Properties of Light Nuclei from Lattice QCD

I. Magnetic structure of nuclei

II. Axial structure

(III. Parton structure)

William Detmold, MIT

Lattice nuclear structure

- NPLQCD collaboration
- Pioneering the study of nuclei in LQCD
  - Spectroscopy and binding
    PRD 80 (2009) 074501
    PRL 106 (2011) 162001
    MPLA 26 (2011) 2587-2595
    PRD 85 (2012) 054511
    PRD 87 (2013), 034506
    PRD 91 (2015), 114503
  - Scattering
    PRL. 97 (2006) 012001
    NPA 794 (2007) 62-72
    PRD 81 (2010) 054505
    PRL. 109 (2012) 172001
    PRC 88 (2013), 024003
    PRD 92 (2015),114512

n

- NPLQCD collaboration
- Nuclear structure through LQCD in presence of external fields

1. Nuclear structure: magnetic moments, polarisabilities (A<5)
   PRL **113**, 252001 (2014)
   PRD **92**, 114502 (2015)
   PRL **116**, 112301 (2016)

2. Nuclear reactions: np→dγ
   PRL **115**, 132001 (2015)

3. Gamow-Teller transitions: pp→de**ν**, $g_A(^3H)$ arXiv:1610.04545

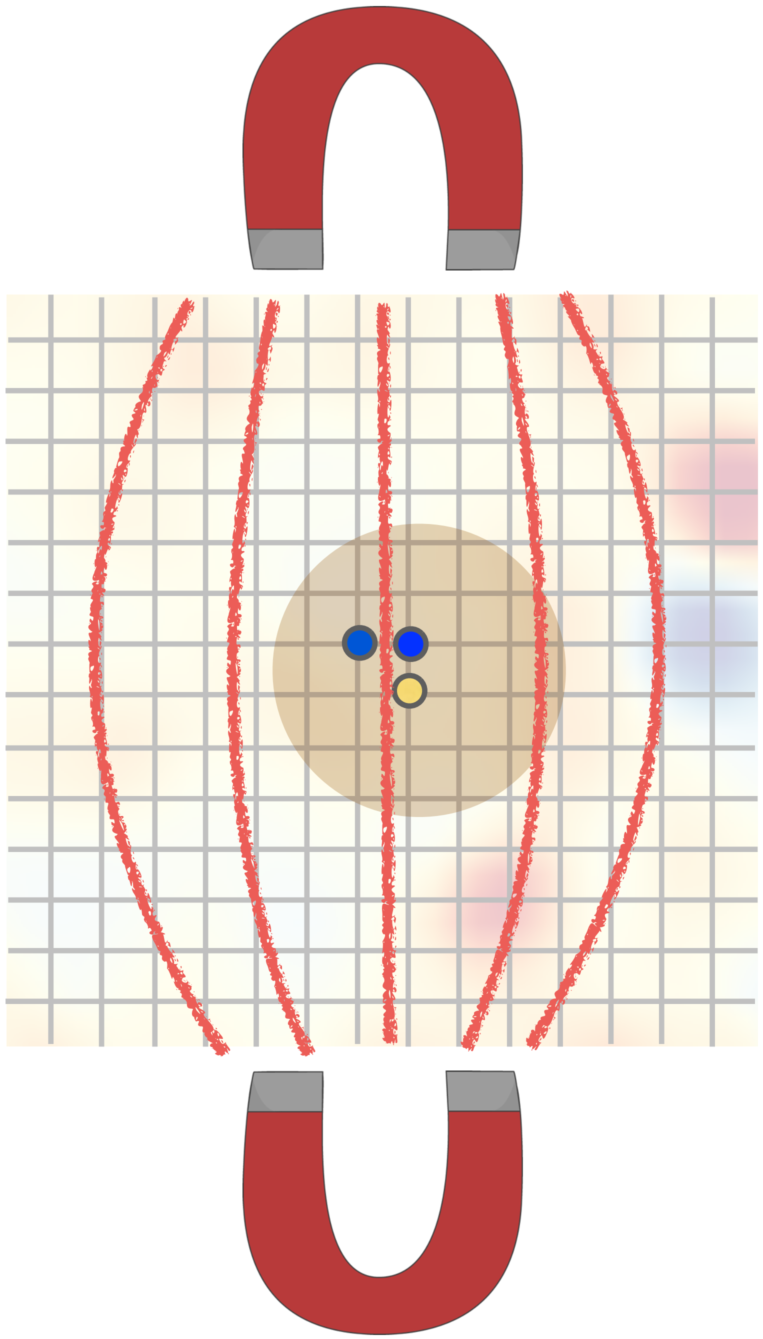
4. Isotensor polarisability (2**ν**ββ decay nn→pp) arXiv:1701.03456

External field method

- Hadron/nuclear energies are modified by presence of fixed external fields
- Eg: fixed B field

$$E_{h;j_z}(\mathbf{B}) = \sqrt{M_h^2 + (2n+1)|Q_h eB|} - \boldsymbol{\mu}_h \cdot \mathbf{B} - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- QCD calculations with multiple fields enable extraction of coefficients of response
  - Magnetic moments, polarisabilities, …
- Not restricted to simple EM fields



[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

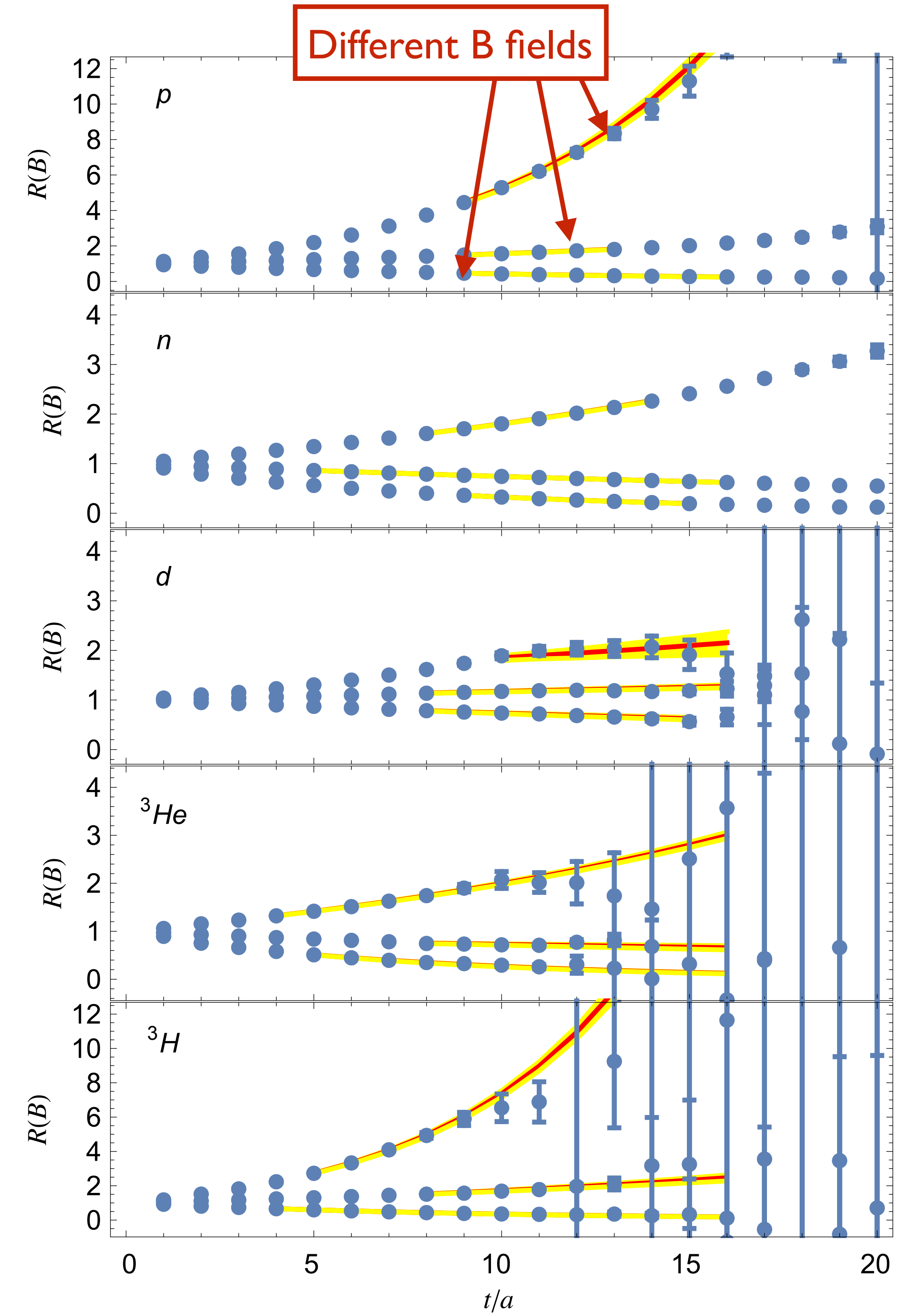
- Magnetic field in z-direction (strength quantised by lattice periodicity)
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E^{(B)}_{+j} - E^{(B)}_{-j} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

$$R(B) = \frac{C^{(B)}_{j}(t)\ C^{(0)}_{-j}(t)}{C^{(B)}_{-j}(t)\ C^{(0)}_{j}(t)} \overset{t\to\infty}{\longrightarrow} Ze^{-\delta E^{(B)}t}$$

