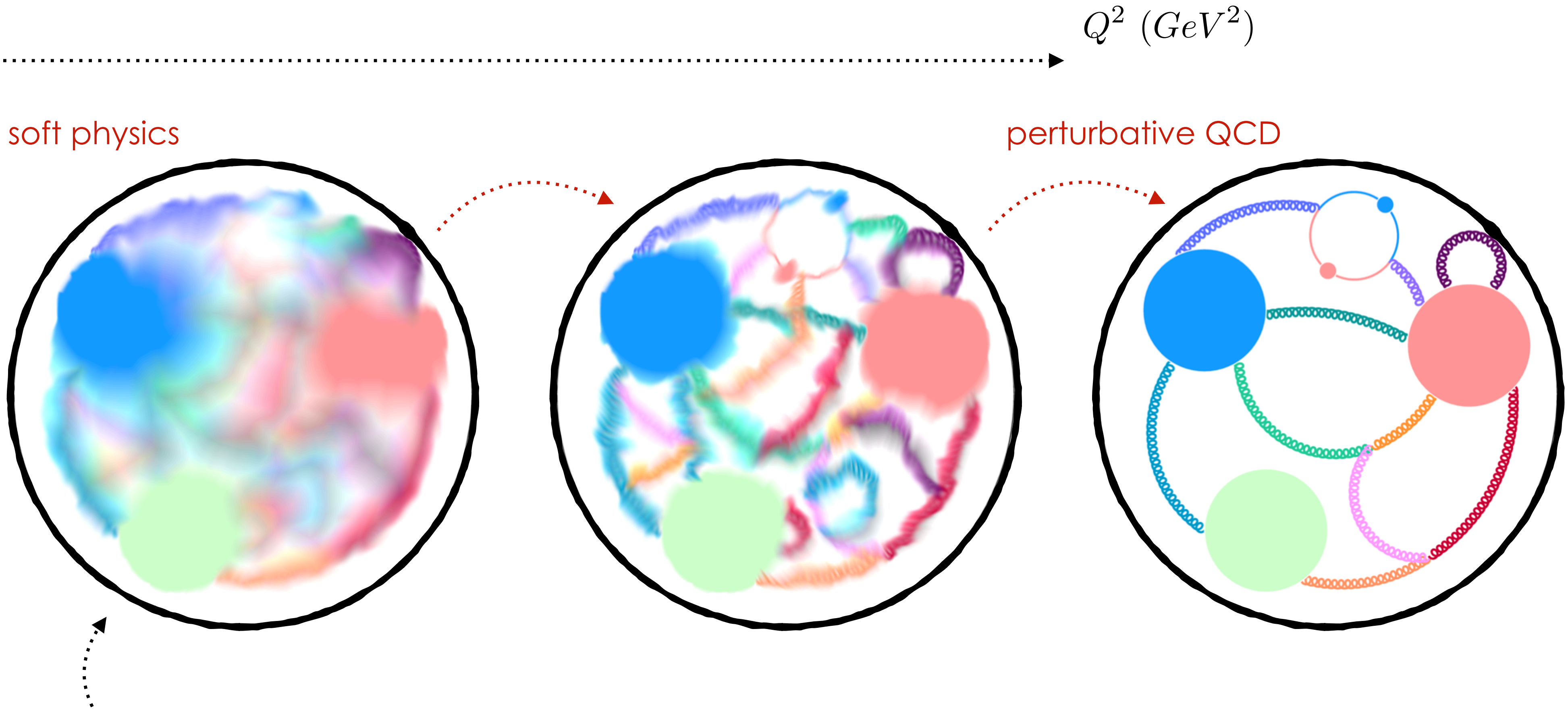
From small to moderate-x: beyond the eikonal approximation
(Drell-Yan and factorization)

Andrey Tarasov

BROOKHAVEN NATIONAL LABORATORY

3 February 2017
7th Workshop of the APS Topical Group on Hadronic Physics

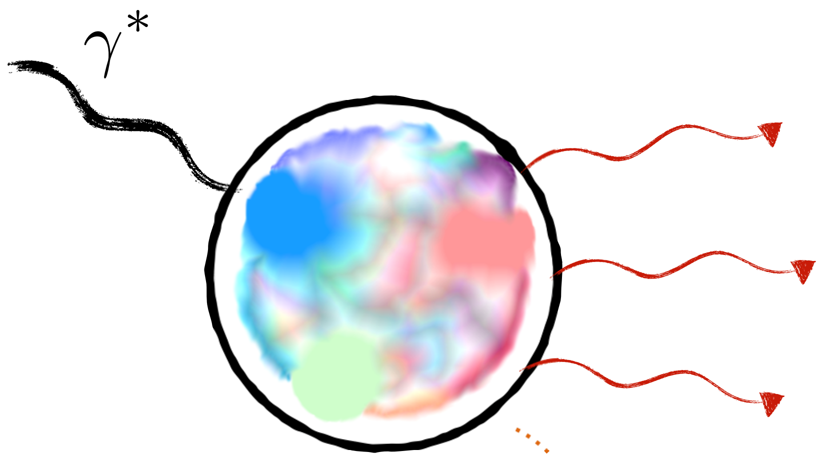
QCD: Hadron at different scales



Quark and gluons don't exist as asymptotic states

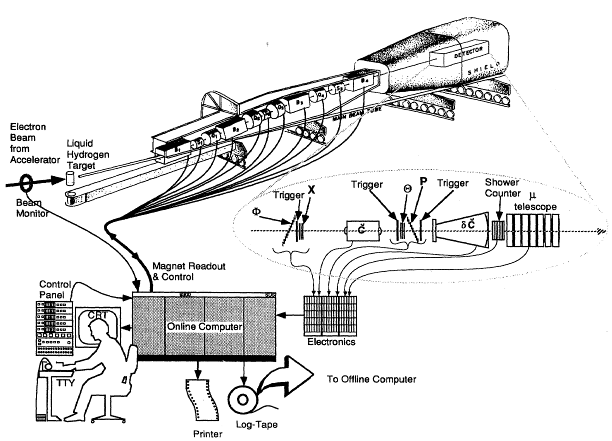
Where does Q^2 come from?

