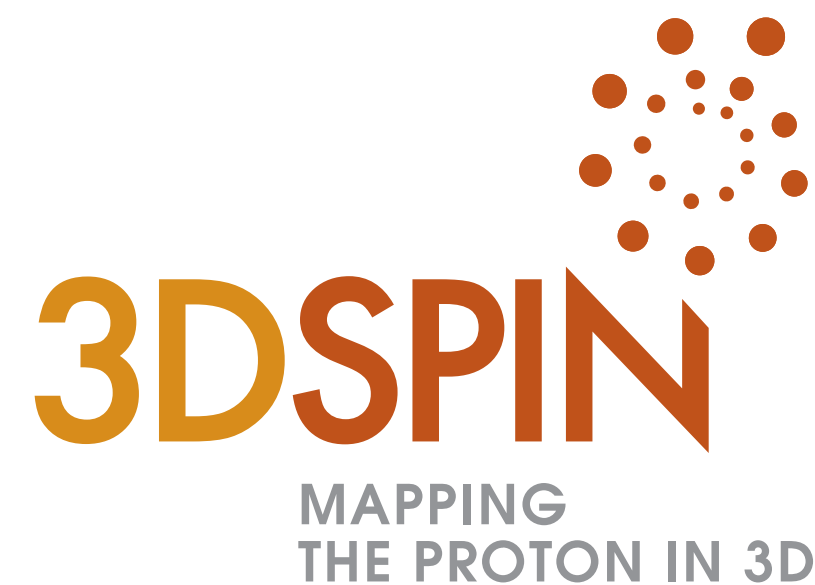


# First attempts at a global fit of unpolarized Transverse Momentum Distributions

---

Alessandro Bacchetta

Funded by

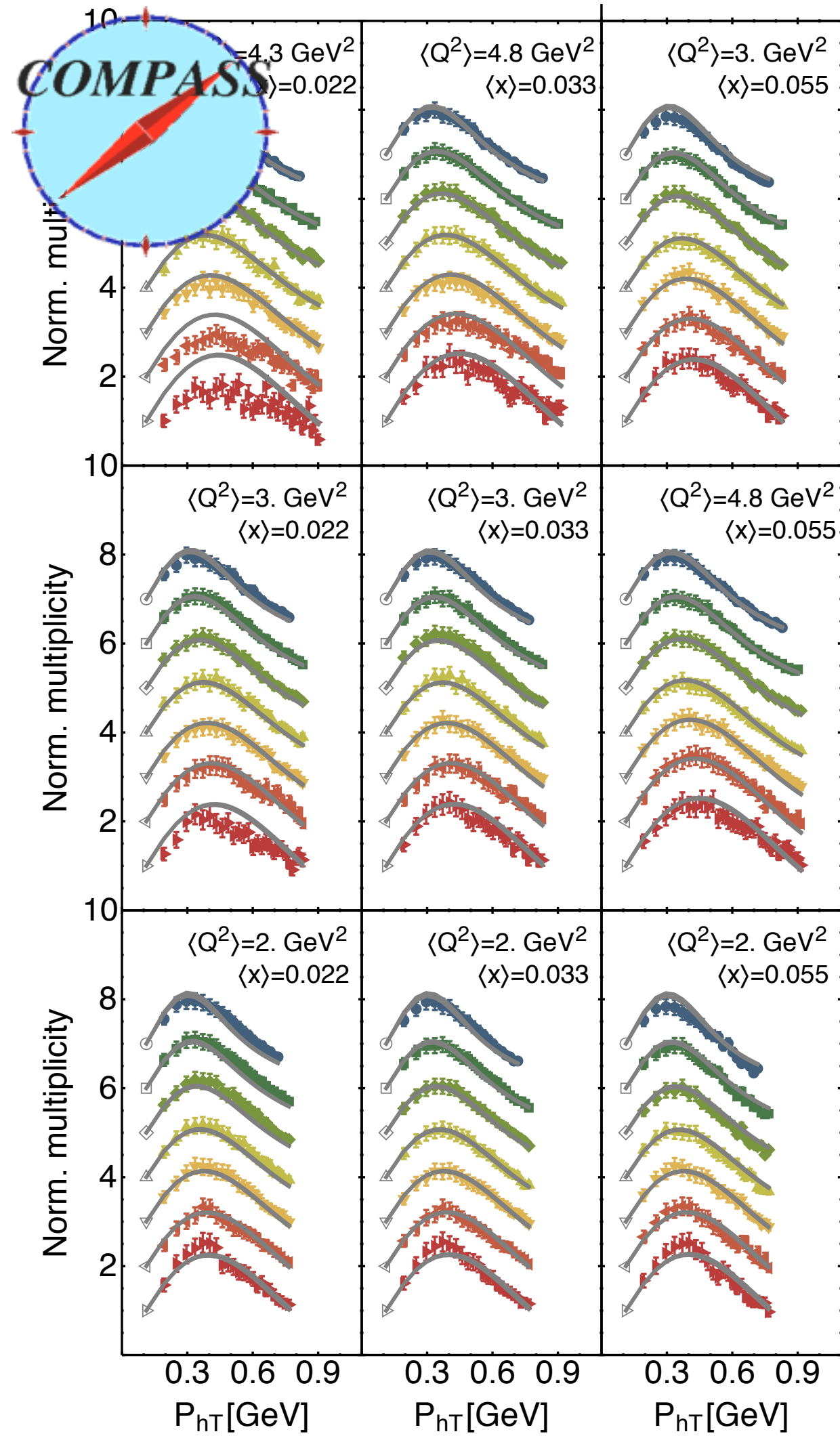


# In collaboration with

---

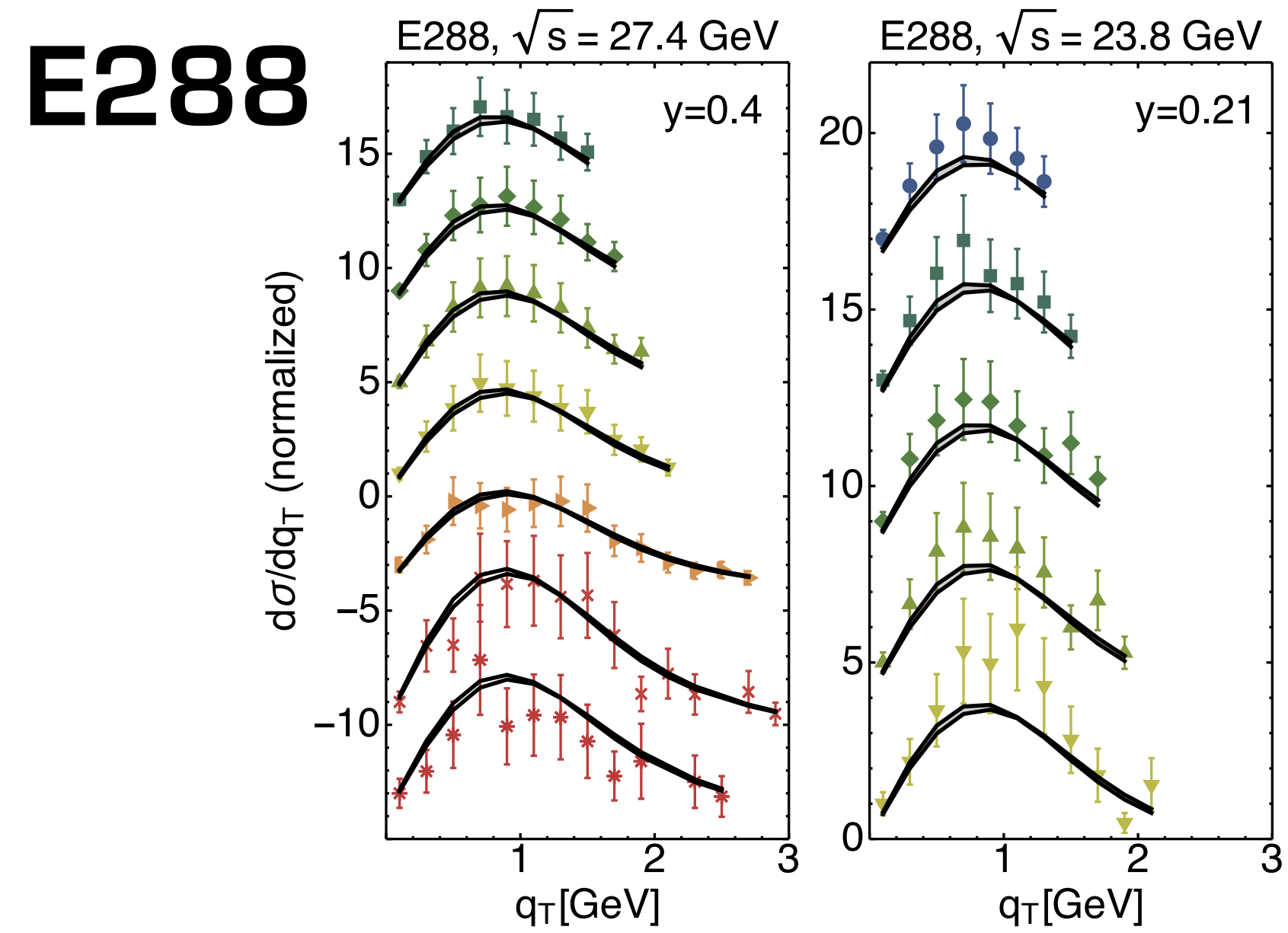
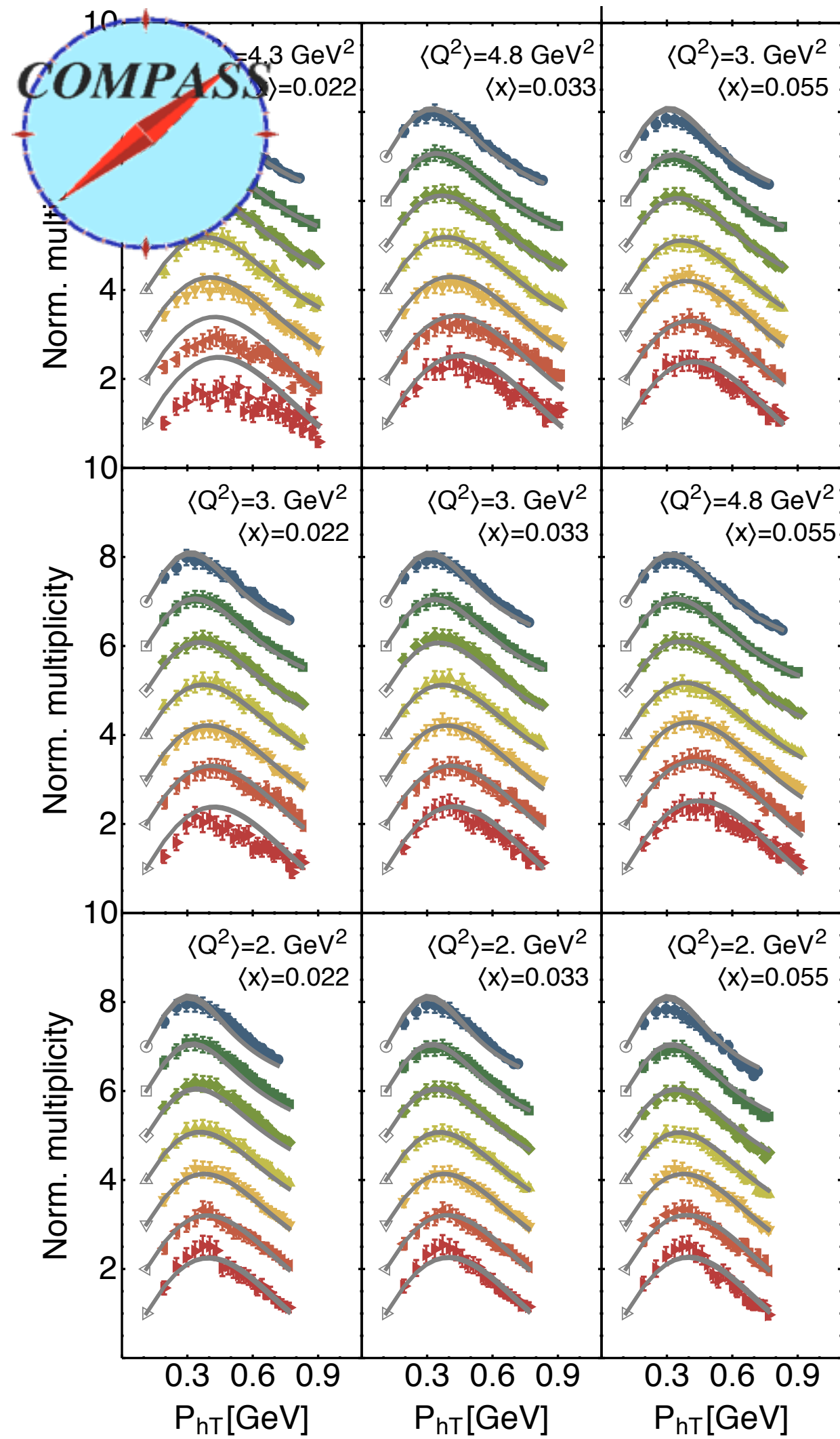
- Filippo Delcarro (PhD student, University of Pavia and INFN Pavia)
- Cristian Pisano (University of Pavia and INFN Pavia)
- Marco Radici (INFN Pavia)
- Andrea Signori (Vrije Universiteit Amsterdam and NIKHEF)