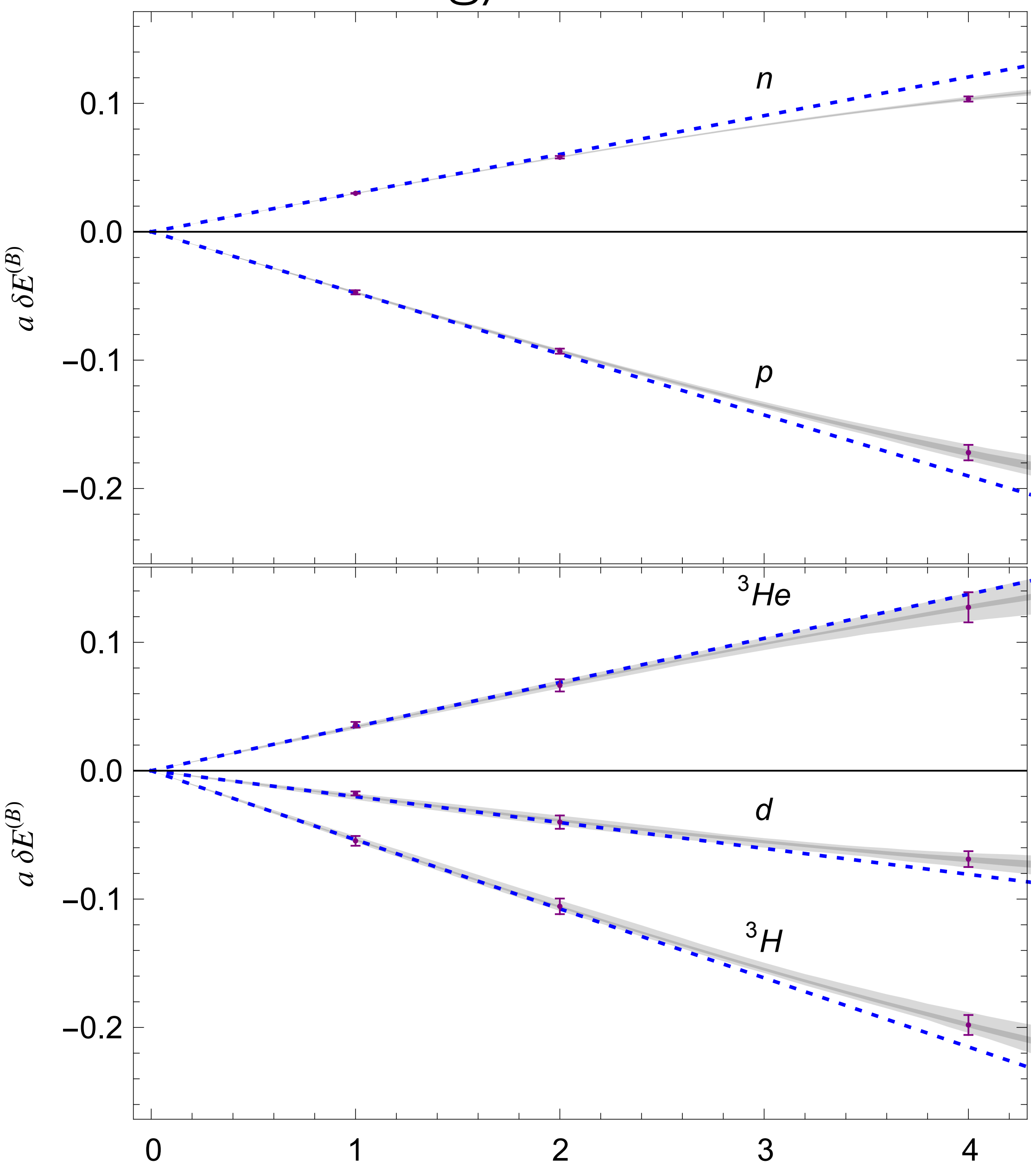
- Careful to be in single exponential region of each correlator



[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

Energy shift vs B

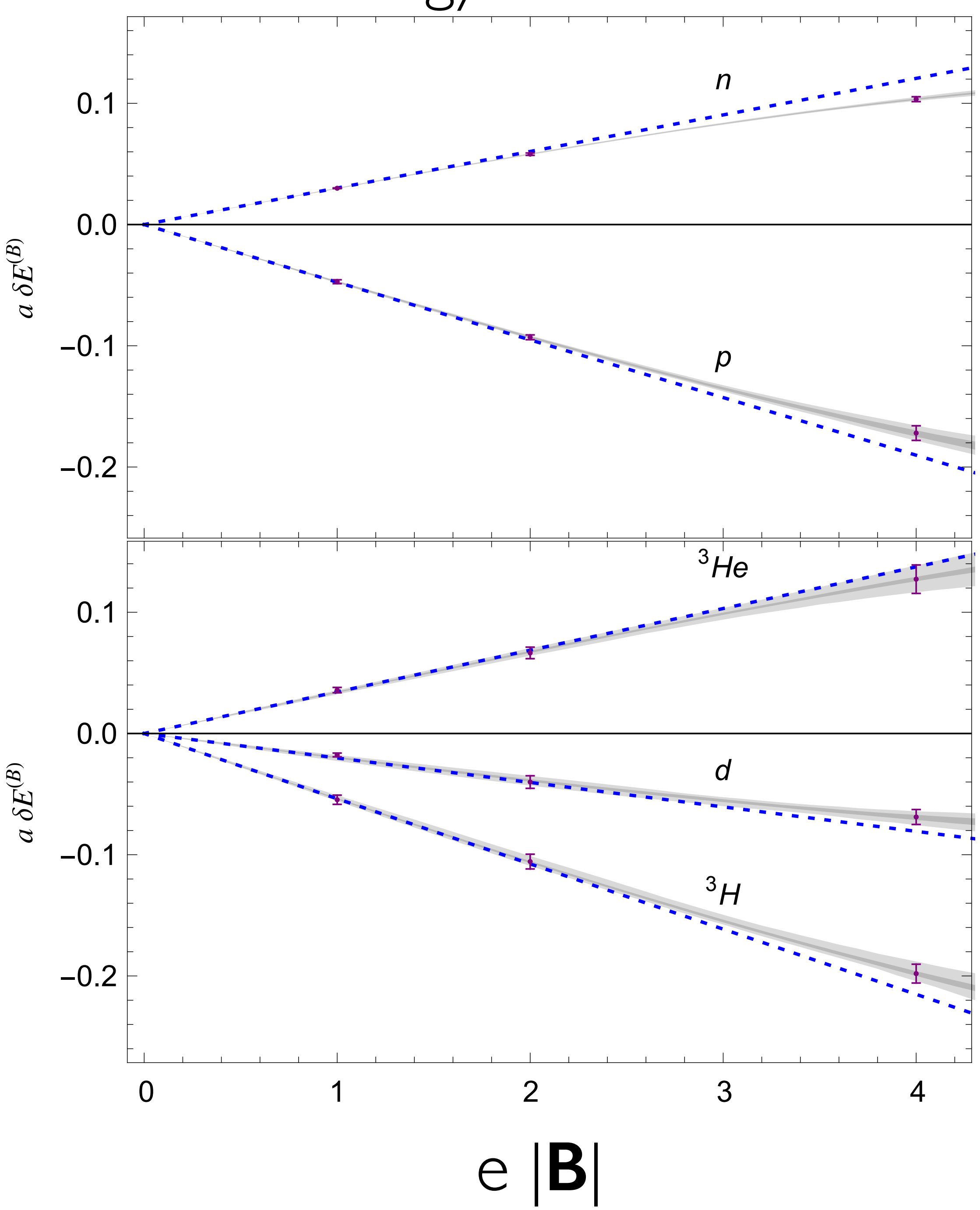


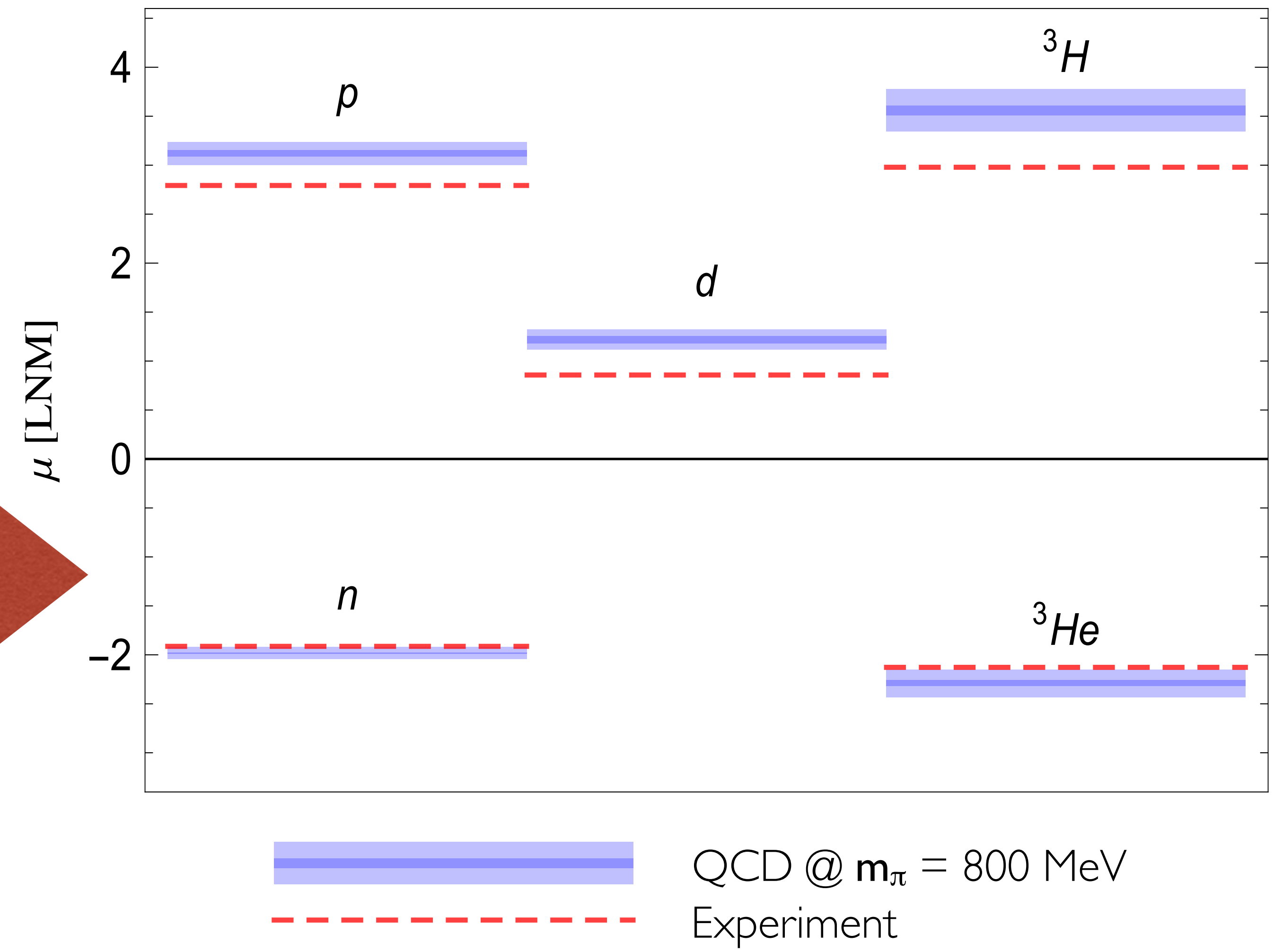
e |**B**|

[NPLQCD PRL **113**, 252001 (2014)]