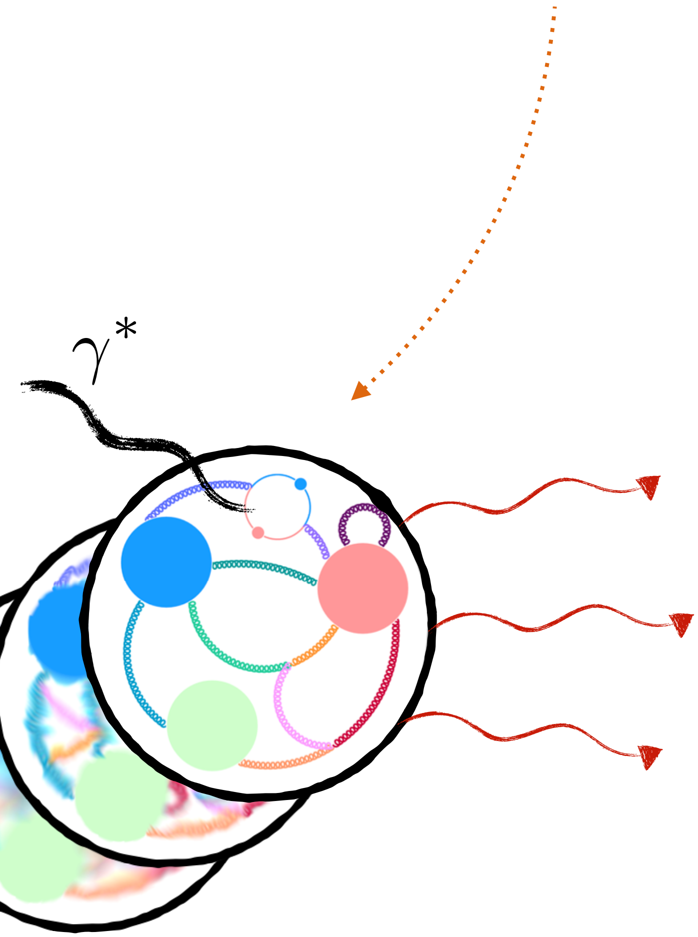
Parton model. Factorization



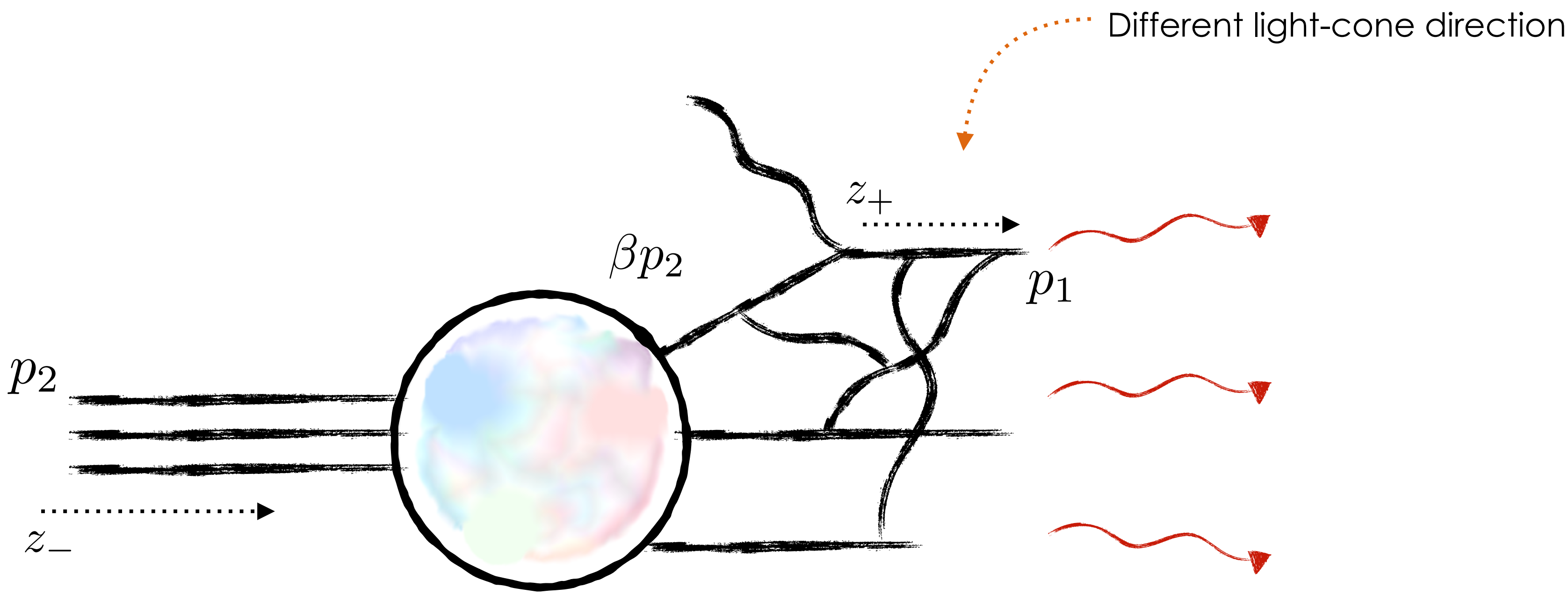
Factorization: separation of different phases

$$d\sigma = \frac{1}{2(S-M^2)} e^4 \frac{1}{Q^4} L^{\mu\nu} 4\pi M W_{\mu\nu} \frac{d^3 l'}{(2\pi)^3 2E'}$$





Kinematic variables



An arbitrary momentum:

$$p = \alpha p_1 + \beta p_2 + p_\perp$$

An arbitrary coordinate:

$$z^\mu = \frac{2}{s} z_+ p_1^\mu + \frac{2}{s} z_- p_2^\mu + z_\perp^\mu$$

Scalar product:

$$p \cdot z = \alpha z_- + \beta z_+ - p_\perp z_\perp$$

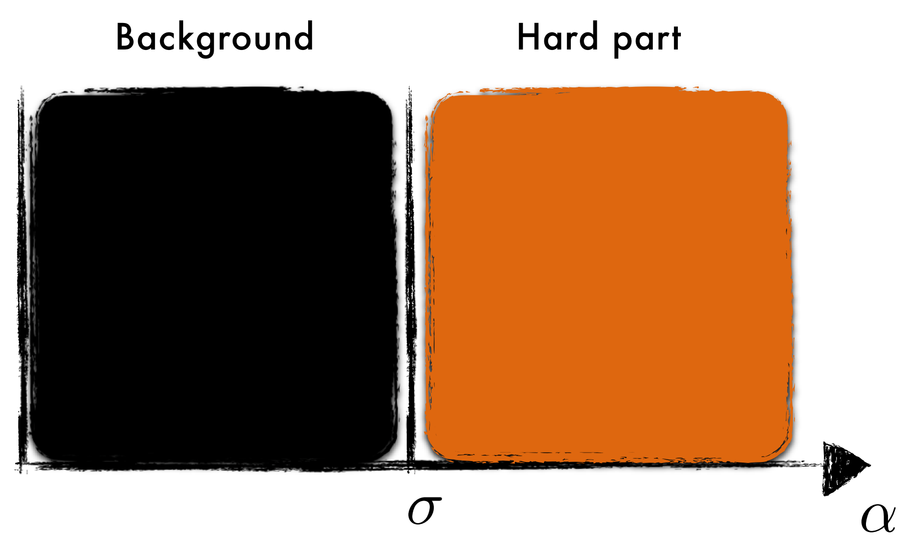
$$p^2 = \alpha\beta s - p_\perp^2$$

Rapidity Factorization. DIS

An arbitrary momentum:

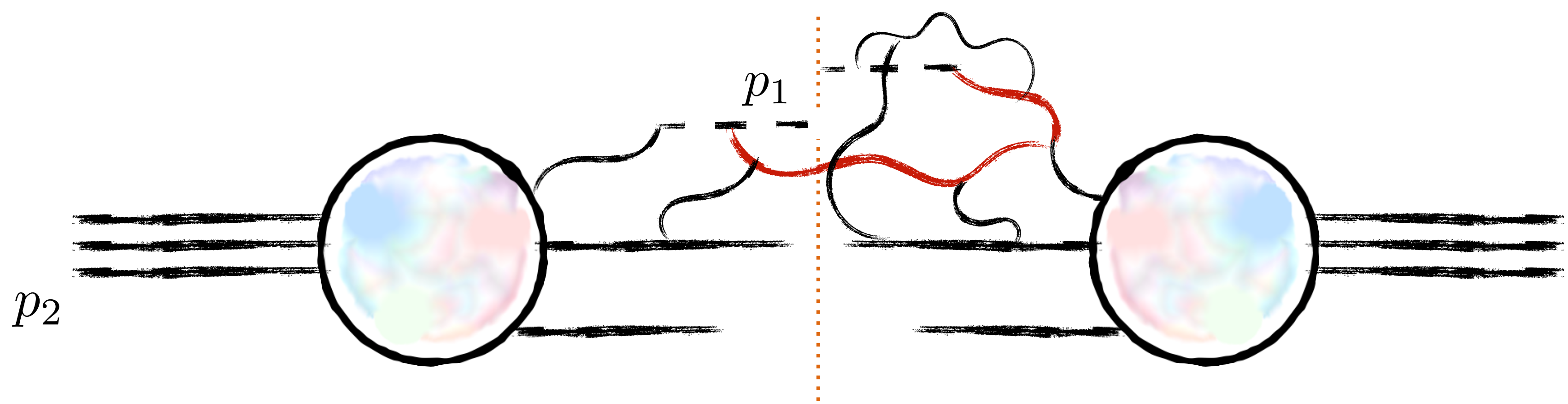
$$p = \alpha p_1 + \beta p_2 + p_\perp$$

Use this parameter to separate phases



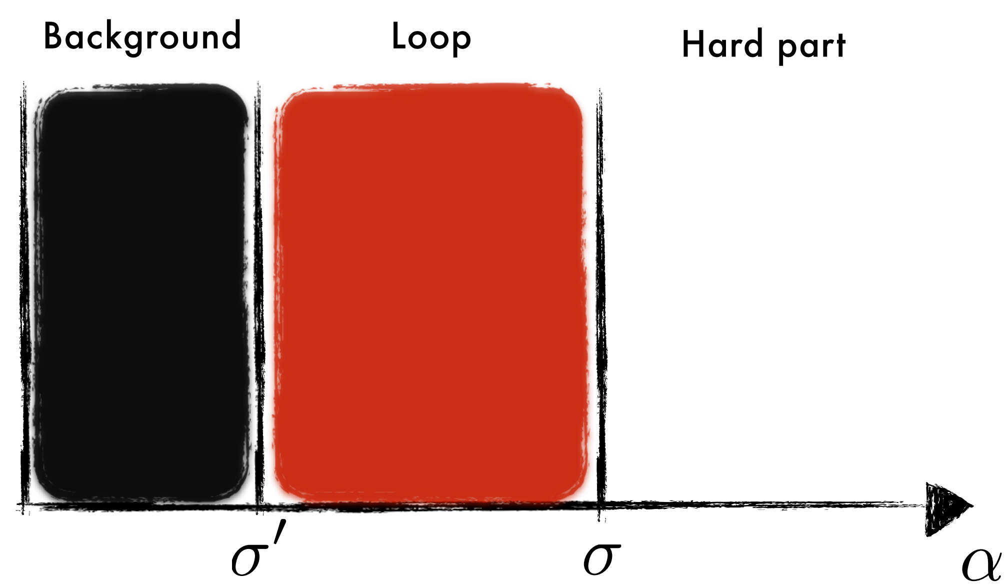
Ian Balitsky (1996)