# In a nutshell



*Bacchetta, Delcarro, Pisano, Radici, Signori, in preparation*

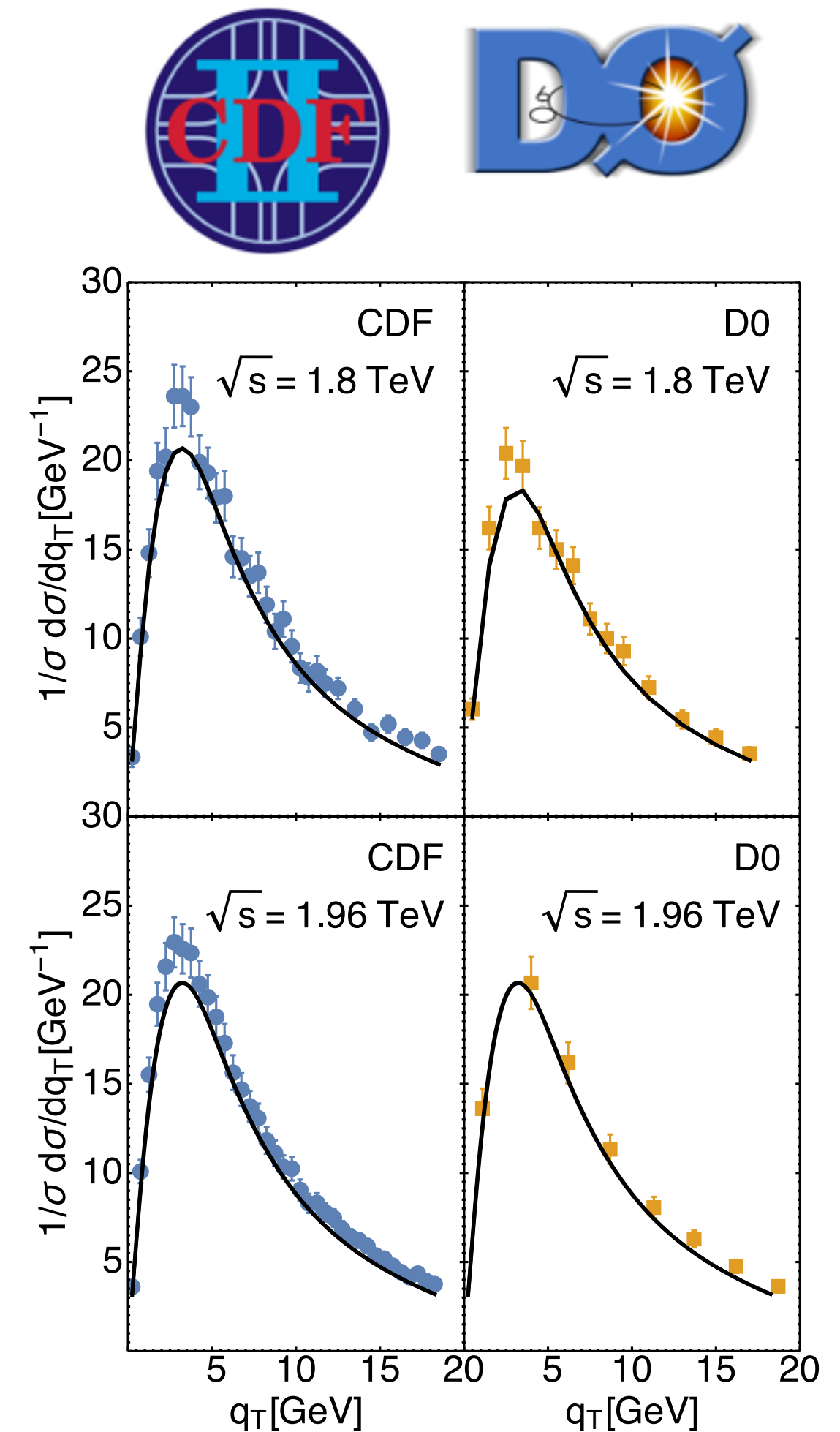
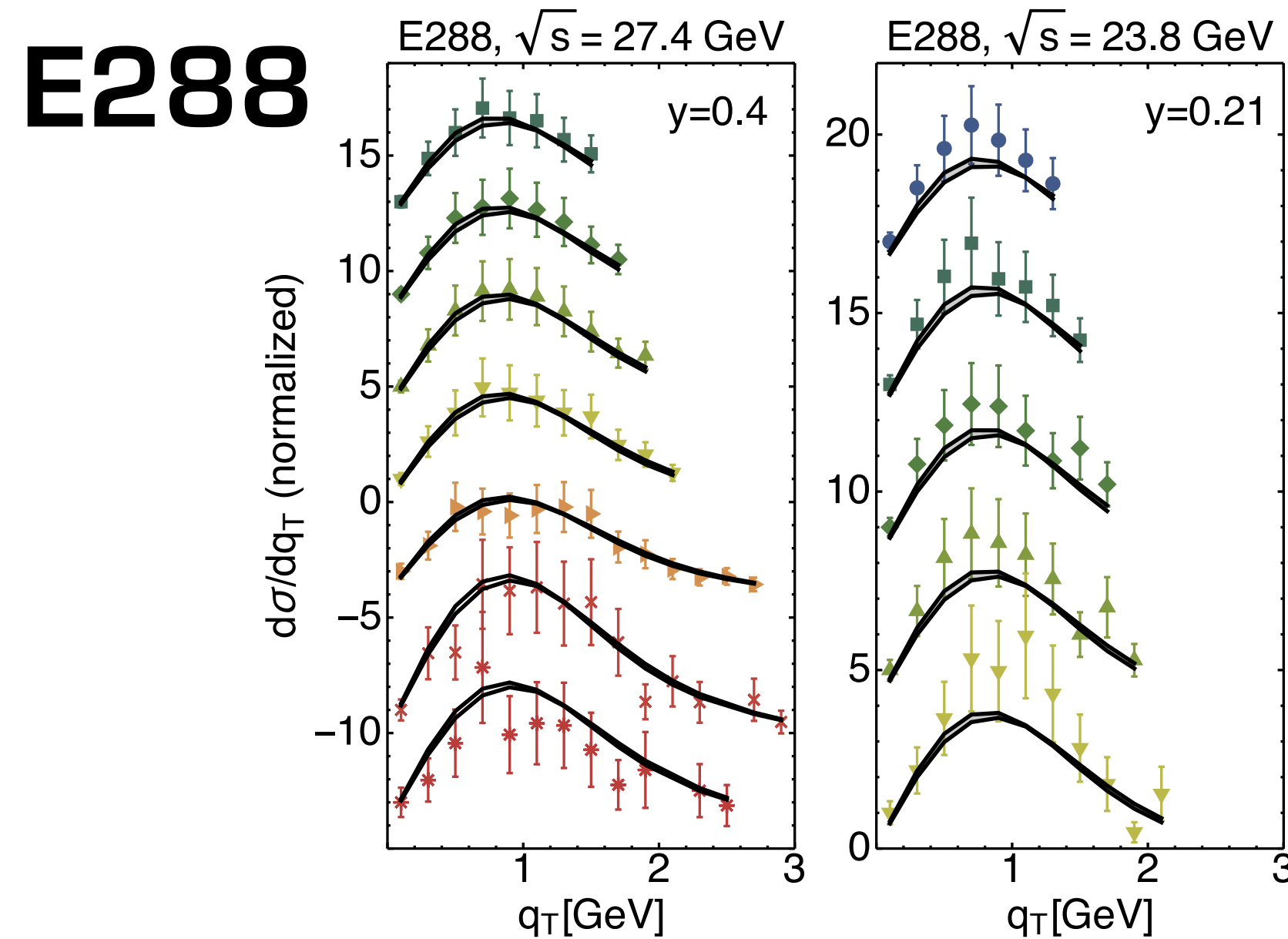
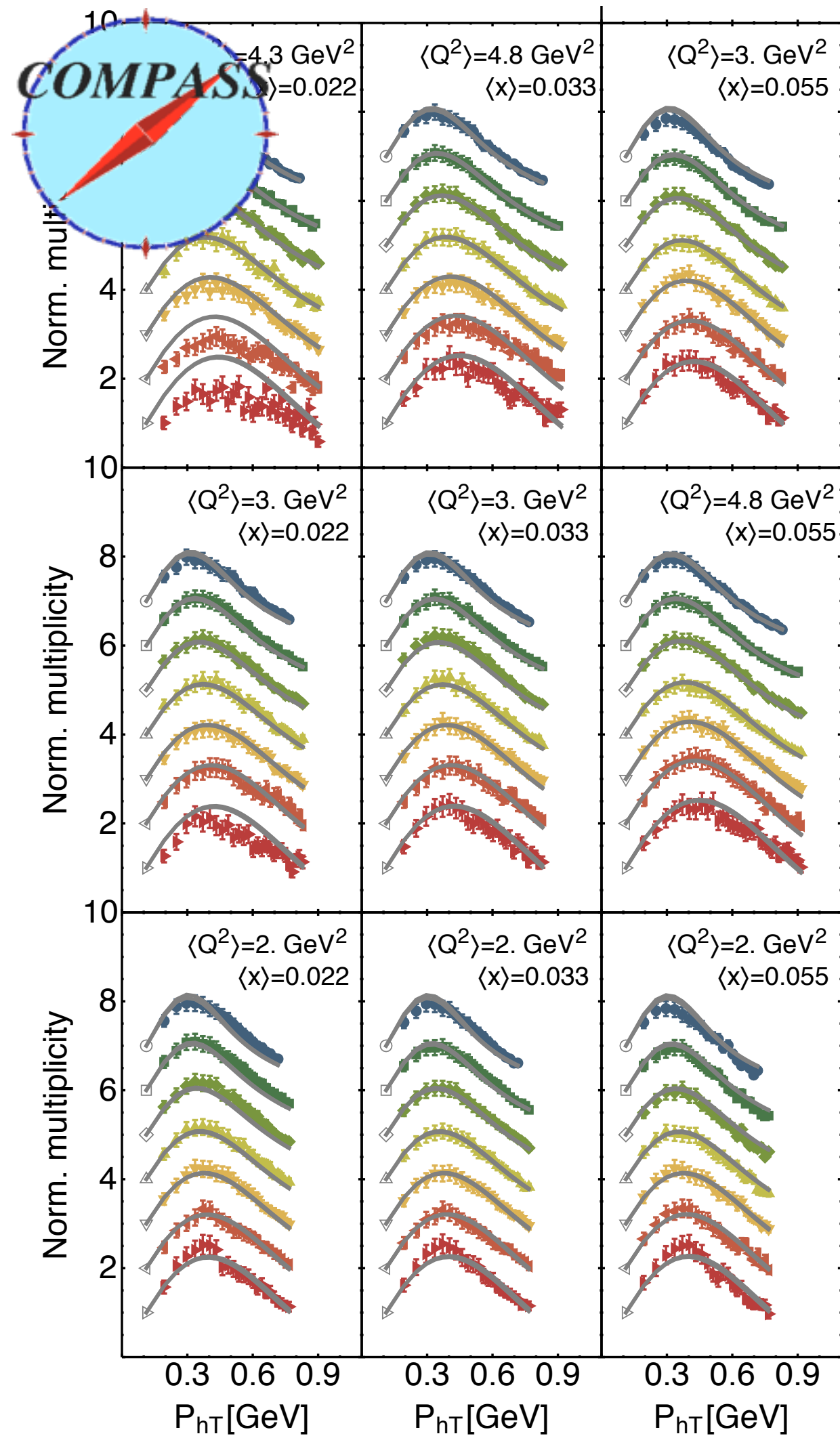
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*Bacchetta, Delcarro, Pisano, Radici, Signori, in preparation*