Magnetic moments of nuclei

Energy shift vs B





	n	p	d	3	3
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

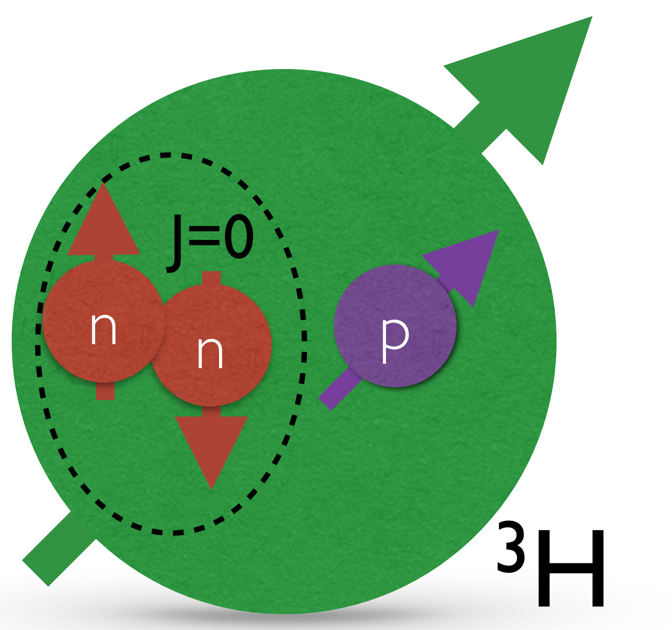
Magnetic moments of nuclei

- Numerical values are surprisingly interesting
- Shell model expectations

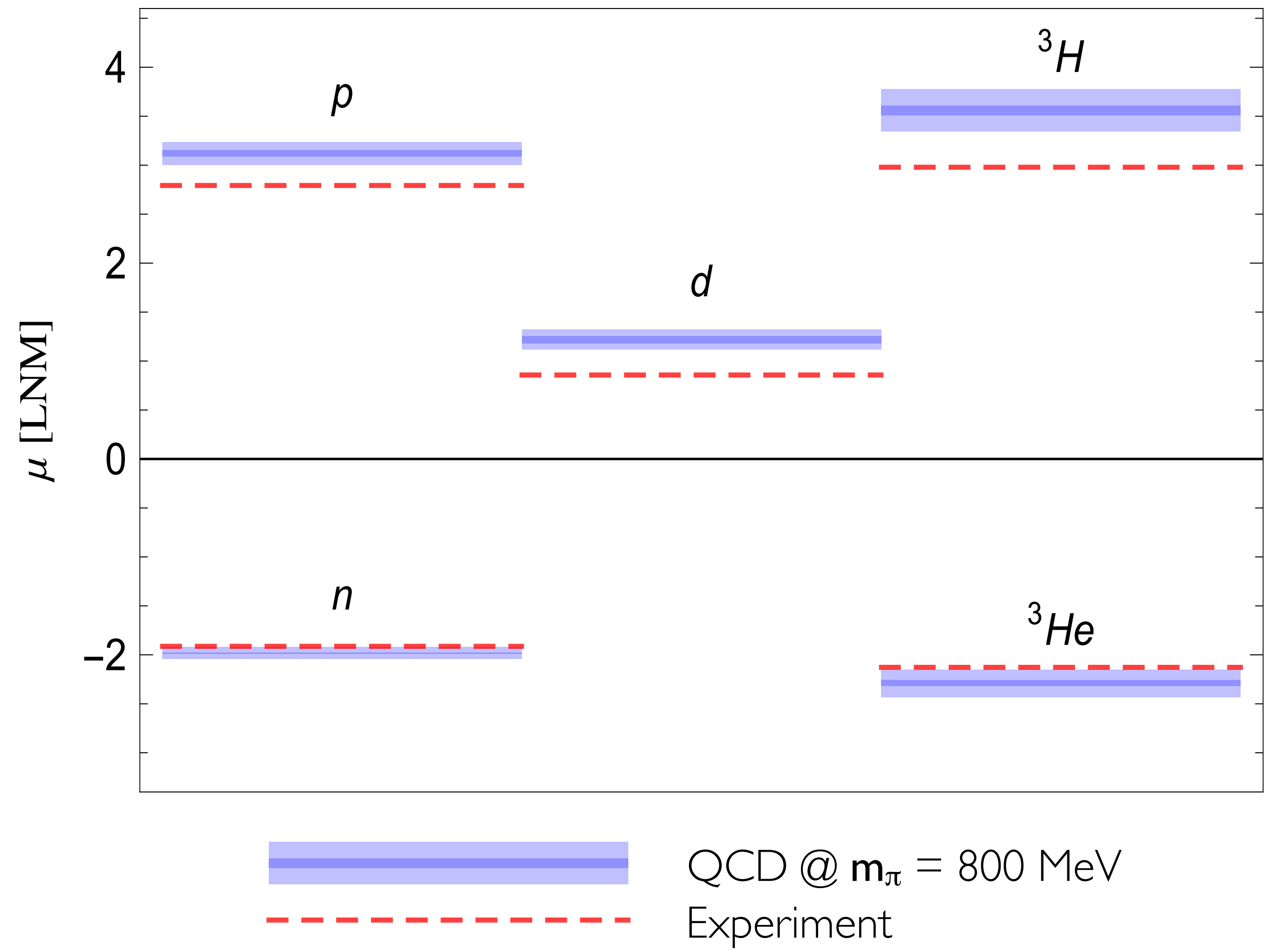
$$\mu_d = \mu_p + \mu_n$$

$$\mu_{^3\mathrm{H}} = \mu_p$$

$$\mu_{^3\mathrm{He}} = \mu_n$$



- Lattice results appear to suggest heavy quark nuclei are shell-model like!



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In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

