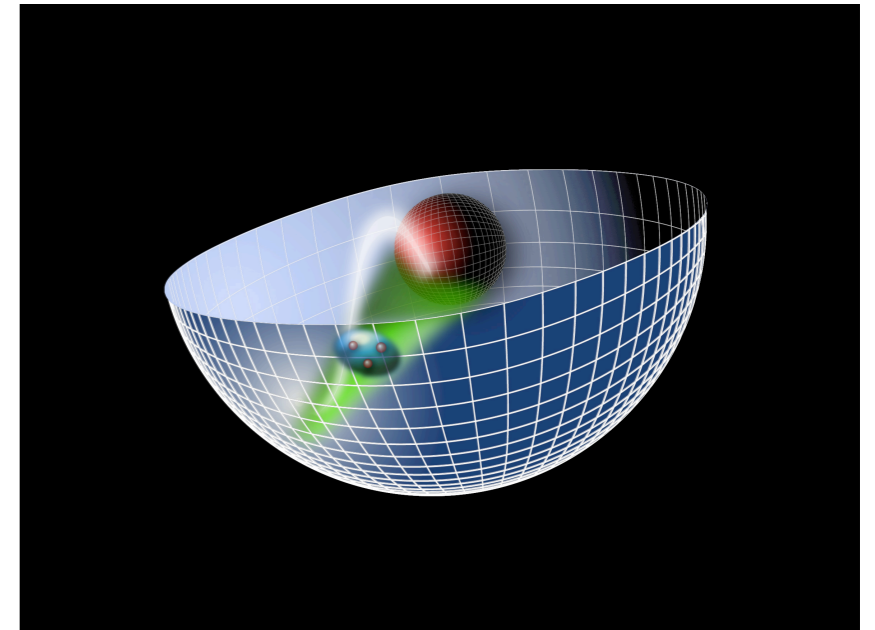
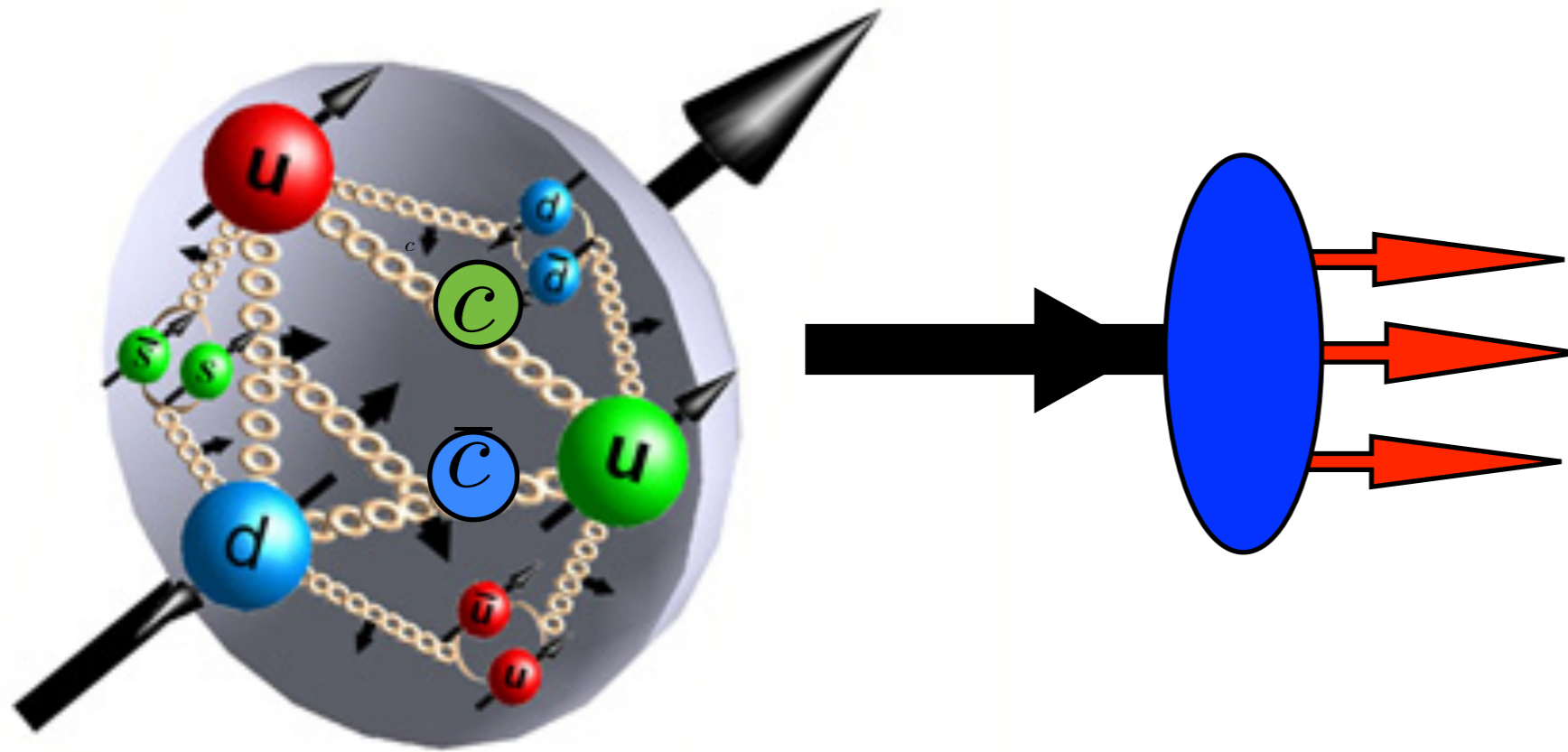


Supersymmetric Meson-Baryon Properties of QCD from Light-Front Holography and Superconformal Algebra



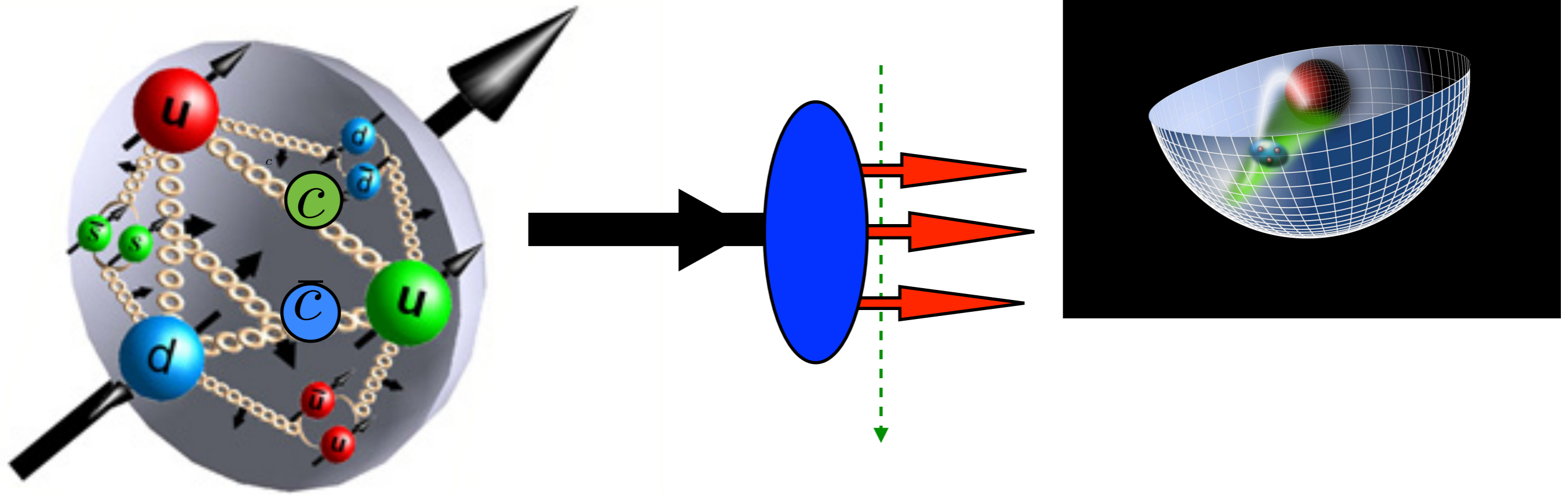
Stan Brodsky



with Guy de Tèramond, Hans Günter Dosch,
C. Lorce, K. Chiu, R. S. Sufian, A. Deur

7th Workshop of the APS Topical Group on Hadronic Physics
Washington D.C., February 3, 2017

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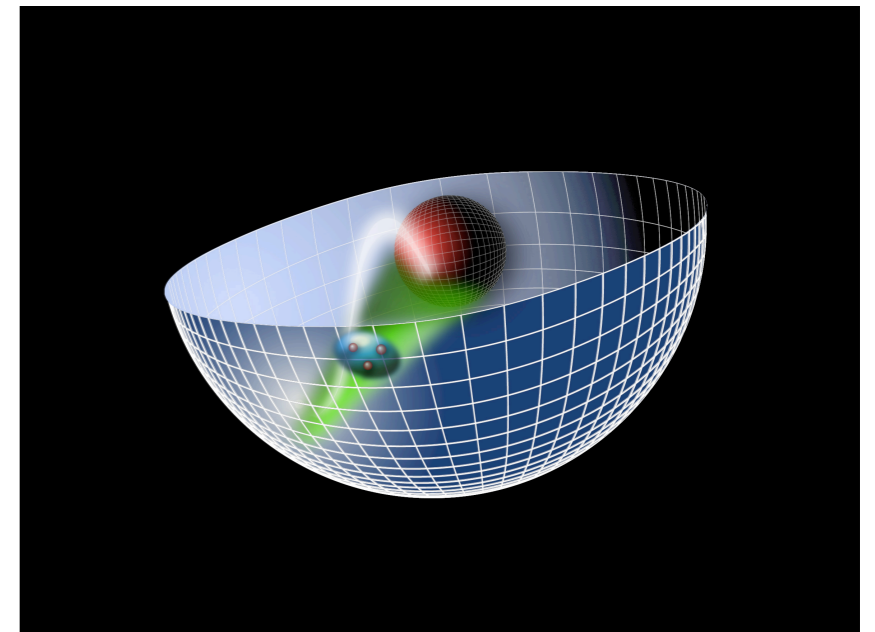
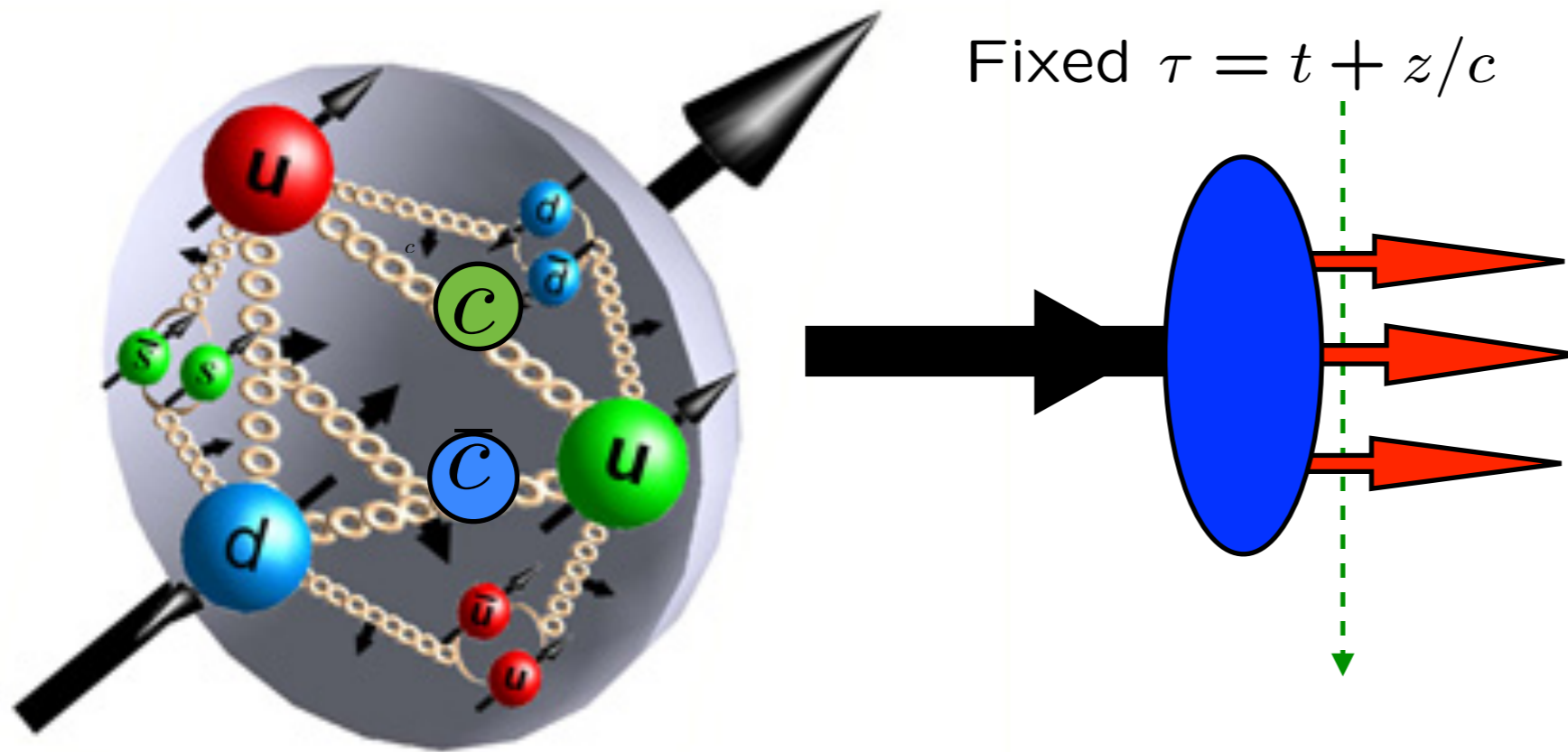
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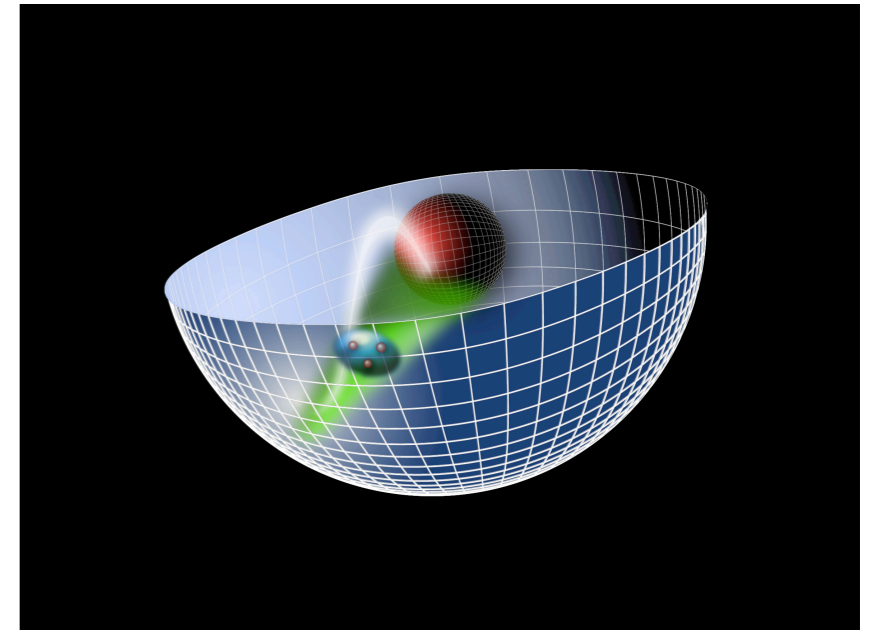
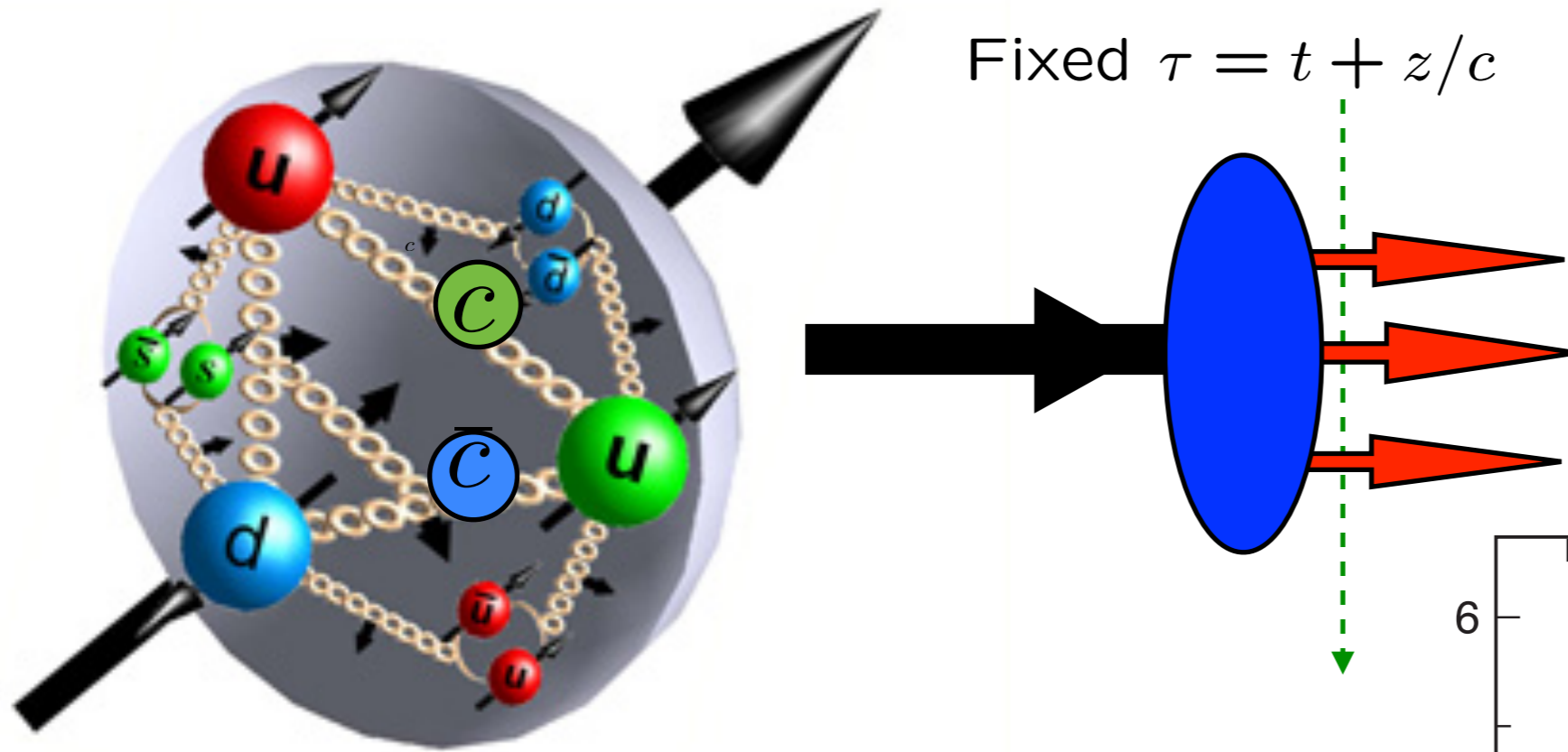
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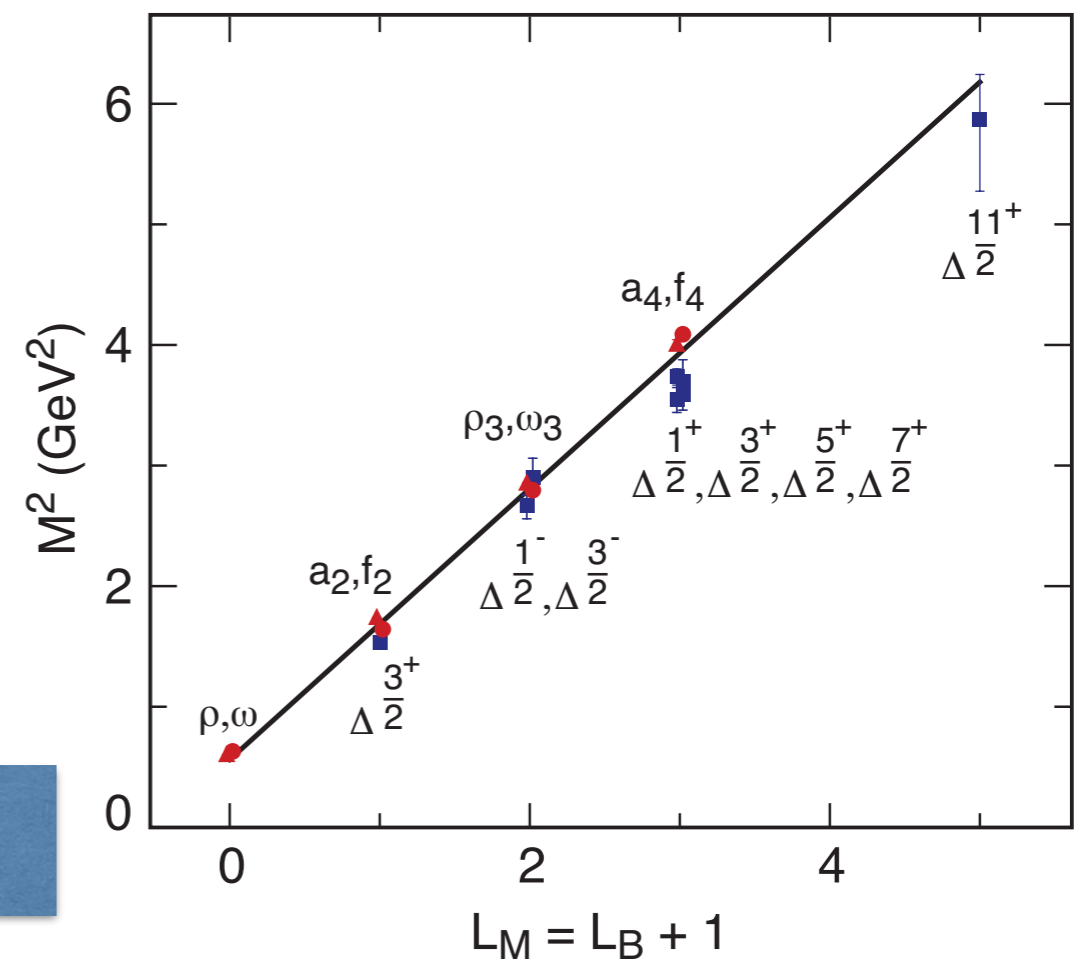
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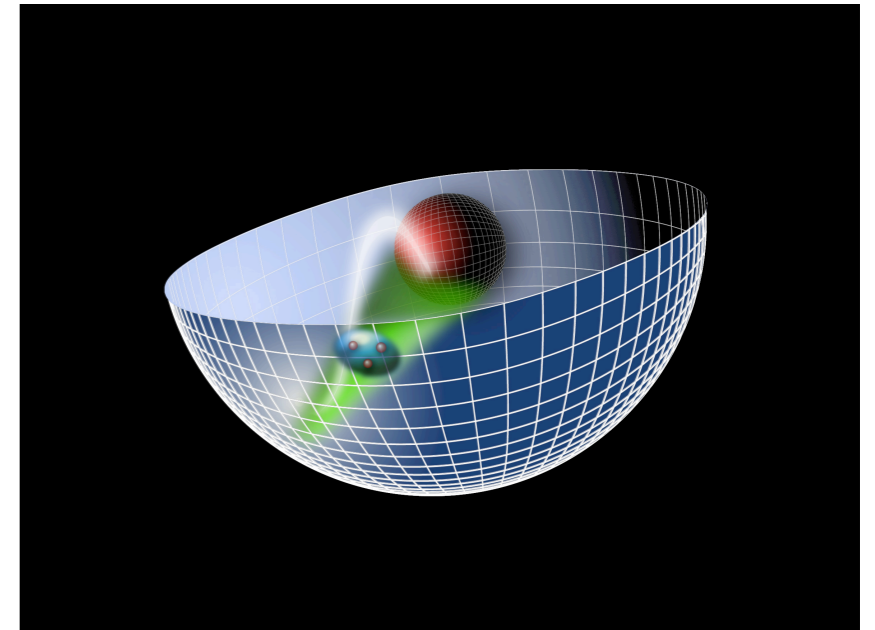
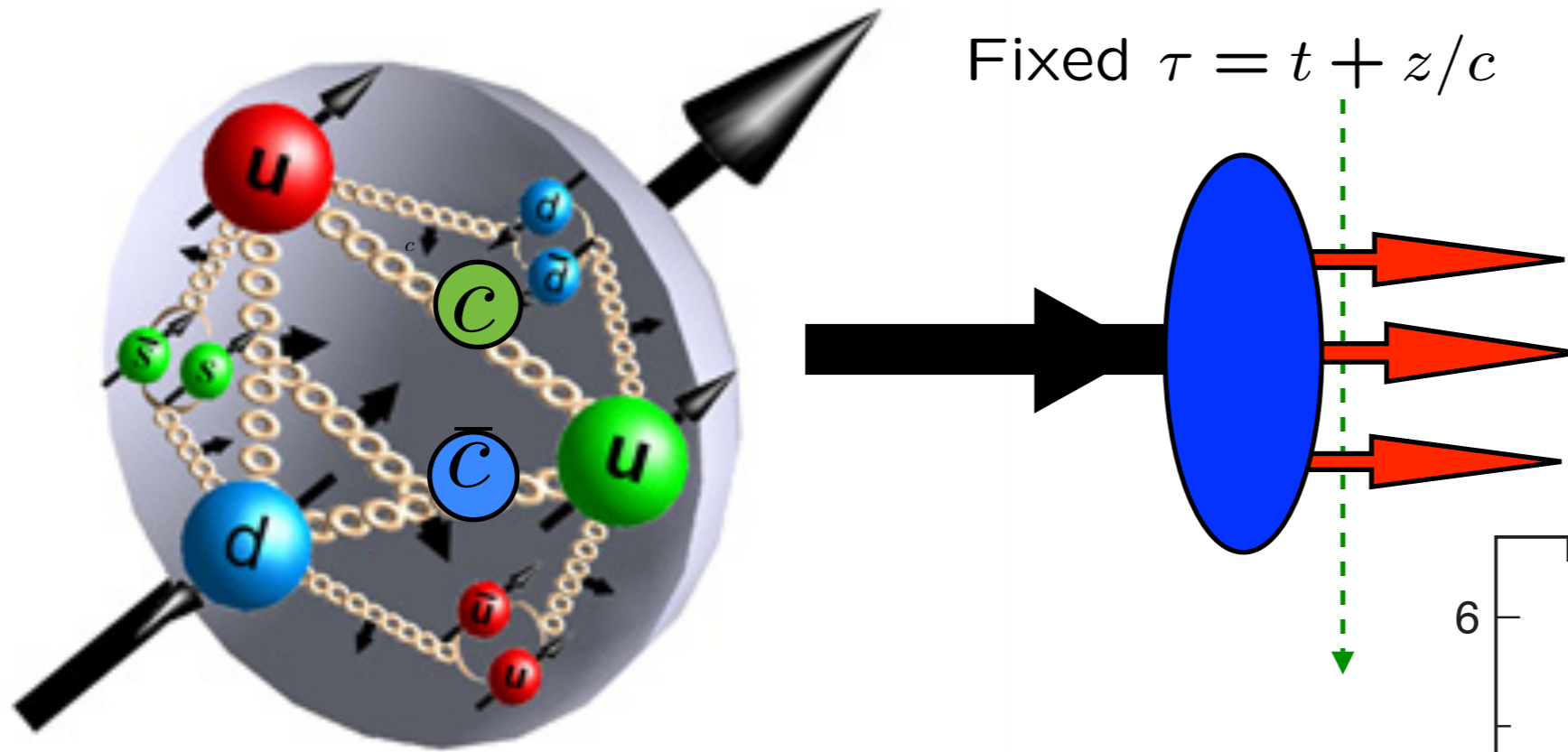


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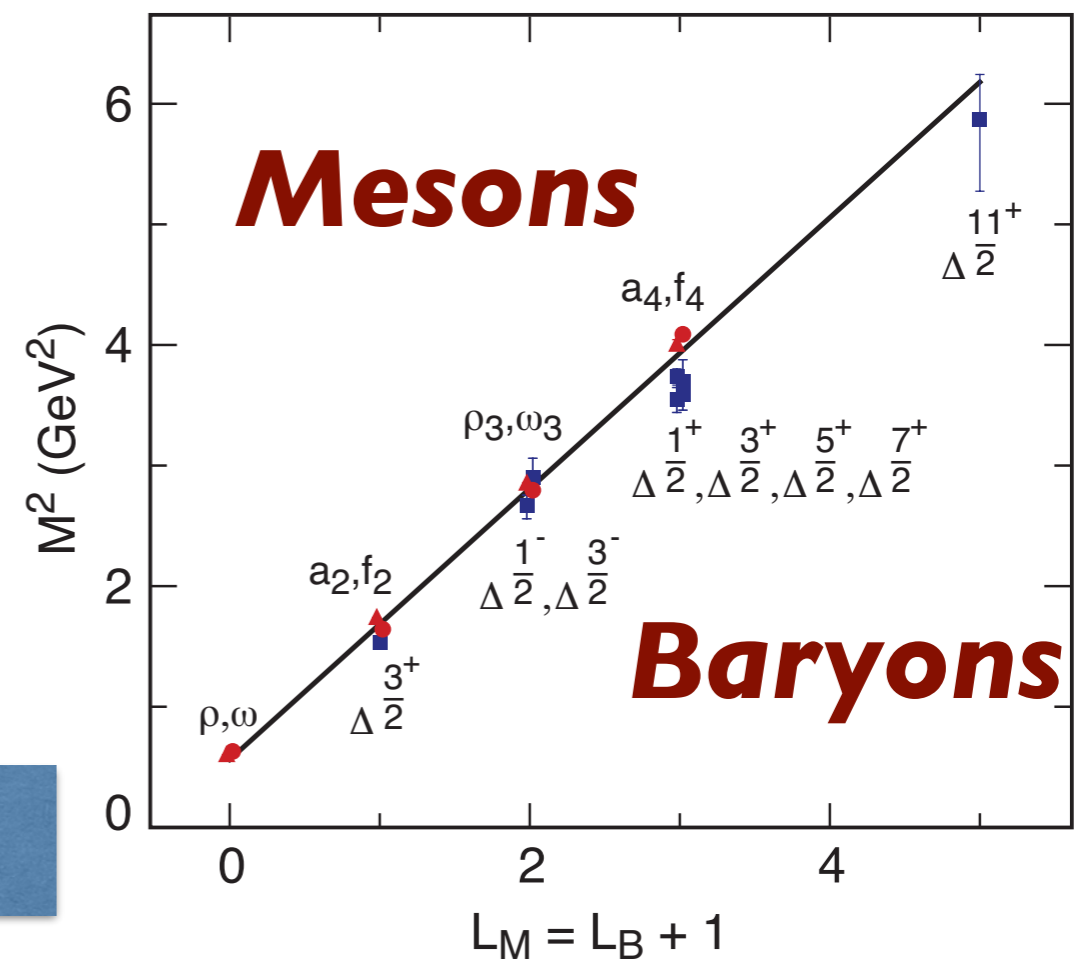
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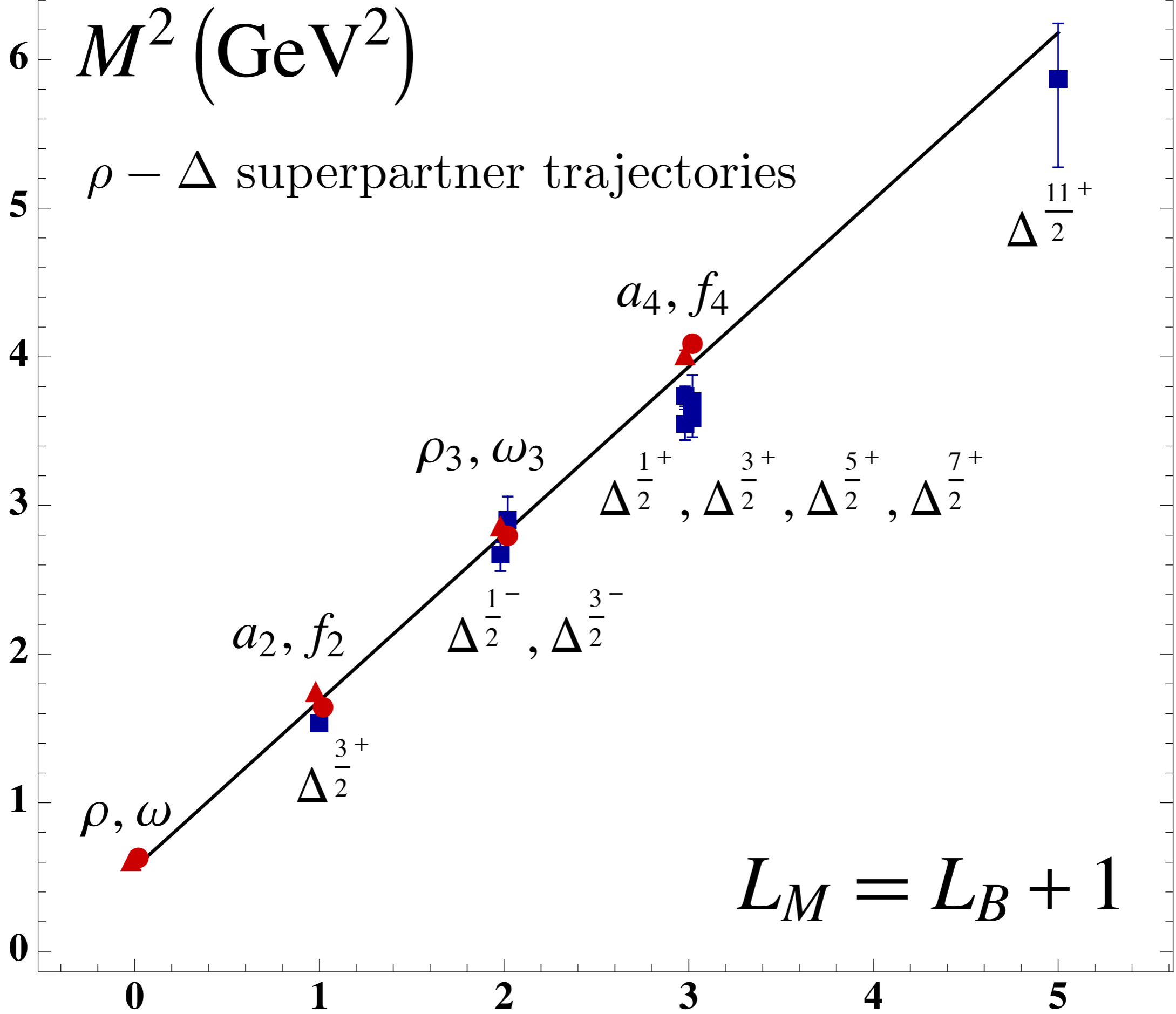
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M^2 (GeV²)

$\rho - \Delta$ superpartner trajectories

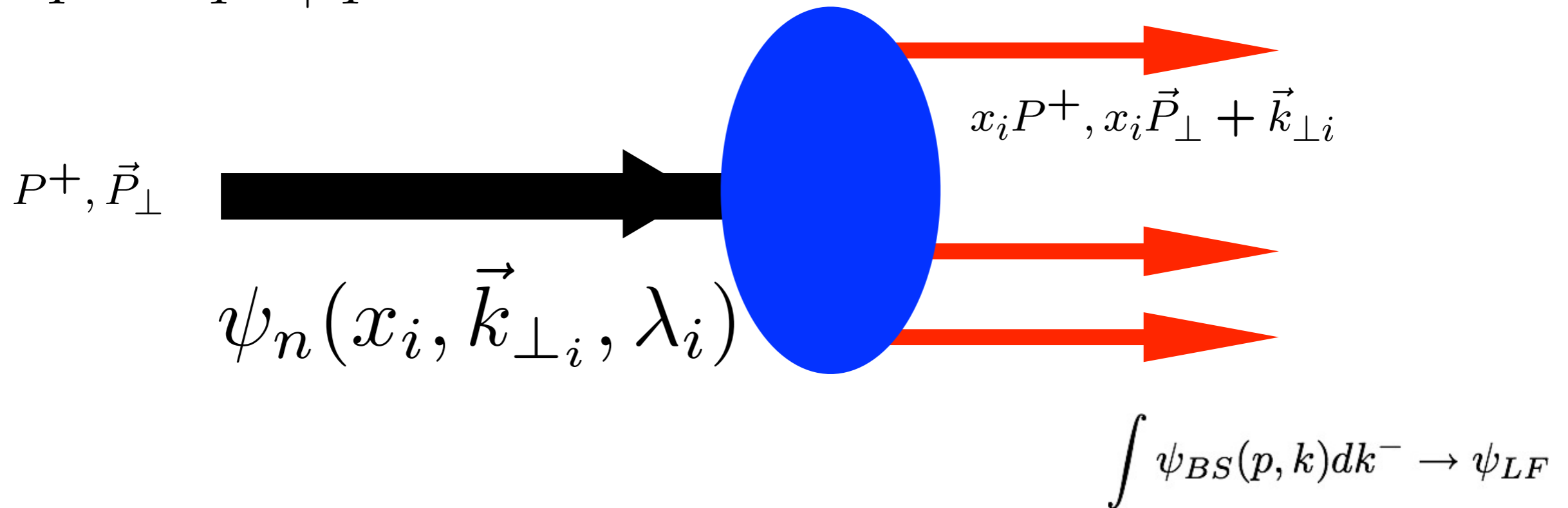


Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



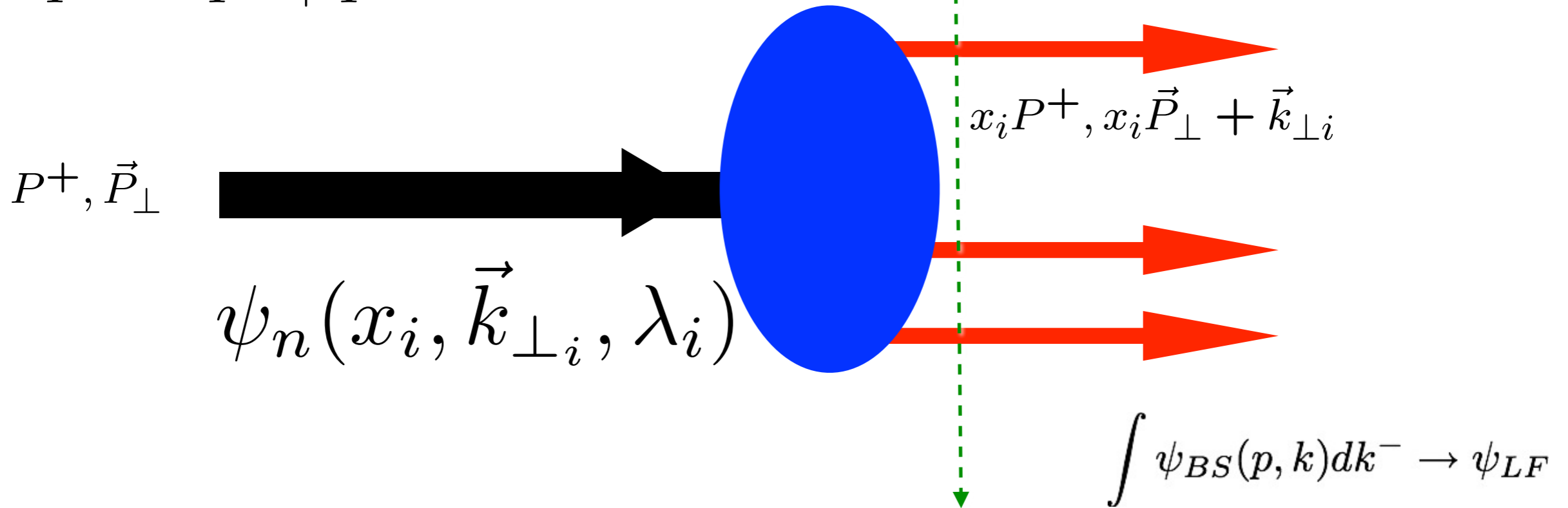
$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

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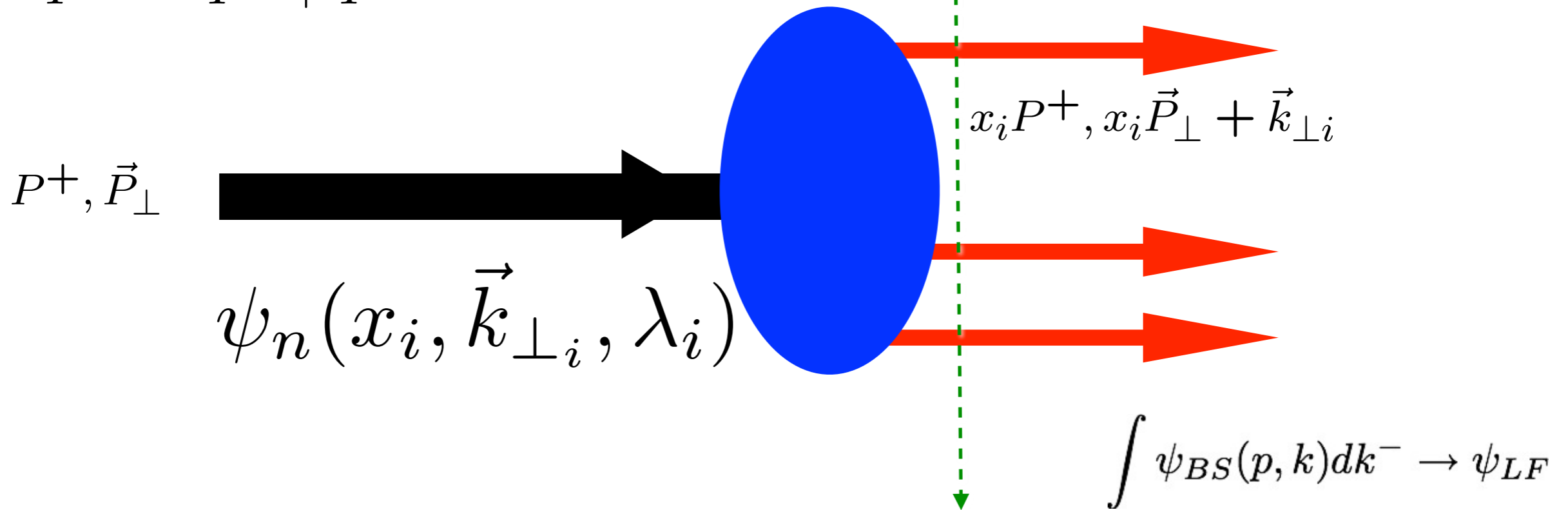
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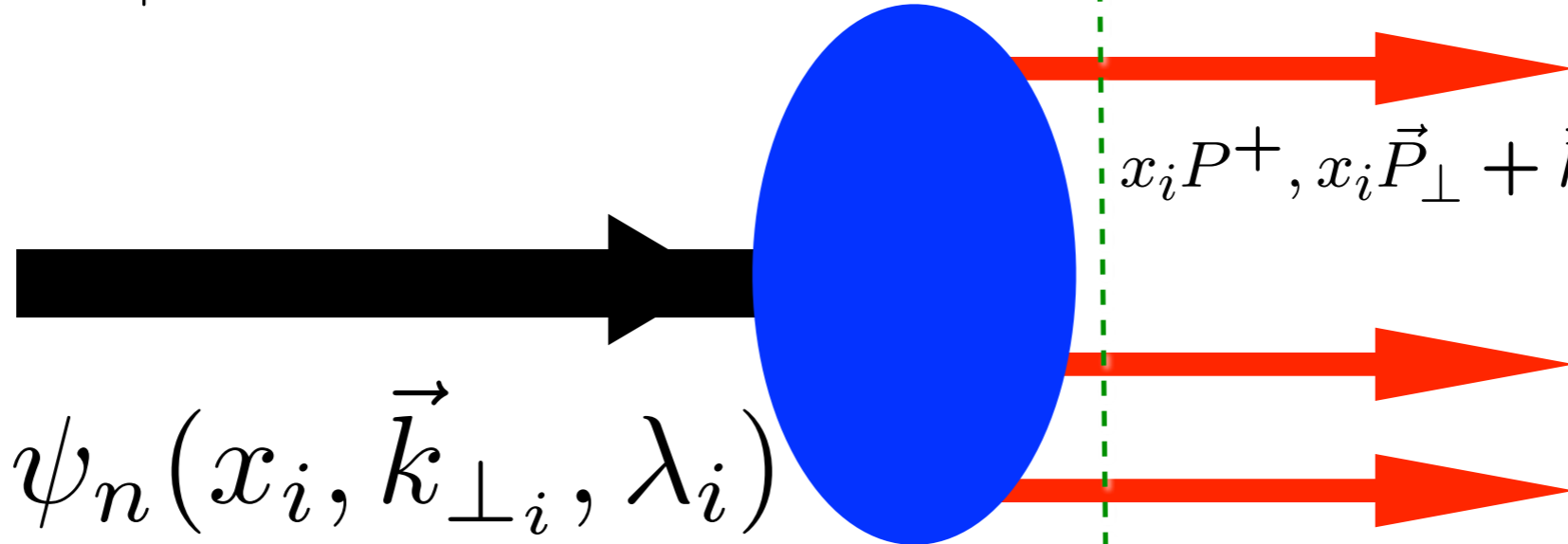
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P^+, \vec{P}_\perp



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\sum_i^n x_i = 1$$

$\psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

$$\int \psi_{BS}(p, k) dk^- \rightarrow \psi_{LF}$$

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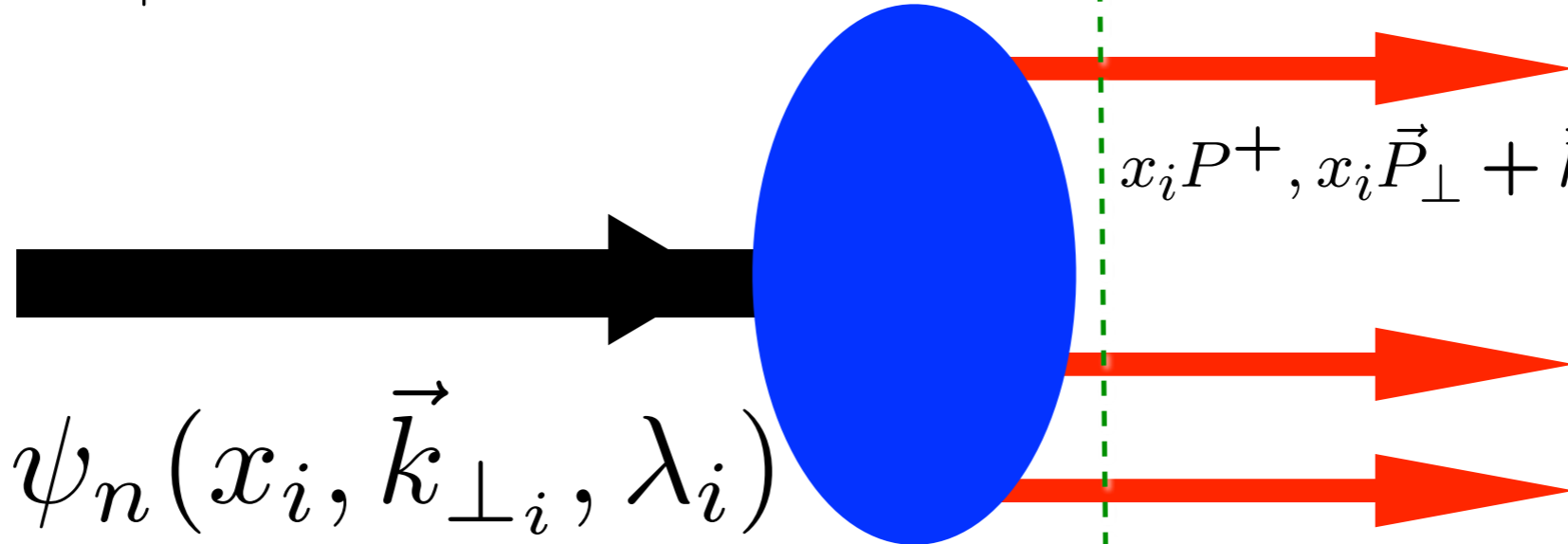
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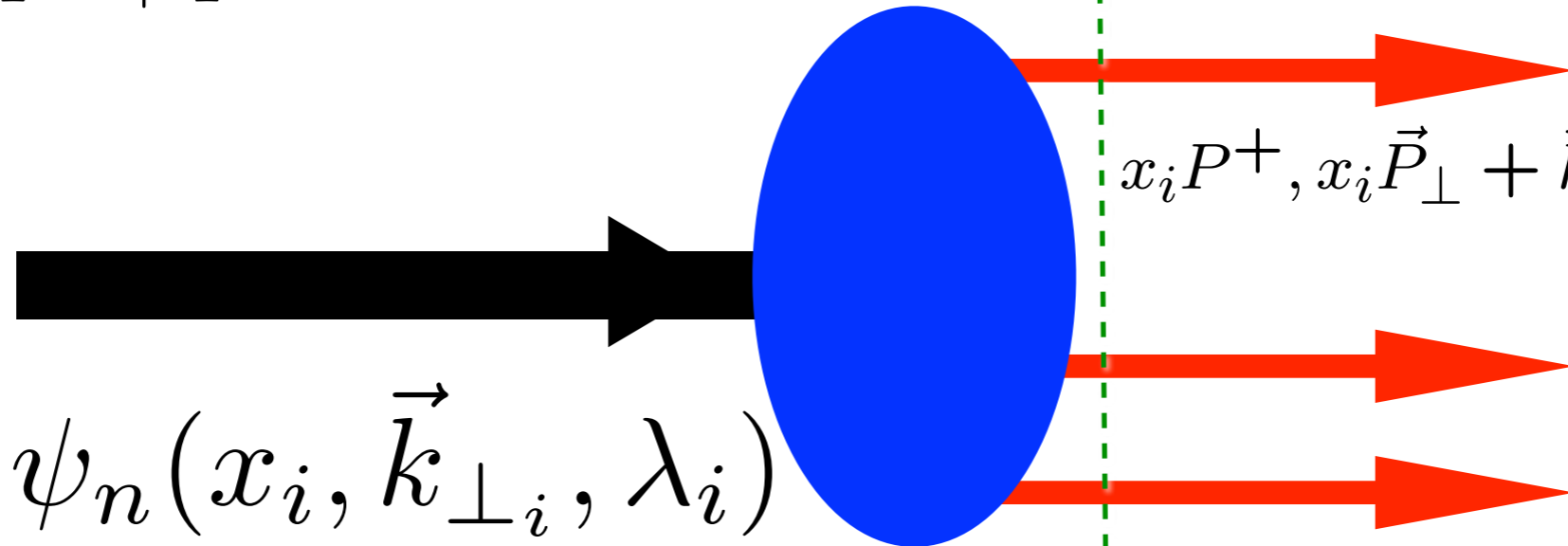
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Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions: **rigorous** representation of composite systems in quantum field theory

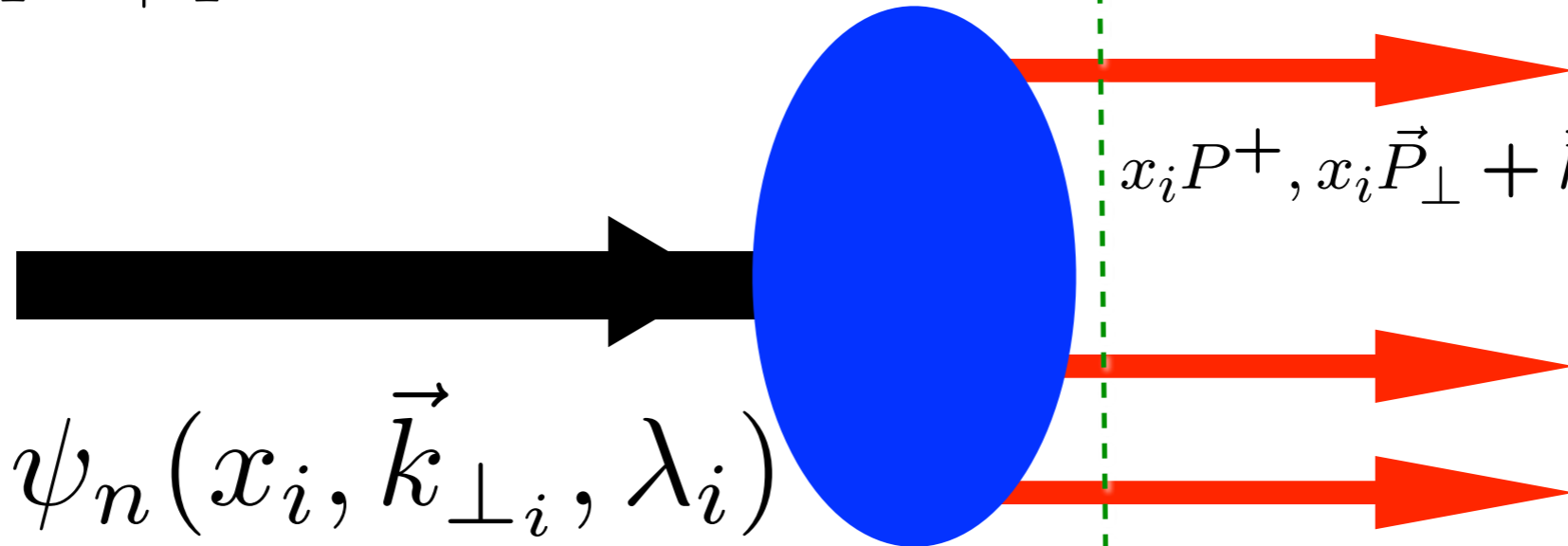
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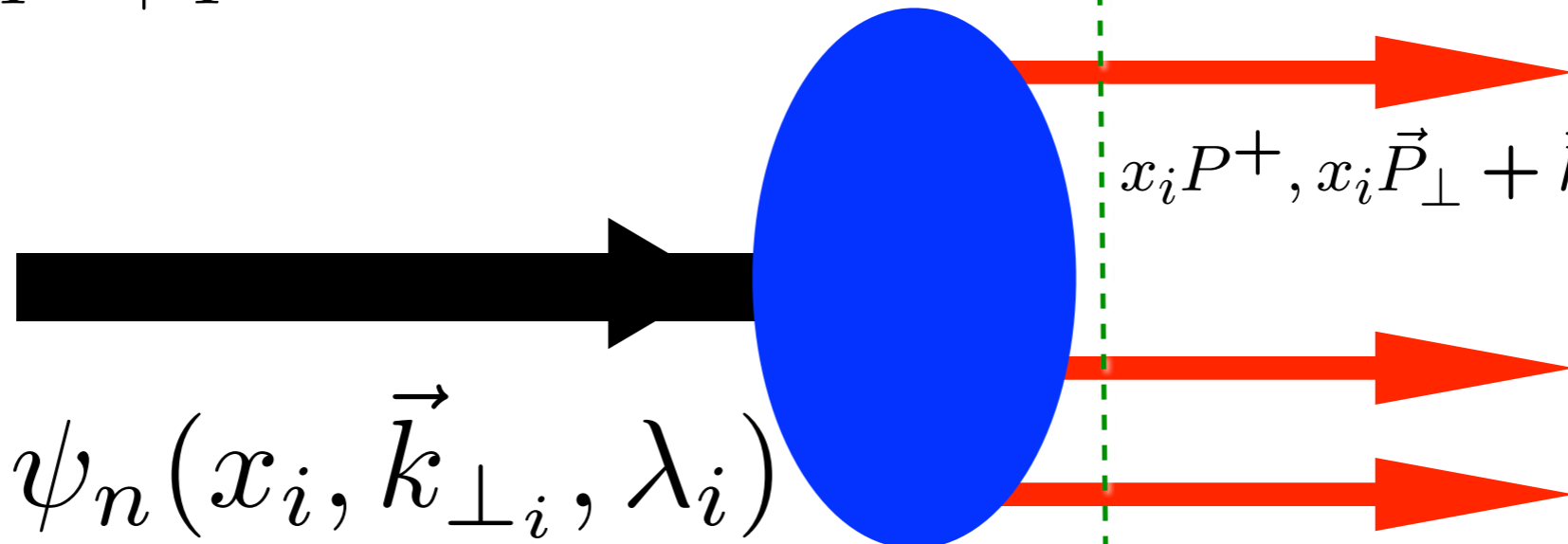
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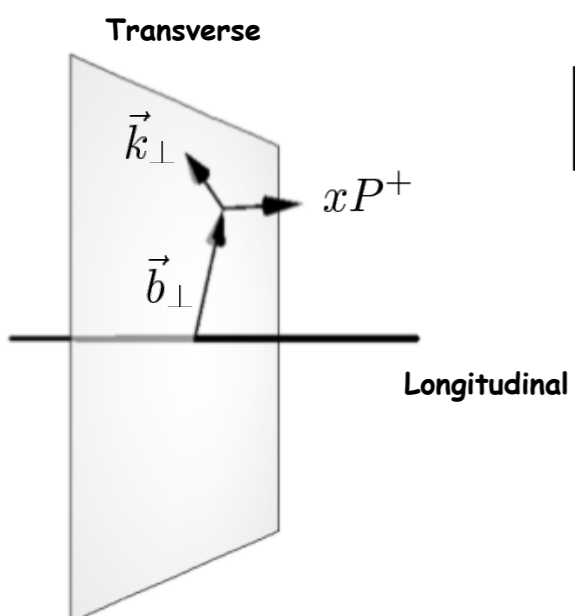
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS



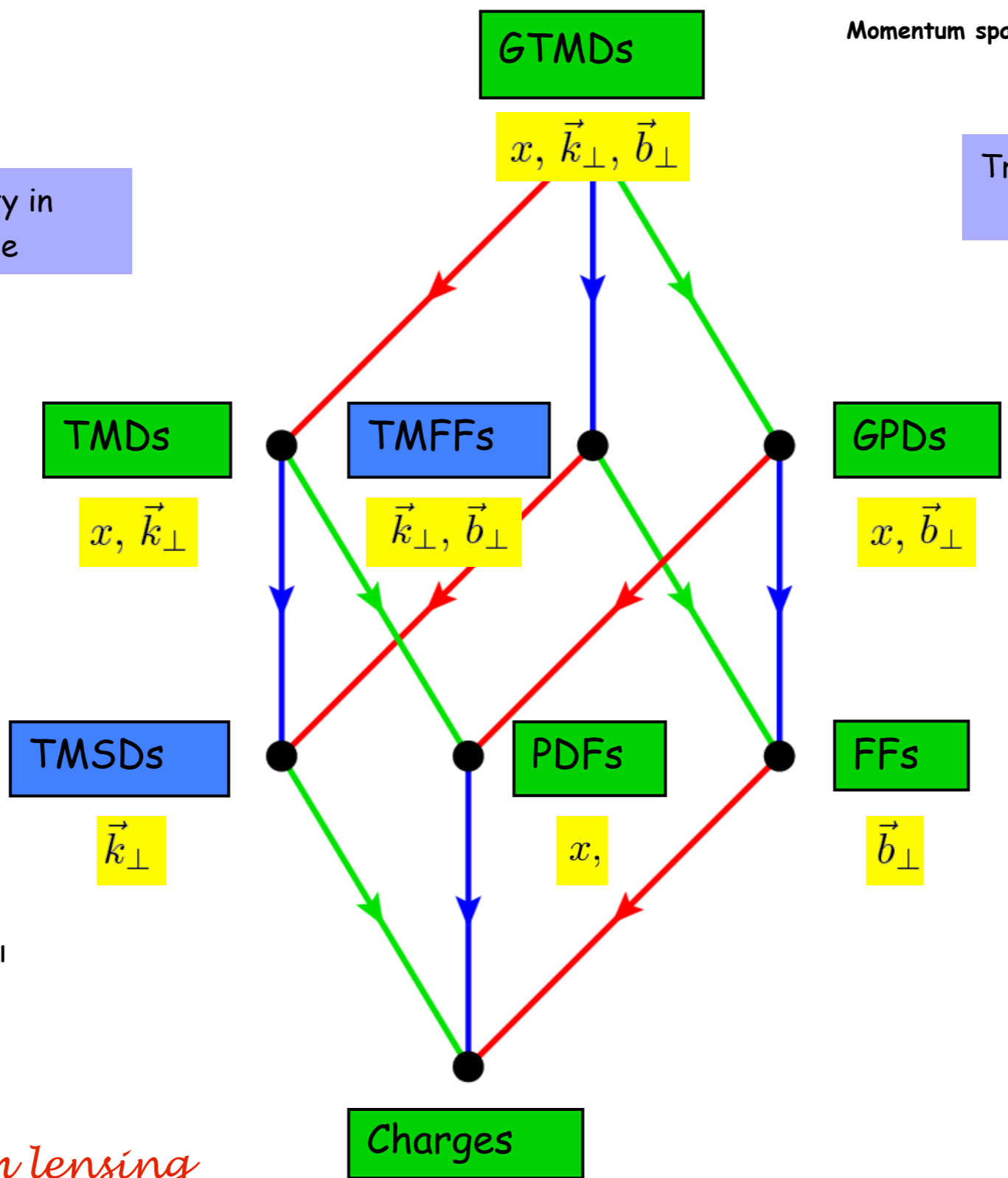
Transverse density in momentum space

Transverse density in position space

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space
 $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

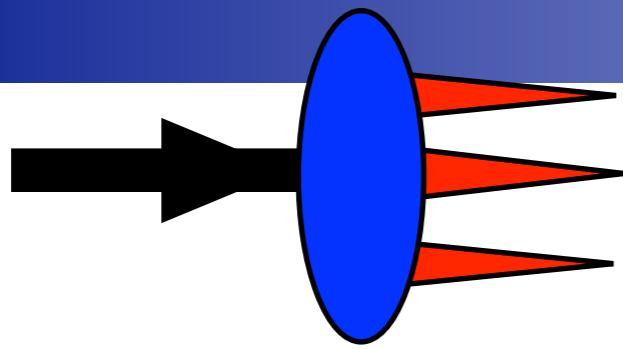


Sivers, T-odd from lensing



Lorce, Pasquini

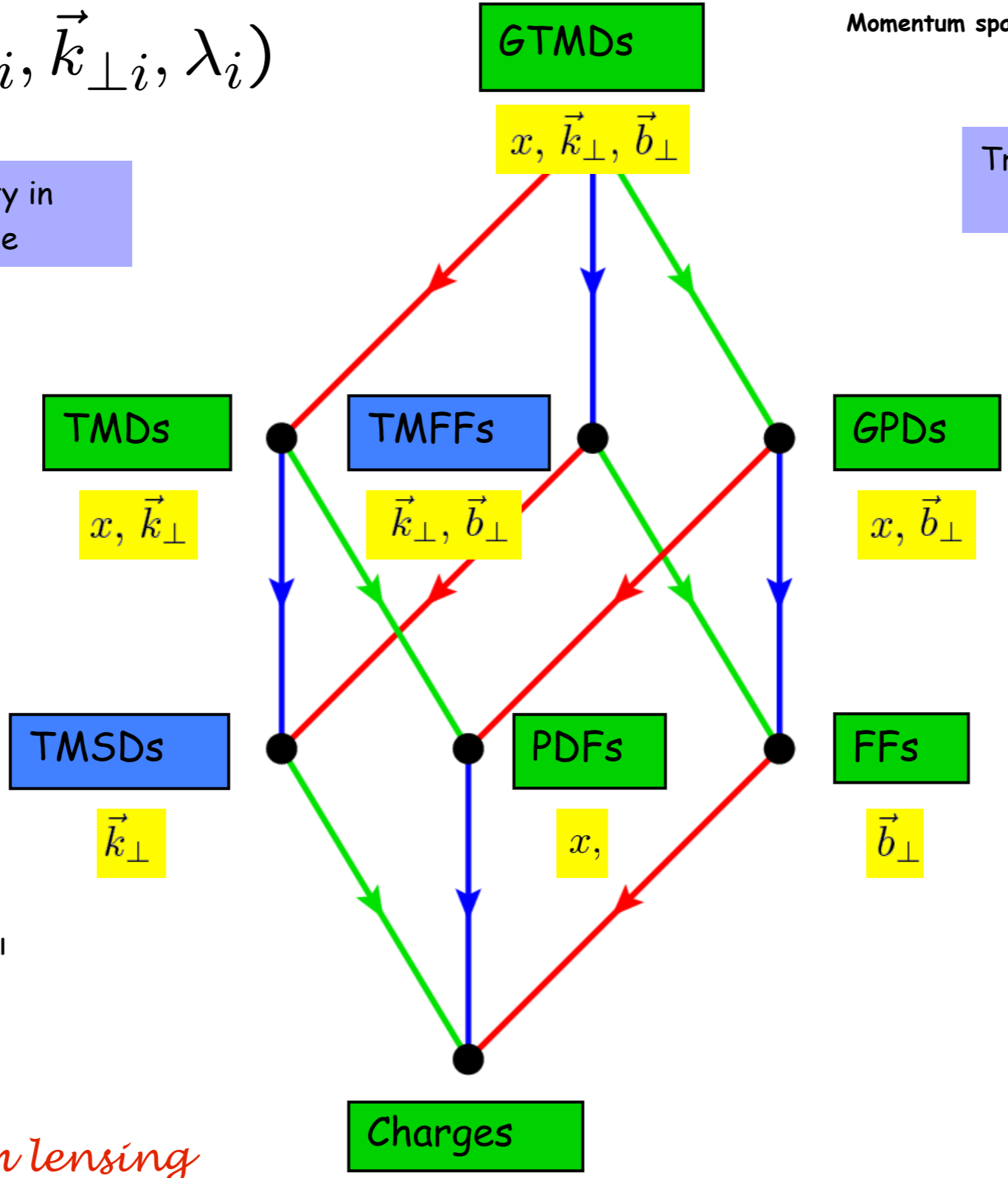
- $\int d^2 b_\perp$
- $\int dx$
- $\int d^2 k_\perp$



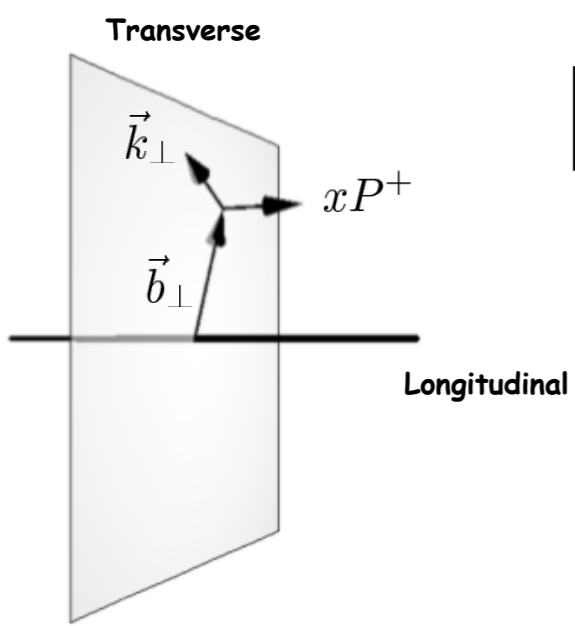
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

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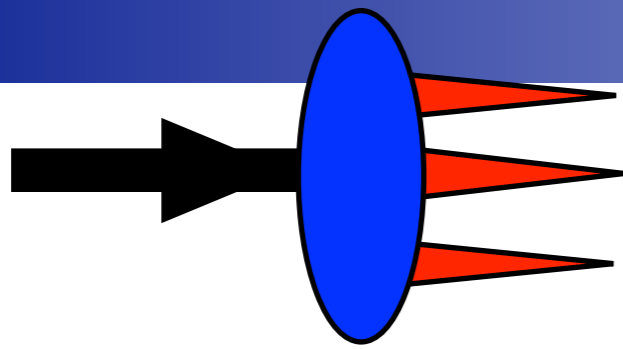


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Transverse density in momentum space

GTMDs

$$x, \vec{k}_{\perp}, \vec{b}_{\perp}$$

Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
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Transverse density in position space

TMDs

$$x, \vec{k}_{\perp}$$

TMFFs

$$\vec{k}_{\perp}, \vec{b}_{\perp}$$

GPDs

$$x, \vec{b}_{\perp}$$

TMSDs

$$\vec{k}_{\perp}$$

PDFs

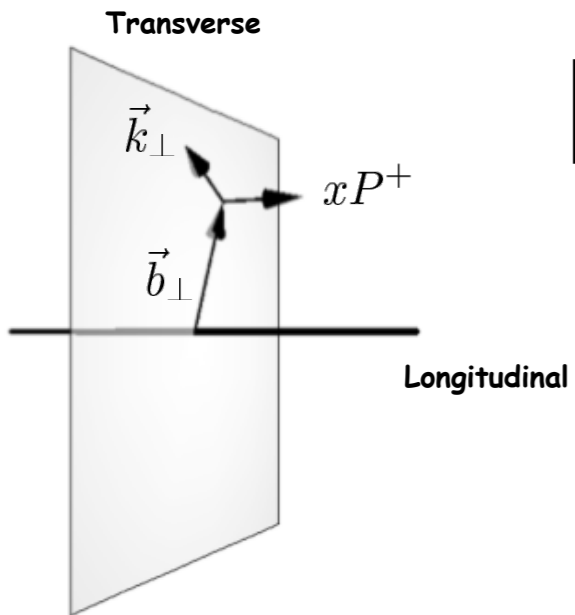
$$x,$$

FFs

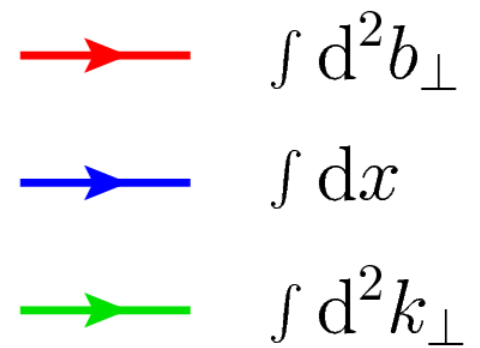
$$\vec{b}_{\perp}$$

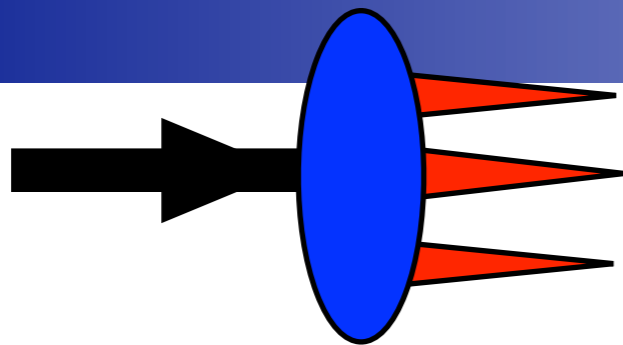
Charges

Lorce, Pasquini



Sivers, T-odd from lensing





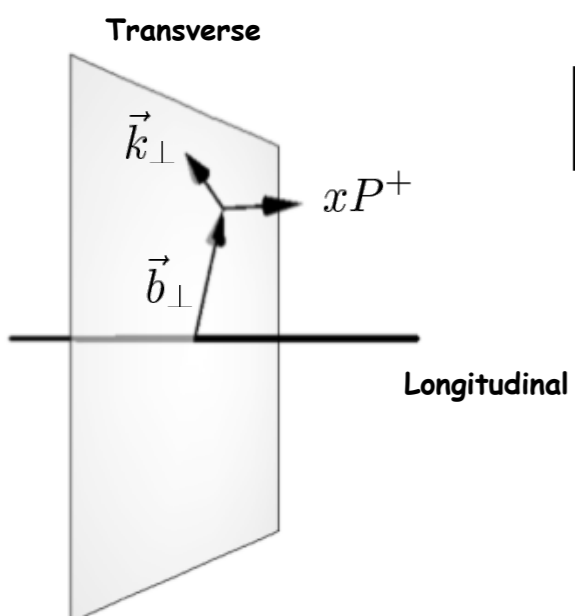
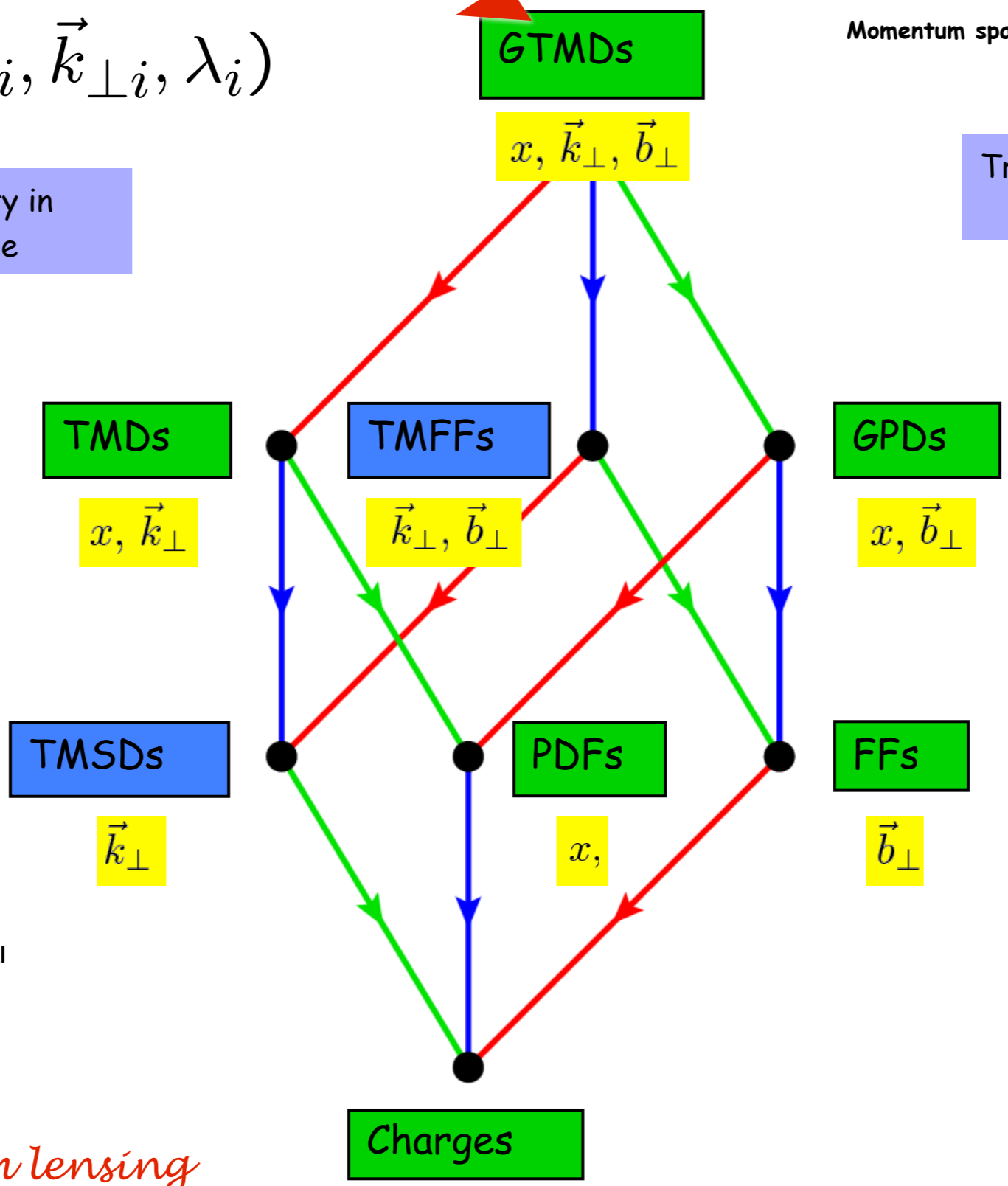
• *Light Front Wavefunctions:*

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Transverse density in momentum space

Transverse density in position space



Lorce, Pasquini

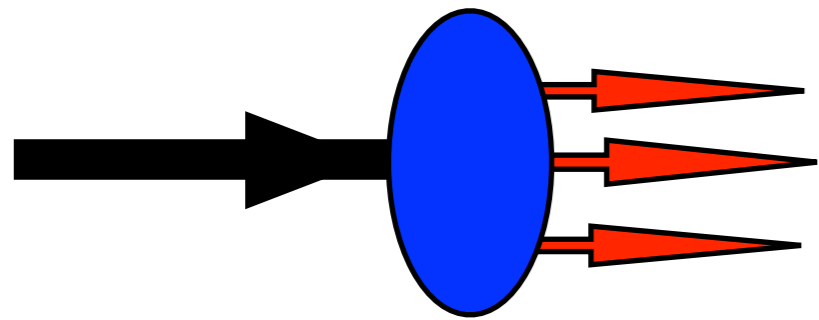
→ $\int d^2 b_{\perp}$
 → $\int dx$
 → $\int d^2 k_{\perp}$

Sivers, T-odd from lensing

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

Direct connection to QCD Lagrangian

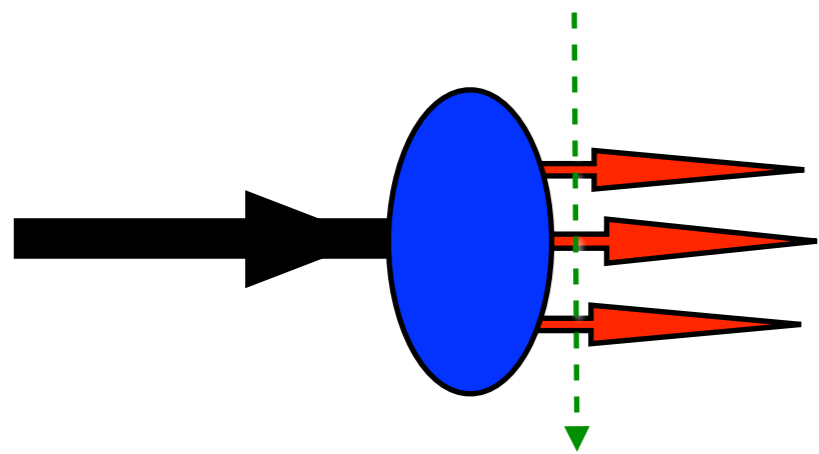
Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

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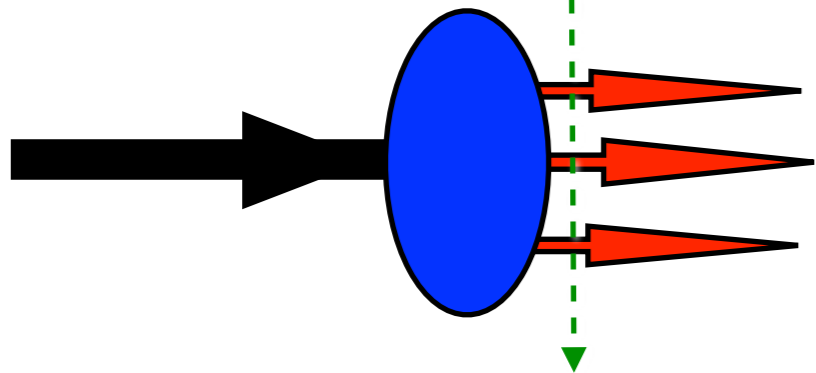
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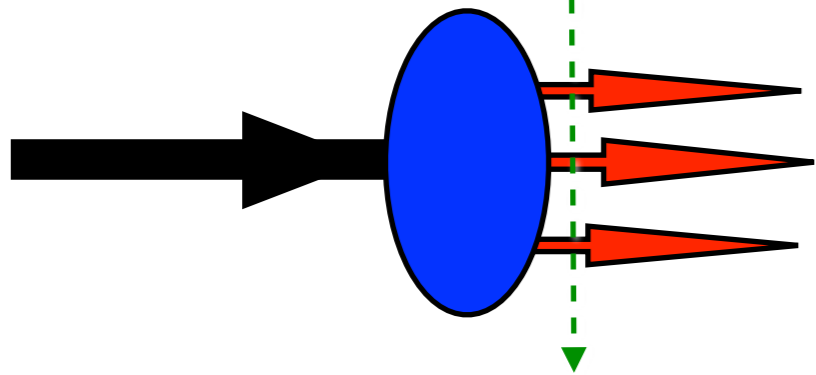
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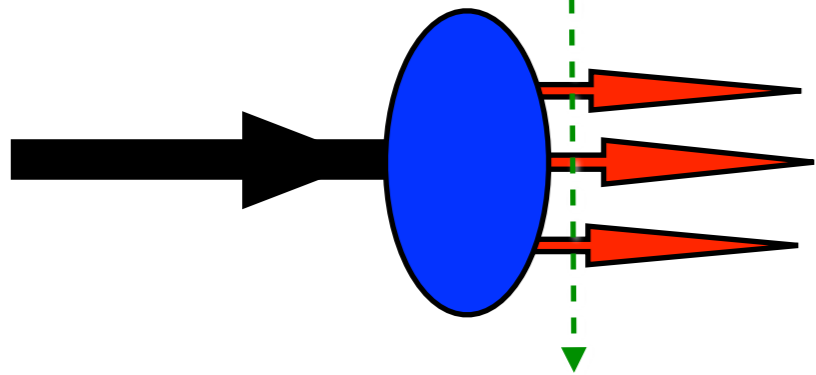
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Exact frame-independent formulation of nonperturbative QCD!