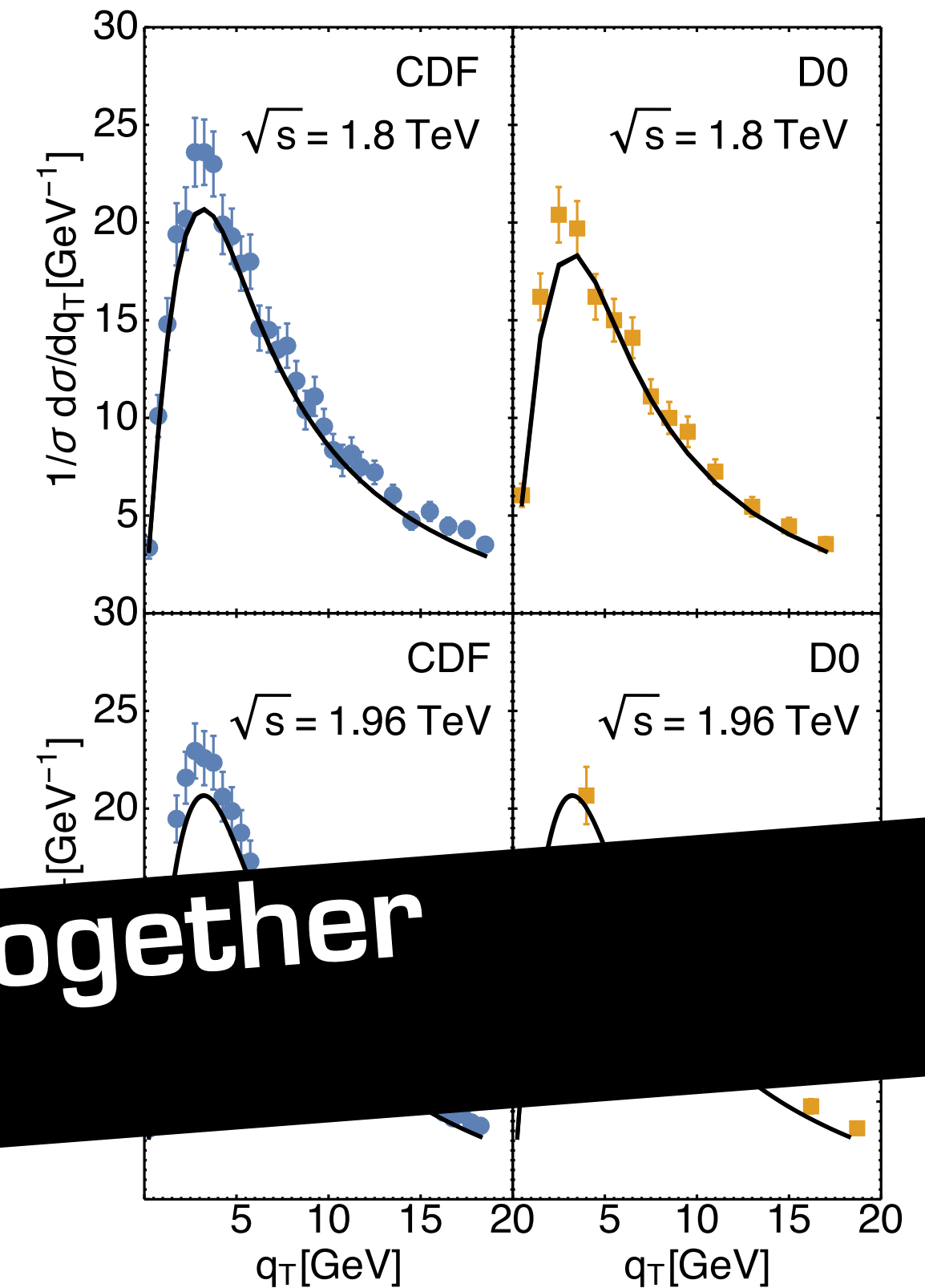
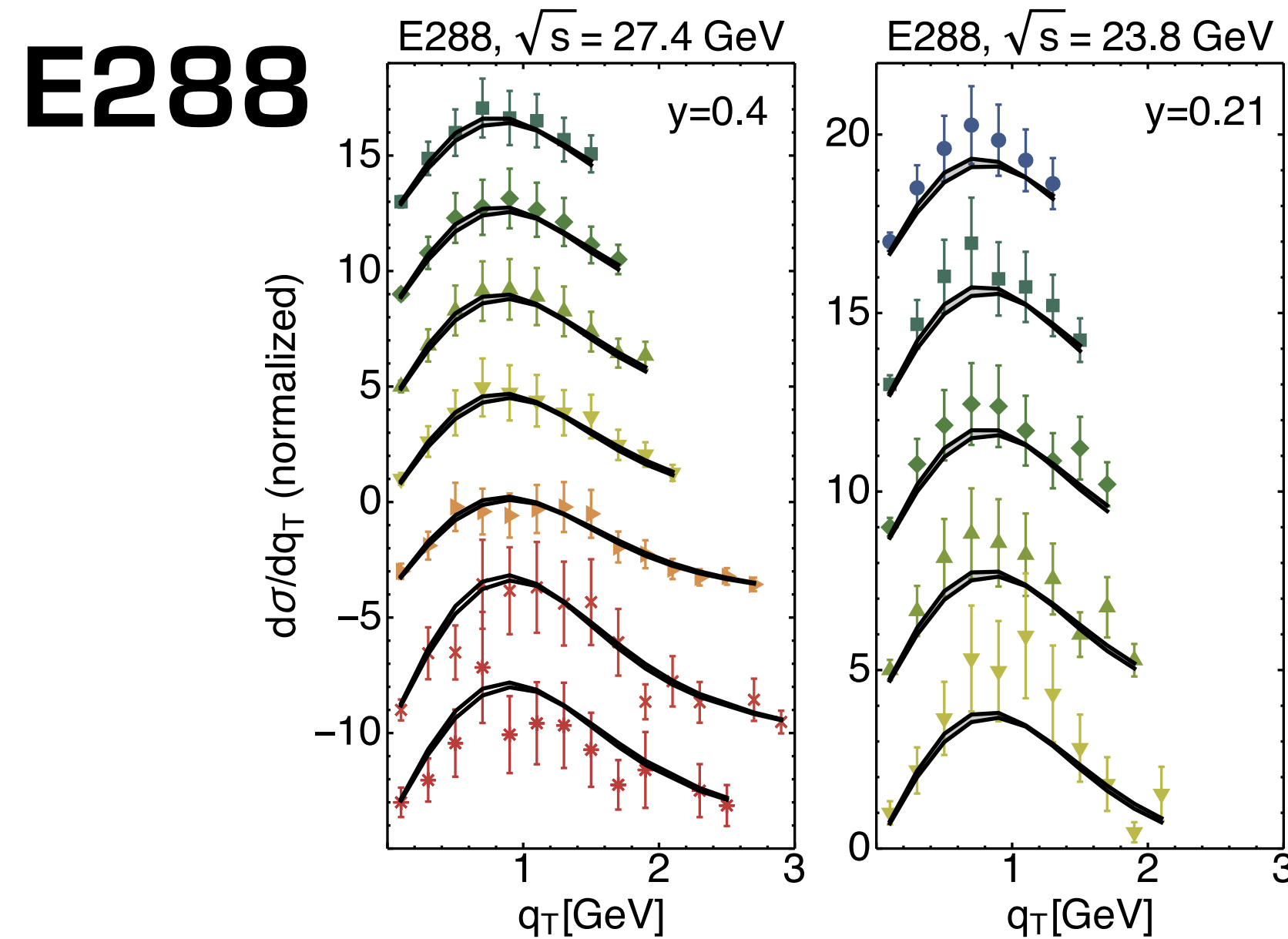
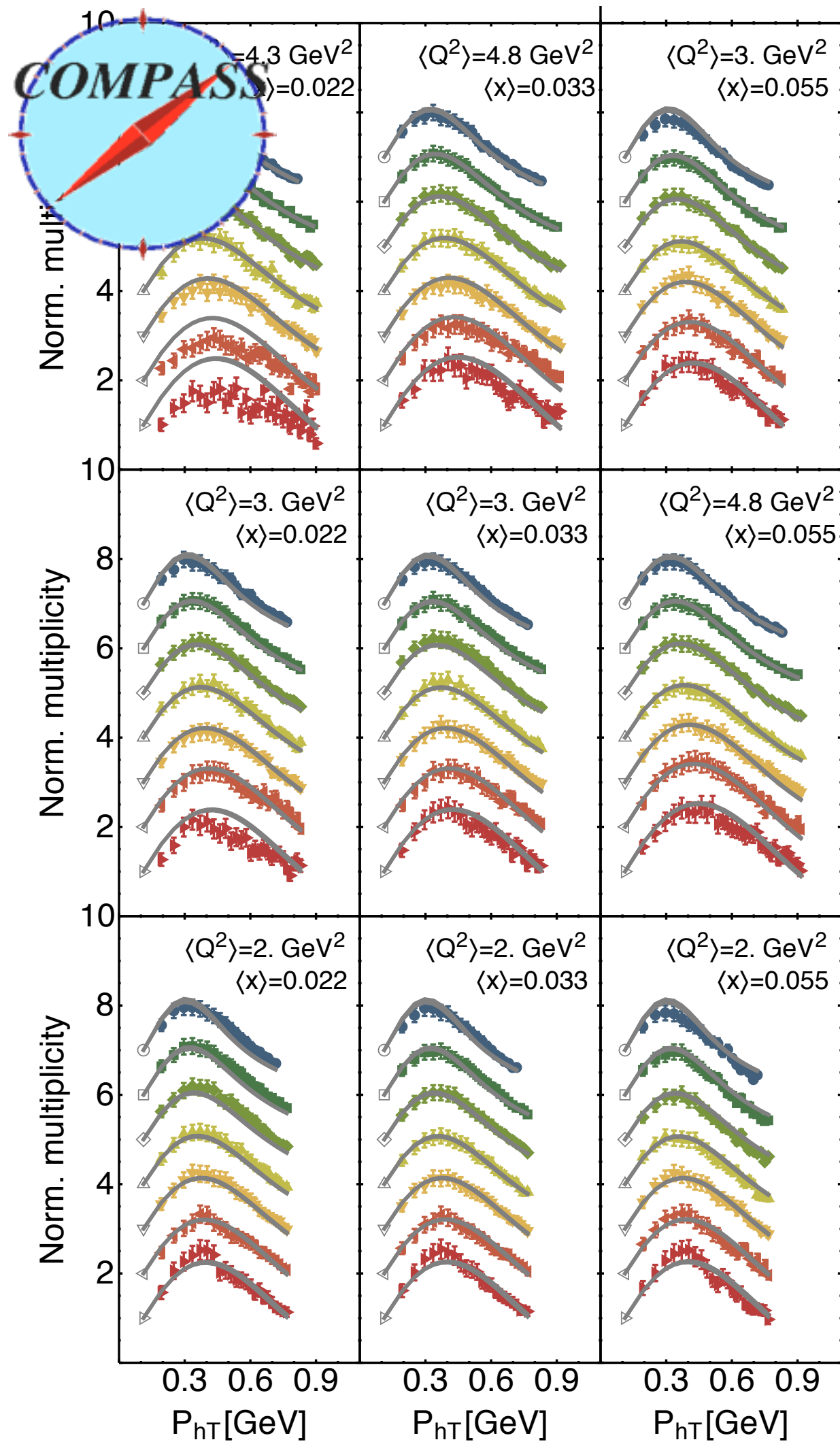


# In a nutshell



Bacchetta, Delcarro, Pisano, Radici, Signori, in preparation

# In a nutshell



**Pavia 2016: first TMD fit putting together  
SIDIS + Drell-Yan + Z production**

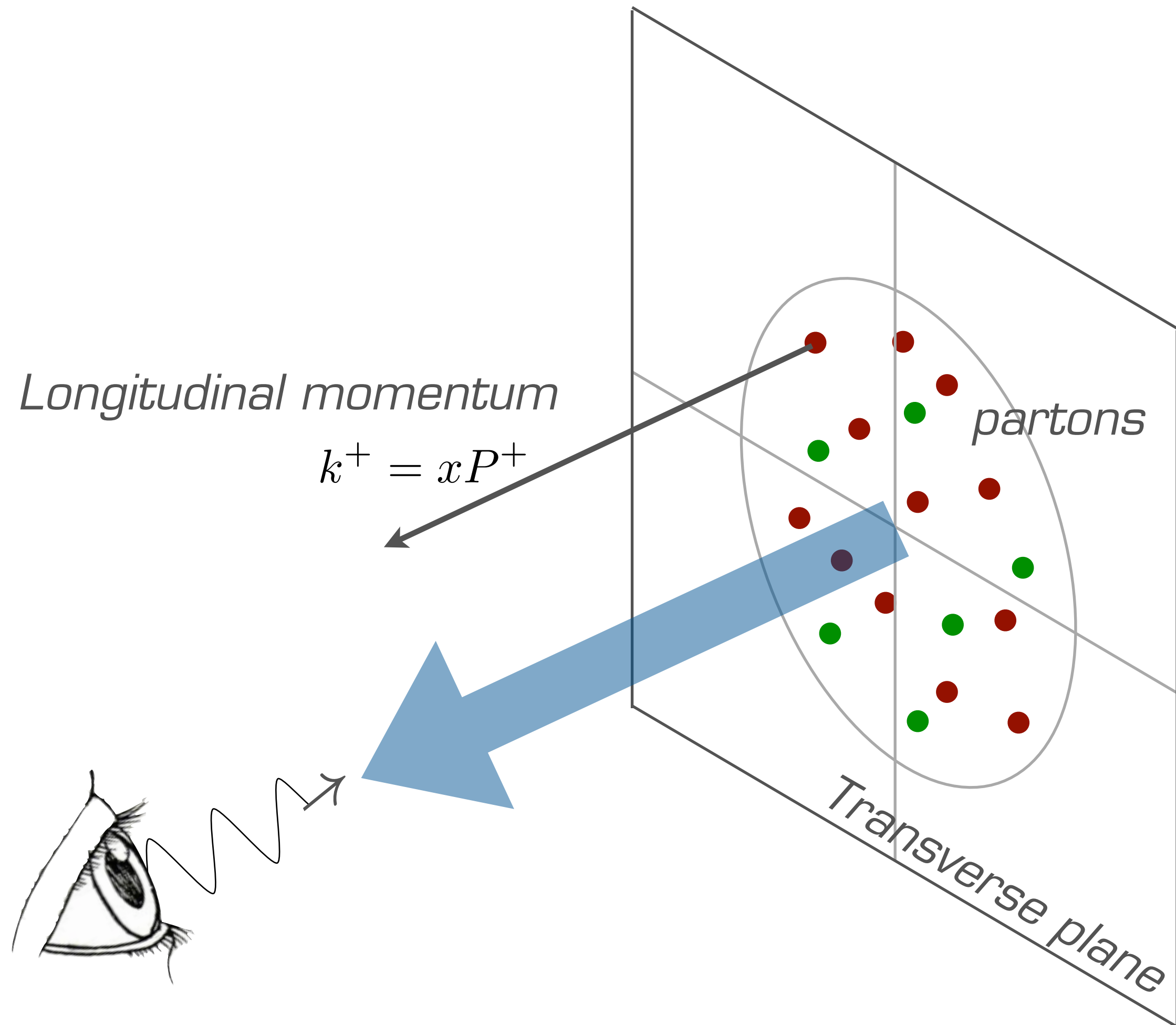
*Bacchetta, Delcarro, Pisano, Radici, Signori, in preparation*