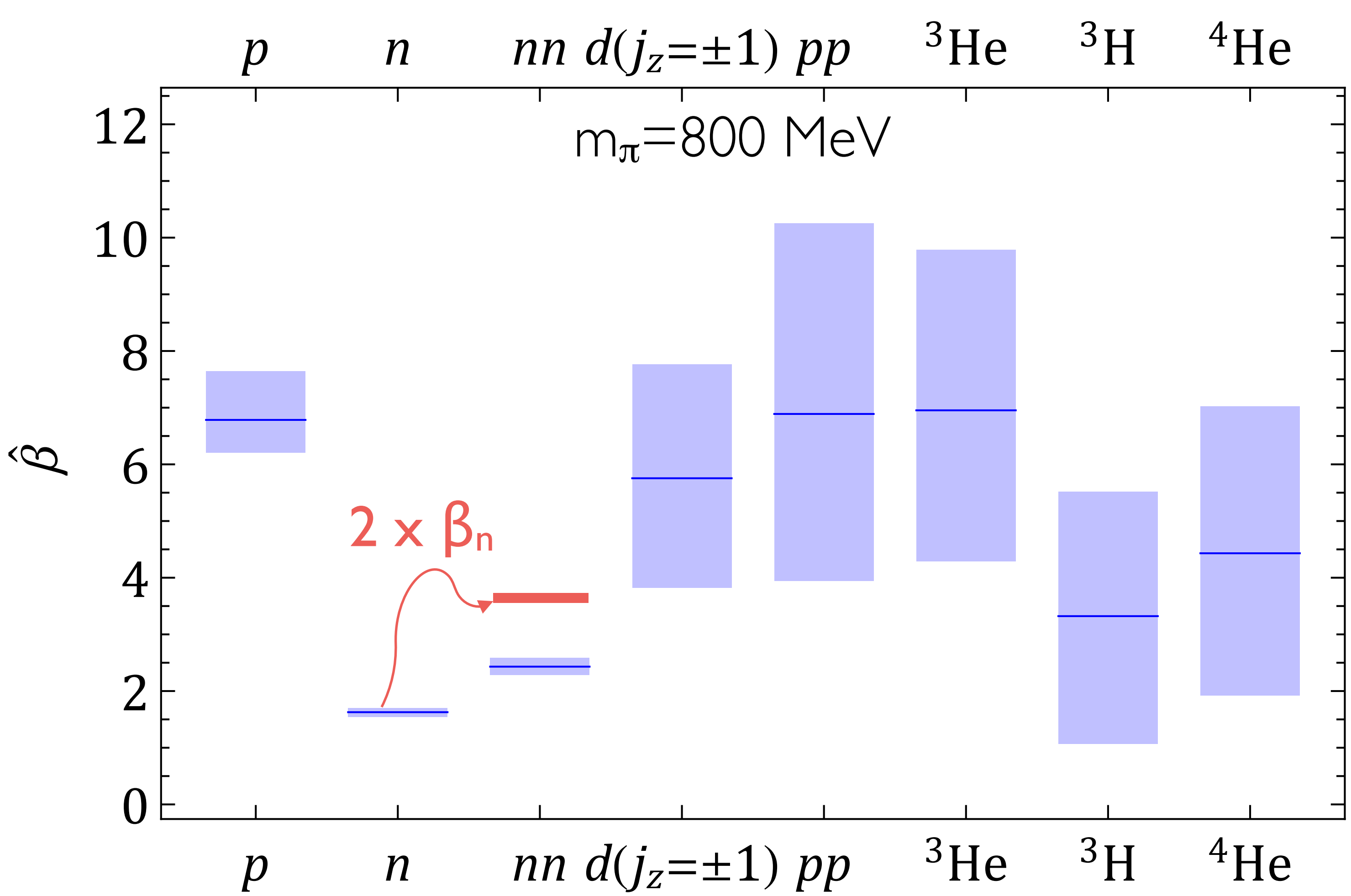
Magnetic Polarisabilities

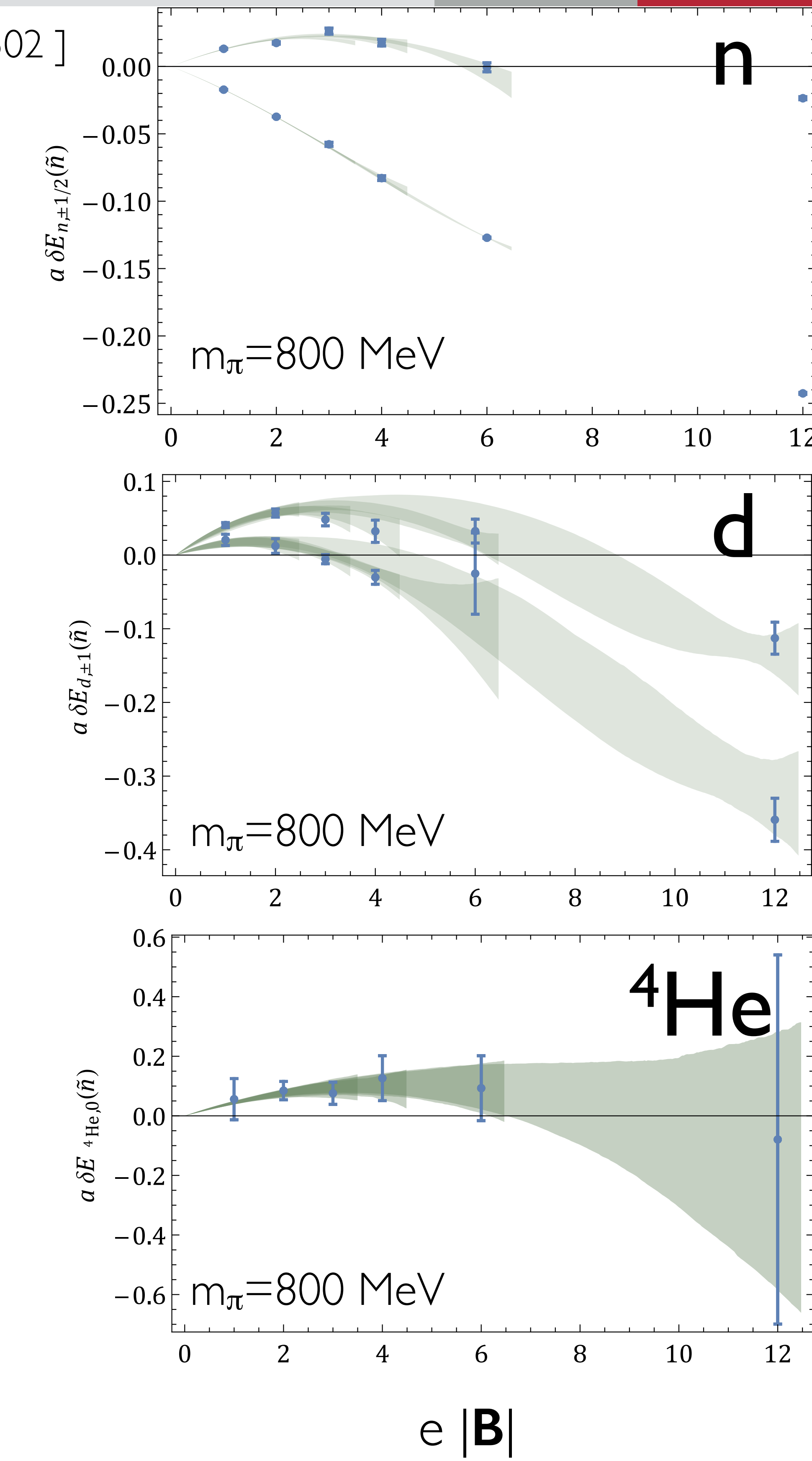
[NPLQCD Phys.Rev. D92 (2015), 114502]

- Second order shifts

$$E_{h;j_z}(\mathbf{B}) \;=\; \sqrt{M_h^2 + (2n+1)|Q_h e B|} - \boldsymbol{\mu}_h \cdot \mathbf{B} \\ \quad - 2\pi\beta_h^{(M0)}|\mathbf{B}|^2 - 2\pi\beta_h^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle + \ldots$$

- Care required with Landau levels
- Polarisabilities (dimensionless units)

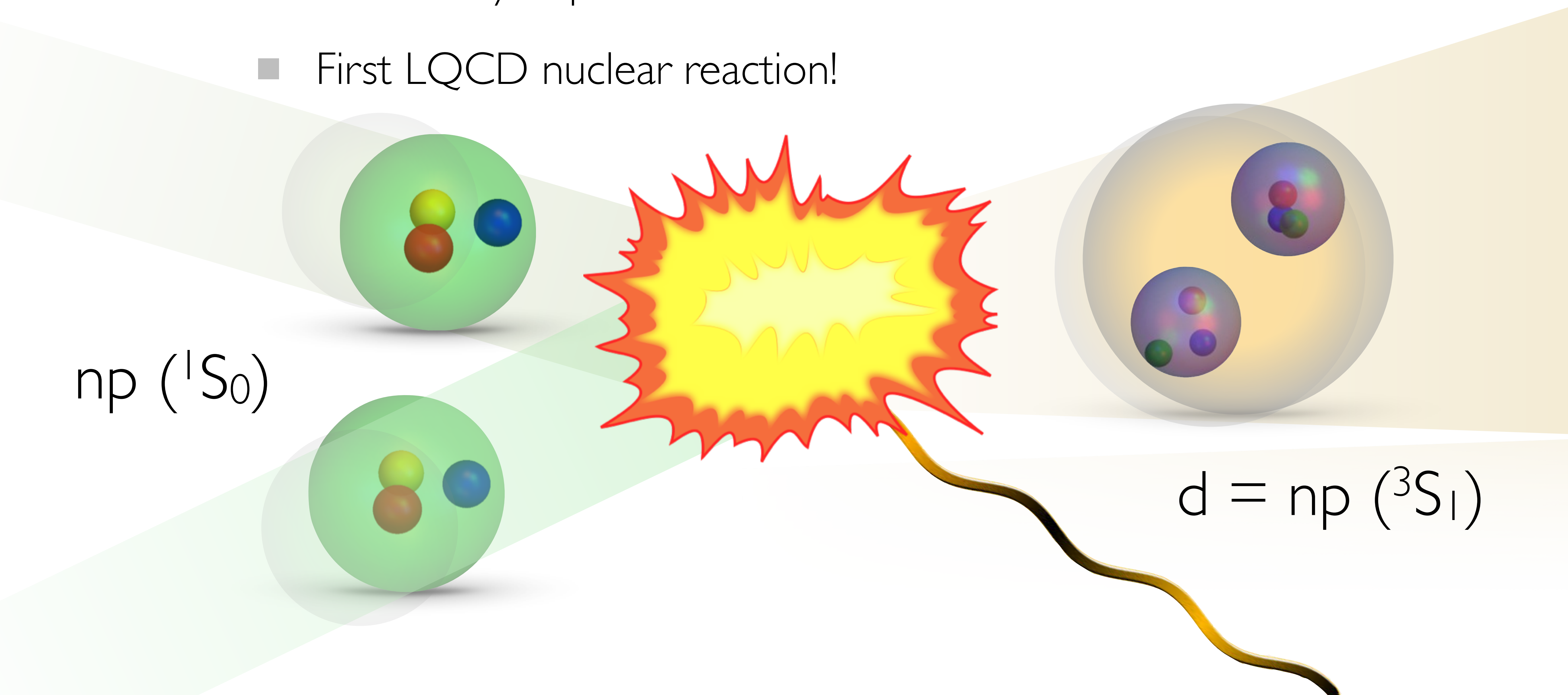




Thermal Neutron Capture Cross-Section

[NPLQCD PRL 115, 132001 (2015)]

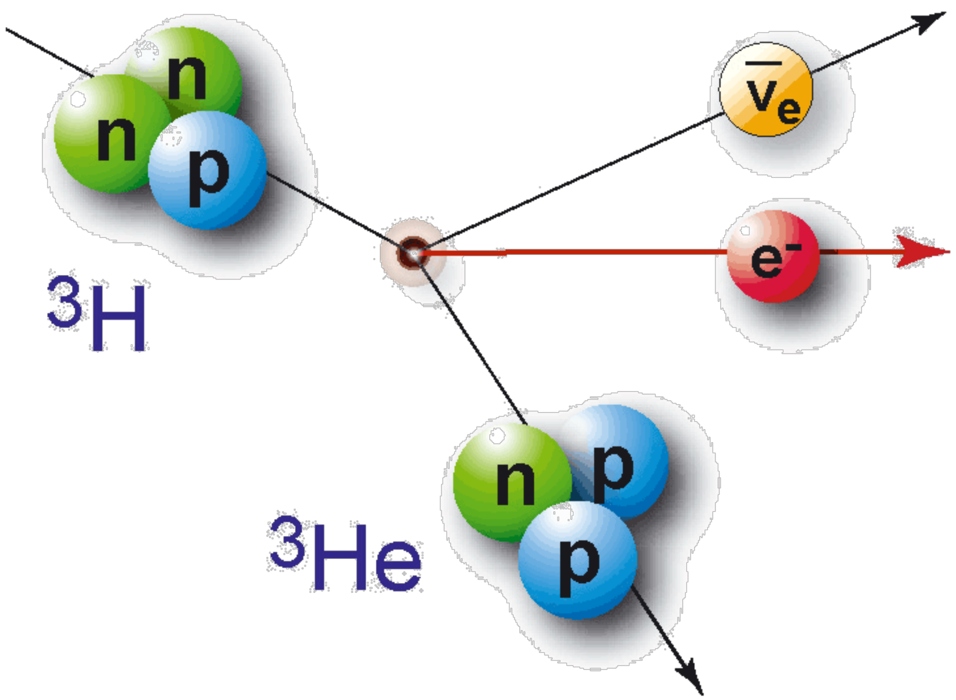
- Thermal neutron capture cross-section: np→dγ
  - Critical process in Big Bang Nucleosynthesis
  - Historically important: MEC contributions ~10%
  - First LQCD nuclear reaction!



