

Exposing the Renormalization Scheme Dependence

Observable in the \mathcal{R}_δ -scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \dots$$

$$\mathcal{R}_0 = \overline{\text{MS}} \ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS} \quad \mu^2 = \mu_{\overline{\text{MS}}}^2 \exp(\ln 4\pi - \gamma_E) \ , \quad \mu_{\delta_2}^2 = \mu_{\delta_1}^2 \exp(\delta_2 - \delta_1)$$

Note the divergent ‘renormalon series’ $n! \beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \text{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$ -term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p . Grouping the different δ_k -terms, one recovers in the $N_c \rightarrow 0$ Abelian limit the dressed skeleton expansion.