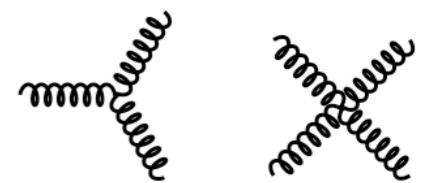
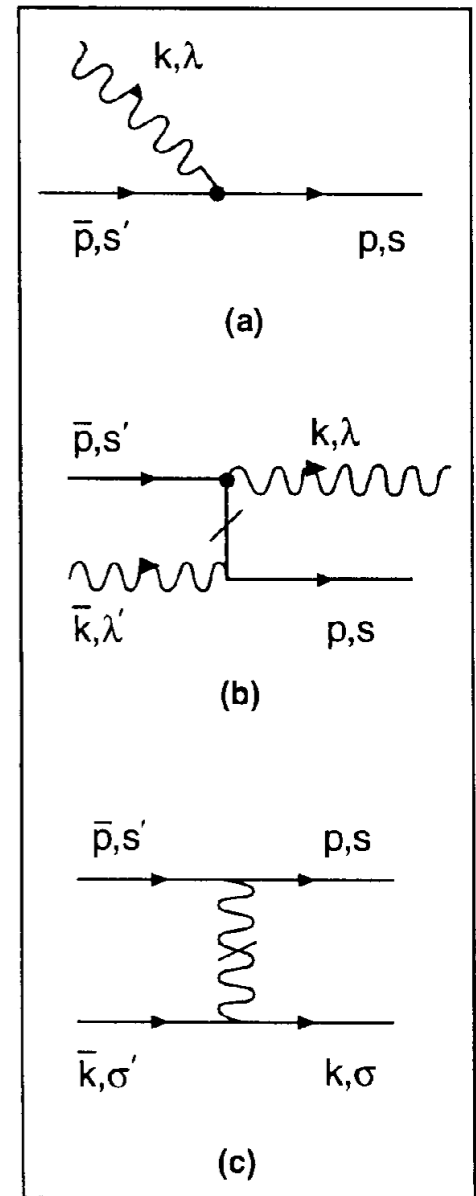
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$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

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H_{LF}^{int}

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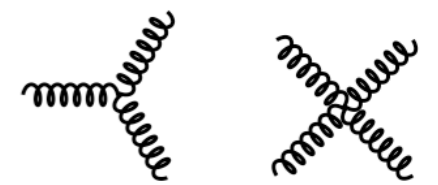
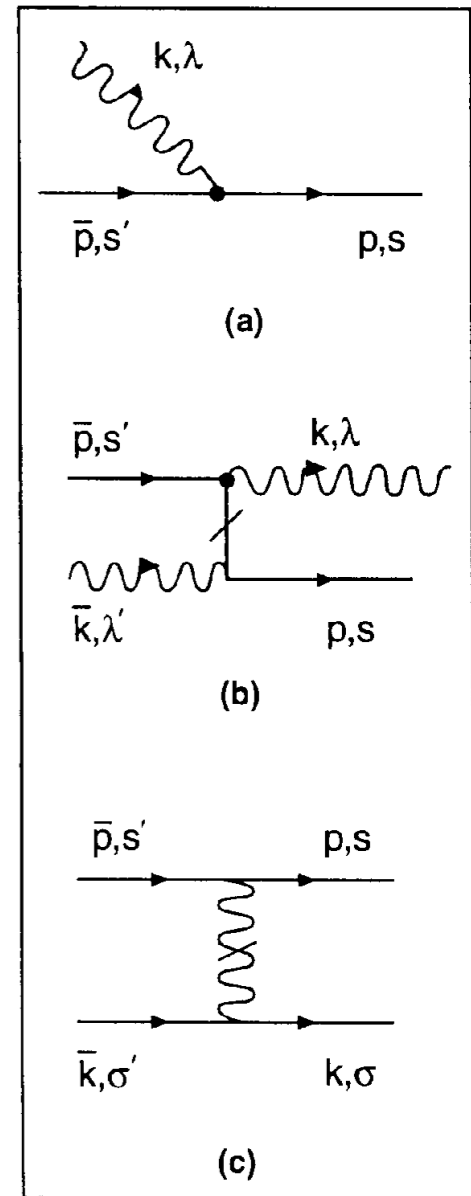
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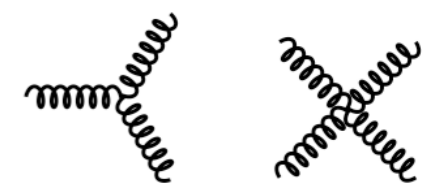
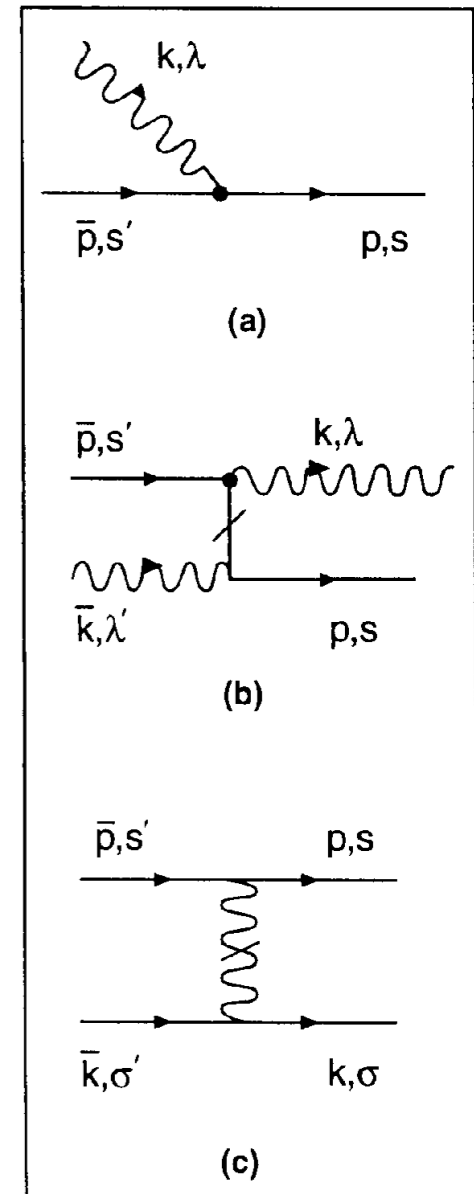
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Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



H_{LF}^{int}

Exact frame-independent formulation of nonperturbative QCD!

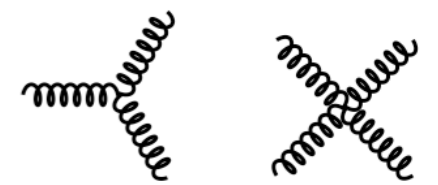
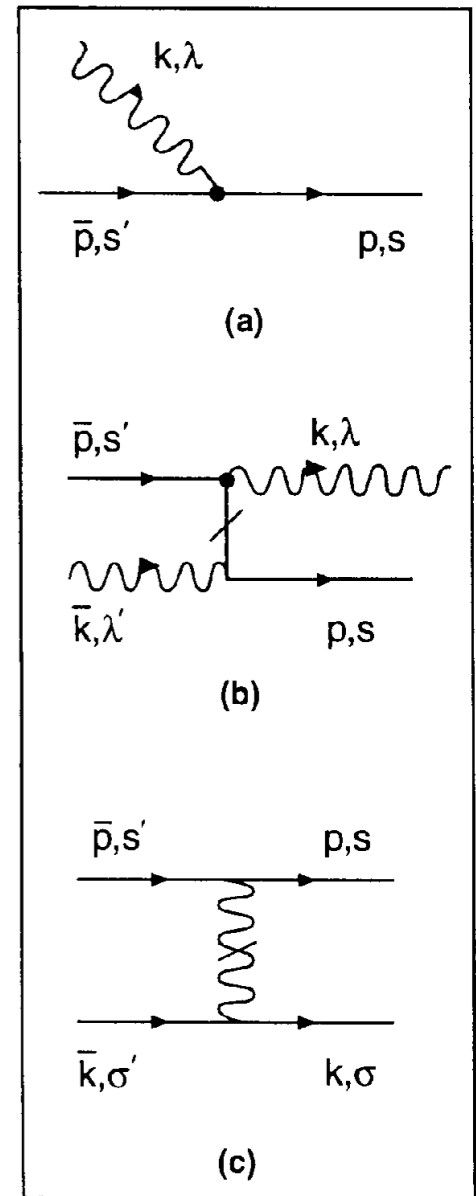
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$



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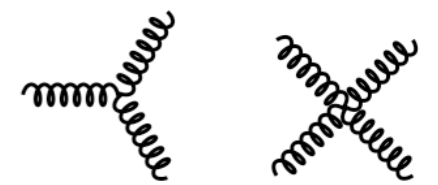
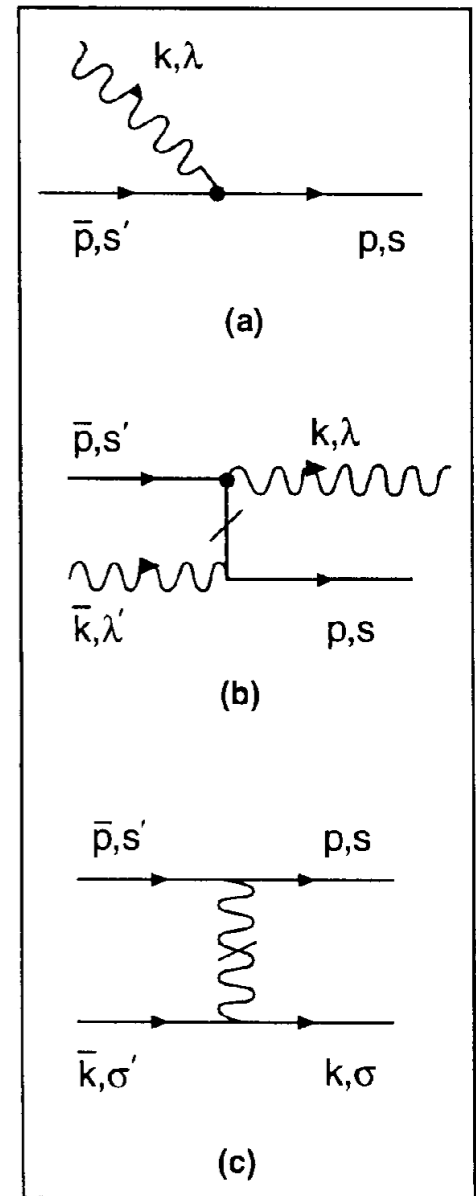
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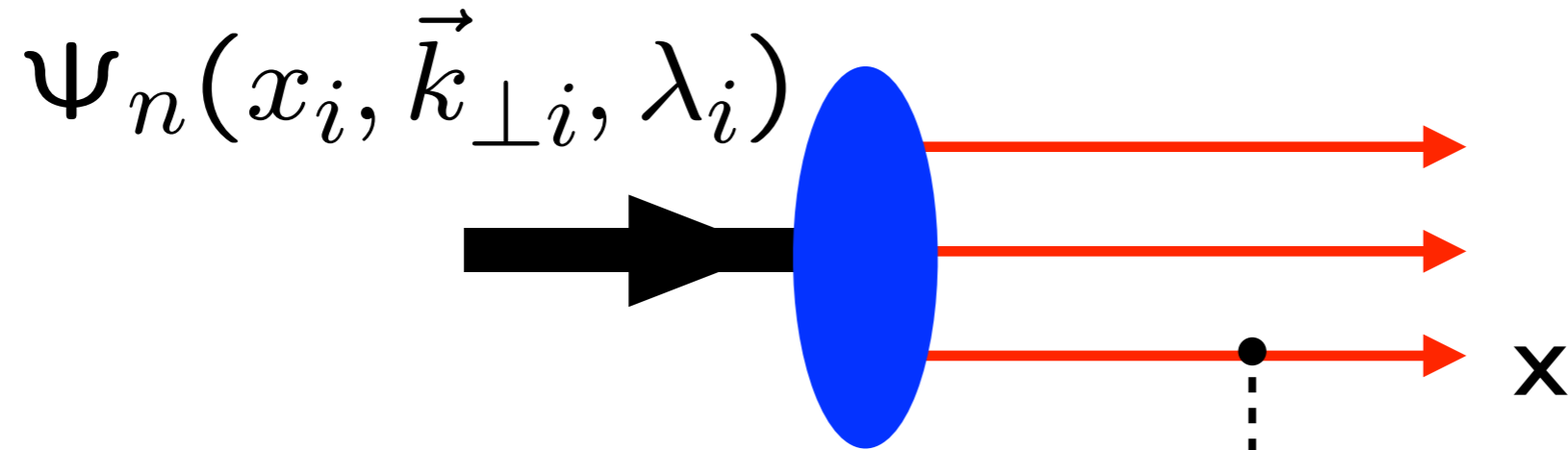
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LFWFs: Off-shell in P- and invariant mass



$$g_q \bar{\psi}_q(x) \psi_q(x) h(x)$$

$$\langle h \rangle$$

Higgs Zero Mode

Yukawa Higgs coupling of confined quark to Higgs zero mode gives

$$\bar{u}u g_q \langle h \rangle = \frac{m_q}{x_q} m_q = \frac{m_q^2}{x_q}$$

$$H_{LF} = \sum_q \frac{k_{\perp q}^2 + m_q^2}{x_q}$$

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sum over states with $n=3, 4, \dots$ constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum P^μ .

The light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

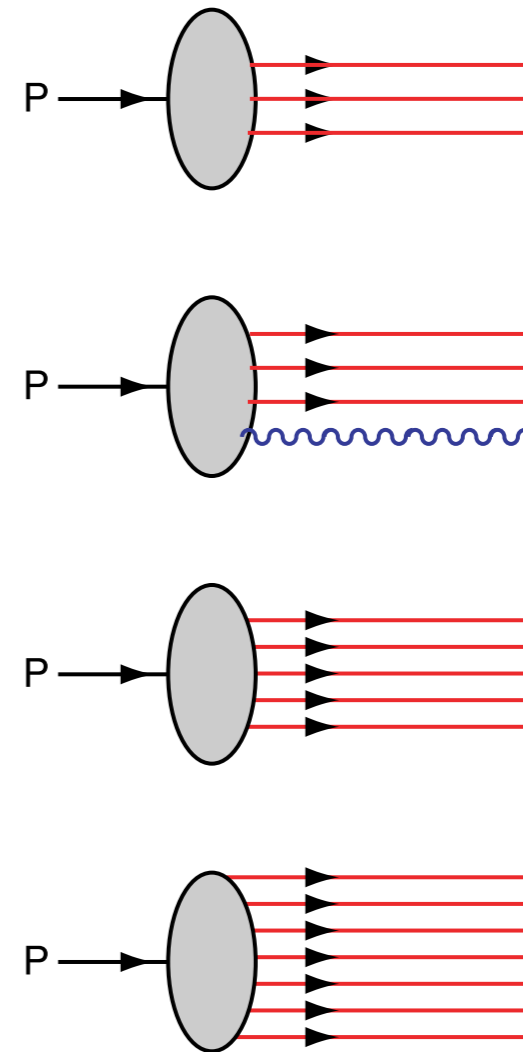
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Intrinsic heavy quarks
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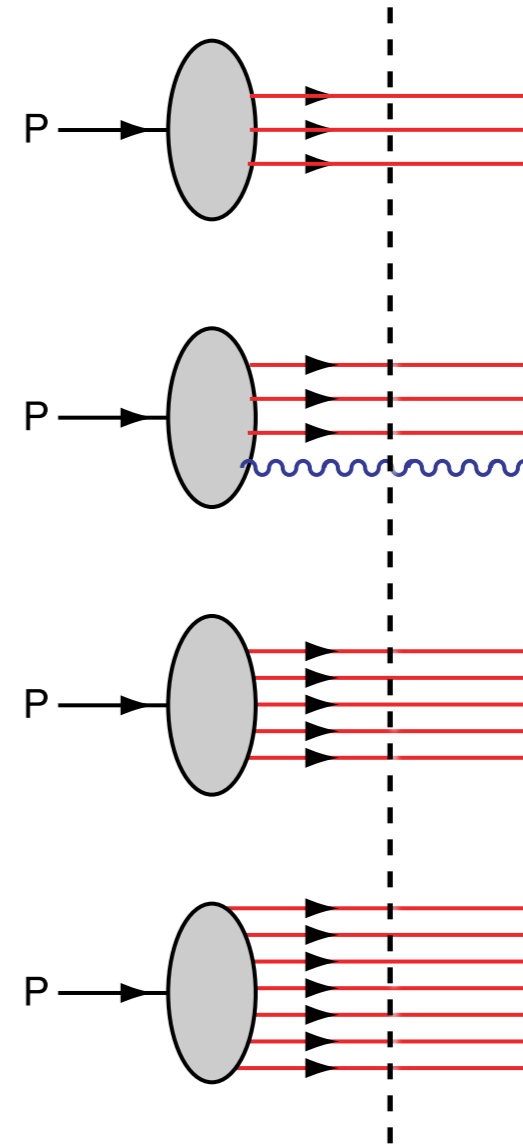
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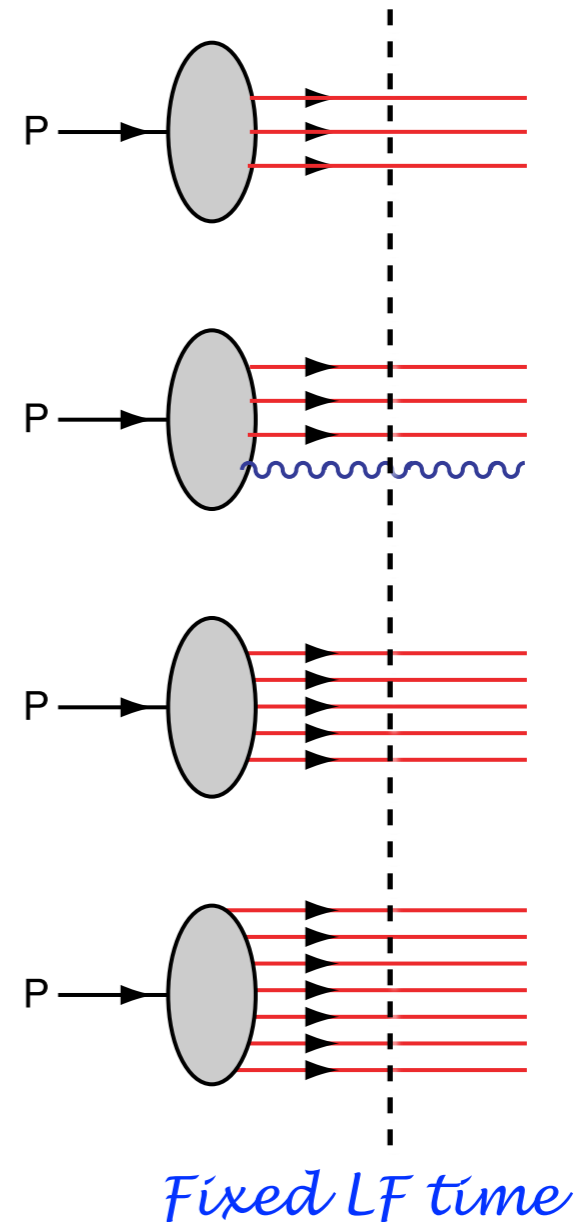
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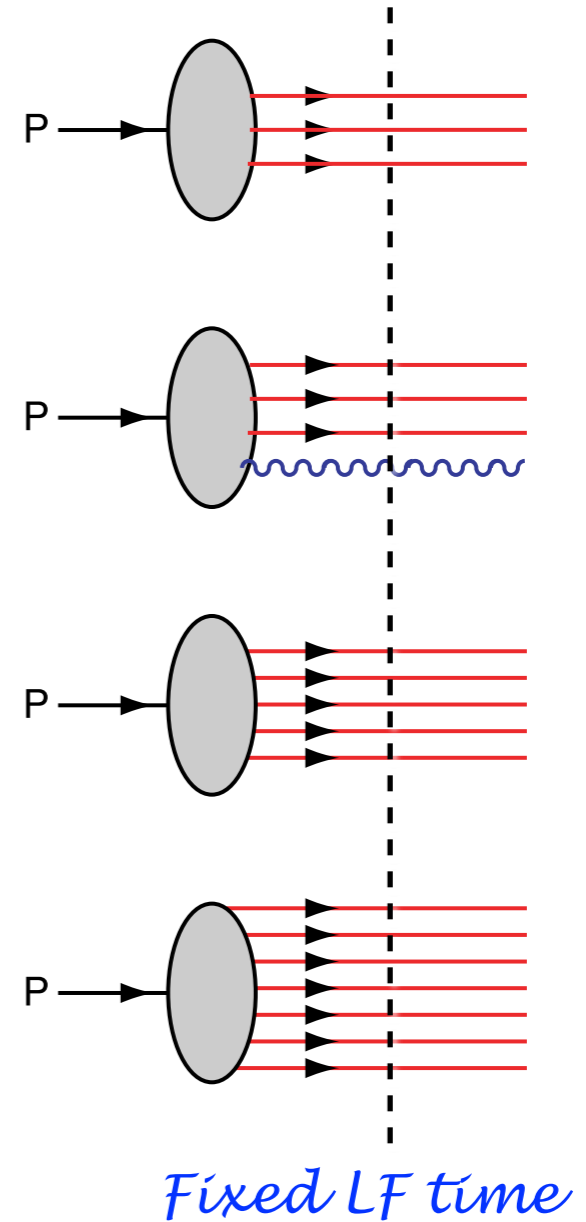
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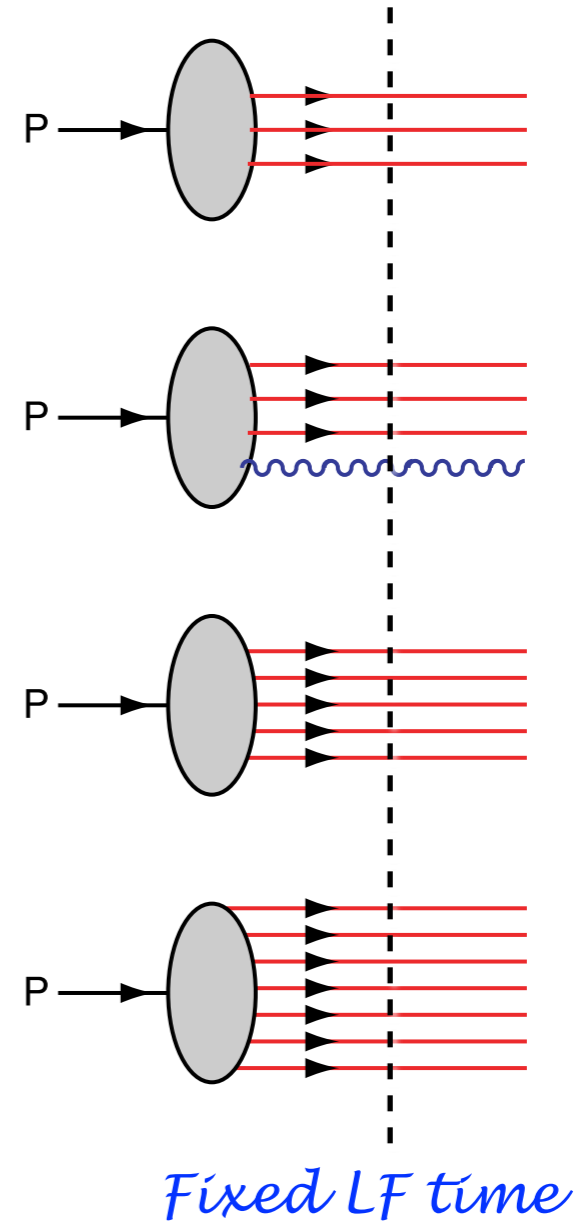
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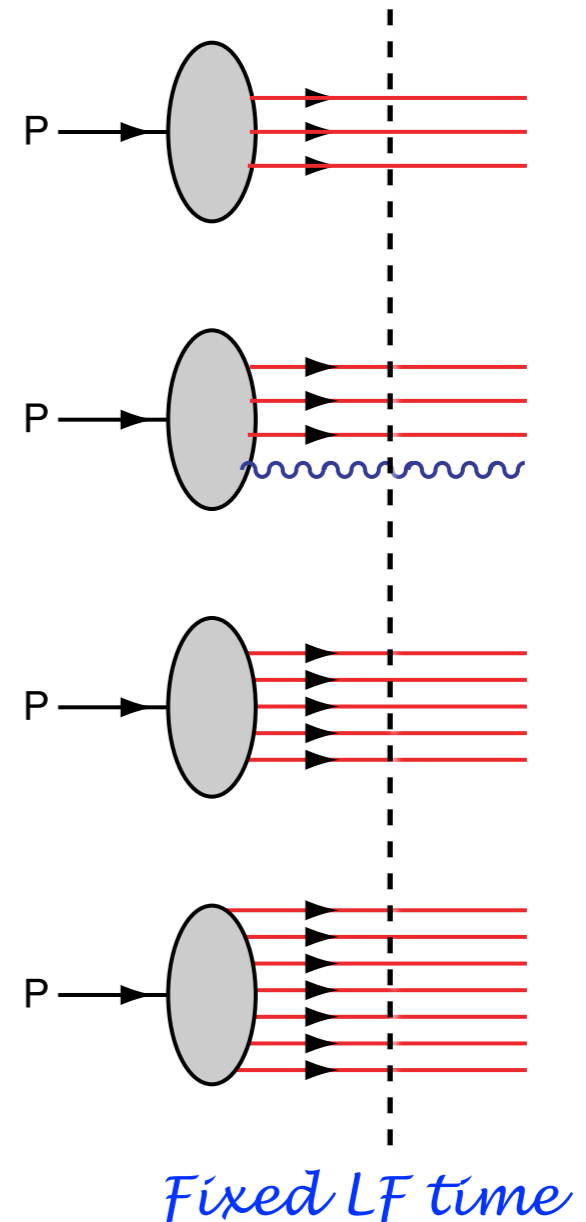
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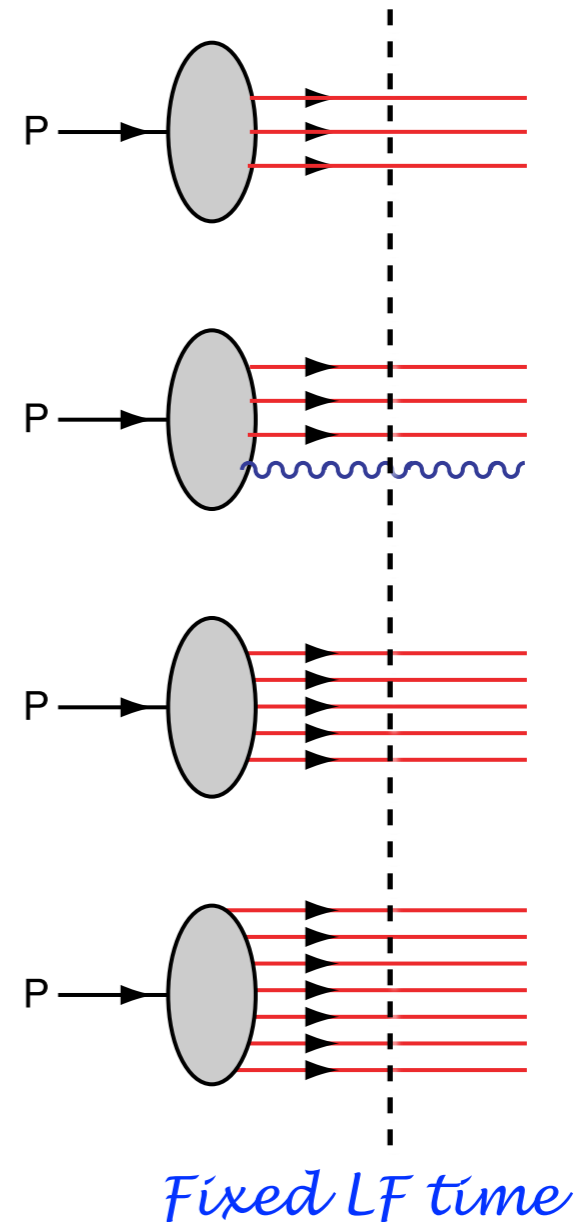
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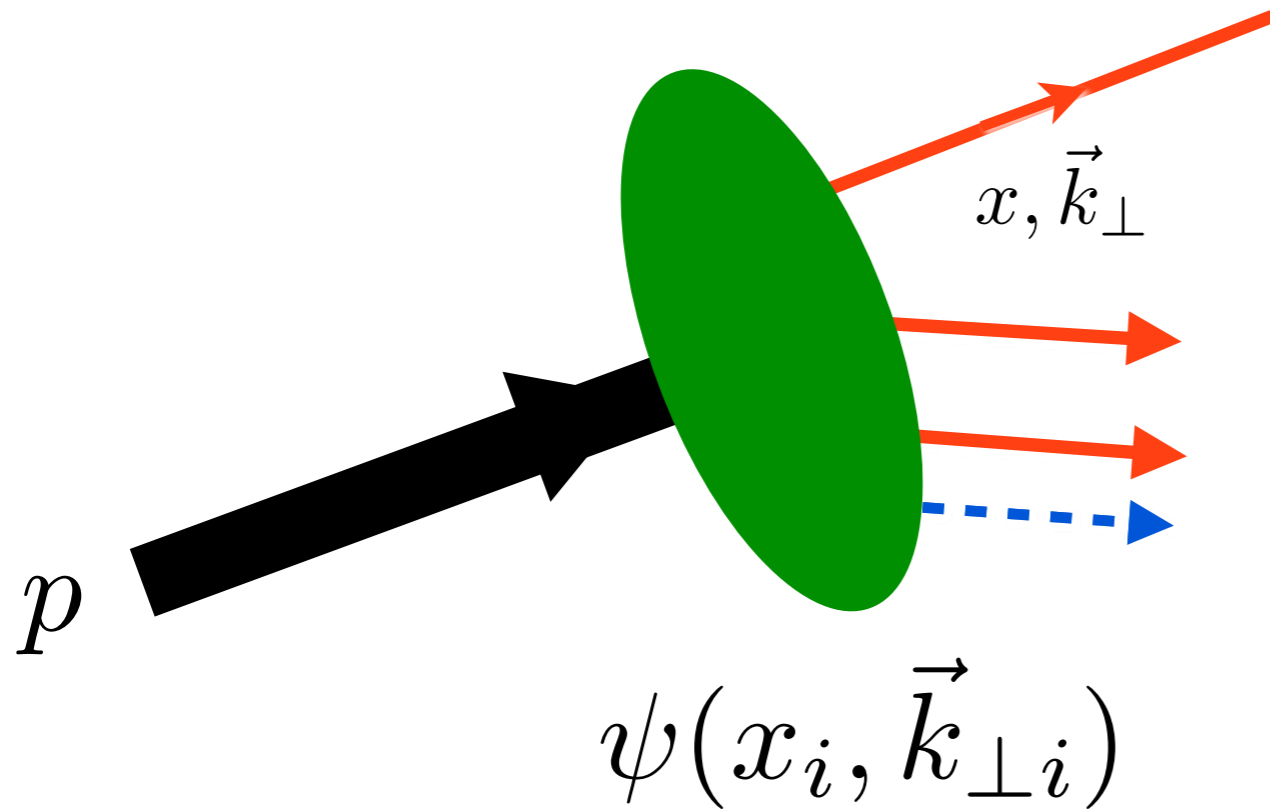
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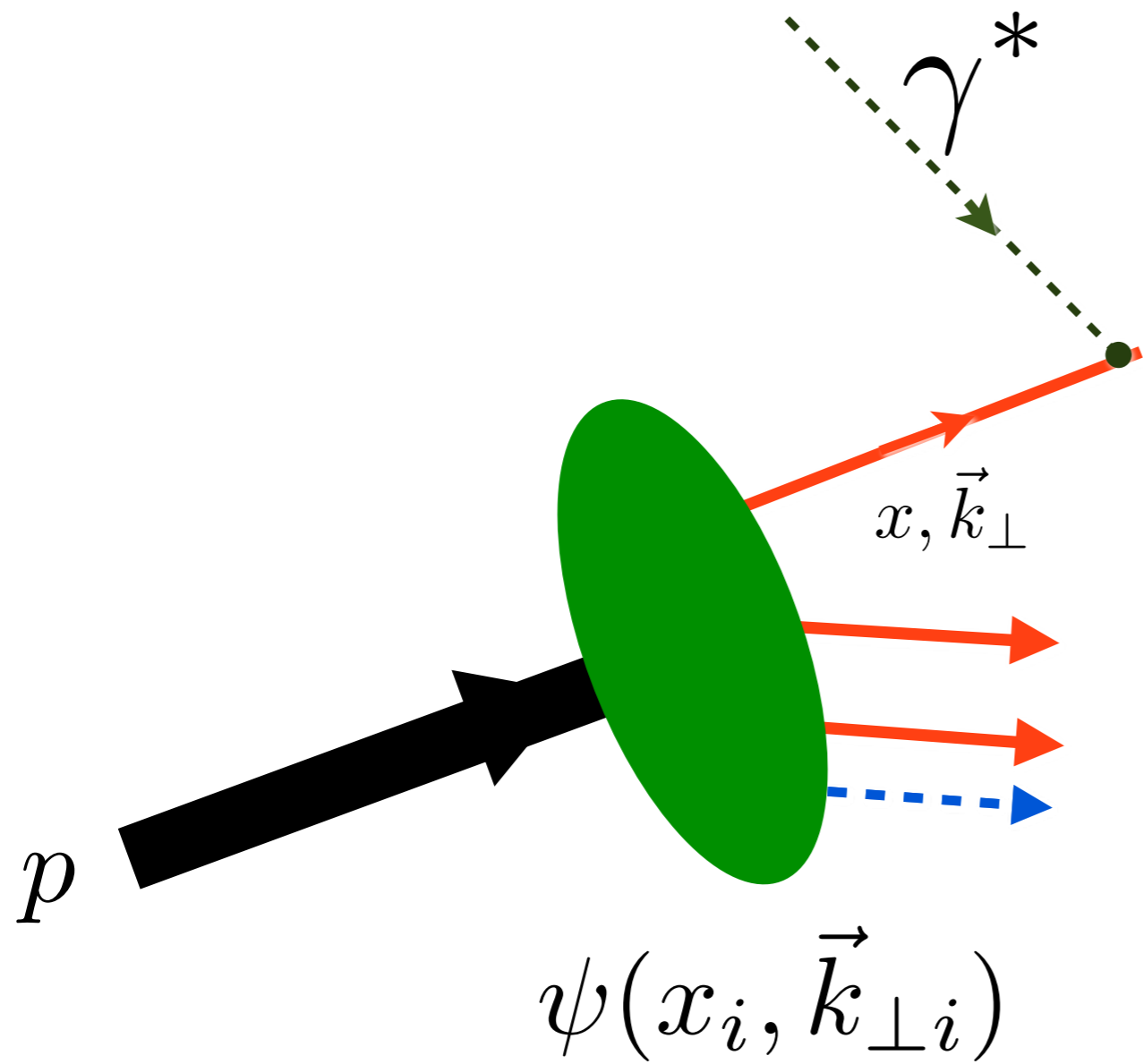
Interaction picture



**Drell & Yan, West
Exact LF formula!**

Drell, sjb

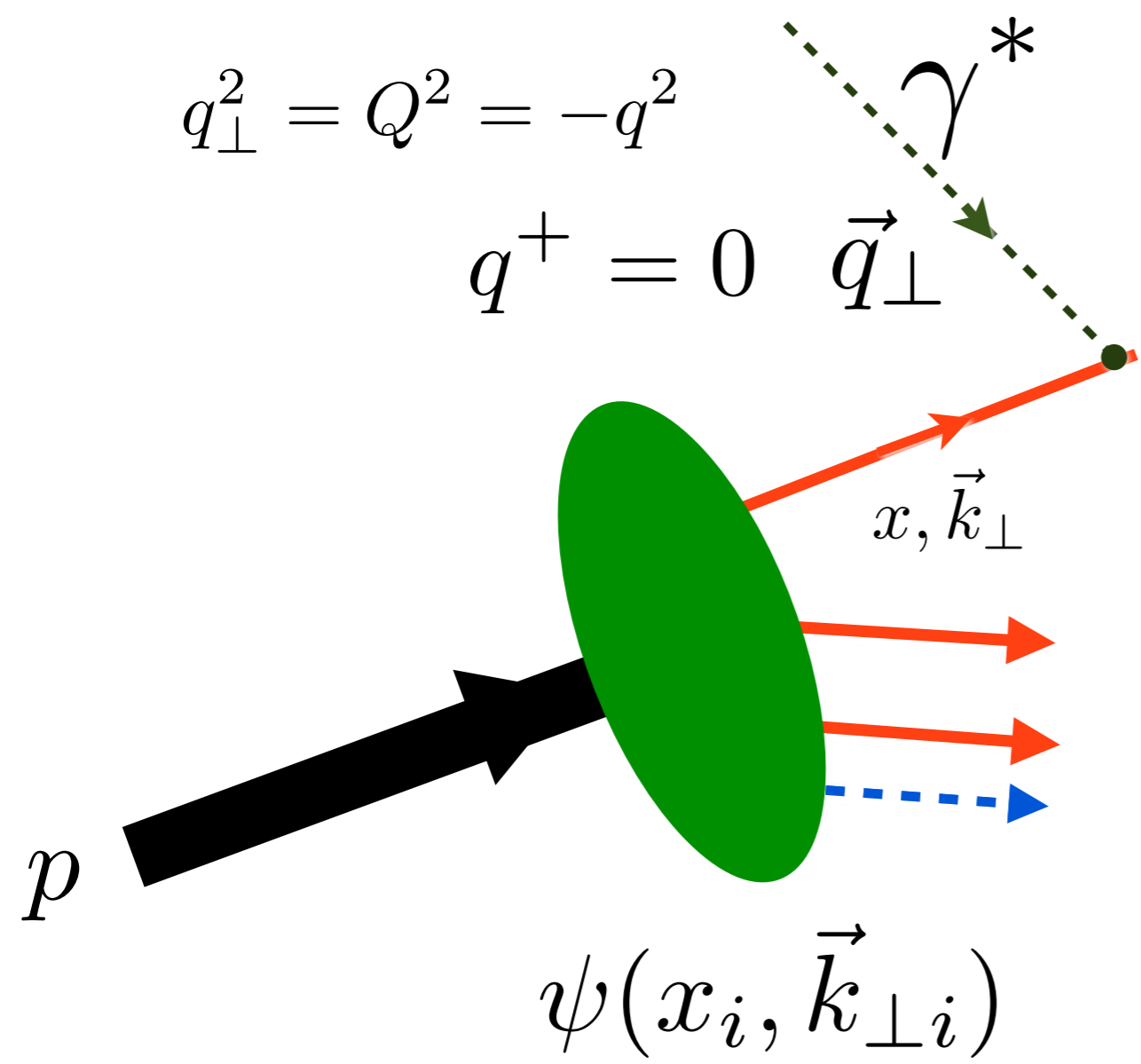
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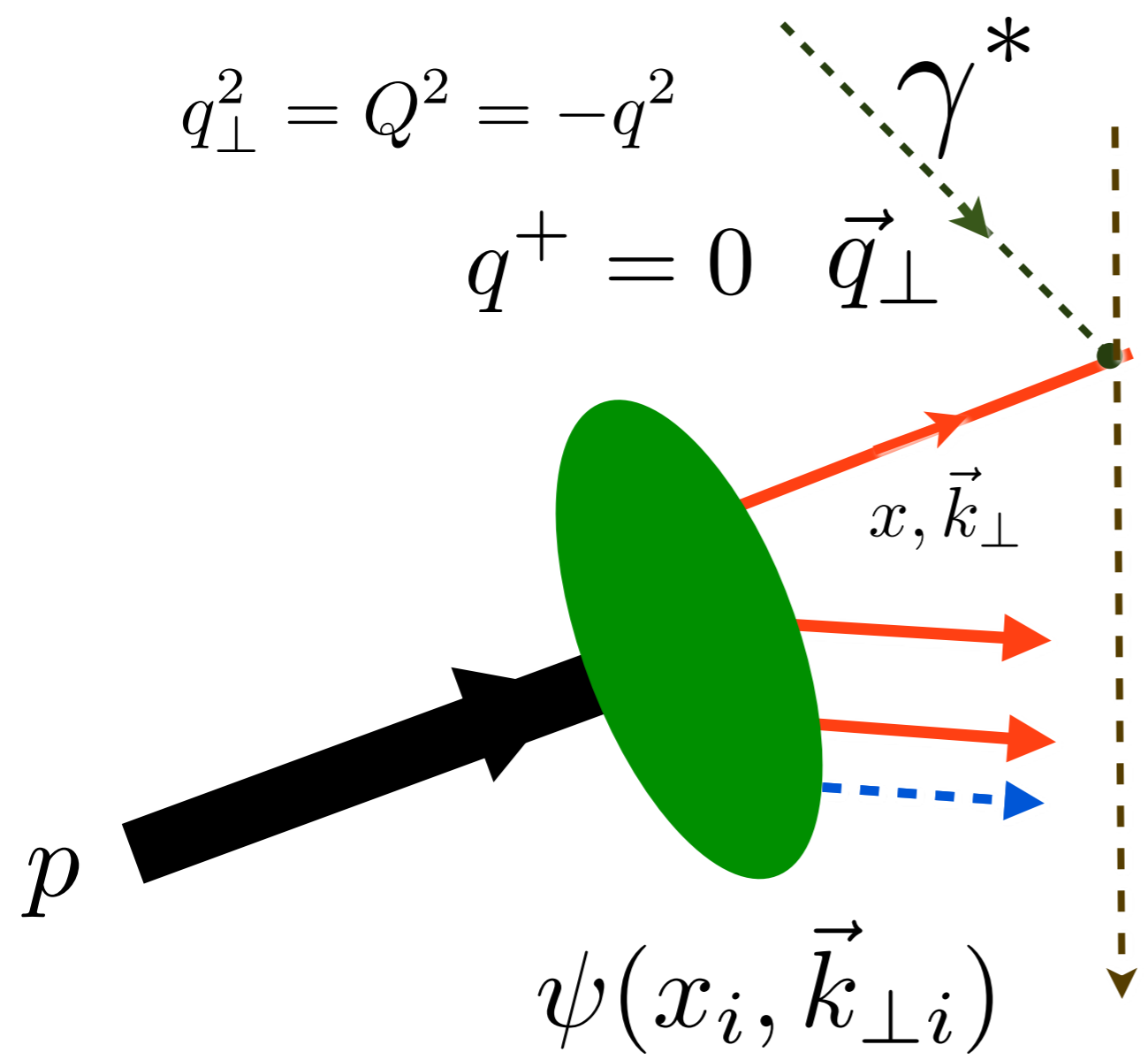
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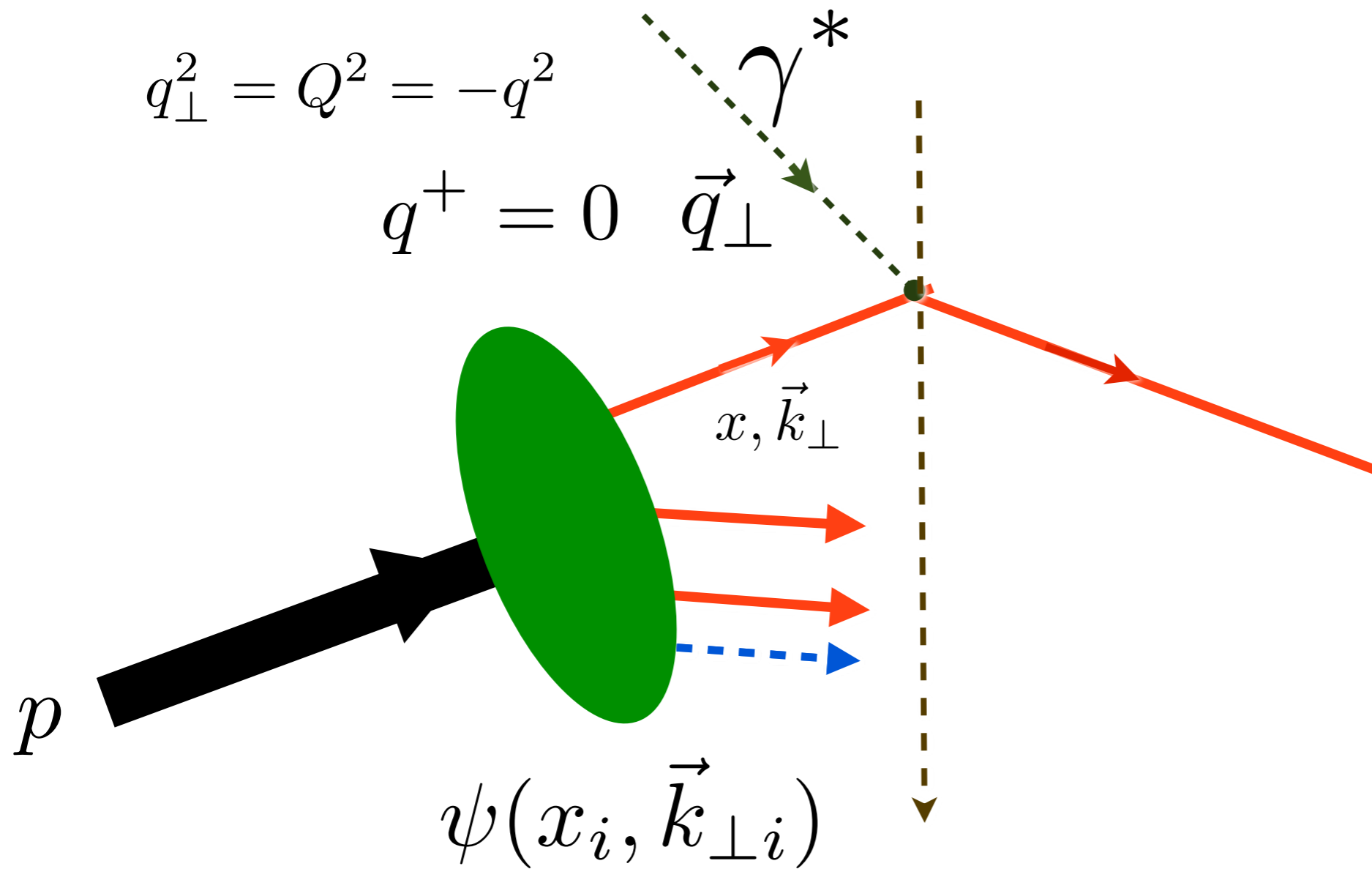
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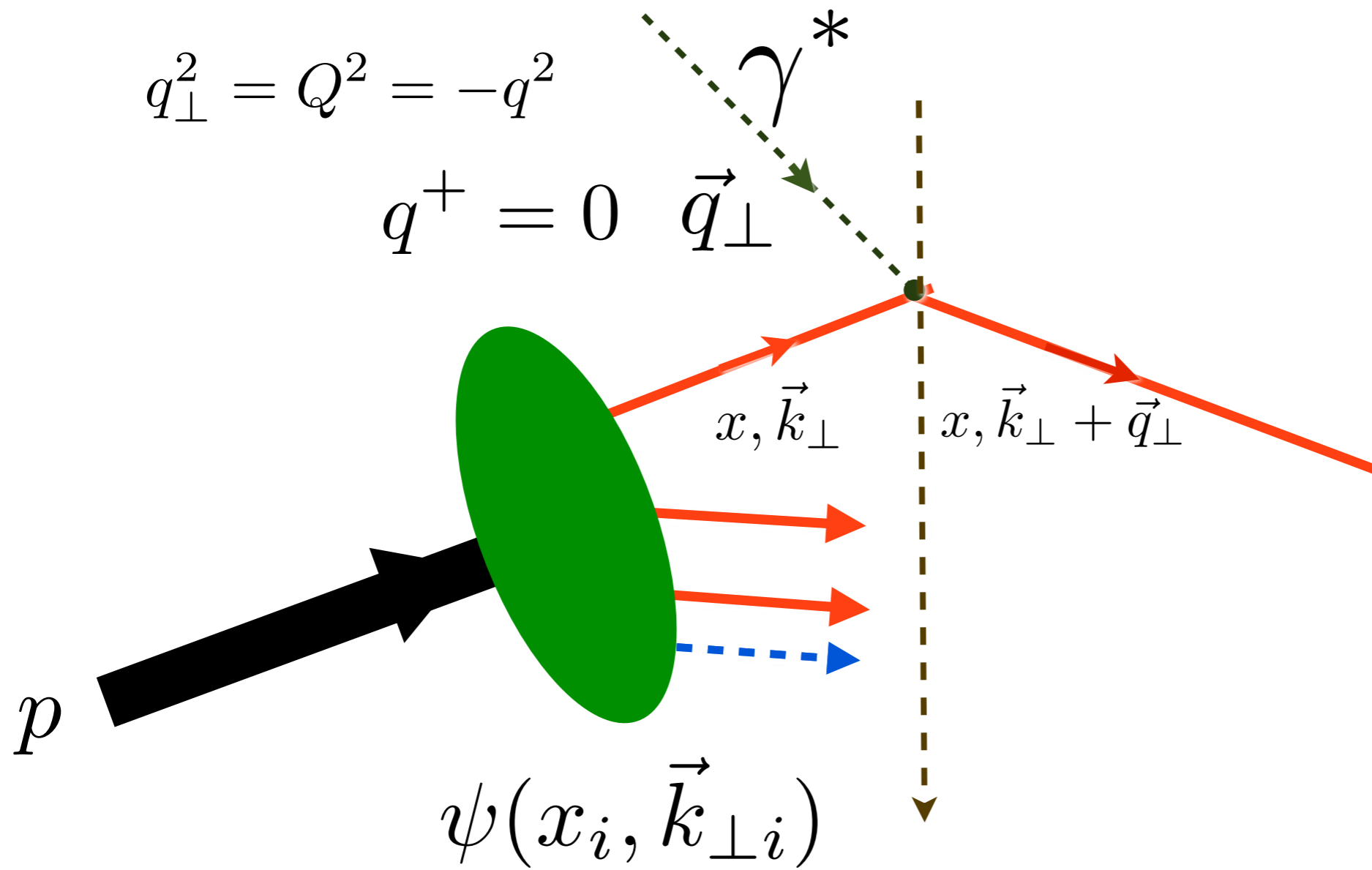
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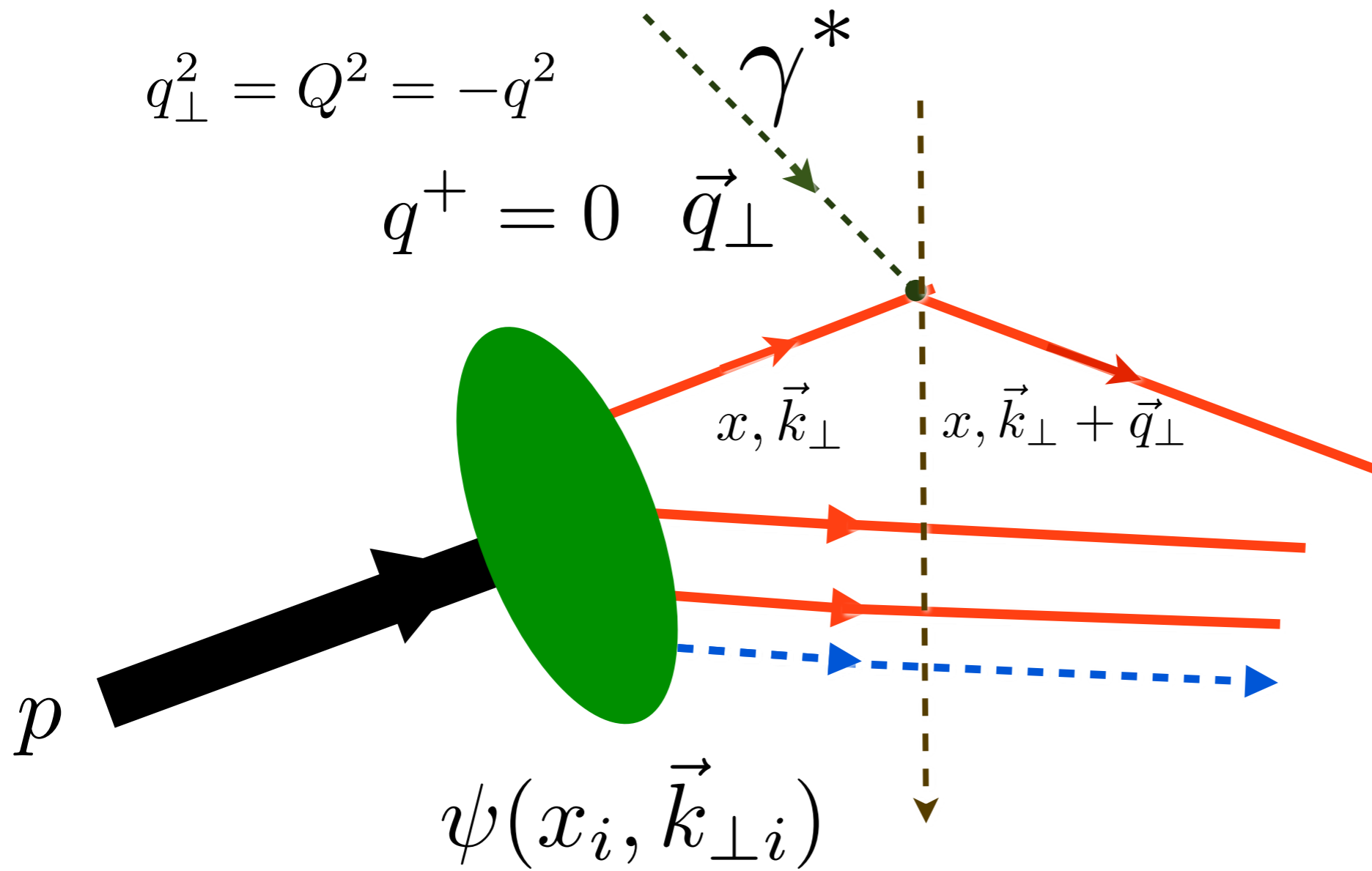
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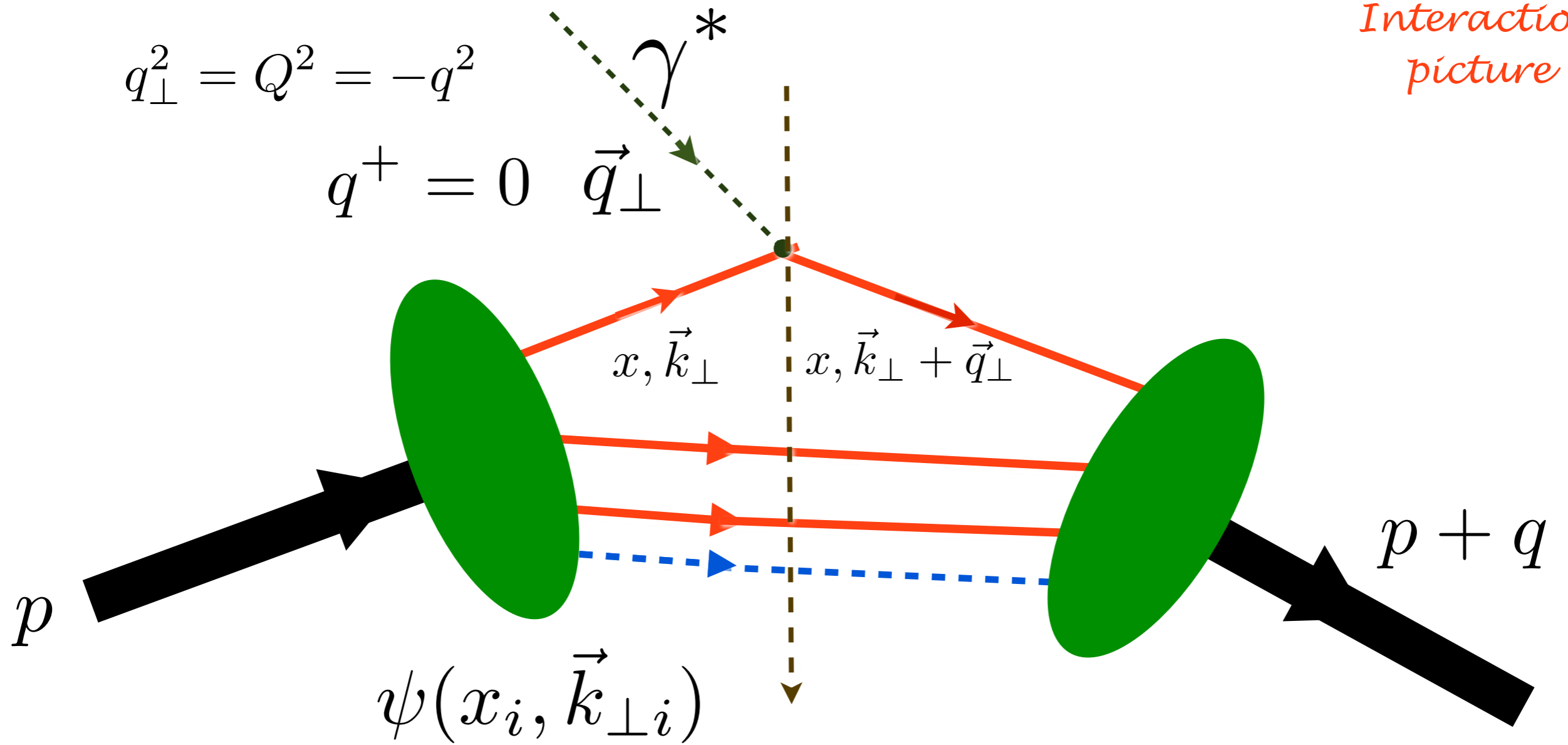
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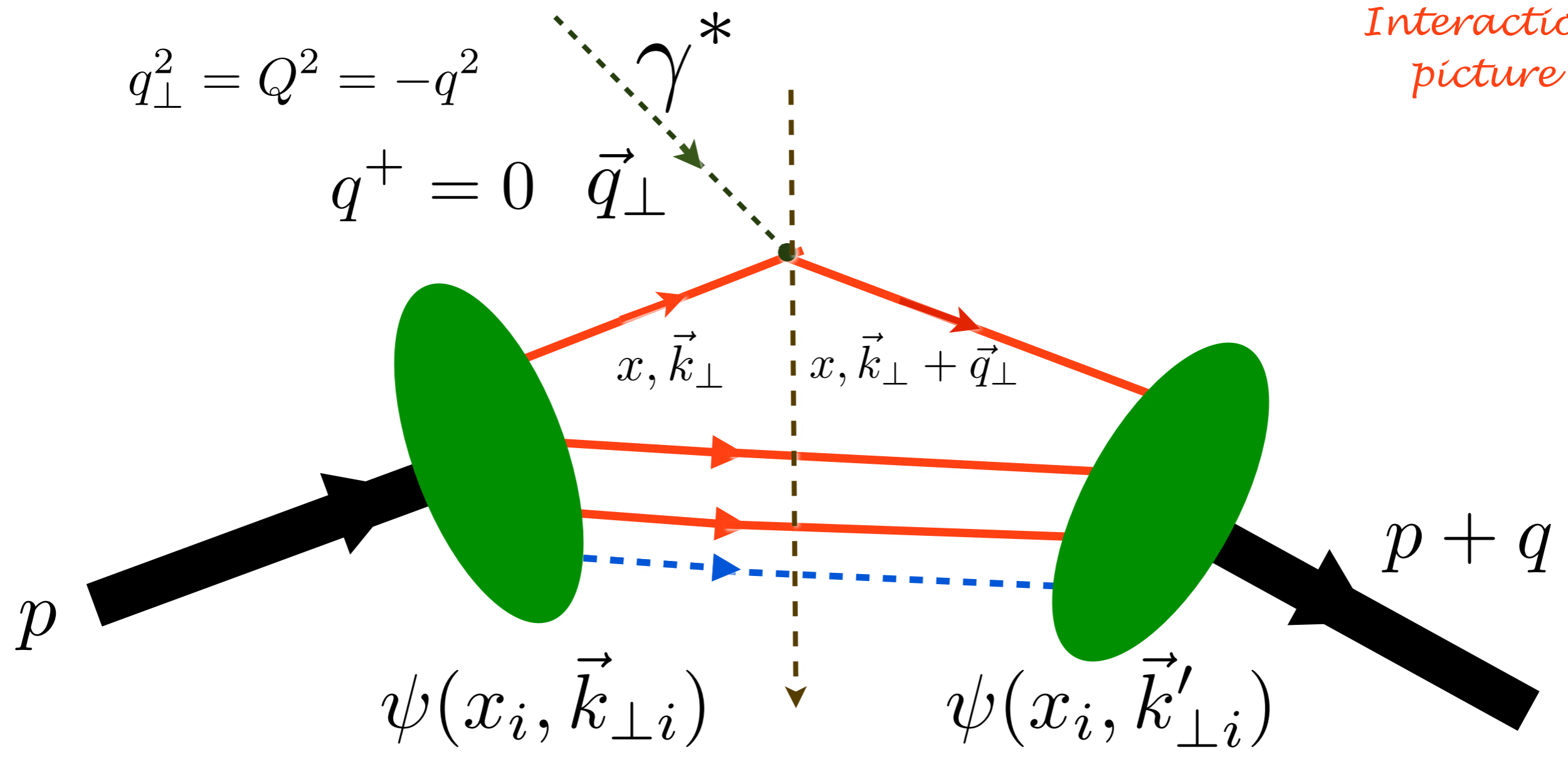
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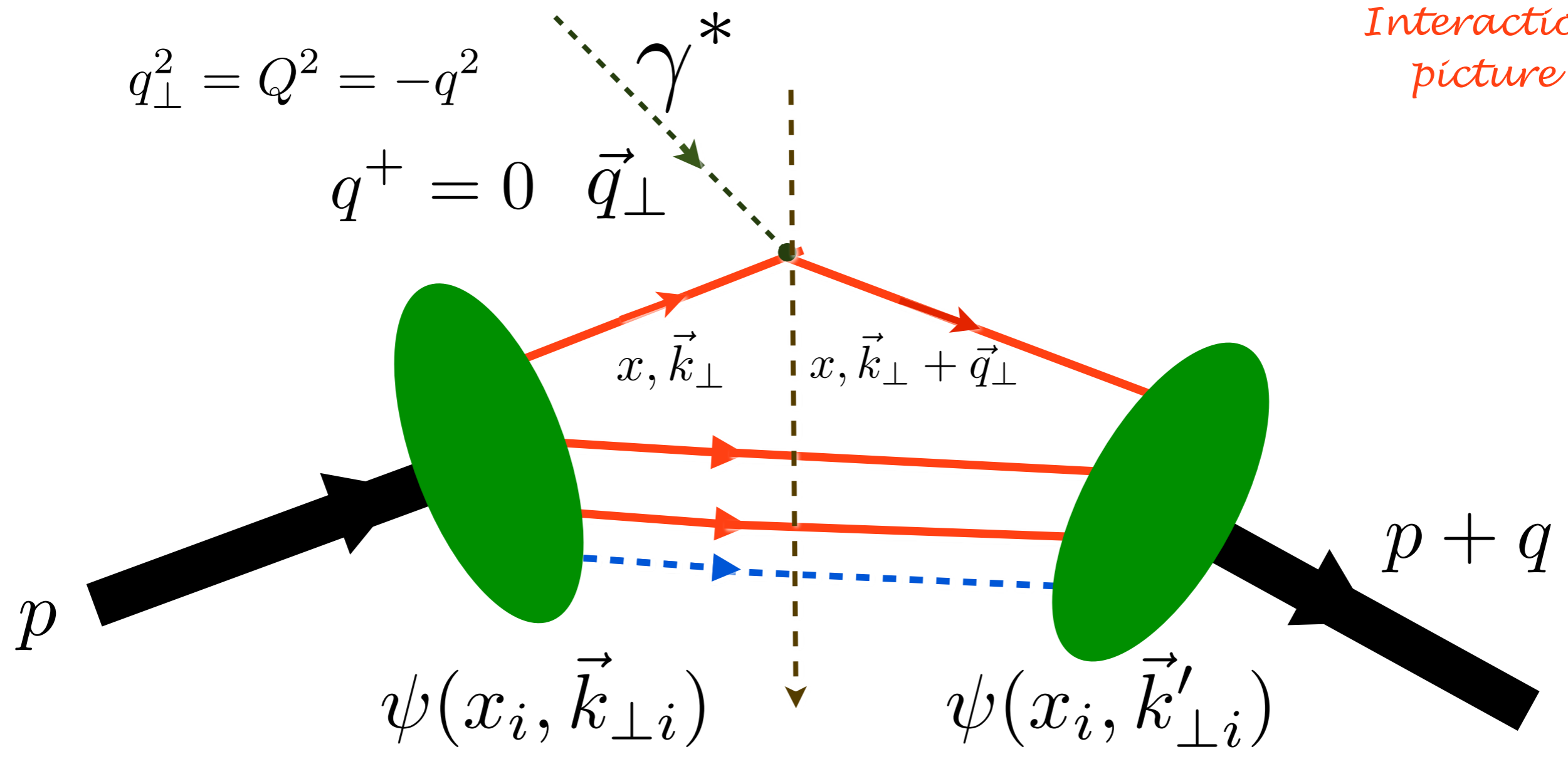
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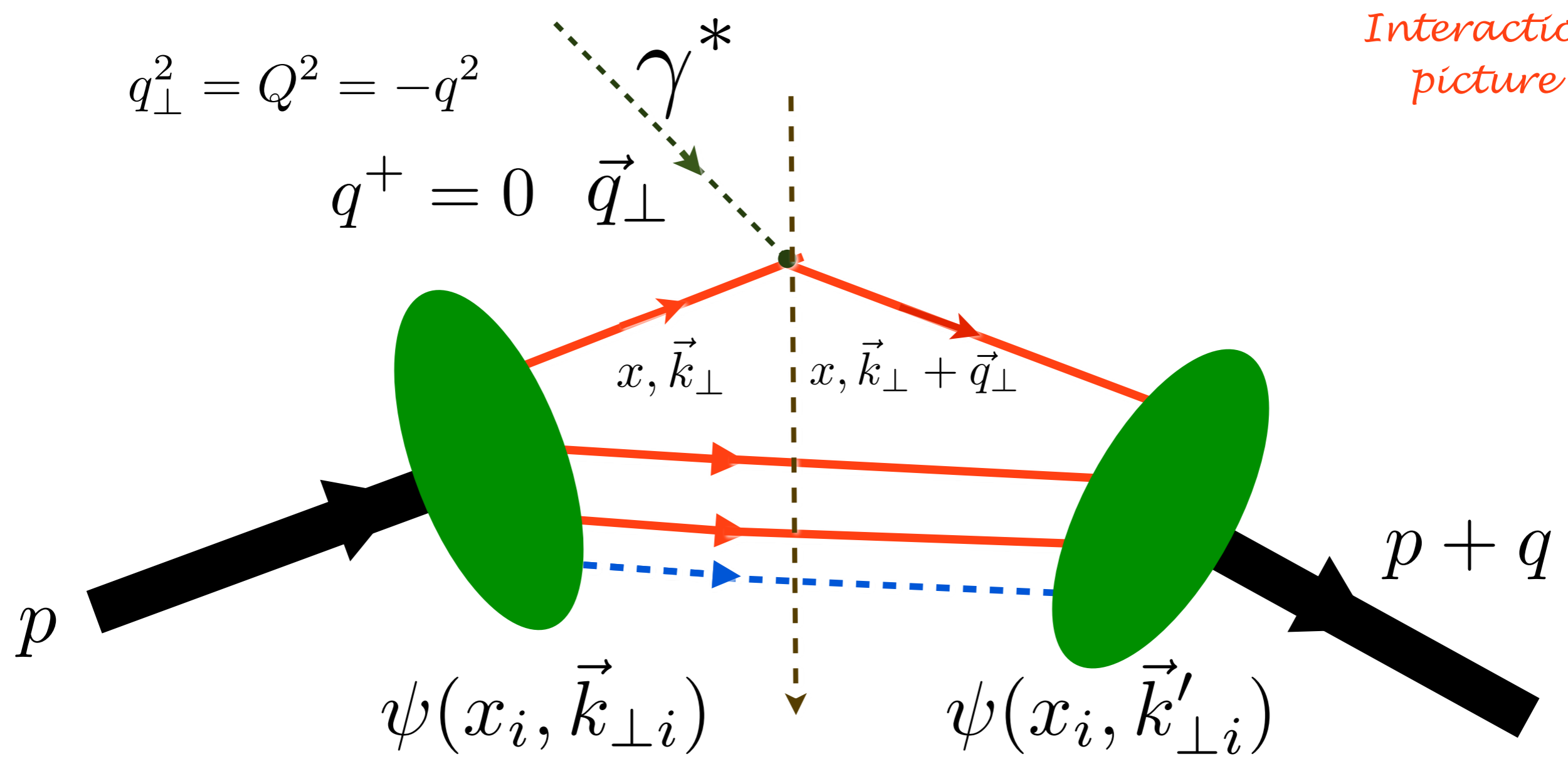
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$$\langle p + q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

