Quark orbital dynamics in the nucleon – from Ji to Jaffe-Manohar orbital angular momentum

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Acknowledgments:
M. Burkardt, S. Liuti,
and members of the Lattice TMD Collaboration

Gauge ensembles provided by:
MILC Collaboration

Proton spin decompositions

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L_q + J_g \qquad \text{(Ji)}$$

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q \mathcal{L}_q + \Delta g + \mathcal{L}_g \qquad \text{(Jaffe-Manohar)}$$

... and many more (in fact, we will see a continuous interpolation between the two ...)

There isn't one unique way of separating quark and gluon orbital angular momentum – the different decompositions have different, legitimate meanings.

Quark orbital angular momentum

Interpreting terms in the energy-momentum tensor:

$$L_q \quad \sim \quad -i\psi^\dagger(\vec{r} \times \vec{D})_z \psi$$

Can be obtained from $L_q = J_q - S_q$, where S_q and J_q can be related to GPDs (Ji sum rule) – this has been used in Lattice QCD.

$$\mathcal{L}_q \quad \sim \quad -i\psi^\dagger(\vec{r} \times \vec{\partial})_z \psi \qquad \text{in light cone gauge}$$

Hitherto not accessed in Lattice QCD.

Direct evaluation of quark orbital angular momentum

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T \, (r_T \times k_T)_3 \, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T)$$

Wigner distribution

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0\, ,\ \Delta_T=0\, ,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0\, ,\ \Delta_T=0\, ,\ z_T\to 0}}$$

n : Number of valence quarks

$p' = P + \Delta_T/2$, $p = P - \Delta_T/2$, P, S in 3-direction, $P \to \infty$

This is the same type of operator as used in TMD studies – generalization to off-forward matrix element adds transverse position information

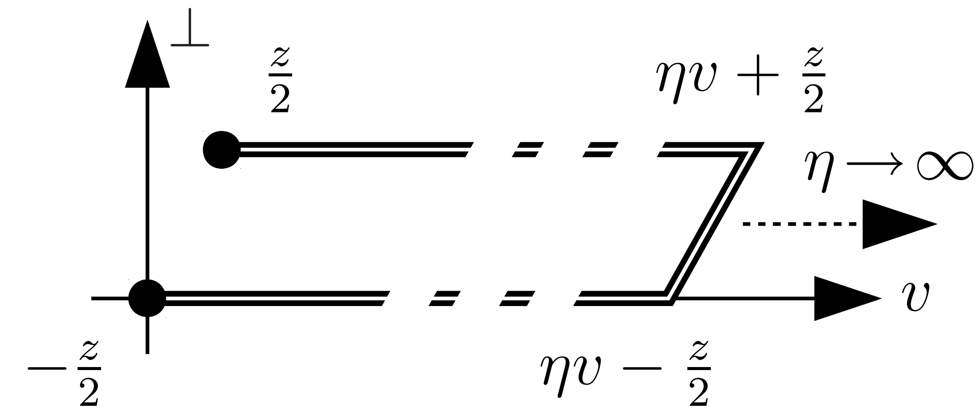
Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Y. Hatta, M. Burkardt:

- Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM
- Difference is torque accumulated due to final state interaction



Direct evaluation of quark orbital angular momentum

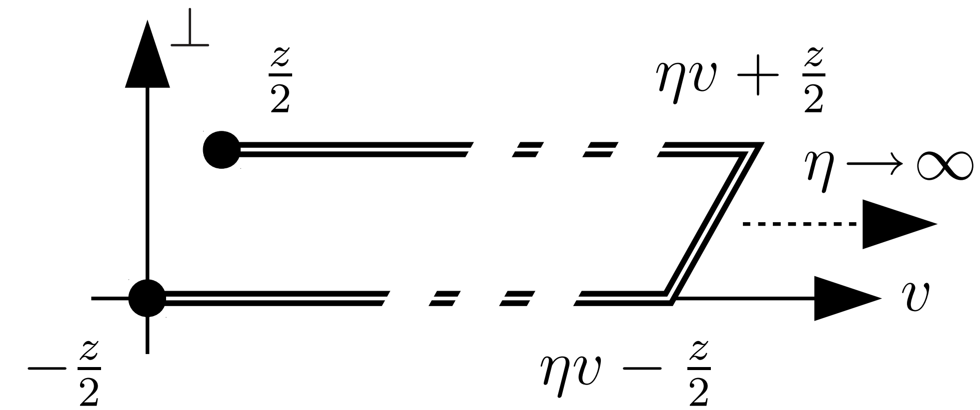
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\hat{\zeta} \equiv \frac{P\cdot v}{|P||v|}$

Are interested in $\hat{\zeta} \longrightarrow \infty$; synonymous with $P \longrightarrow \infty$ in the frame of the lattice calculation $(v = e_3)$