Rapidity Factorization. Evolution



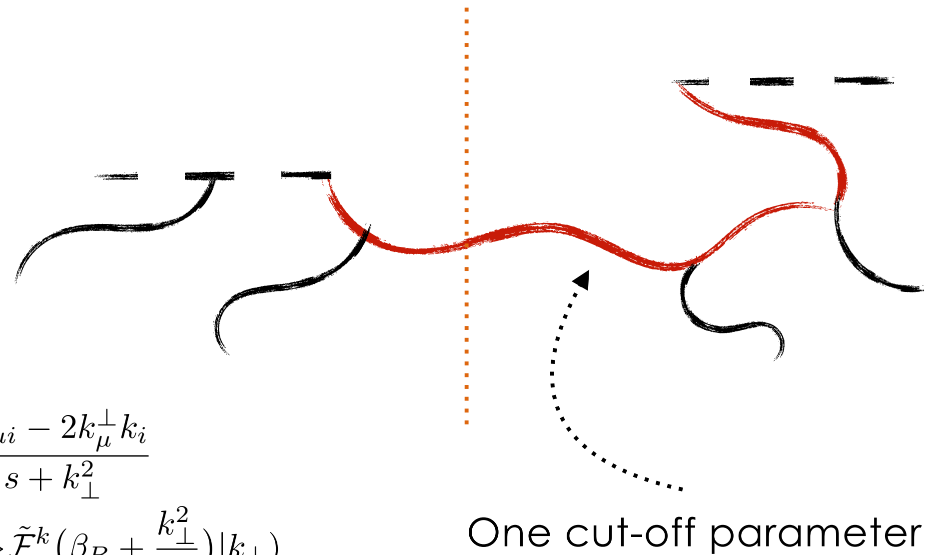
$$p = \alpha p_1 + \beta p_2 + p_\perp$$

Use this parameter to separate phases



Evolution

Can construct general equations interpolating between different kinematic limits



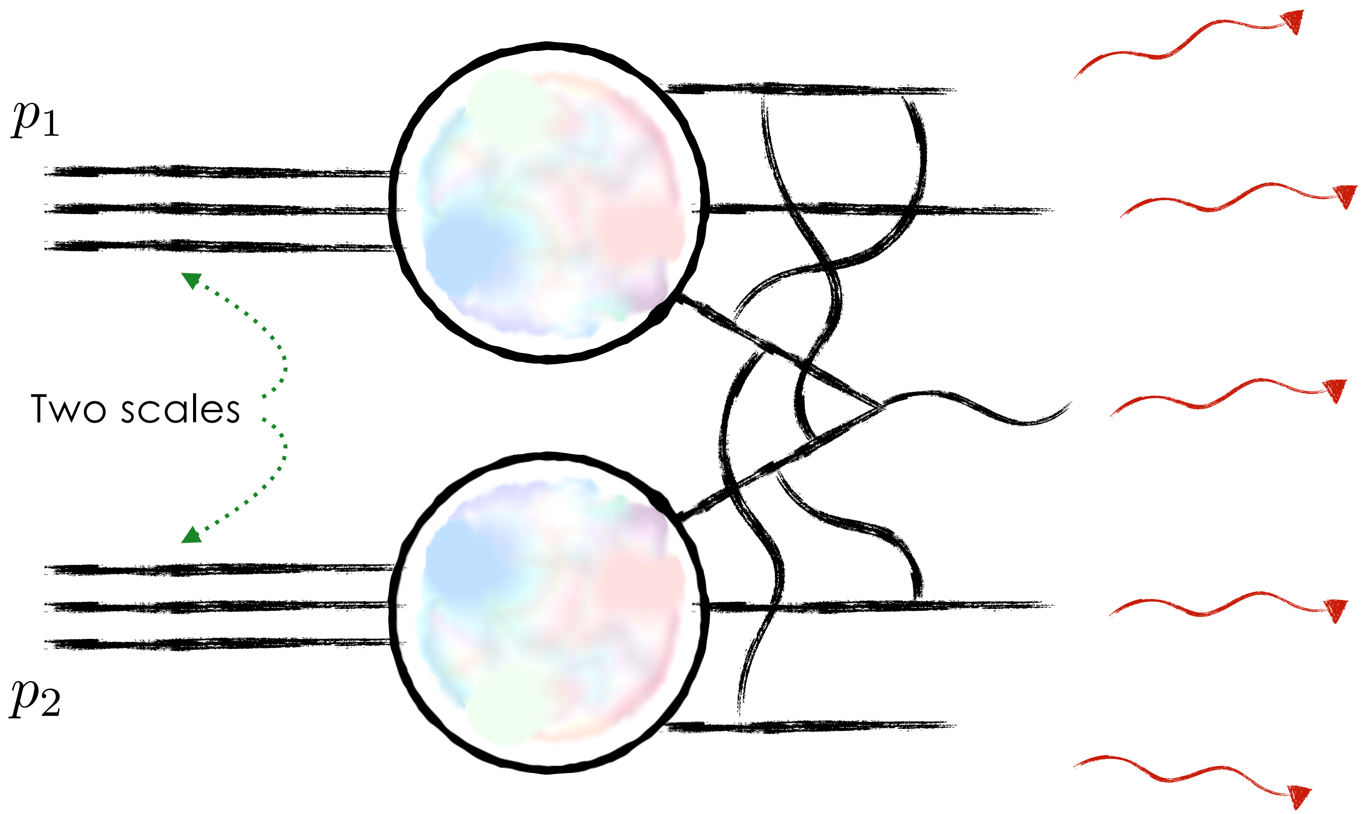
$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\tilde{\mathcal{F}}^a_i(\beta_B,x_\perp)\mathcal{F}^a_j(\beta_B,y_\perp)\\
=\;& -\alpha_s \mathrm{Tr}\Big\{\int d\!\!{}^-{}^2k_\perp (x_\perp|\Big\{U^\dagger\frac{1}{\sigma\beta_B s+p_\perp^2}(Uk_k+p_kU)\frac{\sigma\beta_B s g_{\mu i}-2k^\perp_\mu k_i}{\sigma\beta_B s+k_\perp^2}\\
-\;& 2k^\perp_\mu g_{ik}U^\dagger\frac{1}{\sigma\beta_B s+p_\perp^2}U-2g_{\mu k}U^\dagger\frac{p_i}{\sigma\beta_B s+p_\perp^2}U+\frac{2k^\perp_\mu}{k_\perp^2}g_{ik}\Big\}\tilde{\mathcal{F}}^k(\beta_B+\frac{k_\perp^2}{\sigma s})|k_\perp)\\
\times\;& (k_\perp|\mathcal{F}^l(\beta_B+\frac{k_\perp^2}{\sigma s})\Big\{\frac{\sigma\beta_B s\delta^\mu_j-2k^\mu_\perp k_j}{\sigma\beta_B s+k_\perp^2}(k_lU^\dagger+U^\dagger p_l)\frac{1}{\sigma\beta_B s+p_\perp^2}U\\
-\;& 2k^\mu_\perp g_{jl}U^\dagger\frac{1}{\sigma\beta_B s+p_\perp^2}U-\;2\delta^\mu_l U^\dagger\frac{p_j}{\sigma\beta_B s+p_\perp^2}U+2g_{jl}\frac{k^\mu_\perp}{k_\perp^2}\Big\}|y_\perp)\\
+\;& 2\tilde{\mathcal{F}}_i(\beta_B,x_\perp)(y_\perp|\frac{p^m}{p_\perp^2}\mathcal{F}_k(\beta_B)(i\overleftarrow{\partial}_l+U_l)(2\delta^k_m\delta^l_j-g_{jm}g^{kl})U^\dagger\frac{1}{\sigma\beta_B s-p_\perp^2+i\epsilon}U\\
+\;&\mathcal{F}_j(\beta_B)\frac{\sigma\beta_B s}{p_\perp^2(\sigma\beta_B s-p_\perp^2+i\epsilon)}|y_\perp)\\
+\;& 2(x_\perp|-U^\dagger\frac{1}{\sigma\beta_B s-p_\perp^2-i\epsilon}U(2\delta^k_i\delta^l_m-g_{im}g^{kl})(i\partial_k-U_k)\tilde{\mathcal{F}}_l(\beta_B)\frac{p^m}{p_\perp^2}\\
+\;&\tilde{\mathcal{F}}_i(\beta_B)\frac{\sigma\beta_B s}{p_\perp^2(\sigma\beta_B s-p_\perp^2-i\epsilon)}|x_\perp)\mathcal{F}_j(\beta_B,y_\perp)\Big\}\;+\;O(\alpha_s^2)
\end{aligned}
$$

Ian Balitsky, AT (2016)