Front Form

Interaction picture



$$q_{\perp}^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_{\perp}$$

$$x, \vec{k}_{\perp} \quad x, \vec{k}_{\perp} + \vec{q}_{\perp}$$

$$\psi(x_i, \vec{k}_{\perp i})$$

$$\psi(x_i, \vec{k}'_{\perp i})$$

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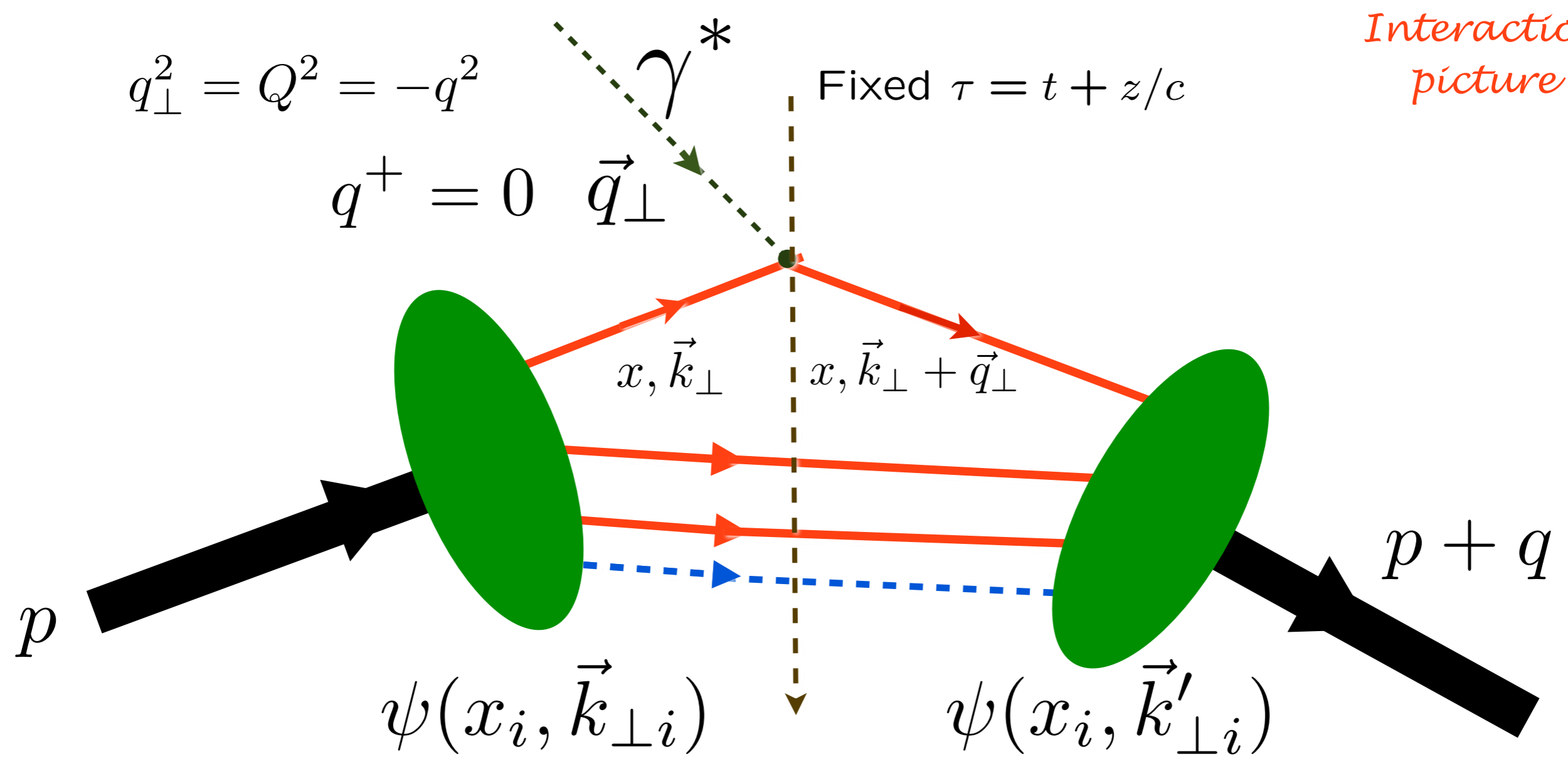
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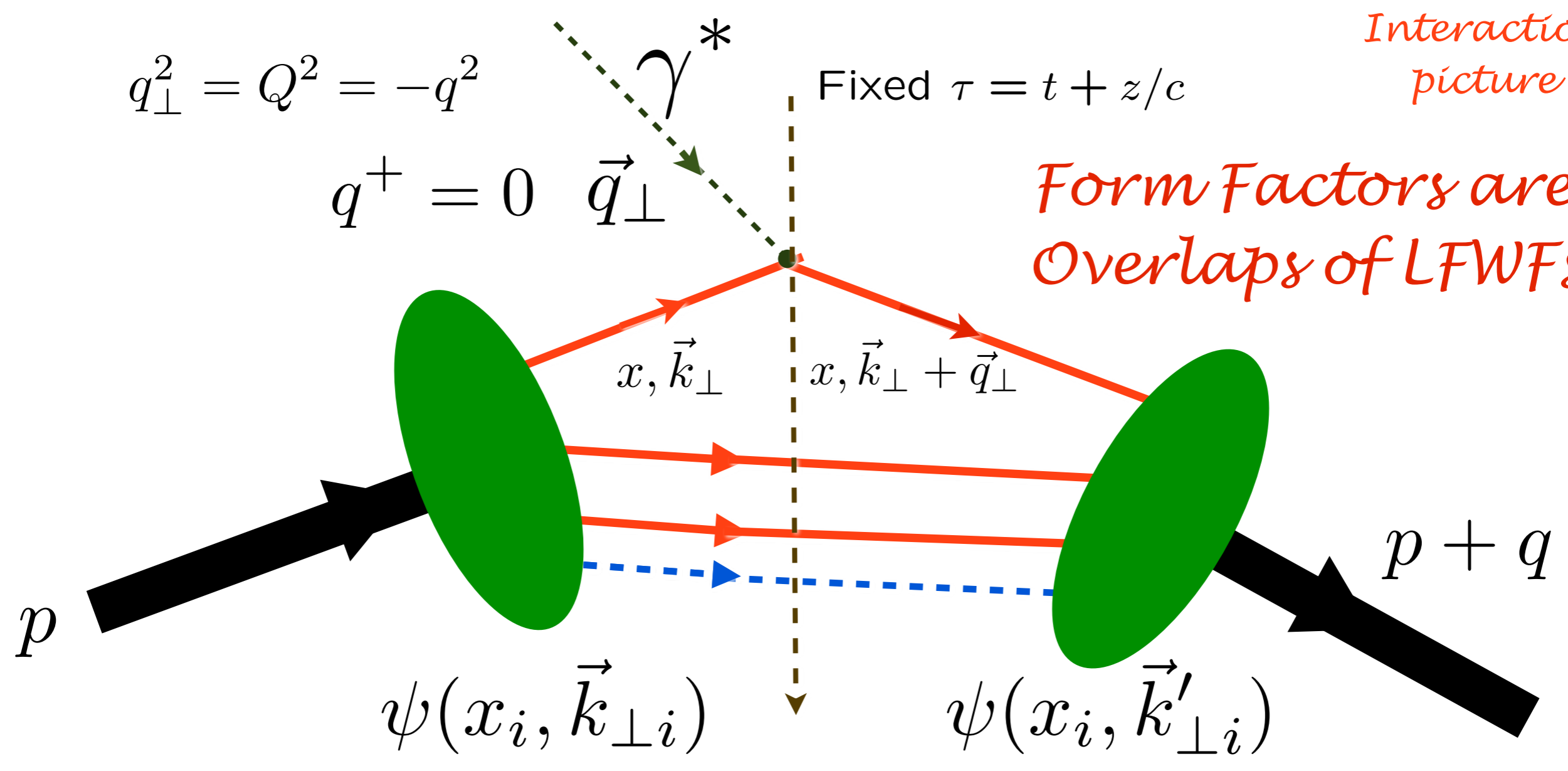
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Form Factors are Overlaps of LFWFs



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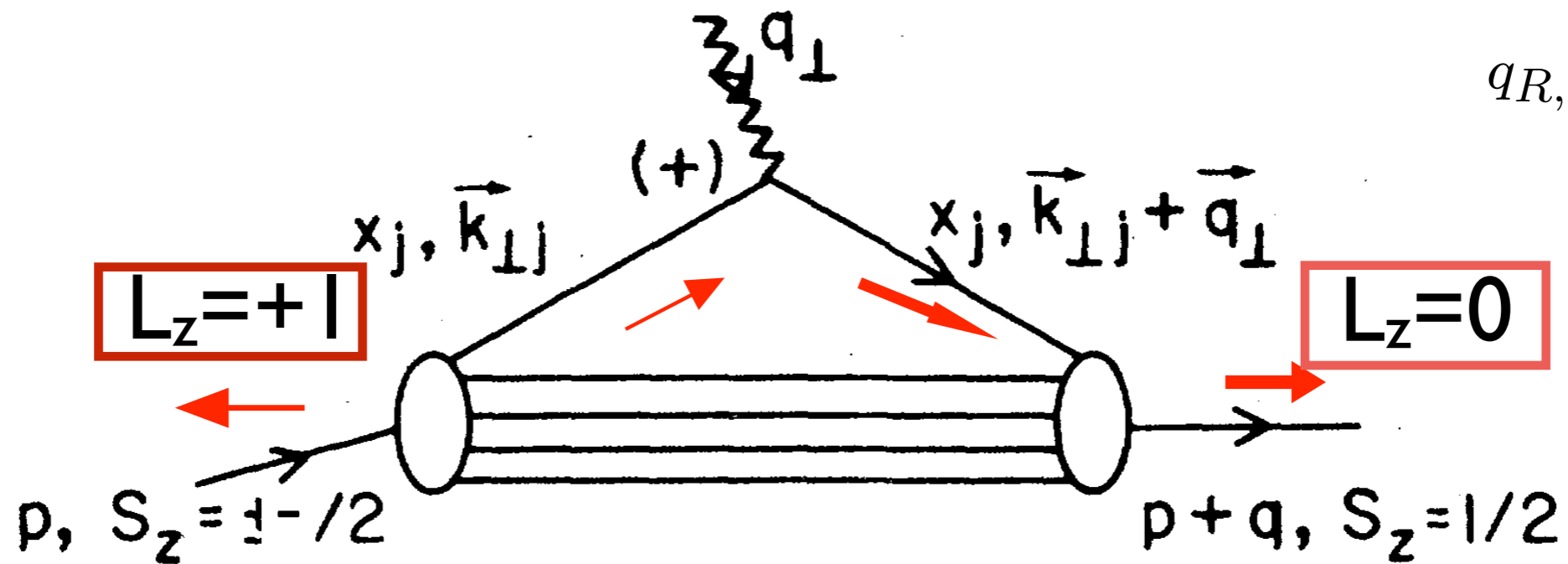
Exact LF Formula for Pauli Form Factor

$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

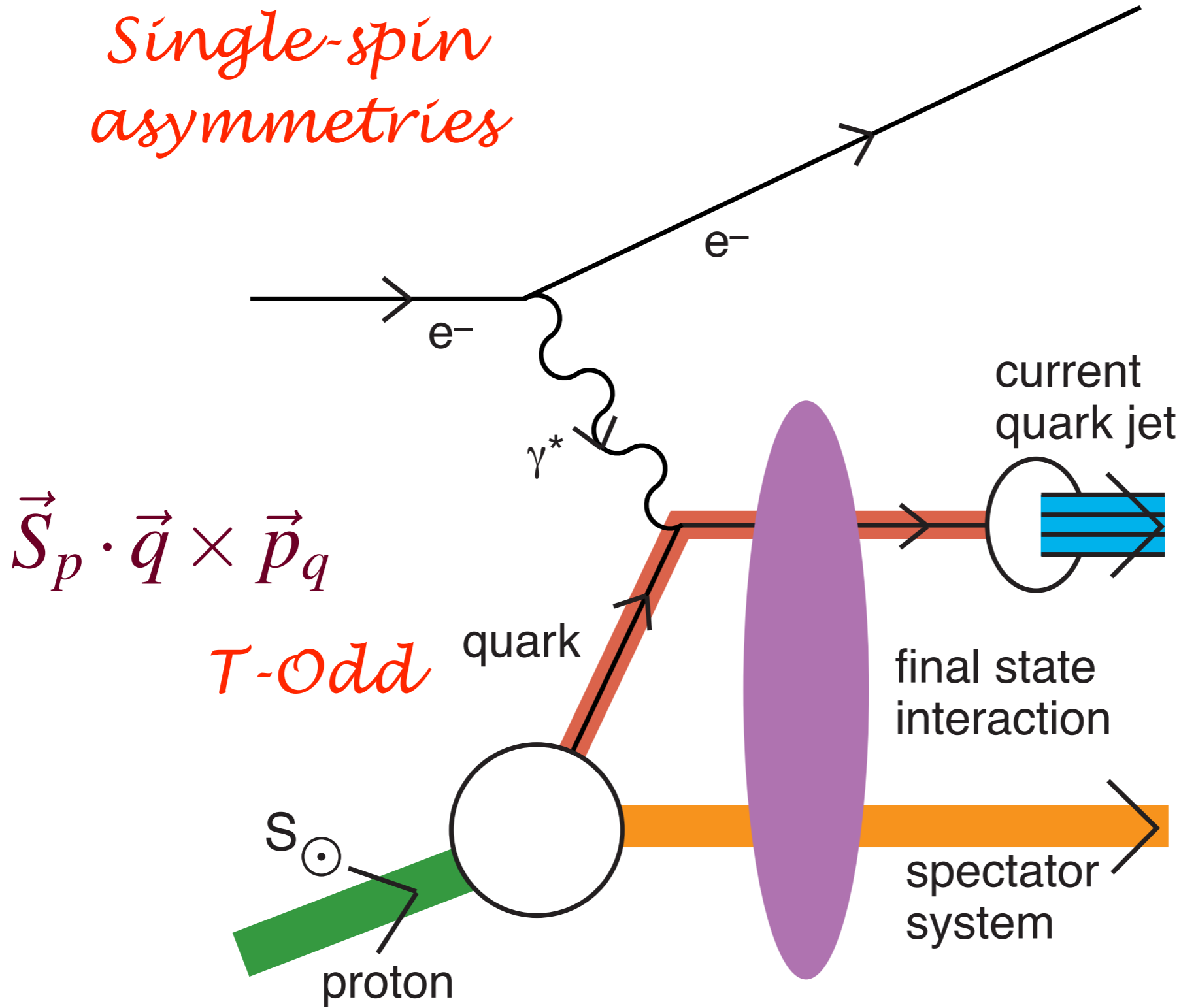


$$q_{R,L} = q^x \pm iq^y$$

Must have $\Delta l_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Single-spin asymmetries



Hwang, Schmidt, sjb

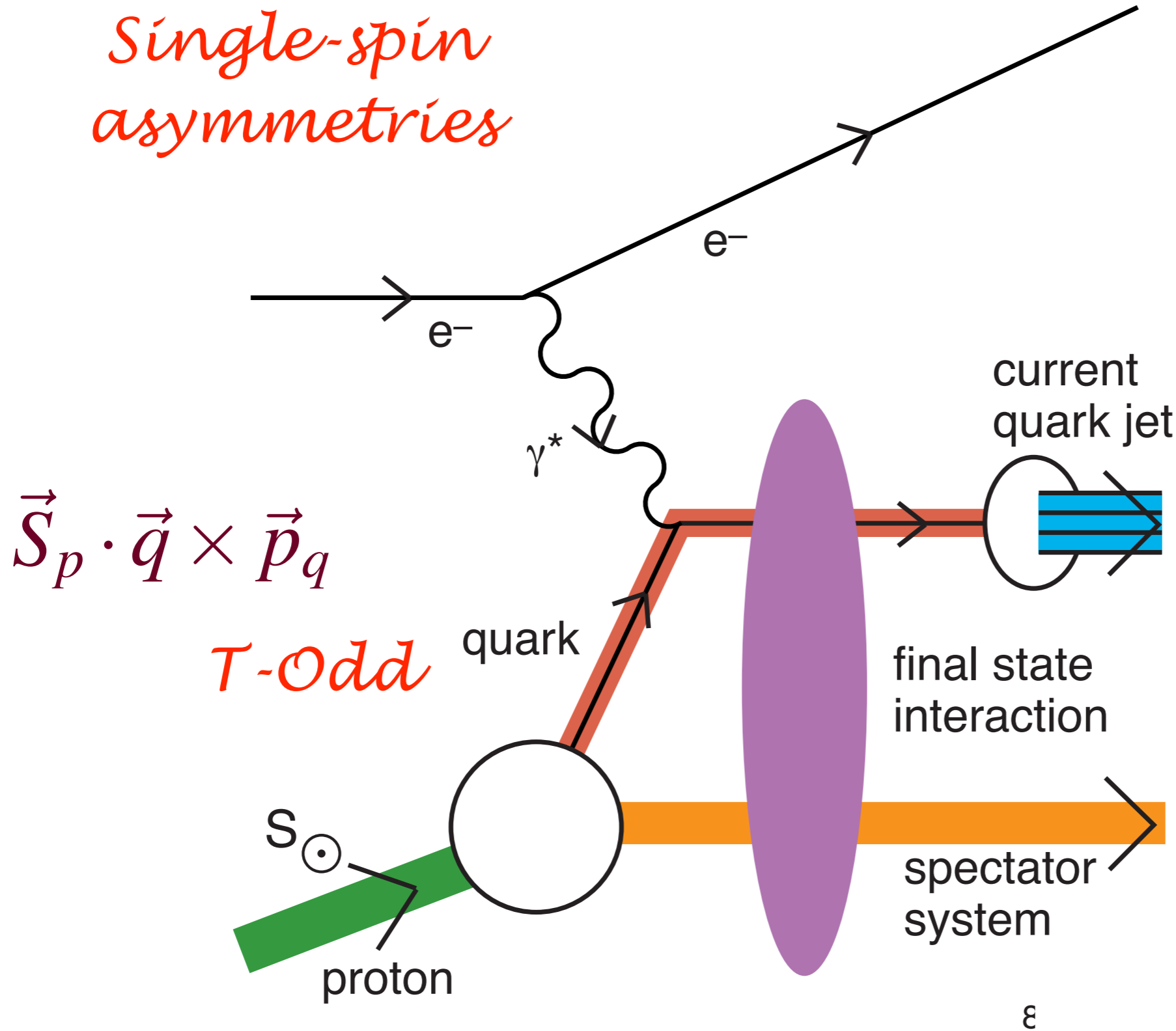
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Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

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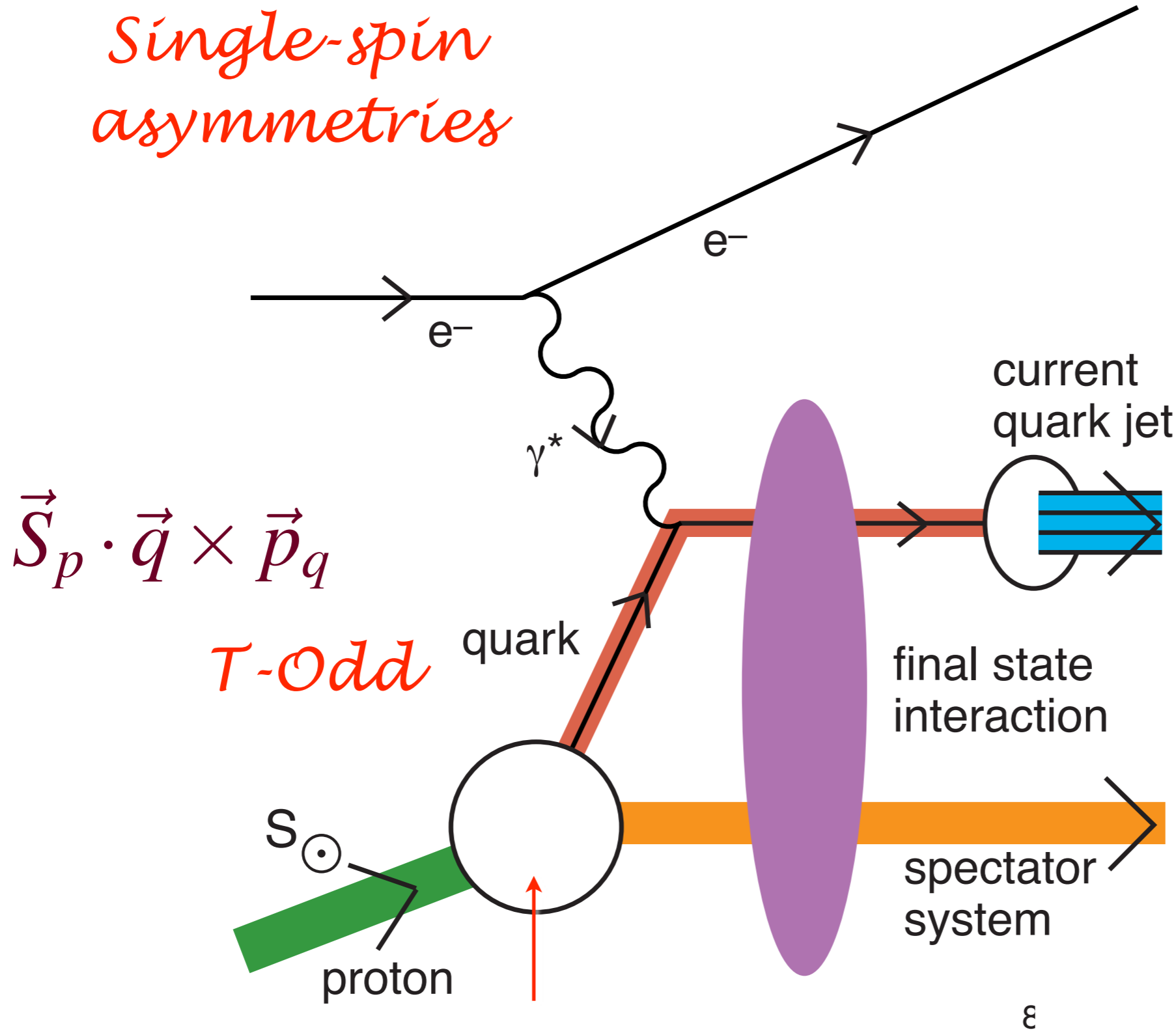


Single-spin asymmetries

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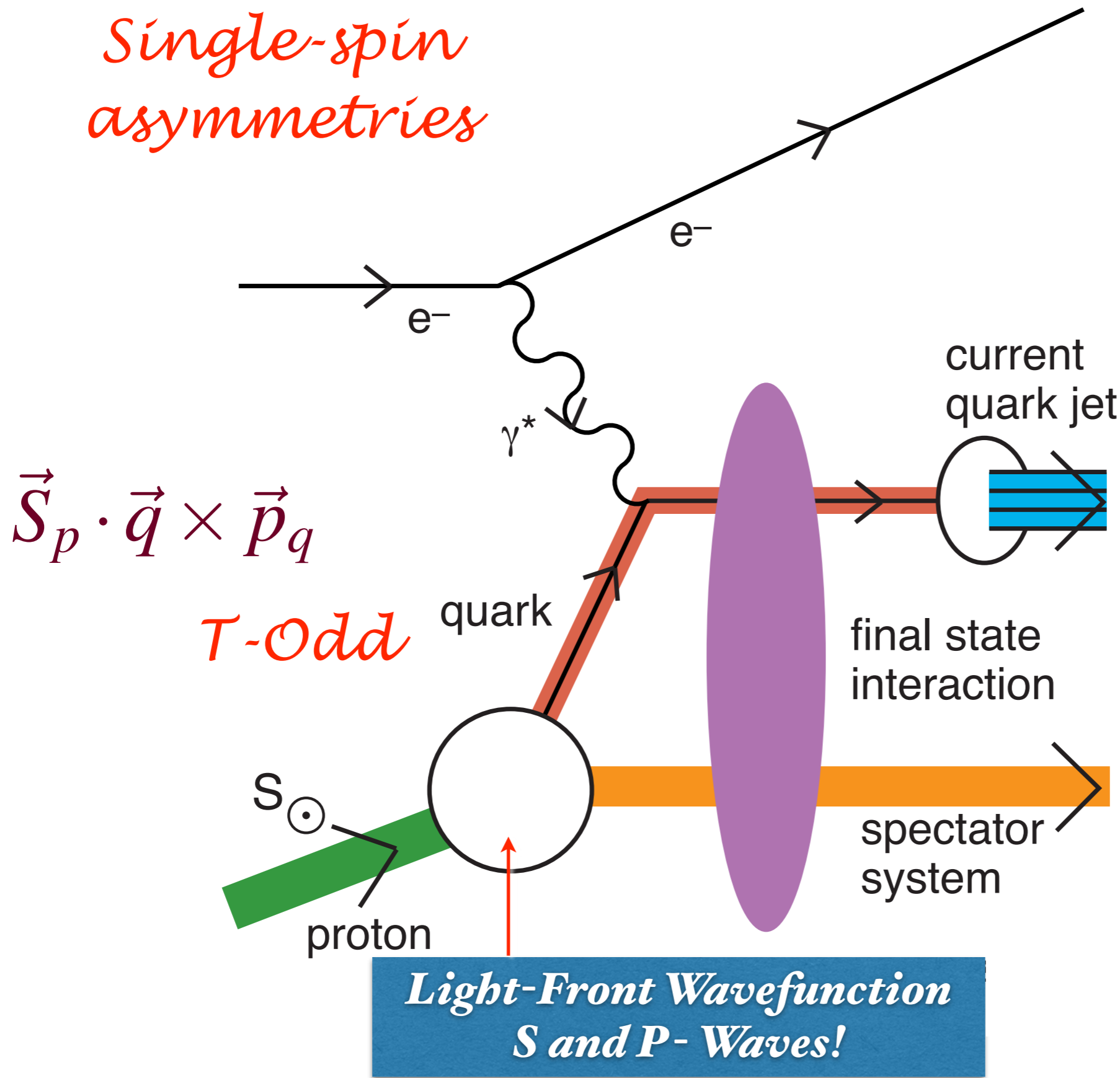


Single-spin asymmetries

**Leading Twist
Sivers Effect**

Hwang, Schmidt,
sjb

Collins, Burkardt, Ji,
Yuan. Pasquini, ...



$$\vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

T-Odd

quark

current
quark jet

final state
interaction

spectator
system

S_{\odot}

proton

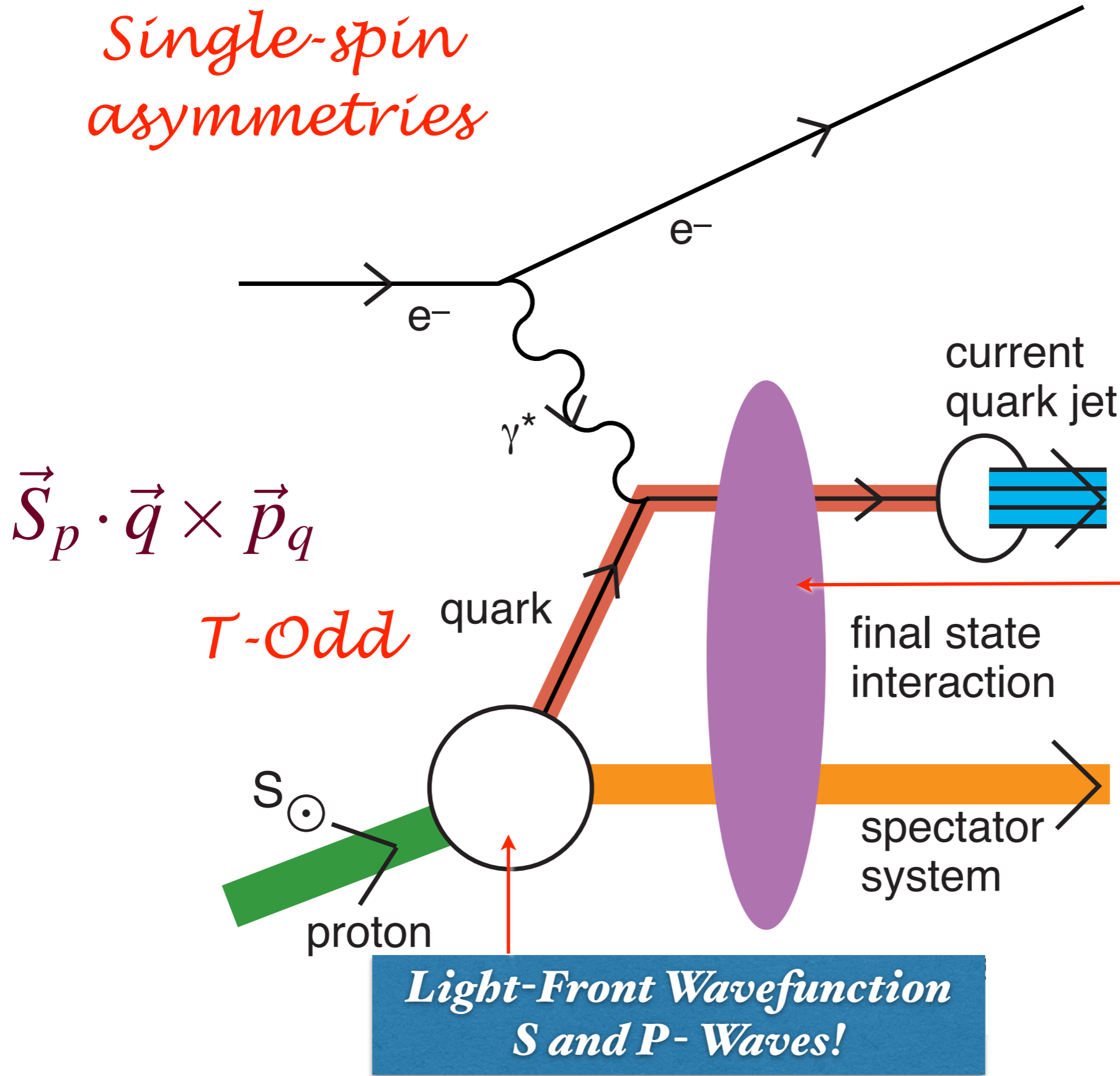
*Light-Front Wavefunction
S and P- Waves!*

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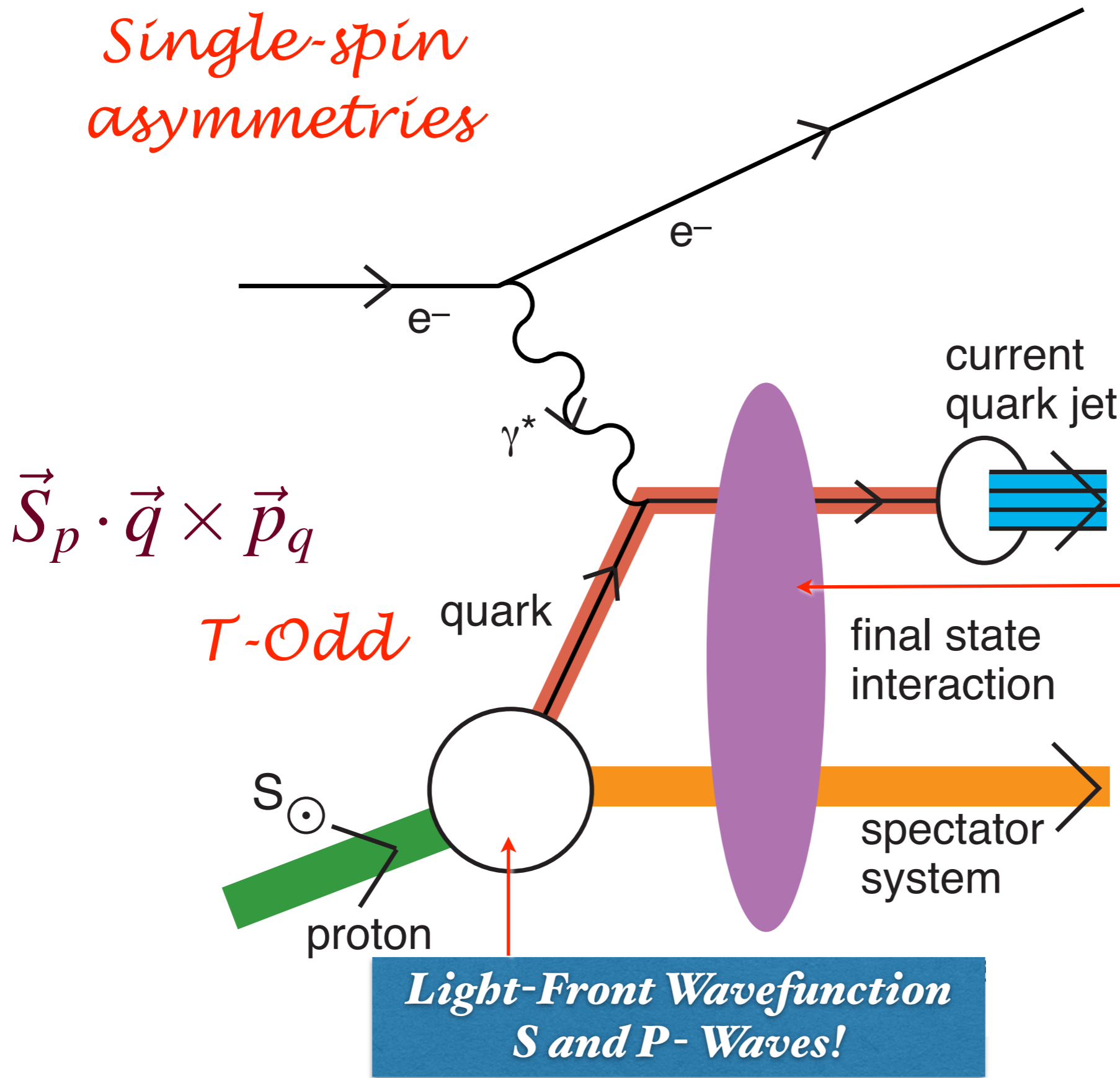
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QCD S- and P-Coulomb Phases --Wilson Line

“Lensing Effect”



Single-spin asymmetries

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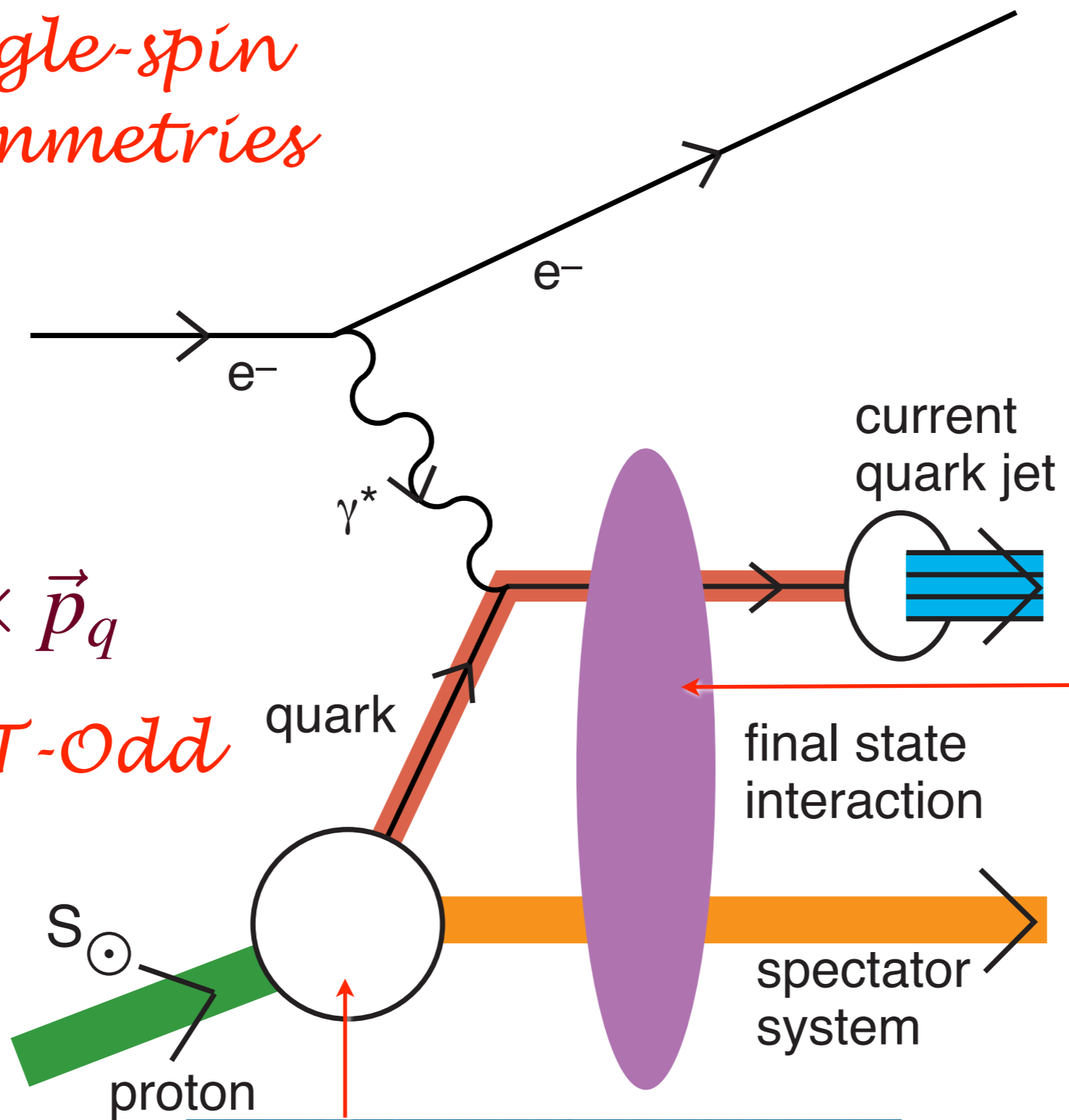
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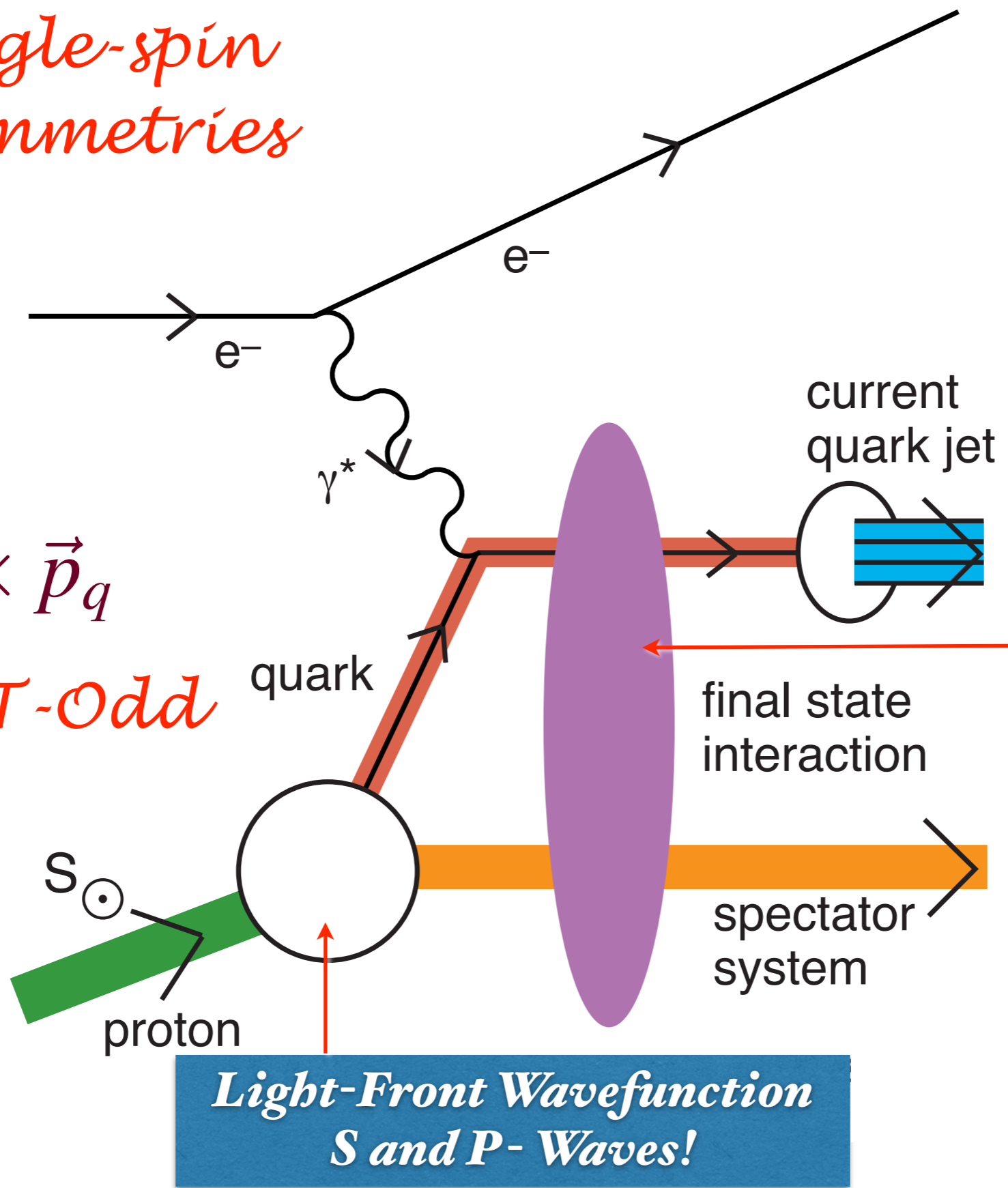
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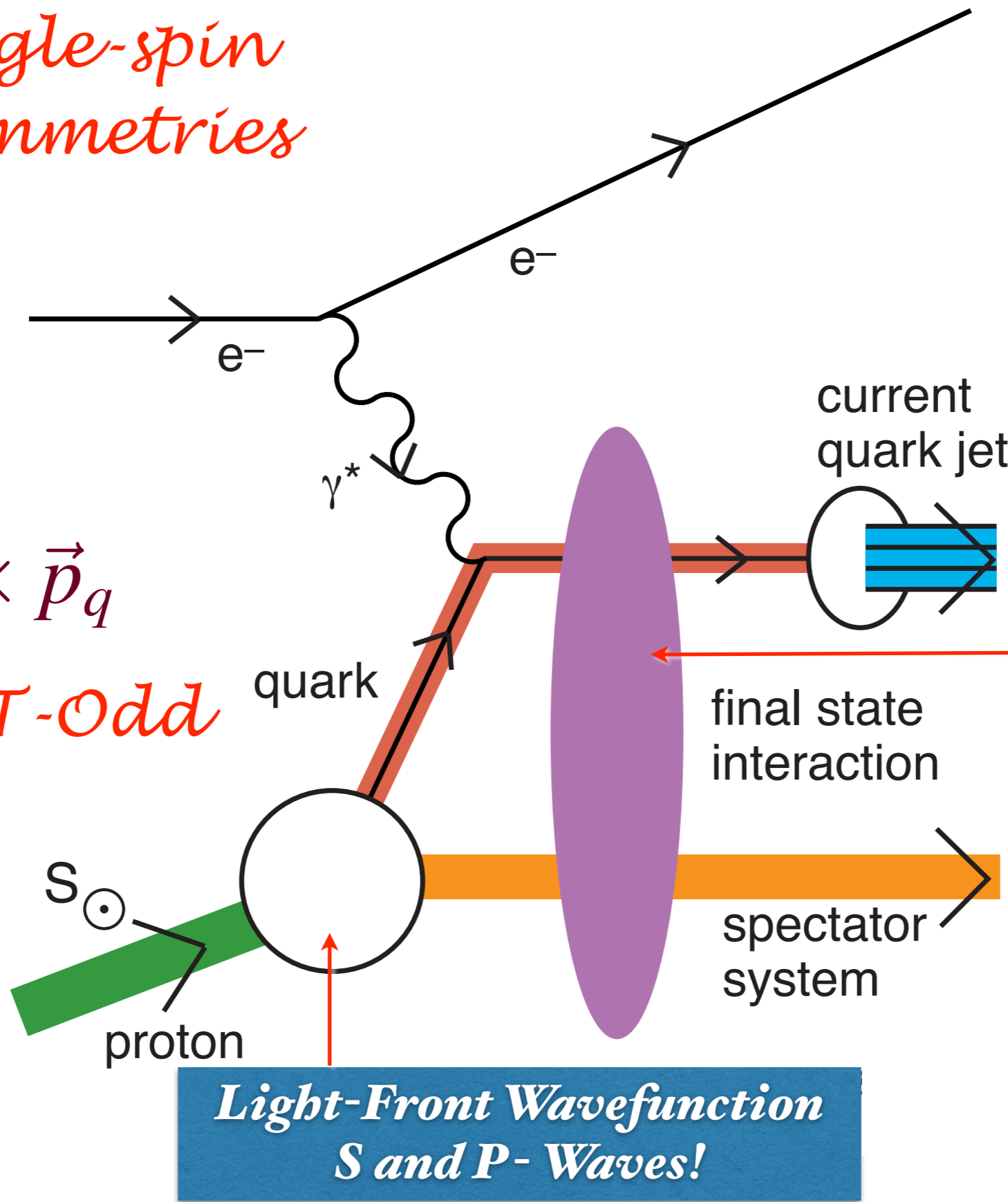
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Leading-Twist Rescattering Violates pQCD Factorization!



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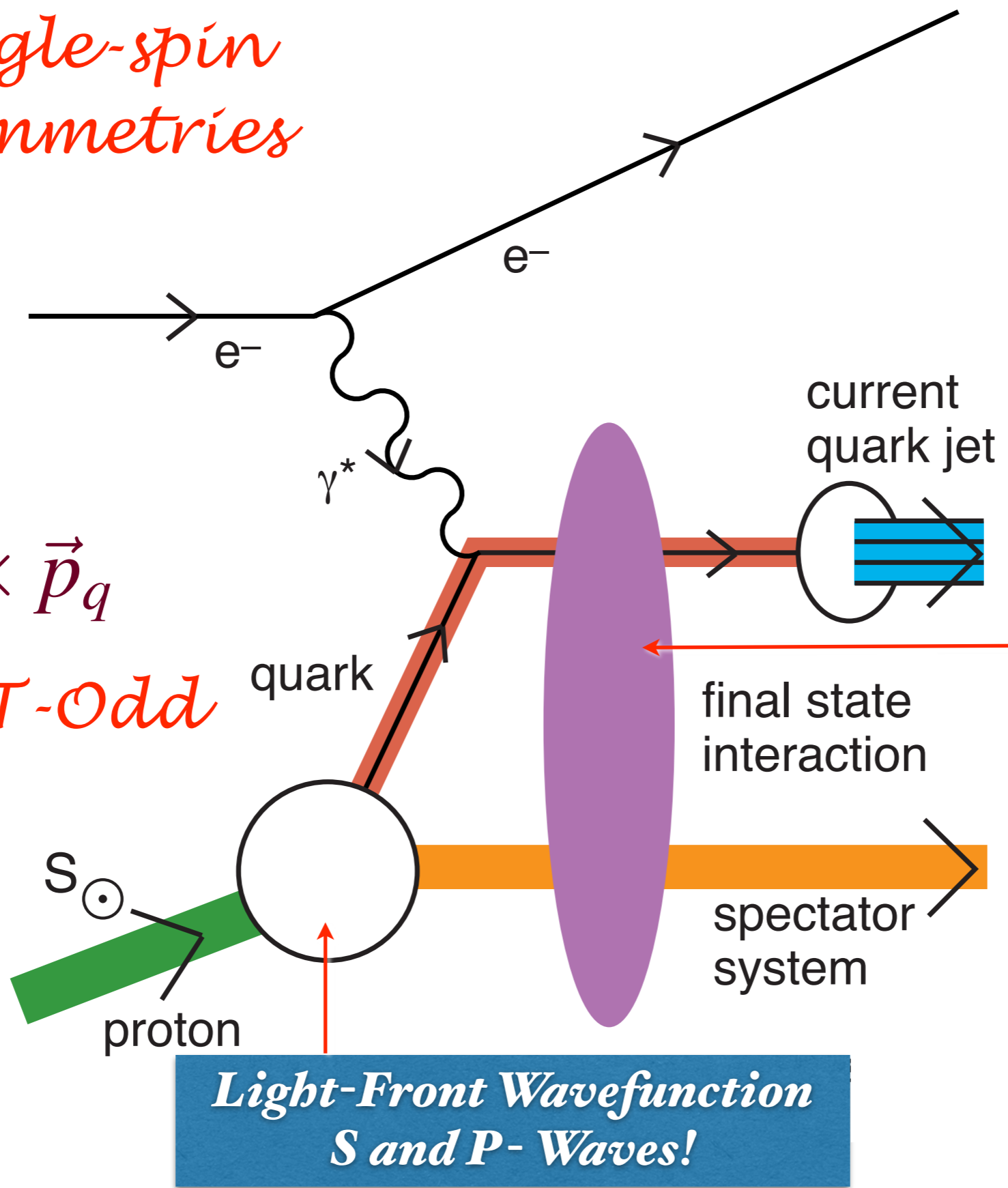
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Sign reversal in DY!

Single-spin asymmetries

Leading Twist Sivers Effect

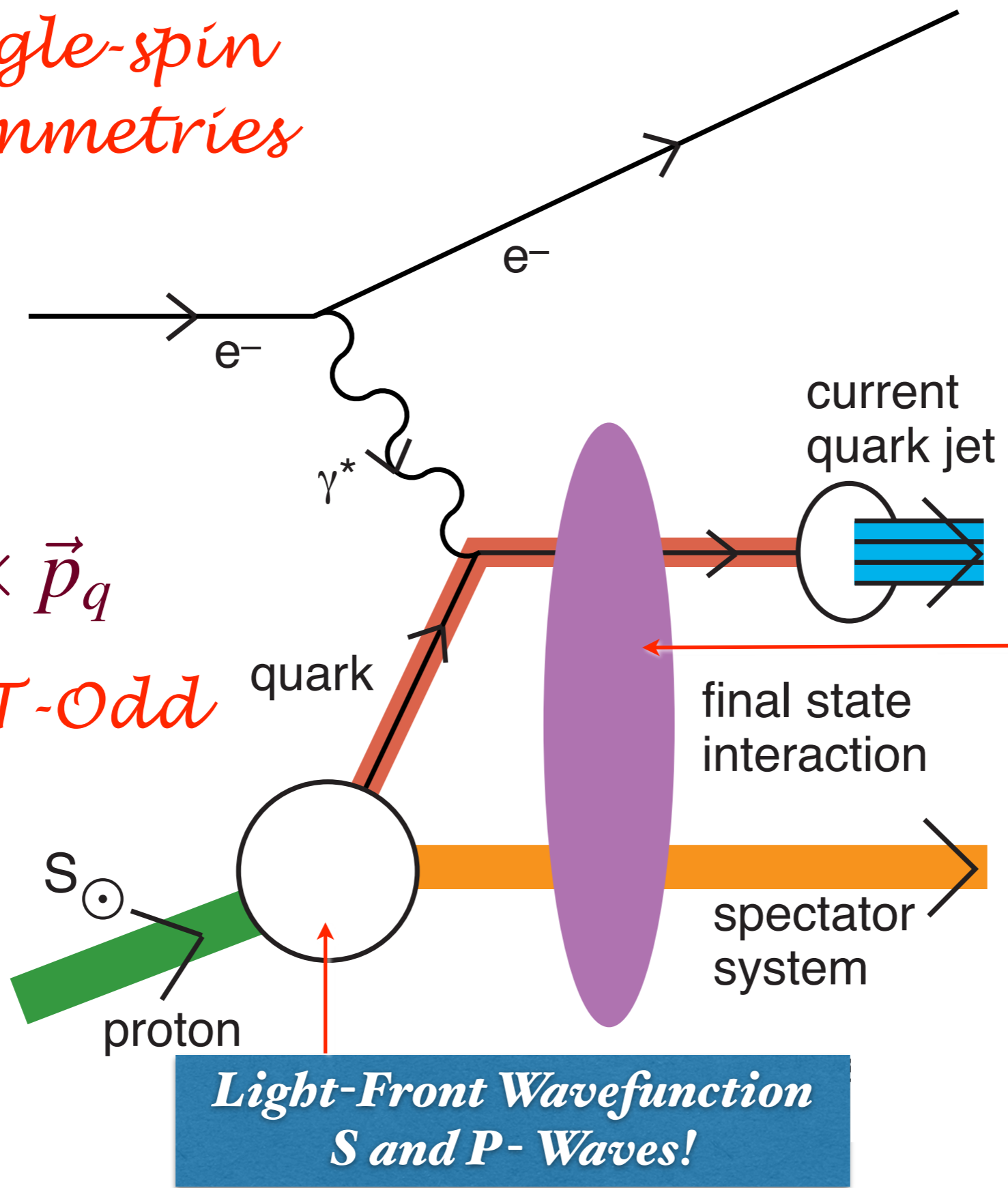
Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

QCD S- and P-Coulomb Phases --Wilson Line

“Lensing Effect”

Leading-Twist Rescattering Violates pQCD Factorization!



$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo-T-Odd

“Lensing” involves soft scales

Light-Front Wavefunction S and P-Waves!

Sign reversal in DY!

Advantages of the Dirac's Front Form for Hadron Physics

Poincare' Invariant



- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent: no boosts, no pancakes!**
- **Same structure function measured at an e p collider and the proton rest frame**
- **No dependence of hadron structure on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no vacuum condensates!**
- **Profound implications for Cosmological Constant**

Terrell, Penrose

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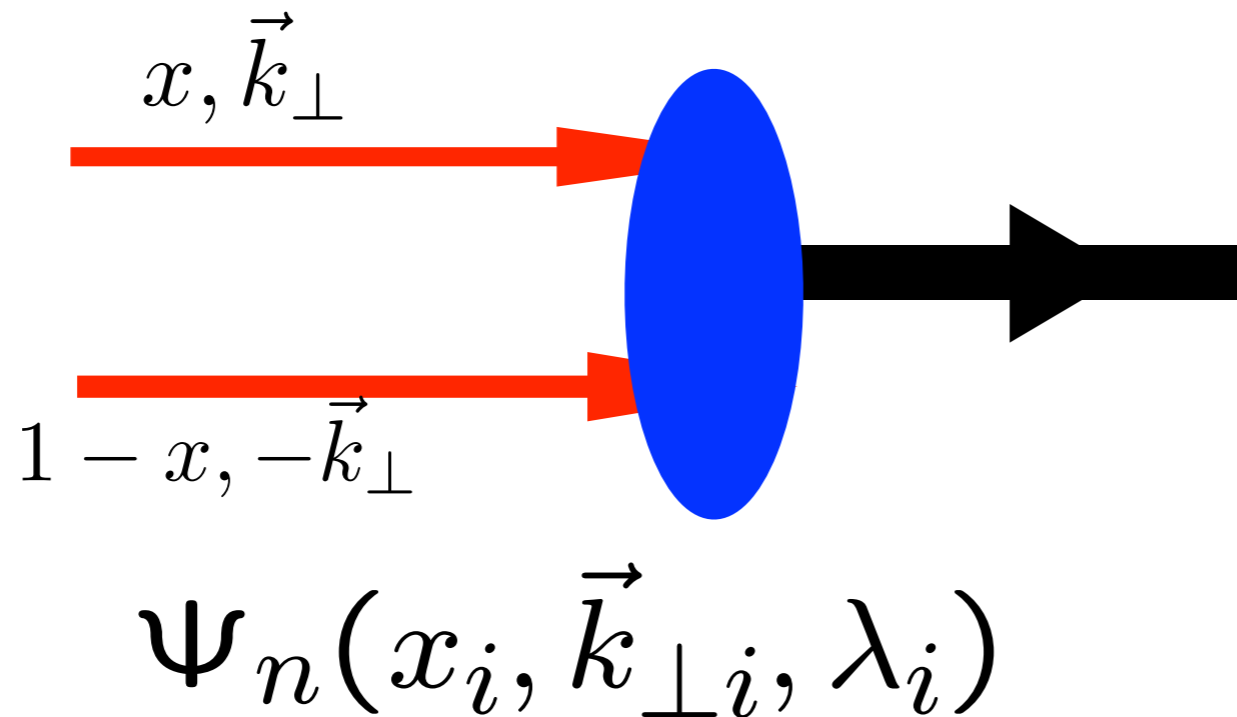


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off-shell in P^- and invariant mass $\mathcal{M}_{q\bar{q}}^2$

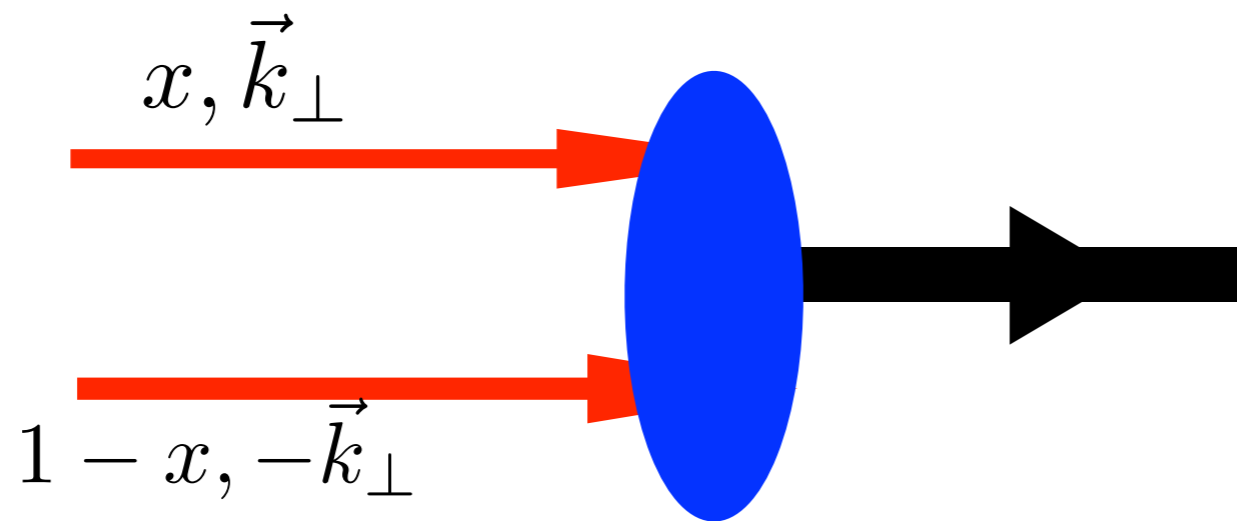


“Hadronization at the Amplitude Level”

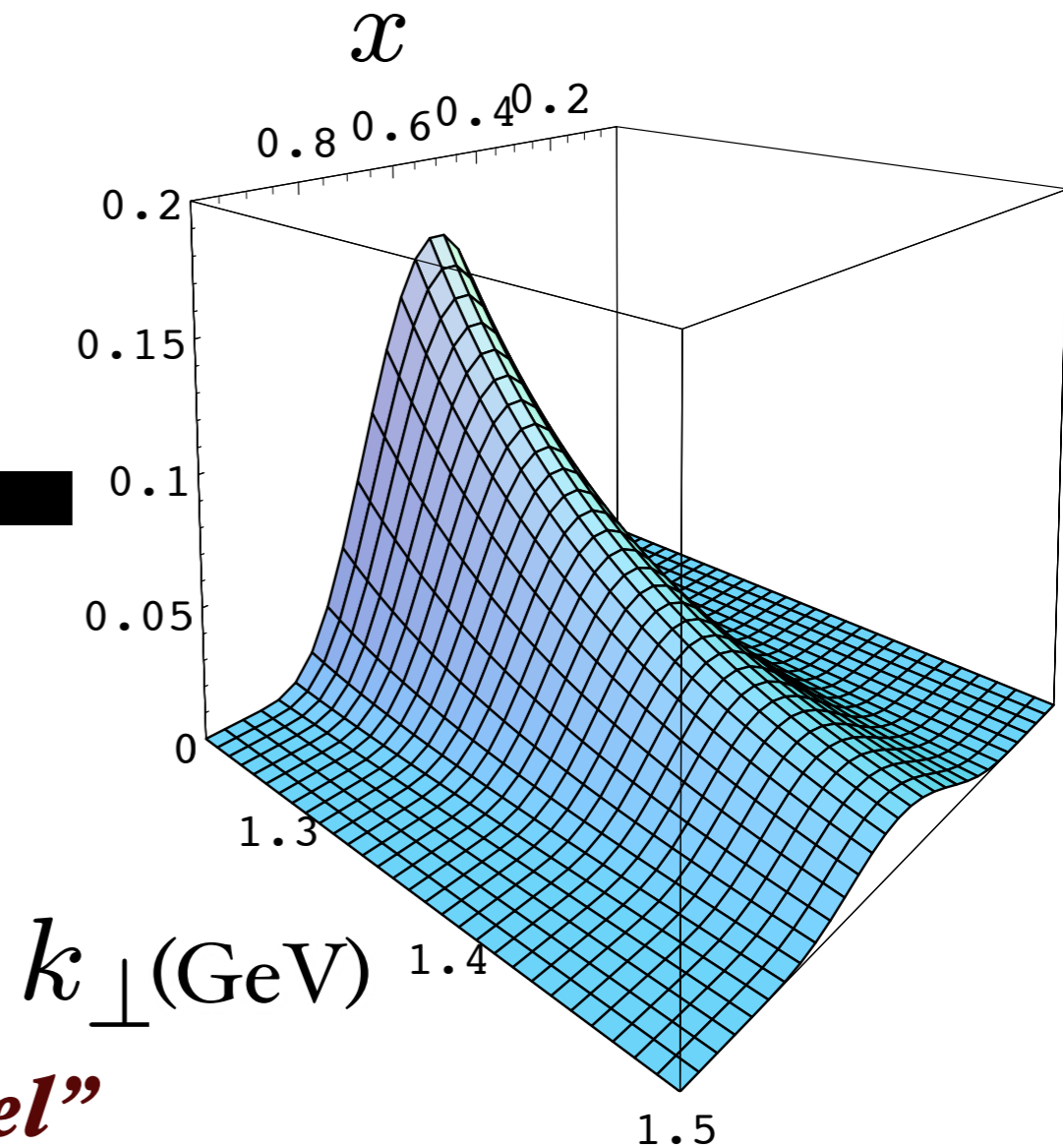
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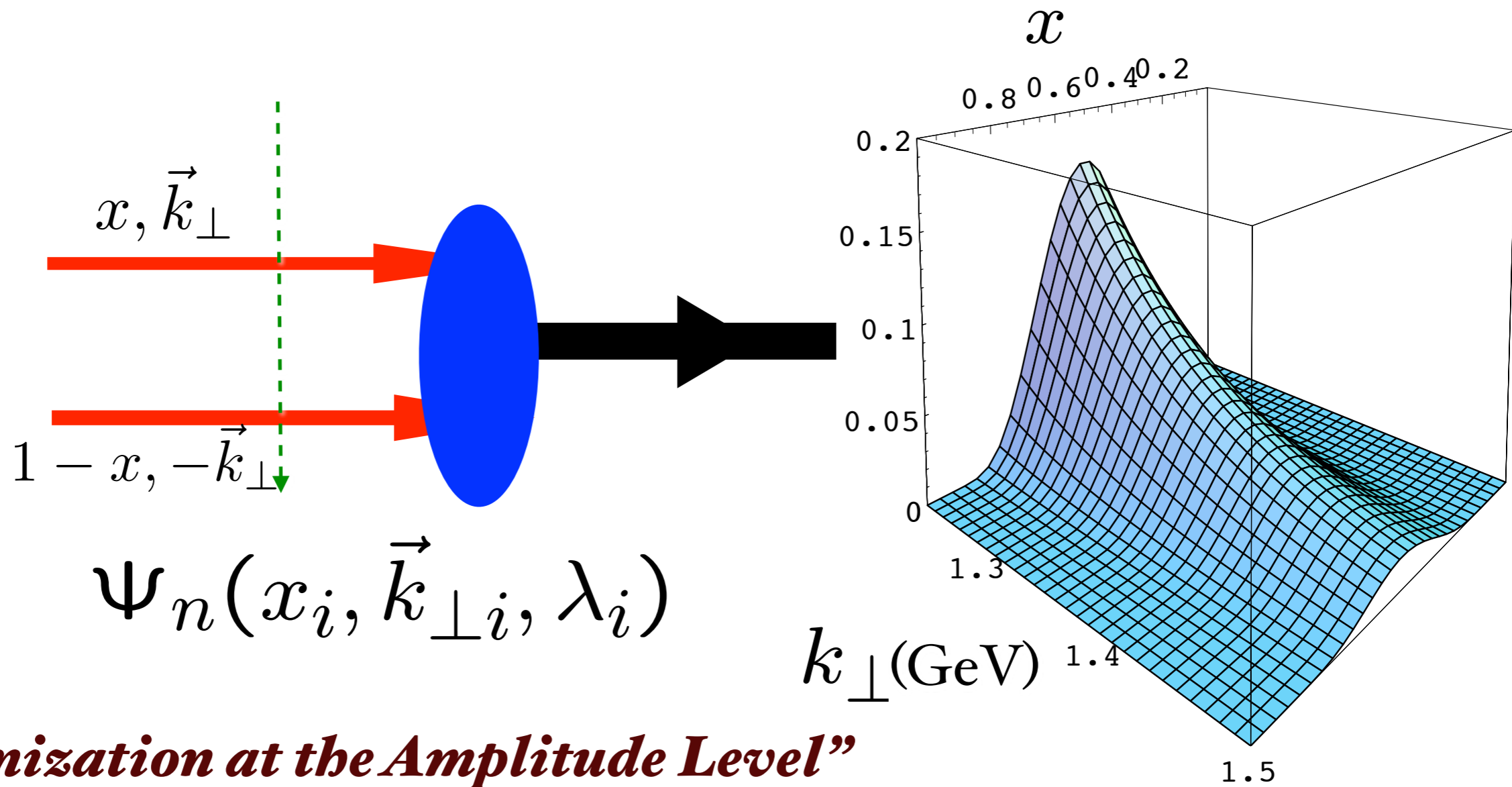


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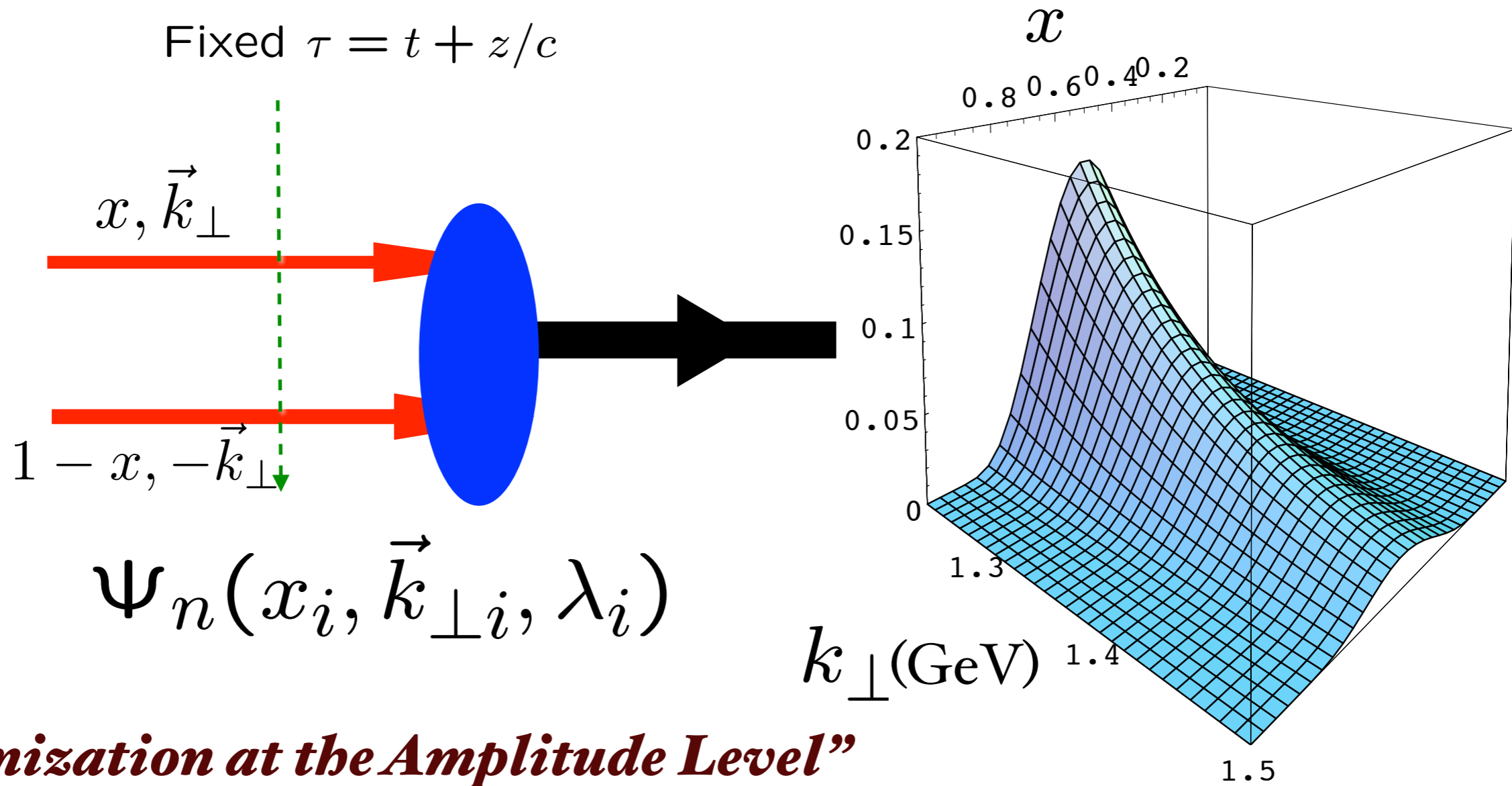


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Need a First Approximation to QCD

*Comparable in simplicity to
Schrödinger Theory in Atomic Physics*

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

*AdS/QCD
Light-Front Holography
Superconformal Algebra*

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

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Unique confinement potential!

$$H_{QED}$$

*QED atoms: positronium
and muonium*

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

$$\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell+1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

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Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

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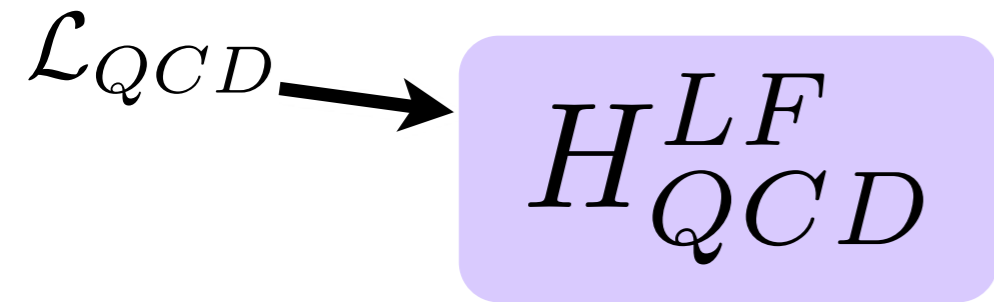


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Bohr Spectrum

Schrödinger Eq.

