.f892", "README.f897", "README.f902", "README.f907", "README.f912", "README.f917", "README.f922", "README.f927", "README.f932", "README.f937", "README.f942", "README.f947", "README.f952", "README.f957", "README.f962", "README.f967", "README.f972", "README.f977", "README.f982", "README.f987", "README.f992", "README.f997", "README.f1002", "README.f1007".

Jefferson Lab Angular Momentum Collaboration

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