Ensemble details

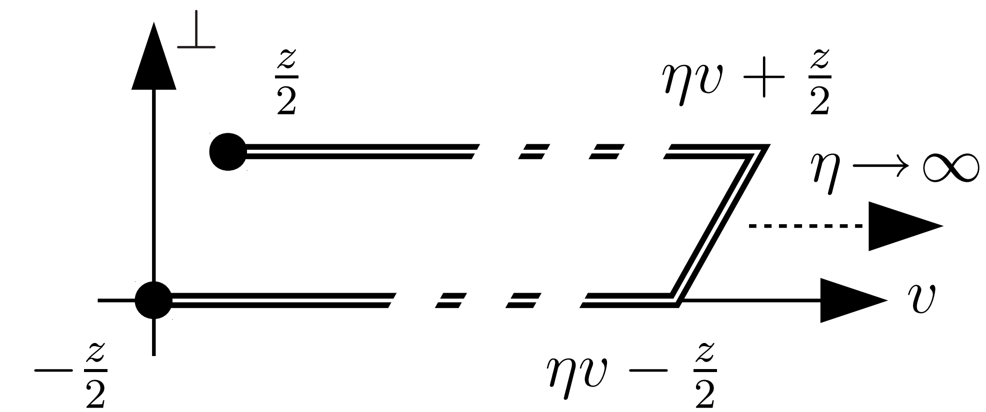
LHPC mixed action scheme: MILC asqtad configurations, domain wall valence quarks

$L^3 \times T$	a(fm)	$am_{u,d}$	am_s	$m_\pi^{\rm DWF}$ (MeV)	$m_N^{\rm DWF}$ (GeV)	#conf.	#meas.
$20^3 \times 64$	0.11849(14)(99)	0.02	0.05	518.4(07)(49)	1.348(09)(13)	486	3888

Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Parameters to consider: Δ, $\hat{\zeta}$, z, η



Direct evaluation of quark orbital angular momentum

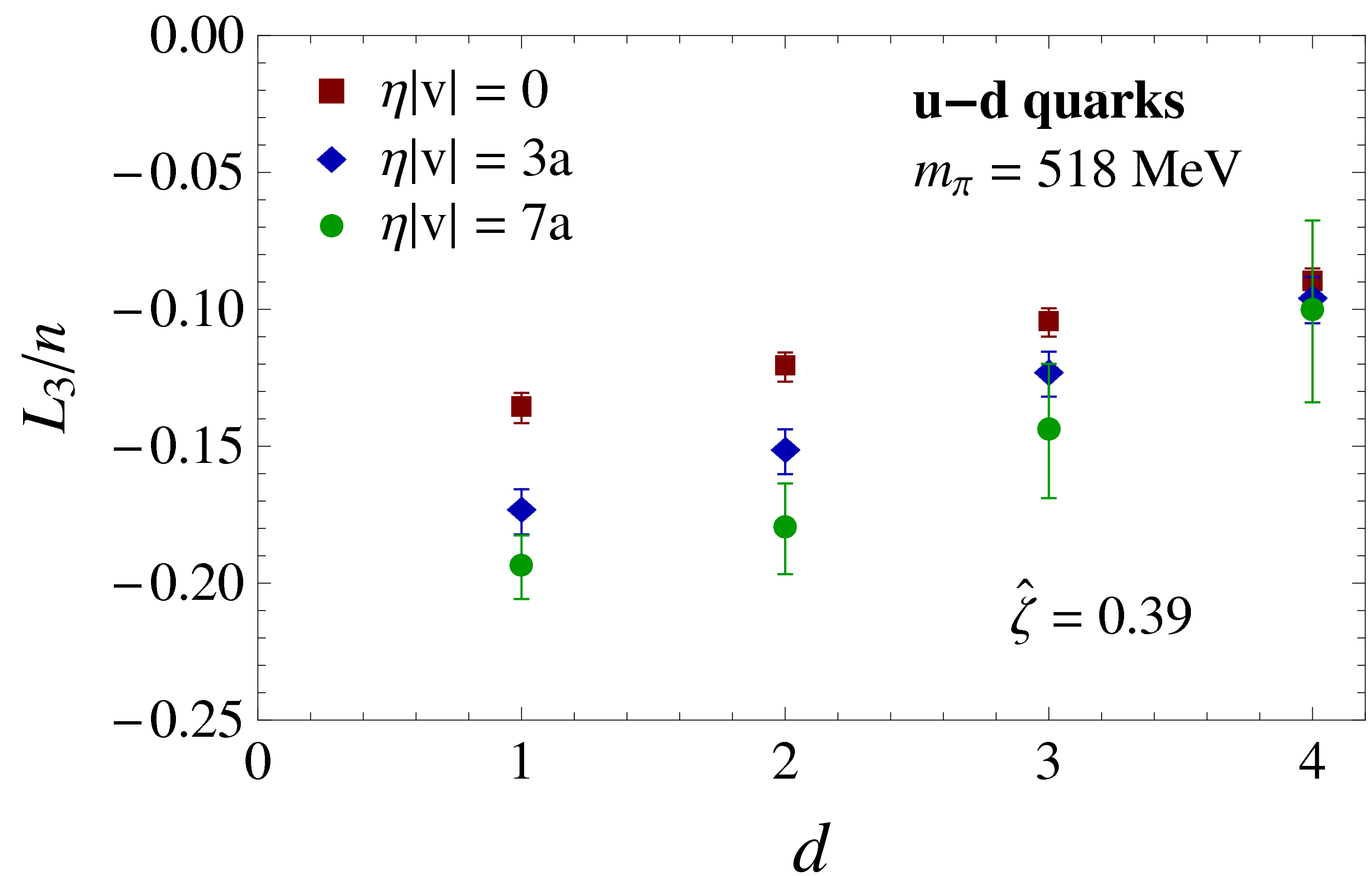
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Dataset contains only one value of $|\Delta_T| = 4\pi/aL \approx 1$ GeV