Light-Front QCD



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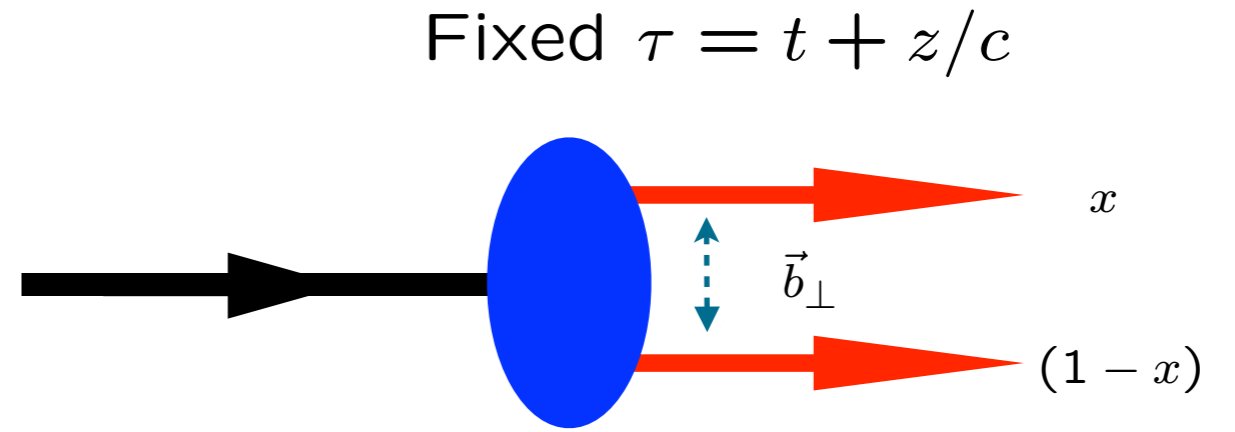
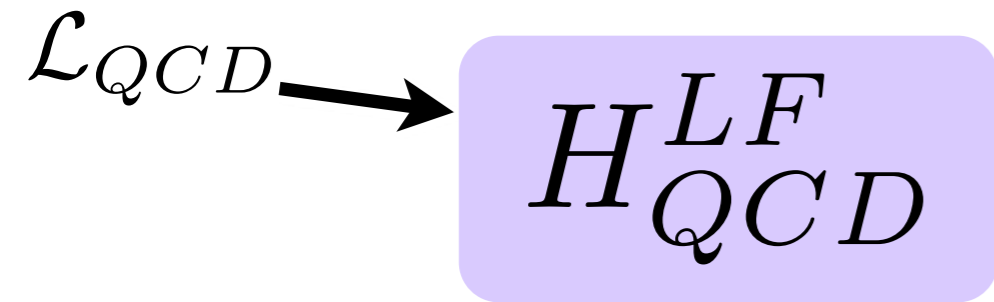
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Azimuthal Basis

$$\zeta, \phi$$

$$m_q = 0$$

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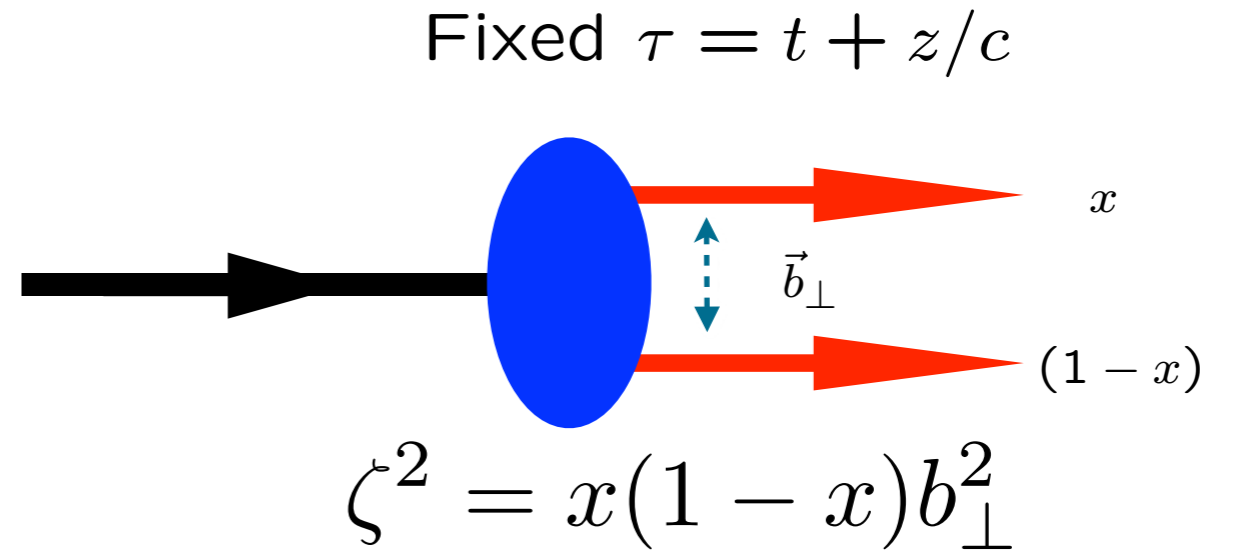
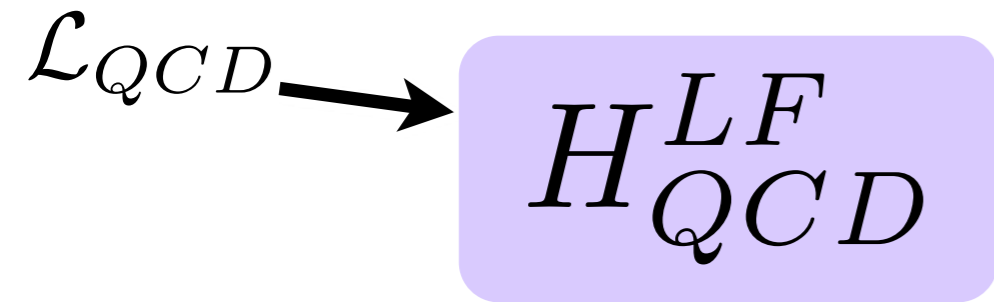
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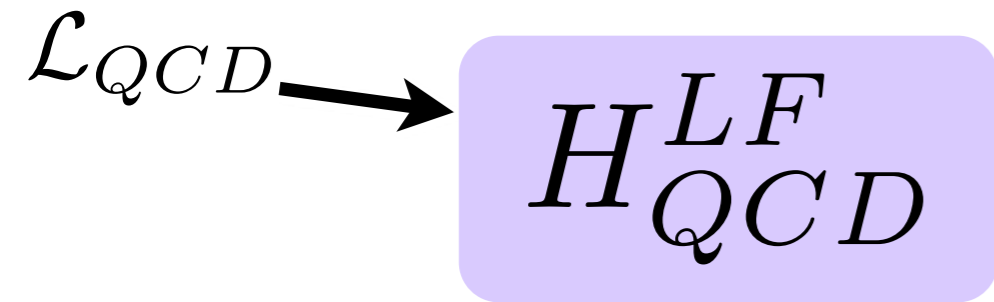
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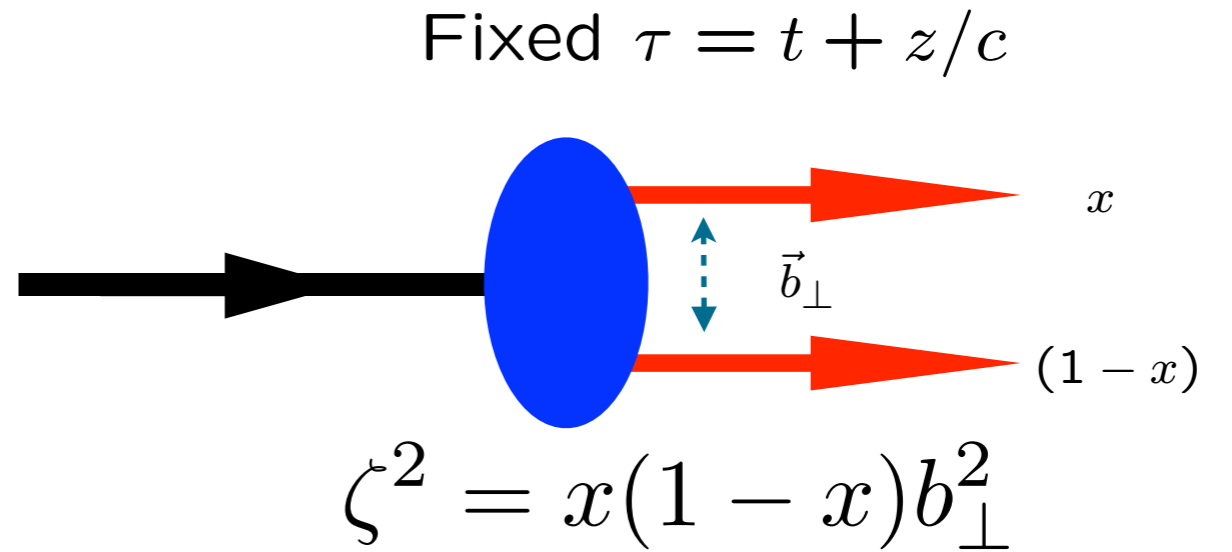
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*Eliminate higher Fock states
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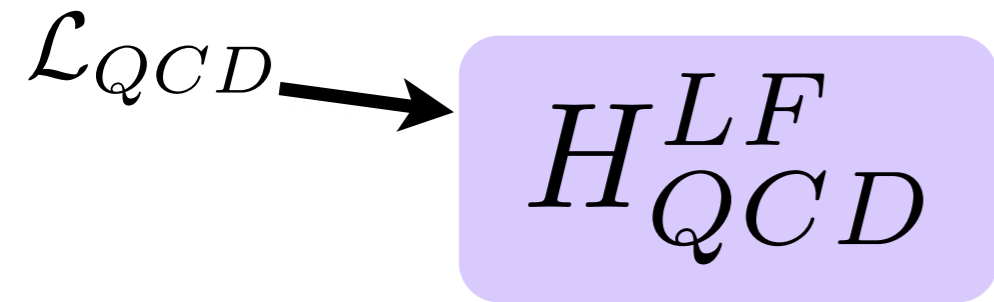
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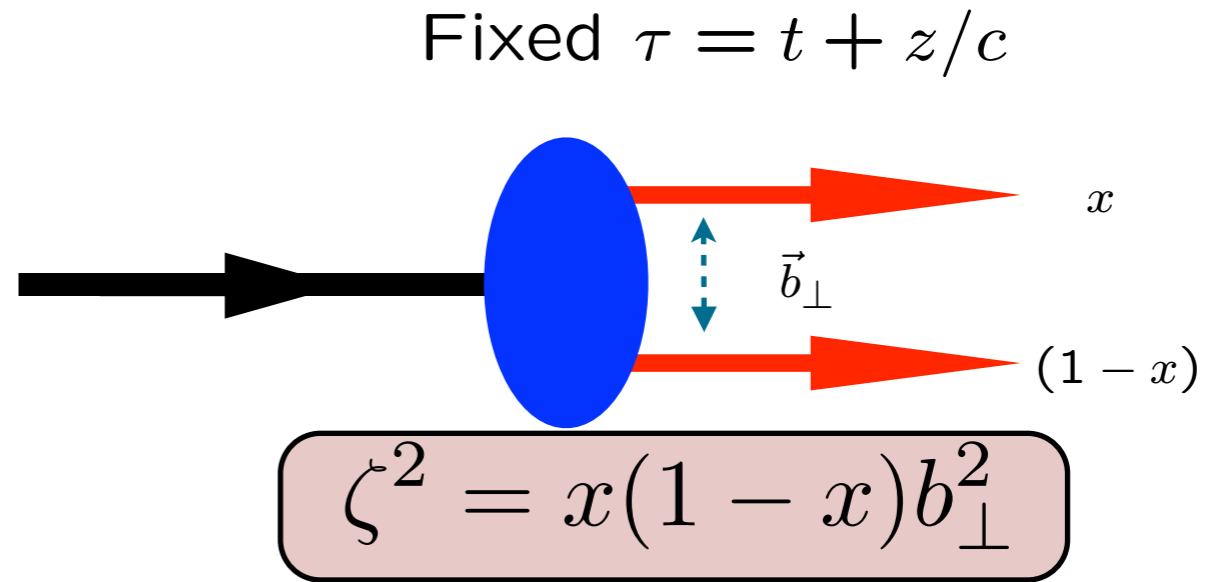
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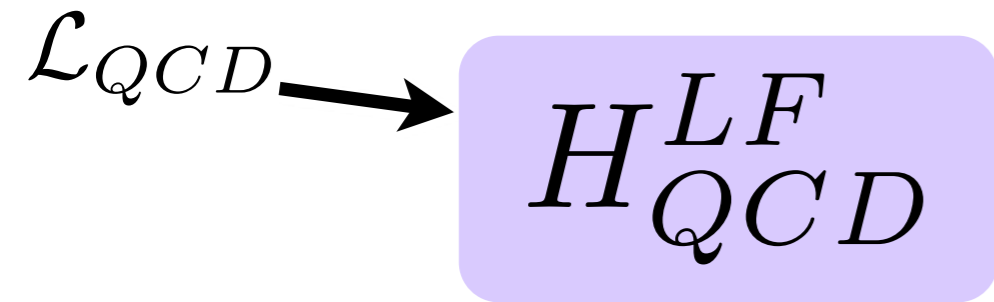
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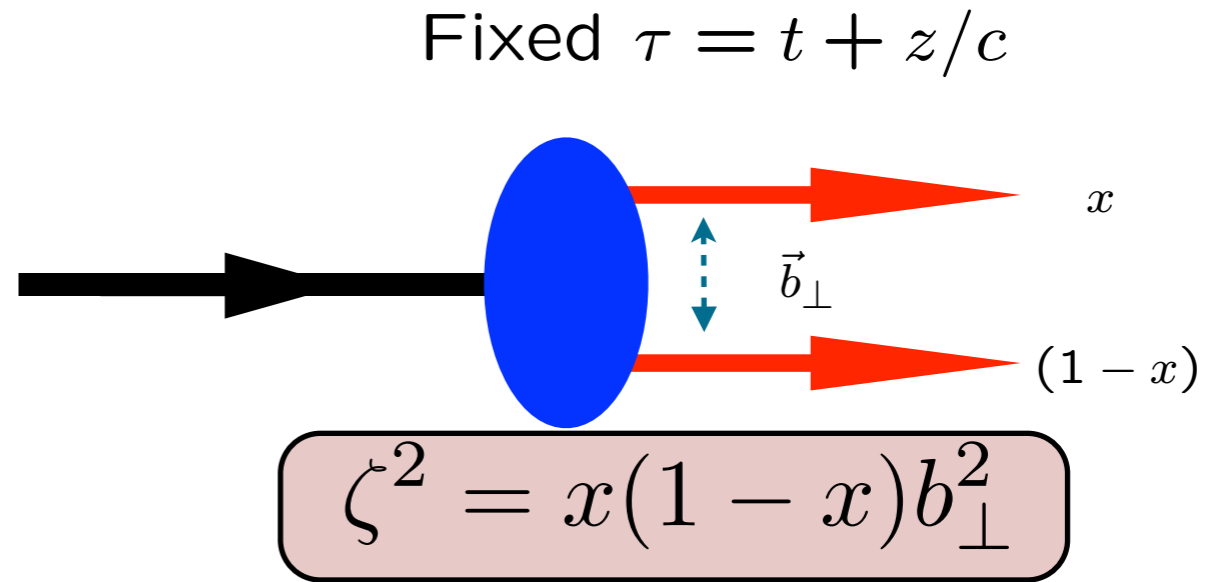


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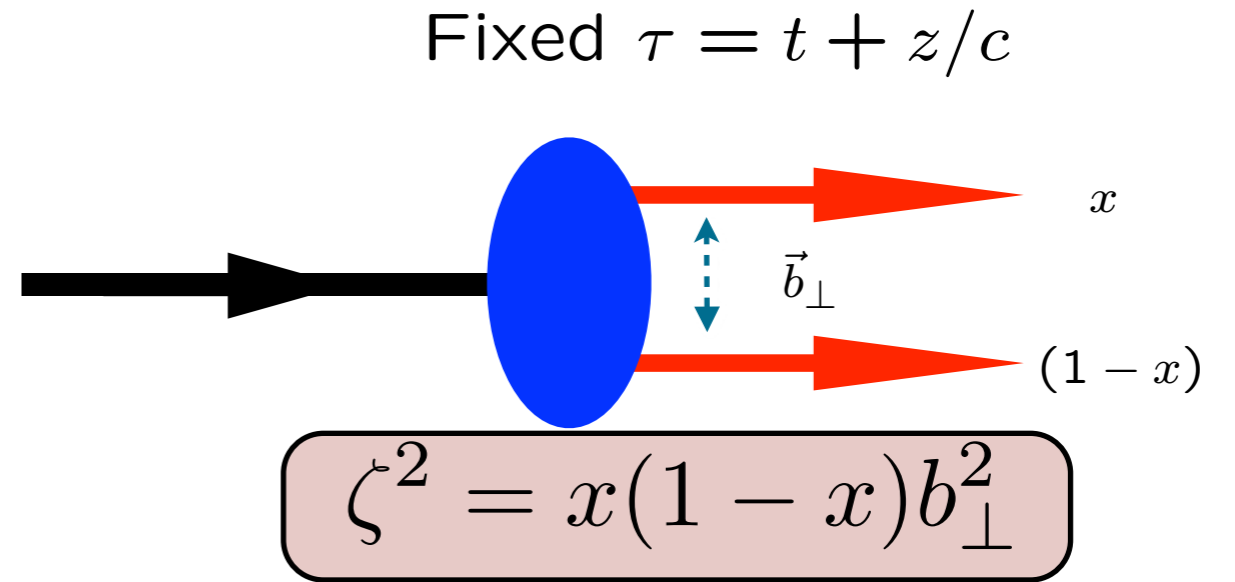
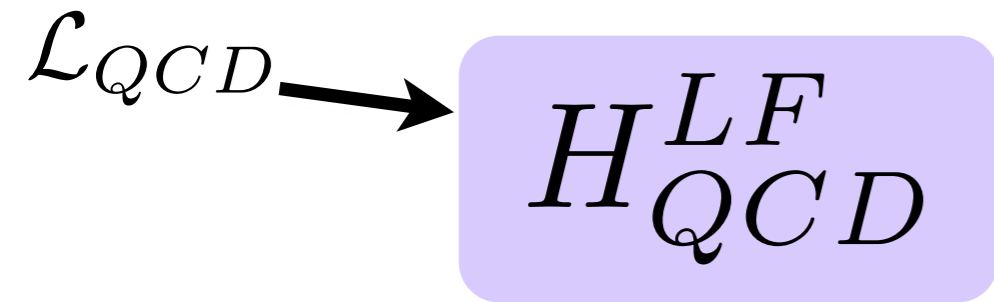
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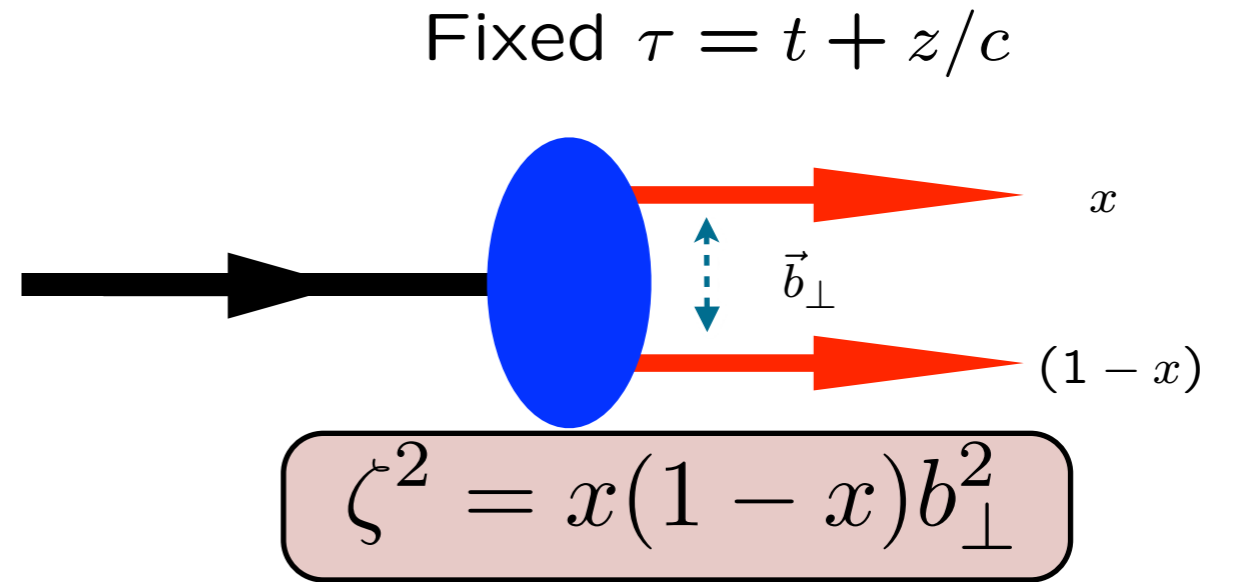
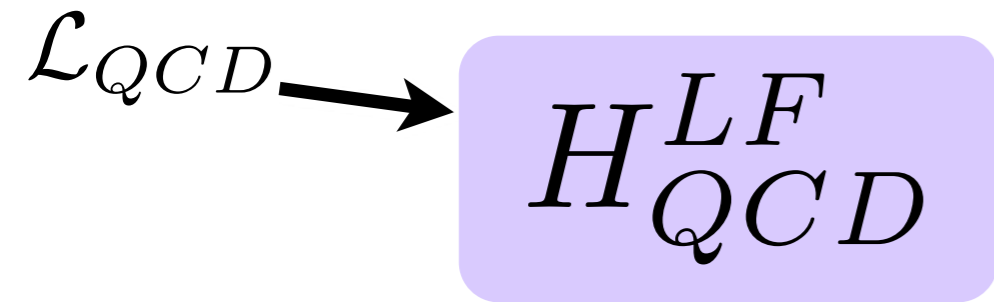
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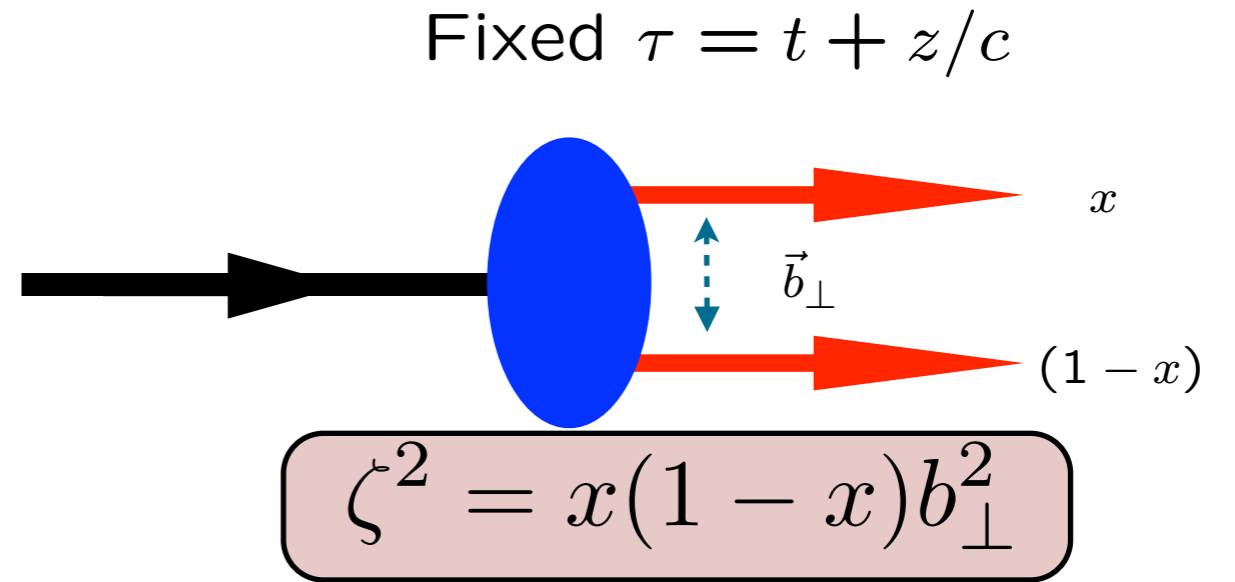
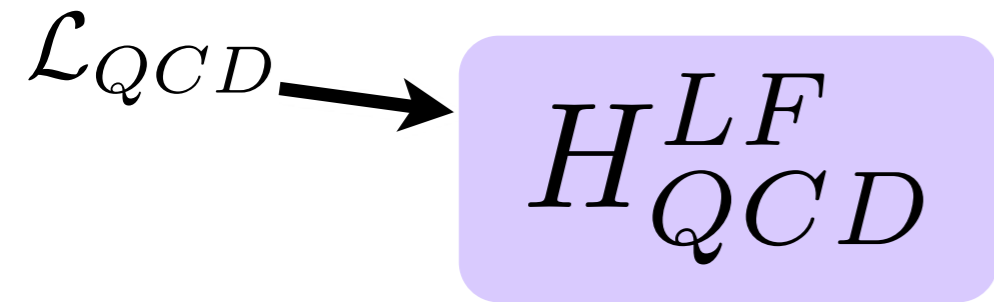
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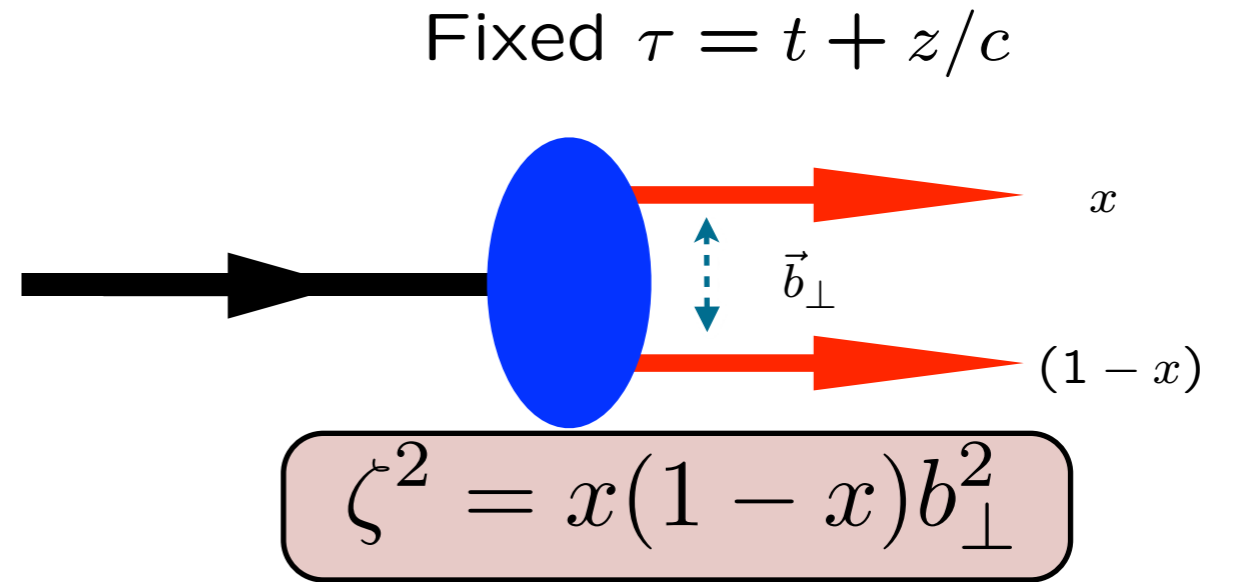
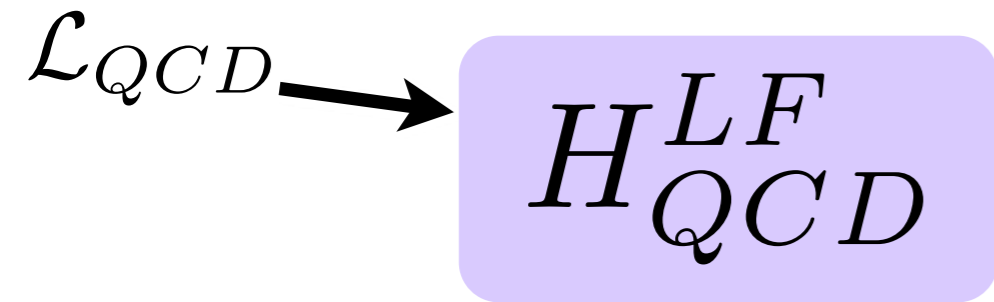
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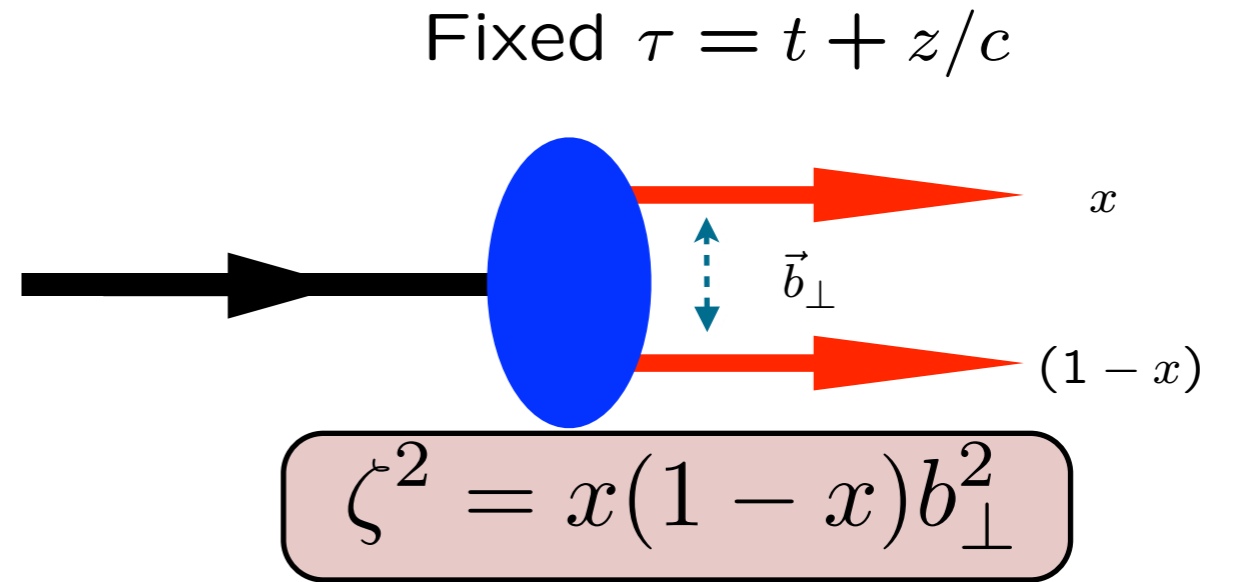
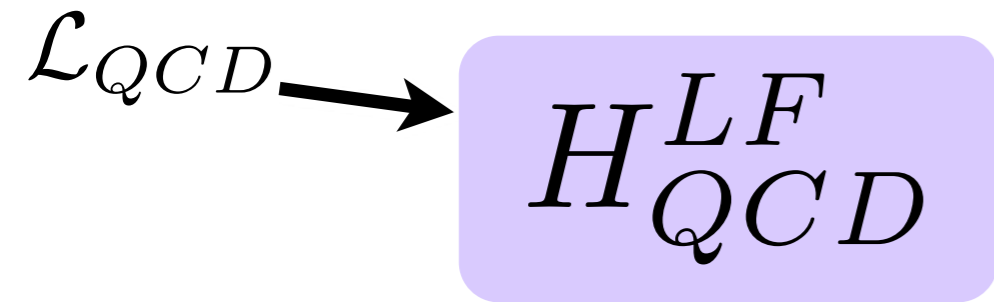
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Confining AdS/QCD potential!

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Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

$$m_q = 0$$

AdS/QCD:

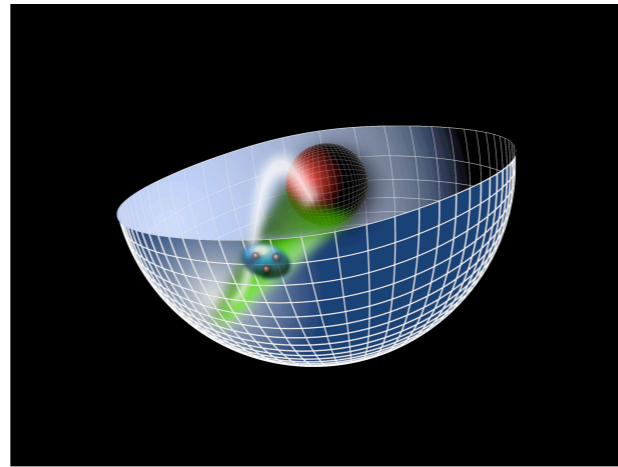
$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

Sums an infinite # diagrams

*AdS/QCD
Soft-Wall Model*



Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

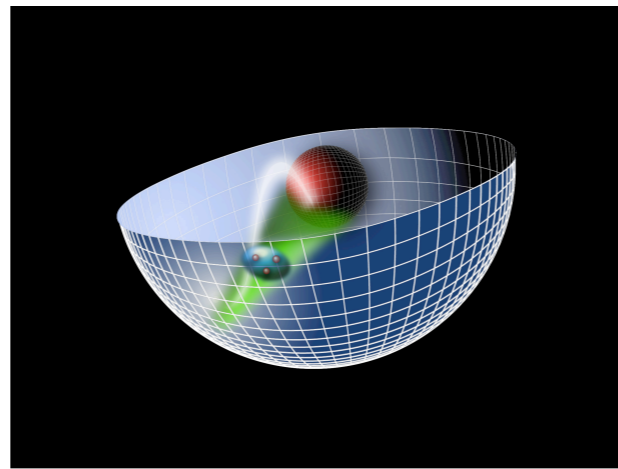
Light-Front Schrödinger Equation



Confinement scale:

$$\kappa \simeq 0.5 \text{ GeV}$$

*AdS/QCD
Soft-Wall Model*



Light-Front Holography

$$\zeta^2 = x(1-x)b_{\perp}^2.$$

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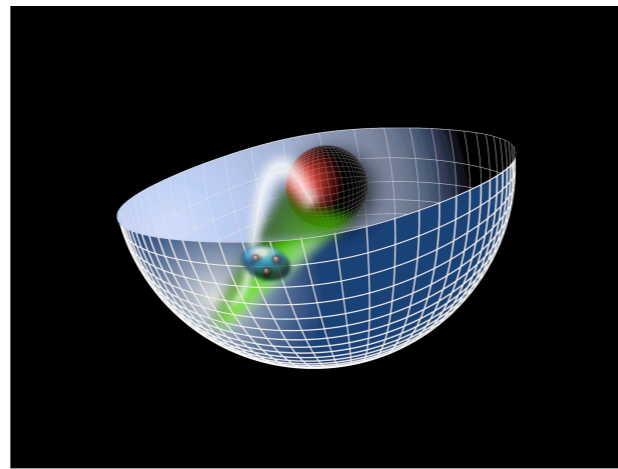
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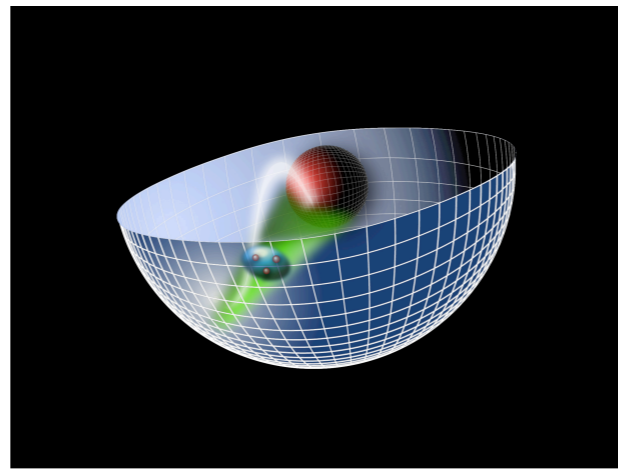
*Conformal Symmetry
of the action*

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*AdS/QCD
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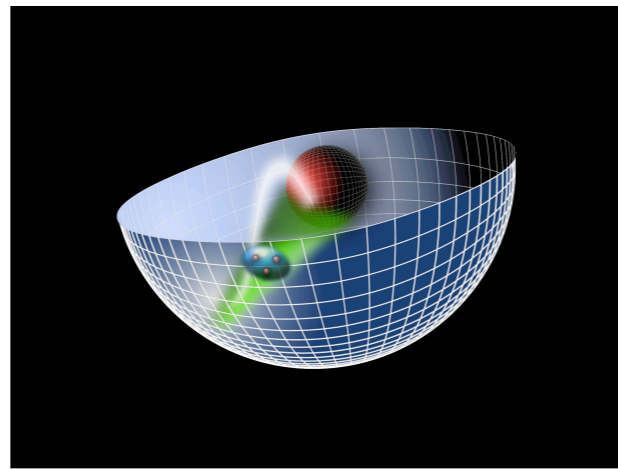
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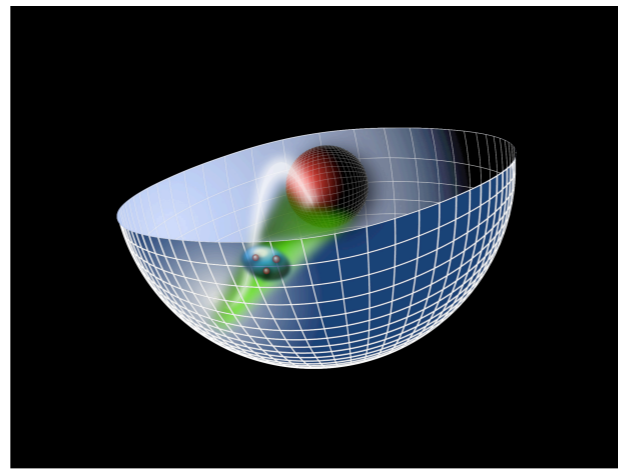
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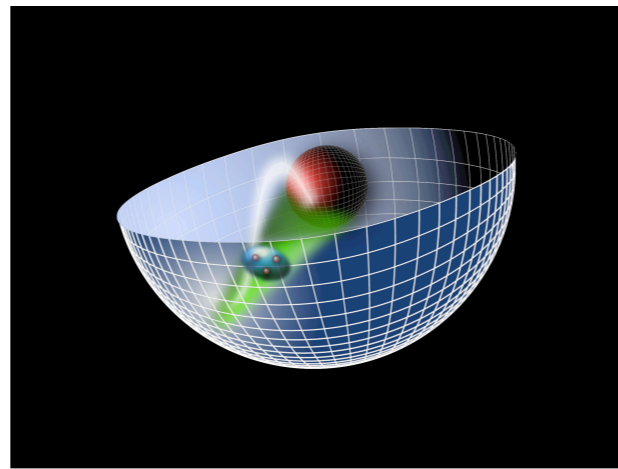
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**Scale can appear in Hamiltonian and EQM
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- **Fubini, Rabinovici**

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● **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)\rangle = i\frac{\partial}{\partial\tau}|\psi(\tau)\rangle$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

● **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Meson Spectrum in Soft Wall Model

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

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- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

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Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Dilaton profile $\varphi(z) = +\kappa^2 z^2$ $z \rightarrow \zeta$

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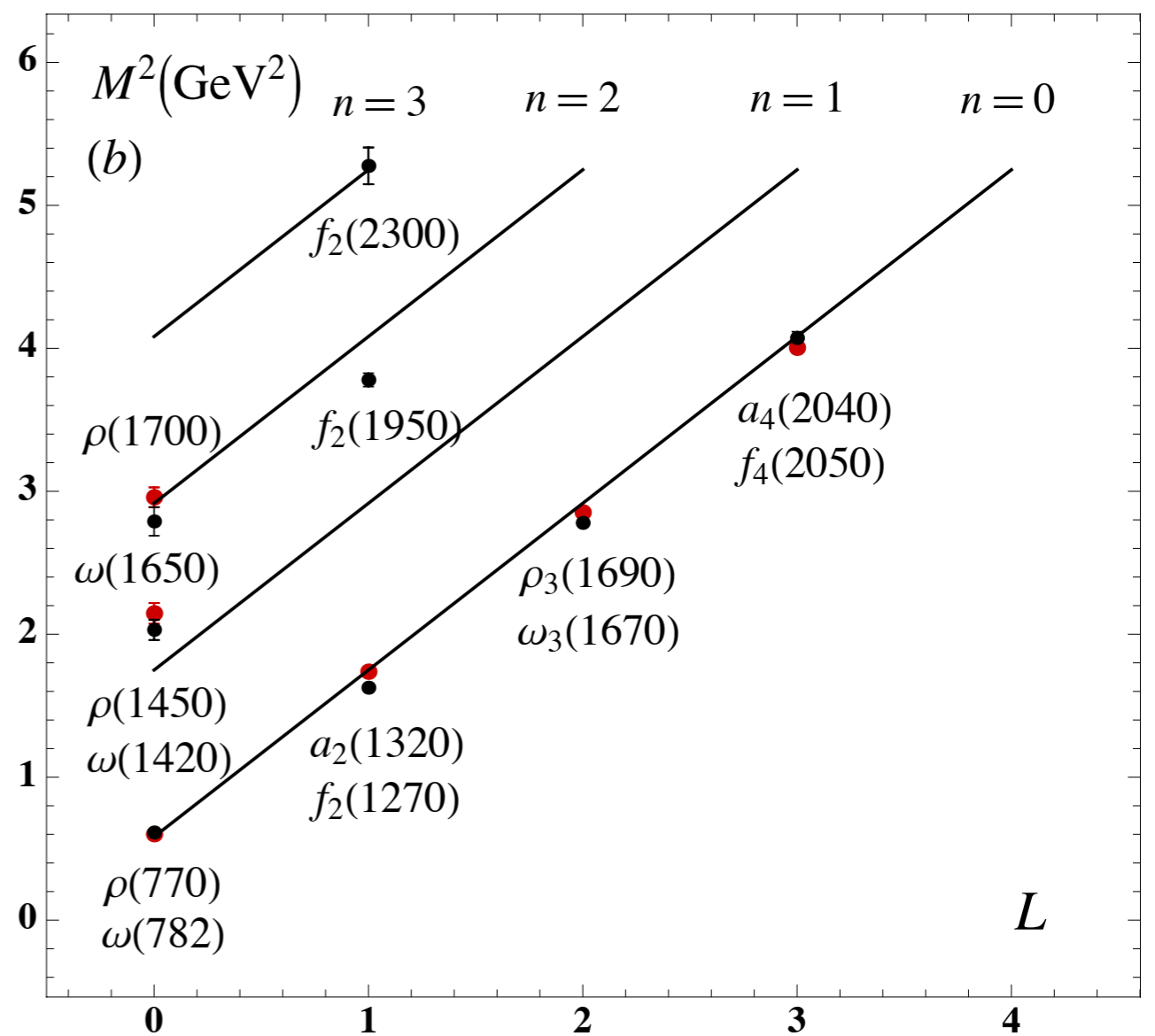
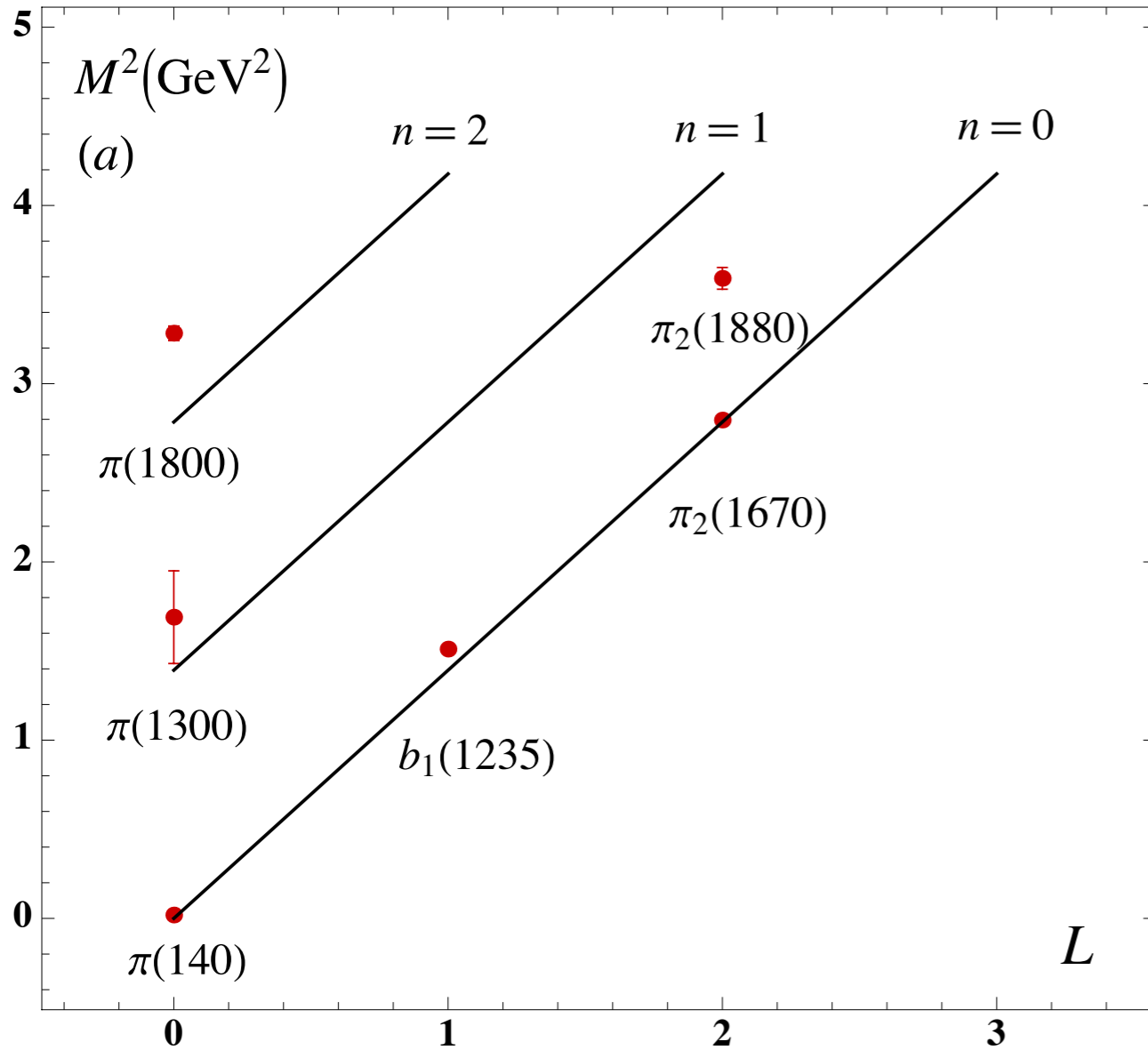
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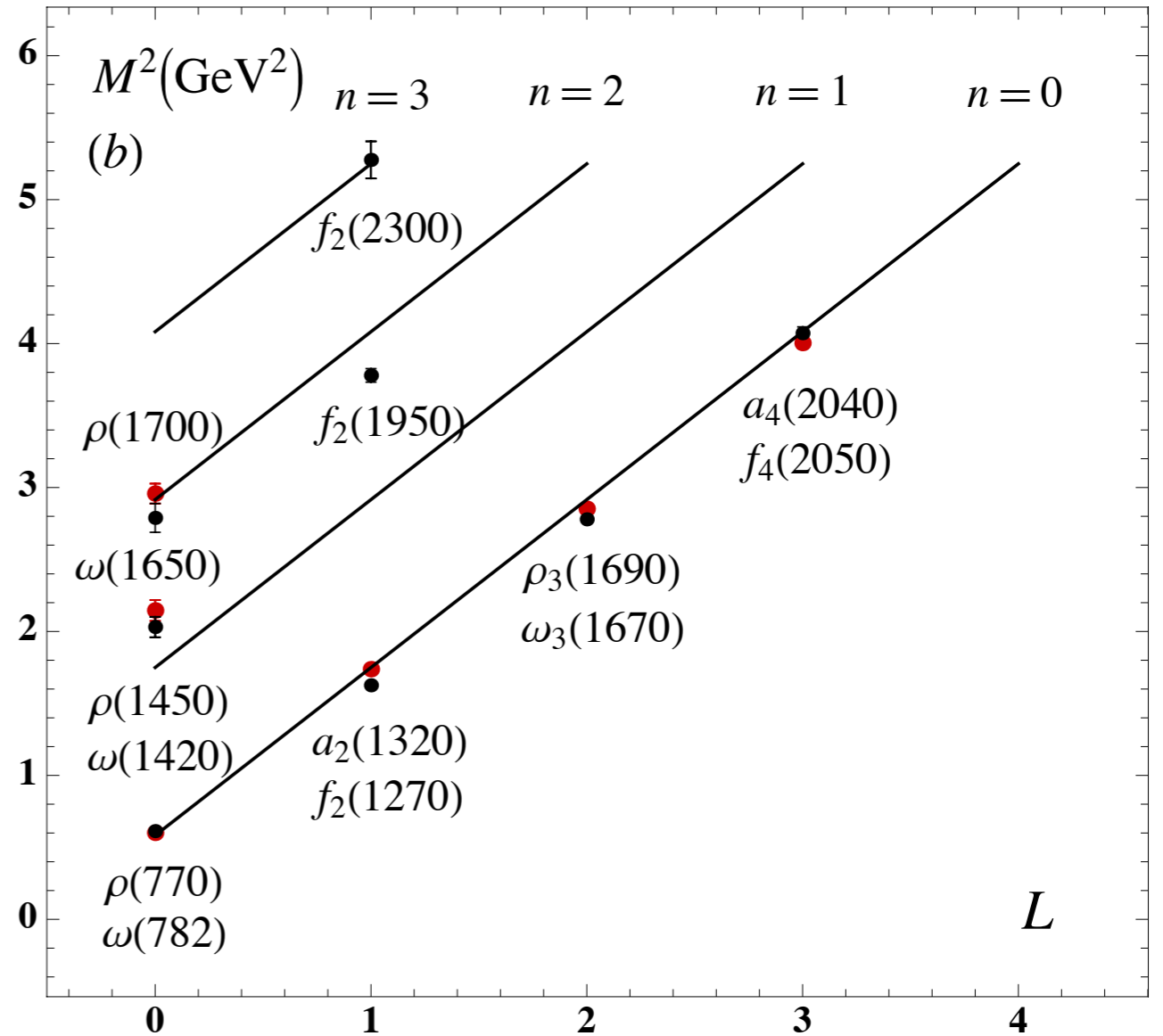
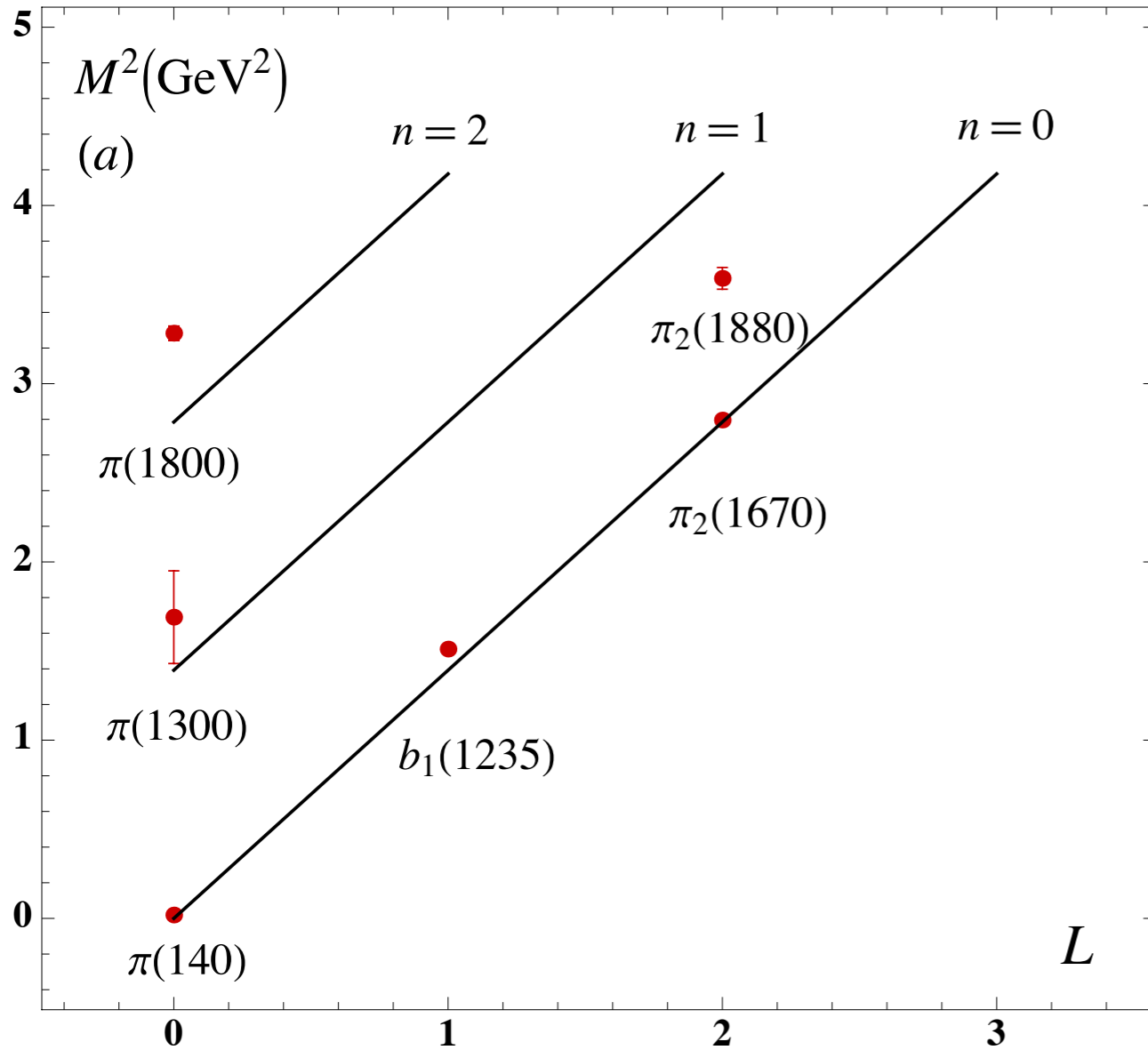
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$$m_u = m_d = 0$$



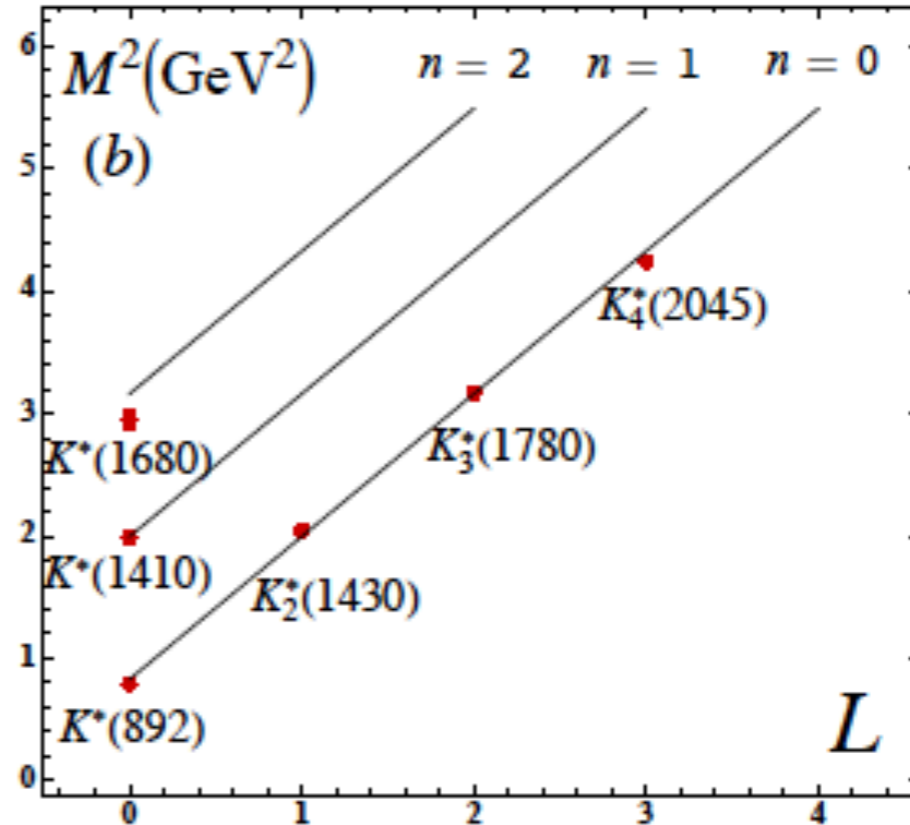
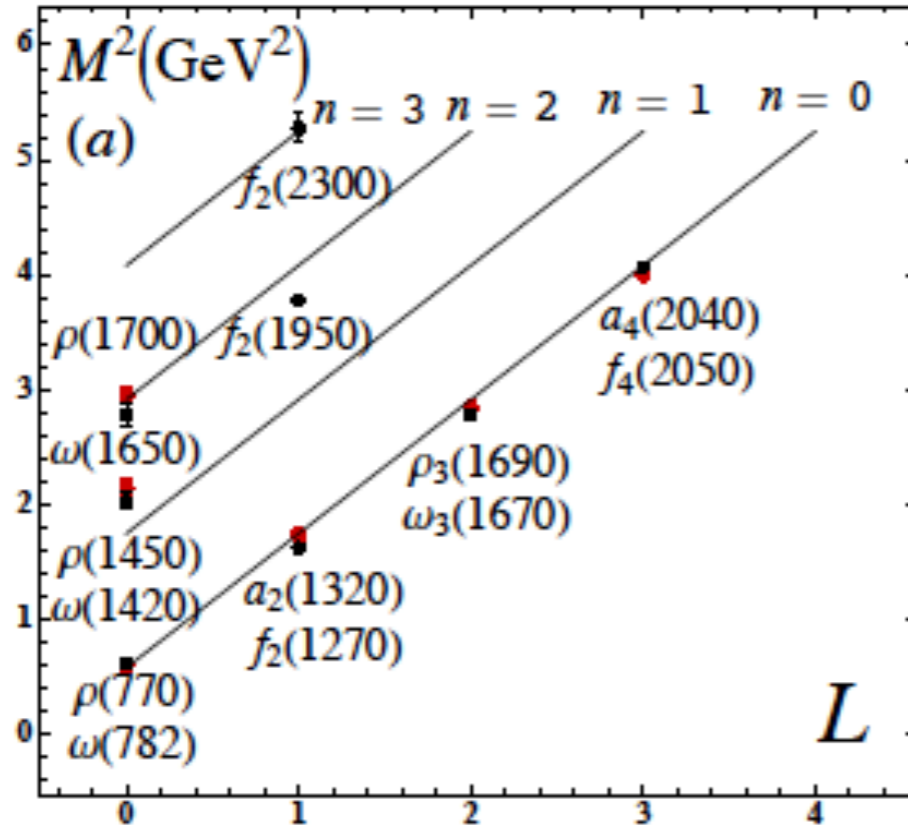
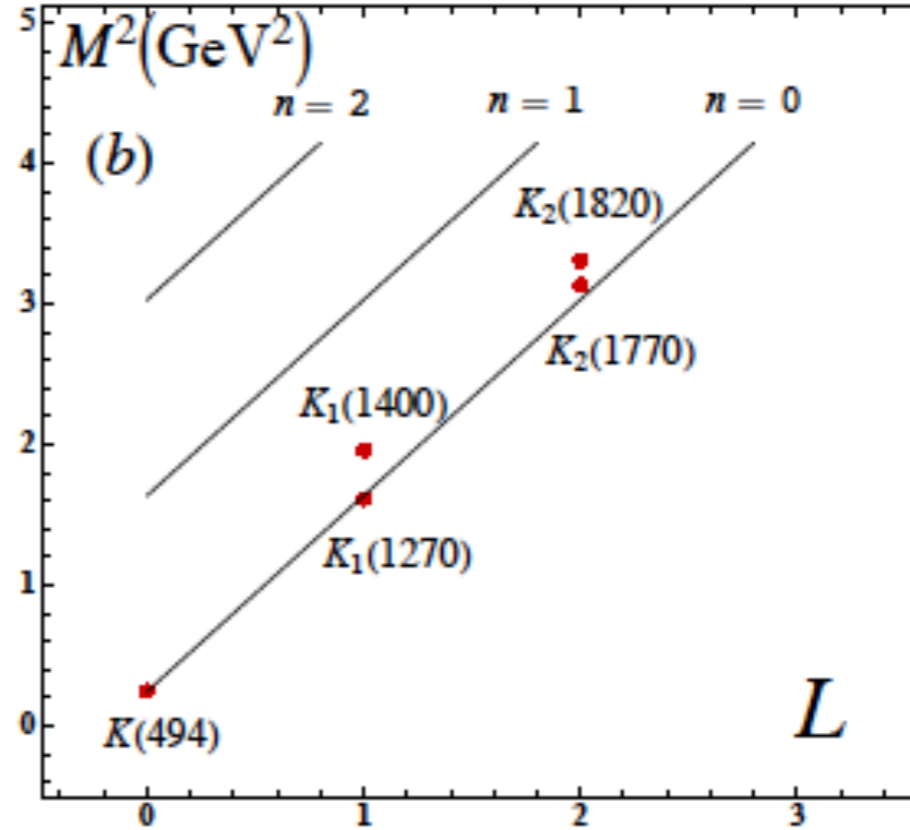
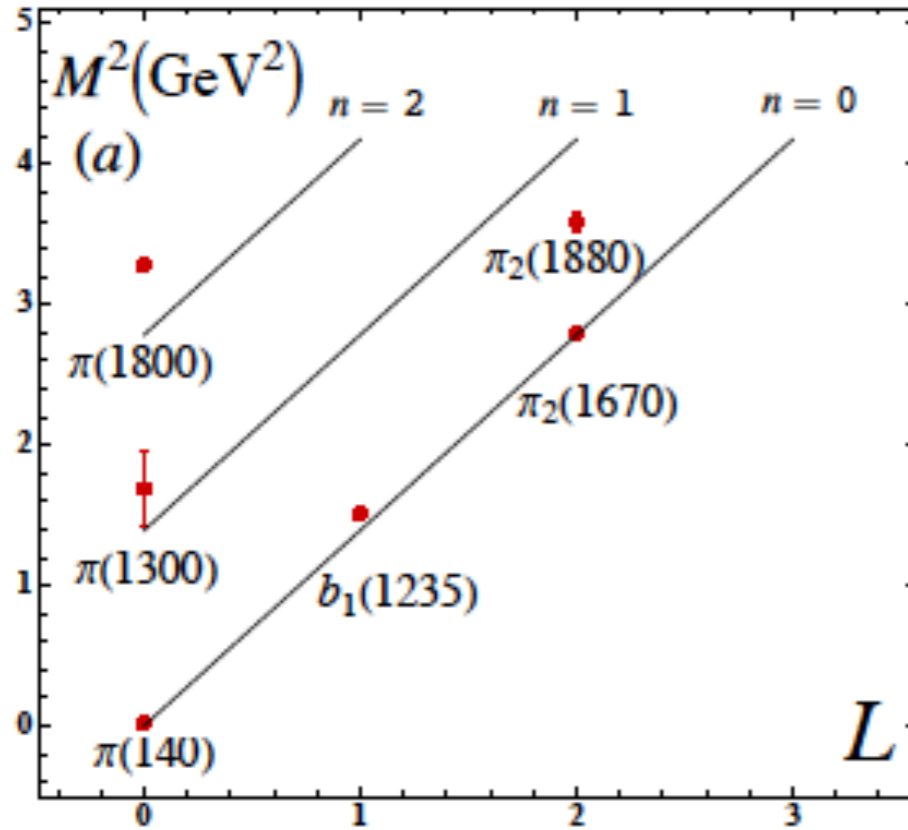
$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

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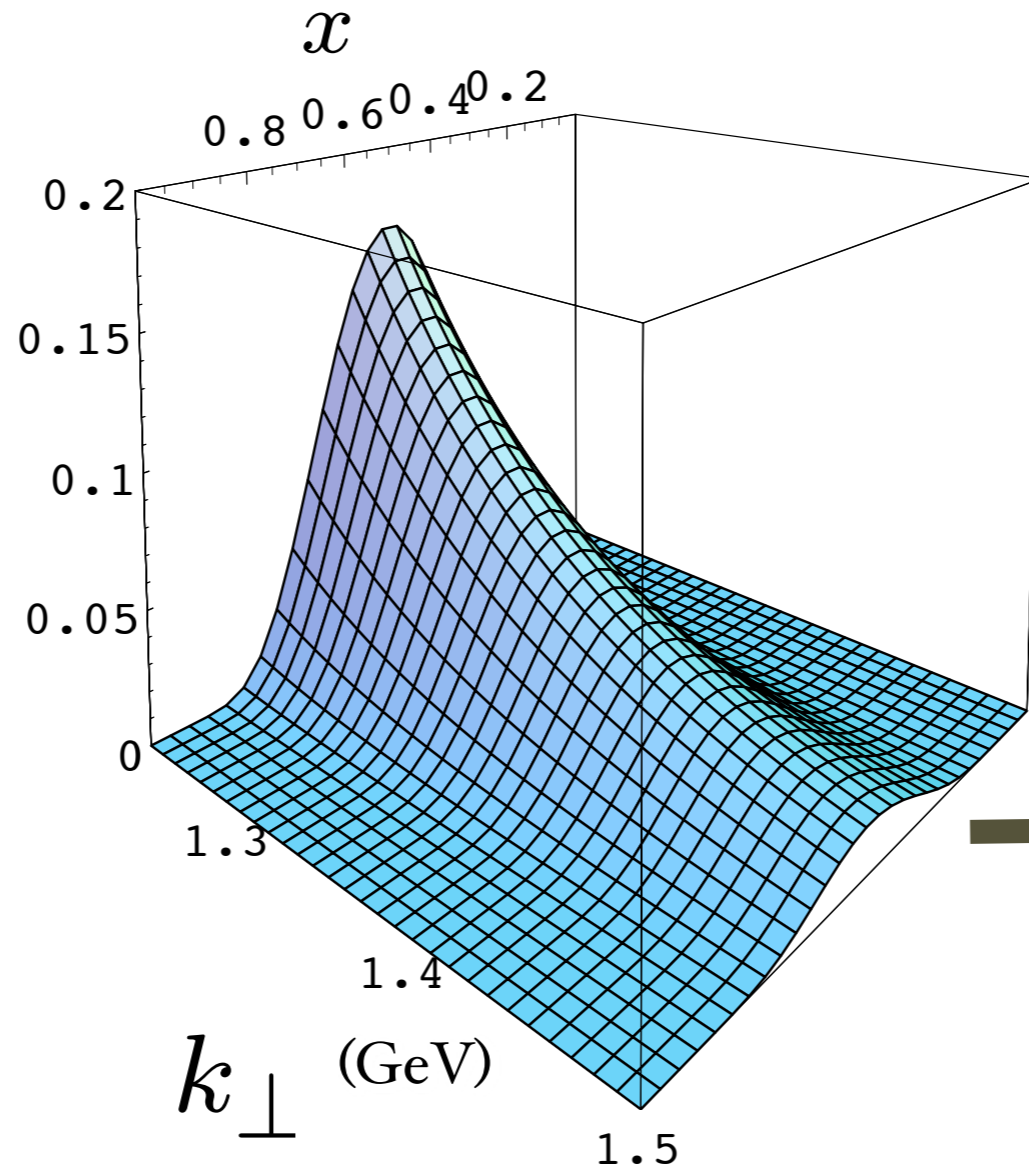
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



Prediction from AdS/QCD: Meson LFWF

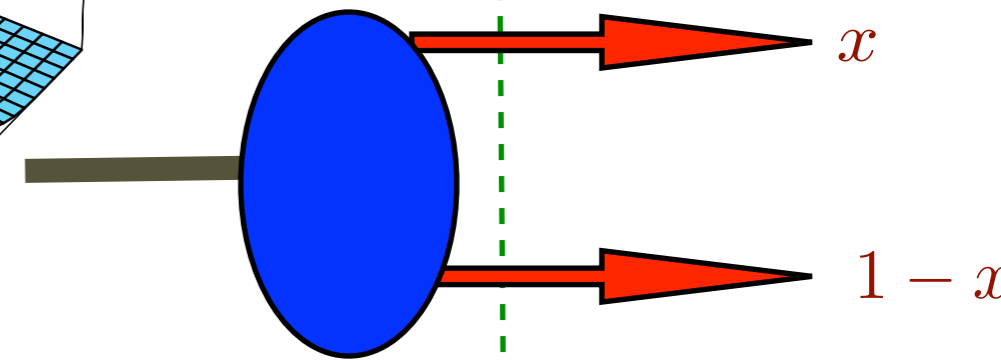
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_{\perp}^2)$$



de Teramond,
Cao, sjb

“Soft Wall”
model



massless quarks

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_{\pi}(x) = \frac{4}{\sqrt{3}\pi} f_{\pi} \sqrt{x(1-x)}$$

$$f_{\pi} = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

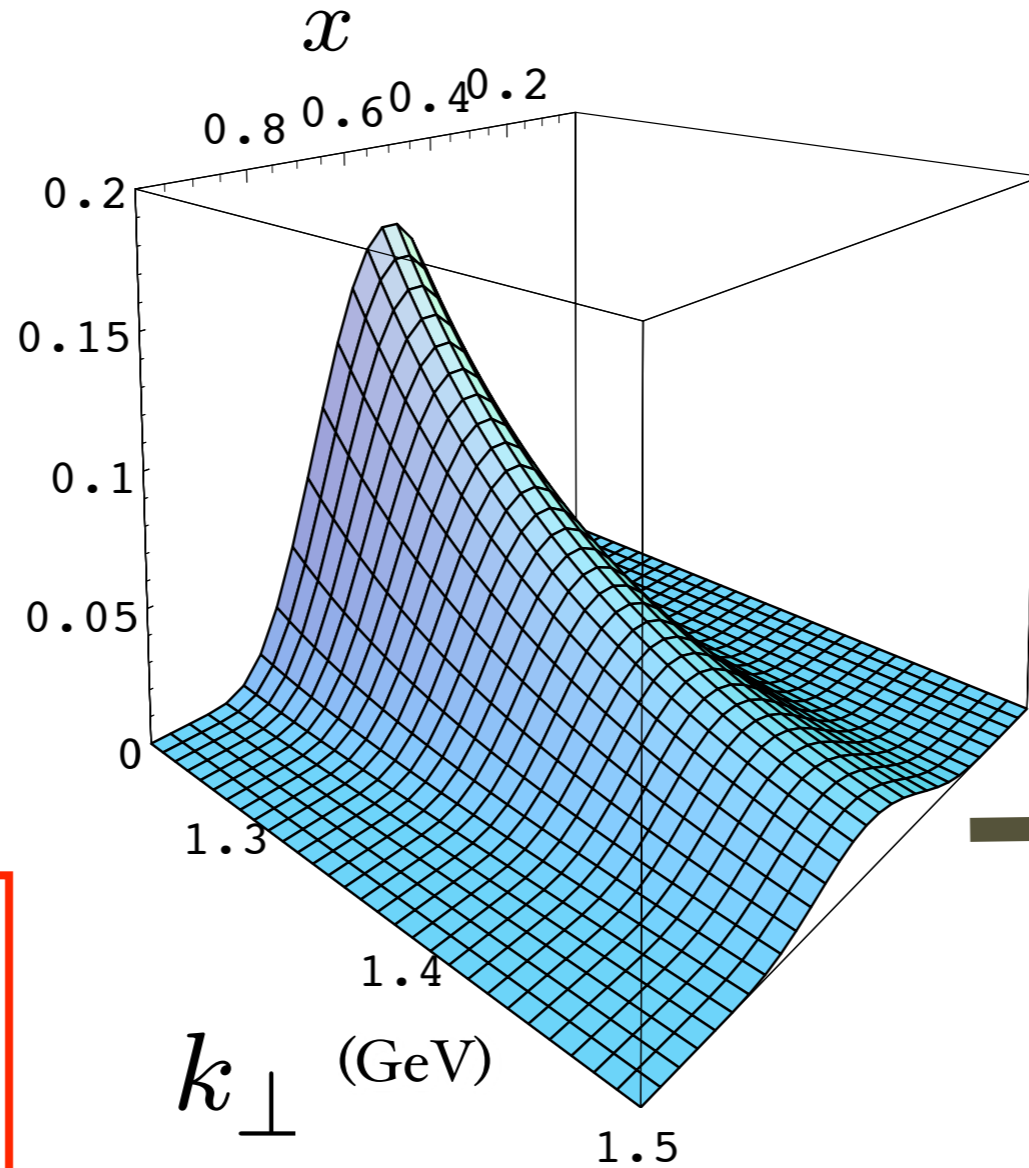
Same as DSE!

