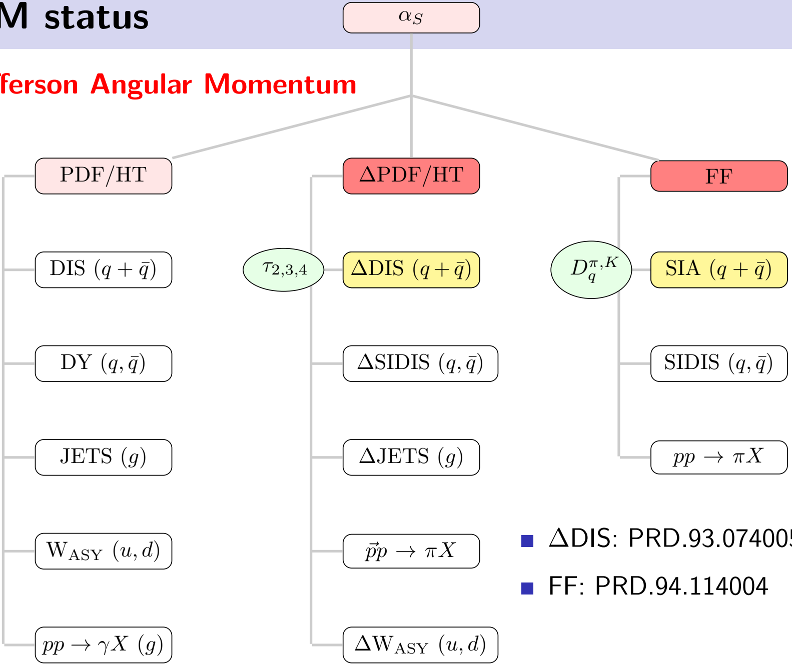
Towards combined QCD global analysis of polarized and unpolarized PDFs and fragmentation functions

Nobuo Sato

7th Workshop of the APS Topical Group on Hadronic Physics
Feb, 2017

JAM status

Jefferson Angular Momentum



- ΔDIS: PRD.93.074005
- FF: PRD.94.114004

JAM in progress

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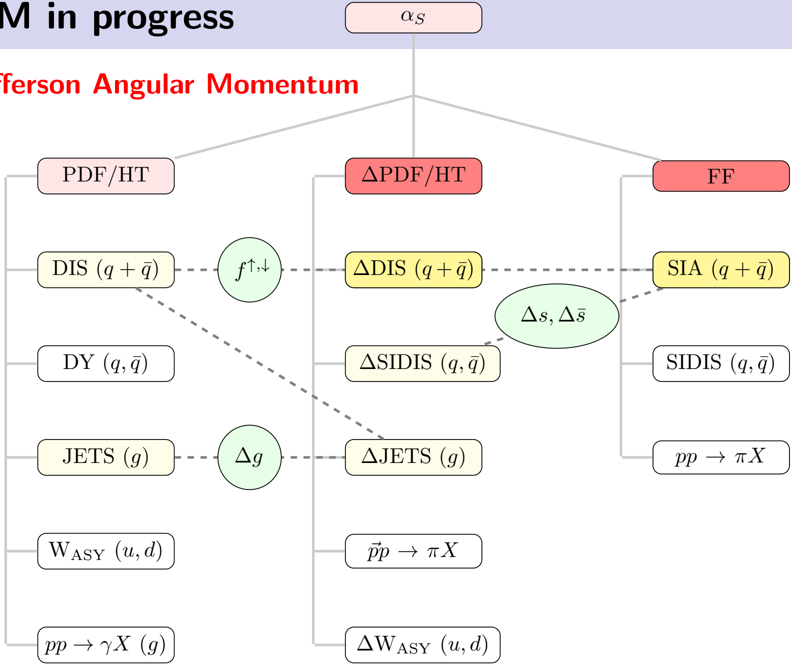

Theory of fitting

The goal is to estimate:

$$\mathrm{E}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; \mathcal{O}(\boldsymbol{a})$$

$$\mathrm{V}[\mathcal{O}] = \int d^n a \;\; \mathcal{P}(\boldsymbol{a}|data) \;\; [\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}]]^2$$

Monte Carlo methods

- $\mathcal{P}(\boldsymbol{a}|data) \rightarrow \{\boldsymbol{a}_k\}$
- $\mathrm{E}[\mathcal{O}] \approx \frac{1}{N}\sum_k \mathcal{O}(\boldsymbol{a}_k)$
- $\mathrm{V}[\mathcal{O}] \approx \frac{1}{N}\sum_k [\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}]]^2$

Maximum Likelihood

- Maximize $\mathcal{P}(\boldsymbol{a}|data) \rightarrow \boldsymbol{a}_0$
- $\mathrm{E}[\mathcal{O}] \approx \mathcal{O}(\boldsymbol{a}_0)$
- $\mathrm{V}[\mathcal{O}] \approx$hessian, $\Delta\chi^2$ envelope,...

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The $\Delta\chi^2$ paradox

- there is a controversy about how to treat errors in global fits
- A particular example occurred in the IC context
  - Jimenez-Delgado et al PRL114 (2015) $\Delta\chi^2 = 1$
  - Brodsky, Gardner PRL116 (2016) $\Delta\chi^2 \neq 1$
- similar issues in other analysis (TMD, FF)
- I will attempt clarify this!!!
- to understand this lets consider an example ...

The $\Delta\chi^2$ paradox: Example

- consider two parameters:

$$\theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2}$$

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- change of variables

$$\theta_{1,2} \quad \rightarrow \quad t_{1,2} = \frac{\theta_{1,2} - \mu_{1,2}}{\sigma_{1,2}} \quad \rightarrow \quad r, \phi$$

$$\begin{aligned} d\theta_1 d\theta_2 p(\theta_1, \theta_2) &= dt_1 dt_2 \frac{1}{2\pi} \exp\left[-\frac{1}{2}(t_1^2 + t_2^2)\right] \\ &= d\phi\ r dr\ p(t_1, t_2) = \frac{d\phi}{2\pi}\ r dr\ \exp\left[-\frac{1}{2} r^2\right] \end{aligned}$$

The $\Delta\chi^2$ paradox: Example

- confidence volume:

$$\mathrm{CV} = \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^R dr\ r\ \exp\left[-\frac{1}{2}r^2\right] = \int_0^R dr\ r\ \exp\left[-\frac{1}{2}r^2\right]$$

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- notice that

$$R^2 = t_1^2 + t_2^2 = \left(\frac{\theta_1 - \mu_1}{\sigma_1}\right)^2 + \left(\frac{\theta_2 - \mu_2}{\sigma_2}\right)^2 \equiv \chi^2$$

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- the *confidence region* for the parameters is

$$\max\left[t_{1,2}\right] = R \quad \to \quad \theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2}\ R$$

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- the *confidence region* for the parameters is

$$\max\left[t_{1,2}\right] = R \quad \to \quad \theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2}\ R$$

- ... but we know that

$$\theta_{1,2} = \mu_{1,2} \pm \sigma_{1,2}$$

- this is the origin of the paradox.

The $\Delta\chi^2$ **paradox**: Example

- **The resolution** $\rightarrow$ **Use calculus!!**

The $\Delta\chi^2$ paradox: Example

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- The expectation value:

$$\begin{aligned}
\mathrm{E}\left[\theta_{1,2}\right] &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\infty} dr \; p(r,\phi) \; \theta_{1,2} \\
&= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\infty} dr \; r \exp\left[-\frac{1}{2} r^2\right] \; \left[\mu_{1,2} + t_{1,2}\sigma_{1,2}\right] \\
&= \mu_{1,2}
\end{aligned}$$

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&= \mu_{1,2}
\end{aligned}$$

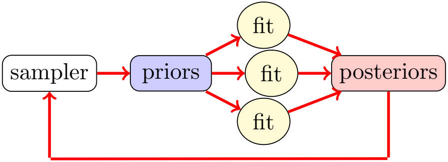
- The variance value:

$$\begin{aligned}
\mathrm{V}\left[\theta_{1,2}\right] &= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\infty} dr\ p(r,\phi)\ \left[\theta_{1,2} - \mu_{1,2}\right]^2 \\
&= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\infty} dr\ r \exp\left[-\frac{1}{2}r^2\right]\ \ \left[t_{1,2}\sigma_{1,2}\right]^2 \\
&= \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\infty} dr\ r \exp\left[-\frac{1}{2}r^2\right]\ \ \left[r\{\cos,\sin\}(\phi)\sigma_{1,2}\right]^2 \\
&= \sigma_{1,2}^2
\end{aligned}$$

The $\Delta\chi^2$ paradox: The resolution

- If:
  - the likelihood function is approximately gaussian in the parameters
  - If the model is well described by a linear approximation
- Then,
  - 1σ CL is characterized by the hessian criterion with $\Delta\chi^2 = 1$ for any arbitrary number of parameters
- Else,
  - Use MC methods

Iterative Monte Carlo analysis (IMC)



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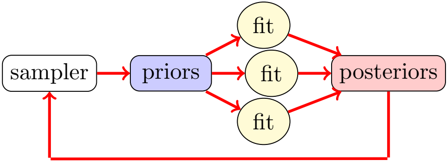

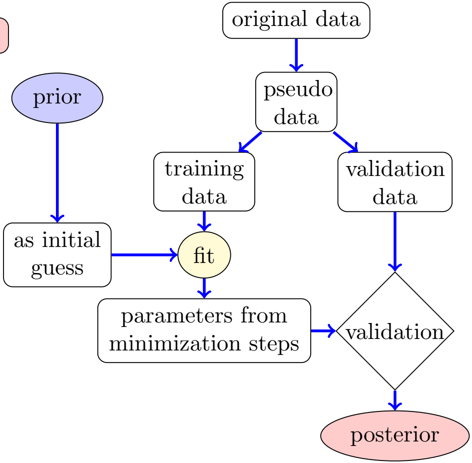

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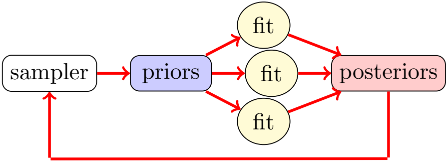
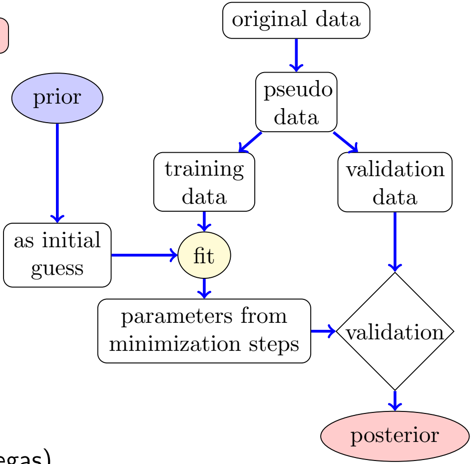
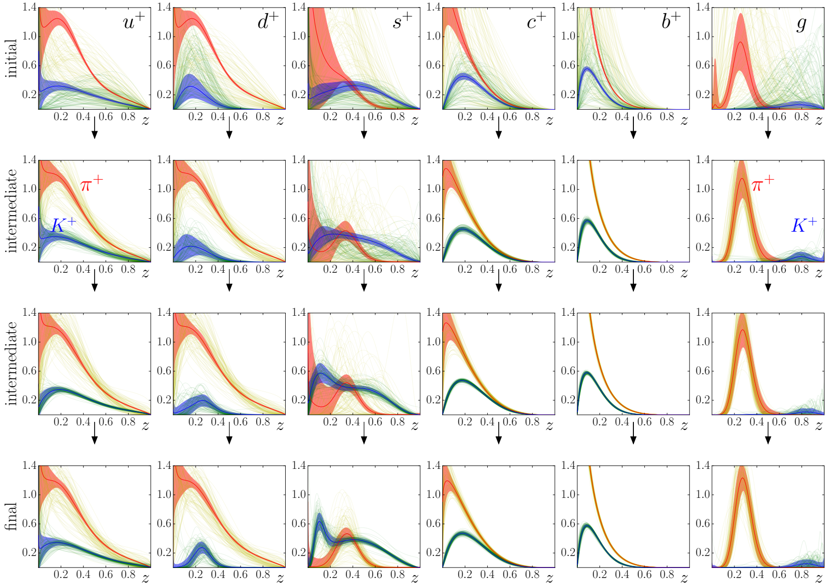




- Use traditional anzat
  $xf(x) = Nx^a(1-x)^b(1 + c\sqrt{x} + dx)$
- Keep all the parameters free.
  No assumptions on the exponents
- Avoid over-fitting by Cross-Validation
- **Iterative procedure**
  $\rightarrow$ Adaptive MC integration (like in Vegas)
- Robust estimation of uncertainties

IMC in action (example in FF case)

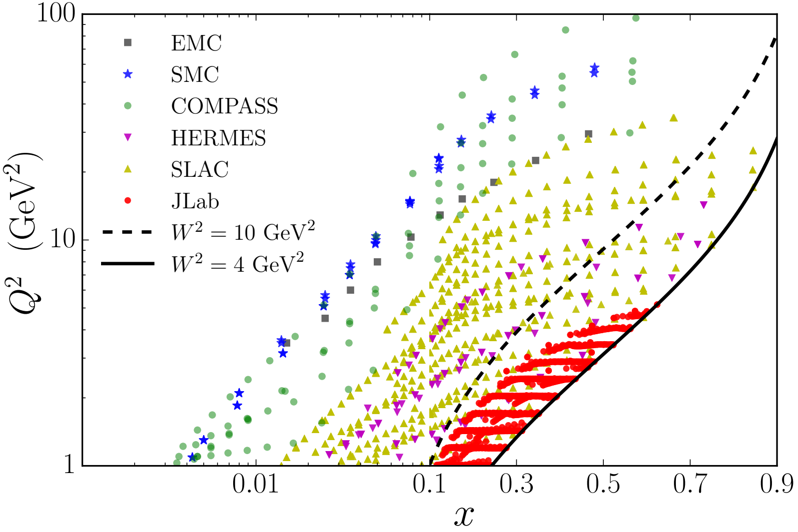


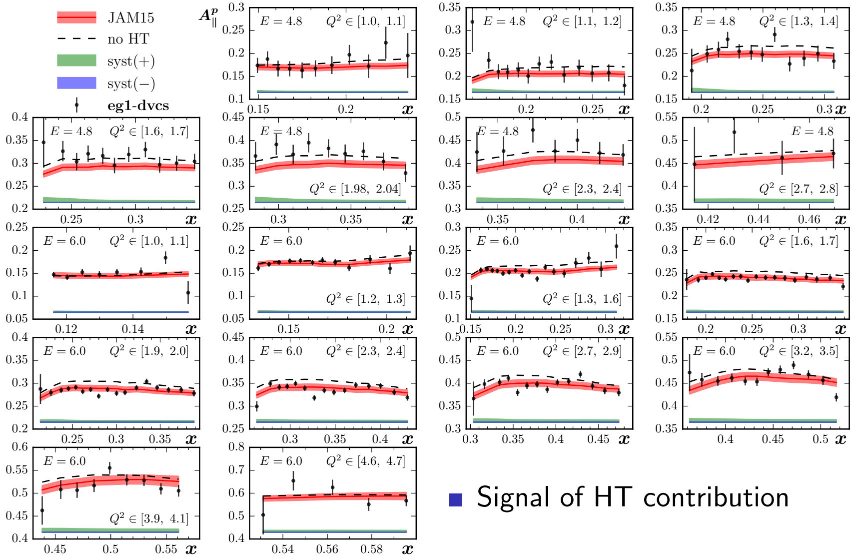
Spin PDFs from polarized DIS

In Collaboration with:

- J. J. Ethier (College of William and Mary)
- W. Melnitchouk (Jefferson Lab)
- A. Accardi (Hampton U. and Jefferson Lab)
- S. E. Kuhn (Old Dominion U.)

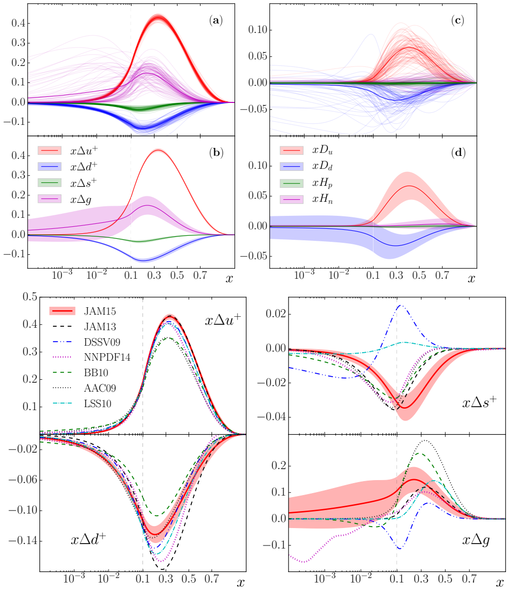
Global polarized DIS data





- Signal of HT contribution

Results



moment	truncated	full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta\Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.011 ± 0.004	0.011 ± 0.004
d_2^n	-0.002 ± 0.002	-0.002 ± 0.002
h_p	-0.000 ± 0.001	0.000 ± 0.001
h_n	0.001 ± 0.002	0.001 ± 0.003

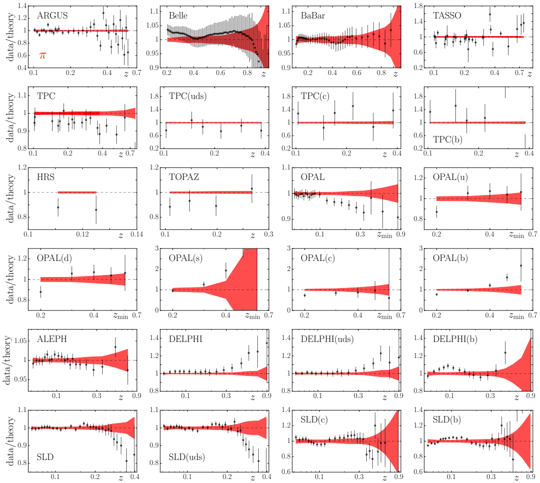
- $\chi^2/N_{npts} = 1.07$
- Sign of τ_3 distributions is the same as τ_2
- **Negative** Δs^+
- Δg compatible with the most recent DSSV fits
- moments of Δg not constrained.

Fragmentation Functions from SIA data

In Collaboration with:

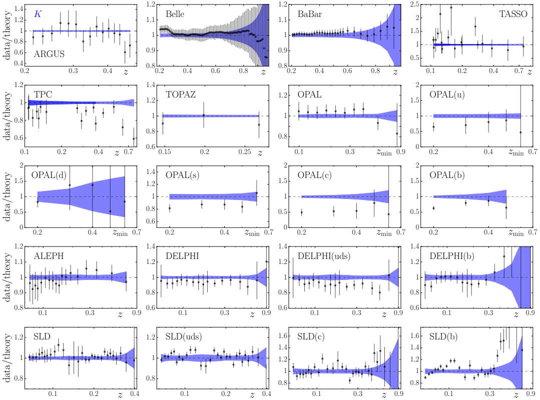
- J. J. Ethier (College of William and Mary)
- W. Melnitchouk (Jefferson Lab)
- A. Accardi (Hampton U. and Jefferson Lab)
- S.Kumano (KEK, J-PARC)
- M.Hirai (Nippon Institute of Technology)

π analysis $(\chi^2/N_{\text{npts}} = 1.31)$



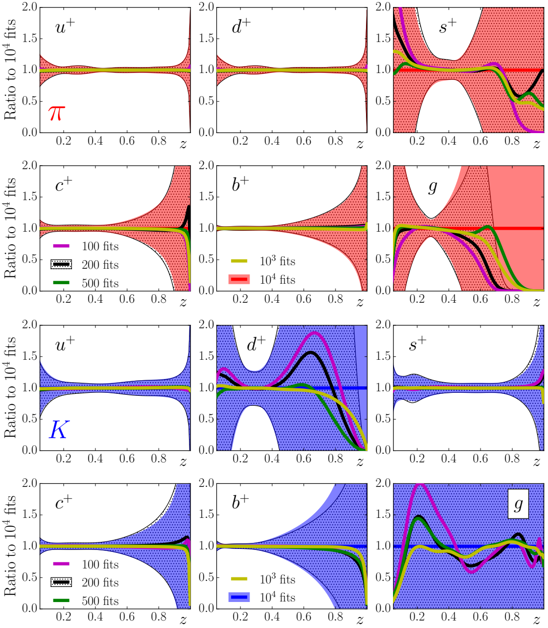
- $z_{\text{cut}} > 0.1$ for low energies
- $z_{\text{cut}} > 0.05$ for high energies
- We use BaBar prompt data set
- Belle data set needs 10% normalization
- Good agreement at low–z for inclusive data sets
- Data inconsistencies at large–z for $Q^2 = M_z^2$

K analysis $\quad (\chi^2/N_{\mathrm{npts}} = 1.01)$



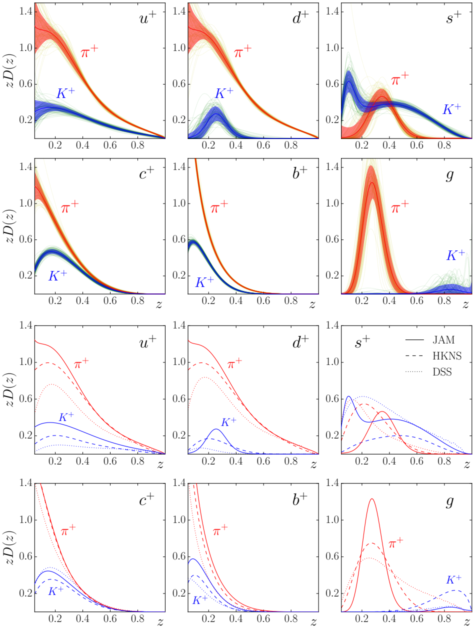
- $z_{\mathrm{cut}} > 0.2$ for low energies to avoid hadron mass corrections
- $z_{\mathrm{cut}} > 0.05$ for high energies
- Smaller χ^2 that π due to larger errors
- Consistent shapes across all z
- Inconsistencies mostly due to normalizations

Fragmentation functions



- Larger constraints on favored FFs than unfavored FFs
- More sensitivity on D_g^π than D_g^K
- D_g^K is unknown
- HQ tagged data provides similar constraints for π and K
- Convergence attained with ~ 200 posteriors

Fragmentation functions



- Similar behavior of unfavored $D^{\pi}_{s^+}$ and $D^{K}_{d^+}$
- In contrast the D^{π}_{g} and D^{K}_{g} behave differently
- The charm and bottom FFs become compatible at large–z
- Favored $D^{\pi}_{u^+}$ and $D^{K}_{s^+}$ have similar shape at large–z
- JAM $D^{K}_{s^+}$ is compatible with DSS. **Will it change the sign of Δs^+?**

Summary and outlook

Polarized DIS data

- New JAM analysis to study impact of all JLab 6 GeV inclusive DIS data at low W and high x
- New extraction of LT & HT distributions
- Constraints on d_2

SIA data

- New study of SIA data including recent Belle and BaBar data
- New extraction of fragmentation functions
- The setup for combined pol DIS, SIA and pol SIDIS is ready!

JAMLIB

- JAM SPDFs and FFs are available at github
- Python, Fortran and LHAPDF interfaces provided

JAM for public