Substantial underestimate of $\partial f/\partial\Delta_T$ by using

$$\left.\frac{\partial f}{\partial \Delta_{T,j}}\right|_{\Delta_{T,j}=0} = \frac{1}{2\Delta_{T,j}}(f(\Delta_{T,j}) - f(-\Delta_{T,j}))$$

Direct evaluation of quark orbital angular momentum

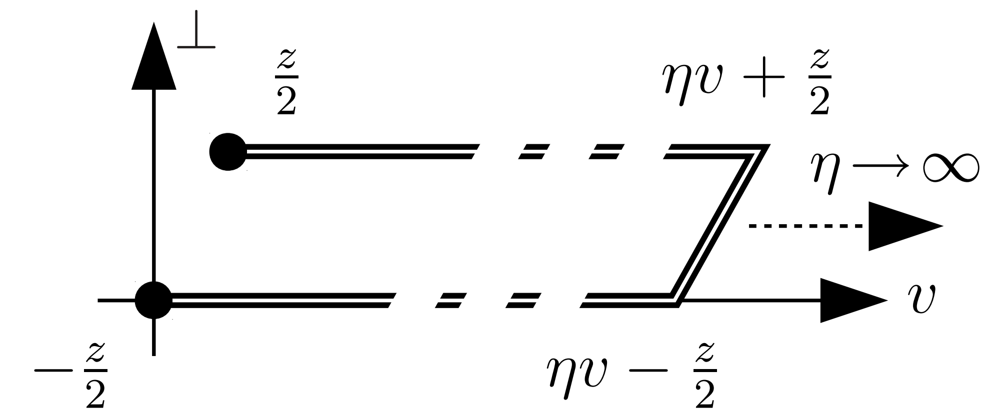


$$\left.\frac{\partial f}{\partial z_{T,i}}\right|_{z_{T,i}=0} = \frac{1}{2da}(f(dae_i) - f(-dae_i))$$

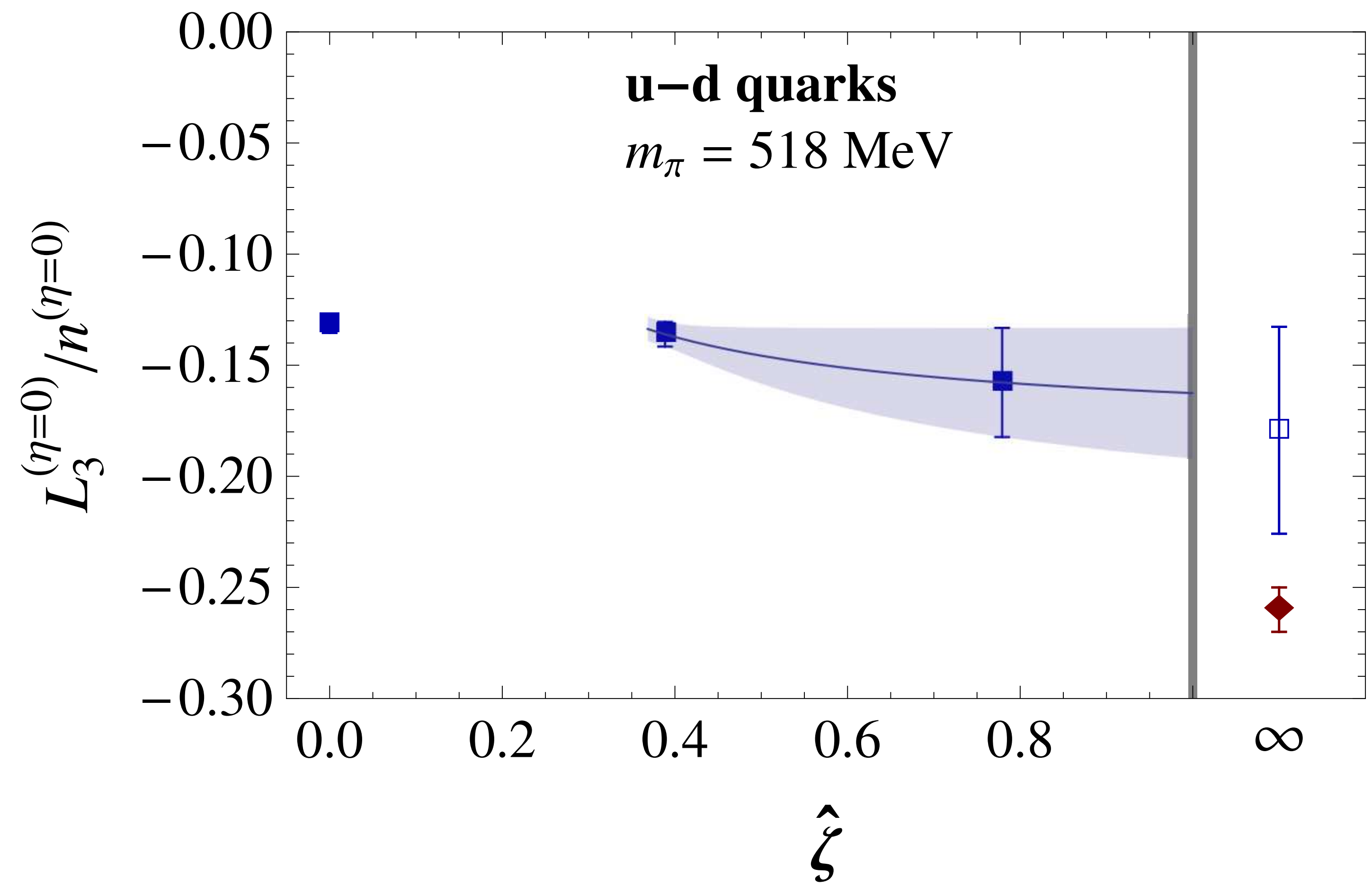
Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Remaining parameters to consider: $\hat{\zeta}$, η