C. D. Roberts et al.

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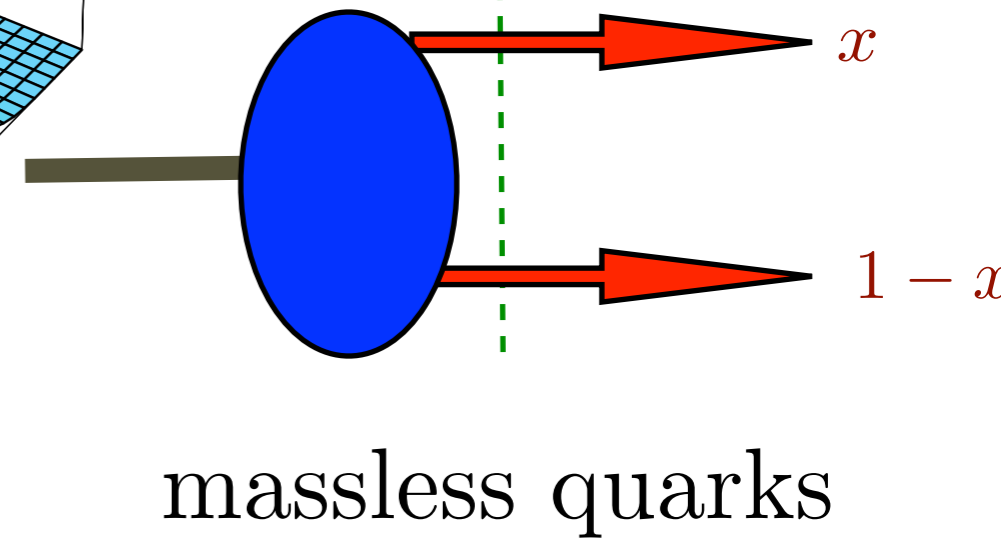
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Note coupling

$$k_{\perp}^2, x$$

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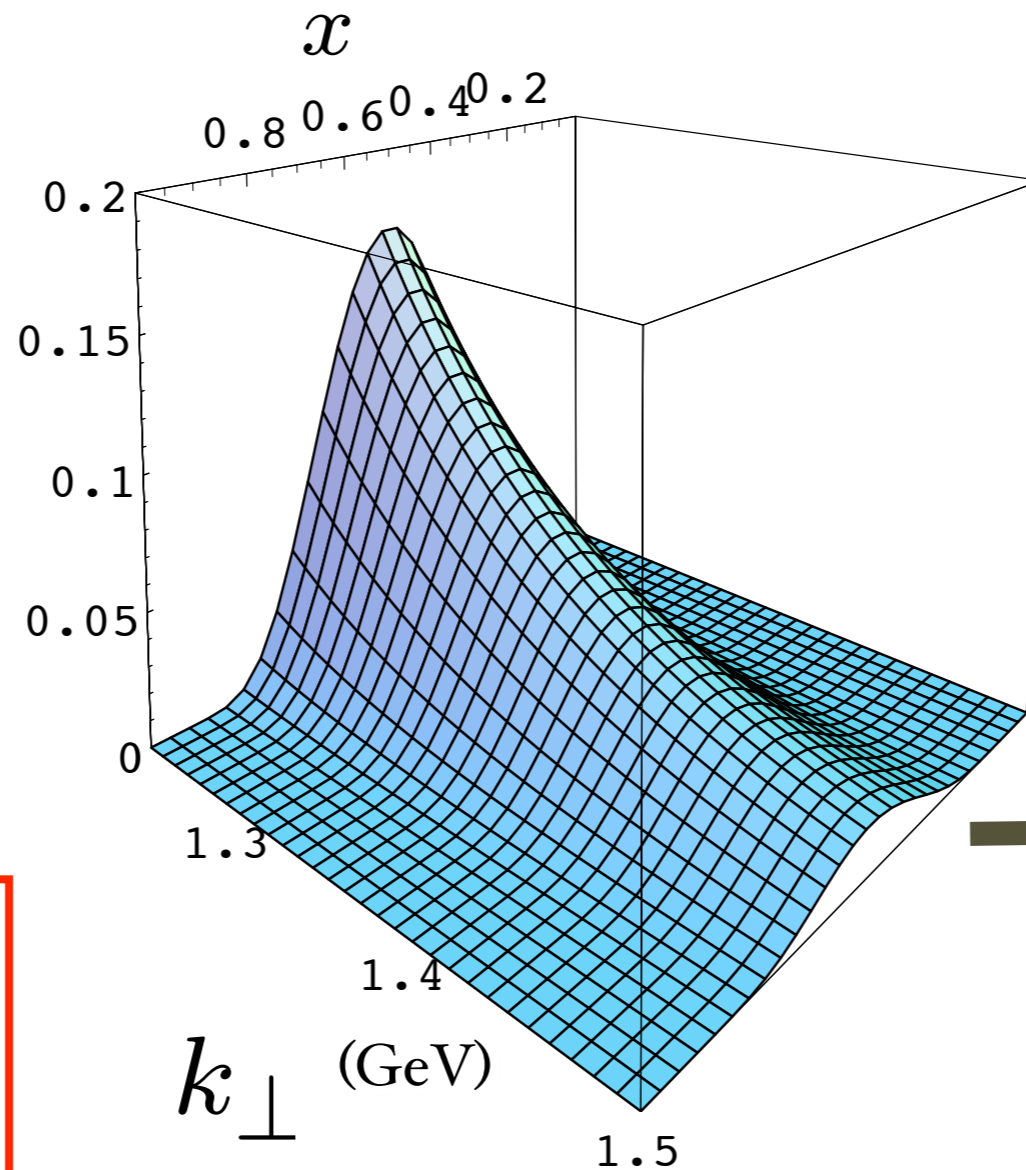
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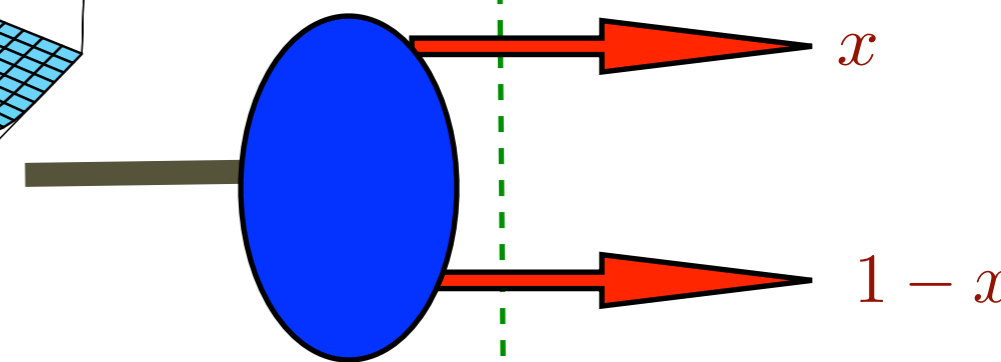
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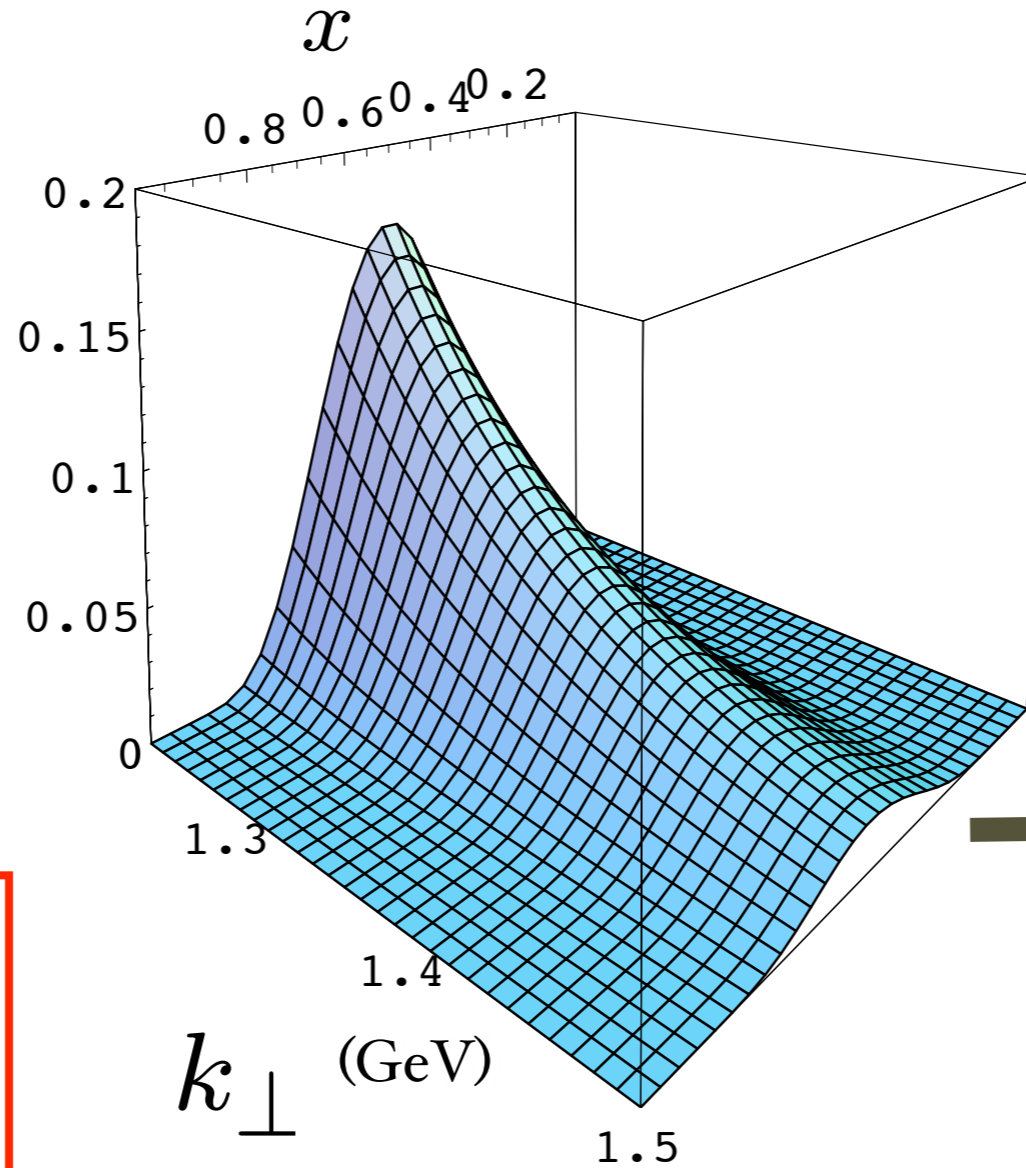
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Provides Connection of Confinement to Hadron Structure

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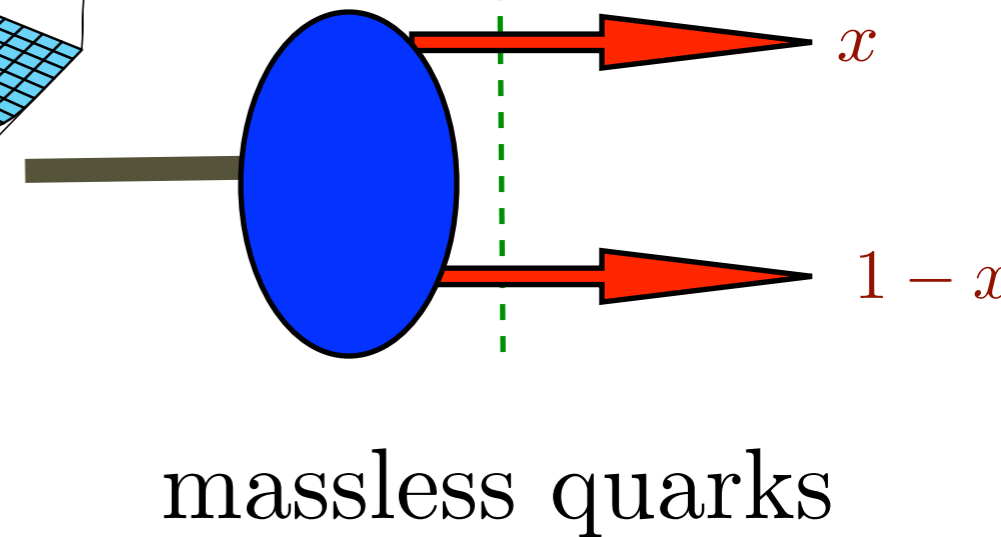
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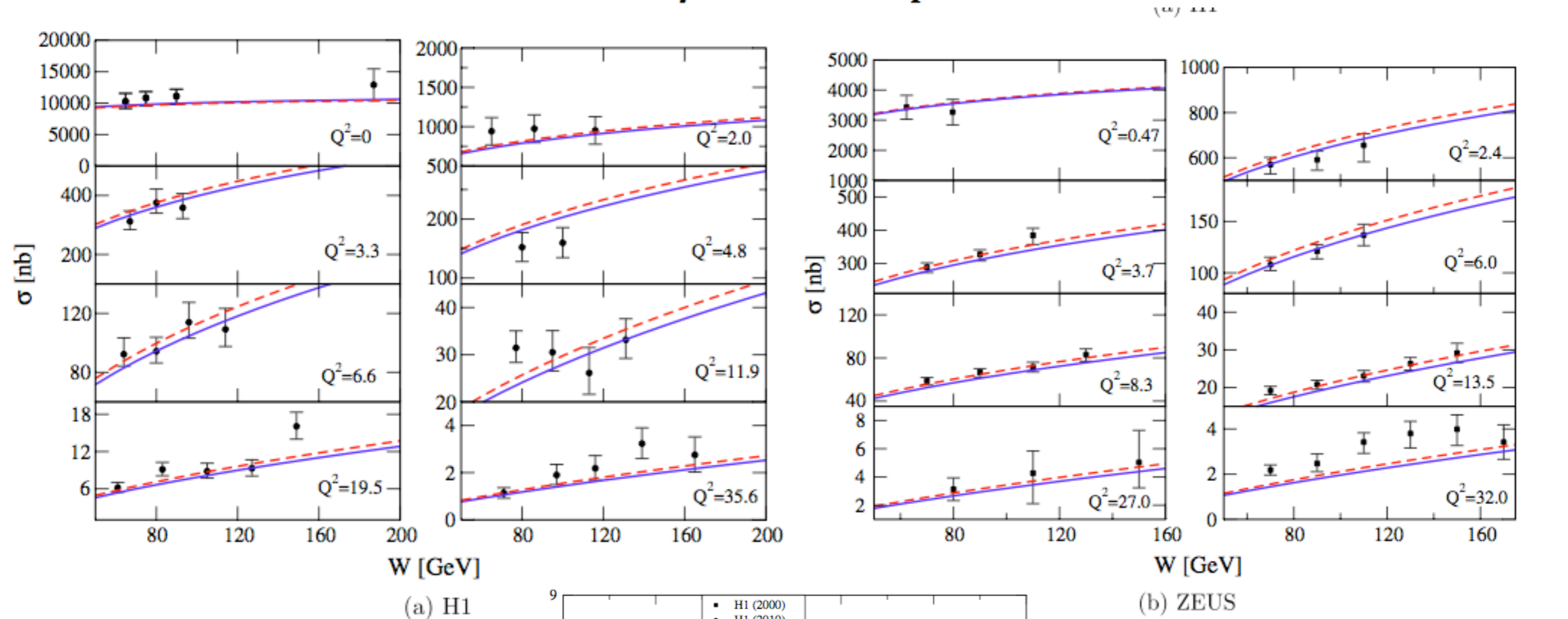
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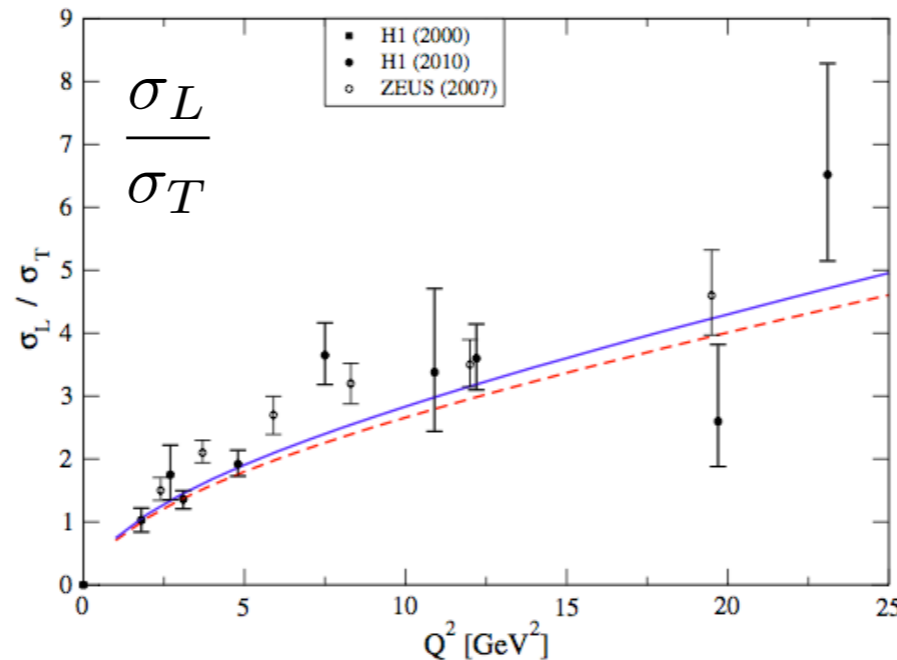
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AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

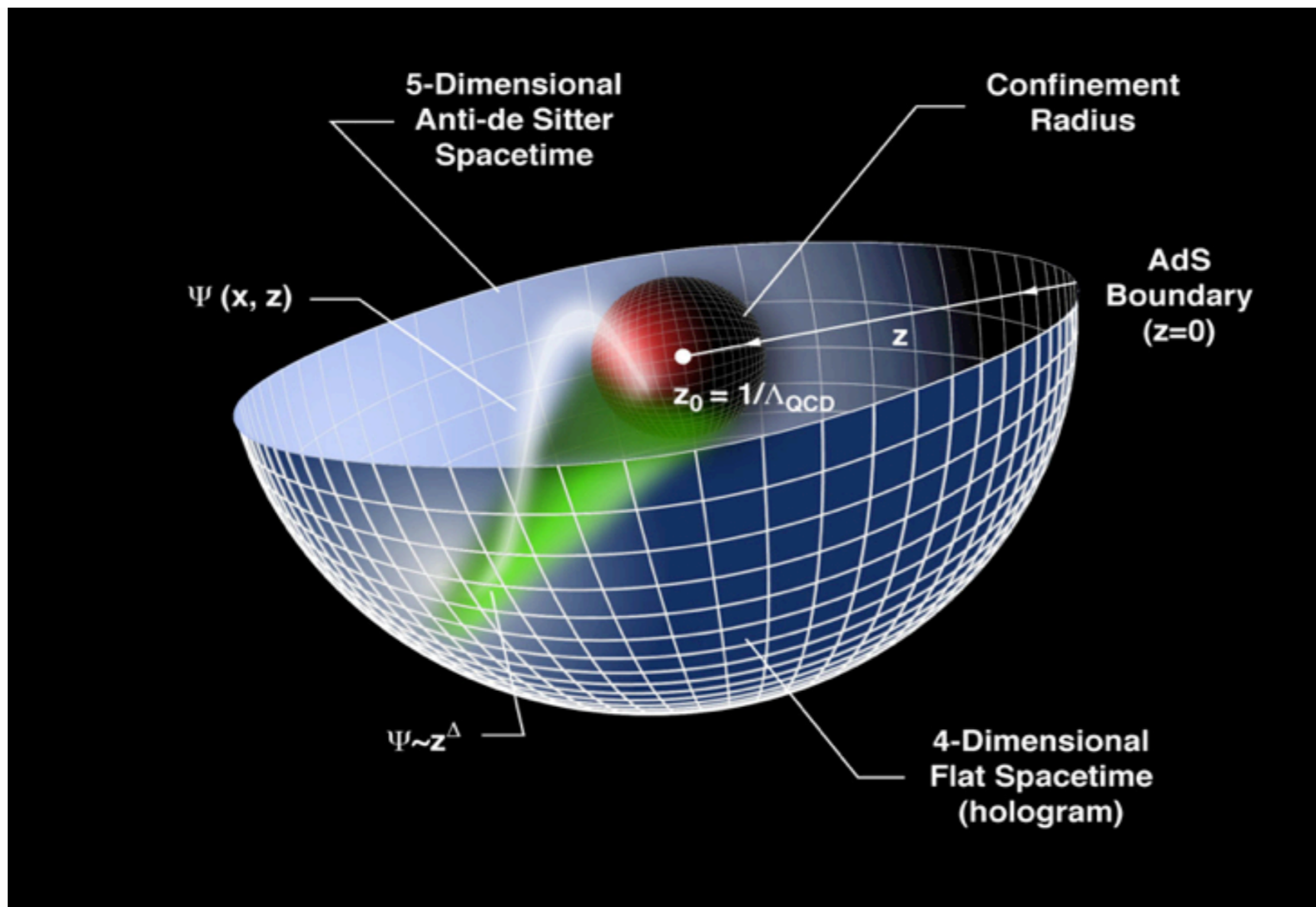


**J. R. Forshaw,
R. Sandapen**

$$\gamma^* p \rightarrow \rho^0 p'$$



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Changes in physical length scale mapped to evolution in the 5th dimension z

AdS₅

8-2007
8685A14

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

APS-GHP Workshop
February 3, 2017

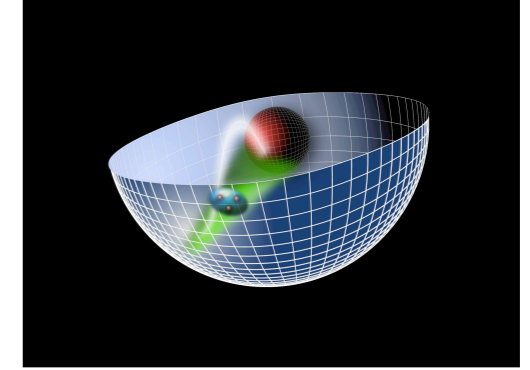
Supersymmetric Features of QCD
from LF Holography

Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY



AdS₅



- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

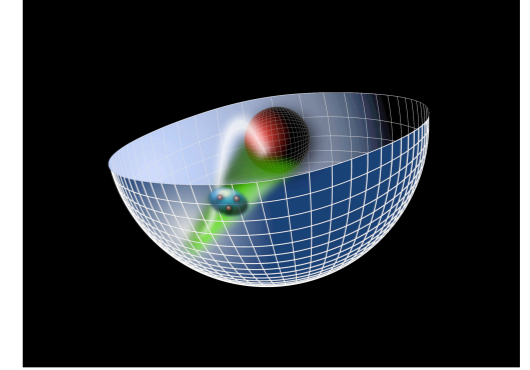
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$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

AdS/CFT

AdS₅



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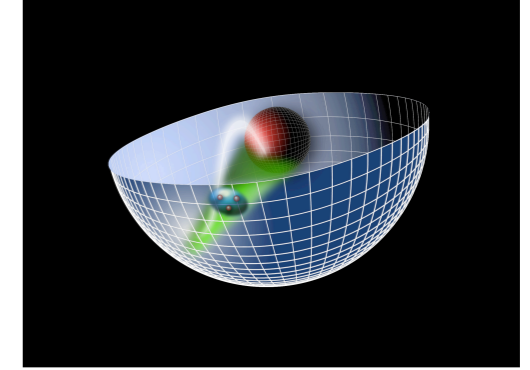
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AdS₅



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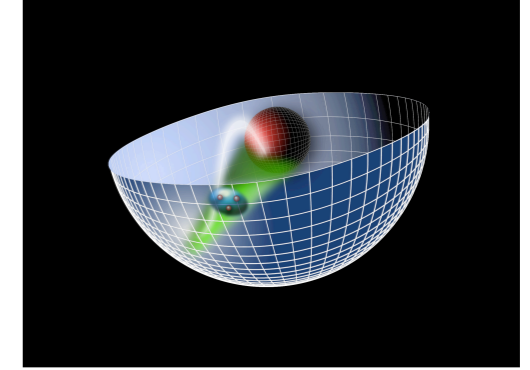
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- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

AdS/CFT

AdS₅



- Isomorphism of $SO(4, 2)$ of **conformal QCD** with the group of **isometries** of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure ←

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

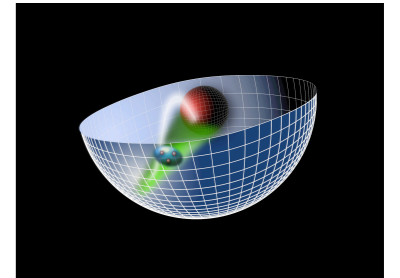
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AdS/CFT

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS₅ as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

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Identical to Light-Front Bound State Equation!

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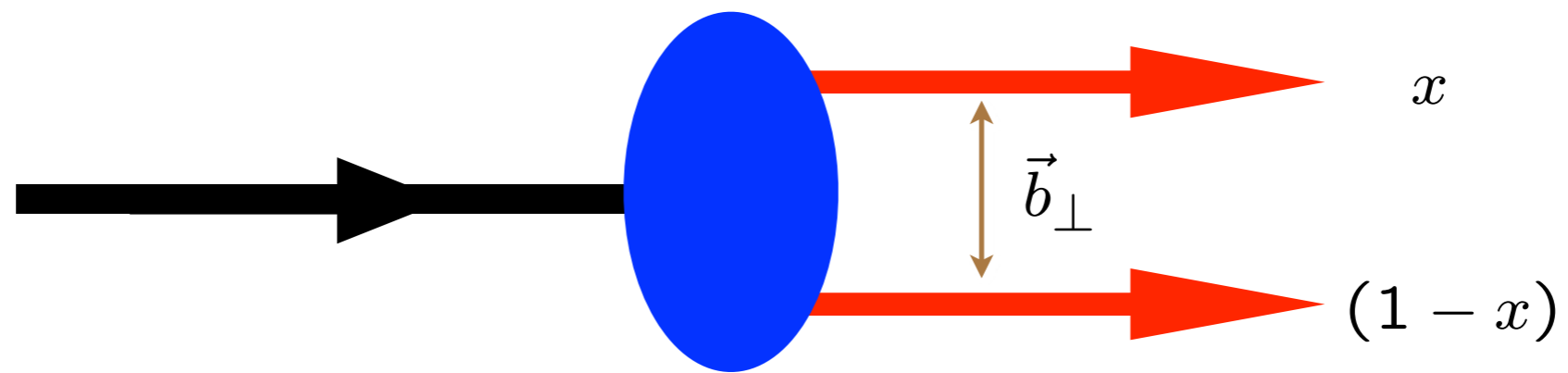
Identical to Light-Front Bound State Equation!

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

$LF(3+1) \longleftrightarrow AdS_5$

de Teramond, sjb

Light-Front Holographic Dictionary



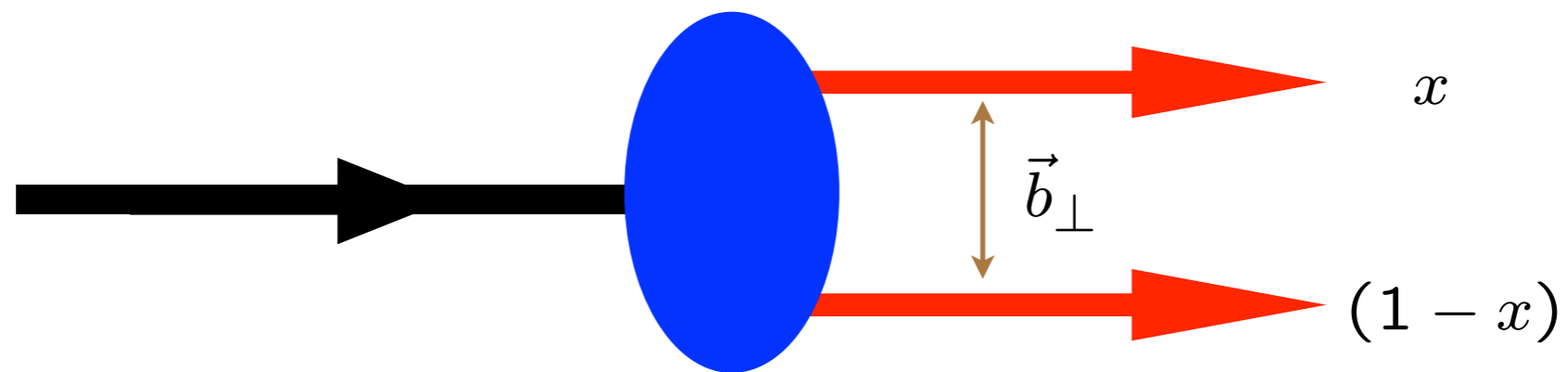
$$(\mu R)^2 = L^2 - (J - 2)^2$$

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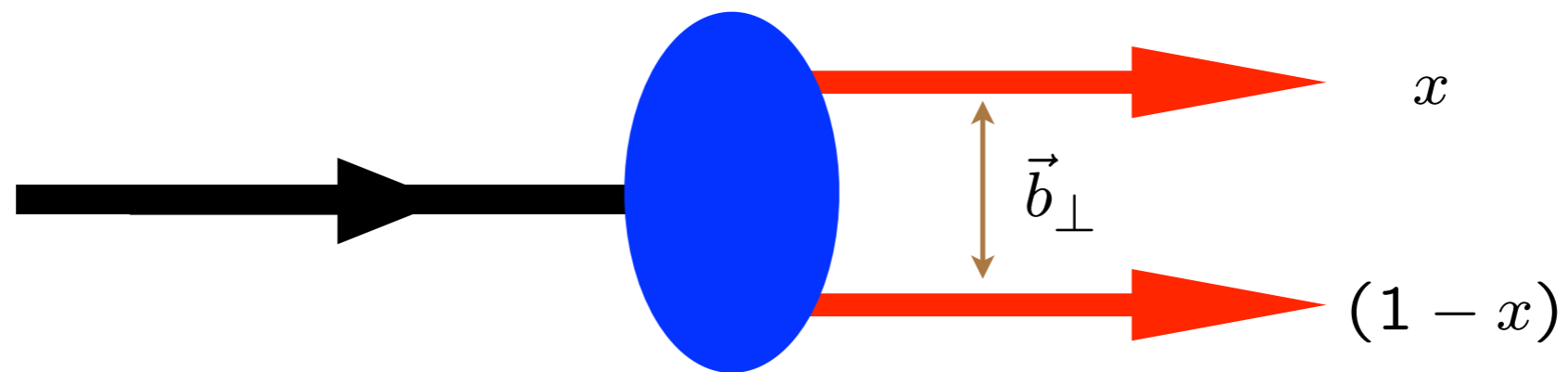
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Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp)$$



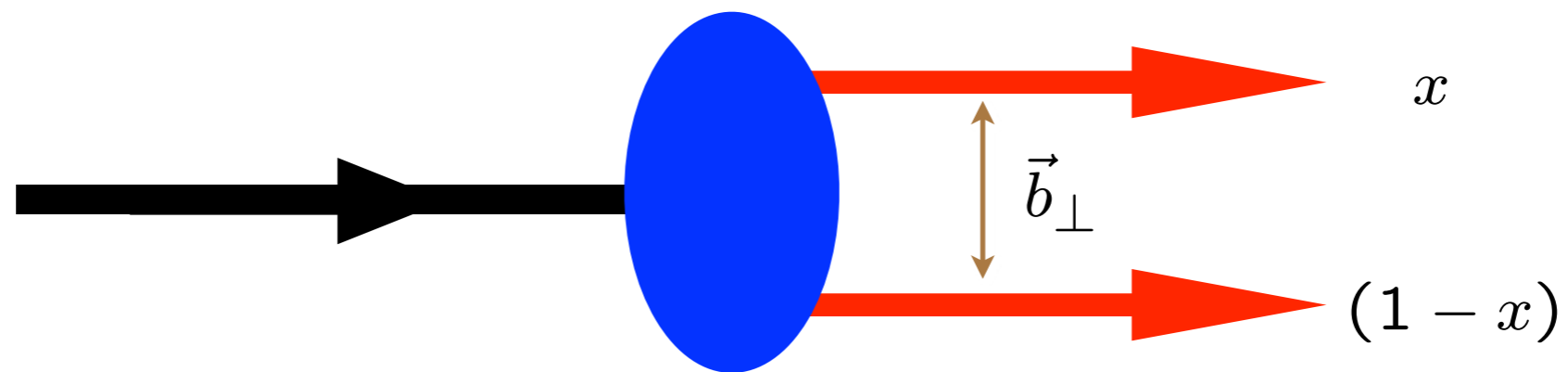
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Light-Front Holographic Dictionary

 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$


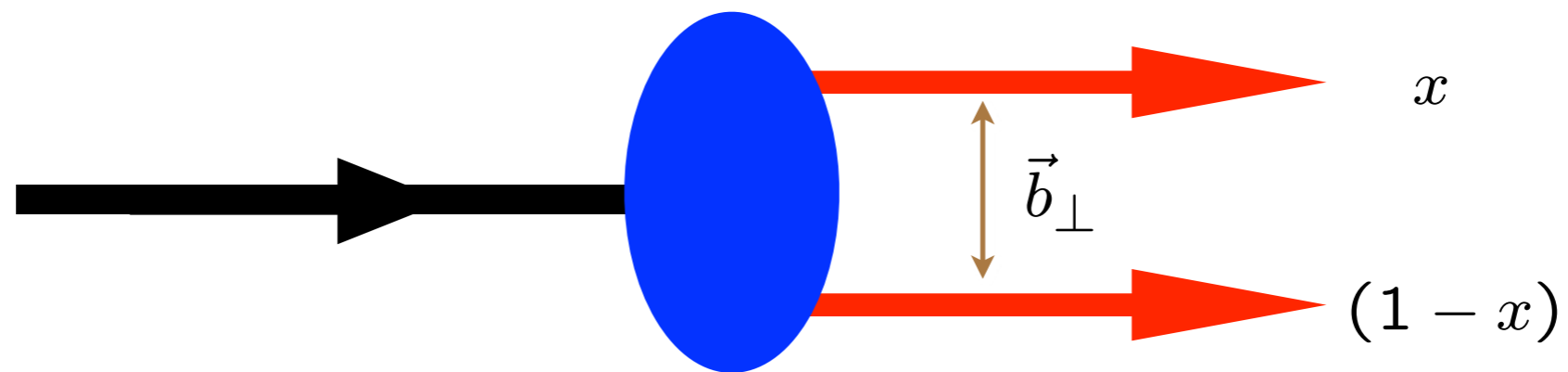
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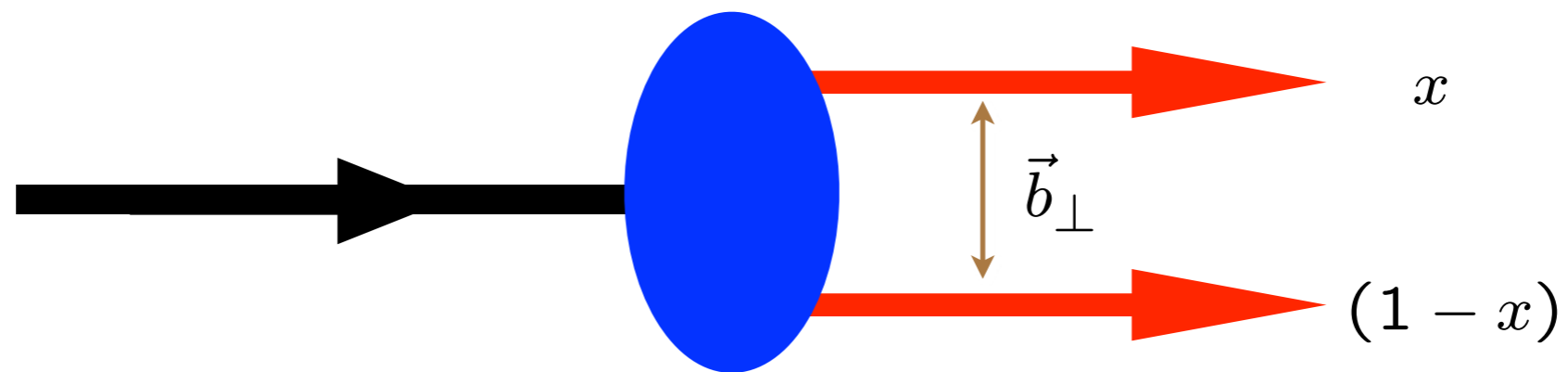
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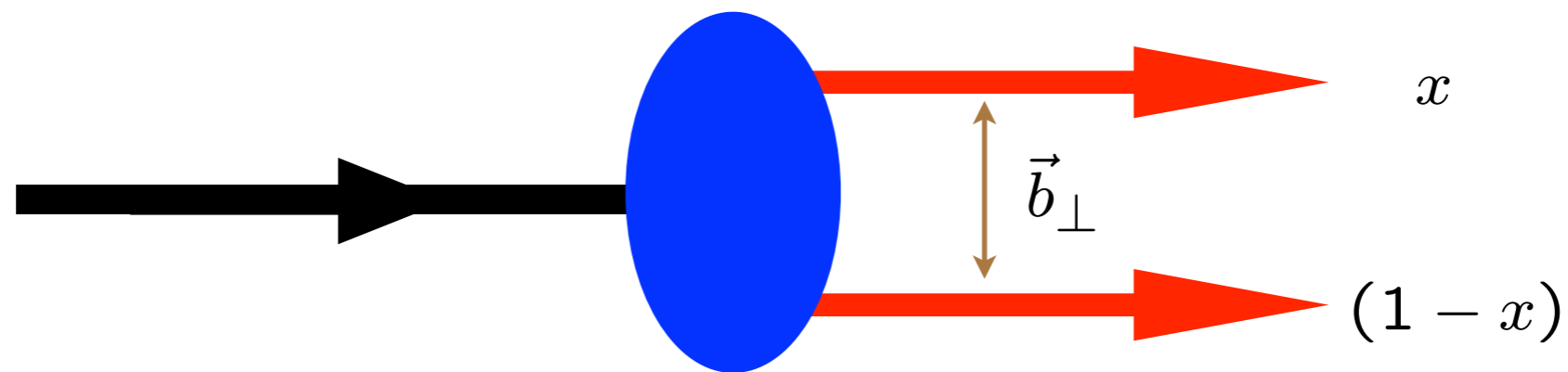
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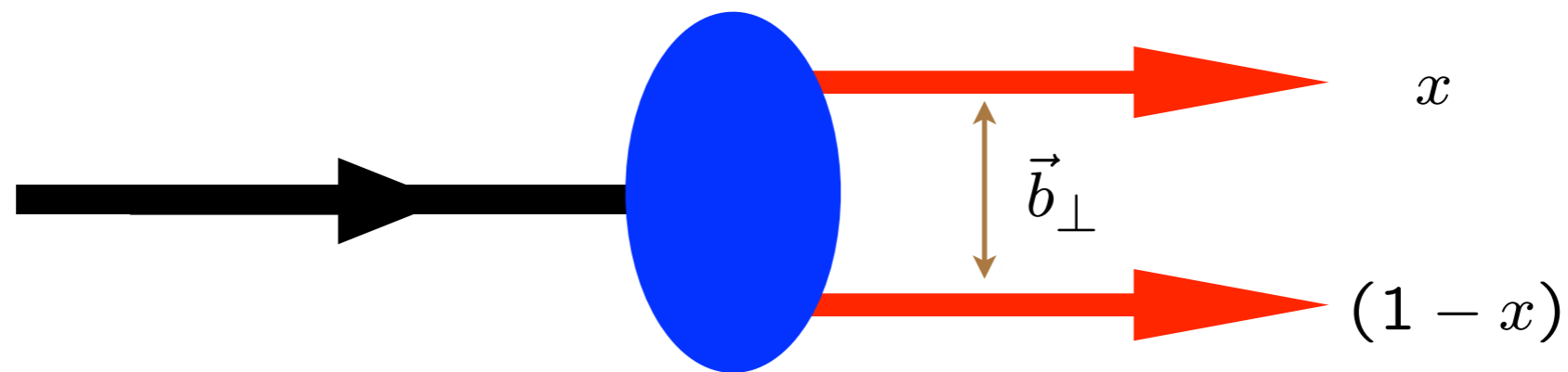
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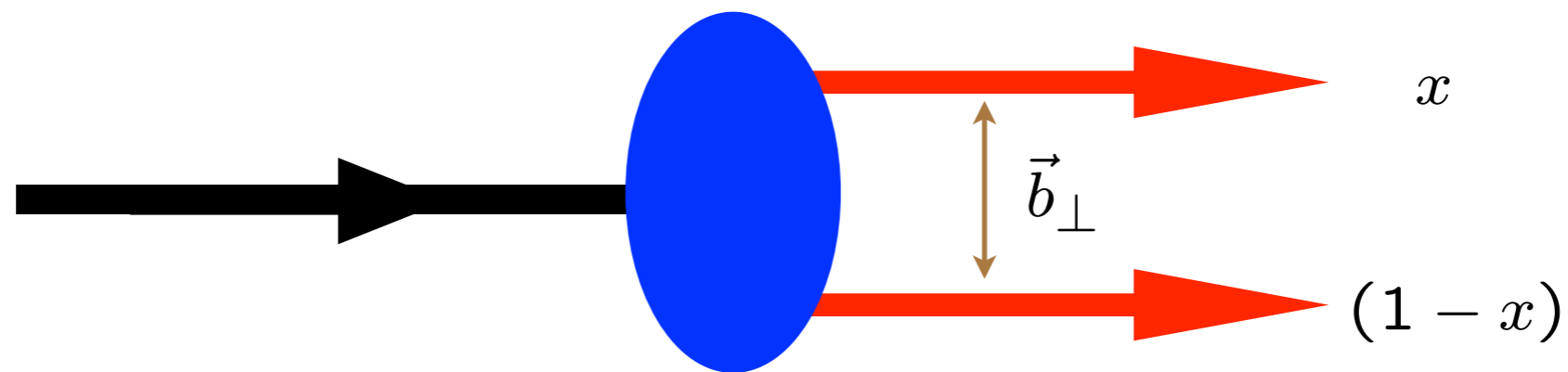
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Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

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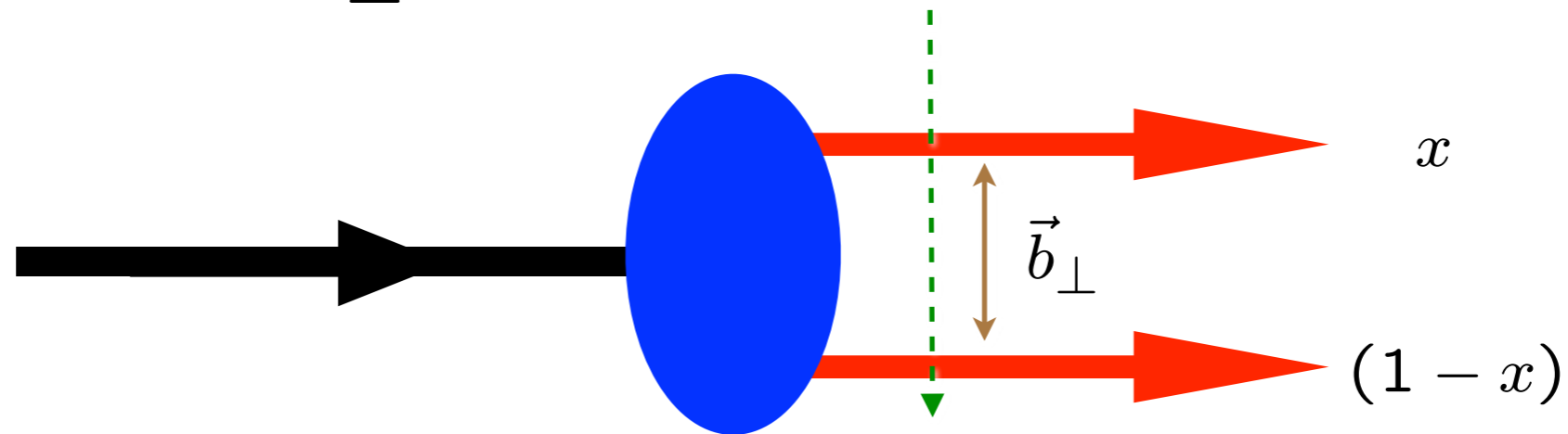
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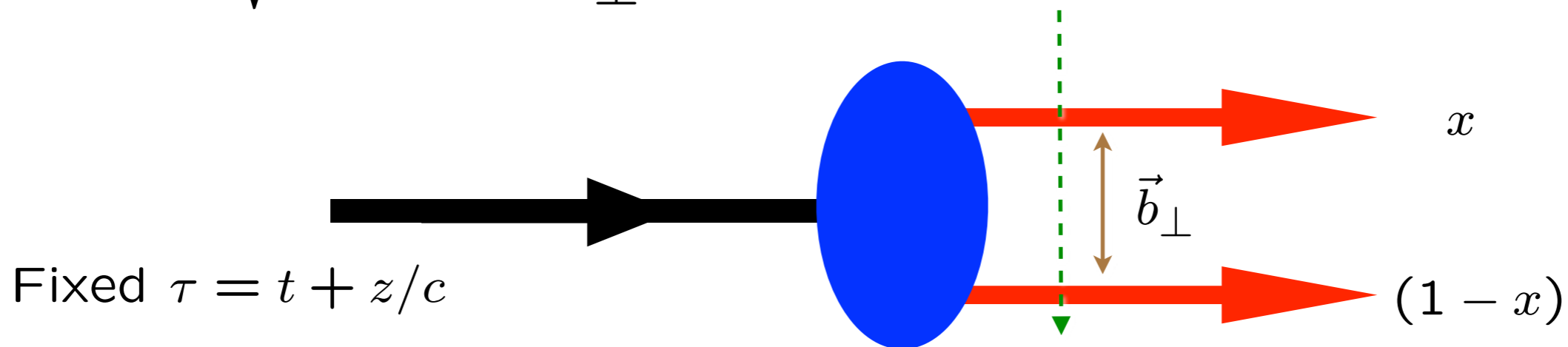
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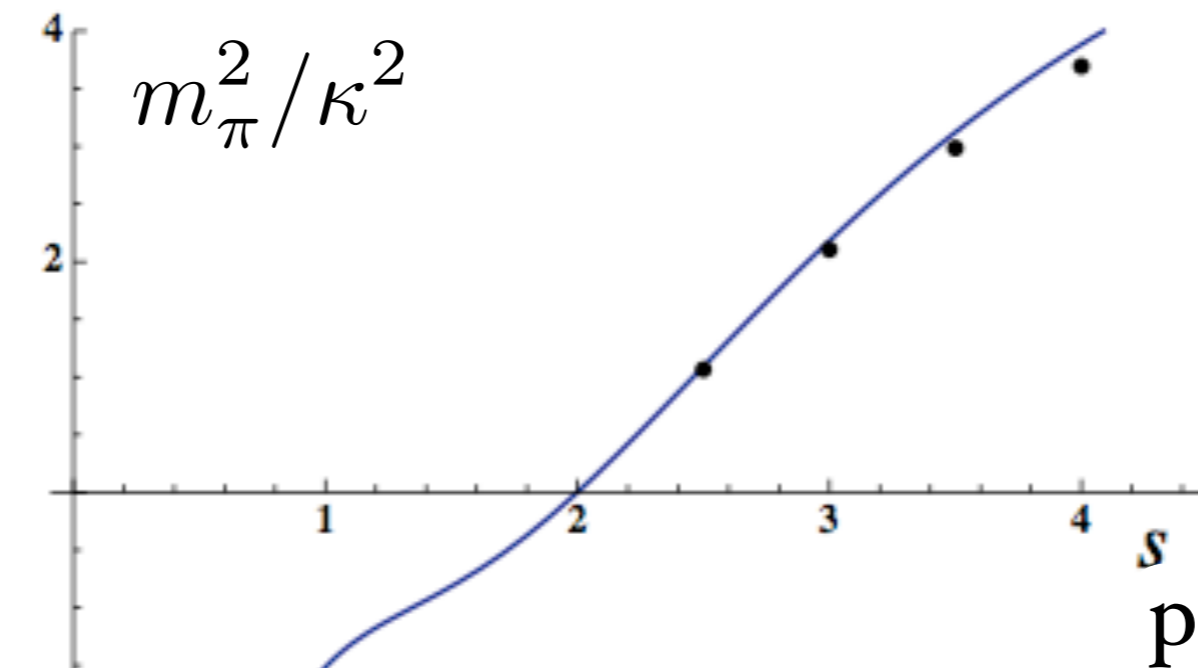
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Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff
 $p=2!$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \quad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \quad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

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Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}$, $S \simeq \sqrt{K}$

Consider $R_w = Q + wS$; w : dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \quad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B$, $w = \kappa^2$

Eigenvalue of G : $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+$$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

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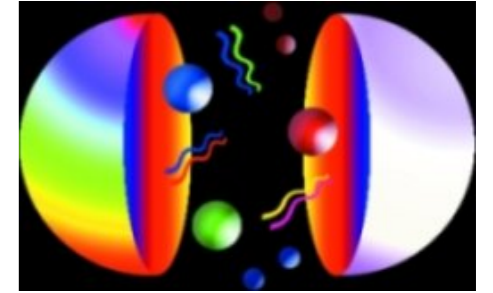
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Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

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- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

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$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

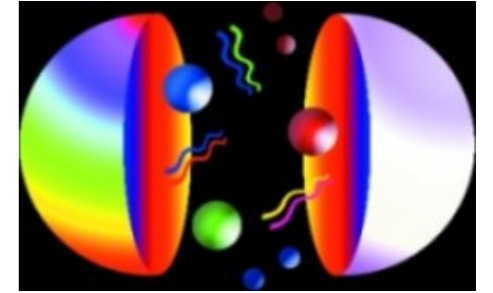
- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

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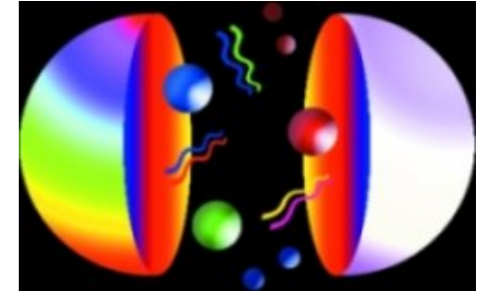
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Symmetry of
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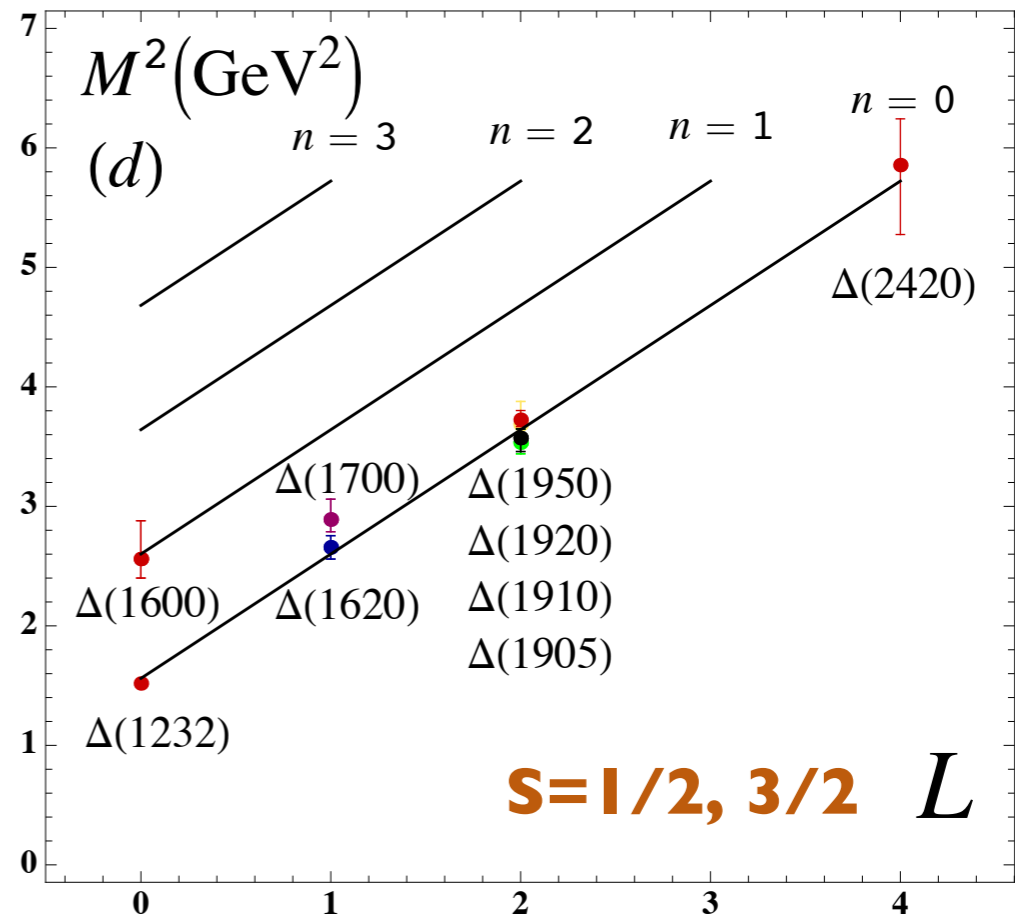
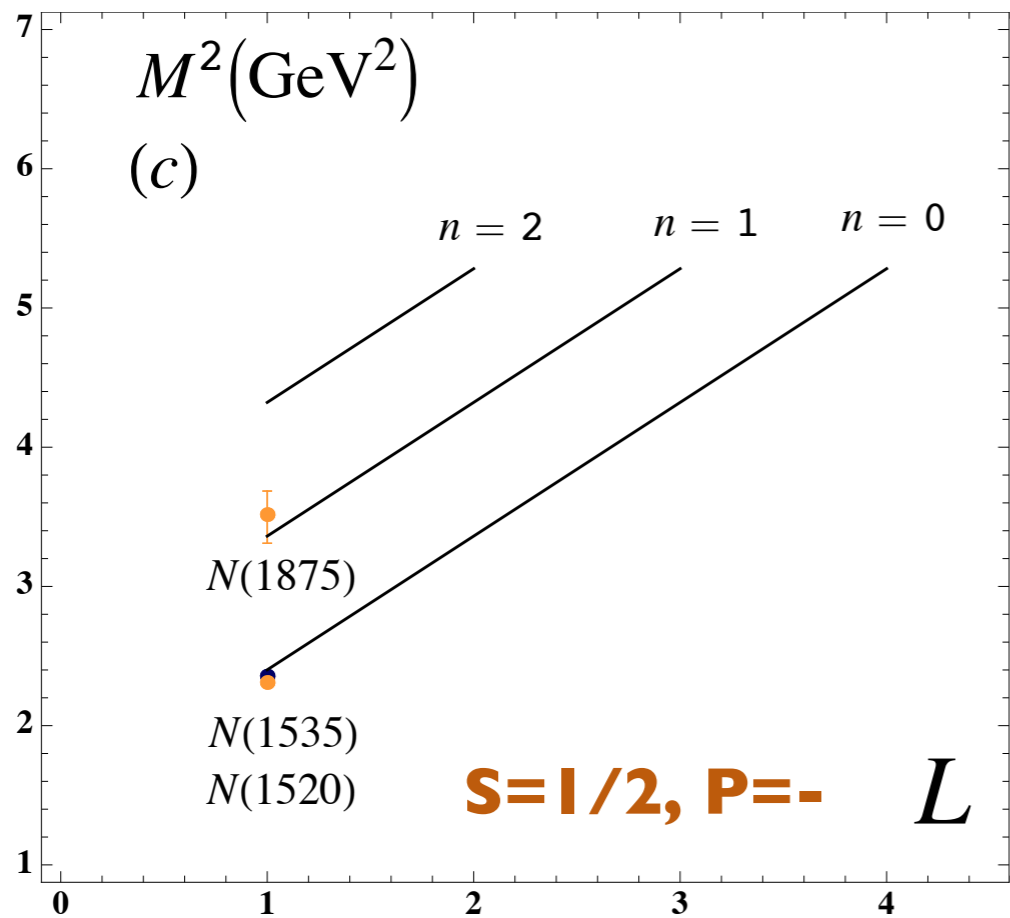
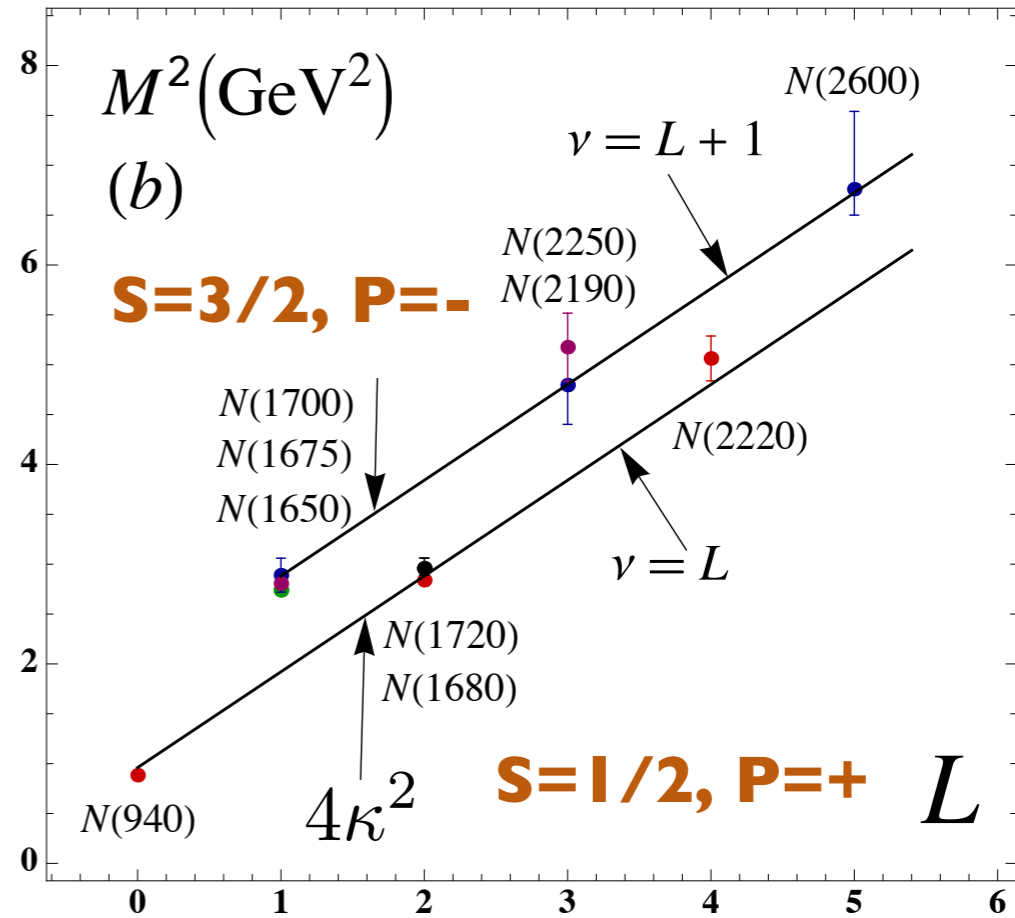
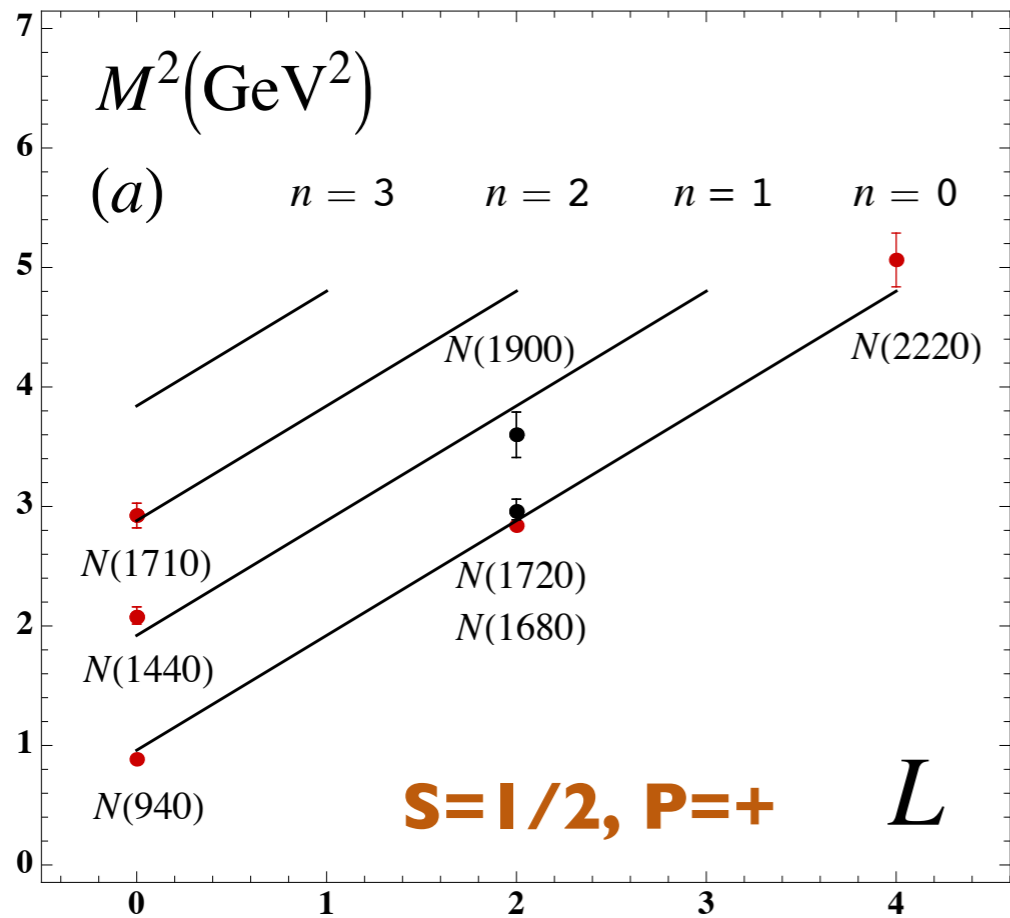
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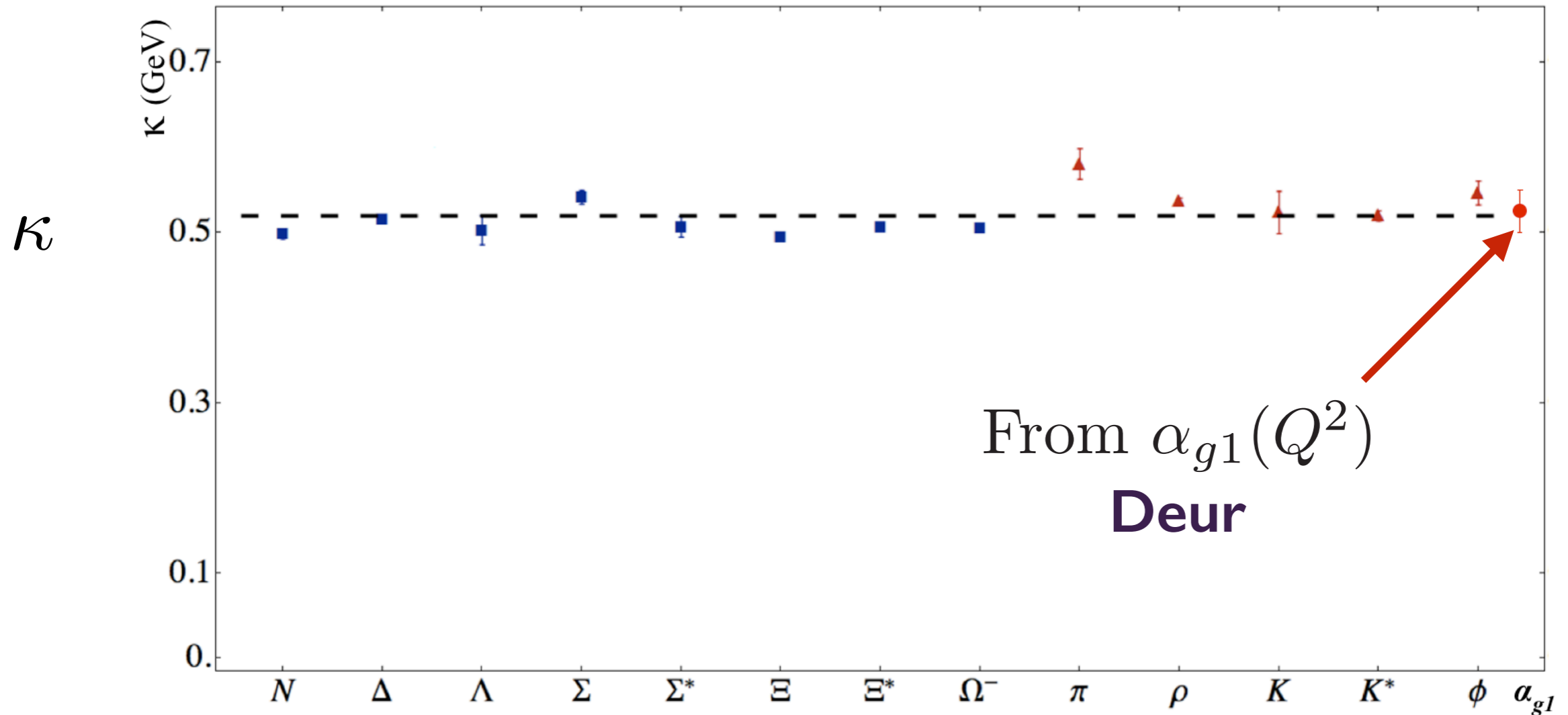
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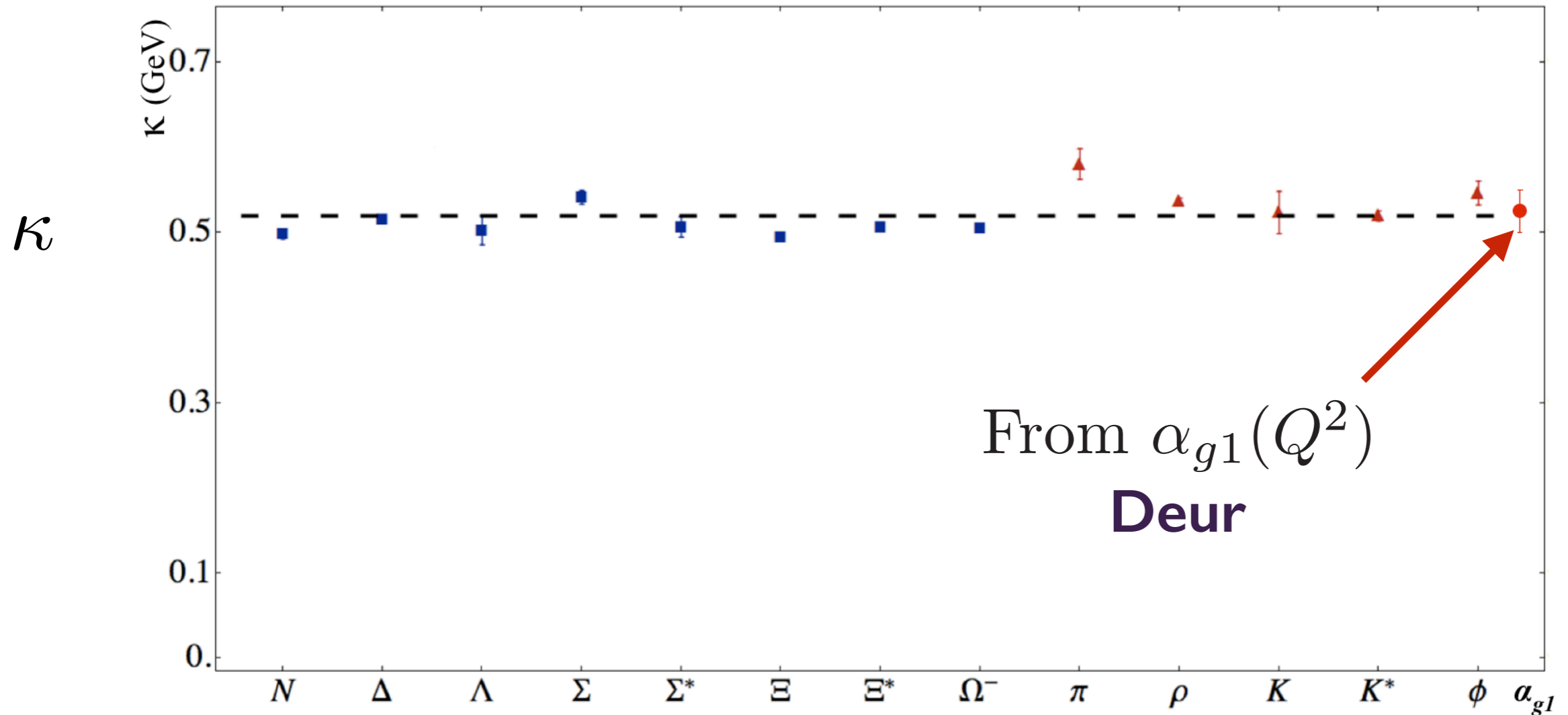
Nucleon: Equal Probability for L=0, 1





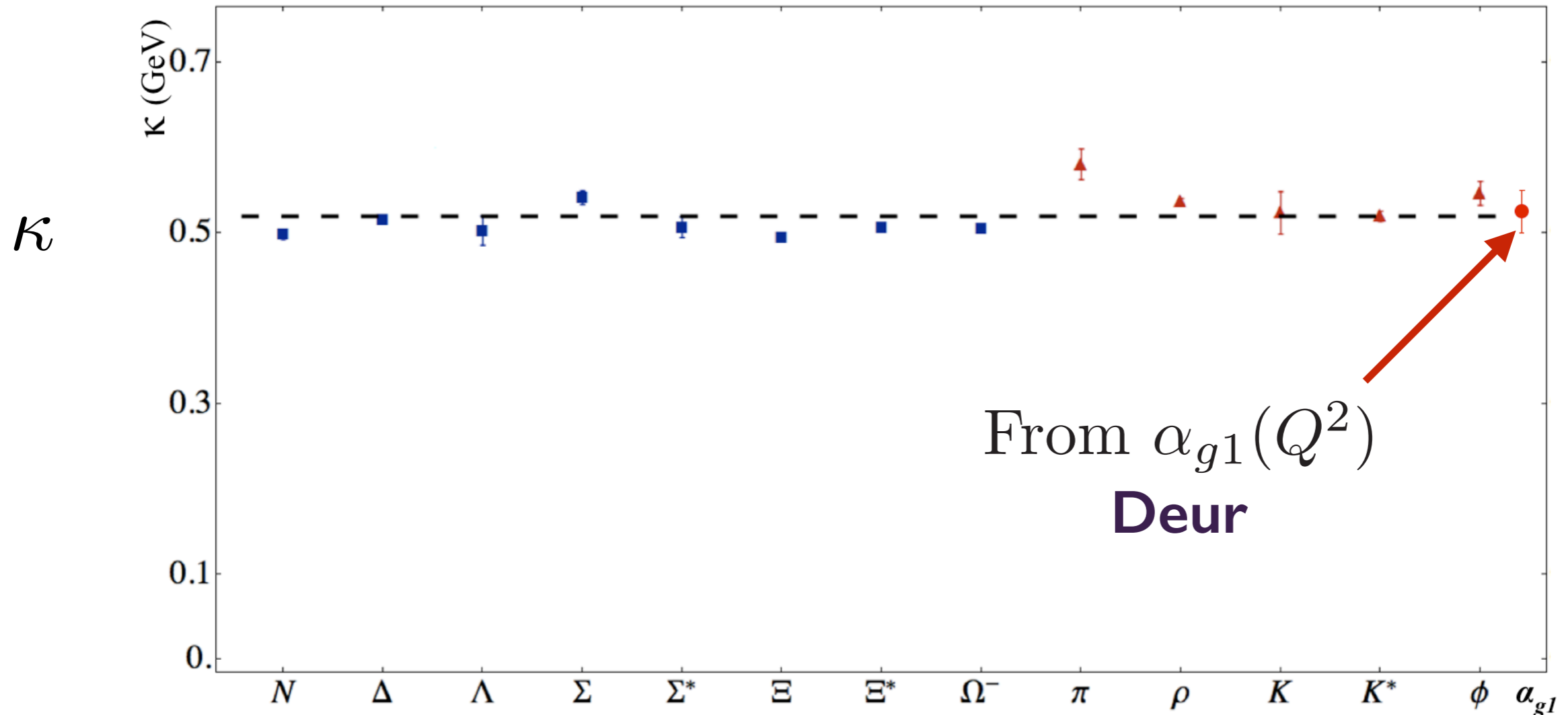
**Fit to the slope of Regge trajectories,
including radial excitations**

$$m_u = m_d = 46 \text{ MeV}, m_s = 357 \text{ MeV}$$



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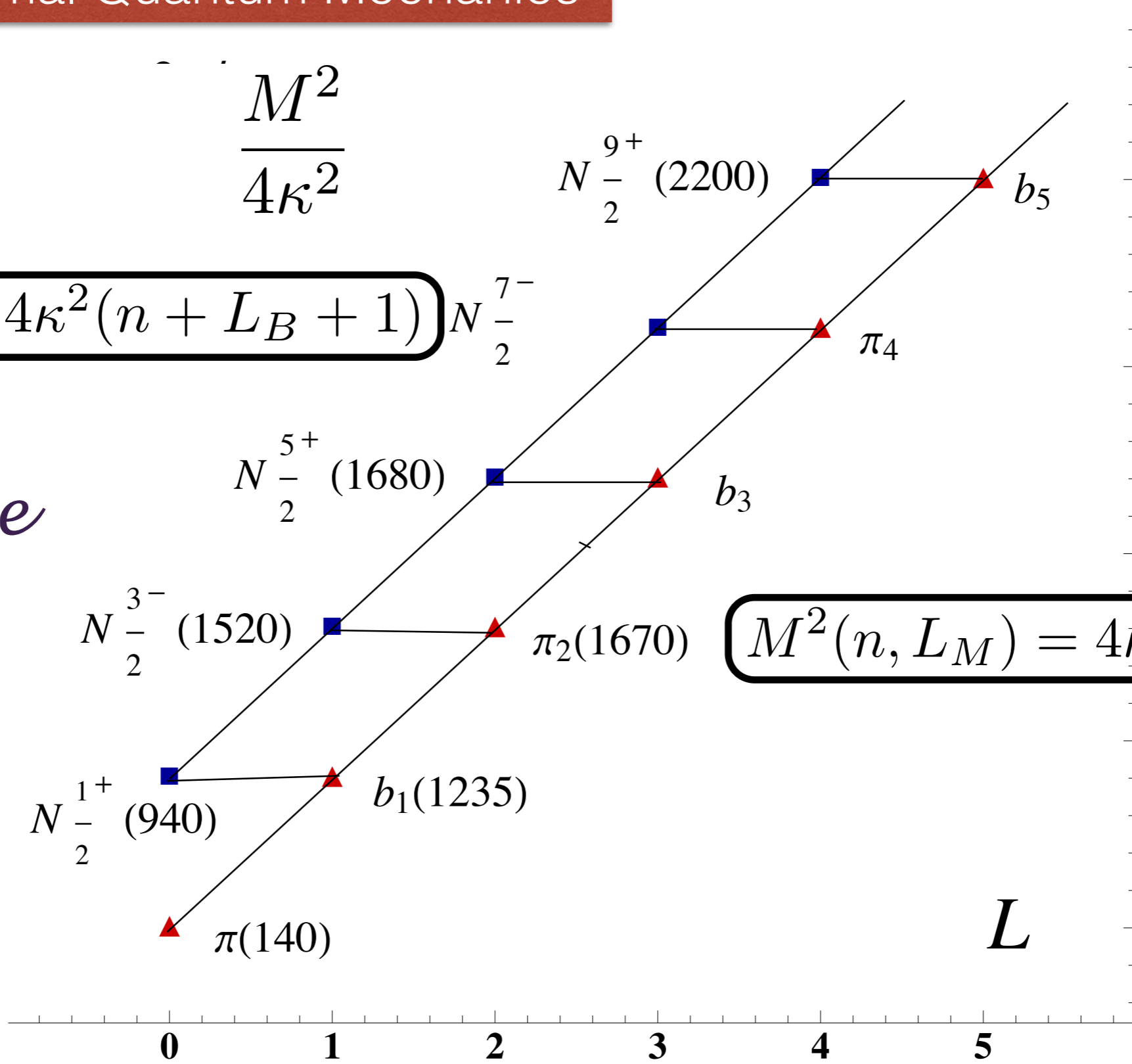
**Fit to the slope of Regge trajectories,
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**Same Regge Slope for Meson, Baryons:
Supersymmetric feature of hadron physics**

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

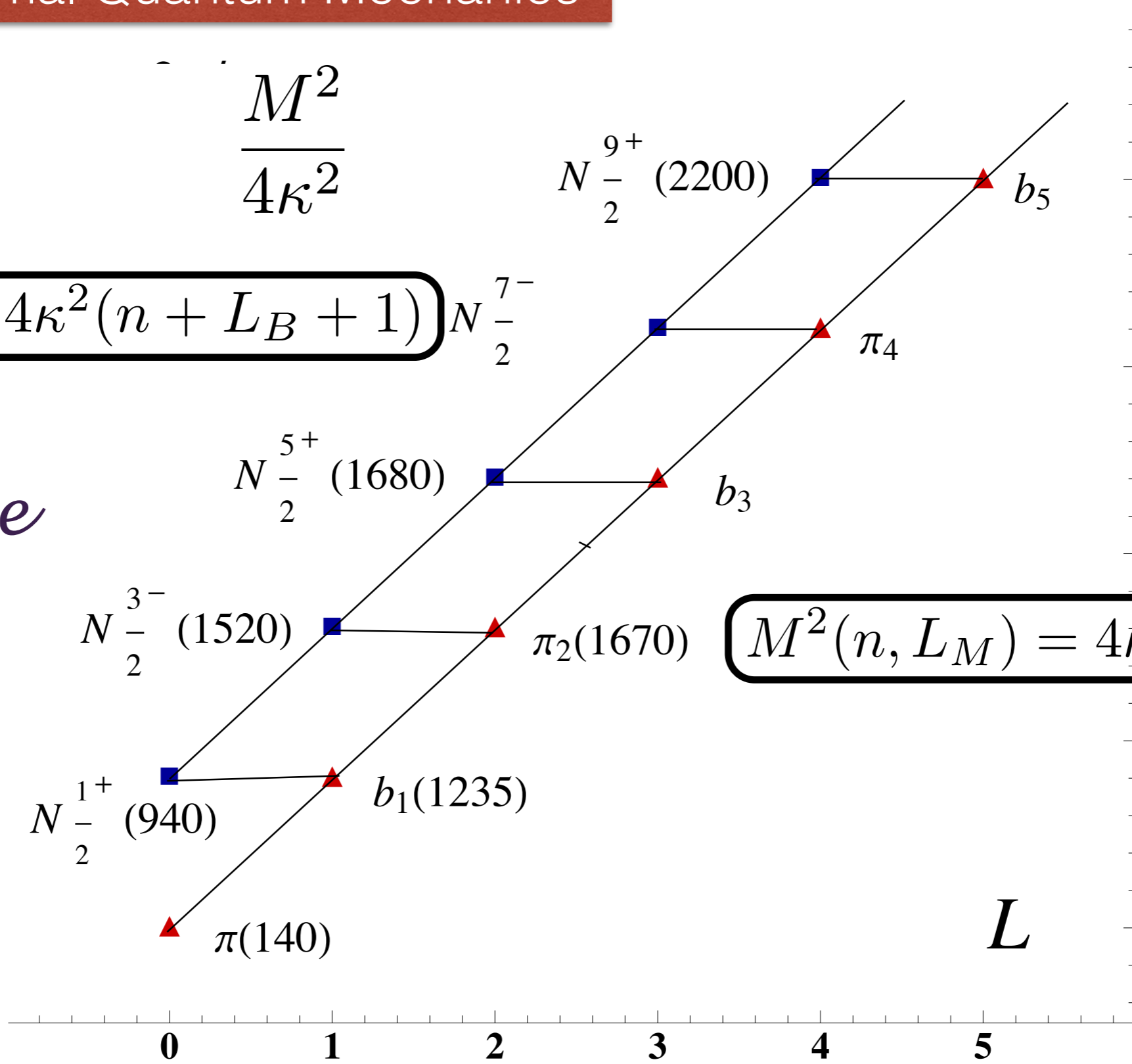
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**Meson-Baryon
Mass Degeneracy
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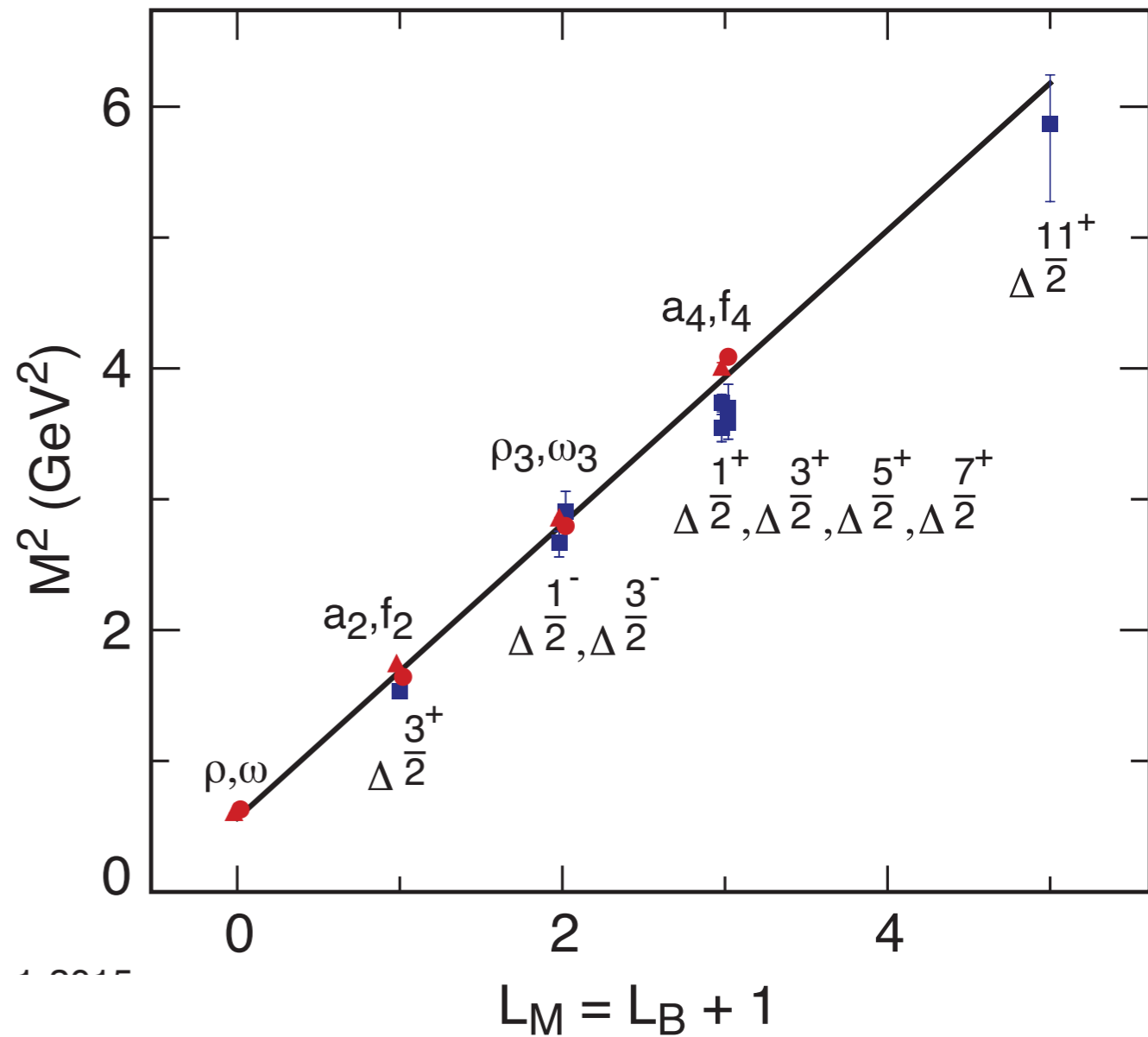
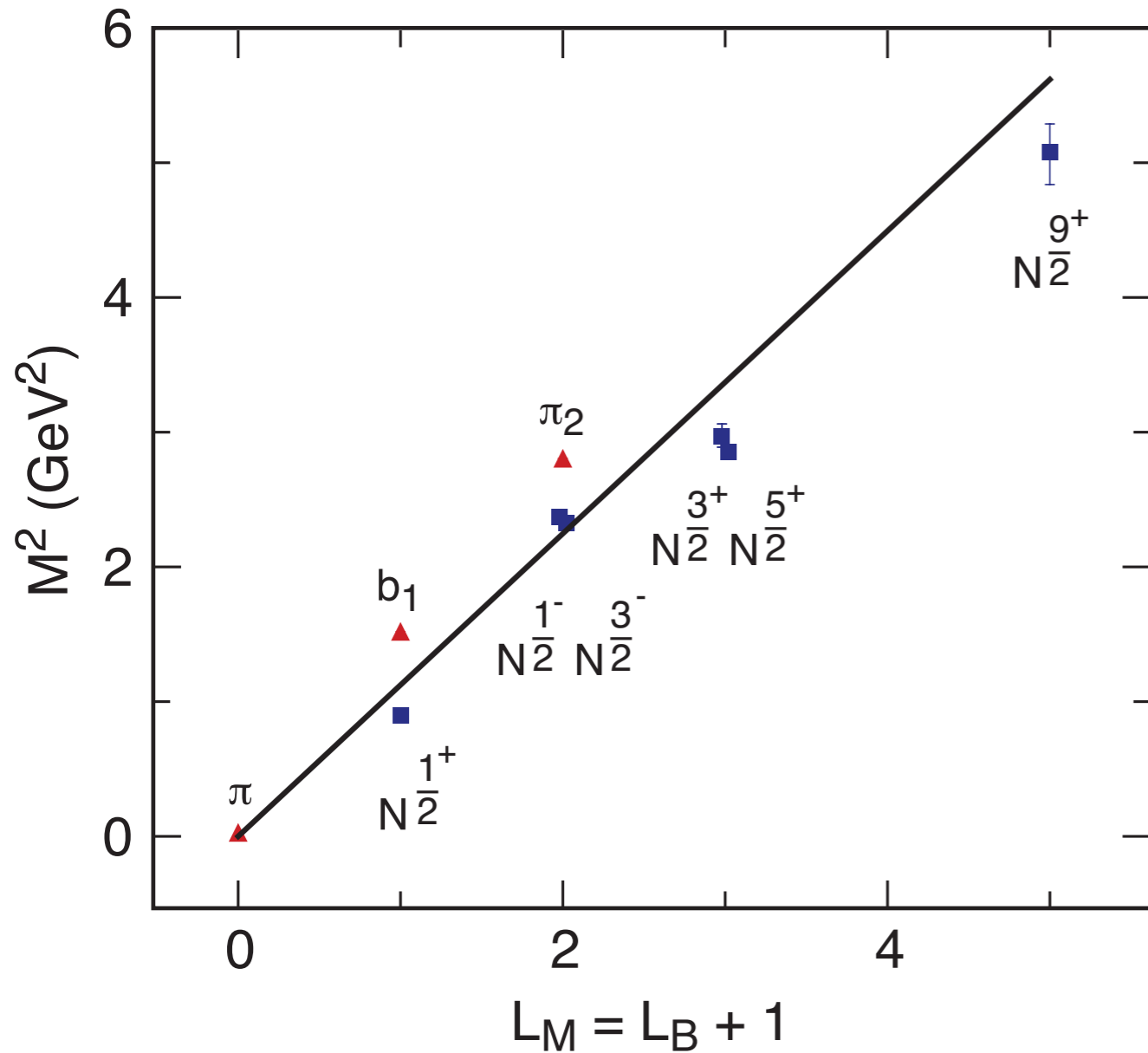