Some introduction

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# Standard parton distribution functions

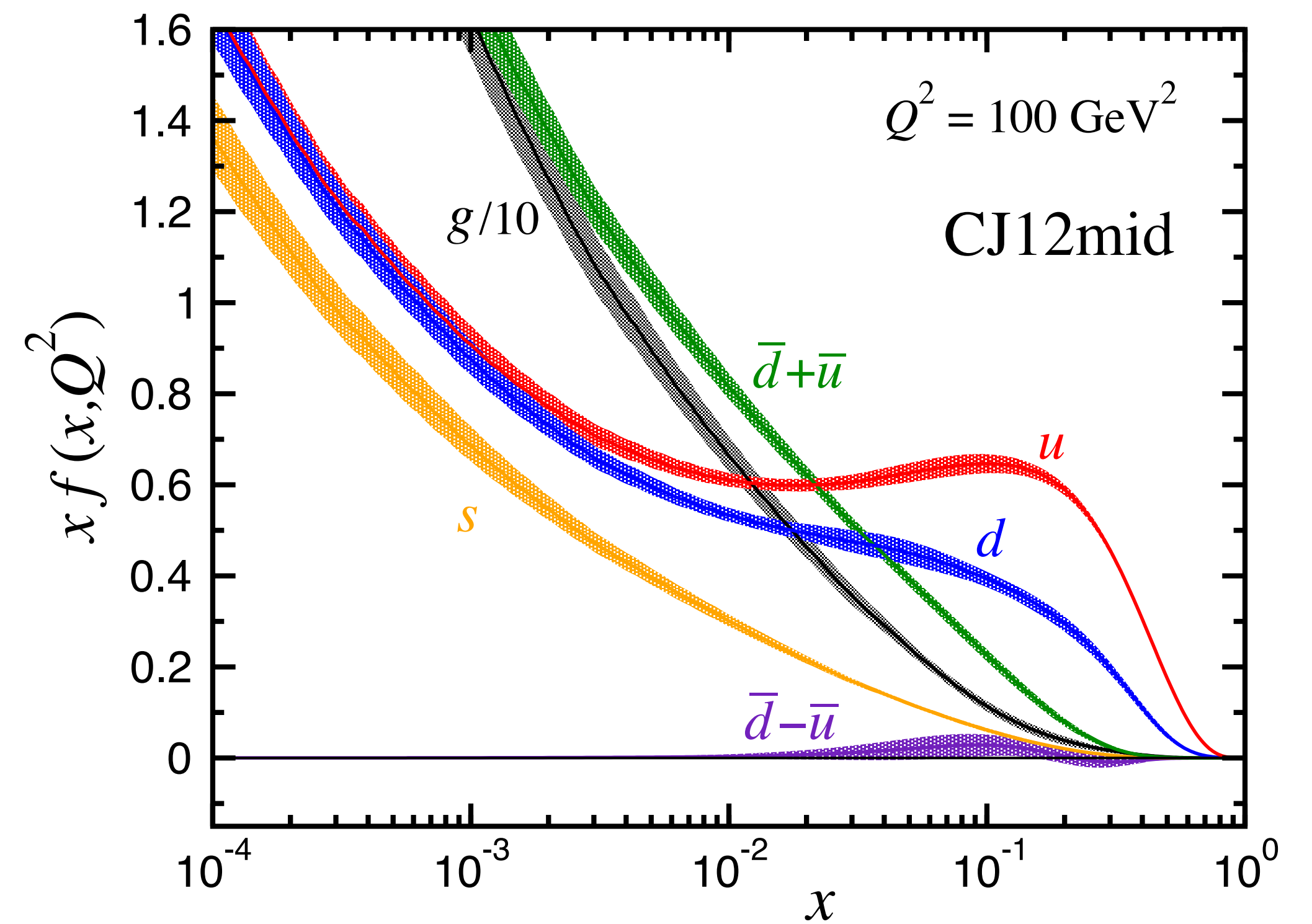
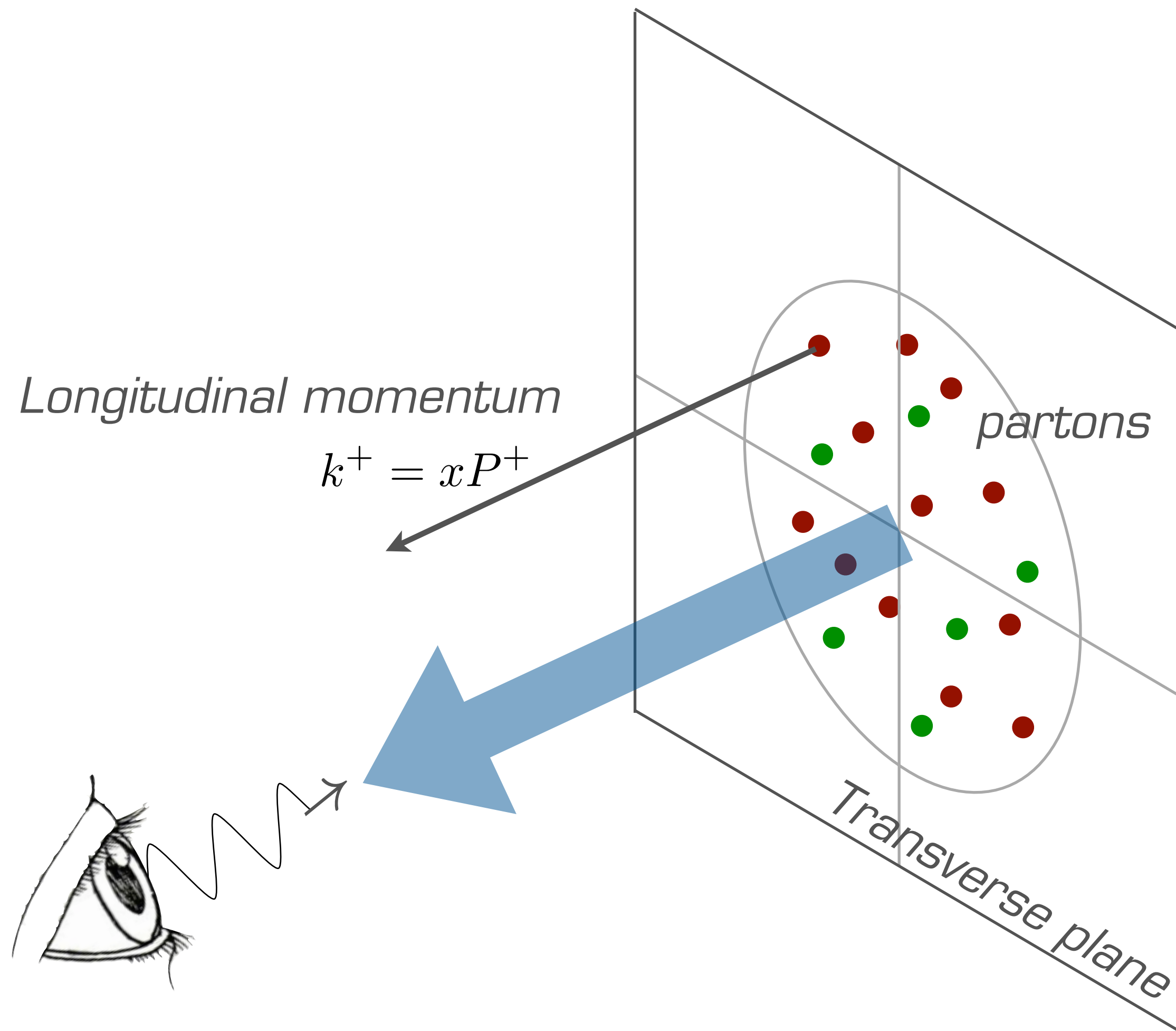
Standard collinear PDFs describe the distribution of partons in one dimension in momentum space





# Standard parton distribution functions

Standard collinear PDFs describe the distribution of partons in one dimension in momentum space

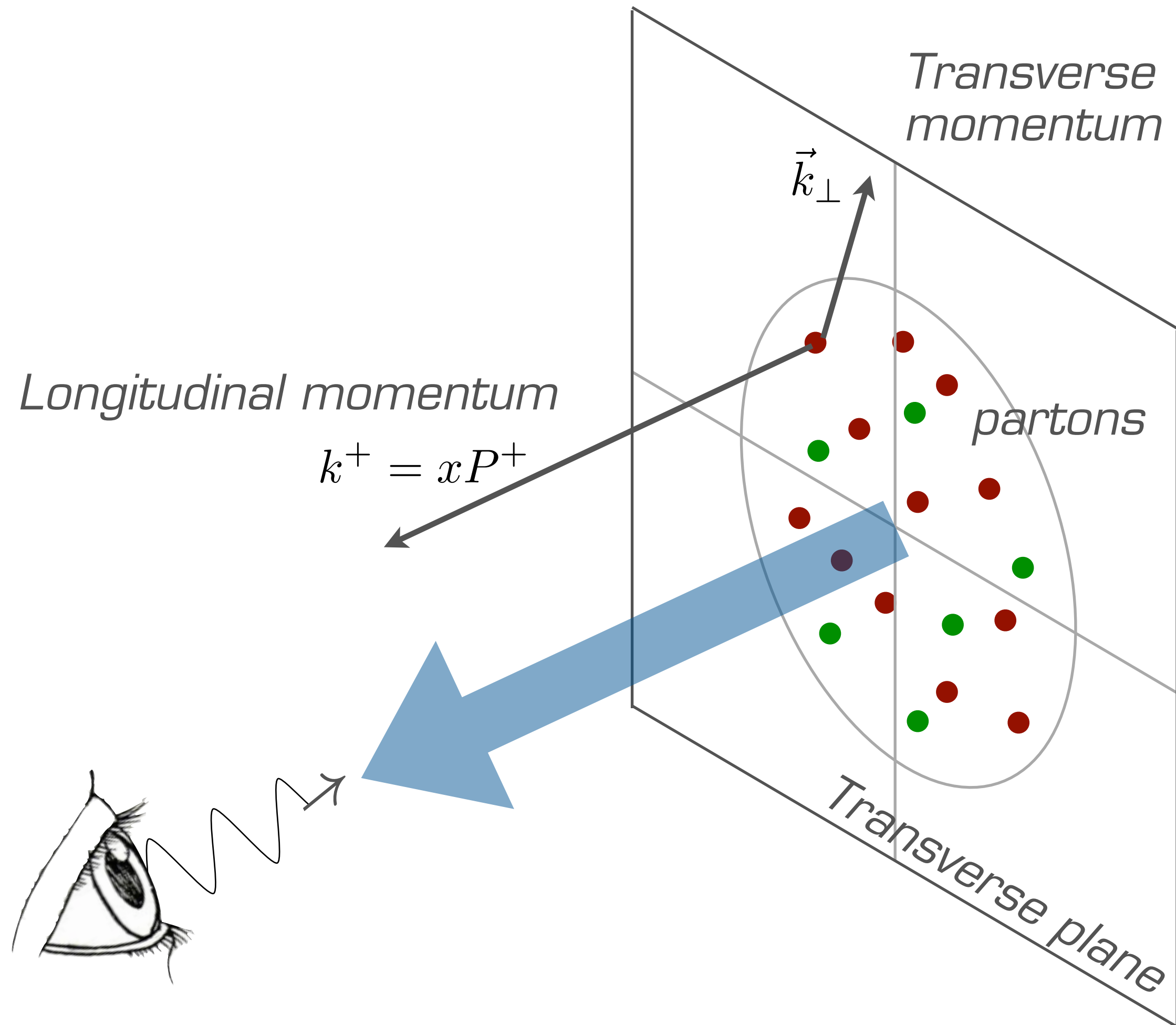


CTEQ-JLAB 12 set, Owens, Accardi, Melnitchouk, PRD87 (13)