Axial Background Field

NPLQCD arXiv:1610.04545

- Background axial field
- Axial coupling to NN system
  - $pp \rightarrow de^{+}\nu$ fusion
  - Muon capture: MuSun @ PSI
  - $d\,\nu \rightarrow n\,n\,e+$: SNO
- Tritium half-life
  - Understand multi-body contributions to $\langle \mathbf{GT} \rangle$: better predictions for decay rates of larger nuclei



Axial Background Field

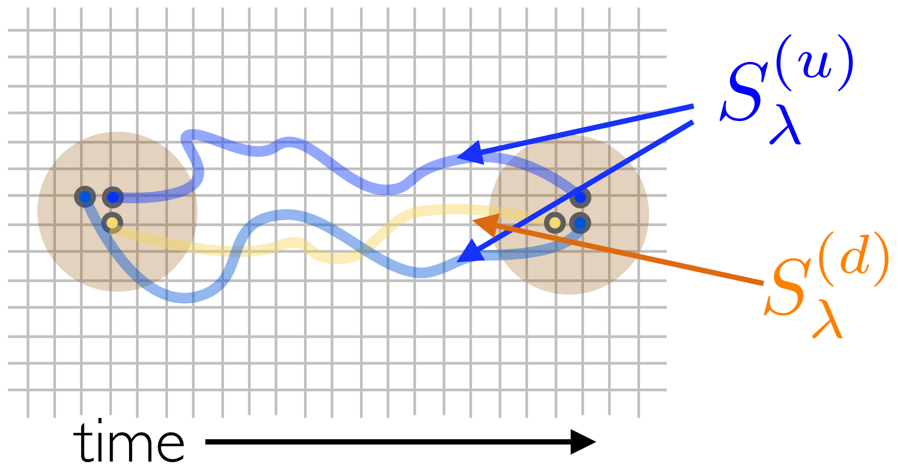
Example: fixed magnetic field → moments, polarisabilities
Axial case: fixed axial background field → axial charges, GT matrix elts.

Construct correlation functions from propagators modified in axial field

compound propagator — **constant**

$$S_\lambda^{(q)}(x,y) = S^{(q)}(x,y) + \lambda_q \int dz\, S^{(q)}(x,z)\,\gamma_3\gamma_5\, S^{(q)}(z,y)$$

$C_{\lambda_u;\lambda_d}(t) =$

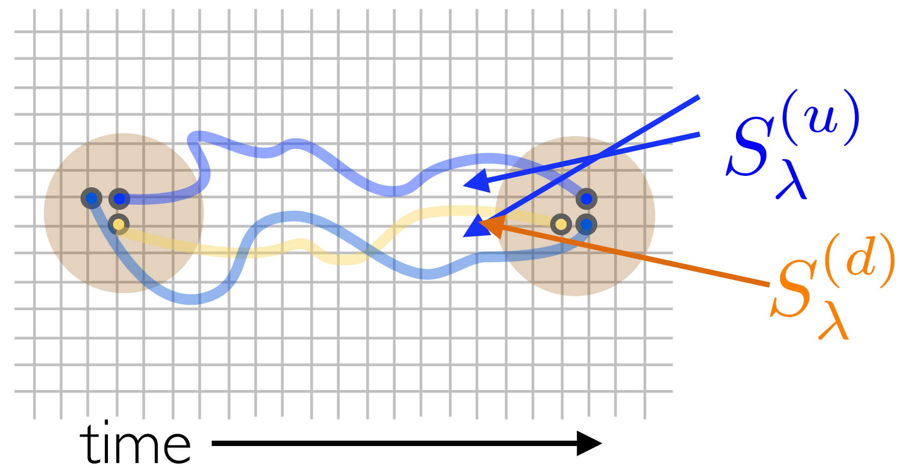


Linear response ↔ axial matrix element

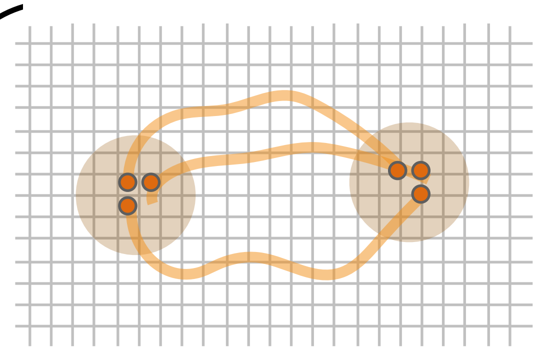
[NPLQCD Nucl. Phys. **A743**, 170 (2004)]

Axial Background Field

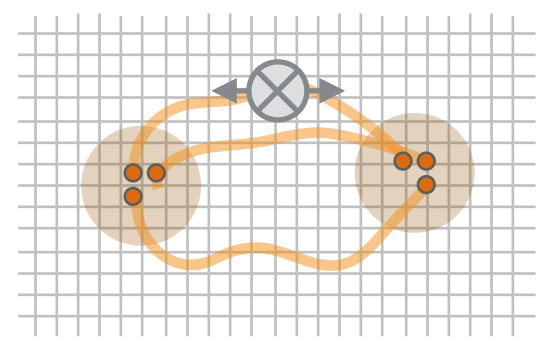
$C_{\lambda_u;\lambda_d}(t) =$



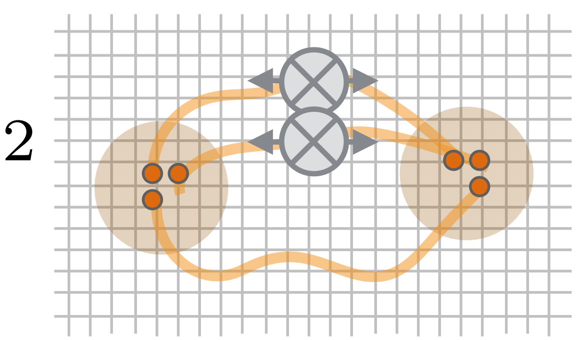
$C_{\lambda_u;\lambda_d}(t) = \Big($



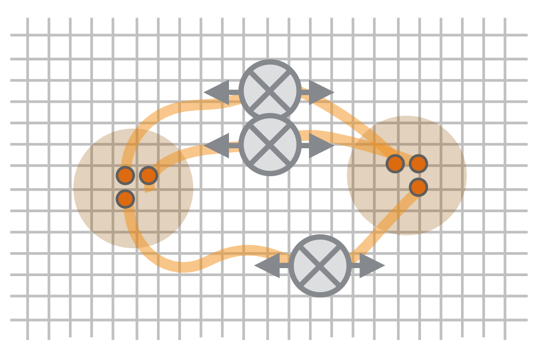
$+ \; \lambda$



$+ \; \lambda^2$

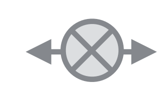


$+ \; \lambda^3$



$\Big)$

Linear response gives axial matrix element



Implicit sum over current insertion times

Tritium β decay

- Tritium decay half life

$$\frac{(1+\delta_R) f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V\, g_A^2 \langle \mathbf{GT} \rangle^2}$$