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$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

**Meson-Baryon
Mass Degeneracy
for L_M=L_B+1**

Solid line: $\kappa = 0.53 \text{ GeV}$



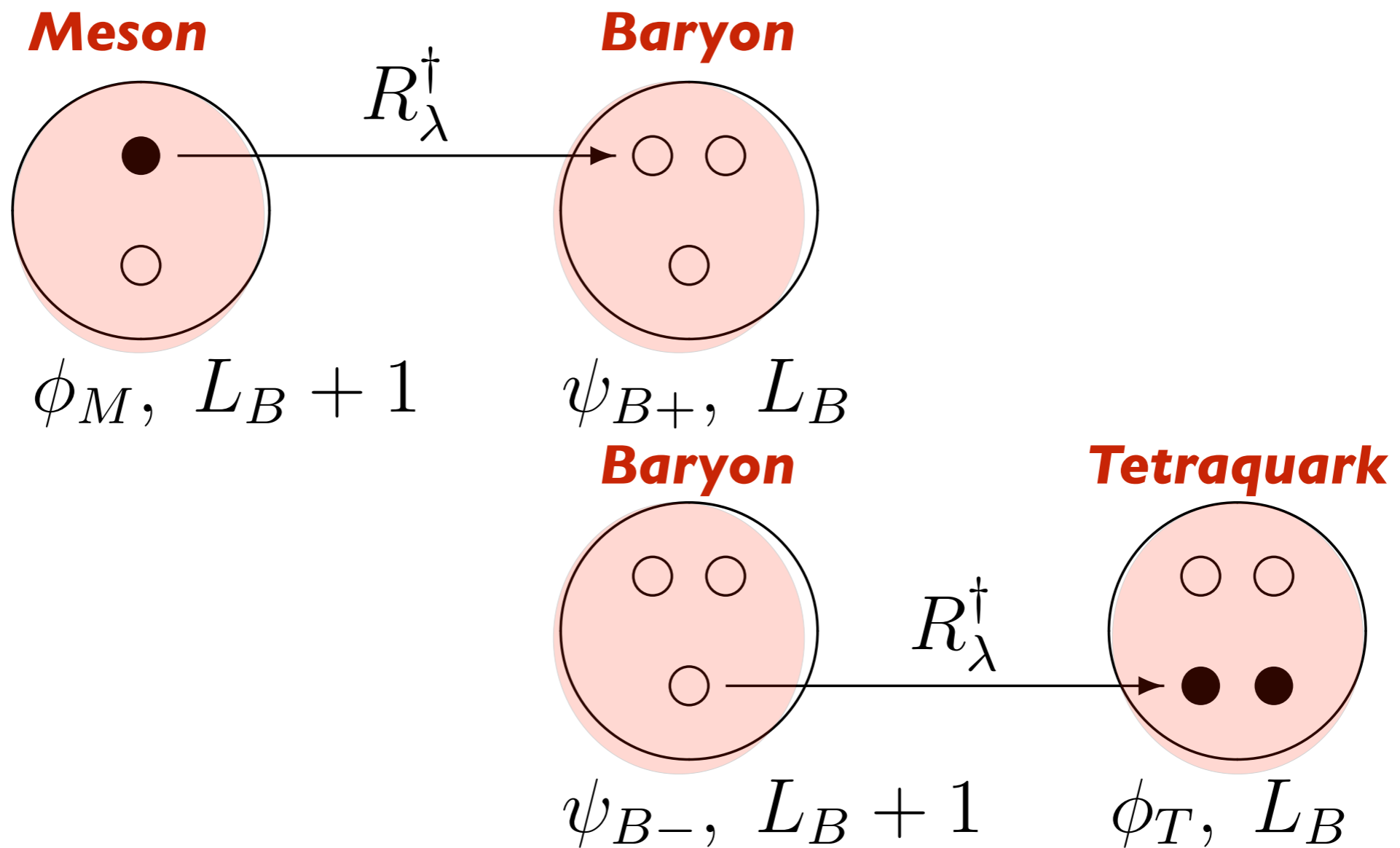
Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: quark + scalar diquark $|q(qq)\rangle$
(Equal weight: $L = 0, L = 1$)

Features of Supersymmetric Equations

- $J = L + S$ baryon simultaneously satisfies both equations of G with L , $L + 1$ with same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2 \quad S^z = \pm 1/2$

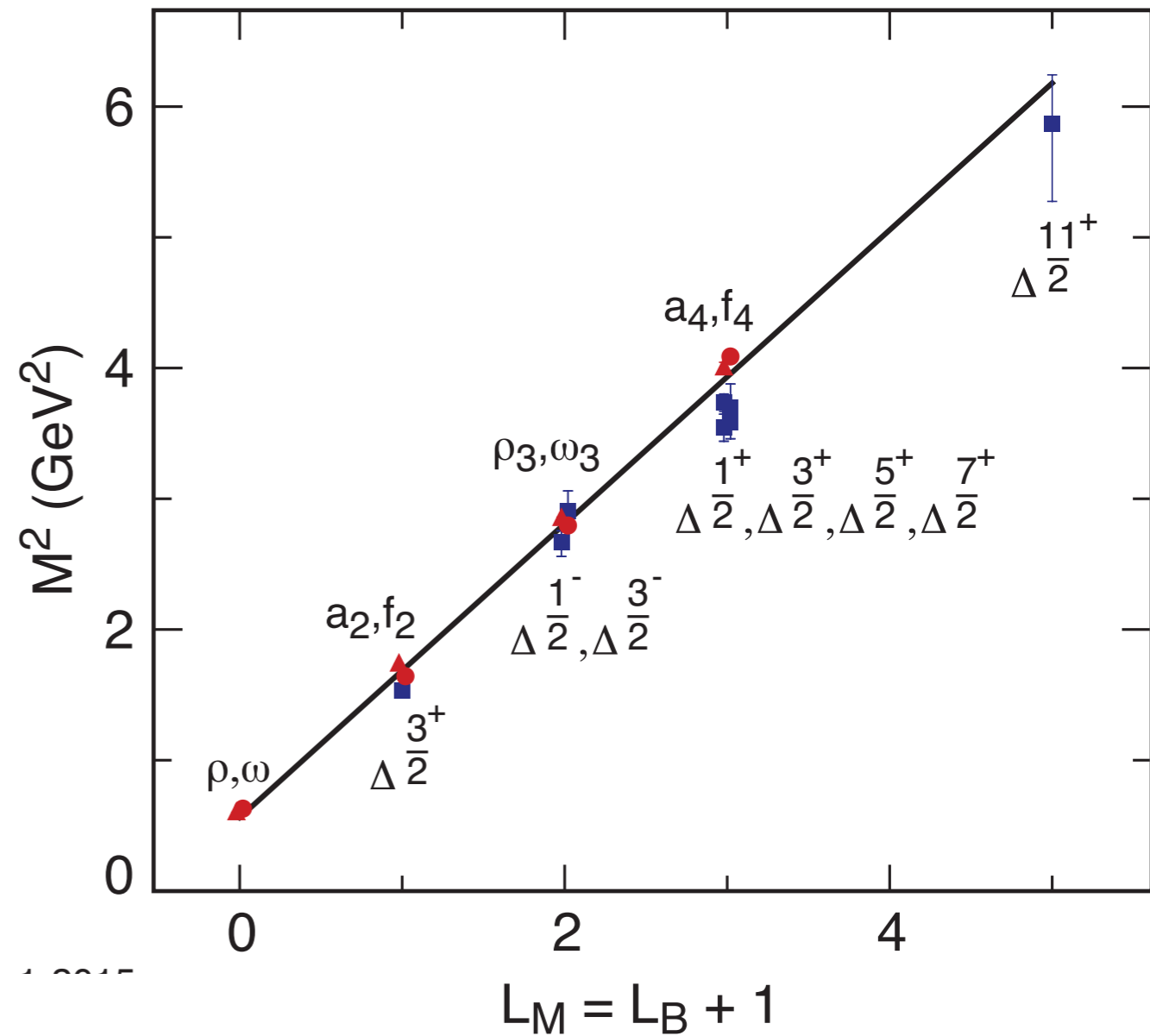
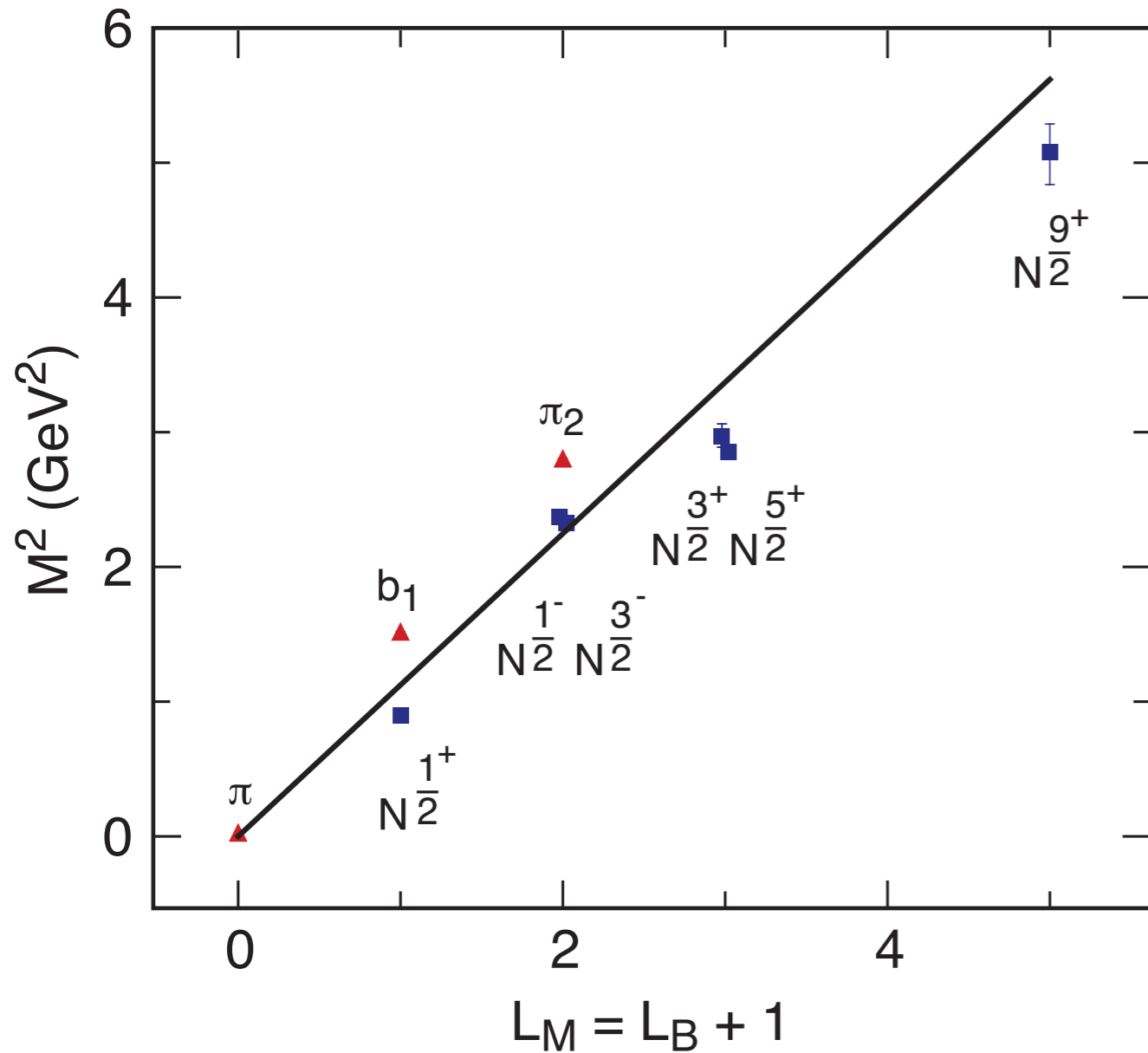
- Proton spin carried by quark L^z

$$\langle J^z \rangle = \frac{1}{2} (S_q^z = \frac{1}{2}, L^z = 0) + \frac{1}{2} (S_q^z = -\frac{1}{2}, L^z = 1) = \langle L^z \rangle = \frac{1}{2}$$

- Mass-degenerate meson “superpartner” with $L_M = L_B + 1$. *“Shifted meson-baryon Duality”*

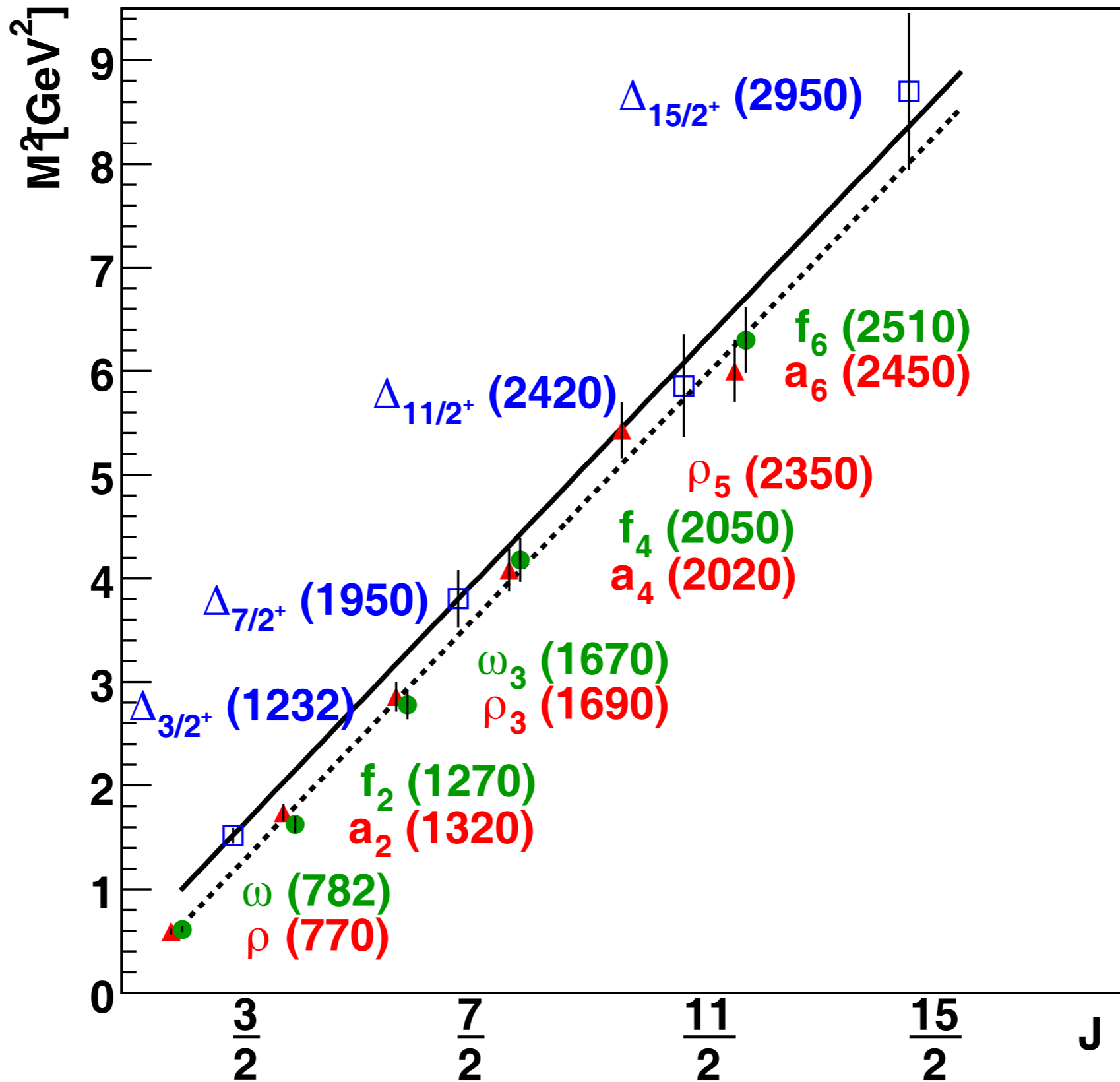
Mesons and baryons have same κ !

Solid line: $\kappa = 0.53 \text{ GeV}$



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The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with $J = L + S$.

Superconformal Algebra

2X2 Hadronic Multiplets

$$\begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi_{B-}(L_B + 1) \\ \psi_{B+}(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$

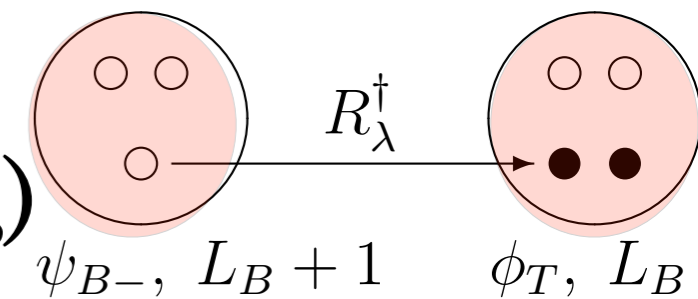
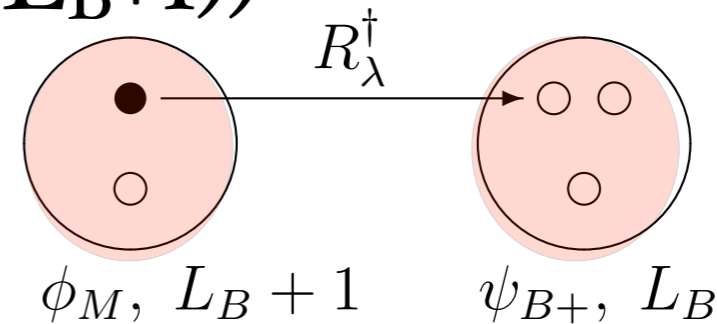
- quark-antiquark meson ($L_M = L_B + 1$)

- quark-diquark baryon (L_B)

- quark-diquark baryon ($L_B + 1$)

- diquark-antidiquark tetraquark ($L_T = L_B$)

- Universal Regge slopes $\lambda = \kappa^2$



$$M_H^2/\lambda = \underbrace{(2n + L_H + 1)}_{\text{kinetic}} + \underbrace{(2n + L_H + 1)}_{\text{potential}} + \underbrace{2(L_H + s) + 2\chi}_{\text{contribution from AdS and superconformal algebra}} + \left\langle \sum_i \frac{m_i^2}{x_i} \right\rangle$$

$$\chi(\text{mesons}) = -1$$

$$\chi(\text{baryons, tetraquarks}) = +1$$

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

Bound!

- Diquark: Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states $\bar{3}_C \times 3_C = 1_C$

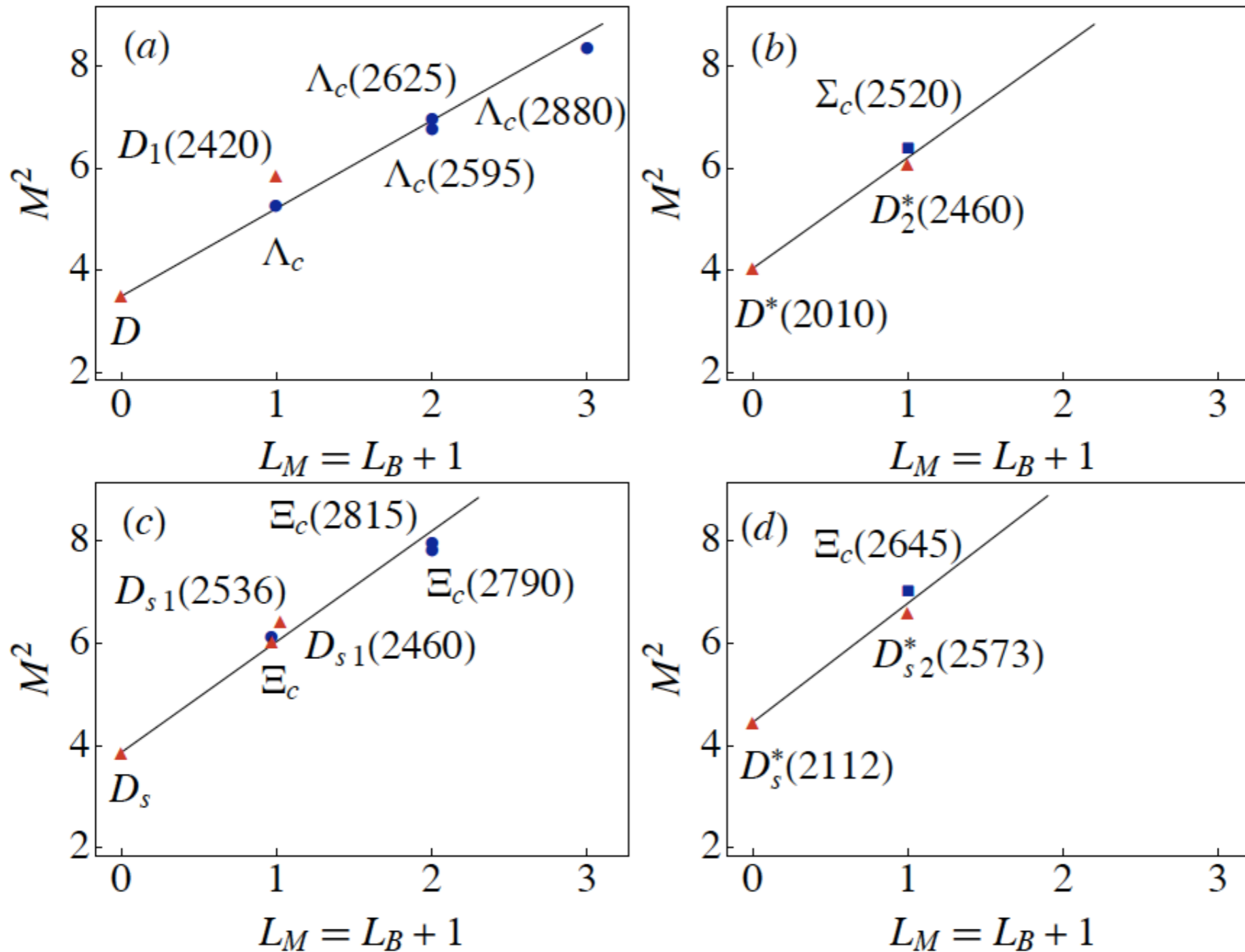
$$\sigma(TN) \simeq 2\sigma(pN) - \sigma(\pi N)$$

$$2[\sigma(\{qq\}N) + \sigma(qN)] - [\sigma(qN) + \sigma(\bar{q}N)] = [\sigma(\{qq\}N) + \sigma(\{qq\}N)]$$

Candidates $f_0(980)I = 0, J^P = 0^+$, partner of proton

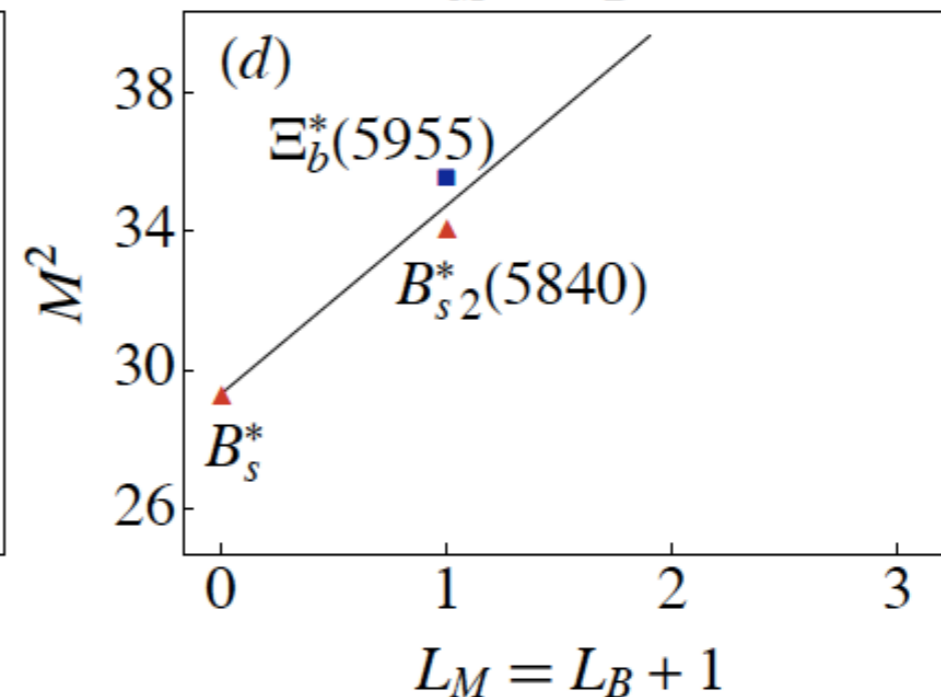
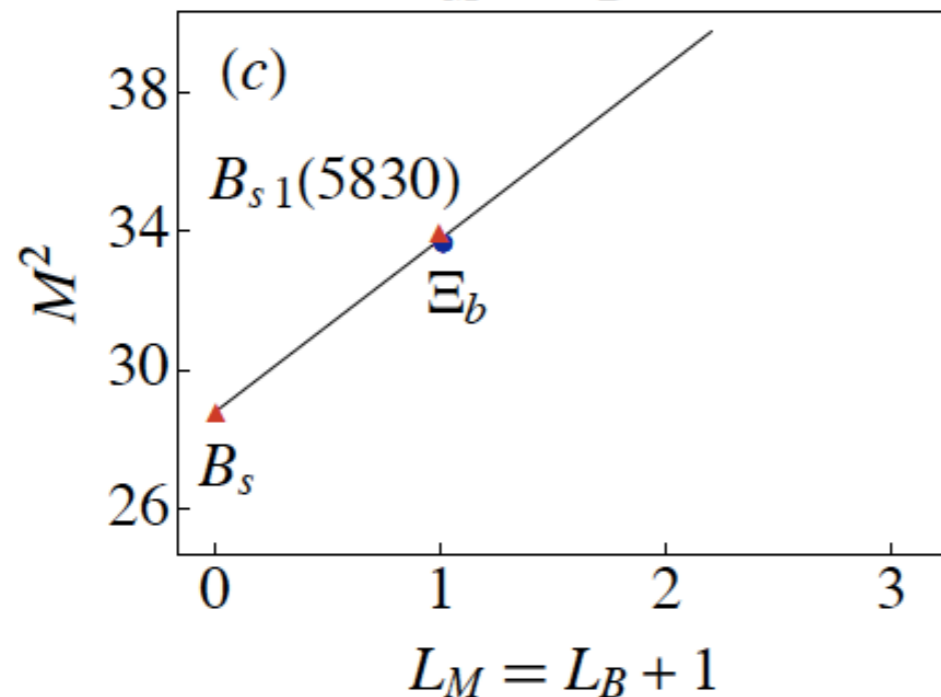
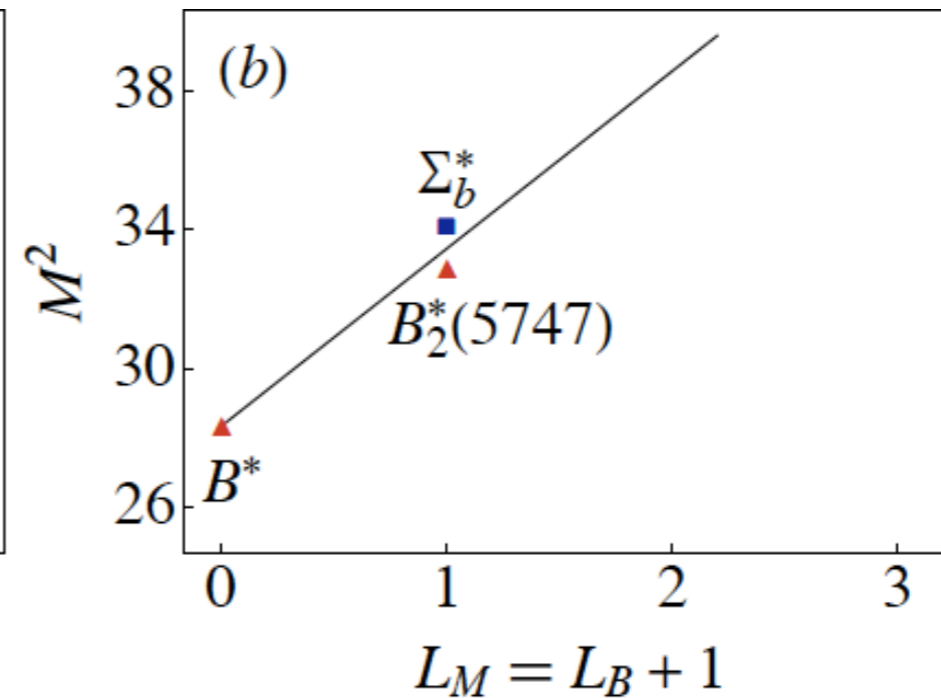
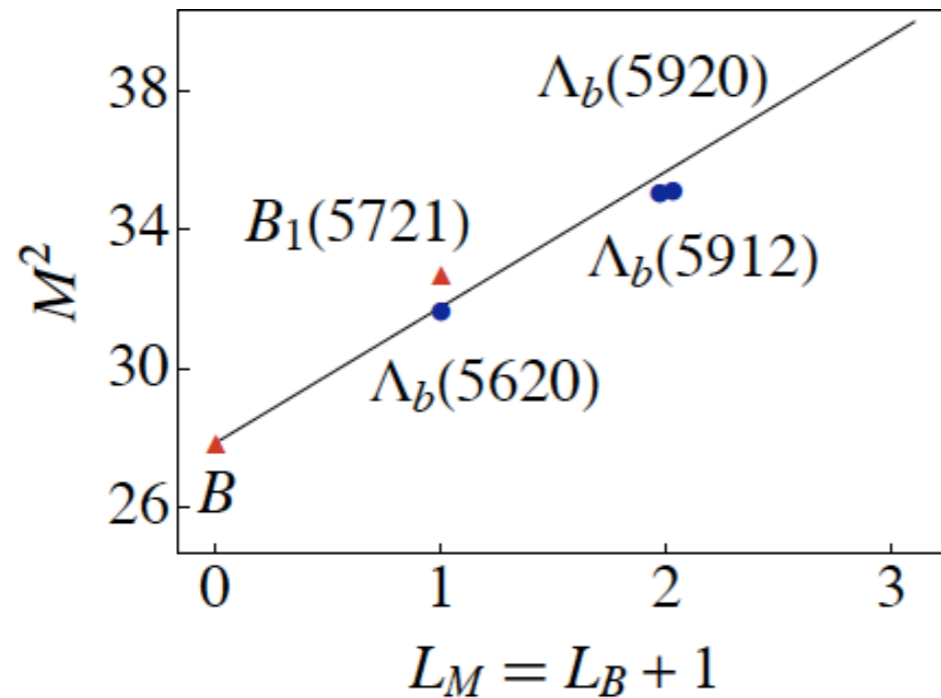
$a_1(1260)I = 0, J^P = 1^+$, partner of $\Delta(1233)$

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

Foundations of Light-Front Holography

- **The QCD Lagrangian for $m_q = 0$ has no mass scale.**
- **What determines the hadron mass scale?**
- **DAFF principle: add terms linear in D and K to Conformal Hamiltonian: Mass scale κ appears, but action remains scale invariant \rightarrow unique harmonic oscillator potential**
- **Apply DAFF to the Poincare' invariant LF Hamiltonian: Unique color-confining potential**
- **Fixes AdS_5 dilaton: predicts Spin and Spin-Orbit Interactions**
- **Apply DAFF to Superconformal representation of the Lorentz group**
- **Predicts Meson, Baryon, Tetraquark spectroscopy, dynamics**
- **Supersymmetric Features of Spectrum**

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$
 $J^z = +1/2 : \langle L^z \rangle = 1/2, \langle S_q^z \rangle = 0$
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$.**

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No mass-degenerate parity partners!

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2} \quad \text{from dilaton } e^{\kappa^2 z^2}$$

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Bjorken sum rule defines effective charge

$$\alpha_{g1}(Q^2)$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large Q^2**
- **Computable at large Q^2 in any pQCD scheme**
- **Universal β_0, β_1**

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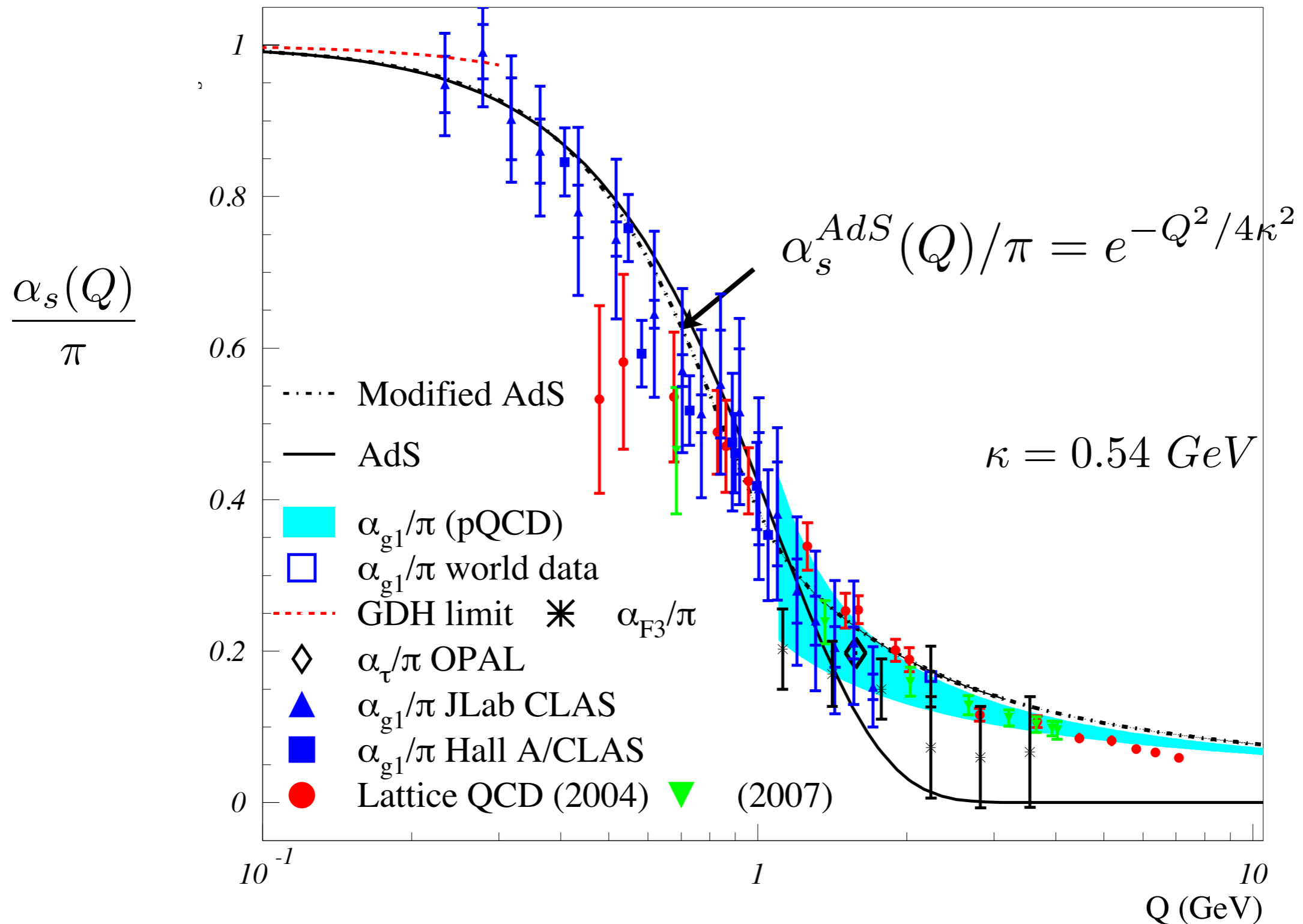
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Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for $Q < 1 \text{ GeV}$

$$e^\varphi = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

All-Scale QCD Coupling

Fit to Bj + DHG Sum Rules:
 $\kappa = 0.513 \pm 0.007 \text{ GeV}$

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$

Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016 \text{ GeV}$$

$$e^{-\frac{Q^2}{4\kappa^2}}$$

Q_0

**Perturbative QCD
(Asymptotic Freedom)**

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$$

Transition scale Q_0

$$Q_0 = 0.87 \pm 0.08 \text{ GeV}$$

\overline{MS} scheme

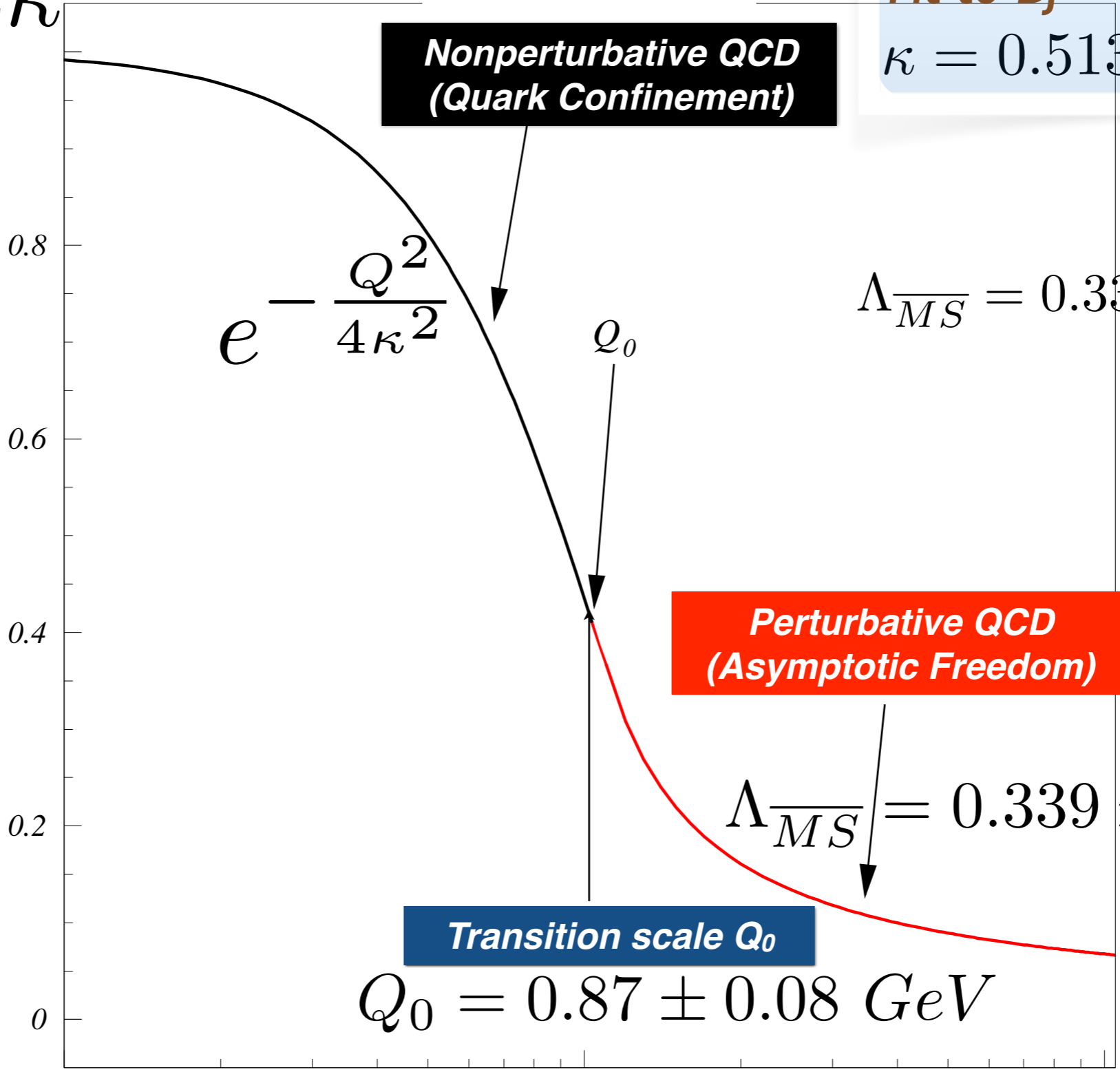
$$\lambda \equiv \kappa^2$$

10^{-1}

1

10

Q (GeV)



$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

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World Data:

$$\Lambda_{\overline{MS}} = 0.332 \pm 0.019 \text{ GeV}$$

**Perturbative QCD
(Asymptotic Freedom)**

Prediction

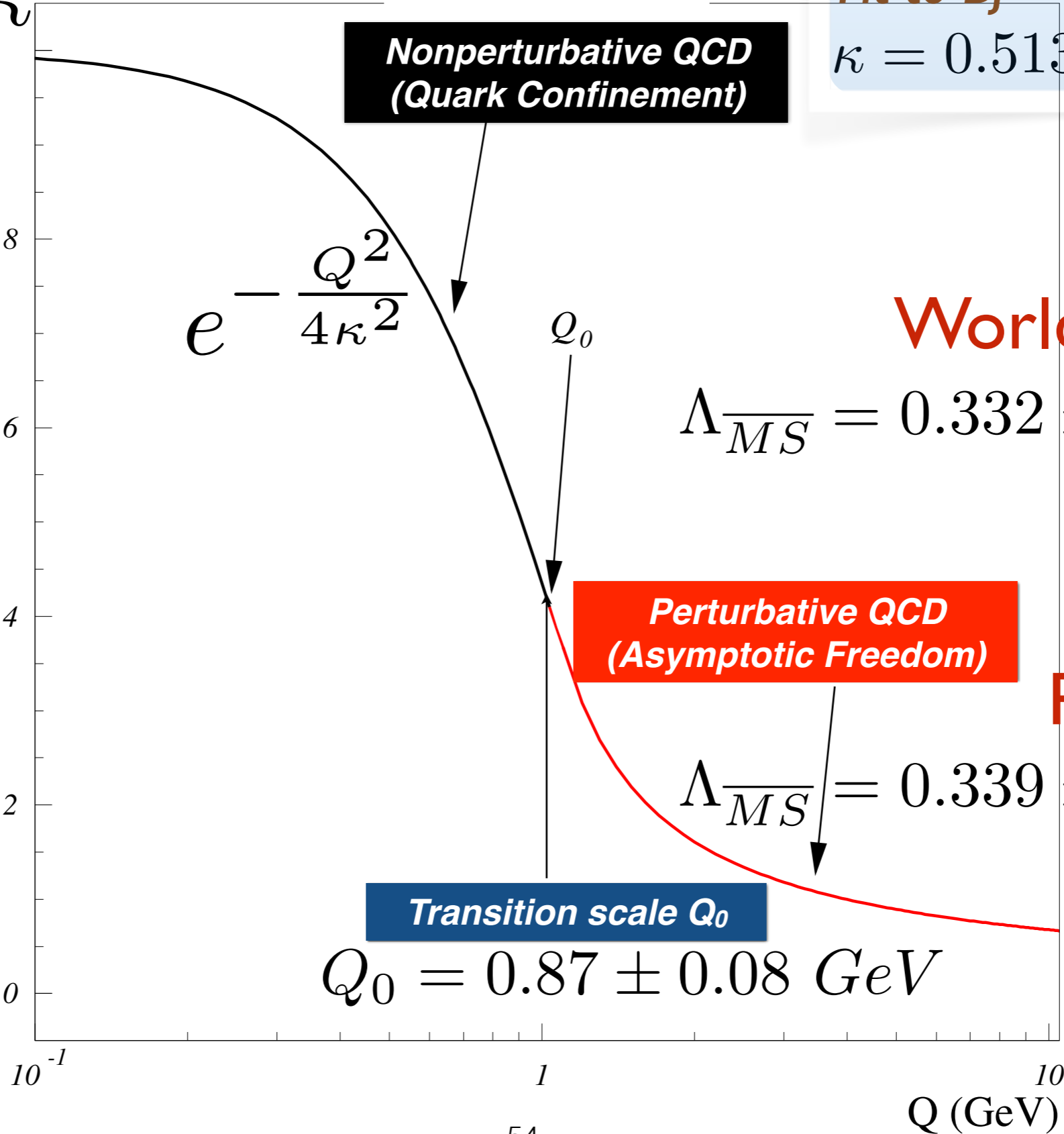
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Use Q_0 for starting DGLAP and ERBL Evolution

Nonperturbative QCD (Quark Confinement)

Perturbative QCD (Asymptotic Freedom)

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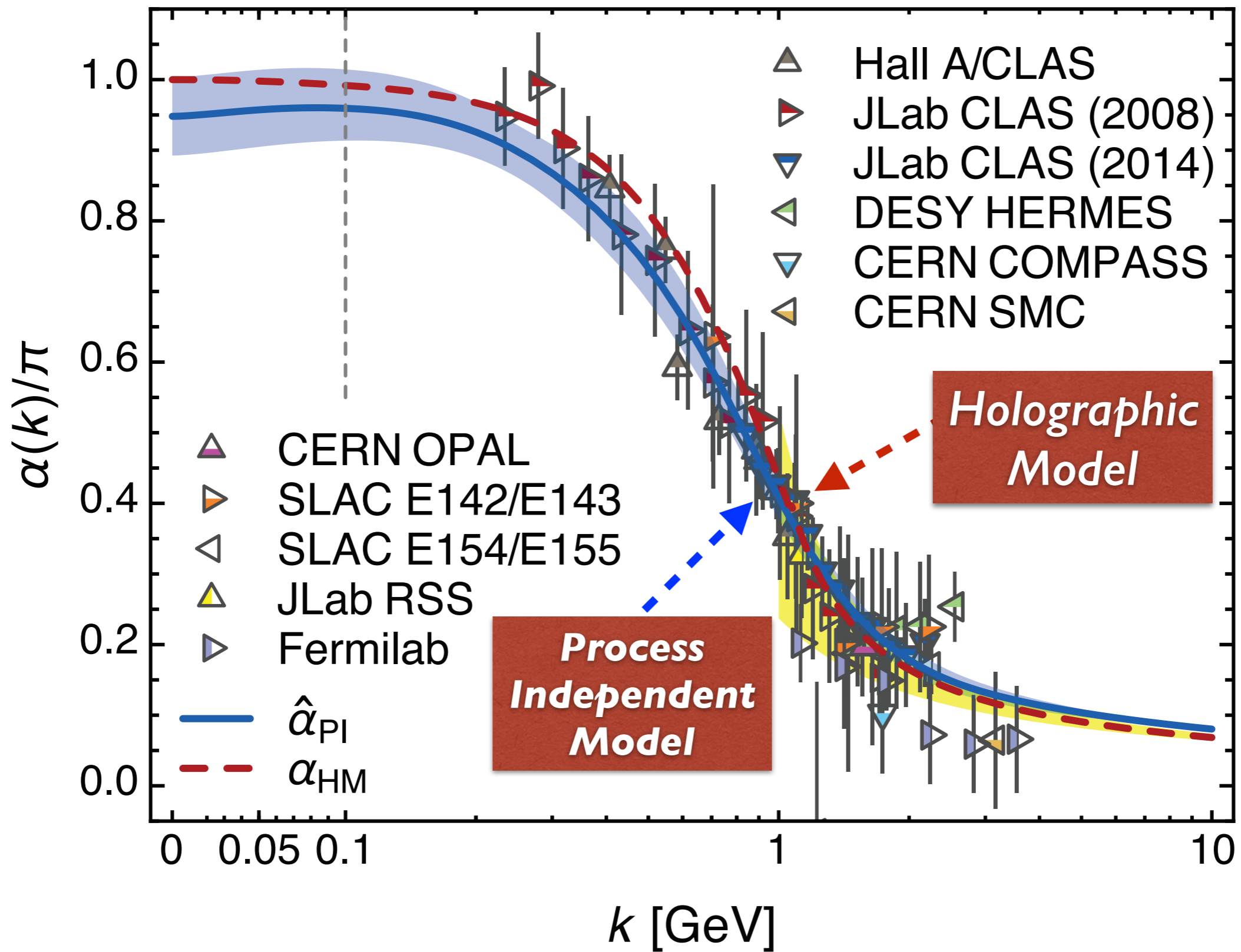
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Q (GeV)

$$\lambda \equiv \kappa^2$$



Process-independent strong running coupling

Features of LF Holographic QCD

- **Regge spectroscopy—same slope in n, L for mesons, baryons**
- **Chiral features for $m_q=0$: $m_\pi=0$, chiral-invariant proton**
- **Hadronic LFWFs**
- **Counting Rules**
- **Connection between hadron masses and $\Lambda_{\overline{MS}}$**

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

“In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Similarly for m_ρ .

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π , the field theory community will hail you as a conquering hero who has solved QCD exactly.”

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$$m_p \simeq 3.21 \Lambda_{\overline{MS}}$$

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$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

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$$(m_q = 0)$$

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Fundamental Hadronic Features of Hadrons

- Partition of the Proton's Mass: Potential vs. Kinetic Contributions
- Color Confinement
- Role of Quark Orbital Angular Momentum in the Proton
- Quark-Diquark Structure
- Quark Mass Contribution
- Baryonic Regge Trajectory
- Mesonic Supersymmetric Partners
- Proton Light-Front Wavefunctions and Dynamical Observables
- Form Factors, Distribution Amplitudes, Structure Functions
- Non-Perturbative - Perturbative QCD Transition
- Dimensional Transmutation: $M_p/\Lambda_{\overline{MS}}$

APS-GHP Workshop
February 3, 2017

Supersymmetric Features of QCD
from LF Holography

Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY



Fundamental Hadronic Features of Hadrons

Partition of the Proton's Mass: Potential vs. Kinetic Contributions

Virial Theorem

$$\Delta\mathcal{M}_{LFKE}^2 = \kappa^2(1 + 2n + L)$$
$$\Delta\mathcal{M}_{LFPE}^2 = \kappa^2(1 + 2n + L)$$

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- Quark Mass Contribution $\Delta M^2 = \langle \frac{m_q^2}{x} \rangle$ *from the Yukawa coupling to the Higgs zero mode*
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 $\Delta \mathcal{M}_{LFPE}^2 = \kappa^2(1 + 2n + L)$
- Role of Quark Orbital Angular Momentum in the Proton Equal $L=0, 1$
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- Quark Mass Contribution $\Delta M^2 = \langle \frac{m_q^2}{x} \rangle$ *from the Yukawa coupling to the Higgs zero mode*
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- Dimensional Transmutation: $m_p \simeq 3.21 \Lambda_{\overline{MS}}$ $m_\rho \simeq 2.2 \Lambda_{\overline{MS}}$

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks



Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

where $F_1^p(0) = 1$, $F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}(\kappa^2 z^2) e^{-\kappa^2 z^2/2}$$

- Normalization ($F_1^p(0) = 1$, $V(Q=0, z) = 1$)

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

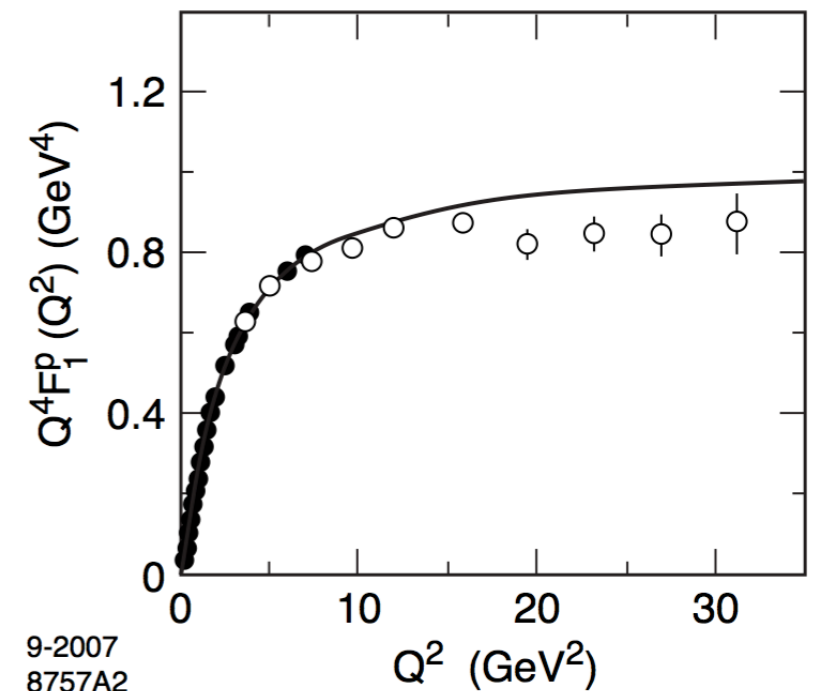
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

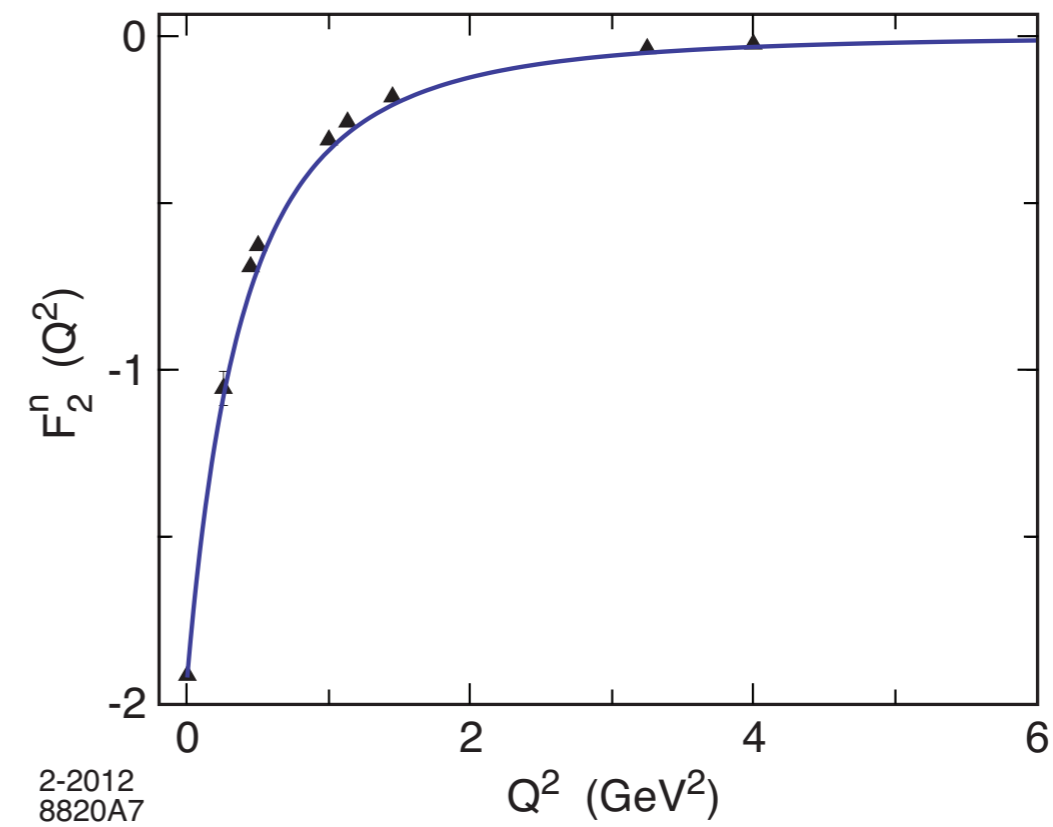
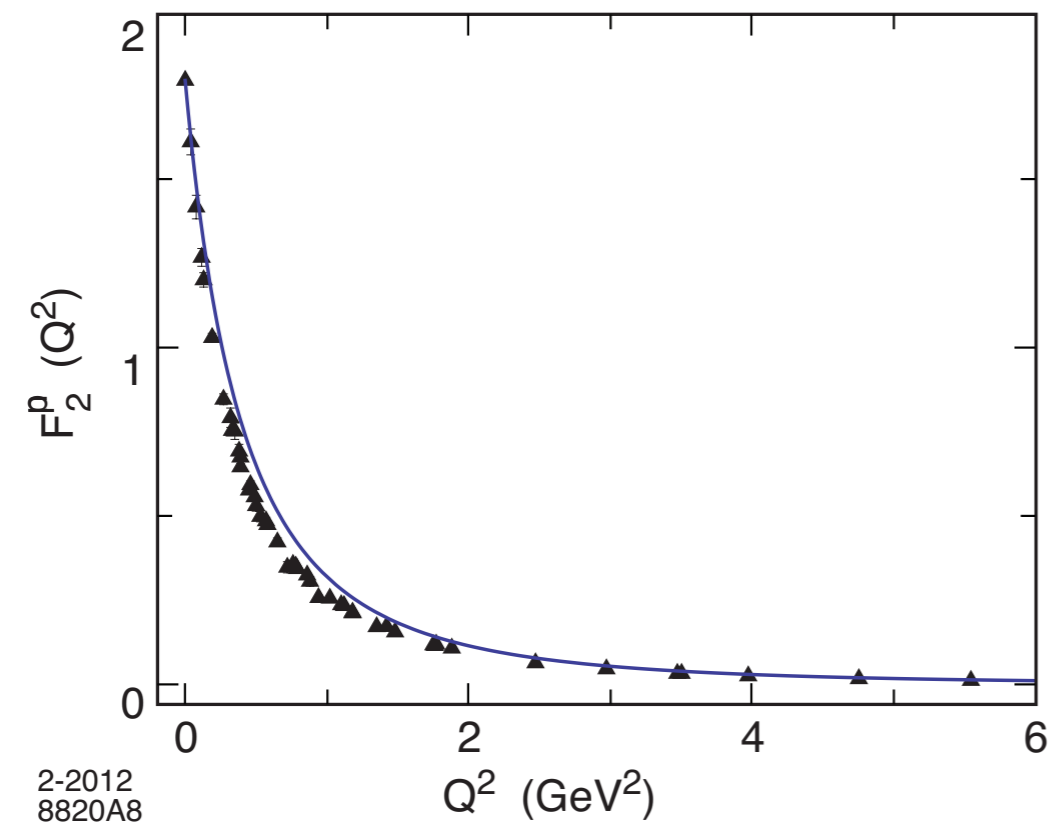
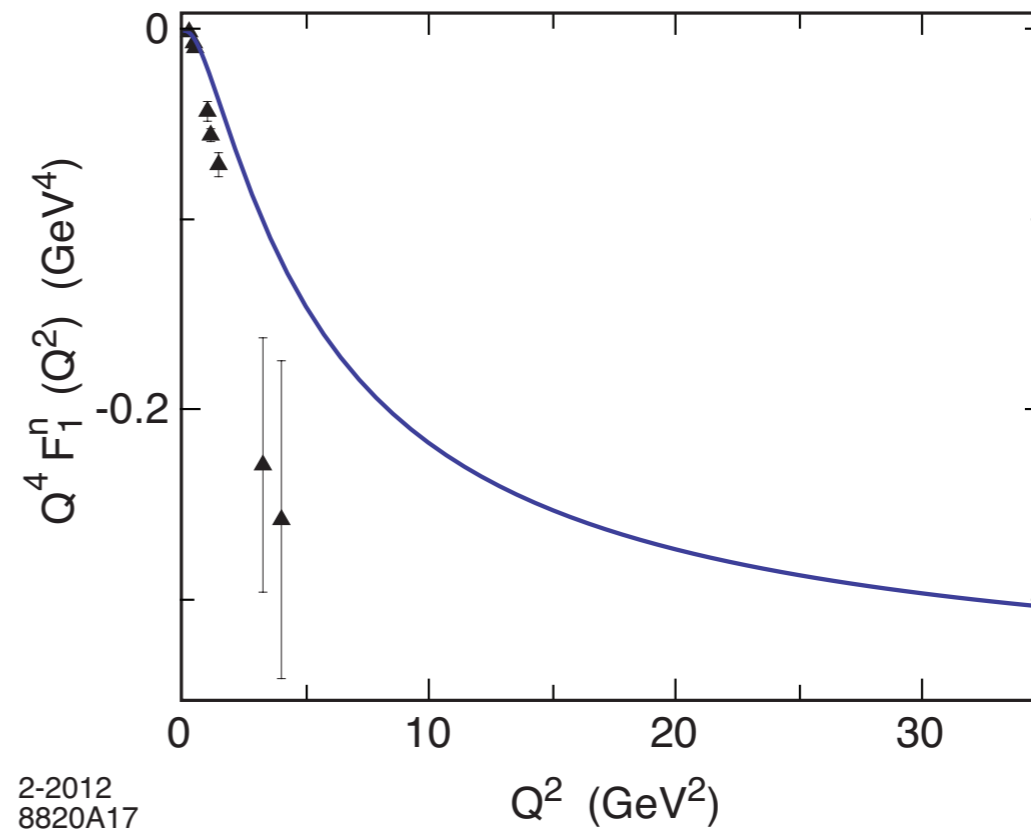
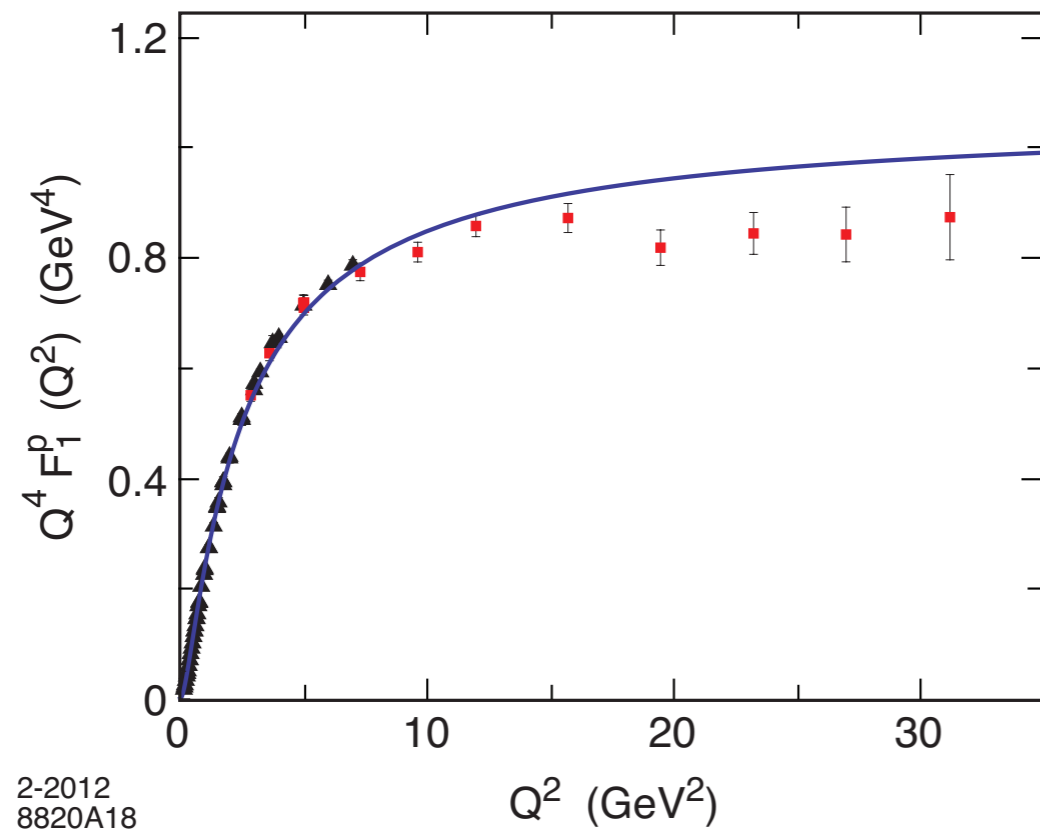
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

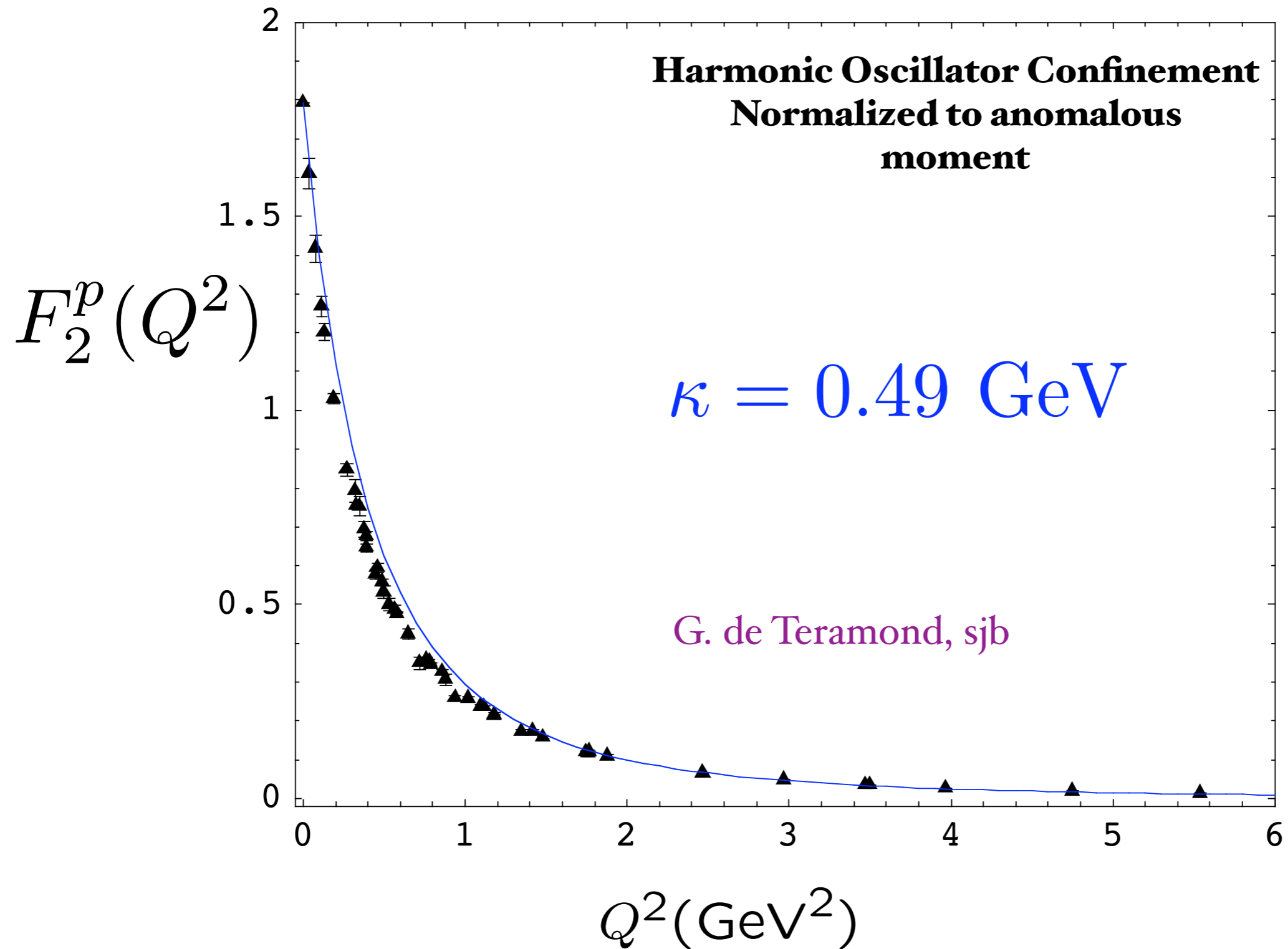


Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \rightarrow N^*(1440)$: $\Psi_+^{n=0,L=0} \rightarrow \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1N \rightarrow N^*}^p(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q, z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $(F_{1N \rightarrow N^*}^p(0) = 0, \quad V(Q=0, z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