Factorization. Drell-Yan

$$p = \alpha p_1 + \beta p_2 + p_\perp$$

Can not use one variable



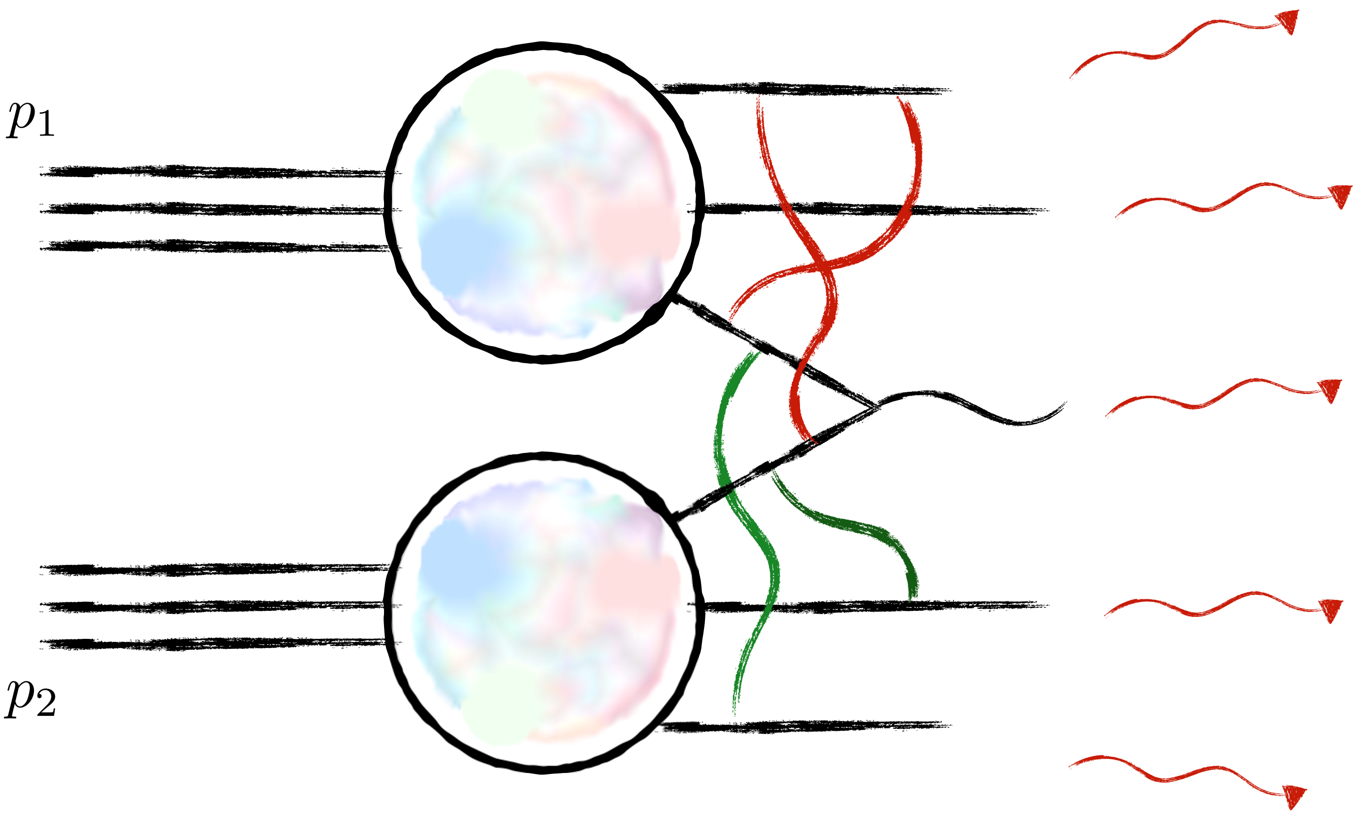
Factorization. Drell-Yan

$$[-\infty, x_+]\bar{\psi}(x)\psi(x)[x_-, -\infty]$$

Factorization is not obvious

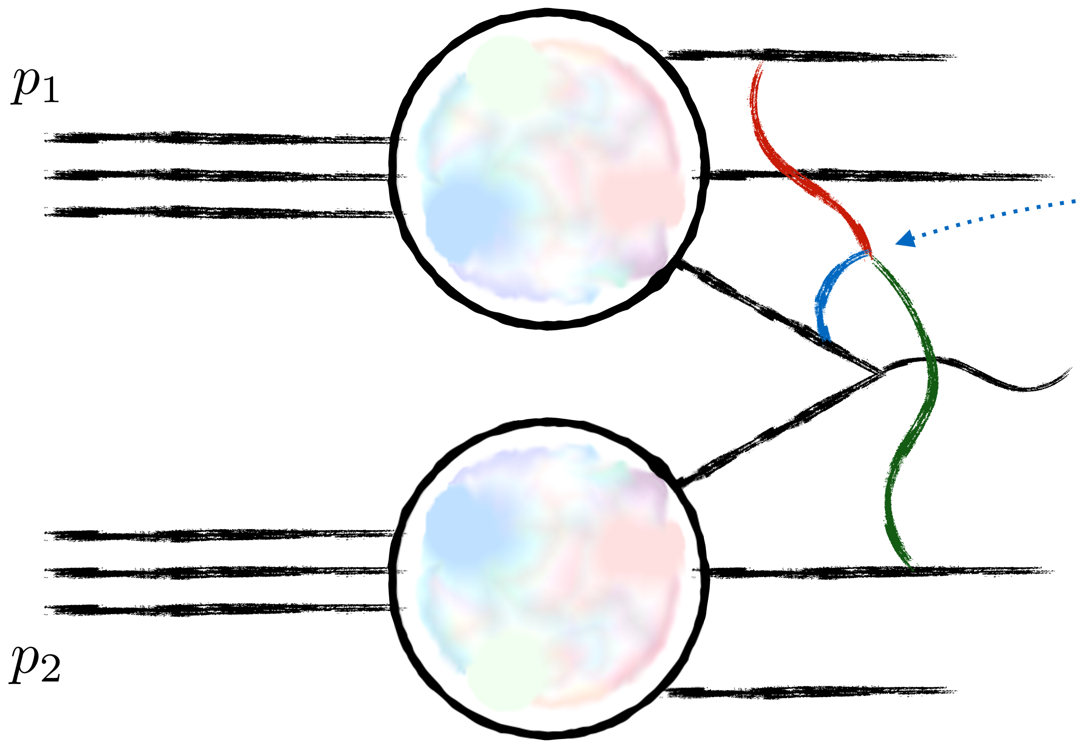
$$p = \alpha p_1 + \beta p_2 + p_\perp$$

Have to include two types of gluons



Factorization. Three-gluon vertex

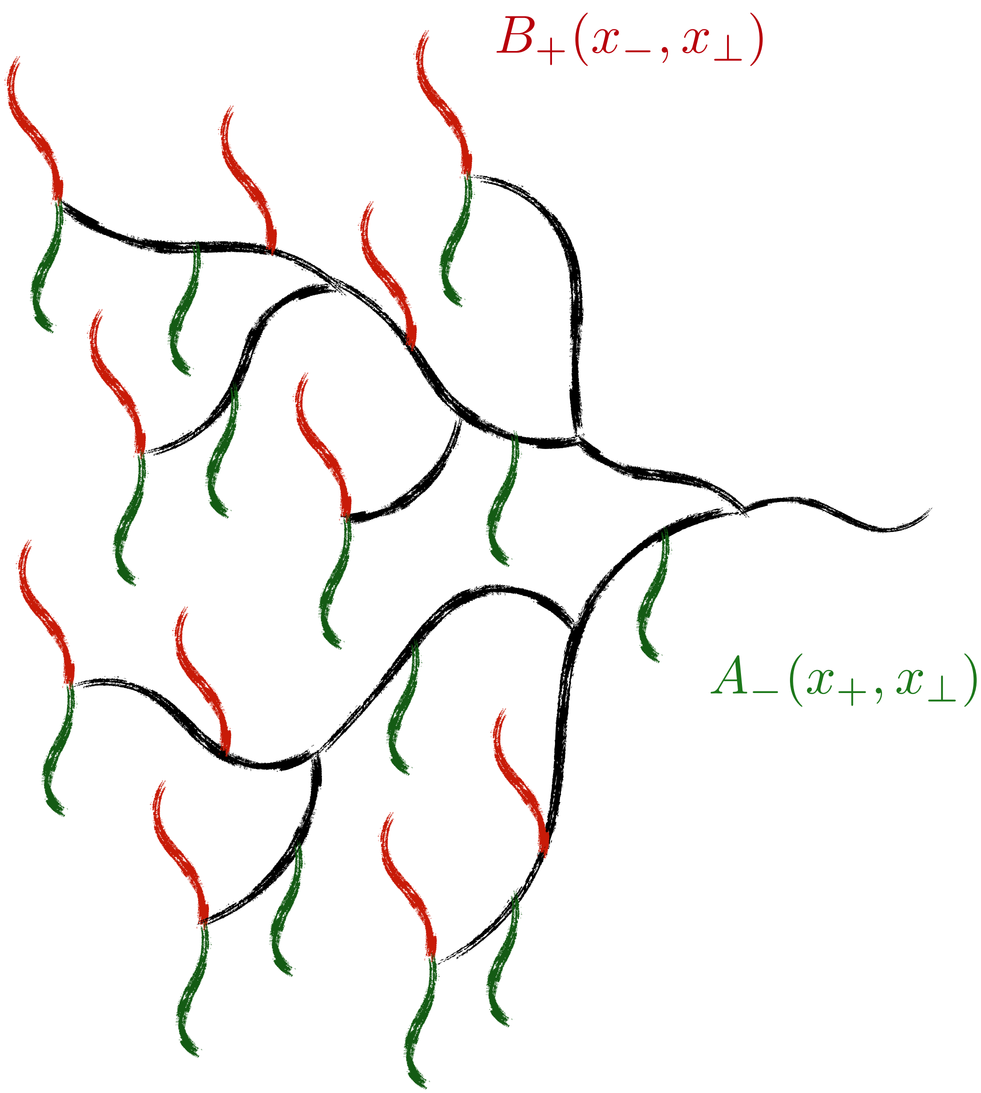
J. C. Collins, D. E. Soper, and G. Sterman, Phys. Lett. 109B, 388 (1982);
C. Collins, D. E. Soper, and G. Sterman, Phys. Lett. 126B, 275 (1983);
G.T. Bodwin, Phys. Rev. D 31, 2616 (1985)



Key ingredient. Easy to check in the leading order

What about resummation in all orders of perturbation theory?

Factorization. Tree diagrams

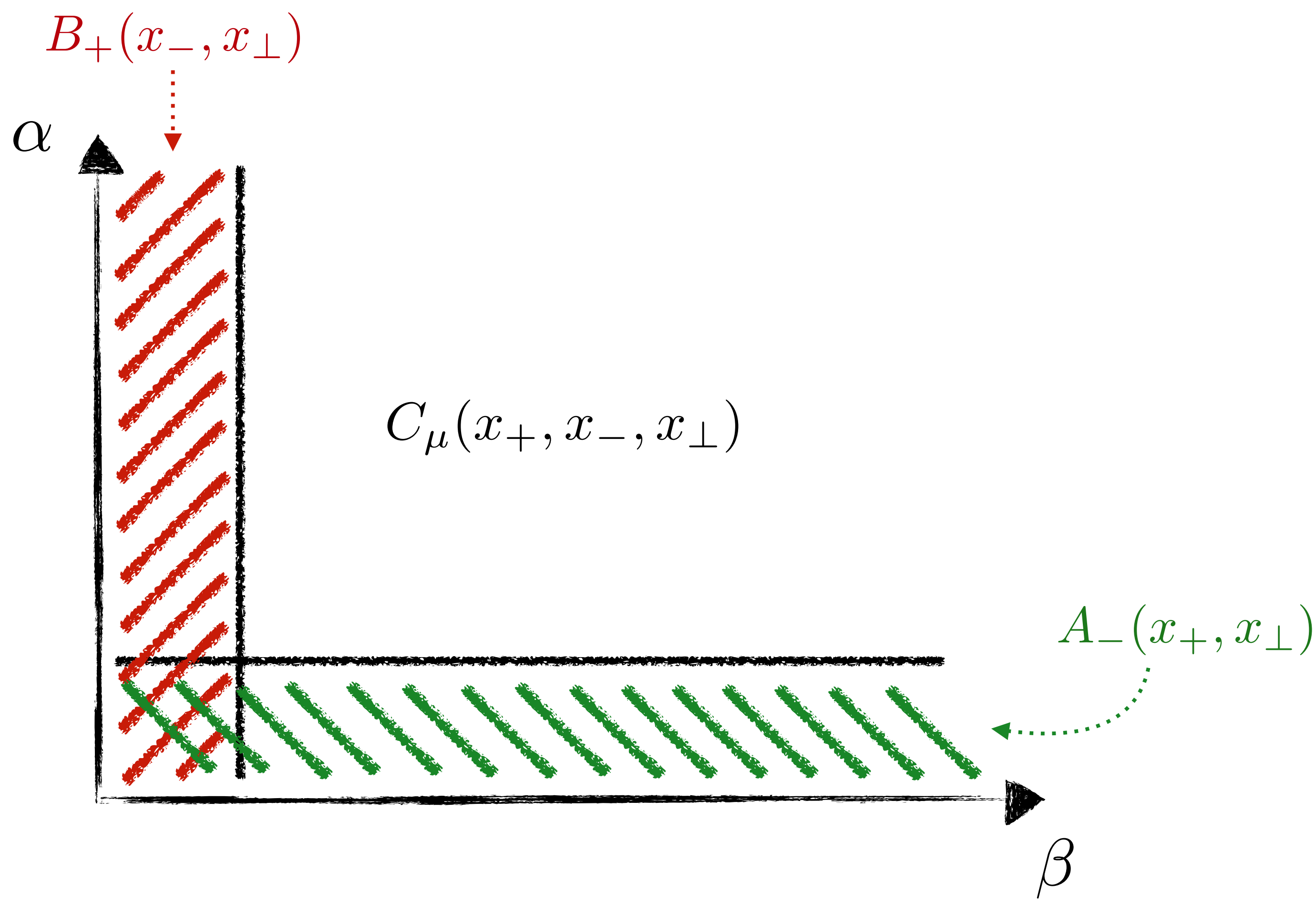


I. Balitsky, Phys. Rev. Lett. 81, 2024 (1998)
Yu. V. Kovchegov and A. H. Mueller, Nucl. Phys. B529, 451 (1998)
Yu. V. Kovchegov and K. Tuchin, Phys. Rev. D 65, 074026 (2002)
A. Dumitru and L. D. McLerran, Nucl. Phys. A700, 492 (2002)
J.P. Blaizot, F. Gelis, and R. Venugopalan, Nucl. Phys. A743, 13 (2004)
F. Gelis, T. Lappi, and R. Venugopalan, Phys. Rev. D 78, 054019 (2008)

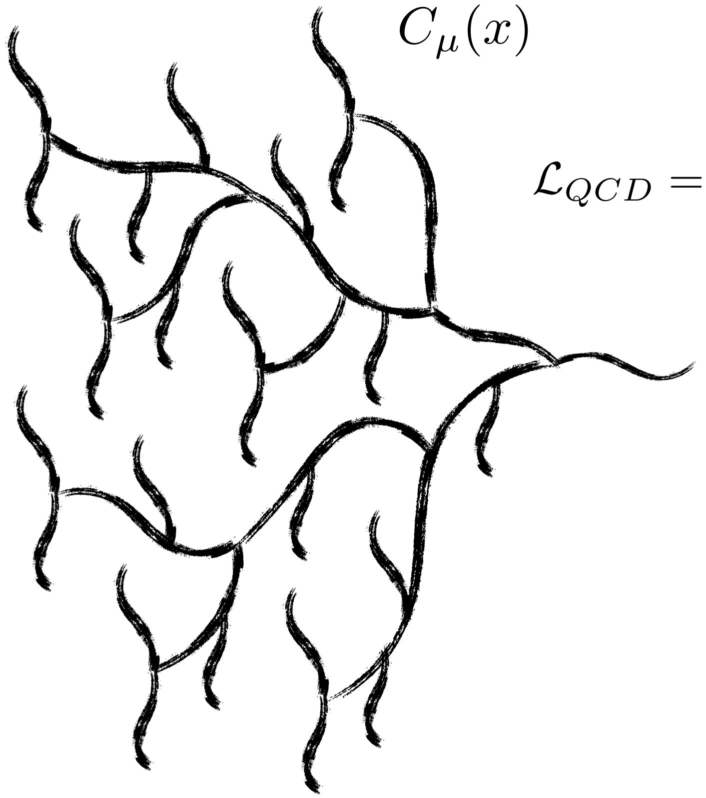
Feynman diagrams in two background fields

We perform resummation of this type of diagrams

Background fields



Shift



$$\mathcal{L}_{QCD} = -\frac{1}{4}(F^a_{\mu\nu})^2$$

$A_-(x_+, x_\perp)$

$B_+(x_-, x_\perp)$

Shift of the field

$$C^a_\mu \to C^a_\mu + A^a_\mu + B^a_\mu$$

Covariant derivative contains interaction with the background fields

$$F_{\mu\nu} = \check{\mathcal{F}}_{\mu\nu} + (\check{\mathcal{D}}_\mu C_\nu) - (\check{\mathcal{D}}_\nu C_\mu) - ig[C_\mu, C_\nu]^{ab}$$

$B_+(x_-, x_\perp)$

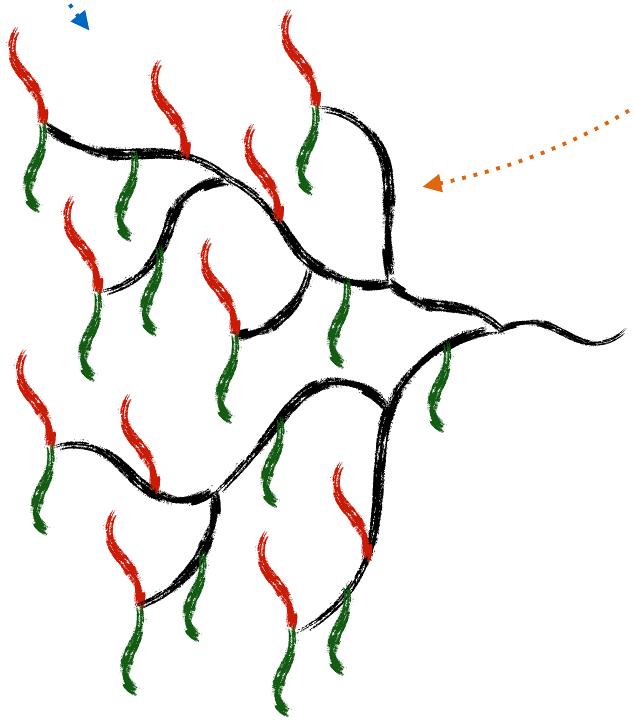
$A_-(x_+, x_\perp)$

Gauge fixing term

$$\mathcal{L}_{QCD} = -\frac{1}{4}(F^a_{\mu\nu})^2(C+\breve{A}) - \frac{1}{2}(\{\check{\mathcal{D}}^\mu - ig\breve{C}^\mu\}C^a_\mu)^2$$

QCD in the background

We will define this field later



Gauge fixing term. Compare with the usual choice

$$-\frac{1}{2}(\check{\mathcal{D}}^\mu C^a_\mu)^2$$

Using non-standard gauge fixing term effectively resum standard diagrams

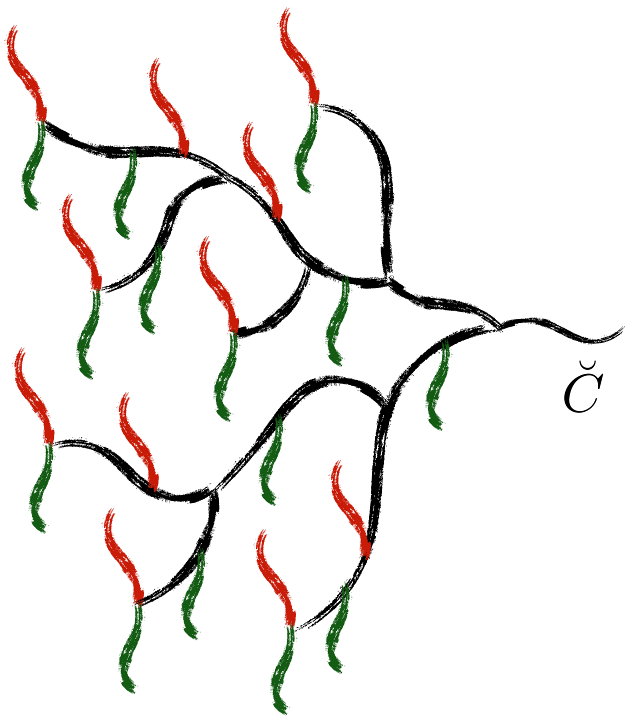
Definition of the field

$$\mathcal{L}_{QCD} = -\frac{1}{2}C^a_\mu \Box^{\mu\nu}_{ab} C^b_\nu + \check{\mathcal{D}}^\mu \check{\mathcal{F}}^a_{\mu\nu} C^{a\nu} - \frac{1}{4}\check{\mathcal{F}}^a_{\mu\nu}\check{\mathcal{F}}^{a\mu\nu} - g f^{abc} C^{b\mu} C^{c\nu} \check{\mathcal{D}}_\mu C^a_\nu - \frac{1}{4} g^2 f^{abc} f^{ade} C^b_\mu C^c_\nu C^{d\mu} C^{e\nu}$$

Non-standard propagator

Field separation

$$C \to C + \check{C}$$



$$\mathcal{L}_{QCD} = C^a_\mu \Big\{ -(g^{\mu\nu}\check{\mathcal{P}}^2_{ab} + 2ig\check{\mathcal{F}}^{\mu\nu}_{ab})\check{C}^b_\nu + \check{\mathcal{D}}_\nu \check{\mathcal{F}}^{a\nu\mu}$$
$$+ g f^{abc}(2\check{C}^{b\nu}\check{\mathcal{D}}_\nu \check{C}^{c\mu} - \check{C}^{b\nu}\check{\mathcal{D}}^\mu \check{C}^c_\nu) - g^2 f^{abr} f^{cdr} \check{C}^b_\nu \check{C}^{c\mu}\check{C}^{d\nu} \Big\} + \ldots$$

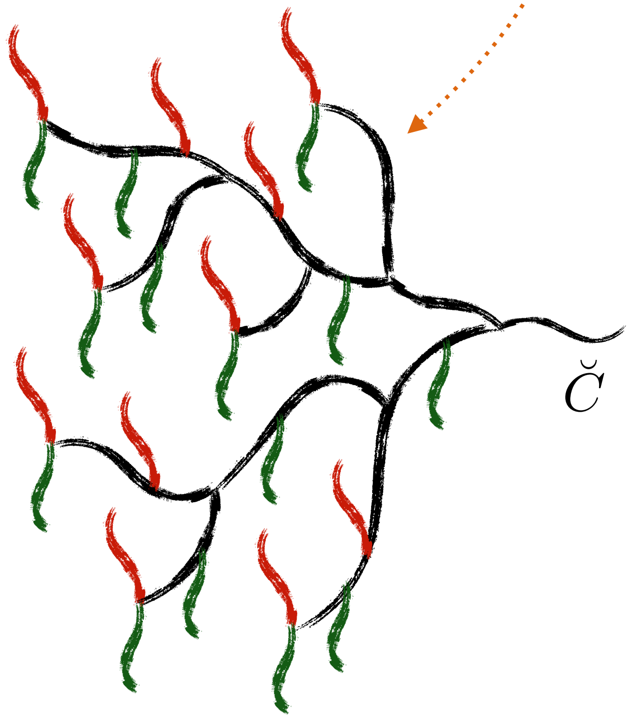
Define the field by cancellation of this term

Master equation

$$(g^{\mu\nu}\check{\mathcal{P}}^2_{ab}+2ig\check{\mathcal{F}}^{\mu\nu}_{ab})\check{C}^b_\nu = \check{\mathcal{D}}_\nu\check{\mathcal{F}}^{a\nu\mu}+gf^{abc}(2\check{C}^{b\nu}\check{\mathcal{D}}_\nu\check{C}^{c\mu}-\check{C}^{b\nu}\check{\mathcal{D}}^\mu\check{C}^c_\nu)-g^2f^{abr}f^{cdr}\check{C}^b_\nu\check{C}^{c\mu}\check{C}^{d\nu}$$

Master equation

1) Neglect transverse momentum



2) Solve by iteration

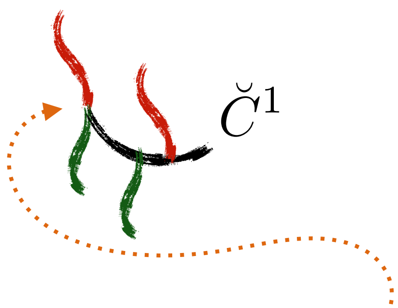
$$\check{C} = \check{C}^1 + \check{C}^2 + \check{C}^3 + \ldots$$

Iterative solution

$$(g^{\mu\nu}\check{\mathcal{P}}^2_{ab}+2ig\check{\mathcal{F}}^{\mu\nu}_{ab})\check{C}^b_\nu = \check{\mathcal{D}}_\nu\check{\mathcal{F}}^{a\nu\mu}+gf^{abc}(2\check{C}^{b\nu}\check{\mathcal{D}}_\nu\check{C}^{c\mu}-\check{C}^{b\nu}\check{\mathcal{D}}^\mu\check{C}^c_\nu)-g^2f^{abr}f^{cdr}\check{C}^b_\nu\check{C}^{c\mu}\check{C}^{d\nu}$$

Master equation

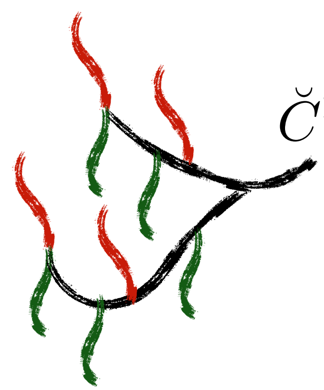
$$\check{C} = \check{C}^1+\check{C}^2+\check{C}^3+\ldots$$



$\check{C}^1$

$$\check{C}^{1a}_+ = \frac{i}{2\check{\mathcal{P}}_-\check{\mathcal{P}}_+}\check{\mathcal{P}}_+\check{\mathcal{F}}^a_{+-}$$

$$\check{C}^{1a}_- = -\frac{i}{2\check{\mathcal{P}}_+\check{\mathcal{P}}_-}\check{\mathcal{P}}_-\check{\mathcal{F}}^a_{+-}$$



$\check{C}^2$

$$\check{C}^{2a}_+ = \frac{ig}{2}\left(\frac{1}{\check{\mathcal{P}}_-\check{\mathcal{P}}_+}\check{\mathcal{P}}_+\right)^{ac}(f^{crb}\check{C}^{1r}_+\check{C}^{1b}_-)$$

$$\check{C}^{2a}_- = -\frac{ig}{2}\left(\frac{1}{\check{\mathcal{P}}_+\check{\mathcal{P}}_-}\check{\mathcal{P}}_-\right)^{ac}(f^{crb}\check{C}^{1r}_+\check{C}^{1b}_-)$$

Iterative solution

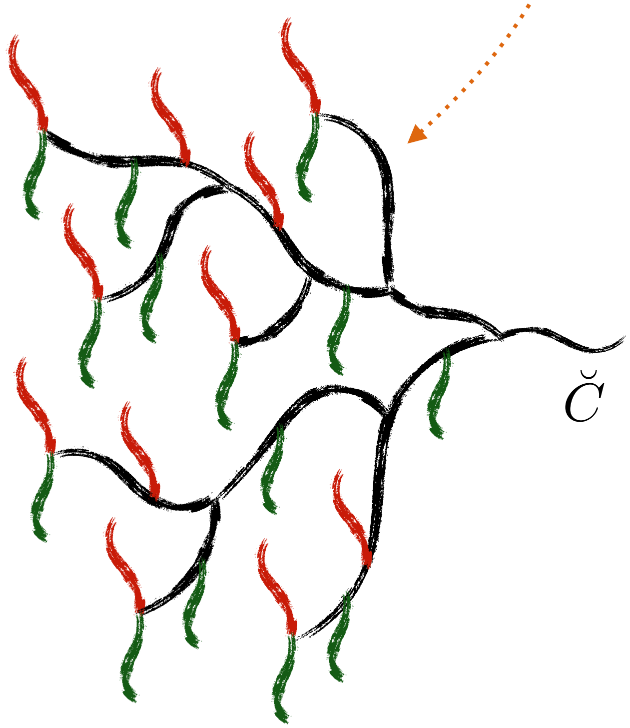
$$(g^{\mu\nu}\check{\mathcal{P}}^2_{ab}+2ig\check{\mathcal{F}}^{\mu\nu}_{ab})\breve{C}^b_\nu = \check{\mathcal{D}}_\nu\check{\mathcal{F}}^{a\nu\mu}+gf^{abc}(2\breve{C}^{b\nu}\check{\mathcal{D}}_\nu\breve{C}^{c\mu}-\breve{C}^{b\nu}\check{\mathcal{D}}^\mu\breve{C}^c_\nu)-g^2f^{abr}f^{cdr}\breve{C}^b_\nu\breve{C}^{c\mu}\breve{C}^{d\nu}$$

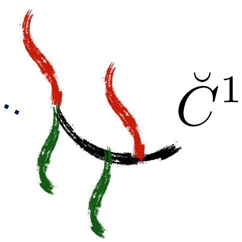
Master equation

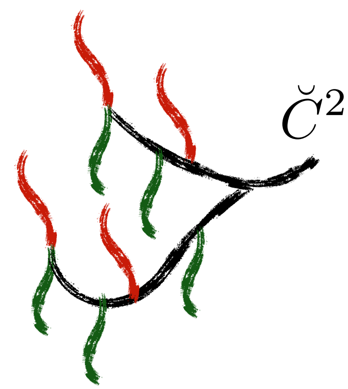
1) Neglect transverse momentum

2) Solve by iteration

$$\breve{C} = \breve{C}^1 + \breve{C}^2 + \breve{C}^3 + \ldots$$







Pure gauge

1) $F^a_{+-}(\check{A} + \check{C}^1) = 0$

2) $F^a_{+-}(\check{A} + \check{C}^1 + \check{C}^2) = 0$

3) $F^a_{+-}(\check{A} + \check{C}^1 + \check{C}^2 + \check{C}^3) = 0$

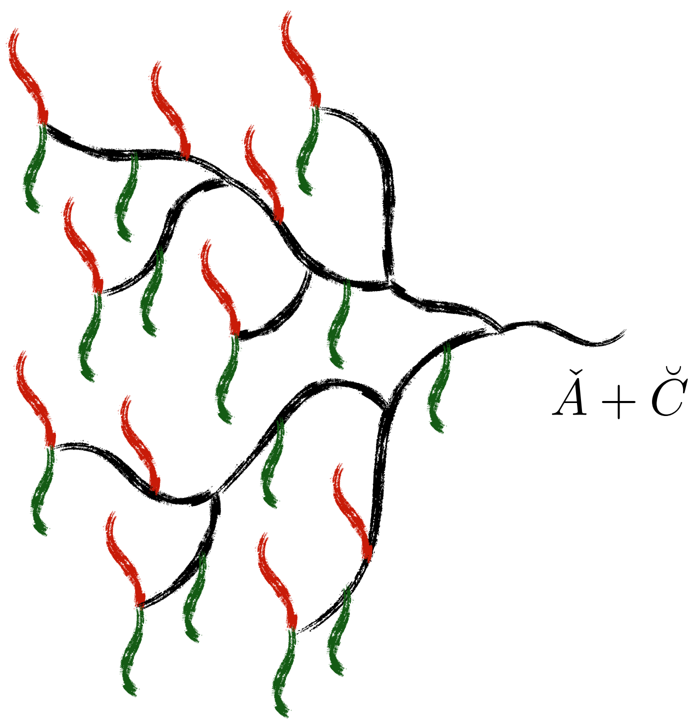
$$F^a_{+-}(\check{A} + \check{C}) = 0$$

Pure gauge

$$g\check{A}_+ + g\check{C}_+ = i\Omega\partial_+\Omega^\dagger$$

$$g\check{A}_- + g\check{C}_- = i\Omega\partial_-\Omega^\dagger$$

We can reconstruct gauge matrix order by order



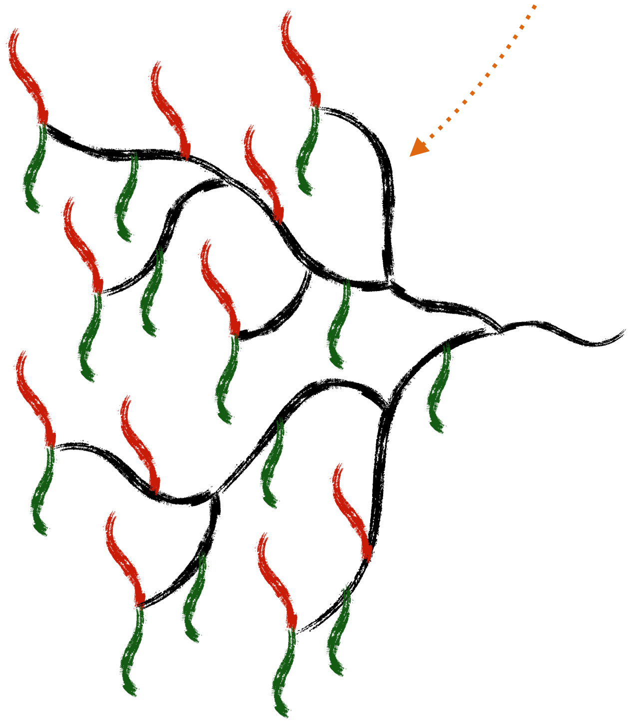
We neglect transverse momentum. Let's calculate correction to this picture

Transverse correction

$$(g^{\mu\nu}\check{\mathcal{P}}^2_{ab}+2ig\check{\mathcal{F}}^{\mu\nu}_{ab})\check{C}^b_\nu = \check{\mathcal{D}}_\nu\check{\mathcal{F}}^{a\nu\mu}+gf^{abc}(2\check{C}^{b\nu}\check{\mathcal{D}}_\nu\check{C}^{c\mu}-\check{C}^{b\nu}\check{\mathcal{D}}^\mu\check{C}^c_\nu)-g^2f^{abr}f^{cdr}\check{C}^b_\nu\check{C}^{c\mu}\check{C}^{d\nu}$$

Master equation

Allow small transverse momentum transition



Correction

$$(\check{\mathcal{P}}+g\check{C})^2_{ab}\check{C}^b_i = i(\check{\mathcal{P}}+g\check{C})^{ab\nu}\partial_i(\check{A}+\check{C})^b_\nu$$

Pure gauge

$$g\check{A}_\mu + g\check{C}_\mu = i\Omega\partial_\mu\Omega^\dagger$$

$$(\Omega p^2\Omega^\dagger)_{ab}\check{C}^b_i = \frac{i}{g}\Omega^{ab}\partial^2(\Omega^\dagger\partial_i\Omega)^b$$