half-life

vector ME

axial ME

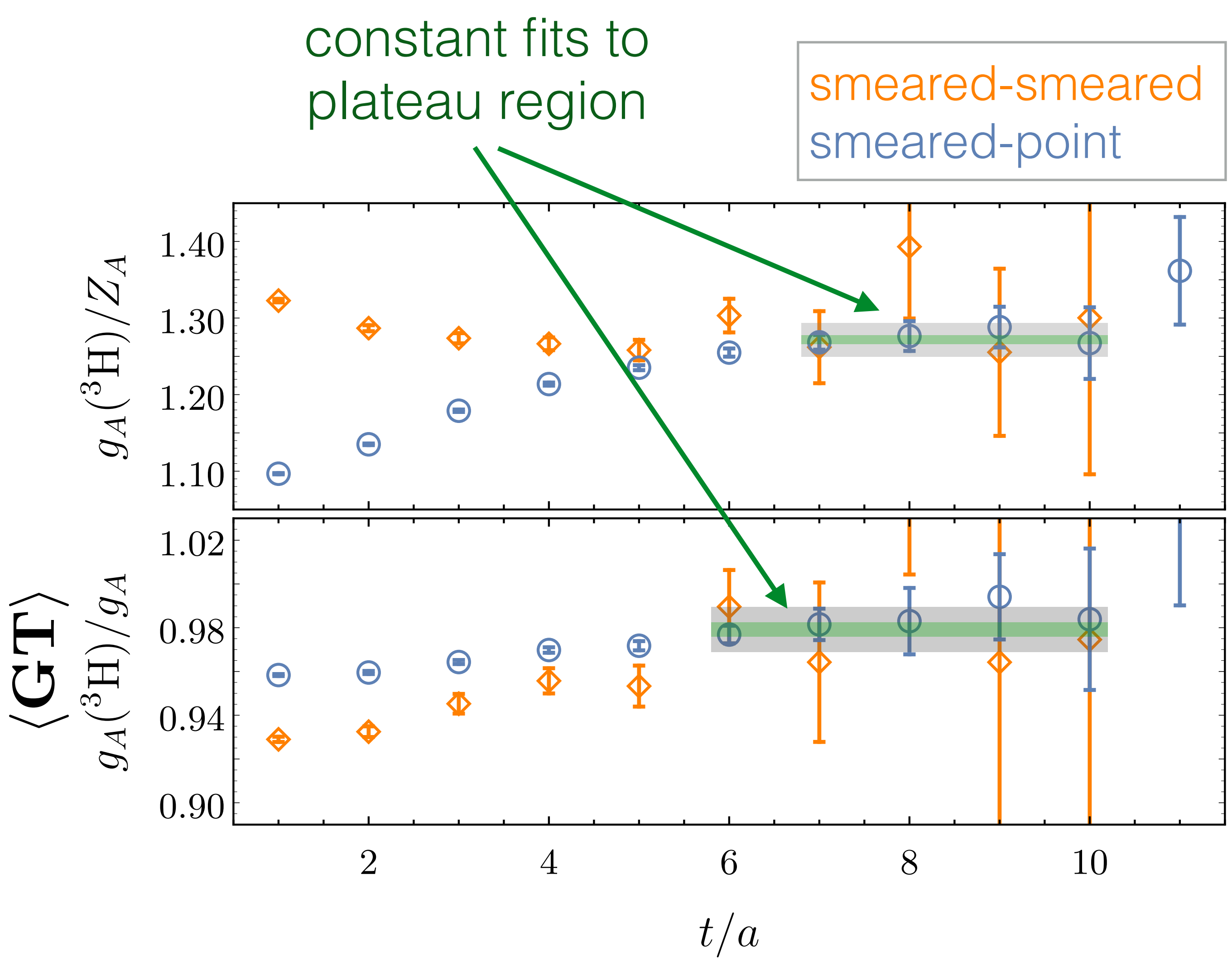
known from theory or expt.

- Biggest uncertainty in

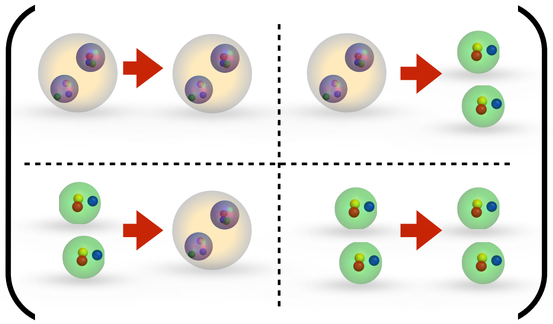
$$g_A \langle \mathbf{GT} \rangle = \langle {}^3\mathrm{He} | \overline{\mathbf{q}} \gamma_{\mathbf{k}} \gamma_{\mathbf{5}} \tau^- \mathbf{q} | {}^3\mathrm{H} \rangle$$

- Form ratios of correlators to cancel leading time-dependence:

$$\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t \to \infty} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT} \rangle$$



- Axial background field mixes $^3S_1$, $^1S_0$ states

H = 

- Extract matrix element through linear response of $^3S_1 \rightarrow {}^1S_0$ correlators to the background field

matrix elt. is linear in λ_u

$$C^{(^3S_1,\,^1S_0)}_{\lambda_u;\lambda_d=0}(t) = \lambda_u \sum_{\tau=0}^{t} \sum_{\mathbf{x}} \langle 0 | \chi^3_{^3S_1}(\mathbf{x},t) A^u_3(\tau) \chi^\dagger_{^1S_0}(0) | 0 \rangle + c_2 \lambda_u^2 + c_3 \lambda_u^3,$$

correlator formed with background field coupling to u quark

irrelevant consts.

- Calculate correlators at multiple values of $\lambda_u,\ \lambda_d$
  → extract matrix element pieces

- Form ratios of compound correlators to cancel leading time-dependence

transition pieces linear in λ_u-λ_d

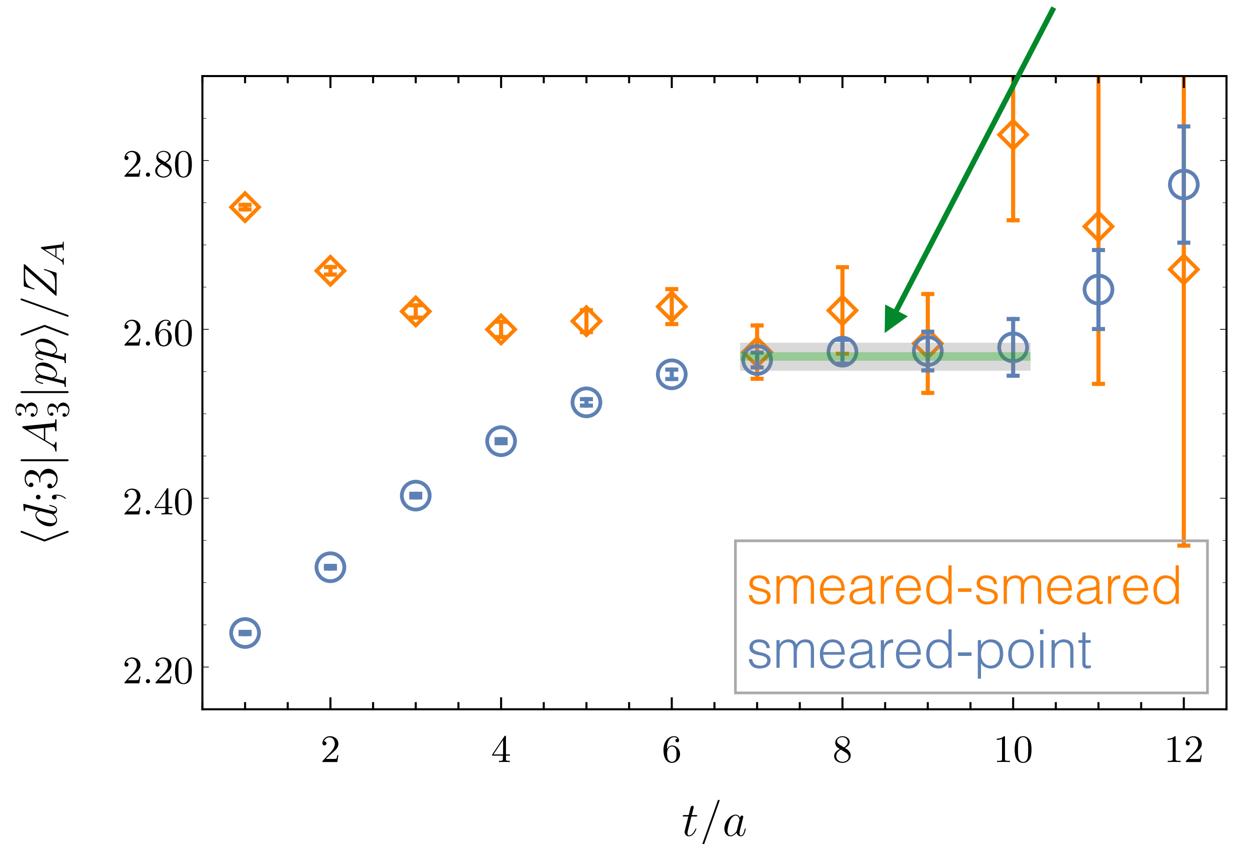
$$R_{^3S_1,^1S_0}(t) = \frac{C^{(^3S_1,^1S_0)}_{\lambda_u,\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} - C^{(^3S_1,^1S_0)}_{\lambda_u=0,\lambda_d}(t)\Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C^{(^3S_1,^3S_1)}_{\lambda_u=0,\lambda_d=0}(t)\,C^{(^1S_0,^1S_0)}_{\lambda_u=0,\lambda_d=0}(t)}}$$

diagonal pieces with no field

- Fit a constant to the 'effective matrix element plot' at late times

$$R_{^3S_1,^1S_0}(t+1) - R_{^3S_1,^1S_0}(t) \xrightarrow{t\to\infty} \frac{\langle ^3S_1; J_z=0|A_3^3|^1S_0; I_z=0\rangle}{Z_A}$$

constant fit to plateau region



pp fusion

- Low-energy cross section for $pp \to de^+\nu$ dictated by the matrix element

$$\left|\langle d; j \,|A_k^-|\, pp\rangle\right| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}}\,\Lambda(p)\,\delta_{jk}$$

C_η	Sommerfield factor
γ	Deuteron binding mtm
$r_1,\ \rho$	Effective ranges
a_{pp}	pp scattering length
$\Gamma(0,\chi)$	Incomplete gamma func.
	$\chi = \alpha M_p/\gamma$

- Relate $\Lambda(0)$ to extrapolated LEC using EFT

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}}\{e^\chi - \gamma a_{pp}[1-\chi e^\chi \Gamma(0,\chi)] + \frac{1}{2}\gamma^2 a_{pp}\sqrt{r_1\rho}\} - \frac{1}{2g_A}\gamma a_{pp}\sqrt{1-\gamma\rho}\,\boxed{L_{1,A}^{sd-2b}}$$

extrapolated lattice value

- Determine $L_{1,A}$ (two body contribution - N²LO $\not\pi$EFT in dibaryon approach)
  - npdγ suggests weak mass dependence of two-body counterterms so extrapolate to physical point

Butler and Chen, Phys. Lett. B520, 87 (2001)
Detmold and Savage, Nucl. Phys. A743, 170 (2004).

pp fusion

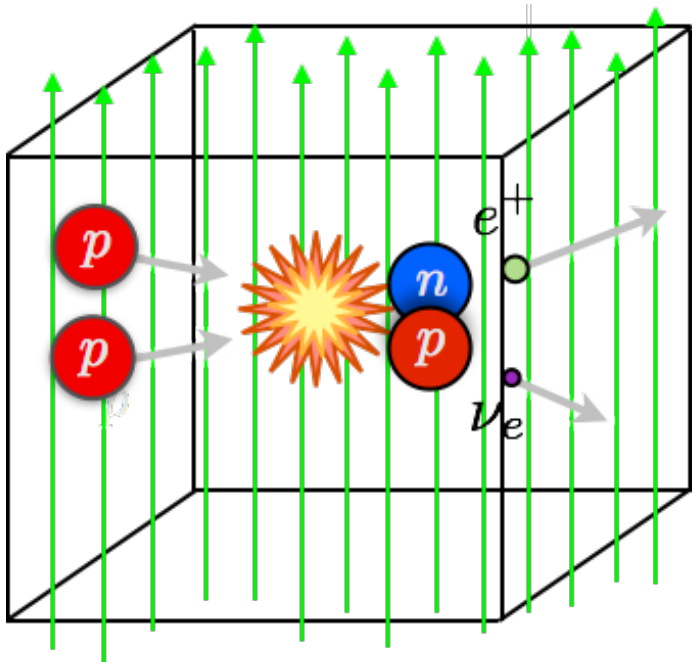


Fig: Z Davoudi

- Fusion cross section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

$$\Lambda(0) = 2.652(2)$$ (models/EFT)

E. G. Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

- Relevant counter-term in EFT

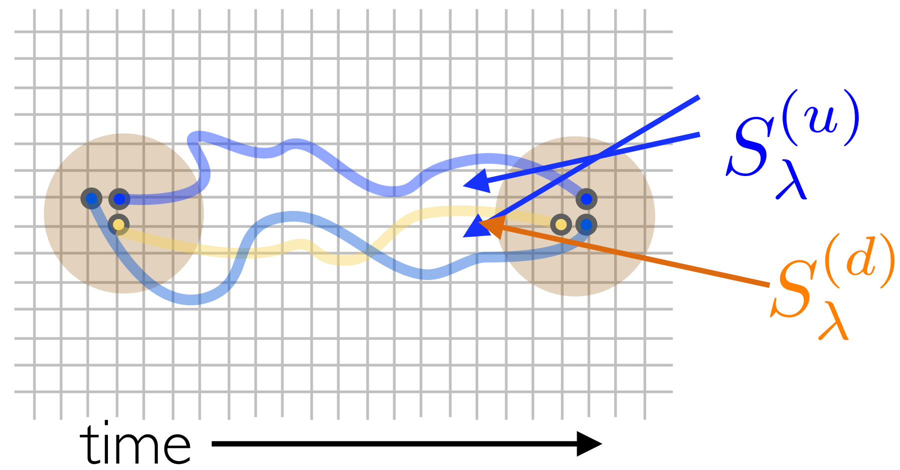
$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \mathrm{fm}^3$$

$$L_{1,A} = 3.6(5.5)\ \mathrm{fm}^3$$ (reactor expts.)

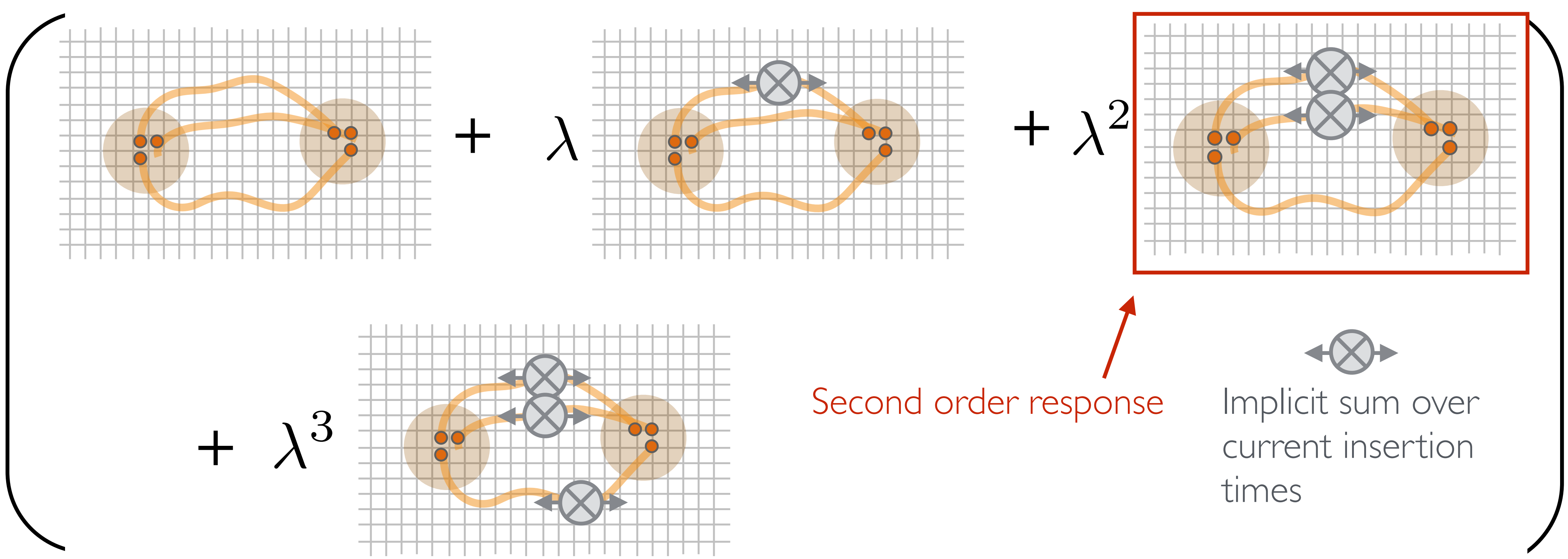
M. Butler, J.-W. Chen, and P. Vogel, Phys. Lett. B549

Axial Background Field

$$C_{\lambda_u;\lambda_d}(t) =$$



$$C_{\lambda_u;\lambda_d}(t) = \Big(\cdots + \lambda \cdots + \lambda^2 \cdots + \lambda^3 \cdots \Big)$$



Second order weak interactions

NPLQCD arXiv:1701.03456, 1702.XXXXX

- Background axial field to second order
  - nn→pp transition matrix element

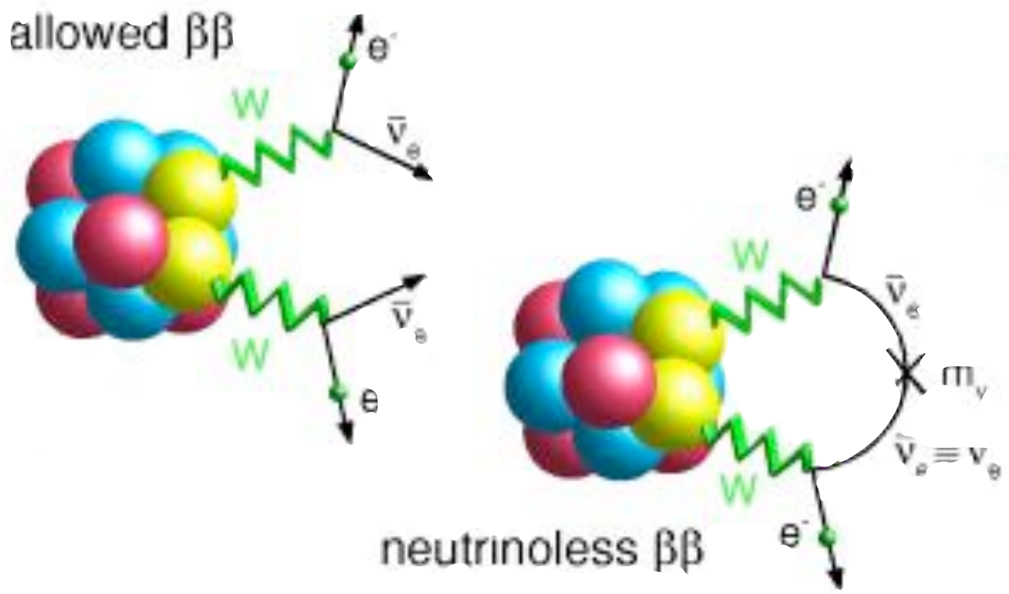
    $$M_{GT}^{2\nu} = 6\int d^4x d^4y \langle pp|T\left[J_3^+(x)J_3^+(y)\right]|nn\rangle$$