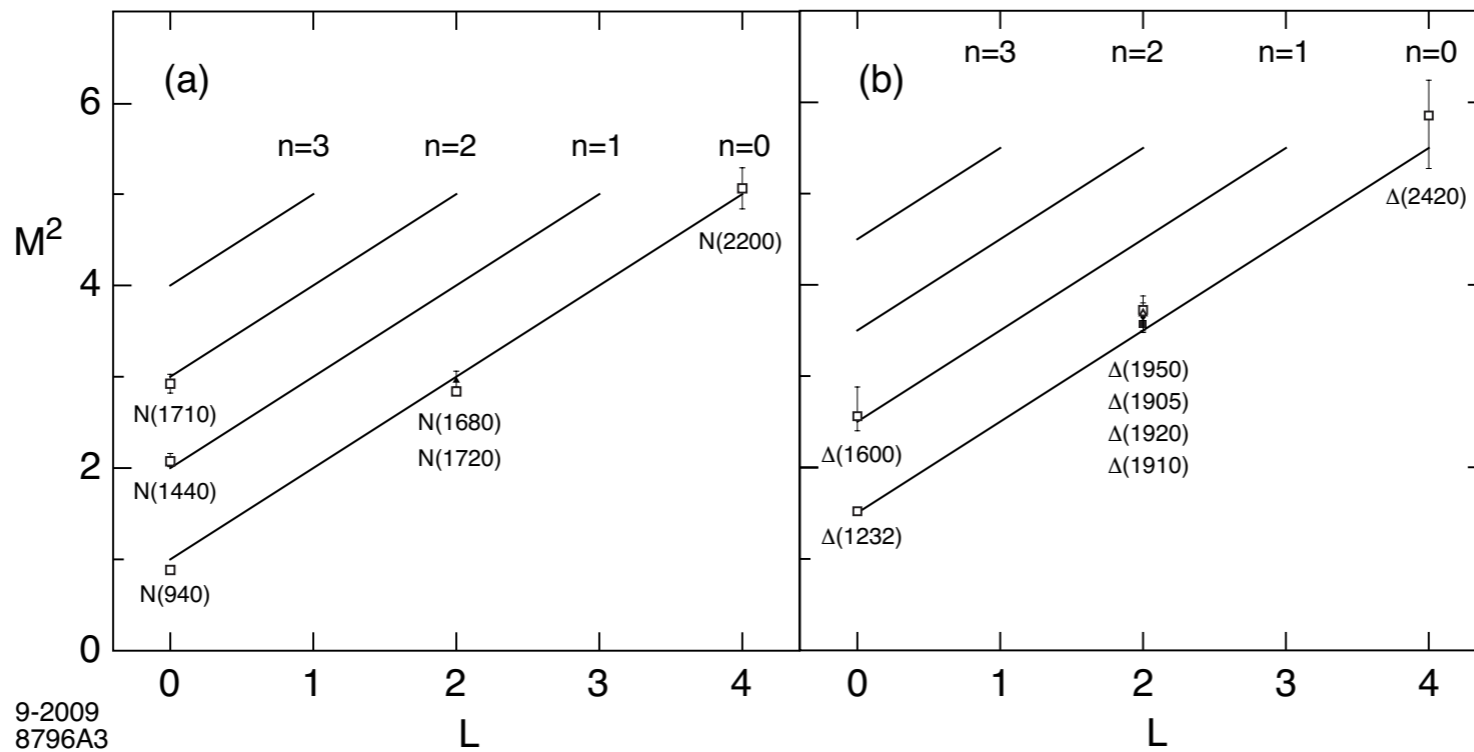
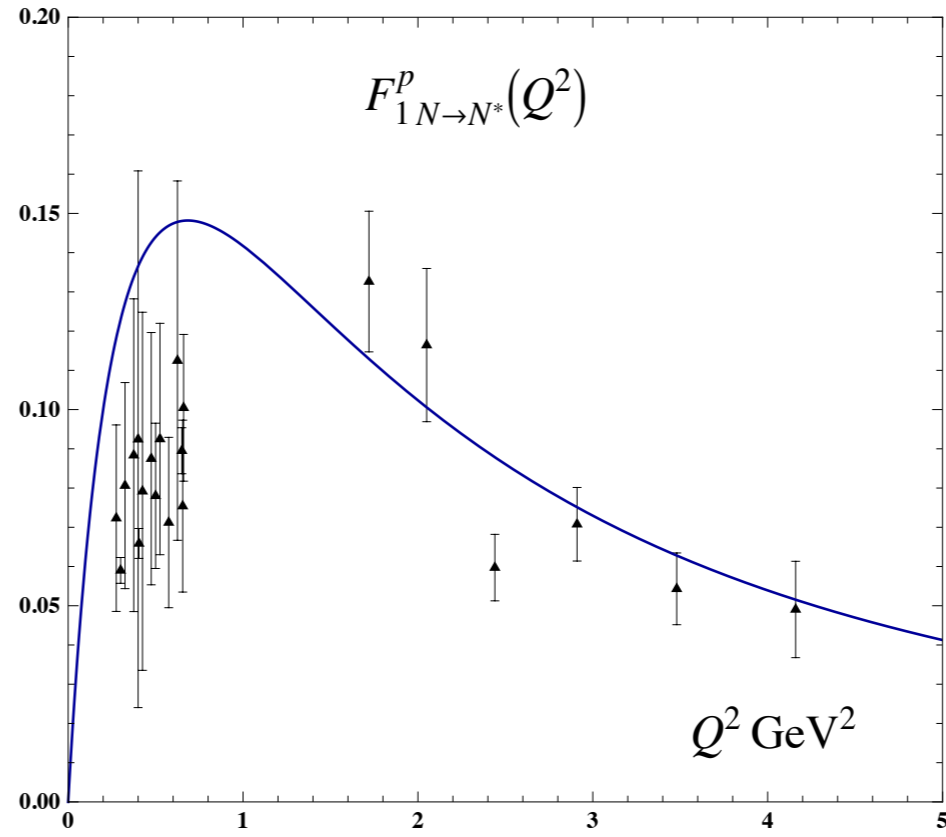
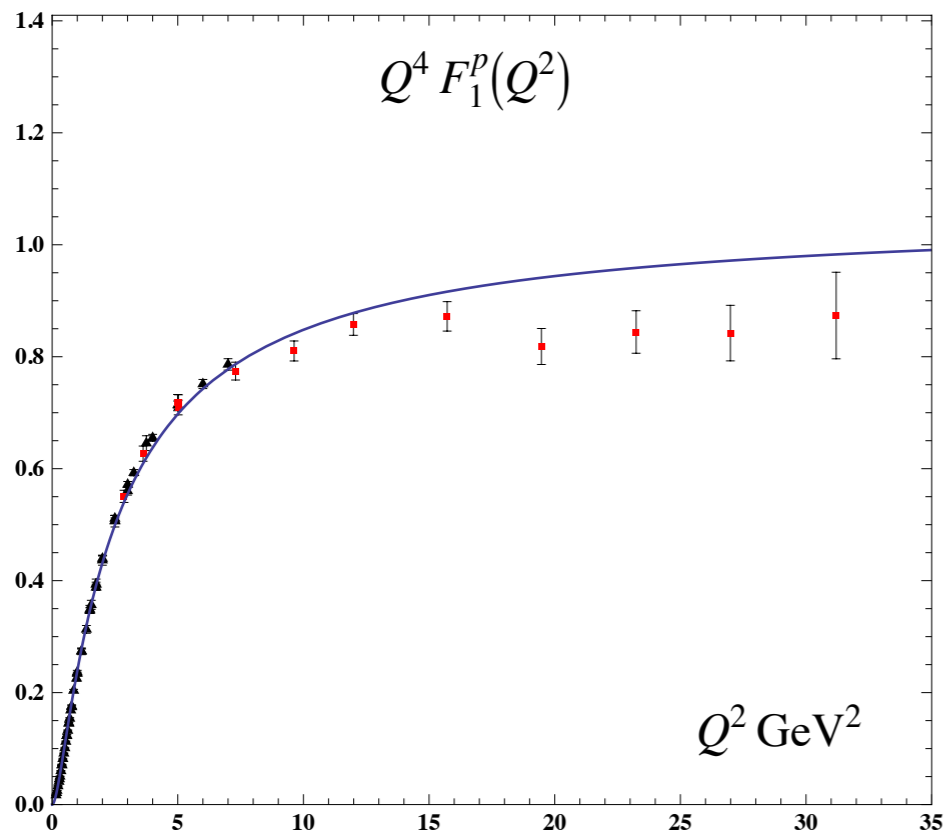
with $\mathcal{M}_{\rho n}^2 \rightarrow 4\kappa^2(n + 1/2)$

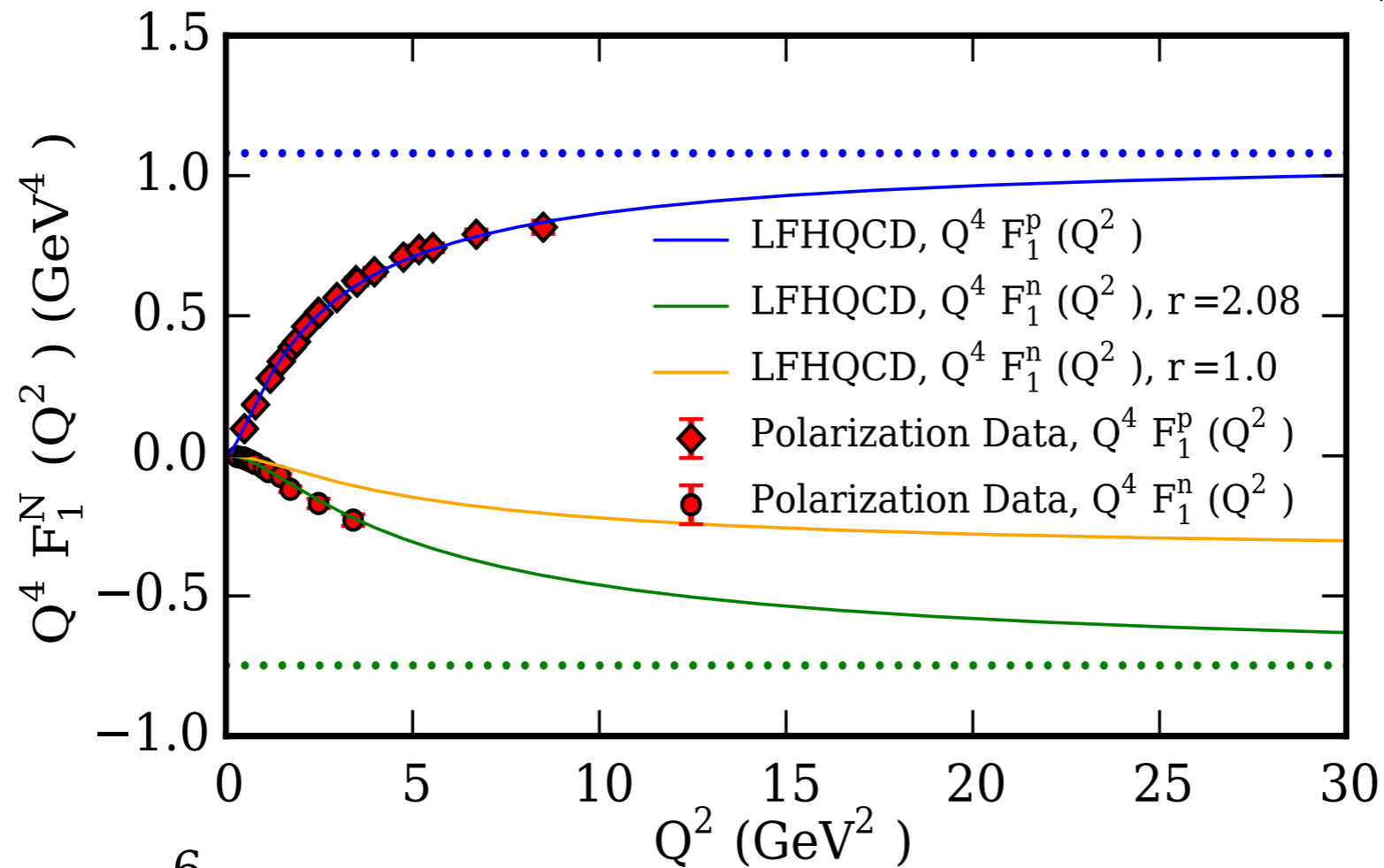
de Teramond, sjb

Consistent with counting rule, twist 3

Excited Baryons in Holographic QCD

G. de Teramond & sjb

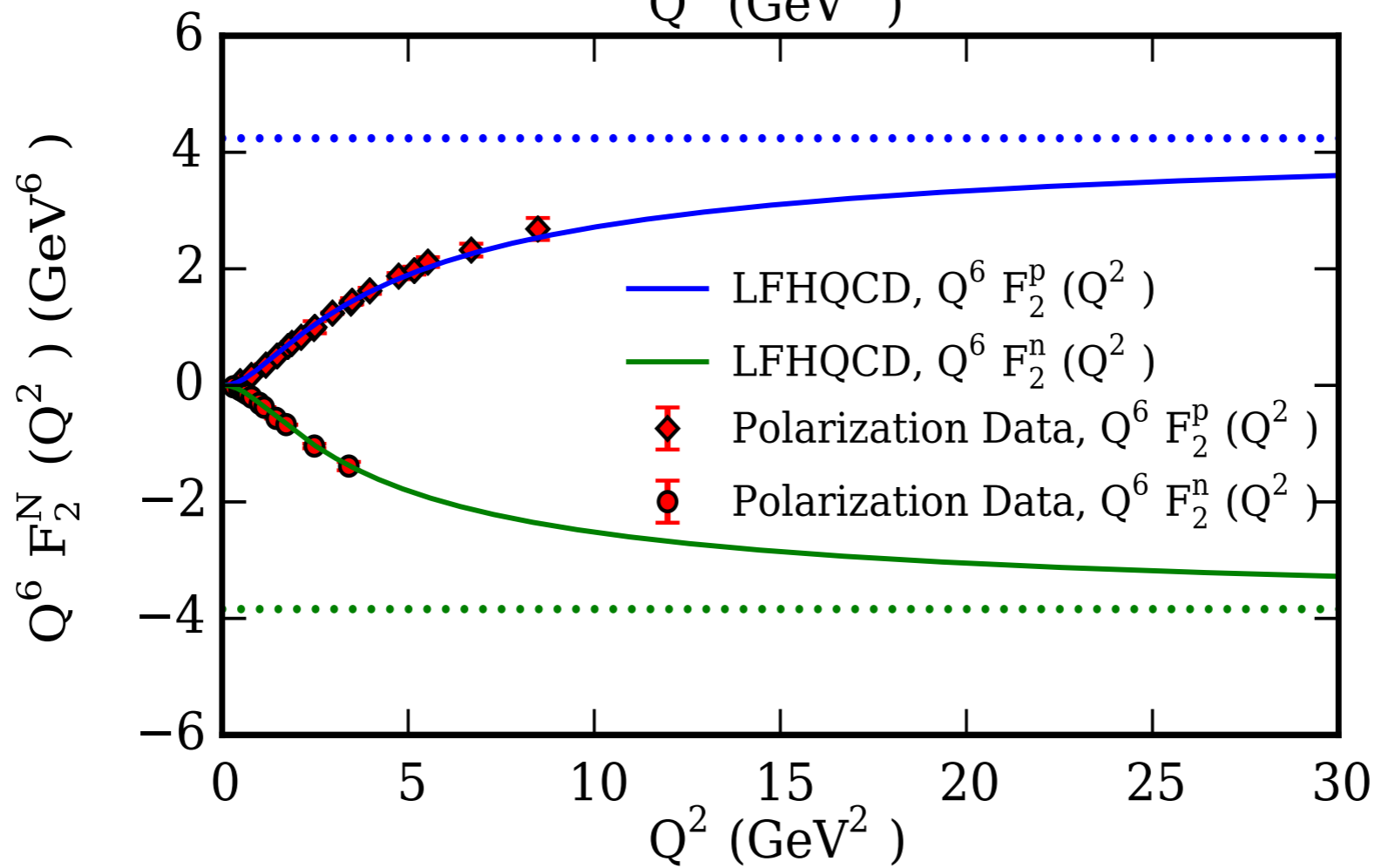




$$Q^4 F_1^p(Q^2)$$

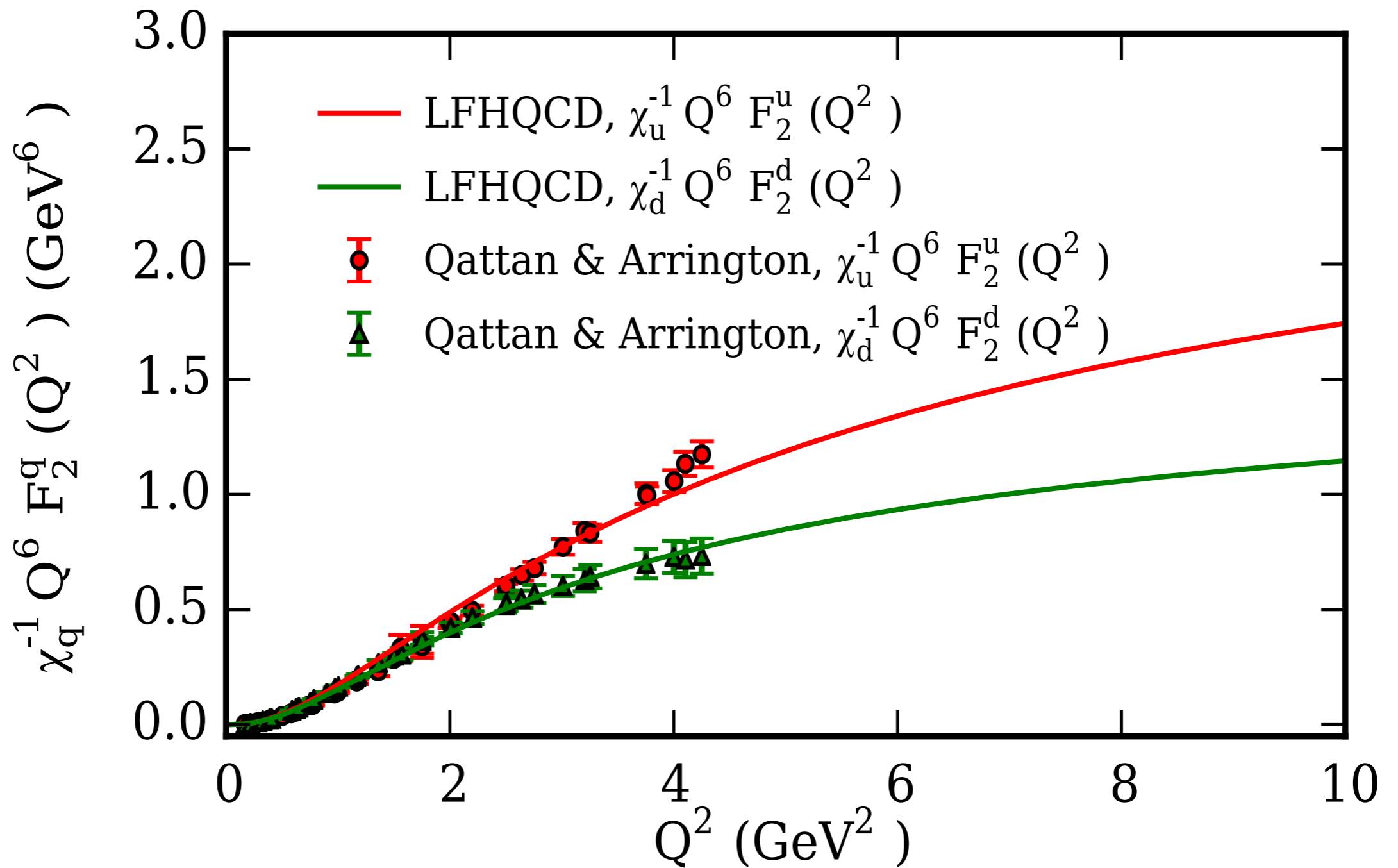
$$Q^4 F_1^n(Q^2)$$

*Includes
5-quark
Fock states*



$$Q^6 F_2^p(Q^2)$$

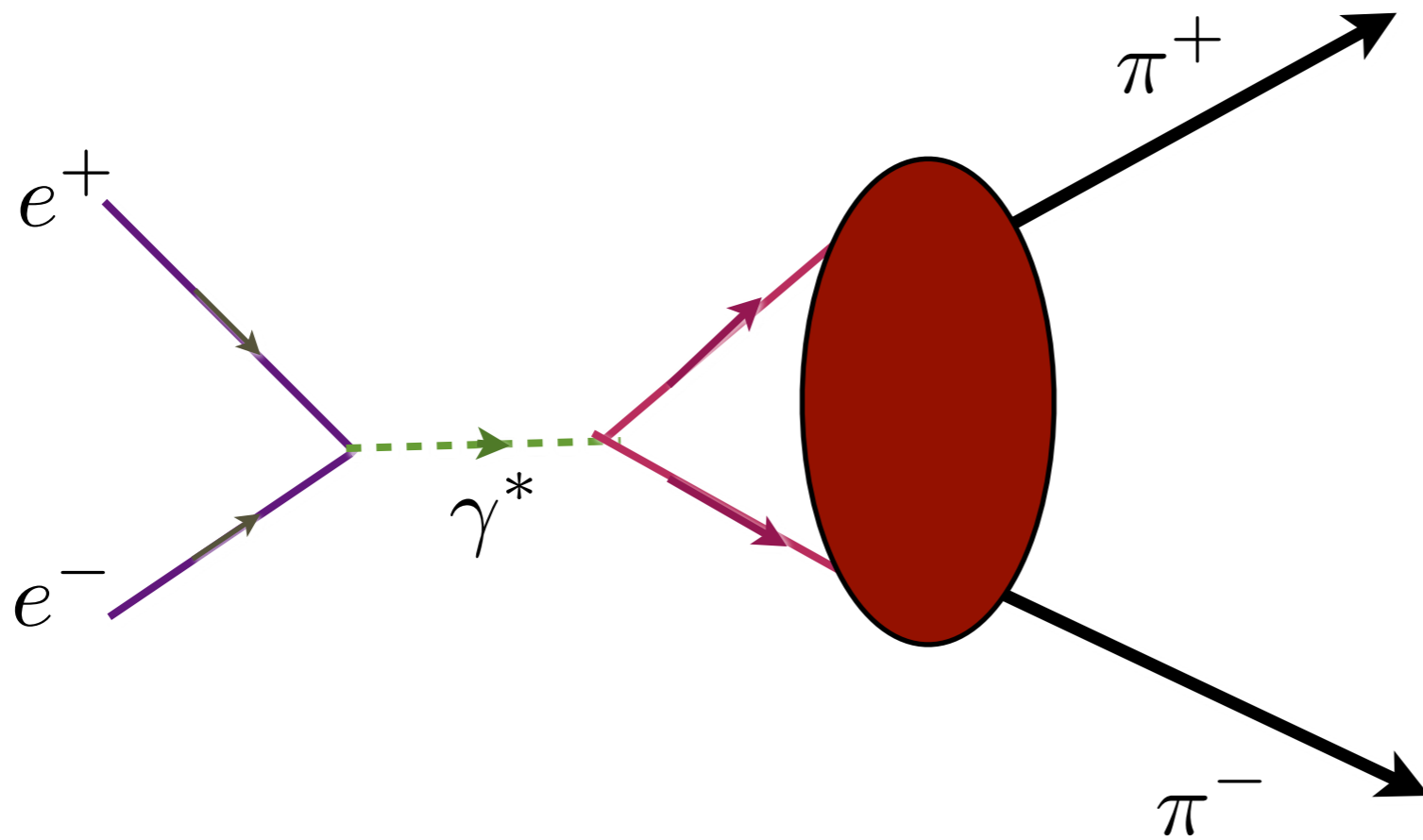
$$Q^6 F_2^n(Q^2)$$



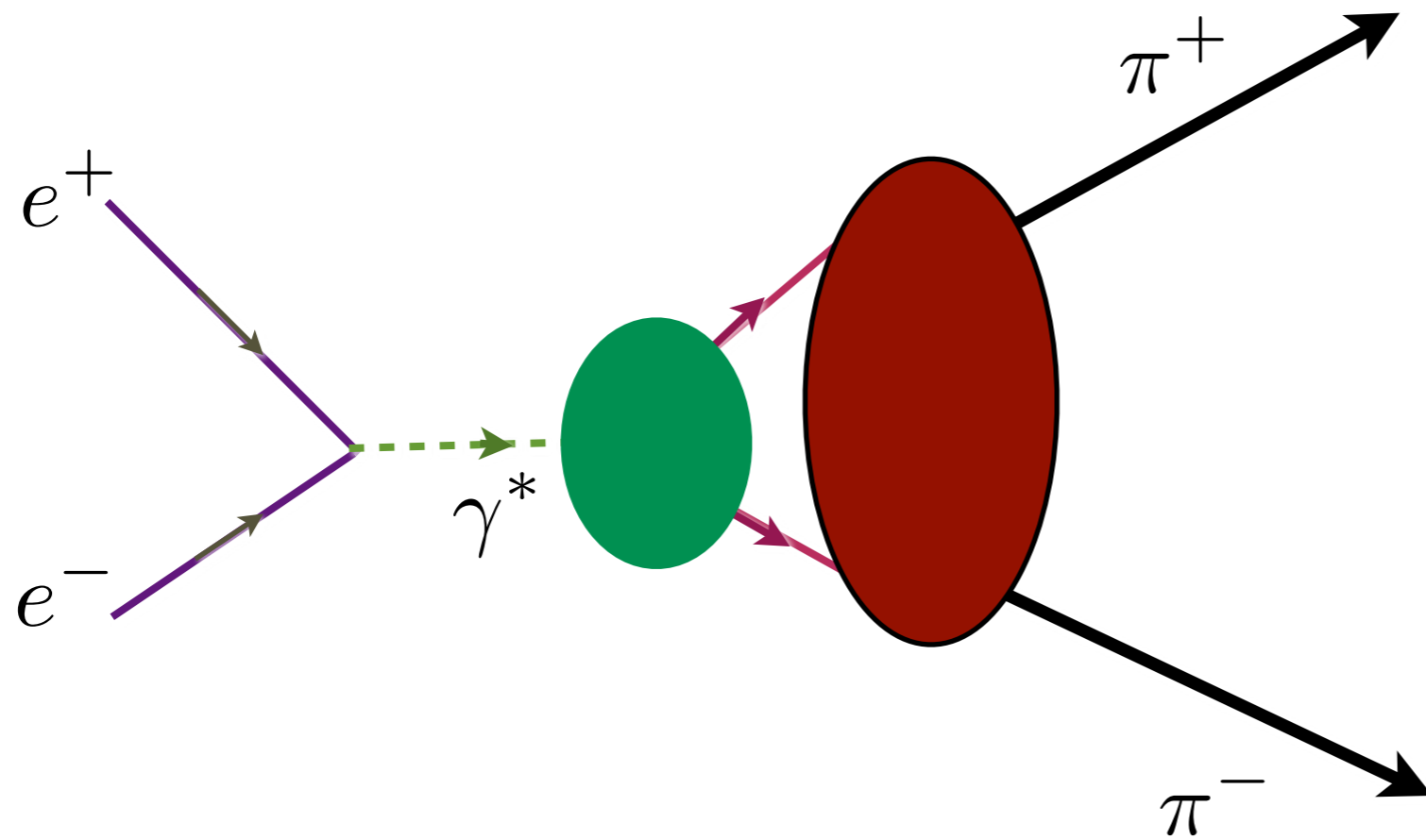
Flavor Dependence of $Q^6 F_2(Q^2)$

Sufian, de Teramond, Deur, Dosch, sjb

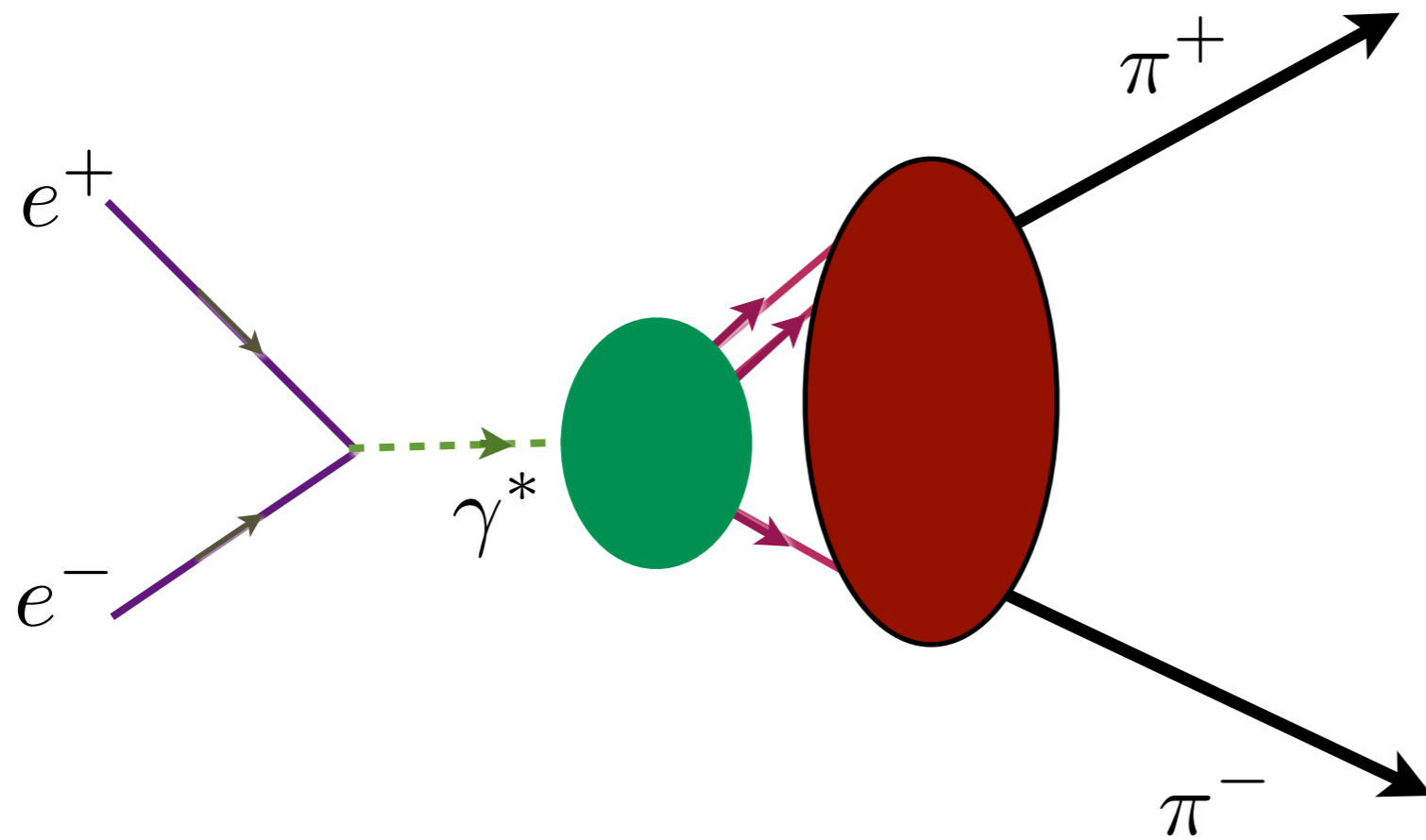
Dressed soft-wall current brings in higher Fock states and more vector meson poles



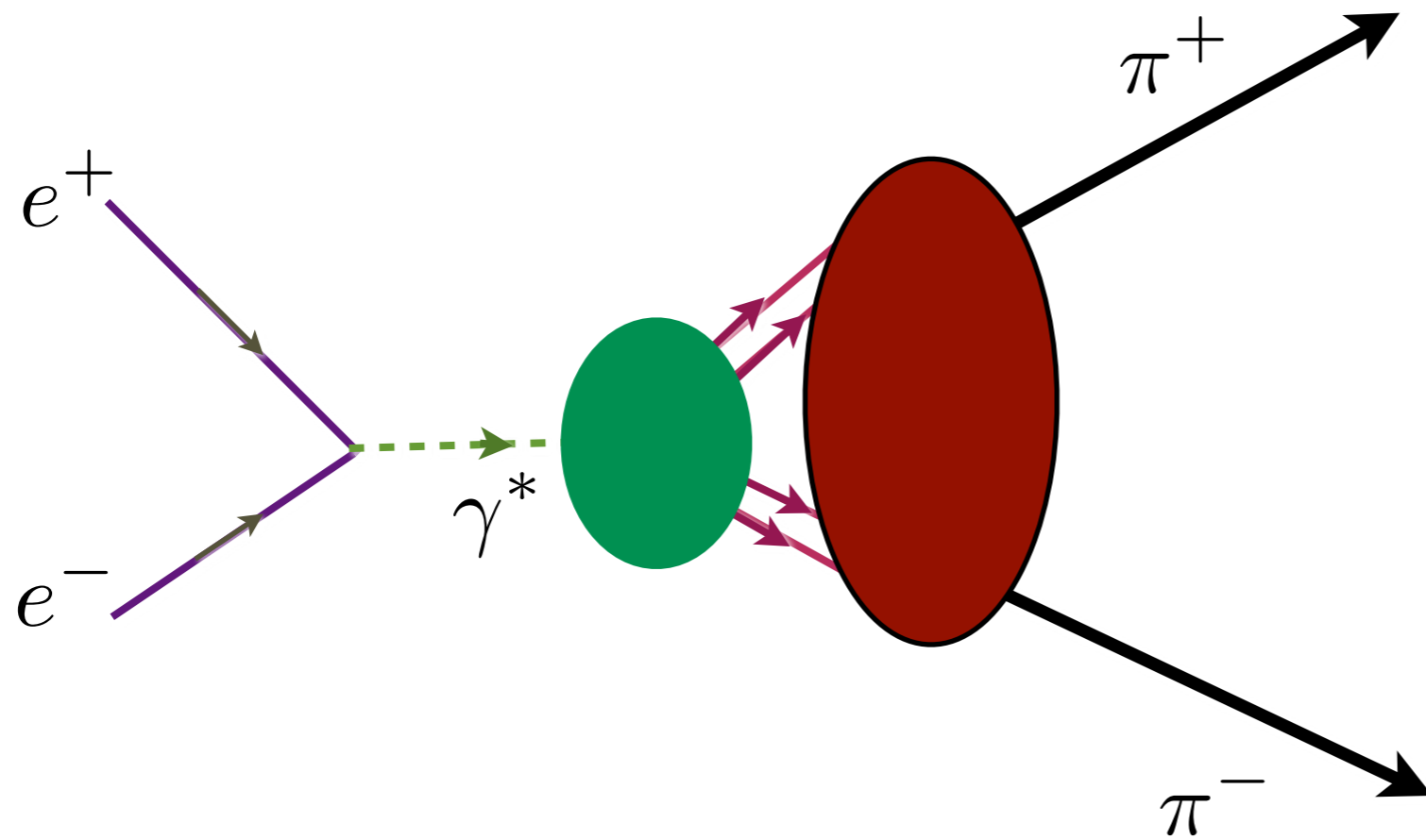
Dressed soft-wall current brings in higher Fock states and more vector meson poles



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Dressed soft-wall current brings in higher Fock states and more vector meson poles



Current Matrix Elements in AdS Space (SW)

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

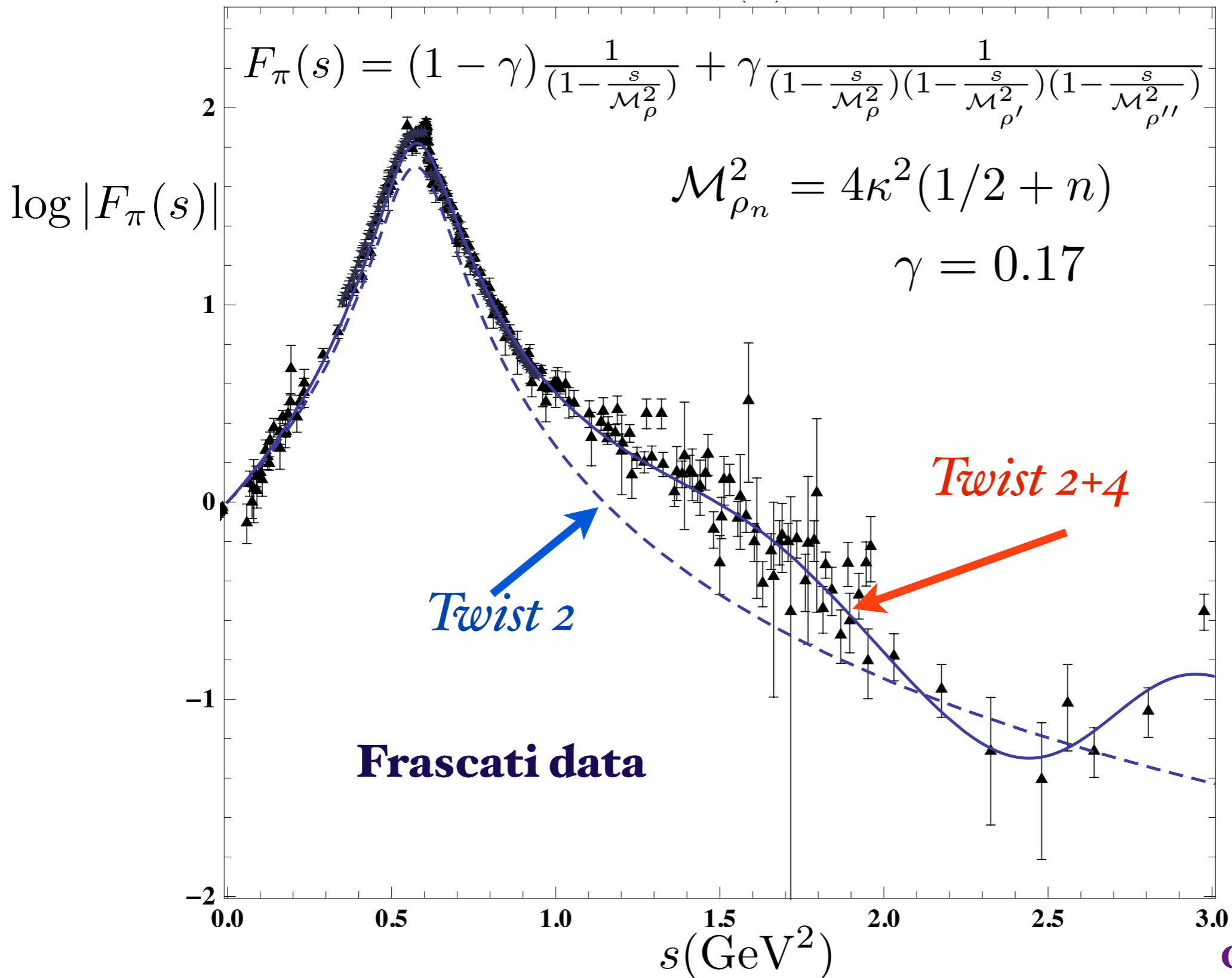
$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Dressed
Current
in Soft-Wall
Model*

**de Tèramond & sjb
Grigoryan and Radyushkin**

Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

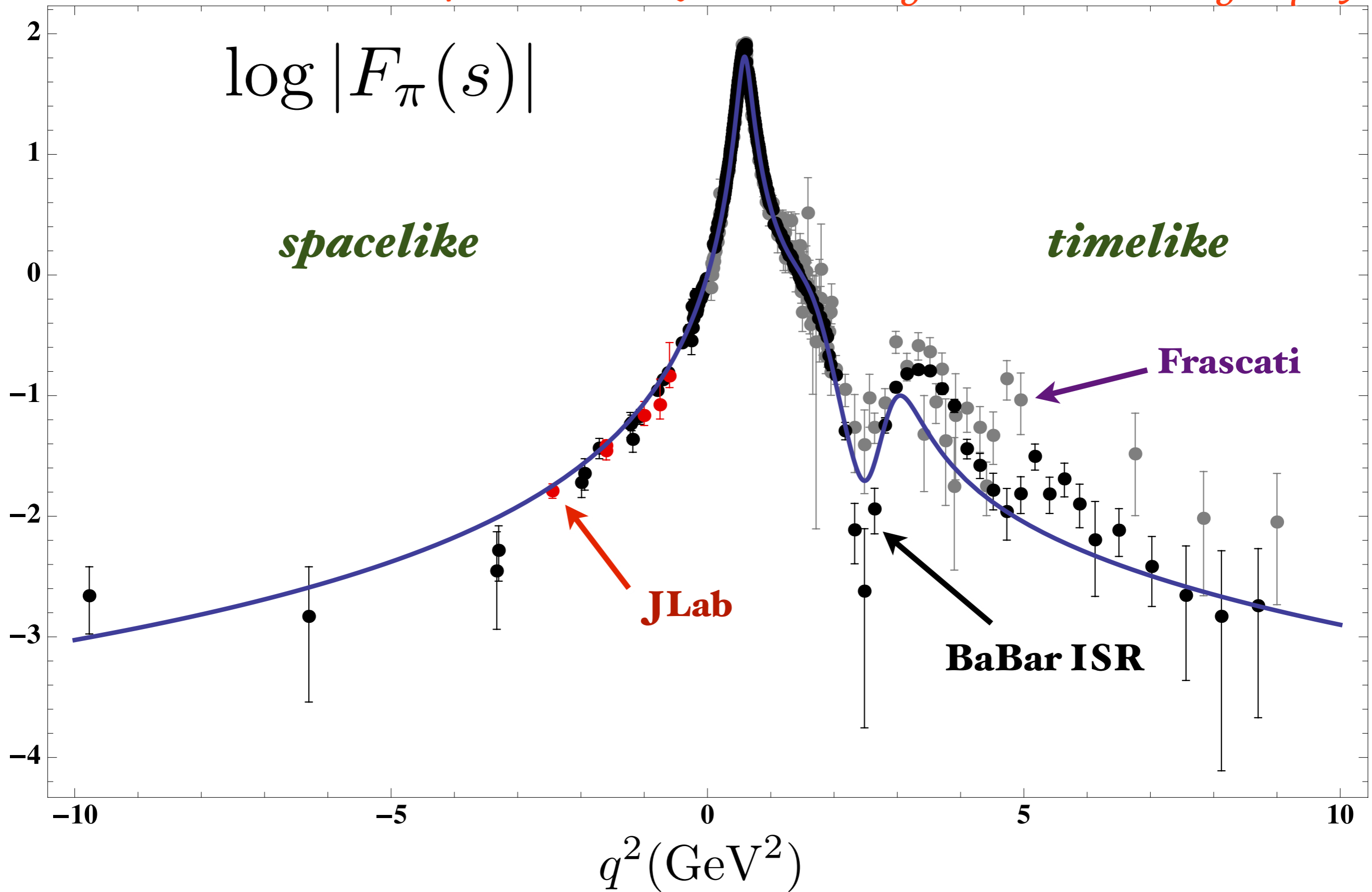


Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

Pion Form Factor from AdS/QCD and Light-Front Holography



Future Directions

de Tèramond, Dosch, Wu, Vary, sjb

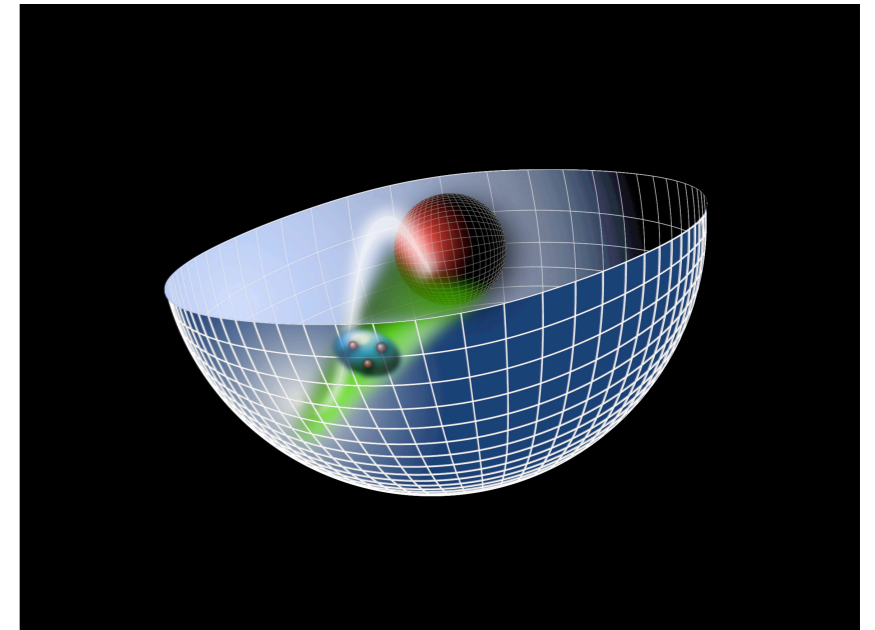
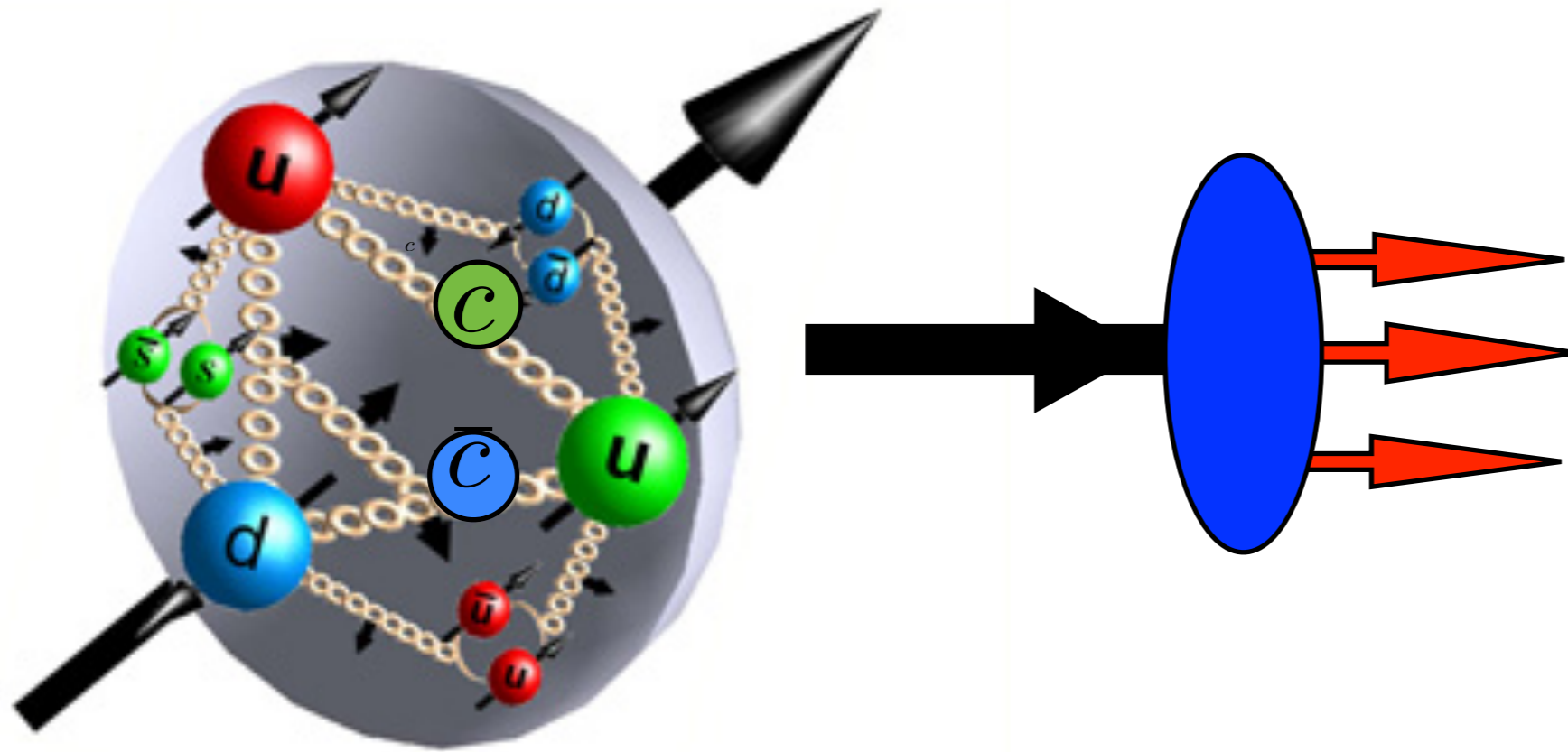
Remarkable similarities with DSE approach of Roberts et al.

- **Hadronization at the Amplitude Level: LFWFs**
- **Running Coupling at all Q^2**
- **Factorization Scale for ERBL, DGLAP evolution: Q_0**
- **Calculate Sivers Effect including FSI and ISI**
- **Eliminate renormalizations scale ambiguity: PMC**
- **Compute Tetraquark Spectroscopy: Sequential Clusters**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization**

Novel QCD

- Flavor-Dependent Anti-Shadowing
- LF Vacuum and Cosmological Constant: No QCD condensates
- Principle of Maximum Conformality (PMC): Eliminate renormalization anomaly; scheme independent
- Match Perturbative and Non-Perturbative Domains
- Hadronization at Amplitude Level
- Intrinsic Heavy Quarks from AdS/QCD: Higgs at high x_F
- Ridge from flux tube collisions
- Baryon-to-meson anomaly at high p_T

Supersymmetric Meson-Baryon Properties of QCD from Light-Front Holography and Superconformal Algebra



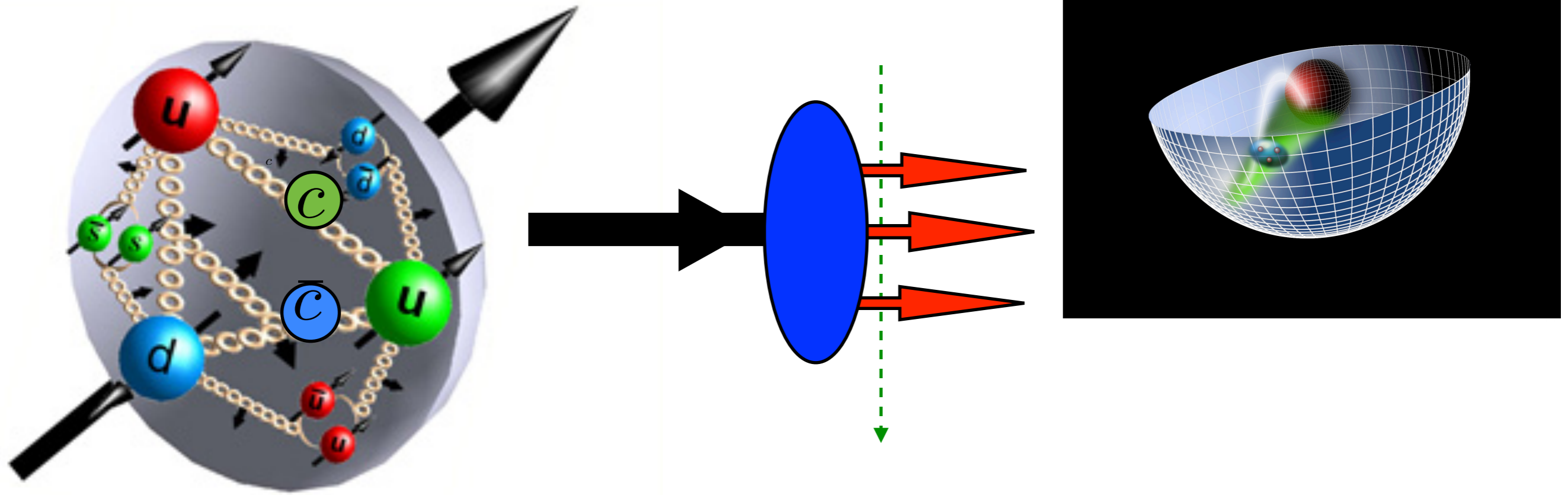
Stan Brodsky



with Guy de Tèramond, Hans Günter Dosch,
C. Lorce, K. Chiu, R. S. Sufian, A. Deur

7th Workshop of the APS Topical Group on Hadronic Physics
Washington D.C., February 3, 2017

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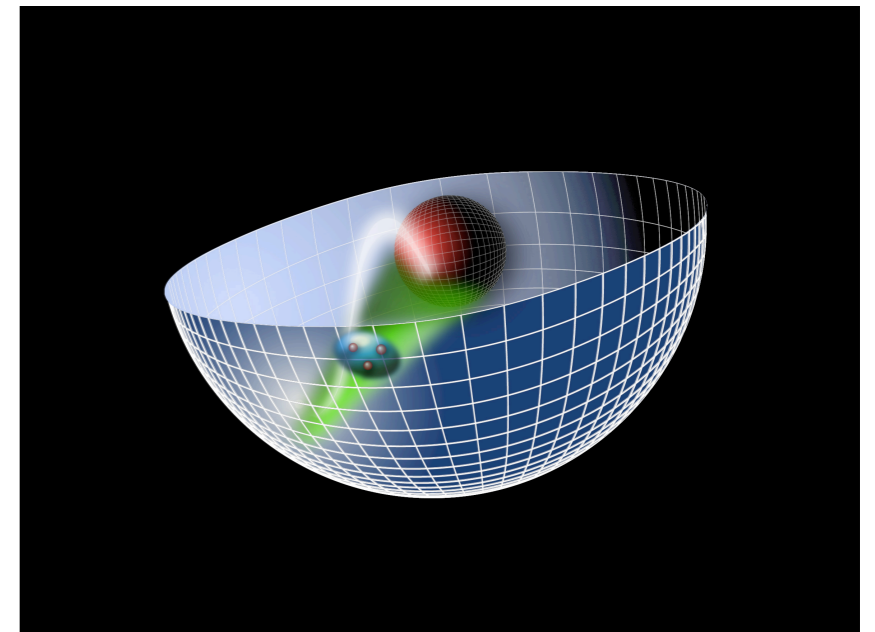
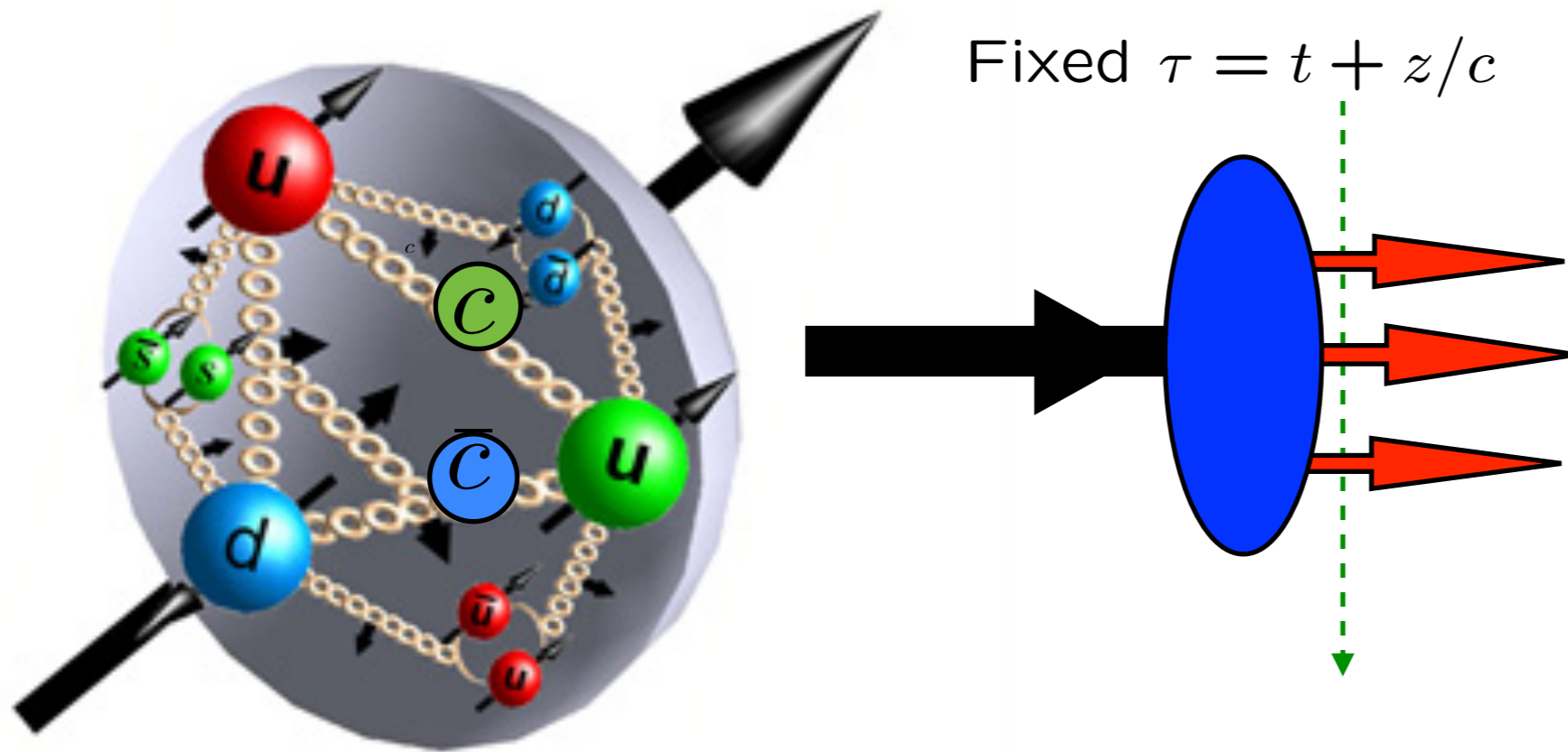
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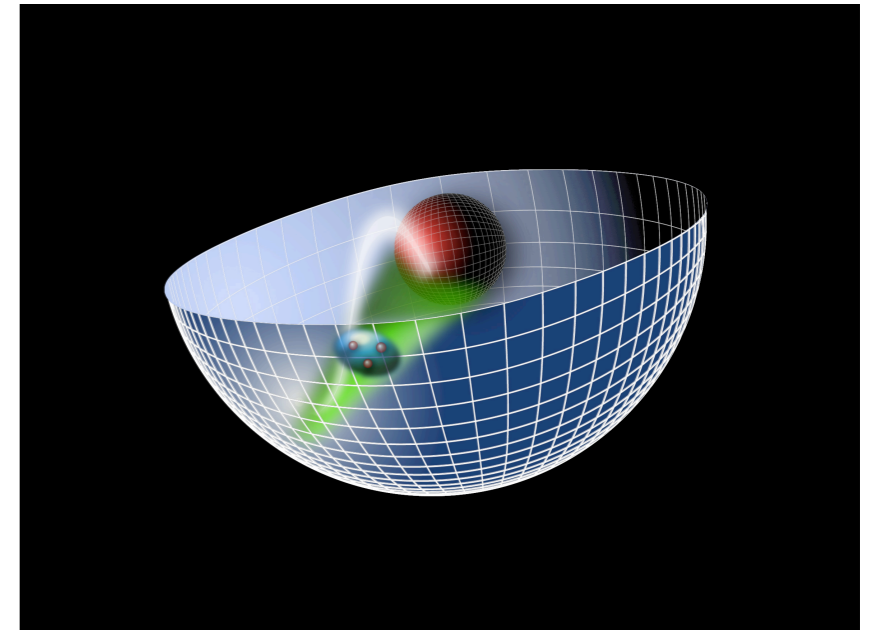
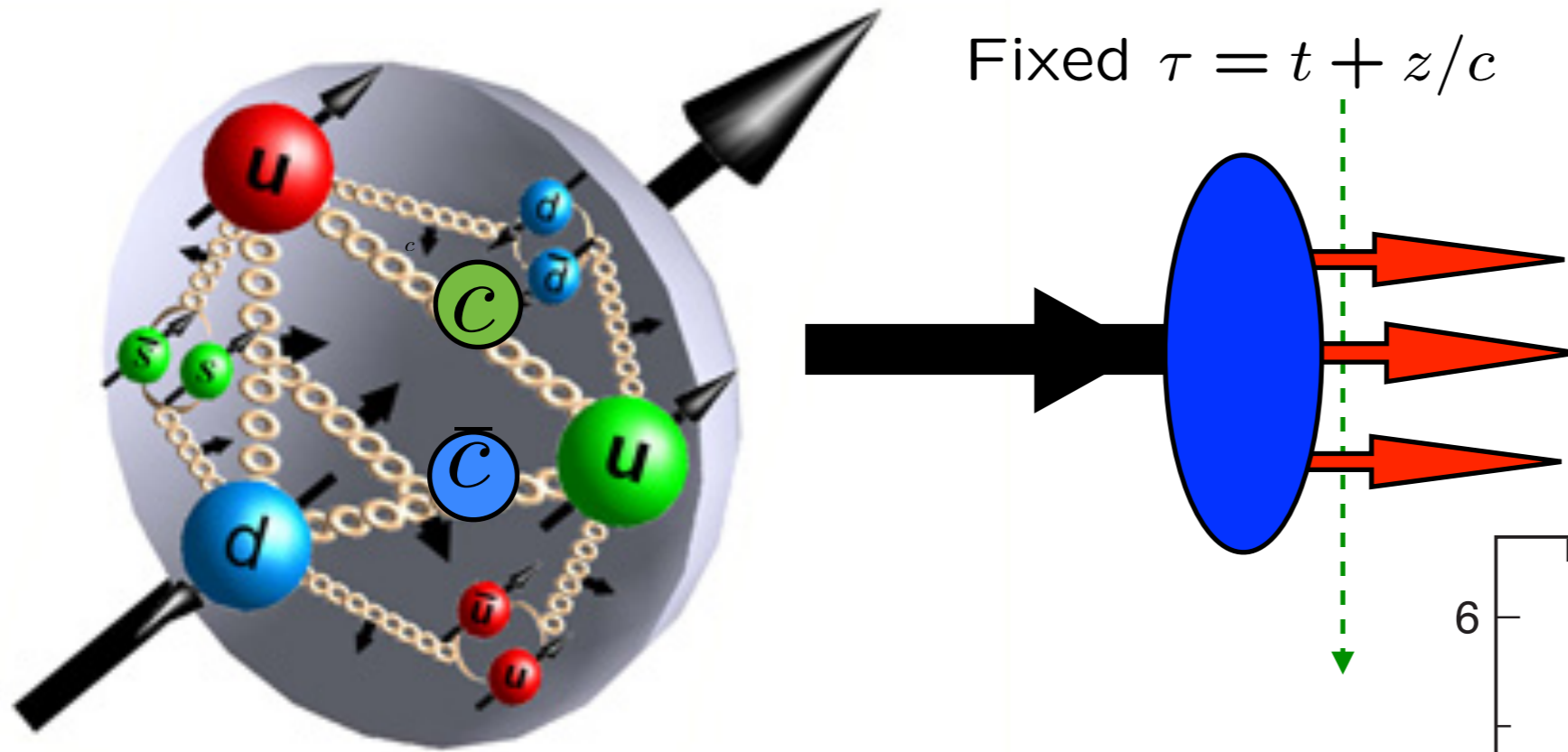
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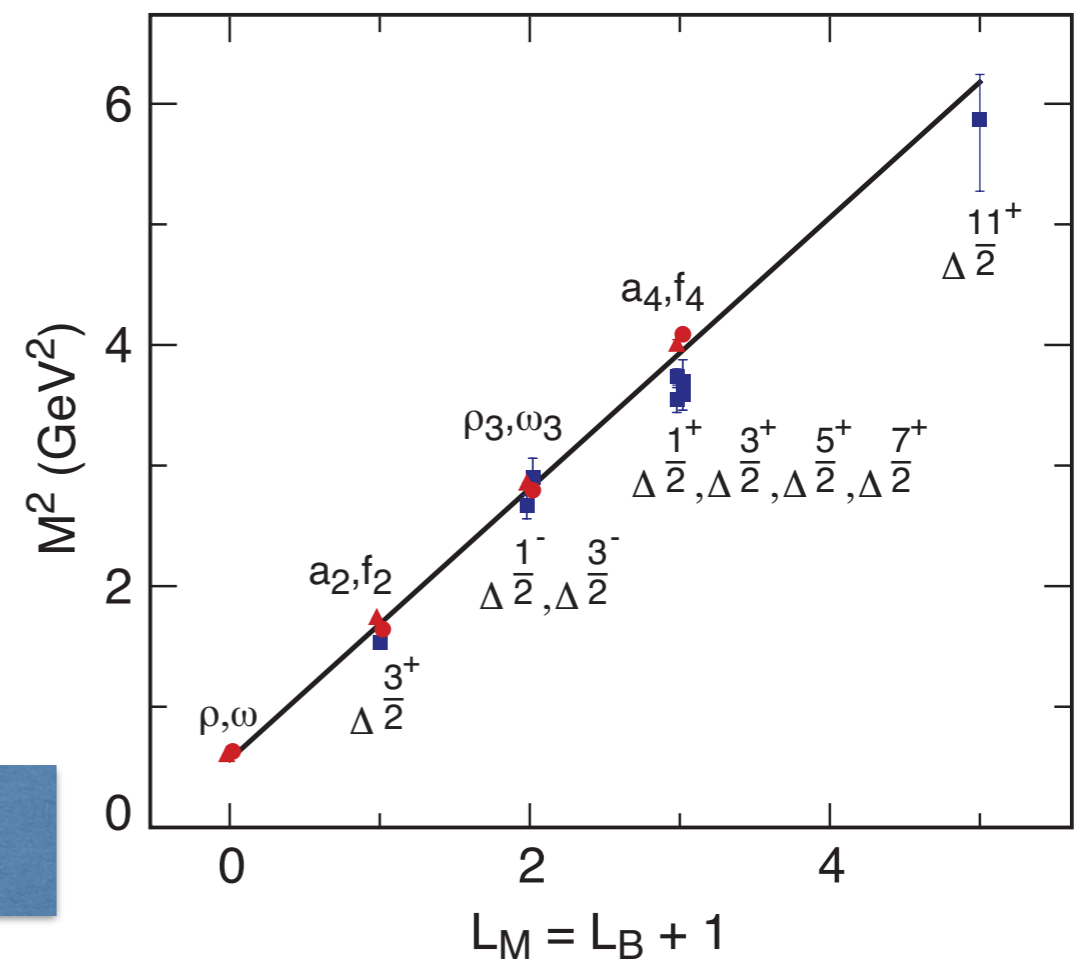
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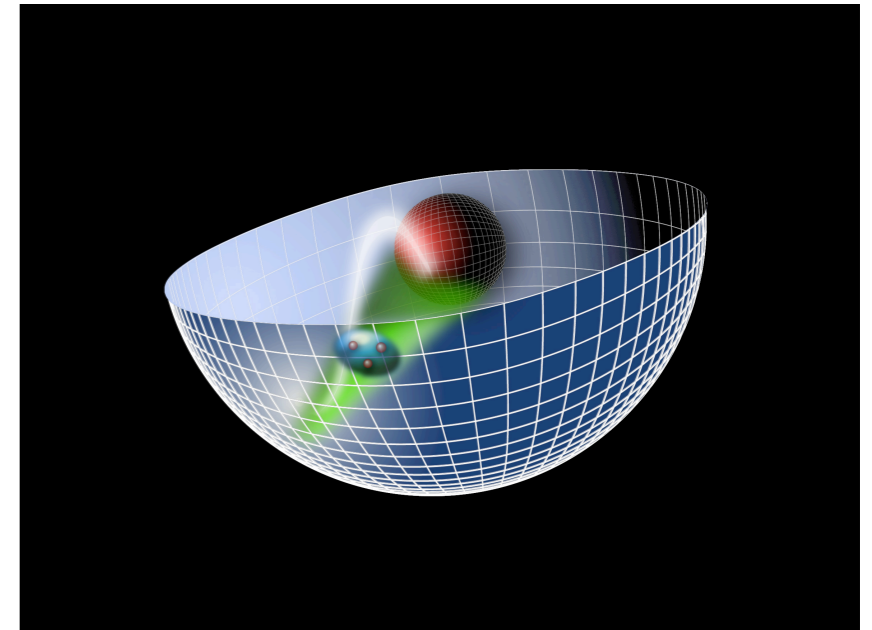
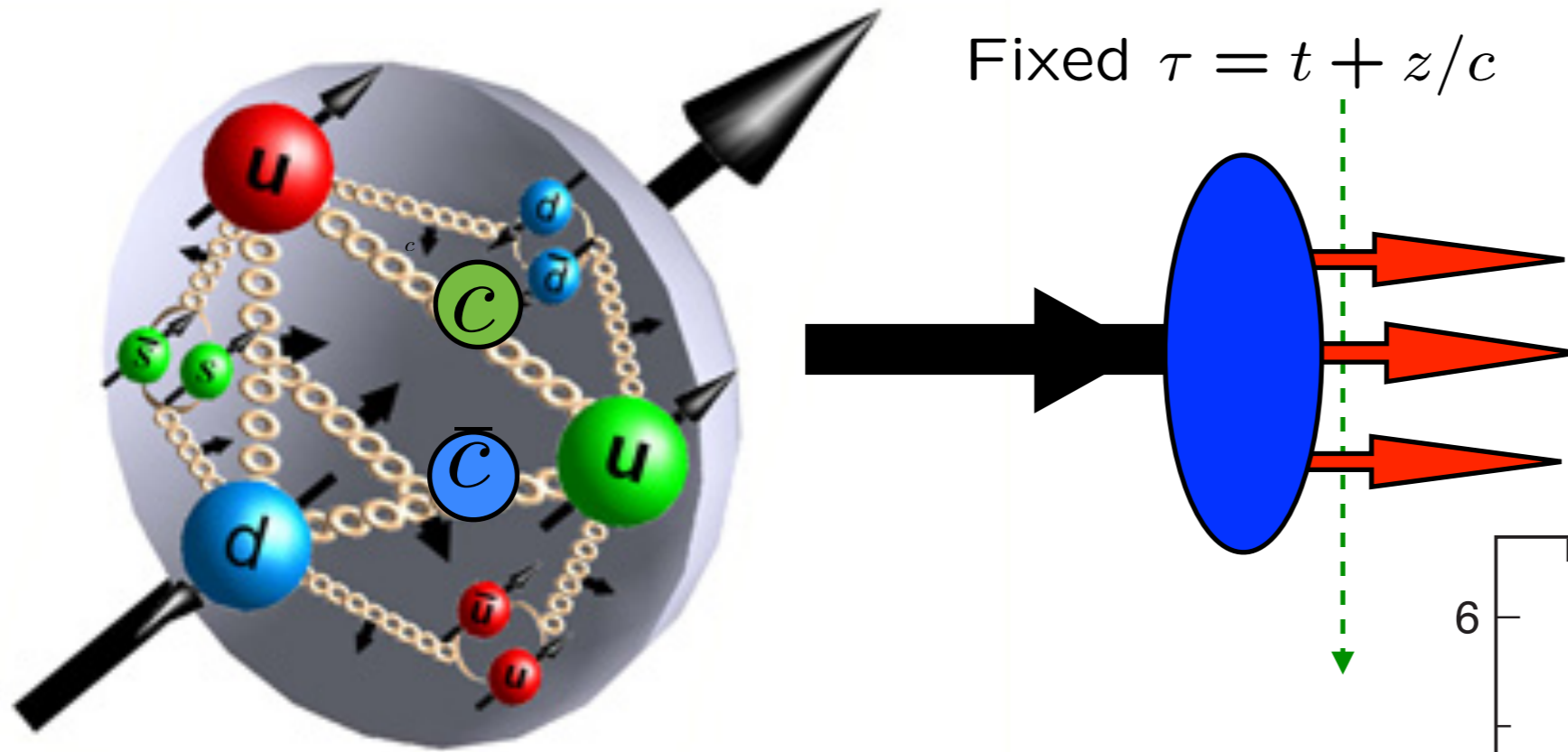


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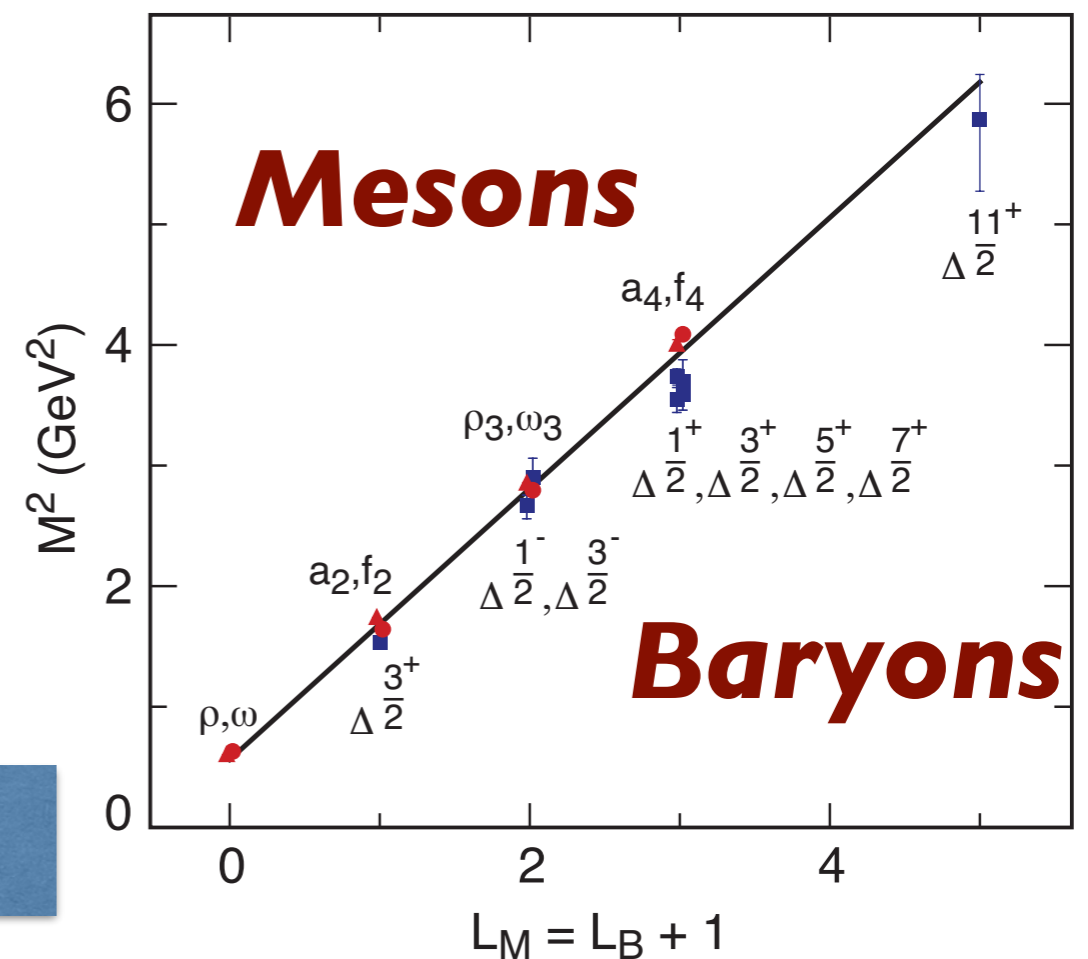
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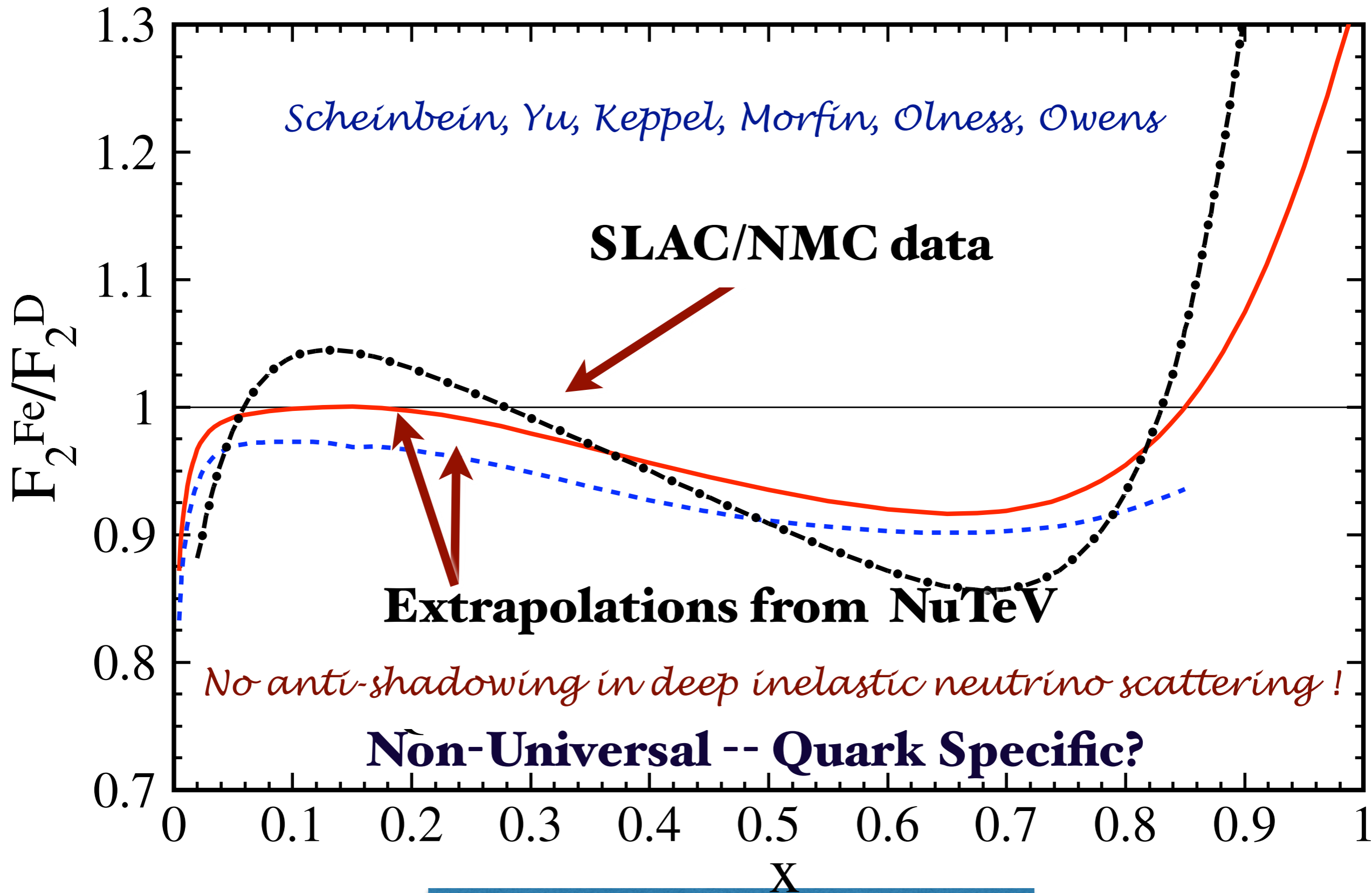


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$$Q^2 = 5 \text{ GeV}^2$$



“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu*

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(\text{expt})$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

“One of the gravest puzzles of theoretical physics”

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Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD “Condensates”

(C) Higgs Light-Front Zero Mode

Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state $M=0$.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: “In-hadron” condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD, EW



Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza*

*CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark
and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

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(Received 13 January 2013; published 10 May 2013)*

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

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Supersymmetric Features of QCD
from LF Holography

Stan Brodsky

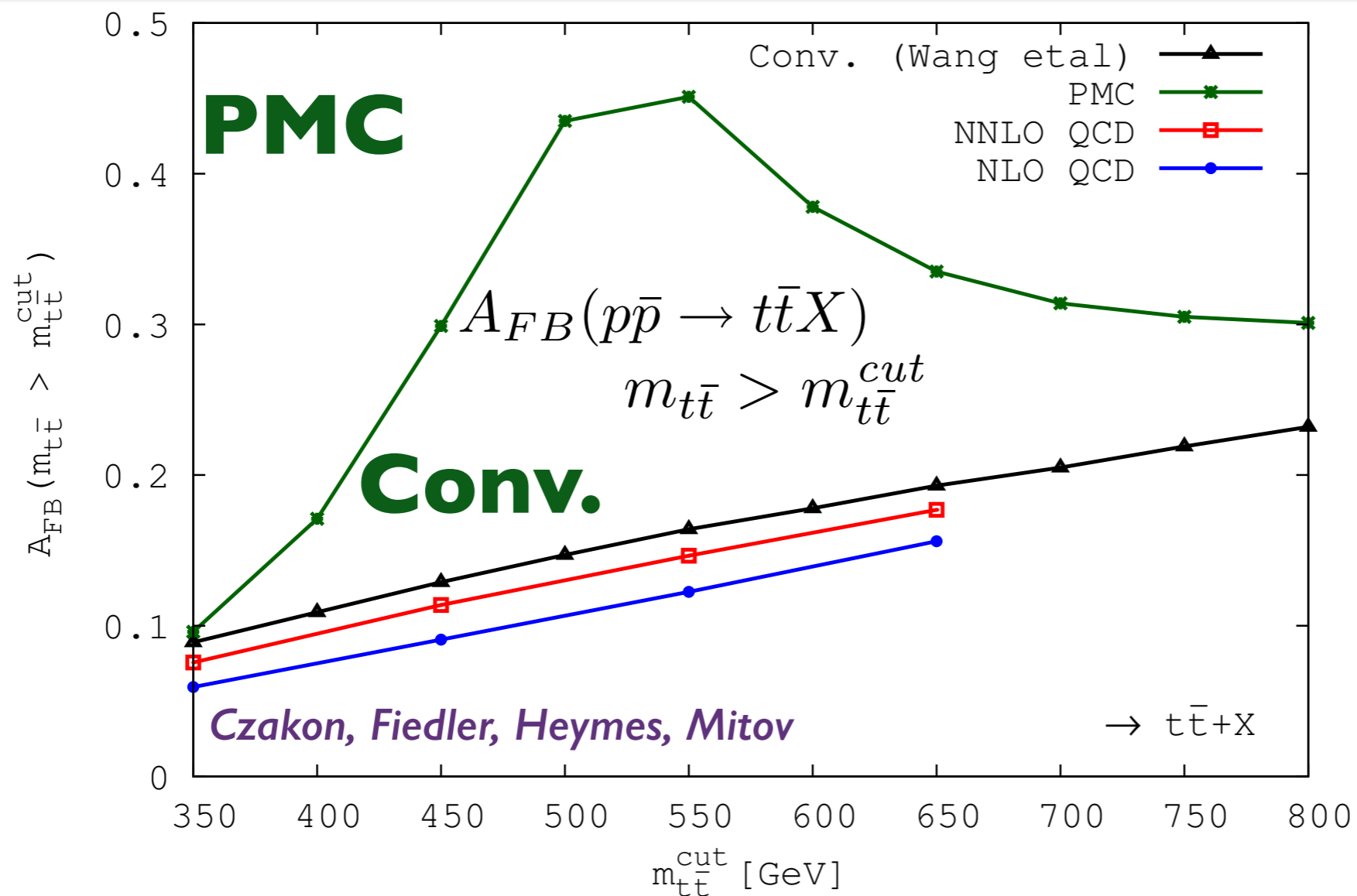
SLAC
NATIONAL ACCELERATOR LABORATORY



Elimination of QCD Scale Ambiguities

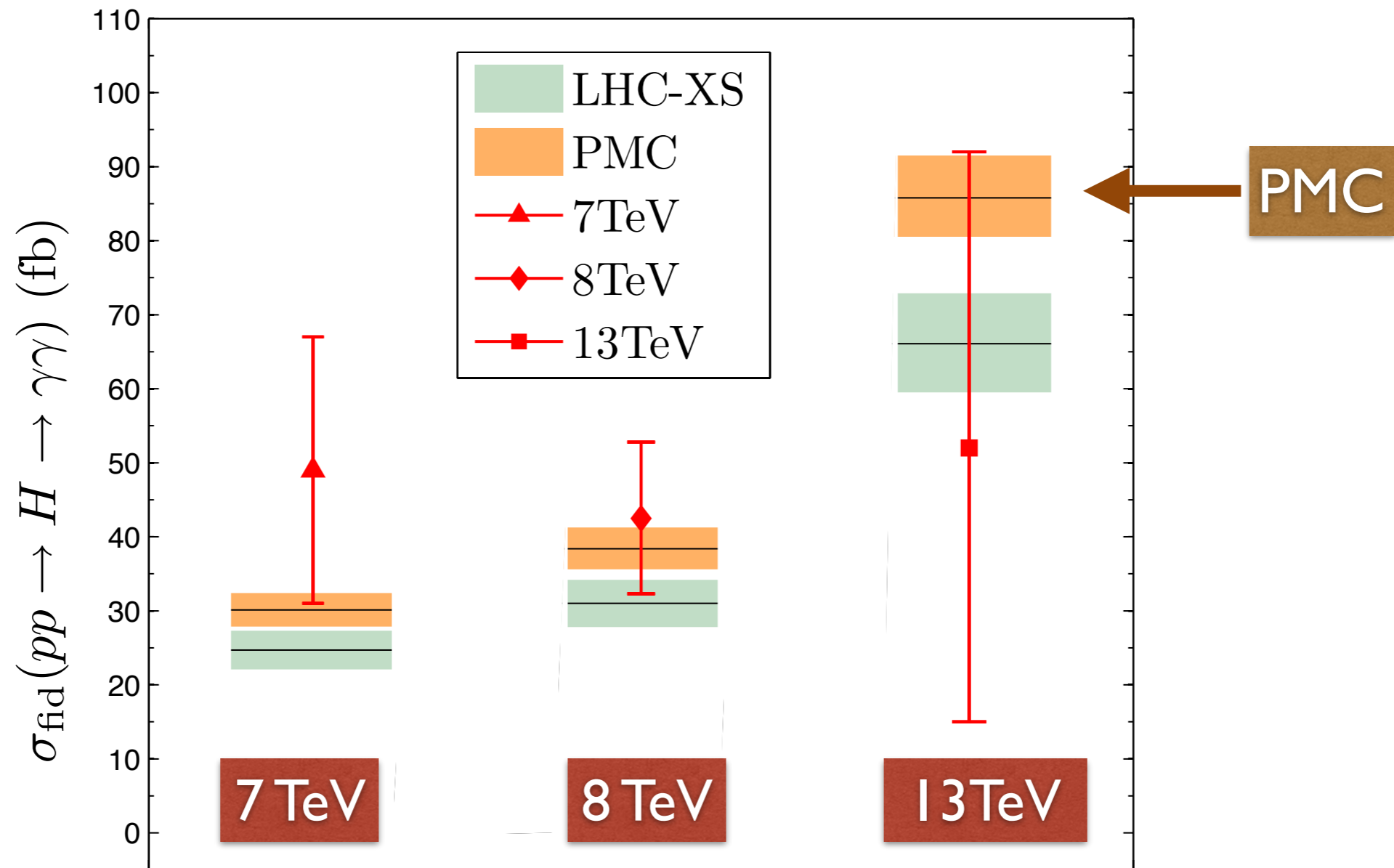
The Principle of Maximum Conformality (PMC)

Applications of PMC renormalization-scale-setting for top, Higgs production, and other processes at the LHC



with Leonardo di Giustino,
Xing-Gang Wu and Martin Mojaza

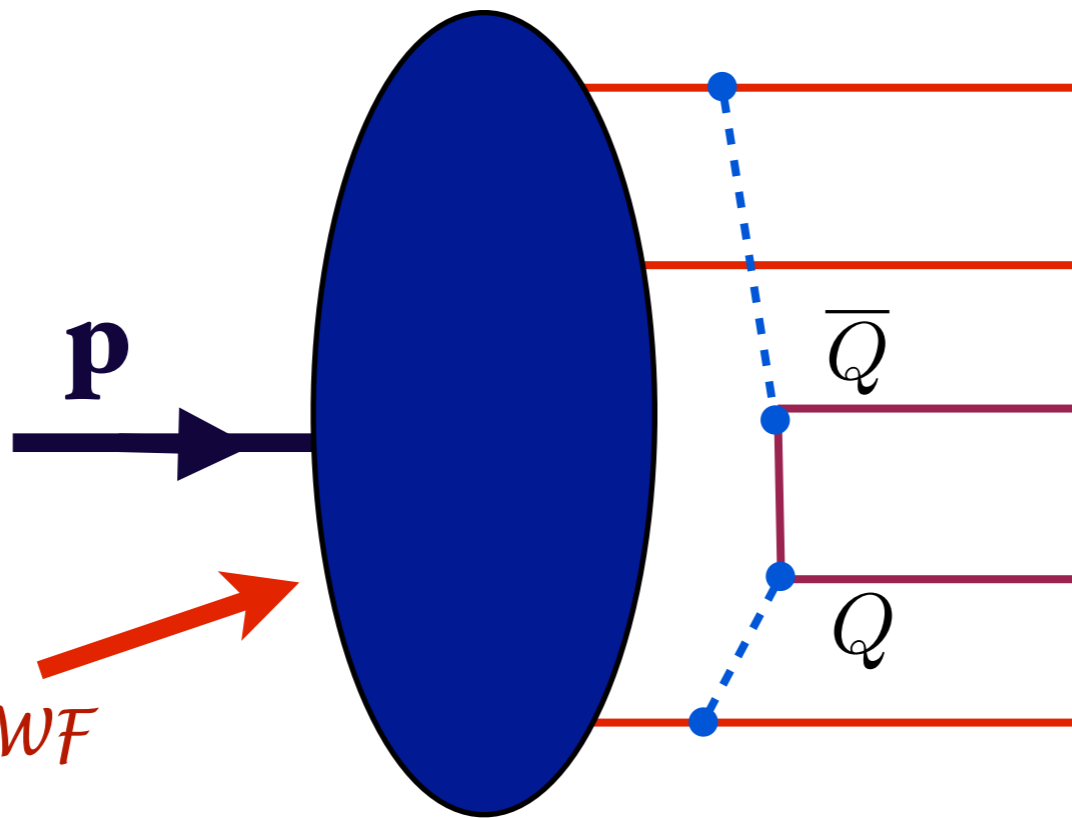
$$\sigma(pp \rightarrow H X \rightarrow \gamma\gamma X)$$



Comparison of the PMC predictions for the fiducial cross section $\sigma_{\text{fid}}(pp \rightarrow H \rightarrow \gamma\gamma)$ with the ATLAS measurements at various collision energies. The LHC-XS predictions are presented as a comparison.