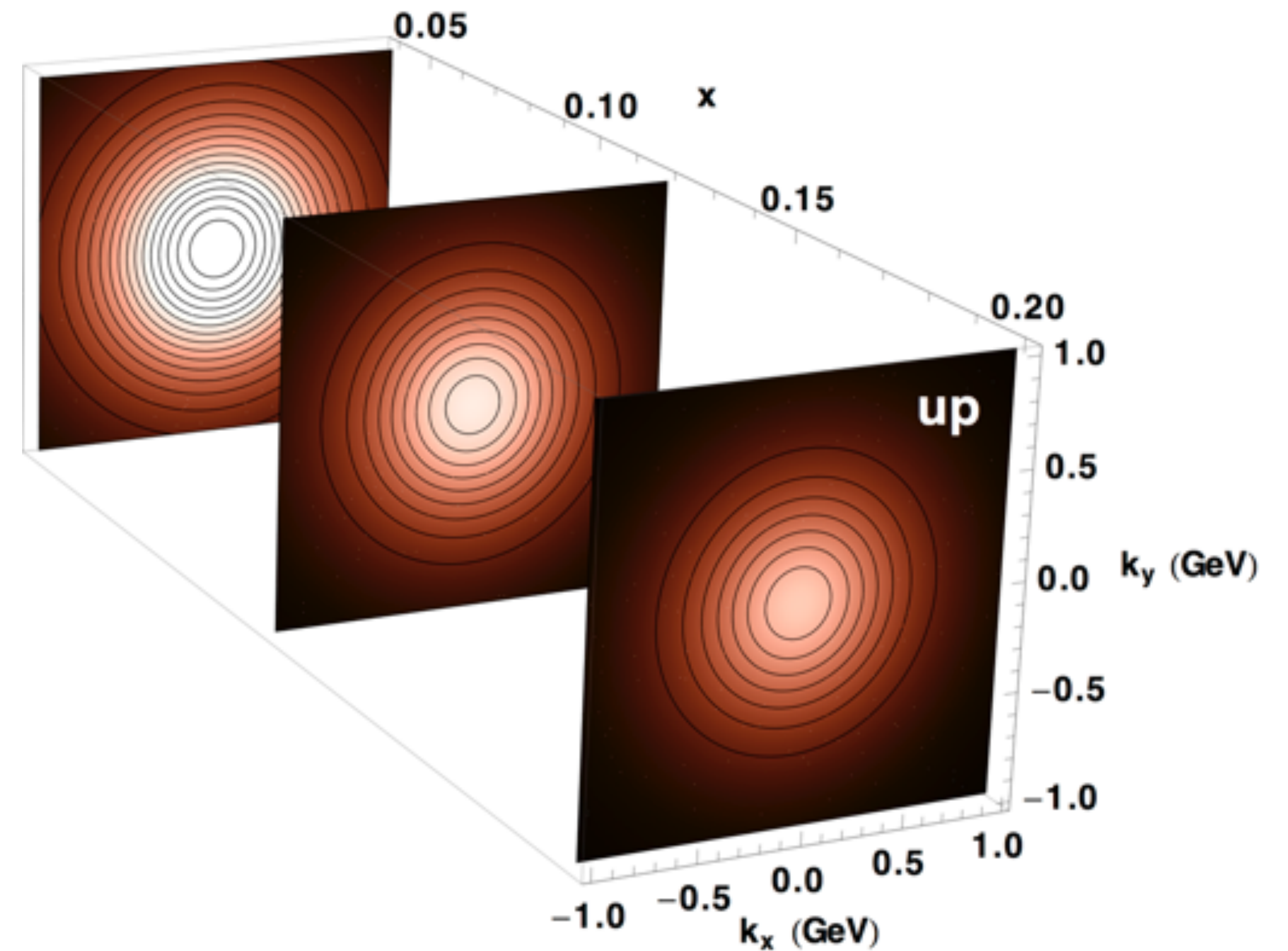
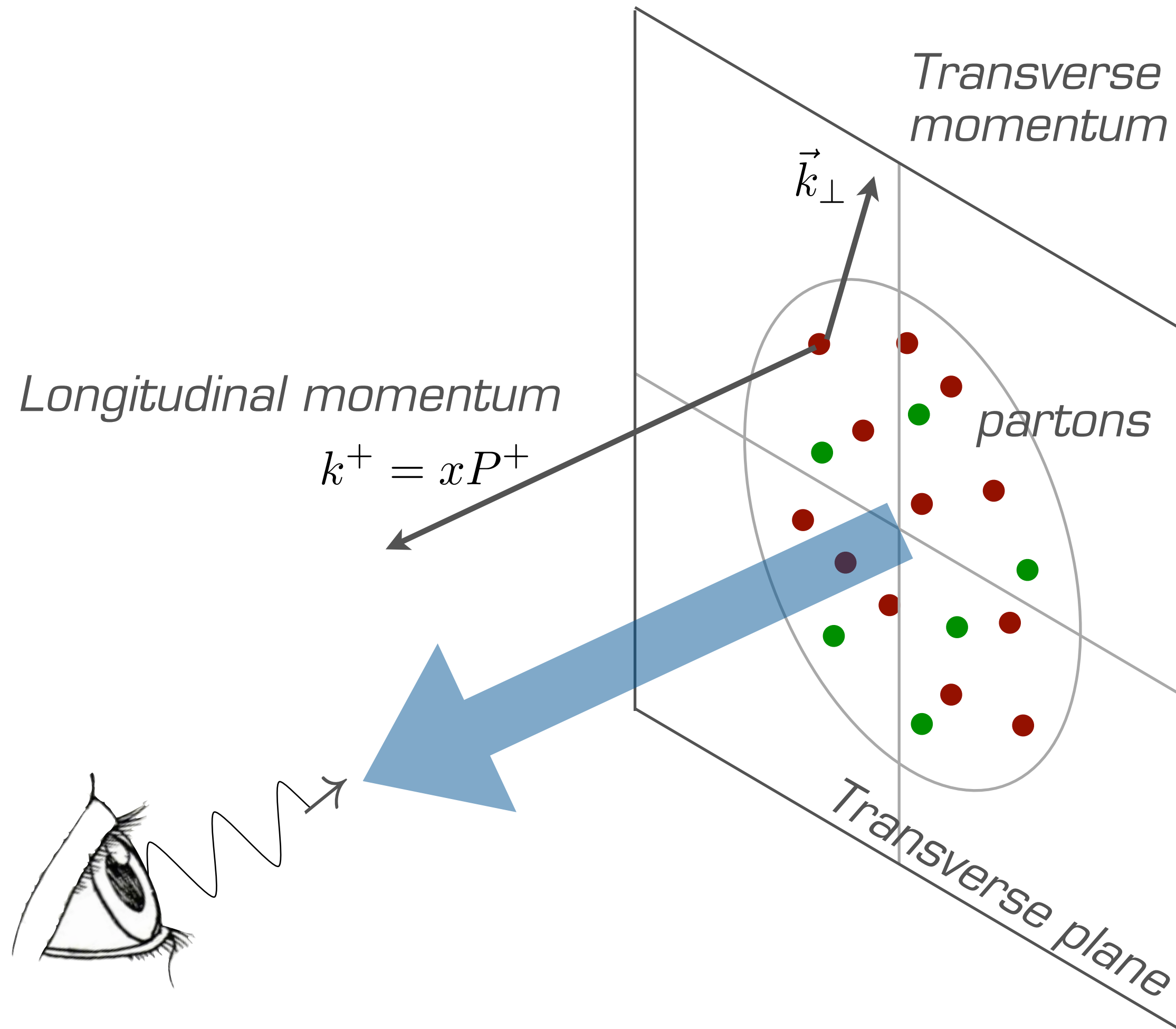
# Transverse Momentum Distributions

TMDs describe the distribution of partons in three dimensions in momentum space

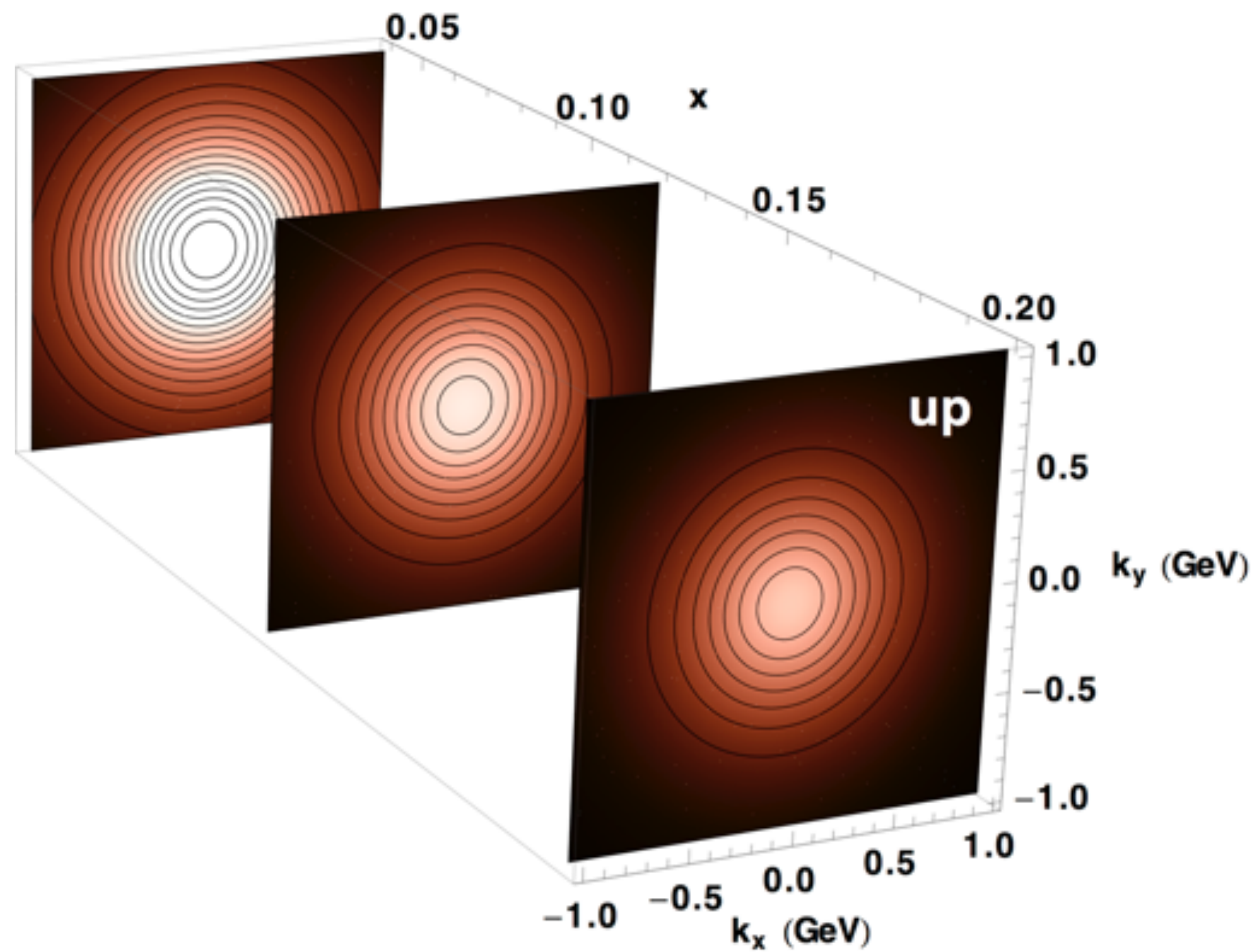


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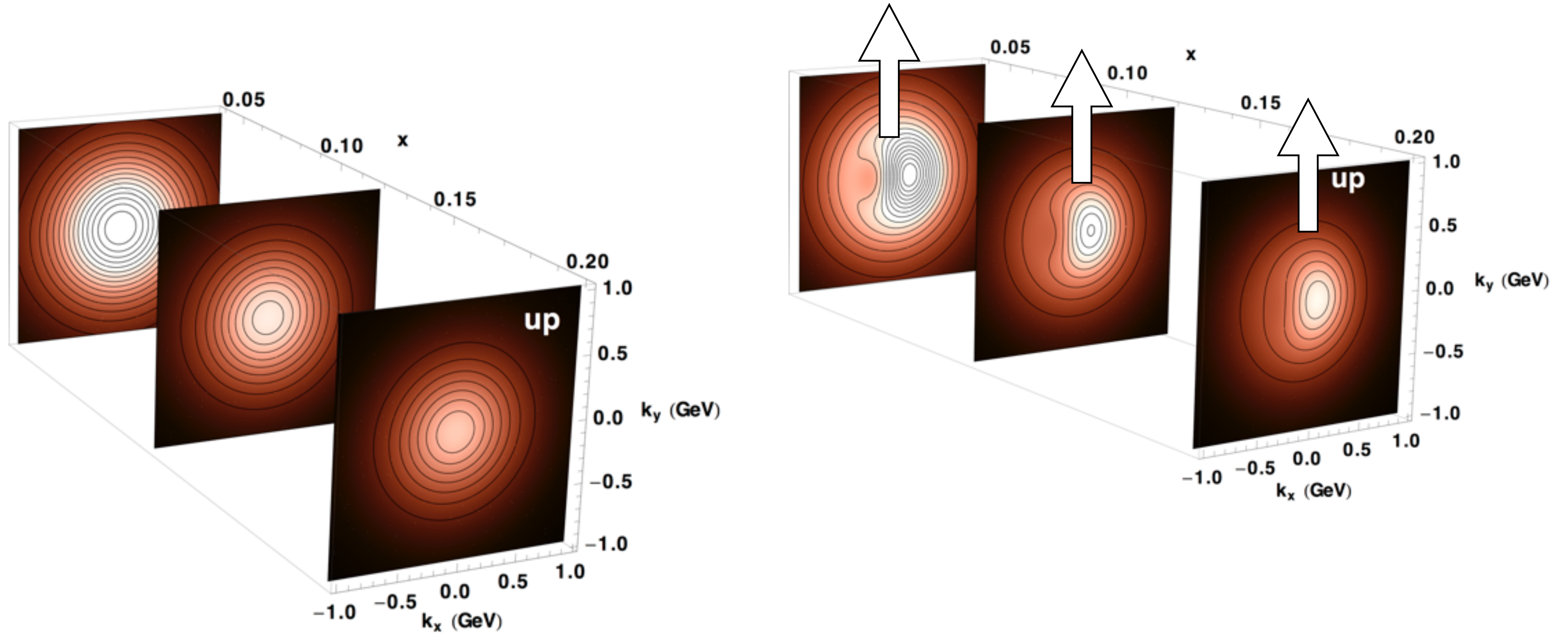
# 3D structure in momentum space