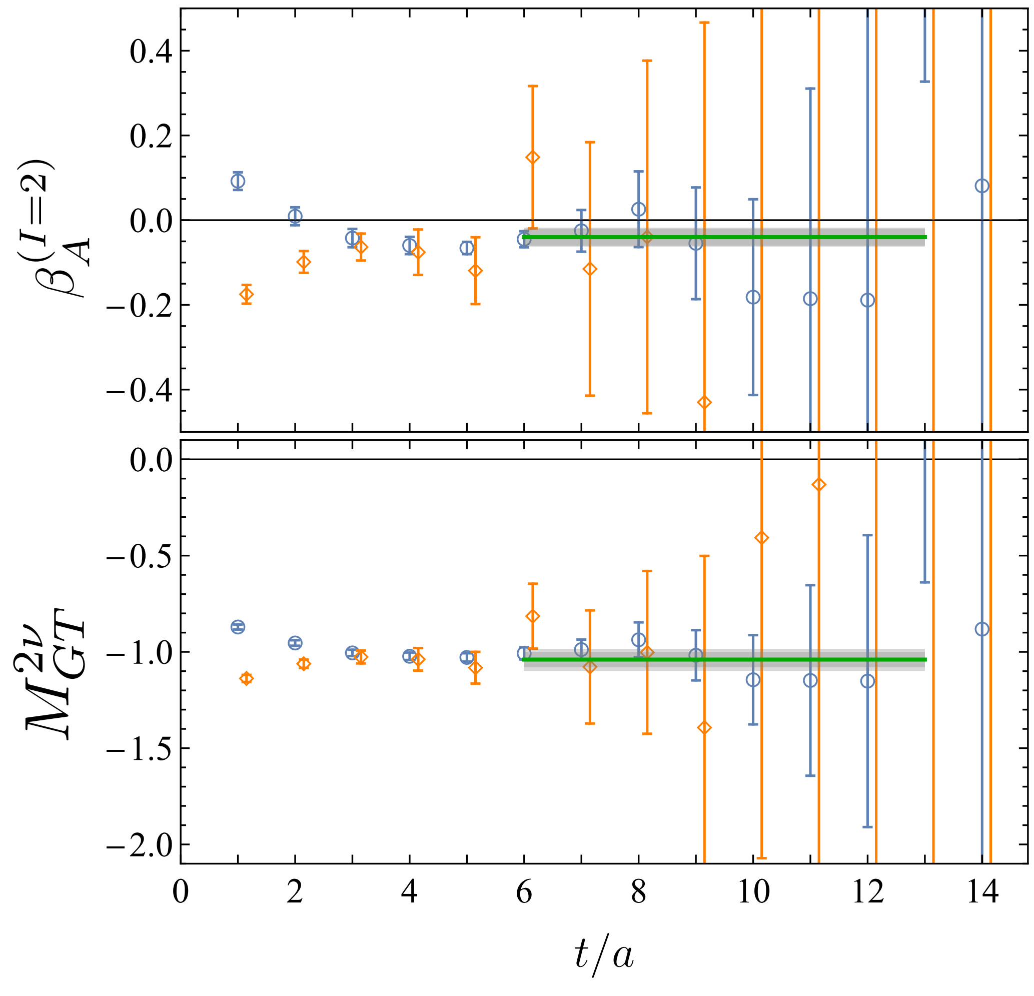
    introduces a host of technical LQCD complications
  - Non-negligible deviation from long distance deuteron intermediate state contribution

    Isotensor axial polarisability

    $$M_{GT}^{2\nu} = -\frac{|M_{pp\to d}|^2}{E_{pp}-E_d} + \beta_A^{(I=2)}$$

  - Quenching of g_A in nuclei is insufficient!
  - TBD: connect to EFT for larger systems





EMC effect

- EFT methods show PDFs of nuclei are factorisable (up to higher order effects)

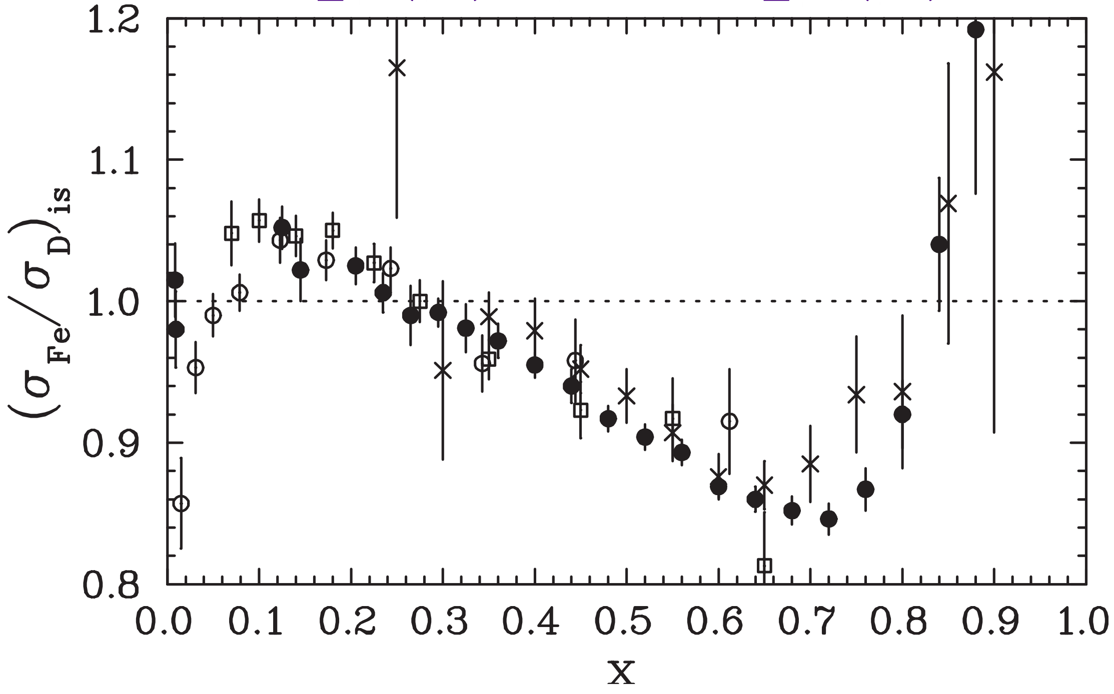
  [Chen, WD 04, Chen, WD, Lynn, Schwenk 16]

$$F_2^A(x) = A\left[F_2(x) + g_2(A) f_2(x)\right]$$

$$\langle x^n \rangle_{q|A} = \langle x^n \rangle_q \left[A + \alpha_n \langle A | (N^\dagger N)^2 | A \rangle\right]$$

- Background twist-2 fields to access moments of PDFs in light nuclei
  - Calculations under way for low moments of quark <u>and gluon</u> PDFs in light nuclei

$$F_2^A(x) \neq A F_2^N(x)$$



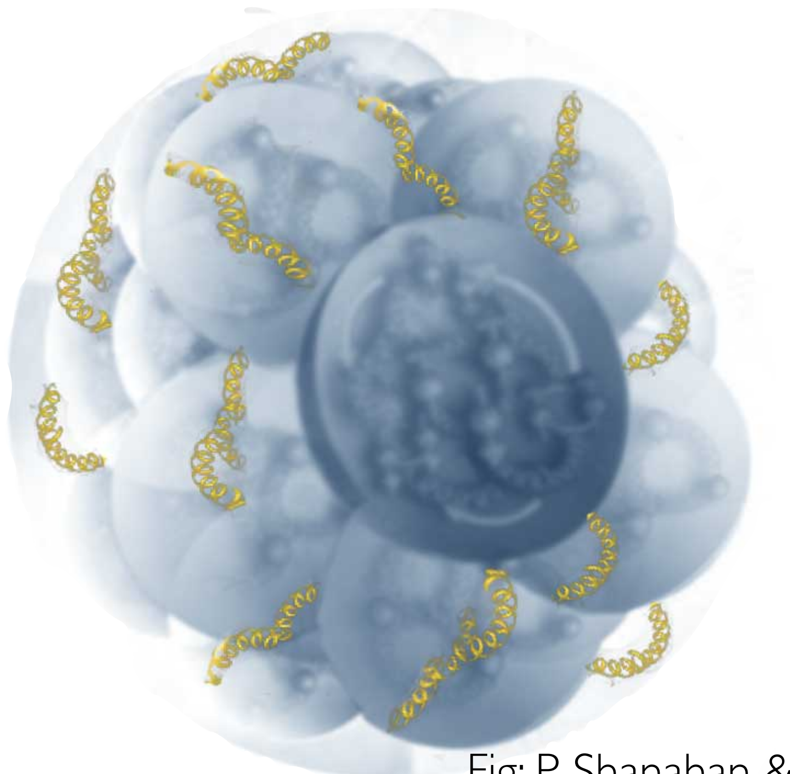
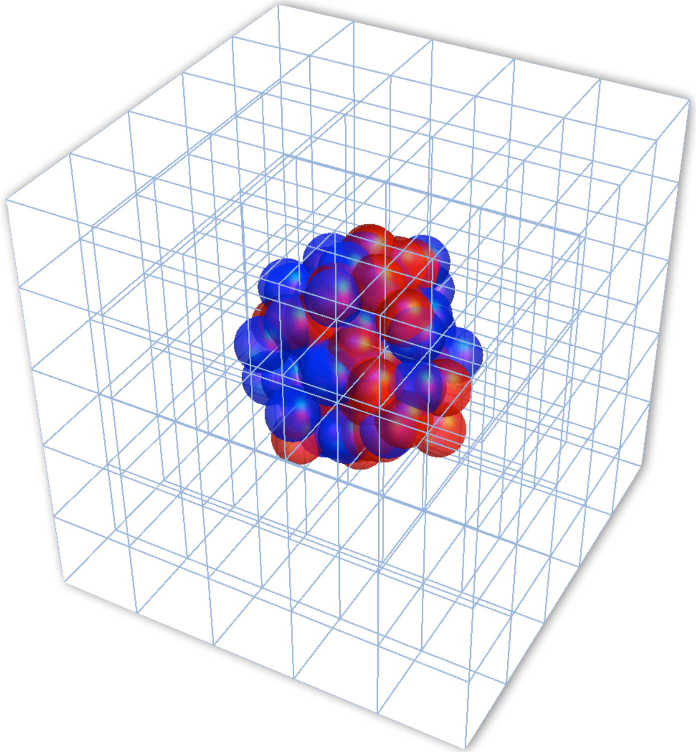


Fig: P Shanahan & EIC

- Nuclei are under serious study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei (strange, charmed, . . .)
  - Structure: magnetic moments and polarisabilities, axial charges
  - Electroweak interactions: thermal capture, *pp* fusion, ββ decay
- Prospect of a quantitative connection to QCD makes this a very exciting time
  - Nuclear matrix elements important to experimental program
  - Learn many interesting things about nuclear physics along the way



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