All effects of the pure gauge are included

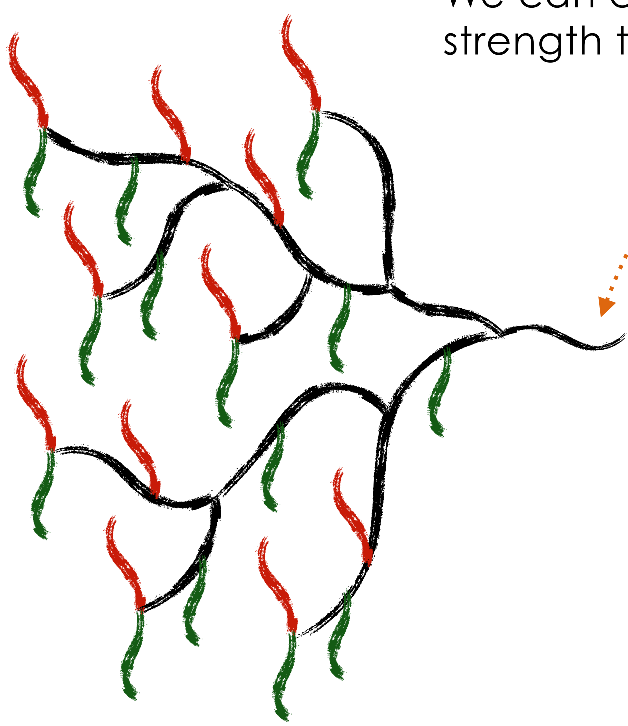
Transverse correction

$$\breve{C}_i^a = -\frac{i}{g}\Omega_x^{ab}(\Omega^\dagger\partial_i\Omega)^b_{-\infty,-\infty,x_\perp} + \frac{i}{g}\Omega_x^{ab}(\Omega^\dagger\partial_i\Omega)^b_{x_-,-\infty,x_\perp} + \frac{i}{g}\Omega_x^{ab}(\Omega^\dagger\partial_i\Omega)^b_{-\infty,x_+,x_\perp} + \frac{i}{g}(\Omega\partial_i\Omega^\dagger)^a$$

Solution in terms of the gauge matrix

We can calculate the strength tensor

$$F^a_{-i}(\breve{A}+\breve{C}) = \frac{i}{g}\Omega_x^{ab}\partial_-(\Omega^\dagger\partial_i\Omega)^b_{-\infty,x_+,x_\perp}$$



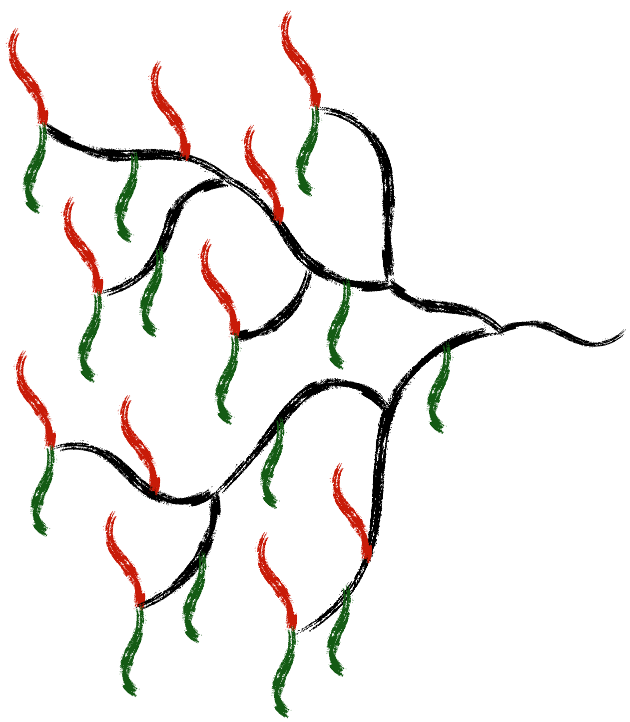
It is easy to reconstruct the gauge matrix at infinity. It is just a semi-infinite Wilson line

Strength tensor

$$F^a_{-i} = \Omega^{ab}_x \check{\mathcal{F}}^c_{-i}(x_+)[x_+, -\infty]^{cb}$$

Gauge matrix, has a complex dependence on background fields

$A_-(x_+, x_\perp)$

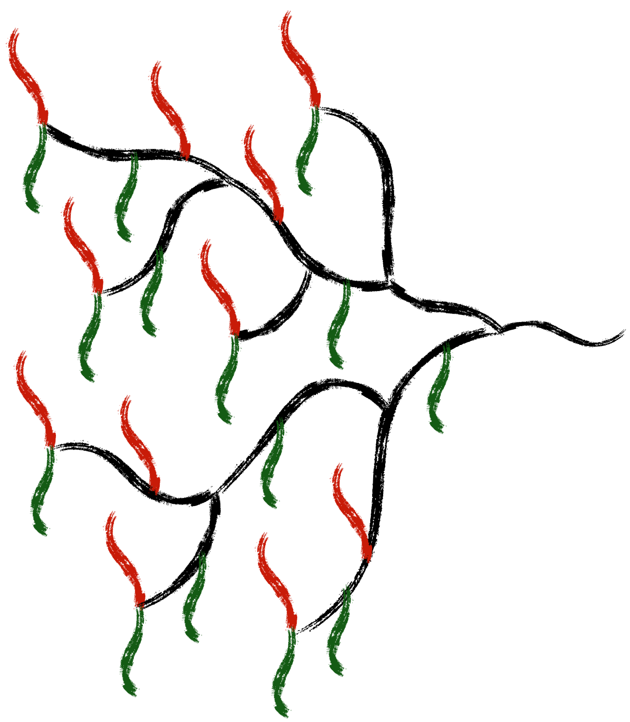


$$F^a_{+i} = \Omega^{ab}_x \check{\mathcal{F}}^c_{+i}(x_-)[x_-, -\infty]^{cb}$$

$B_+(x_-, x_\perp)$

Factorization

$$F^{a}_{-i}(x)F^{a}_{+i}(x) = [-\infty, x_+]^{al}\check{\mathcal{F}}^{l}_{-i}(x_+, x_\perp)\check{\mathcal{F}}^{e}_{+i}(x_-, x_\perp)[x_-, -\infty]^{ea}$$



1) The color is disentangled

2) The expression was obtained in the limit of small momenta

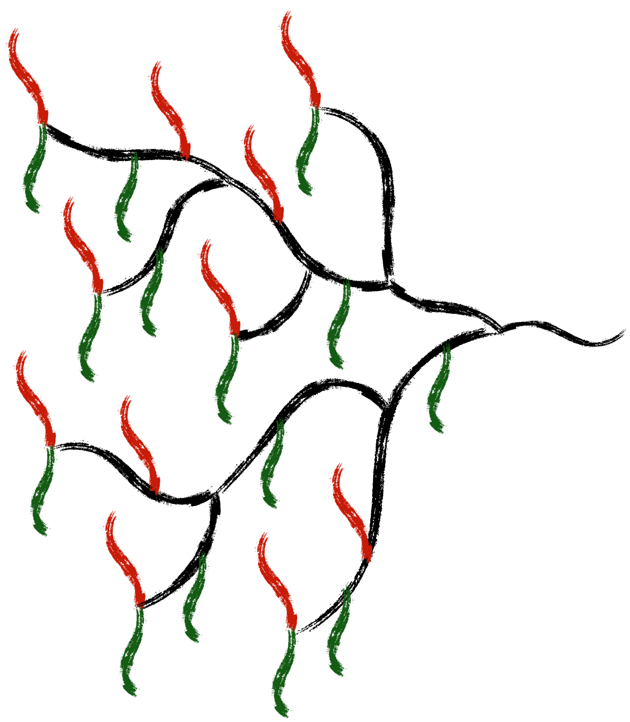
3) The result was obtained through solution of the equation of motion

Factorization. Corrections

$$(g^{\mu\nu}\check{\mathcal{P}}^2_{ab}+2ig\check{\mathcal{F}}^{\mu\nu}_{ab})\check{C}^b_\nu = \check{\mathcal{D}}_\nu\check{\mathcal{F}}^{a\nu\mu}+gf^{abc}(2\check{C}^{b\nu}\check{\mathcal{D}}_\nu\check{C}^{c\mu}-\check{C}^{b\nu}\check{\mathcal{D}}^\mu\check{C}^c_\nu)-g^2f^{abr}f^{cdr}\check{C}^b_\nu\check{C}^{c\mu}\check{C}^{d\nu}$$

$$F^a_{-i}(x)F^a_{+i}(x) = [-\infty,x_+]^{al}\check{\mathcal{F}}^l_{-i}(x_+,x_\perp)\check{\mathcal{F}}^e_{+i}(x_-,x_\perp)[x_-,-\infty]^{ea}$$

Corrections?



1) The color is disentangled

2) The expression was obtained in the limit of small momenta

3) The result was obtained through solution of the equation of motion

4) Can calculate corrections to this result

Ian Balitsky, A.T. (2017) in preparation