Unpolarized TMD: cylindrically symmetric



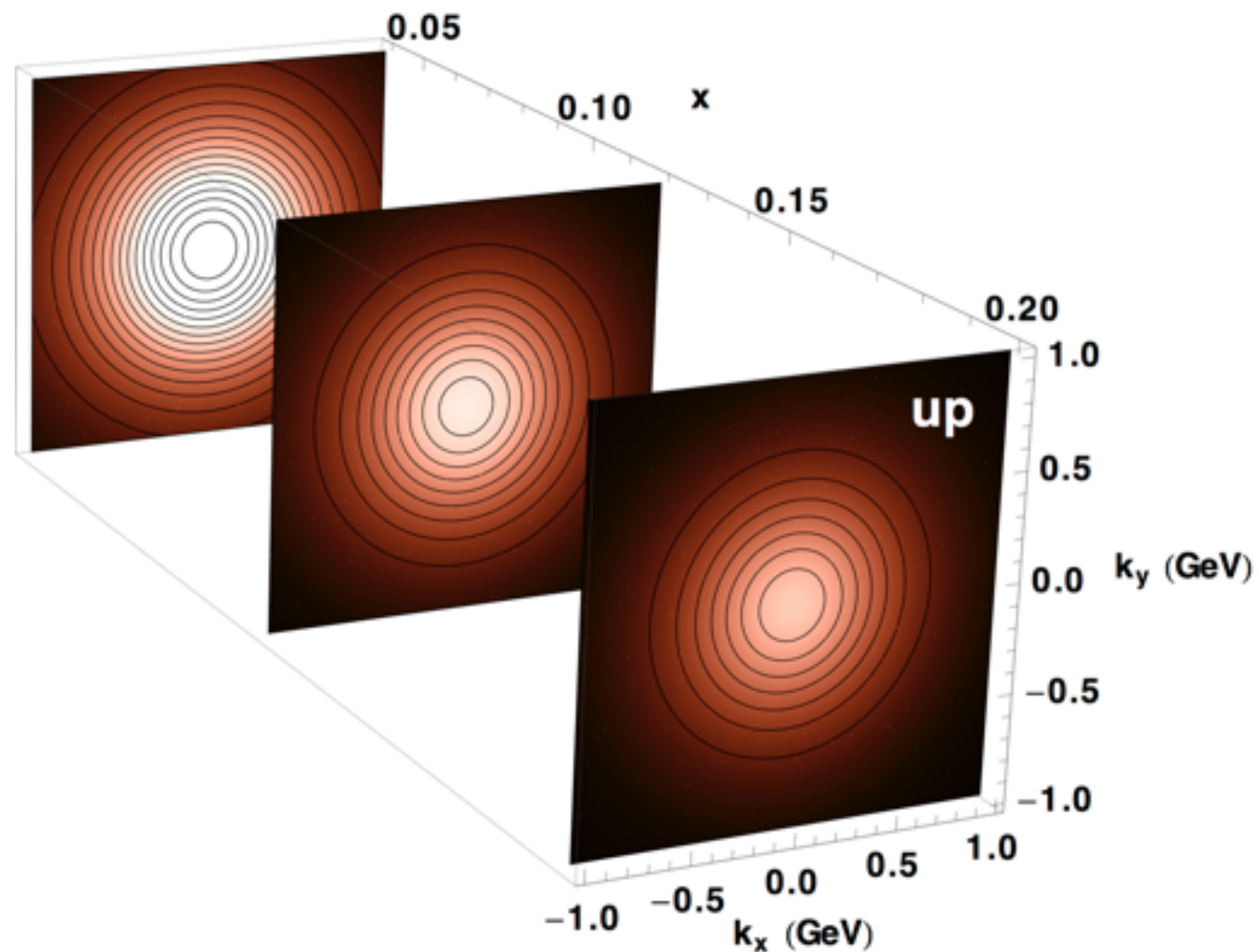
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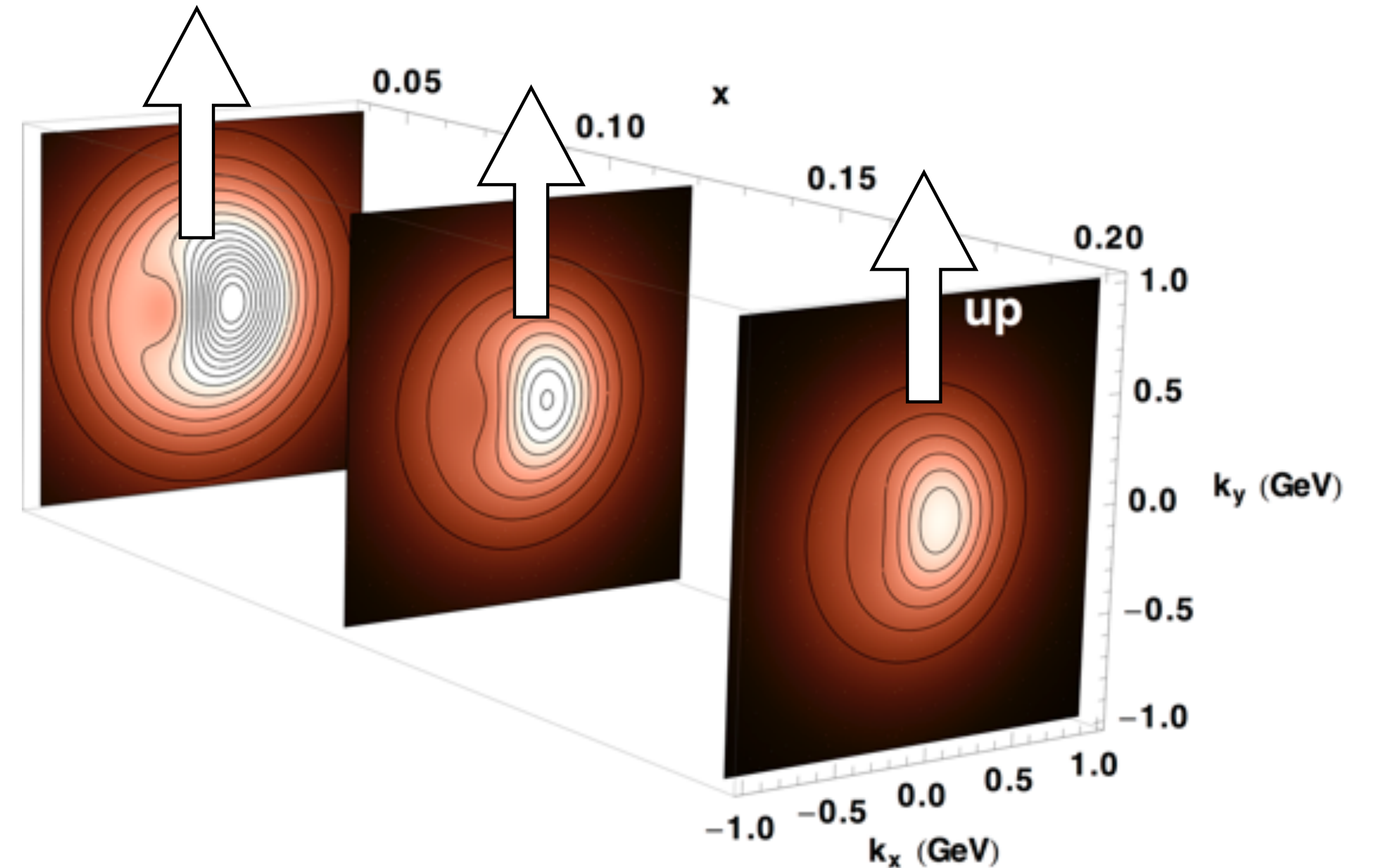
Unpolarized TMD: cylindrically symmetric



# 3D structure in momentum space



Unpolarized TMD: cylindrically symmetric



With transverse spin: distortions can occur,  
encoded in Sivers TMD  
Requires presence of orbital angular momentum



# PDFs


Parton distribution functions ( $x$ )


Transverse-momentum distributions ( $x, \vec{k}_\perp$ )

Impact-parameter distributions ( $x, \vec{b}_\perp$ )

# TMDs

Wigner distributions ( $x, \vec{k}_\perp, \vec{b}_\perp$ )

  $\vec{b}_\perp$  dependence

  $\vec{k}_\perp$  dependence



these two variables are NOT Fourier conjugate

*see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)*

# PDFs

Parton distribution functions ( $x$ )

Transverse-momentum distributions ( $x, \vec{k}_\perp$ )

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Impact-parameter distributions ( $x, \vec{b}_\perp$ )


2D Fourier transform ( $\vec{b}_\perp$ )

Wigner distributions ( $x, \vec{k}_\perp, \vec{b}_\perp$ )

Generalized parton distributions ( $x, \xi = 0, \vec{\Delta}_T$ )

# GPDs

  $\vec{b}_\perp$  dependence

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Form factors ( $\vec{\Delta}_T^2$ )

Integral over  $x$

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# GTMDs

*see talk by A. Metz*

  $\vec{b}_\perp$  dependence

  $\vec{k}_\perp$  dependence



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*see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)*



# Recent review

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*EPJ A (2016) 52*

The European Physical Journal A  
All Volumes & Issues

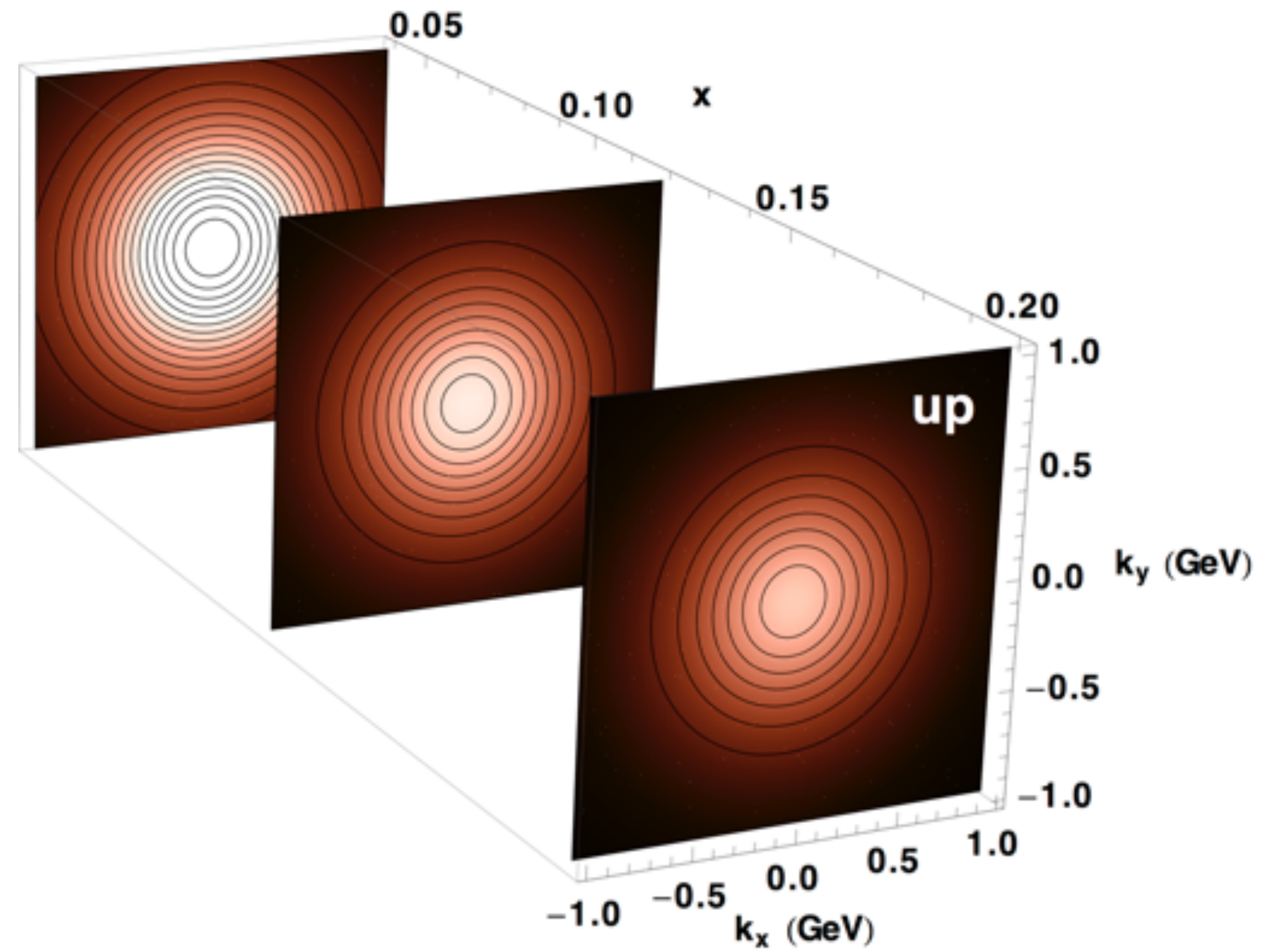
## The 3-D Structure of the Nucleon

ISSN: 1434-6001 (Print) 1434-601X (Online)