$\sigma_{\text{fid}}(pp \rightarrow H \rightarrow \gamma\gamma)$	7 TeV	8 TeV	13 TeV
ATLAS data [48]	49 ± 18	$42.5^{+10.3}_{-10.2}$	52^{+40}_{-37}
LHC-XS [3]	24.7 ± 2.6	31.0 ± 3.2	$66.1^{+6.8}_{-6.6}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.4^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$

*Proton 5-quark Fock State:
Intrinsic Heavy Quarks*



Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_{\perp}^2)^{1/2}$$

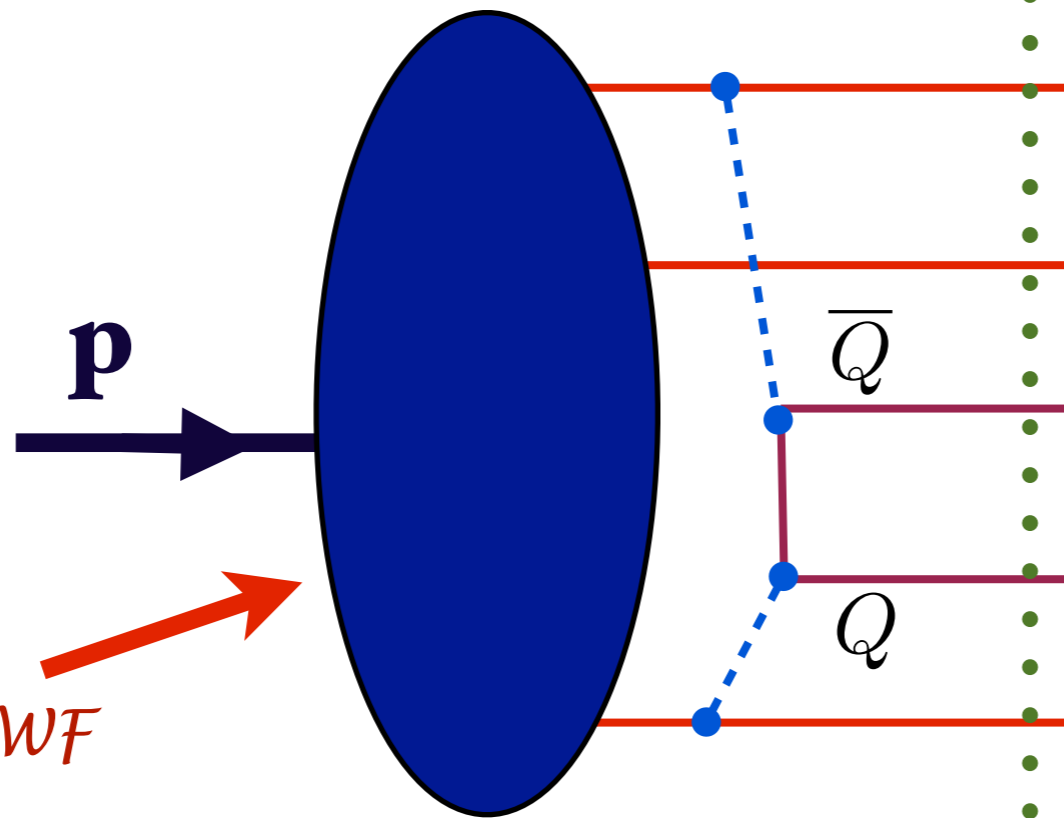
Probability (QED) $\propto \frac{1}{M_{\ell}^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

**Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al. Hoyer, Vogt, et al**

Fixed LF time

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Use AdS/QCD LFWF

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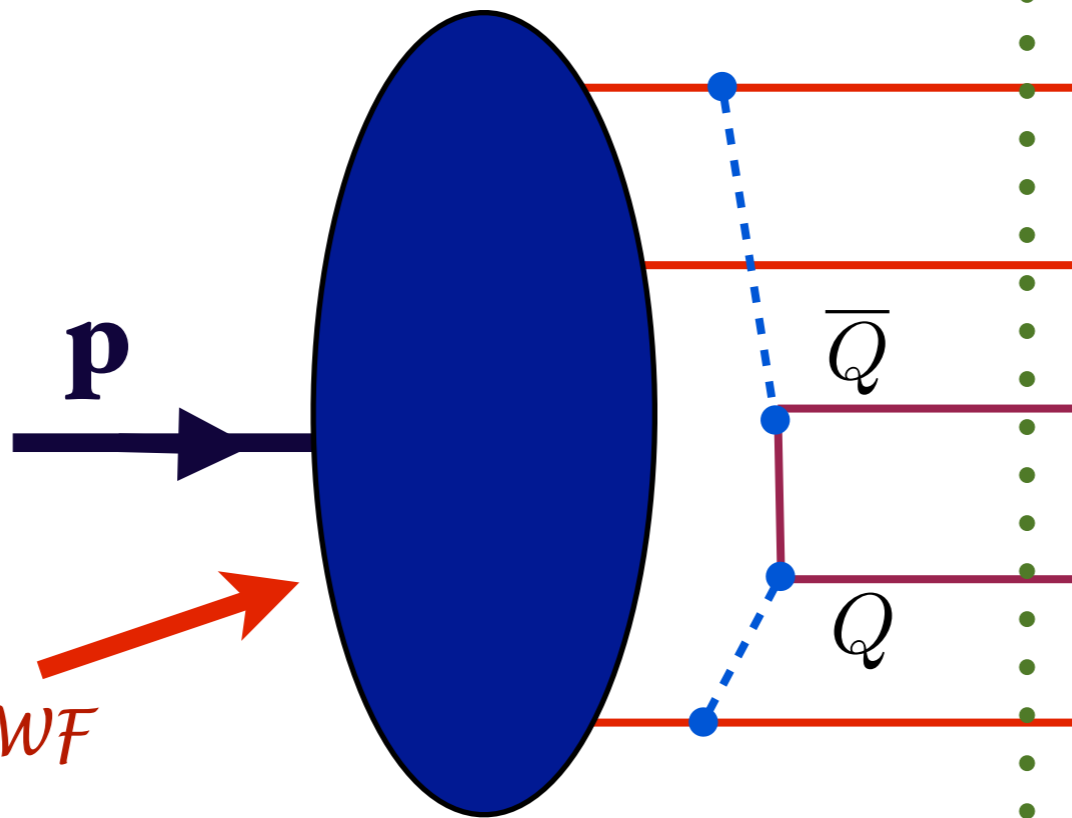
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*Proton 5-quark Fock State:
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Fixed LF time



*QCD predicts
Intrinsic Heavy
Quarks at high x .*

Minimal off-shellness

Use AdS/QCD LFWF

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Probability (QED) $\propto \frac{1}{M_{\ell}^4}$

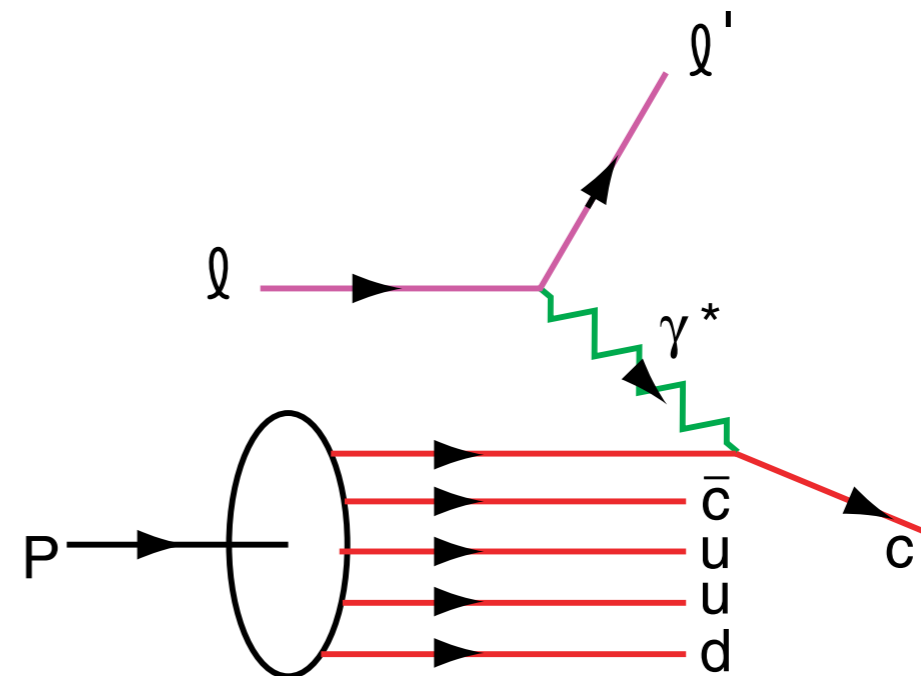
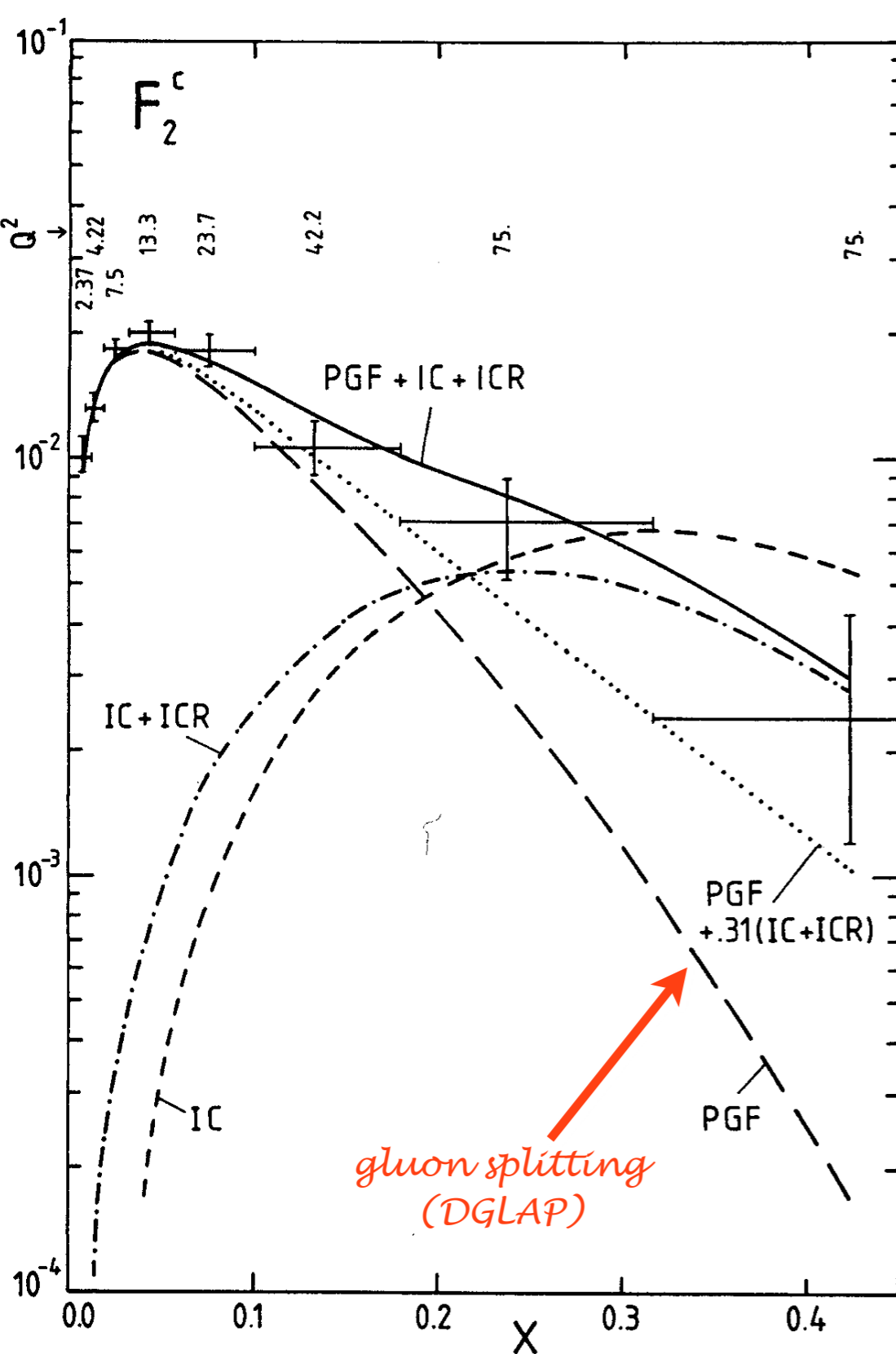
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Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

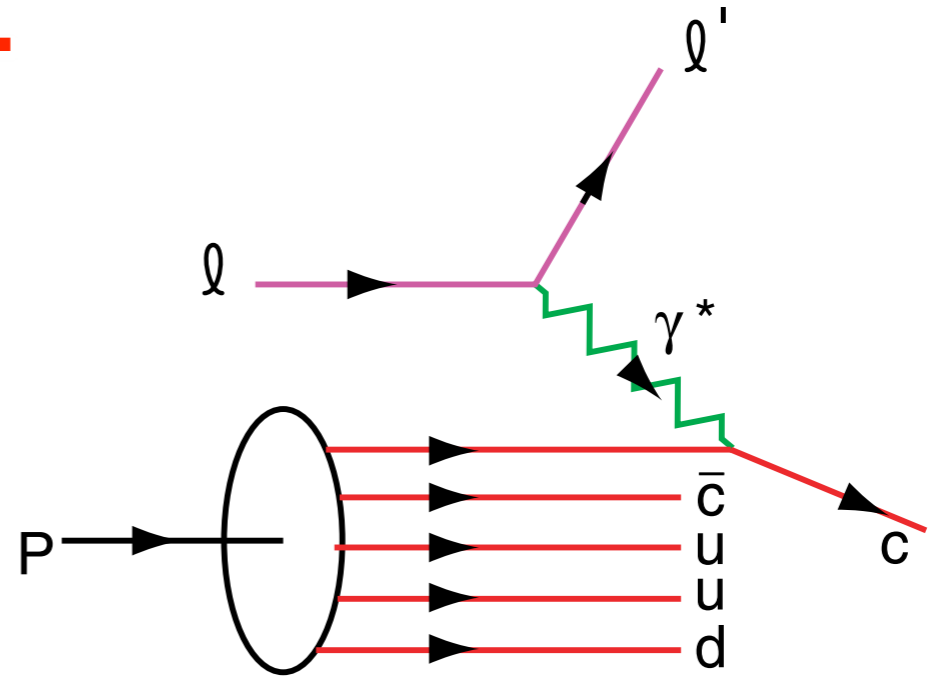
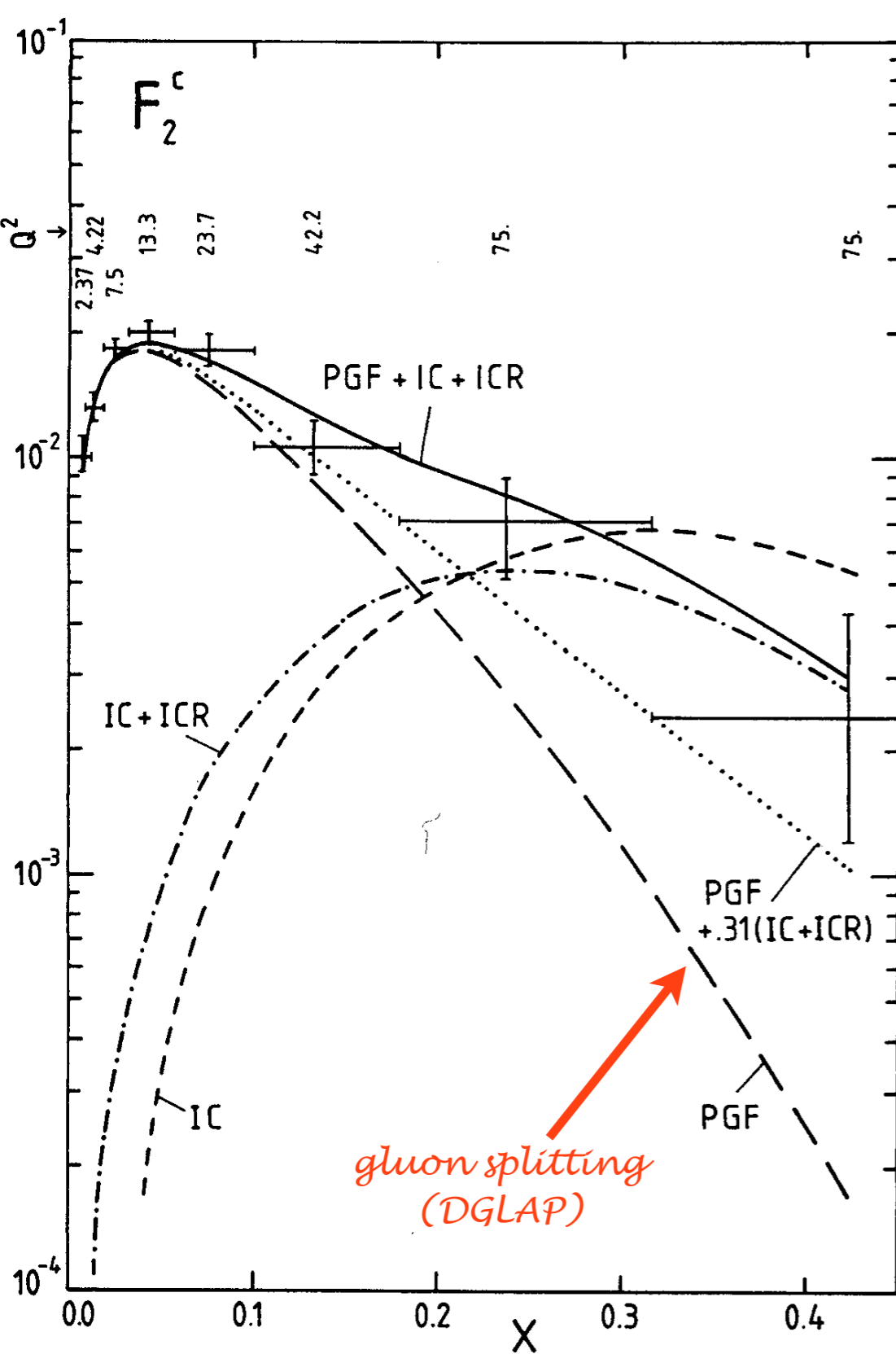
First Evidence for Intrinsic Charm Hoyer, Peterson, Sakai, sjb



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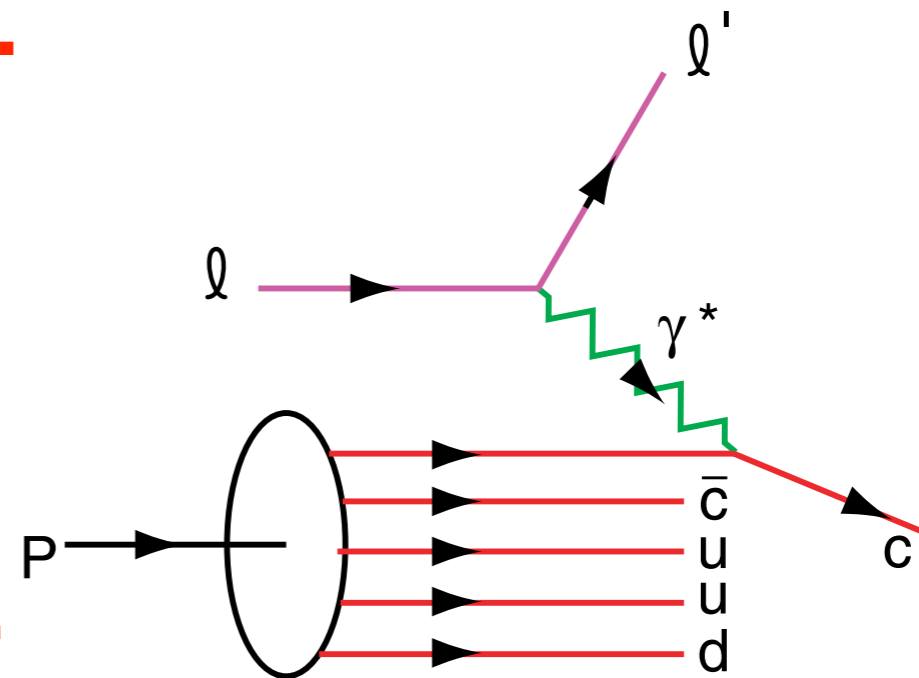
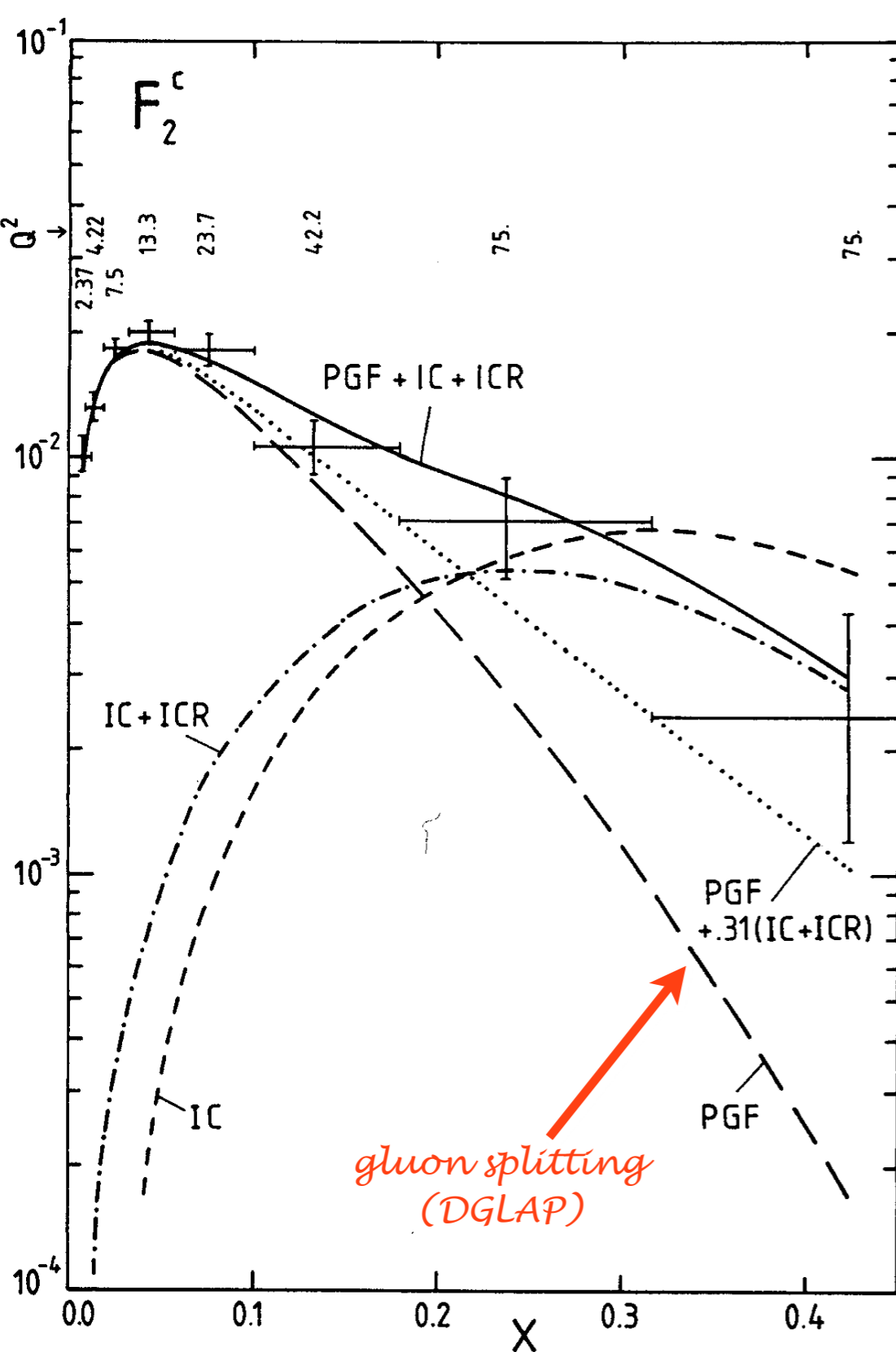
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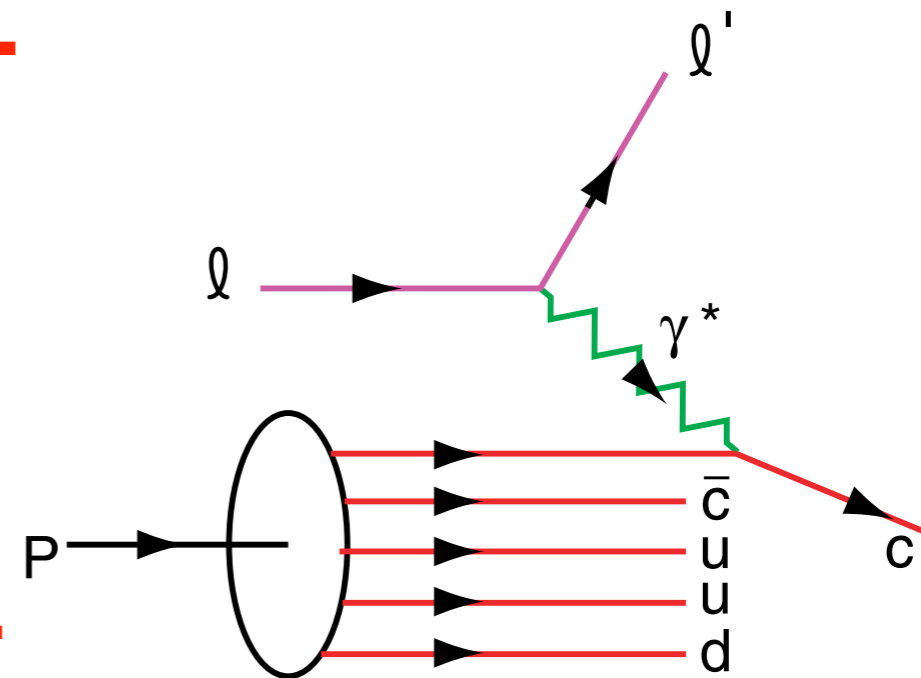
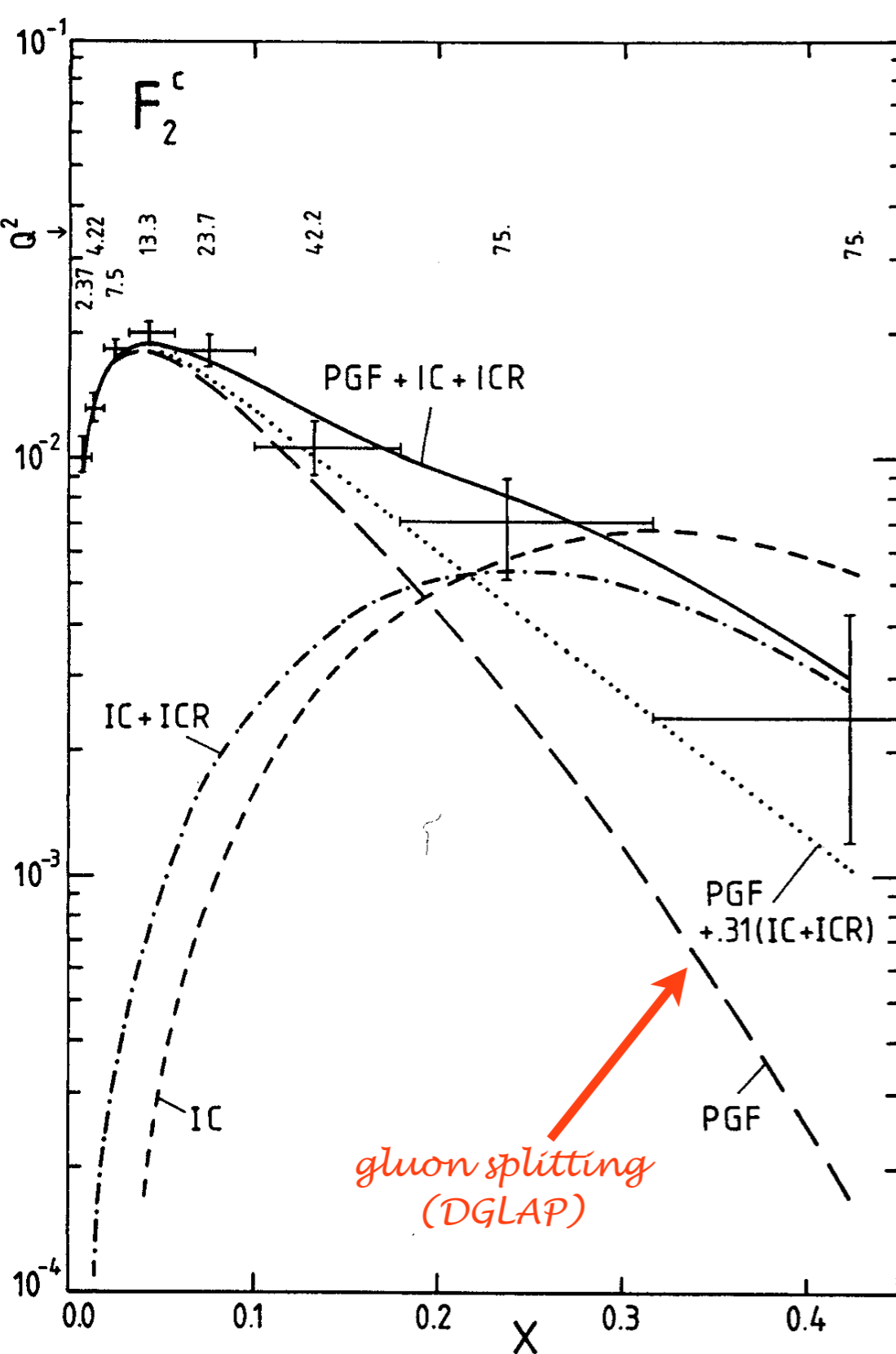
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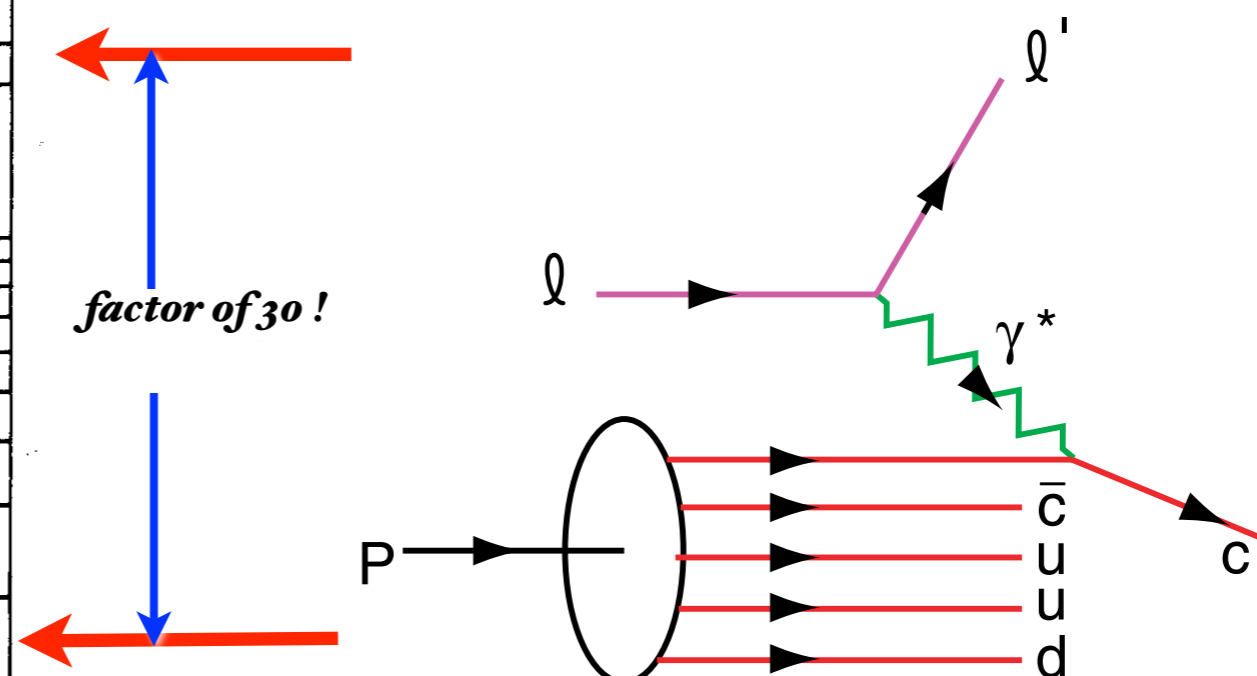
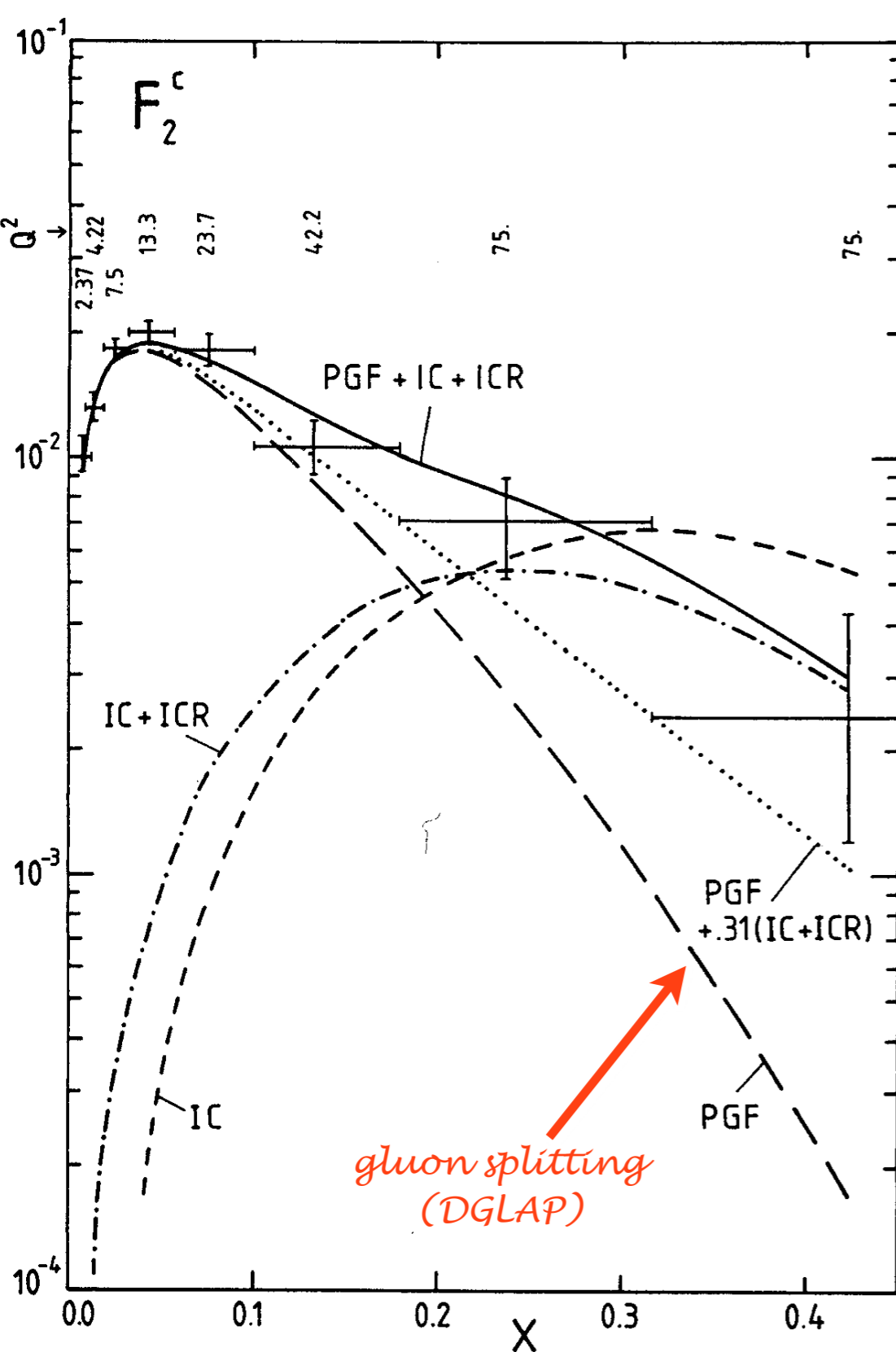


DGLAP / Photon-Gluon Fusion: factor of 30 too small

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factor of 30!

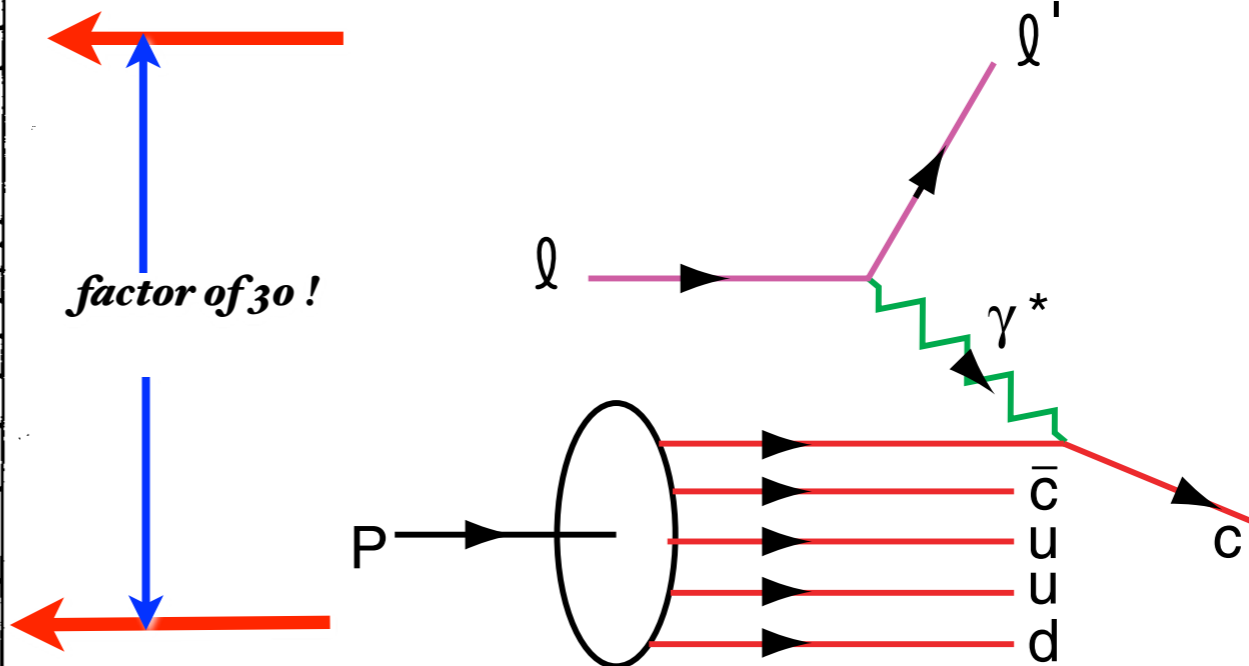
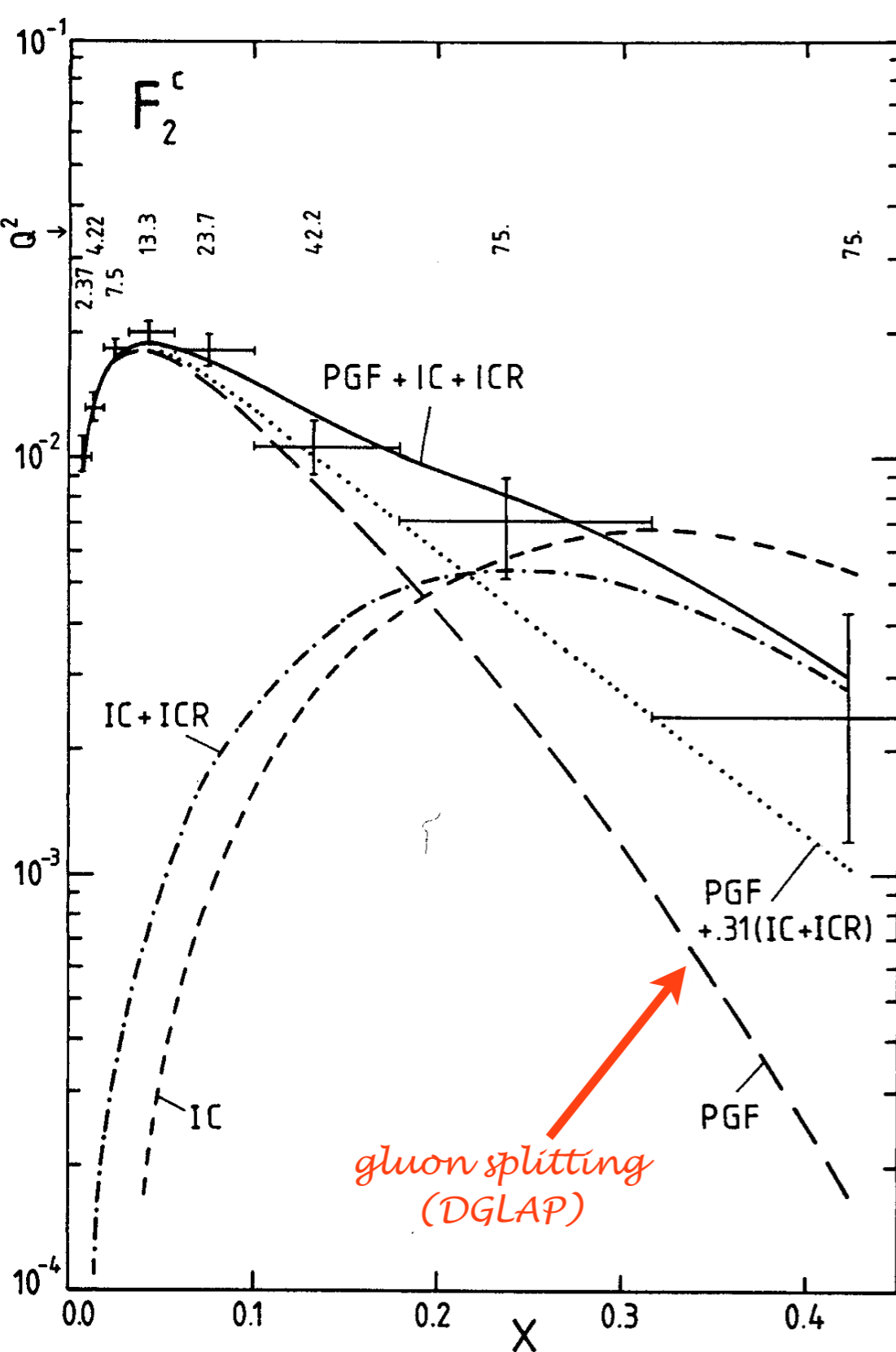
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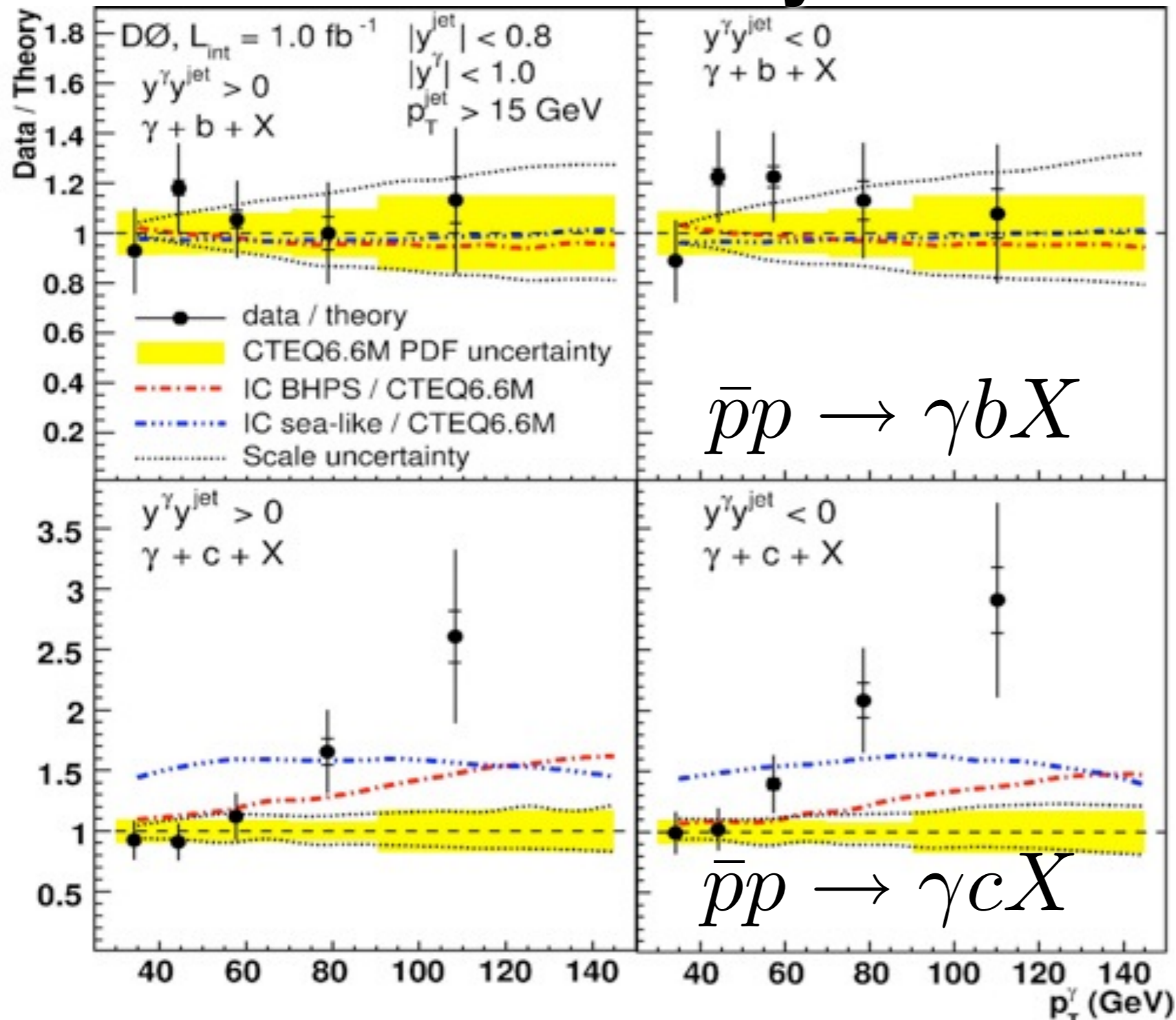
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Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections
in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

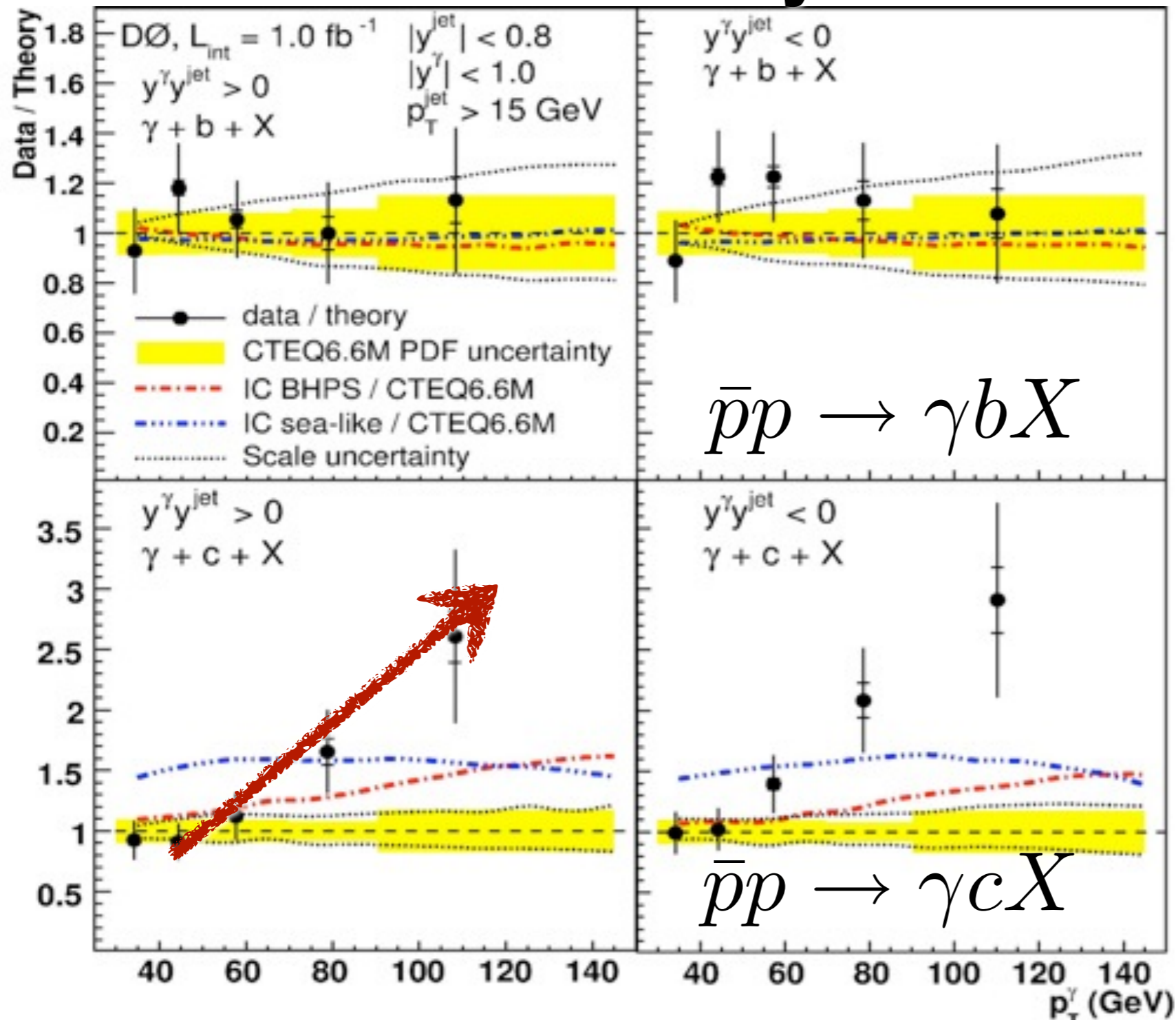
Data/Theory



Consistent with EMC measurement of charm structure function at high x

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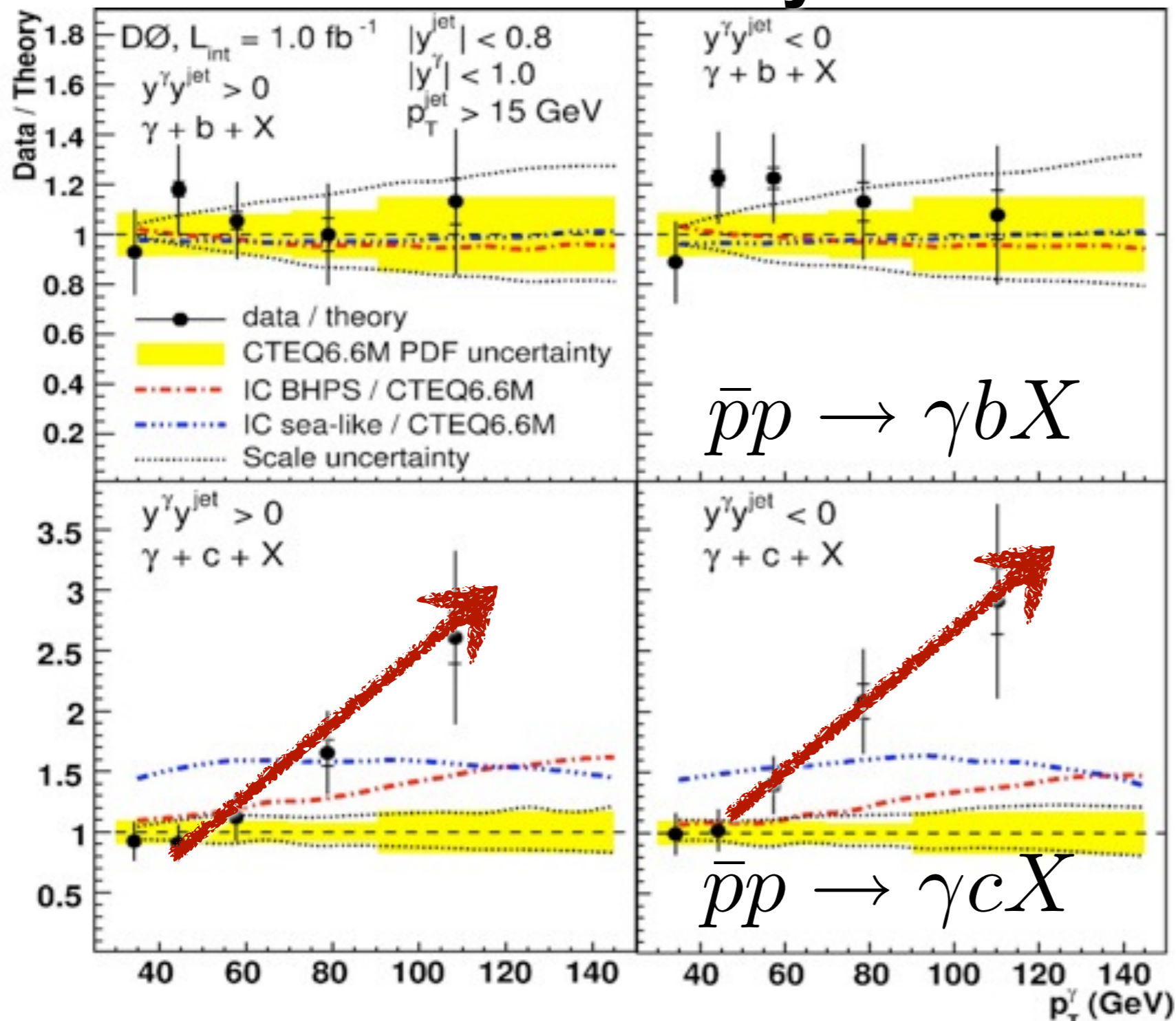
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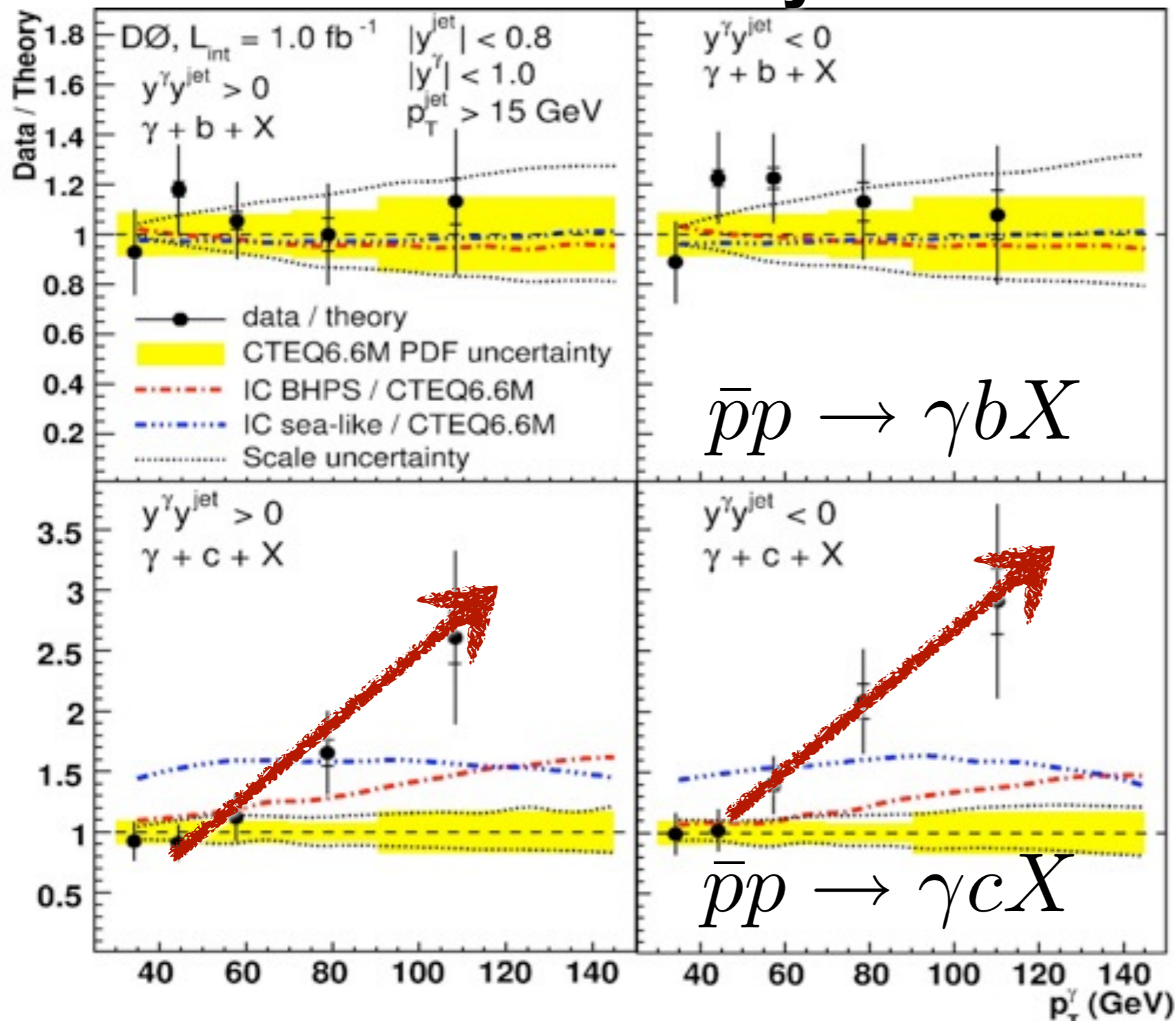
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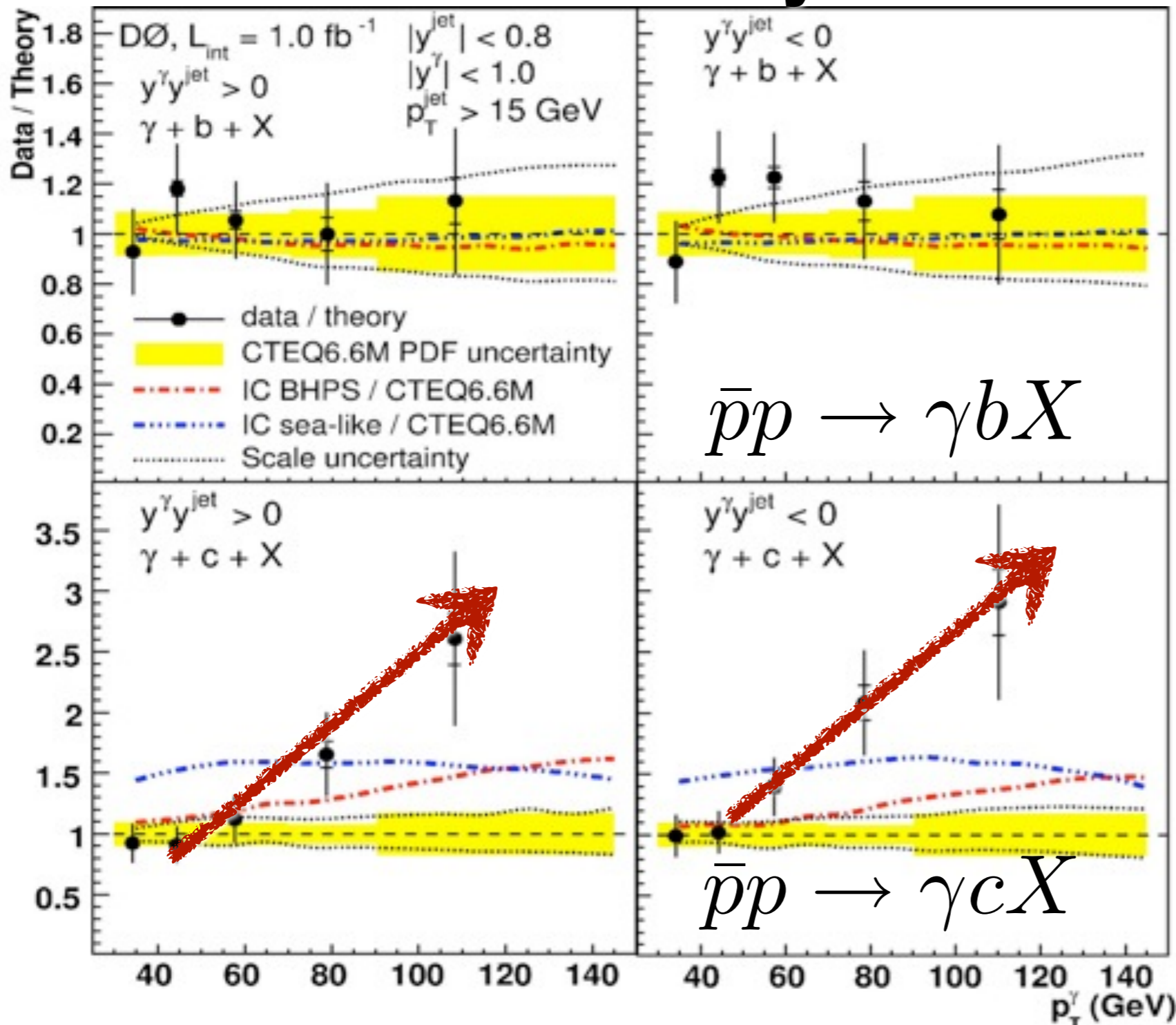


**Signal for significant
IC
at $x > 0.1$**

*Consistent with EMC measurement of charm
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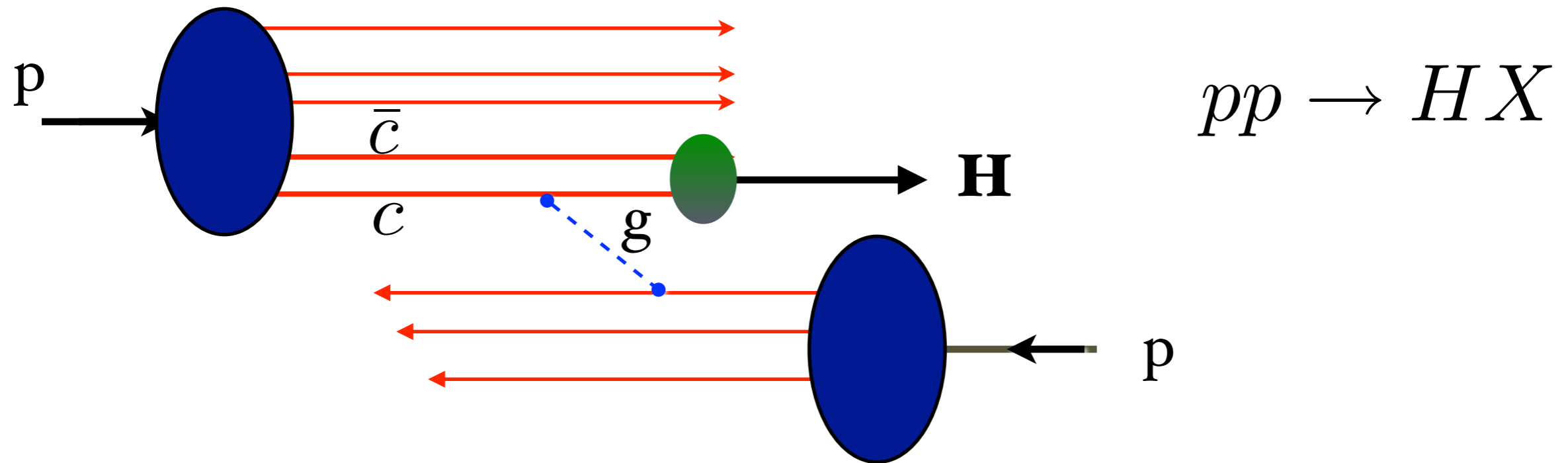
$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

**Ratio insensitive
to gluon PDF,
scales**

**Signal for significant
IC
at $x > 0.1$**

*Consistent with EMC measurement of charm
structure function at high x*

*Intrinsic Charm Mechanism for Inclusive
High- x_F Higgs Production*



Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs