

Special Degeneracy in PQCD

There is nothing special about a particular value for δ , thus for any δ

$$\begin{aligned} \rho(Q^2) = & r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3 \\ & + [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 \end{aligned}$$

According to the **principal of maximum conformality** we must set the scales such to absorb all ‘renormalon-terms’, i.e. **non-conformal terms**

$$\begin{aligned} \rho(Q^2) = & r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \dots) \underline{r_{2,1}} \\ & + (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \dots) \underline{r_{3,2}} + (\beta_0^3 + \dots) r_{4,3} \\ & + r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \dots) \underline{r_{3,1}} \\ & + \dots \end{aligned}$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \dots + \frac{(-1)^n}{n!} \frac{d^{n-1}\beta}{(d \ln \mu^2)^{n-1}} r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \dots$$