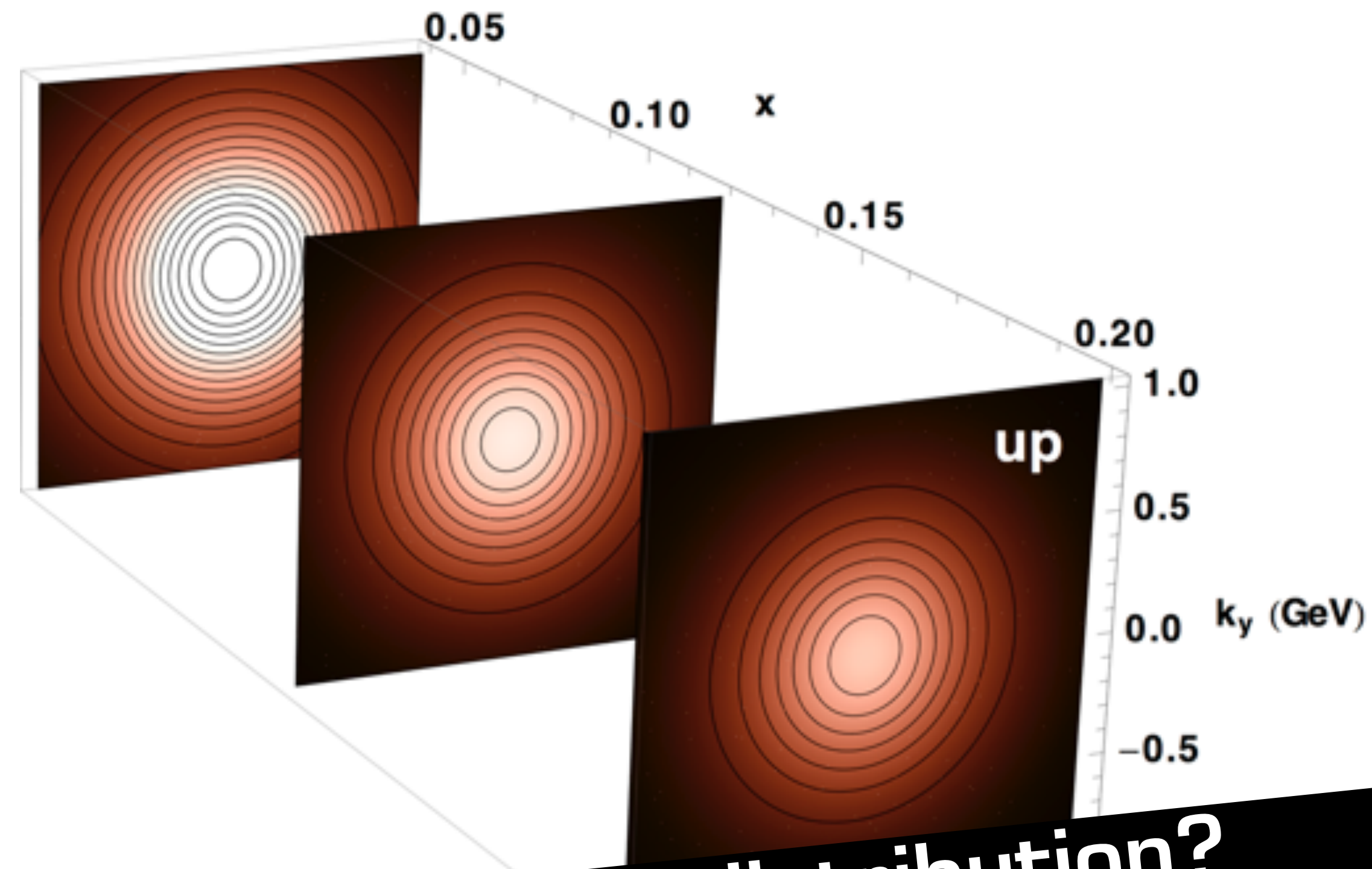
In this topical collection (17 articles)



# The unpolarized TMD



# The unpolarized TMD



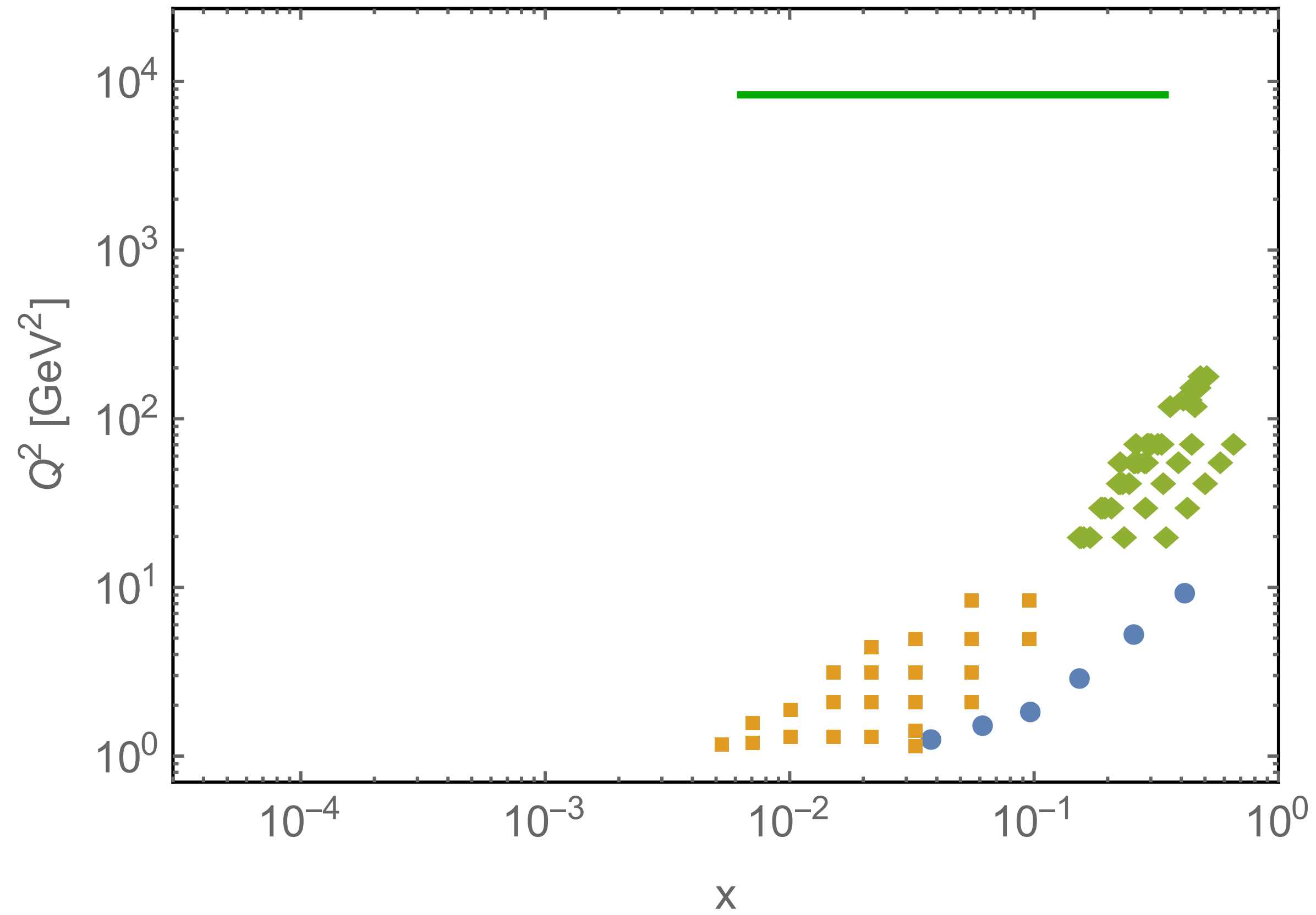
How "wide" is the distribution?  
Is there a difference between flavors?  
Does it get wider at low  $x$ ?