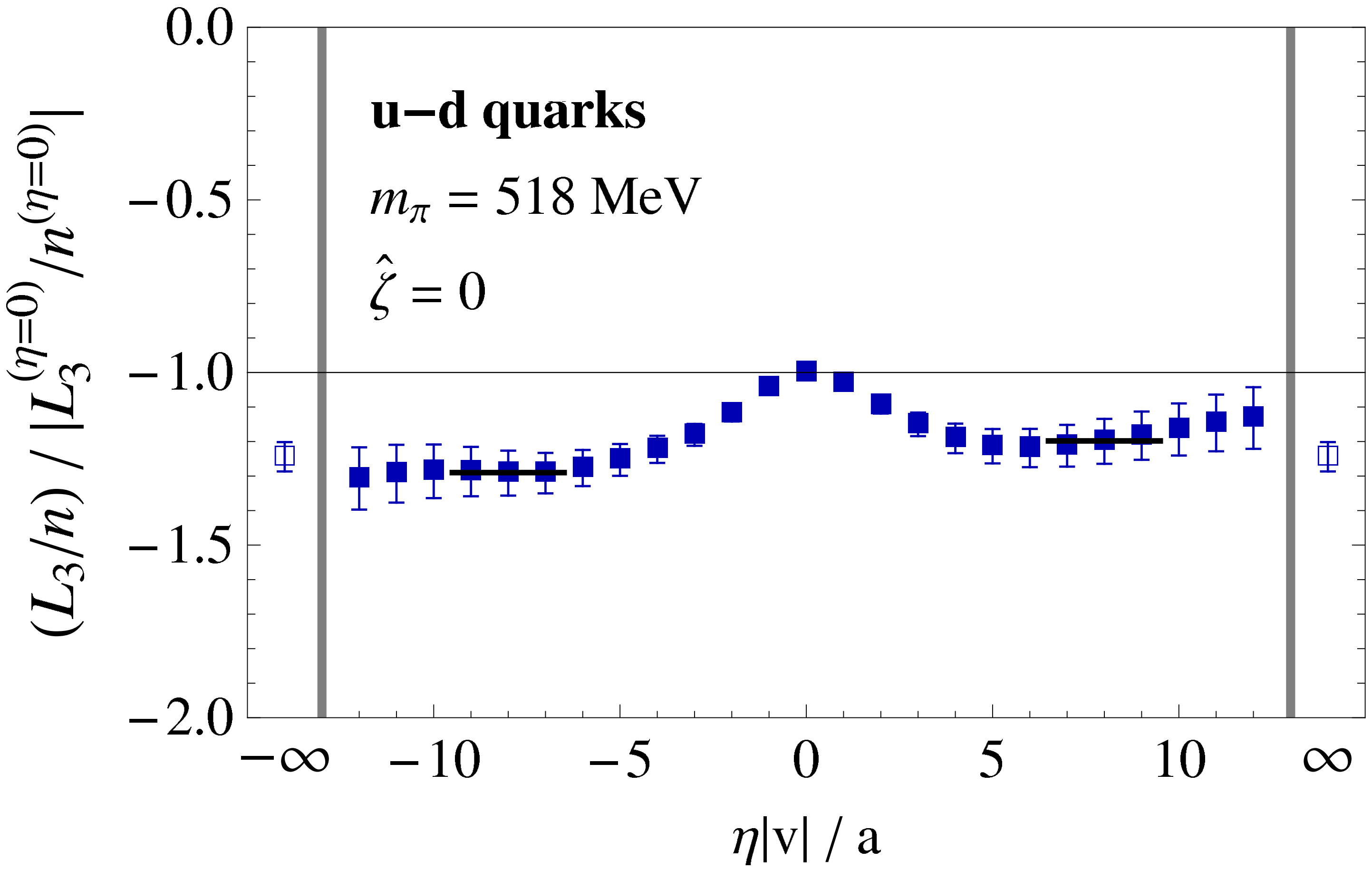


Ji quark orbital angular momentum: $\eta = 0$

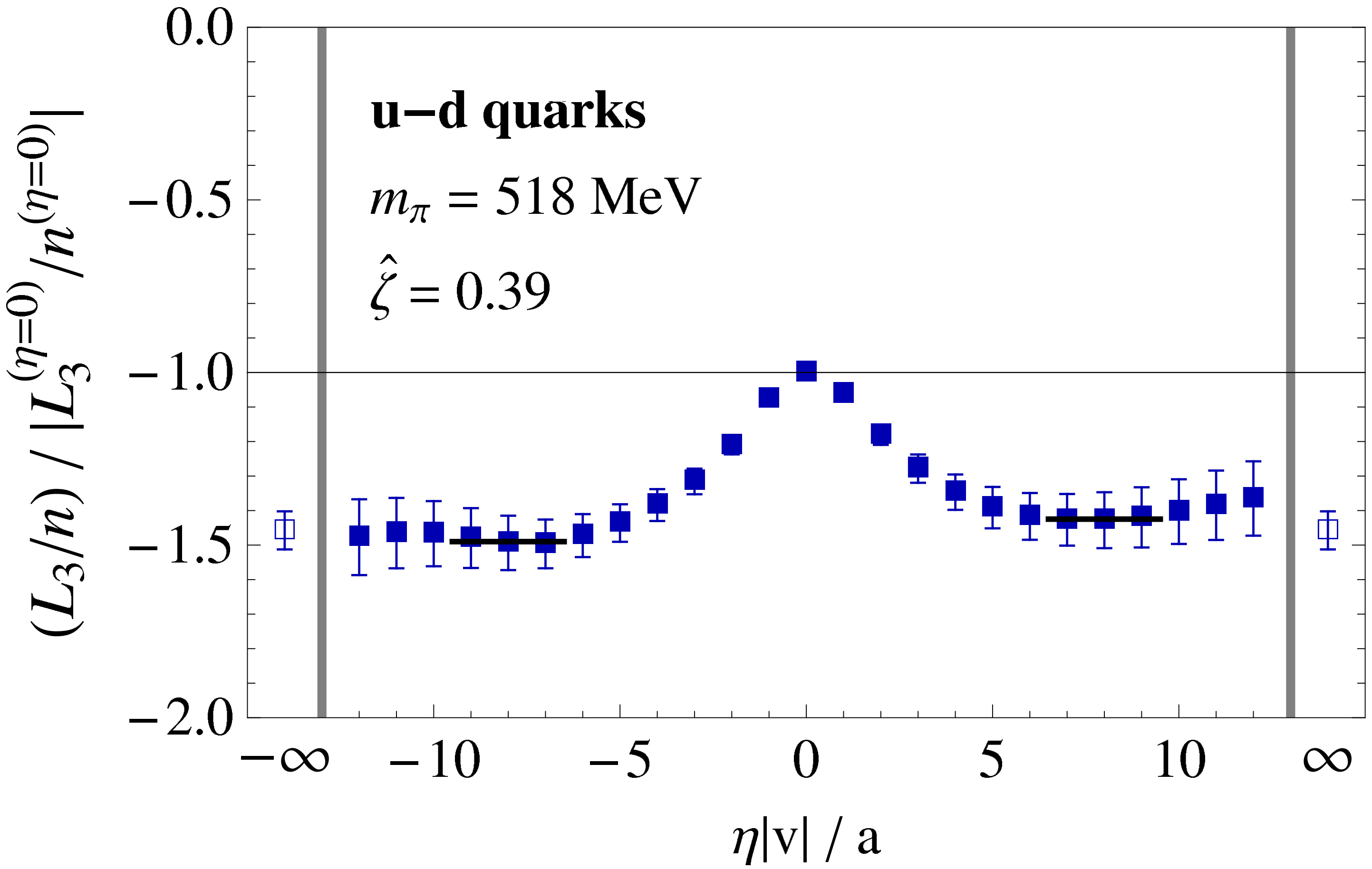


$\longrightarrow$ Signature of underestimate of $\partial f/\partial \Delta_T$

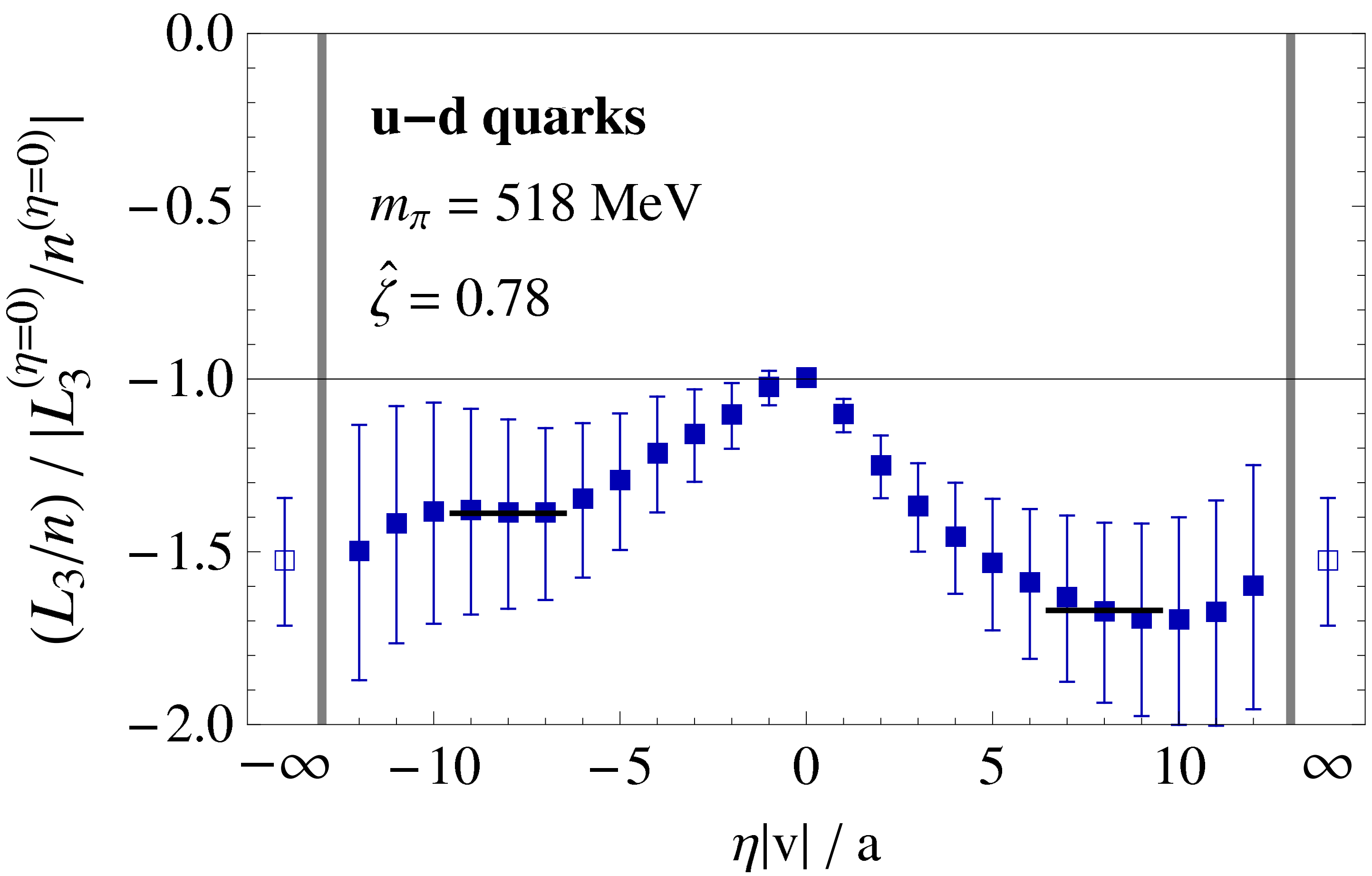
From Ji to Jaffe-Manohar quark orbital angular momentum



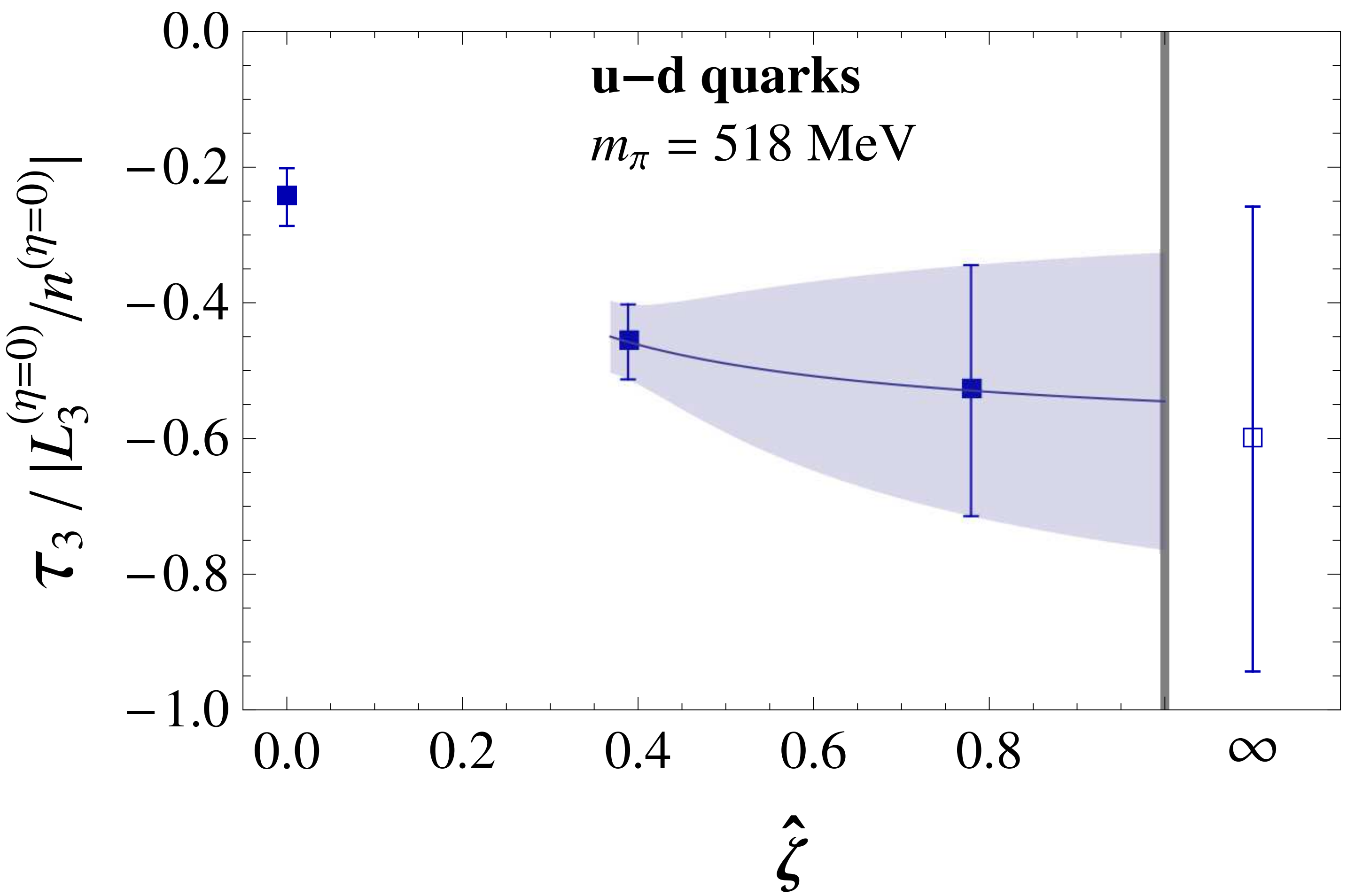
From Ji to Jaffe-Manohar quark orbital angular momentum



From Ji to Jaffe-Manohar quark orbital angular momentum



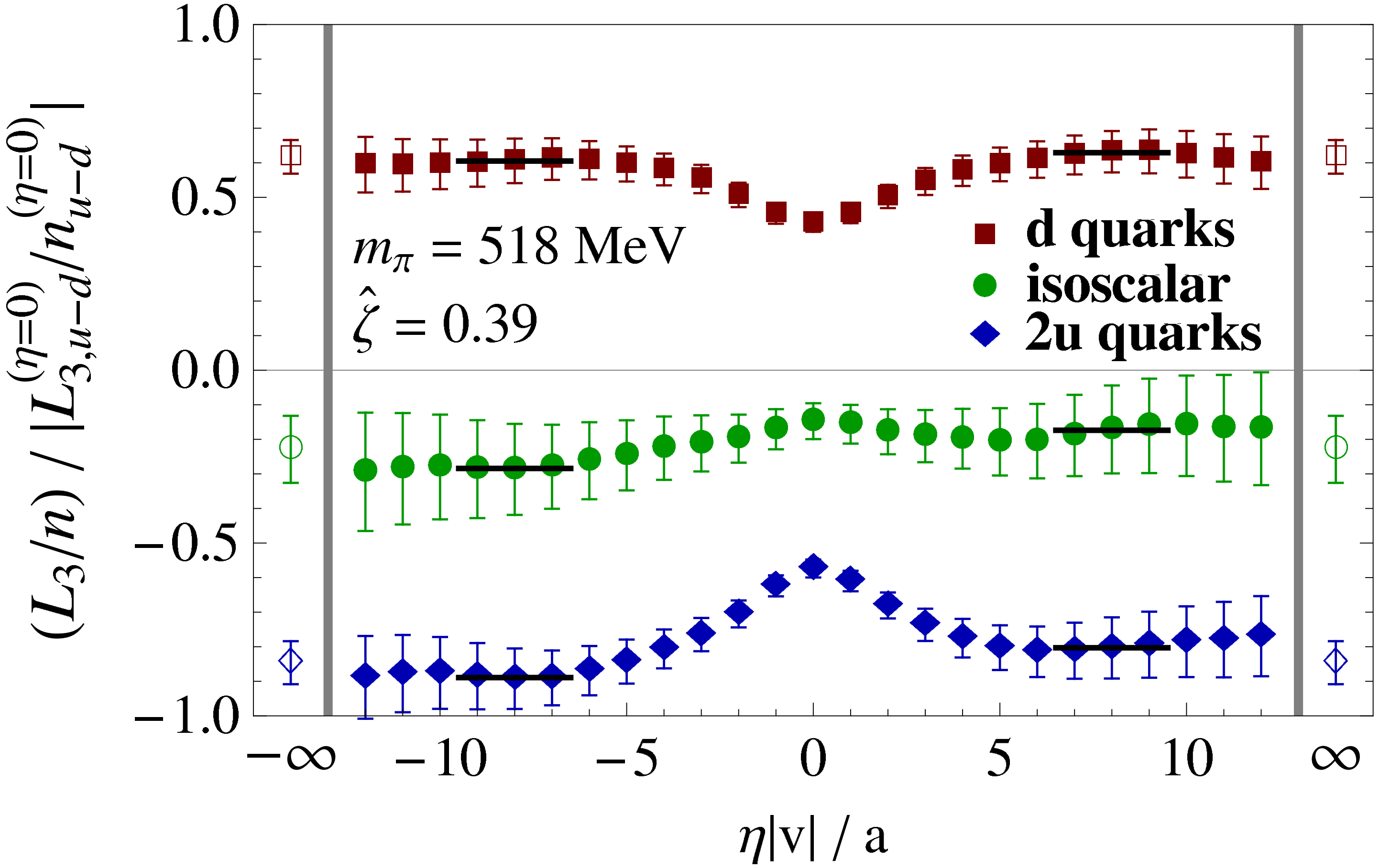
Burkardt's torque – extrapolation in $\hat{\zeta}$



$$\tau_3 = (L_3^{(\eta=\infty)}/n^{(\eta=\infty)}) - (L_3^{(\eta=0)}/n^{(\eta=0)})$$

Integrated torque accumulated by struck quark leaving proton

Flavor separation – from Ji to Jaffe-Manohar quark orbital angular momentum



Connection to GTMDs and twist-3 GPDs

$$L_3^{\mathcal{U}} = \epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}$$

$$= -\int dx \int d^2k_T\, \frac{k_T^2}{m^2}\, F_{14}(x, k_T^2, k_T\cdot\Delta_T, \Delta_T^2)\ \Big|_{\Delta_T = 0}$$

Generalized transverse momentum-dependent parton distribution (GTMD)

Lorentz invariance and equation of motion relations

Straight gauge link

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{\text{straight}} = \widetilde{E}_{2T} + H + E$$

$$-x\widetilde{E}_{2T} = \widetilde{H} - \int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{straight}} + \widetilde{\mathcal{M}}^{\text{straight}}$$

Staple-shaped gauge link

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14}^{\text{staple}} = \widetilde{E}_{2T} + H + E + \mathcal{A}$$

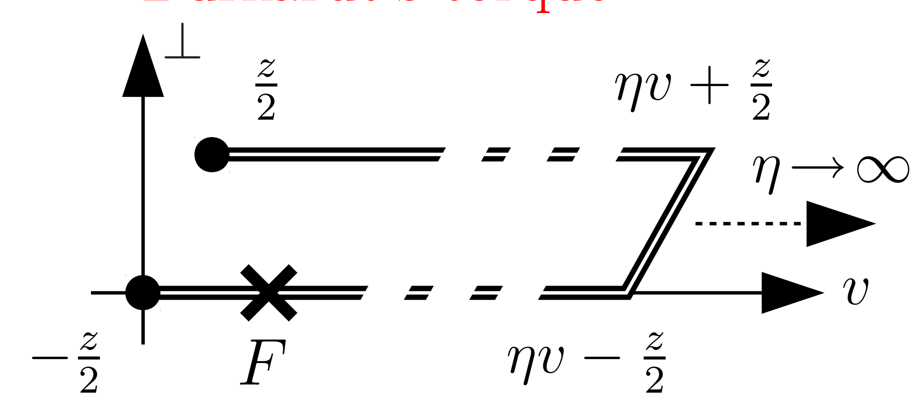
$$-x\widetilde{E}_{2T} = \widetilde{H} - \int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{staple}} + \widetilde{\mathcal{M}}^{\text{staple}}$$

A. Rajan, A. Courtoy, M.E., S. Liuti (arXiv:1601.06117)

$$\mathcal{A} = \frac{d}{dx}\left(\int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{staple}} - \int d^2k_T\,\frac{k_T^2}{M^2}F_{14}^{\text{straight}}\right)$$
$$= \frac{d}{dx}\left(\widetilde{\mathcal{M}}^{\text{staple}} - \widetilde{\mathcal{M}}^{\text{straight}}\right)$$

$\longrightarrow$ Burkardt's torque

(here: $z_T \to 0$ limit)



Conclusions

- Quark orbital angular momentum can be accessed directly in Lattice QCD, continuously interpolating between the Ji and Jaffe-Manohar definitions.
- In the gathered exploratory dataset, the difference between the Ji and Jaffe-Manohar definitions, i.e., the torque accumulated by the struck quark leaving a proton, is clearly resolvable, sizeable ($\sim 50\%$ of the original Ji OAM), and leads to an enhancement of Jaffe-Manohar OAM relative to Ji OAM.
- Principal shortcomings of present dataset are too large momentum transfer, too small proton momentum. These practical issues are planned to be resolved in future work.
- Plan to explore quark orbital angular momentum also via twist-3 GPD $\widetilde{E}_{2T}$.