# Extracting TMDs

---

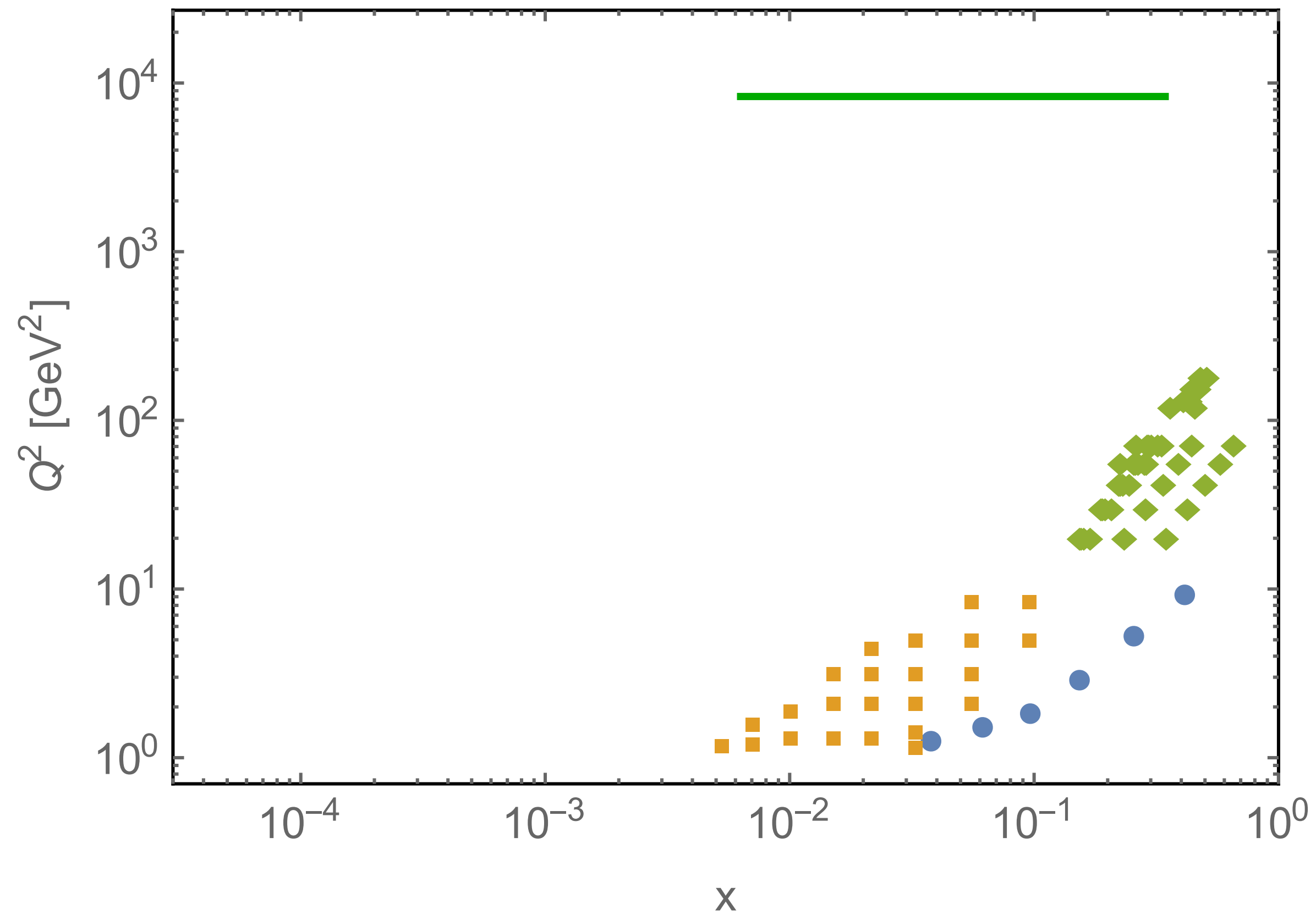
# Experiments

---





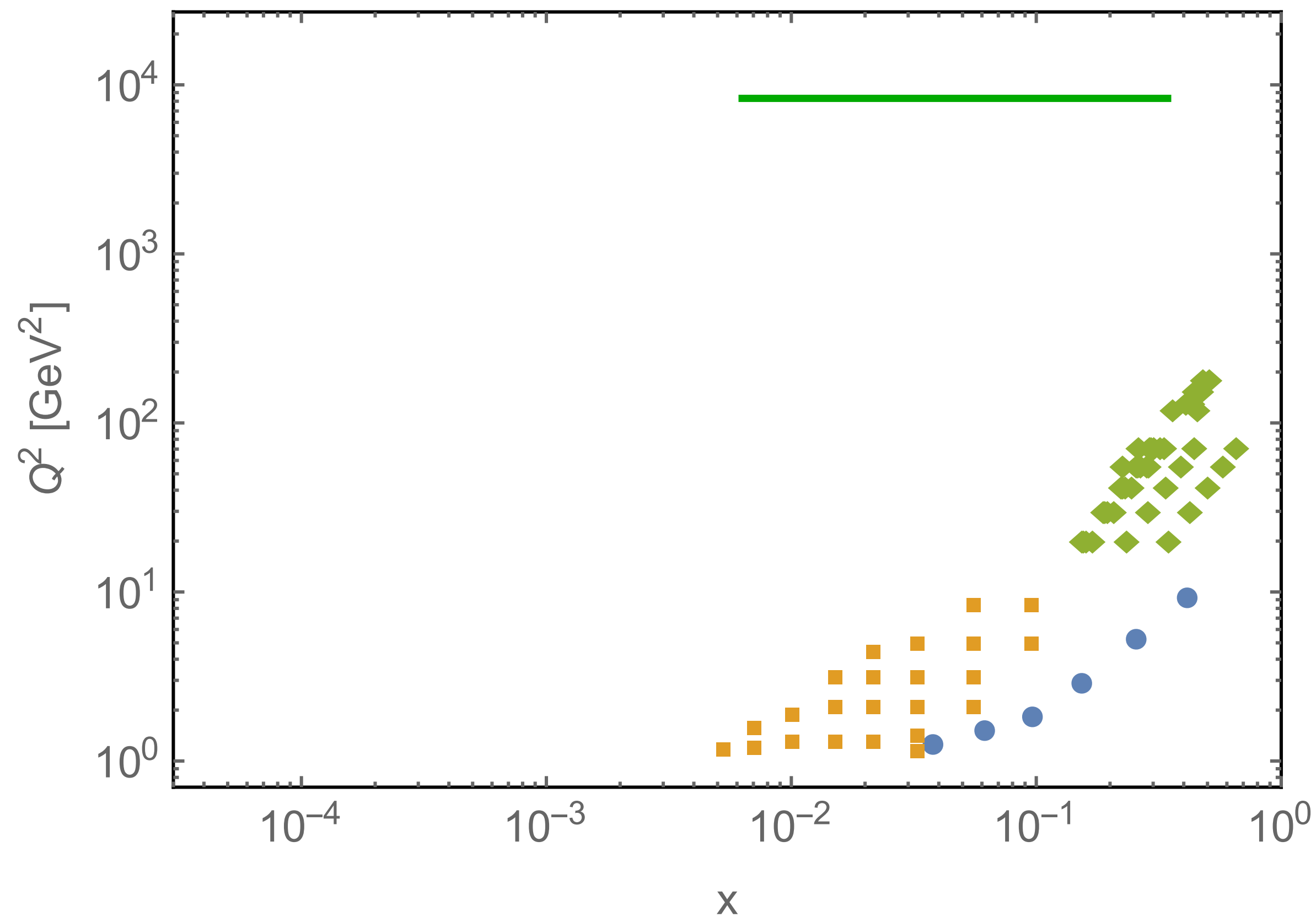
# Experiments



Drell-Yan@  
 Fermilab

*Ito et al., PRD93 (81)*  
*Moreno et al. PRD 43 (91)*  
*Antreyan et al. PRL47 (81)*

# Experiments



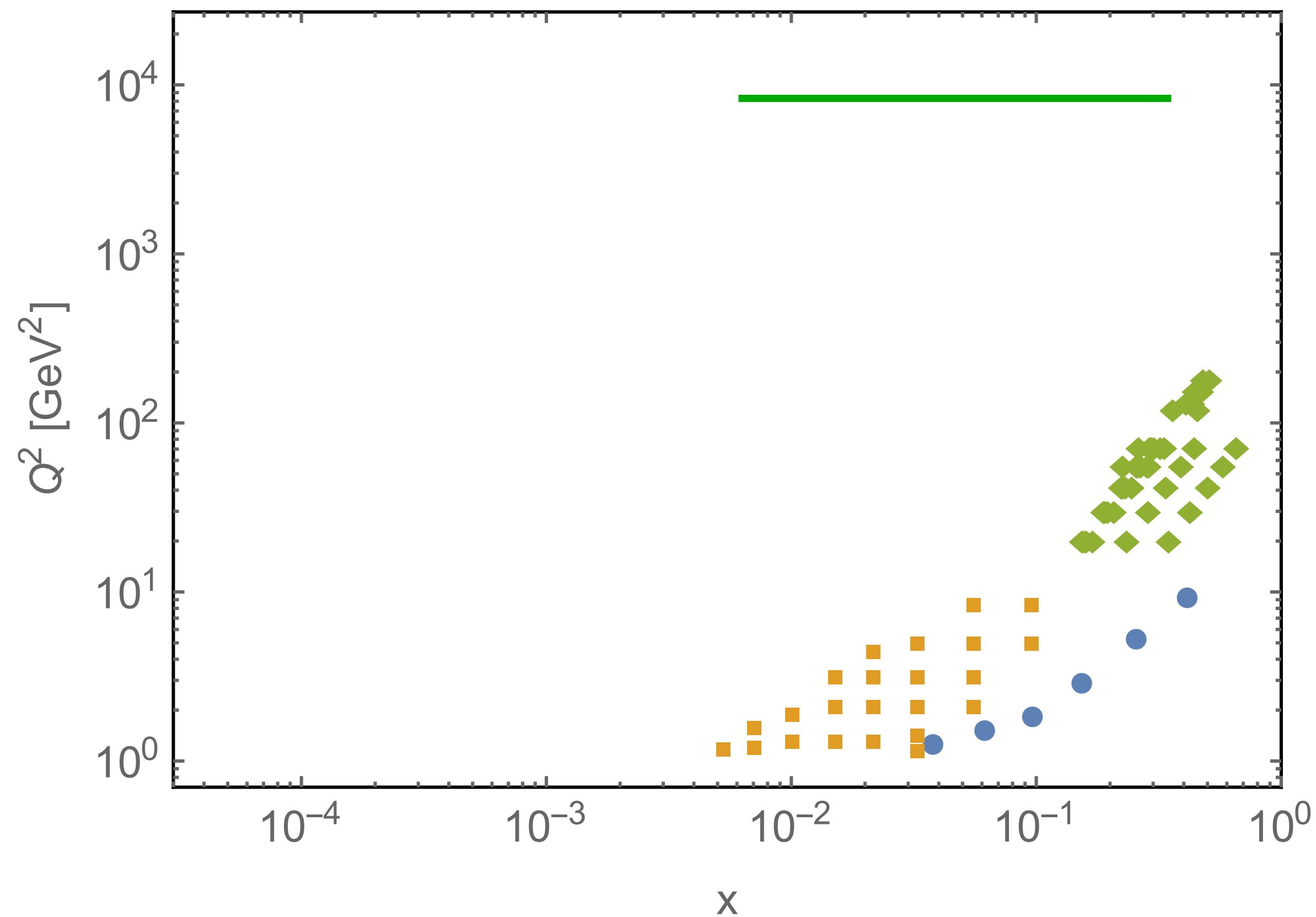
Z production@  
 Fermilab

*Abbot et al. hep-ex/9909020*  
*Affolder et al. hep-ex/0001021*  
*Abazov et al. arXiv:0712.0803*  
*Aaltonen et al. arXiv:1207.7138*

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 Fermilab

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# Experiments



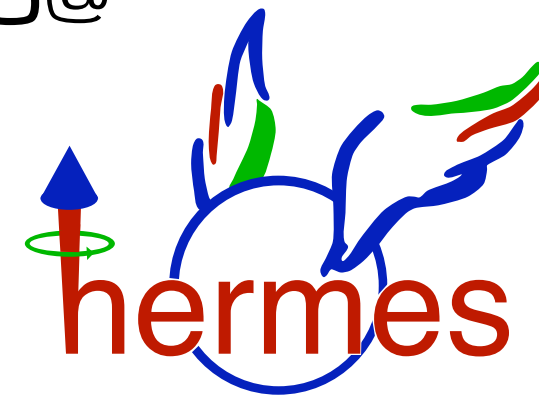
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*Abbot et al. hep-ex/9909020*  
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Drell-Yan@  
 Fermilab

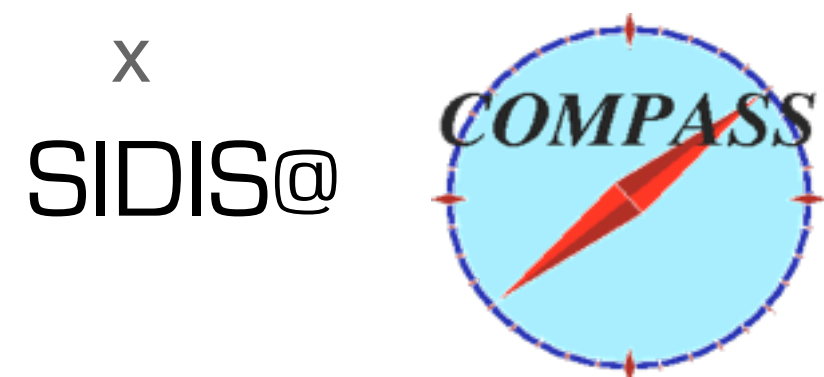
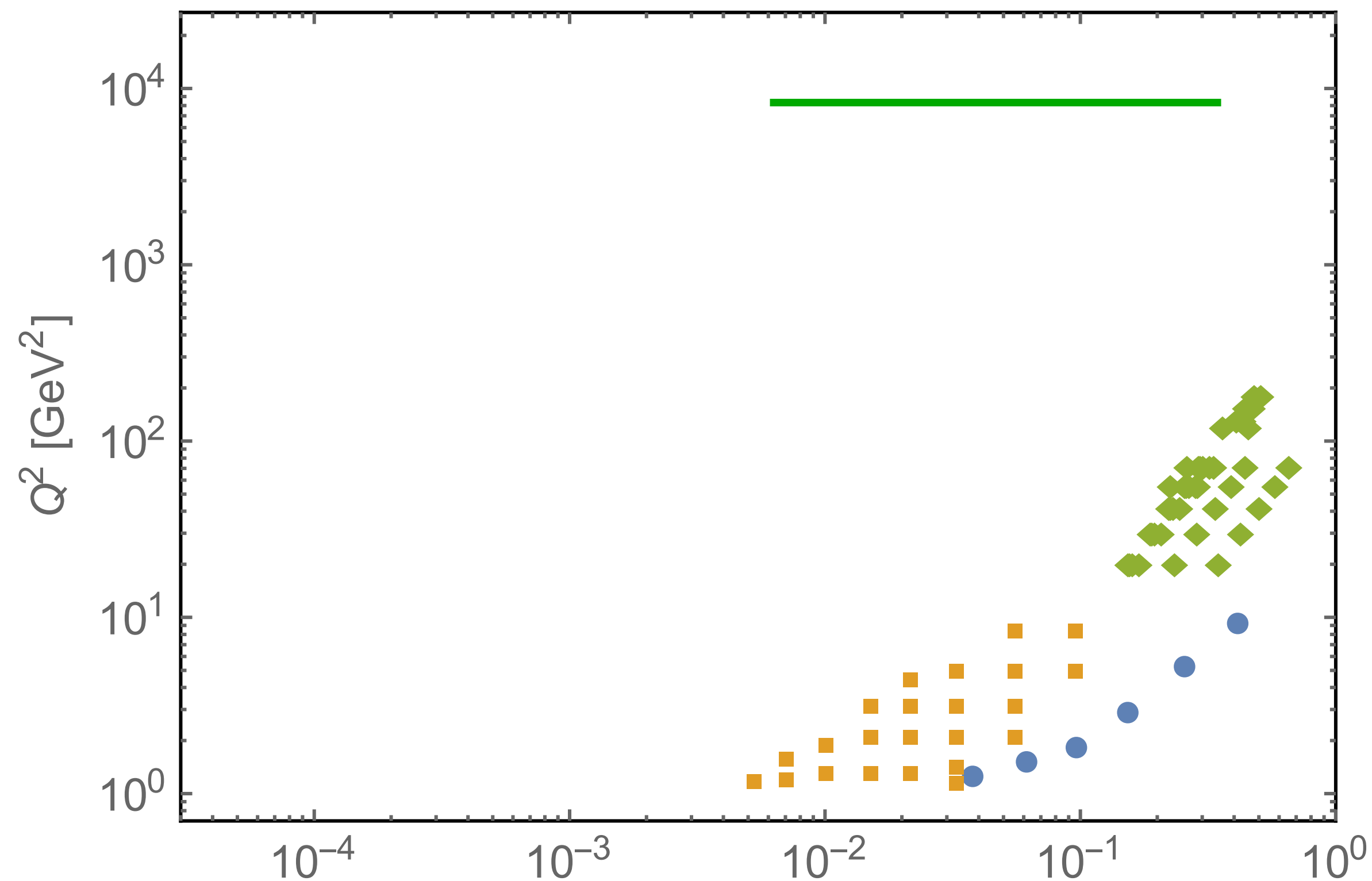
*Ito et al., PRD93 (81)*  
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*Antreyan et al. PRL47 (81)*

SIDIS@



*Airapetian et al., PRD87 (2013)*

# Experiments



*Adolph et al., EPJ C73 (13)*

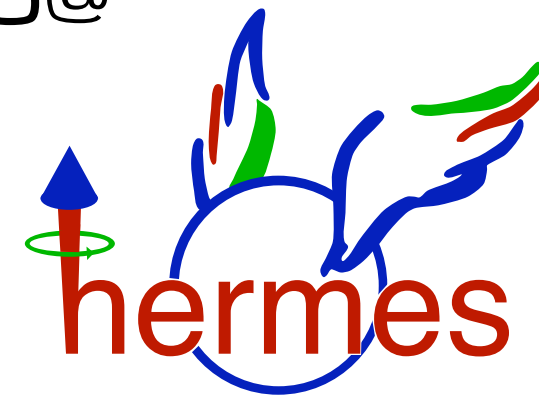
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Drell-Yan@  
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SIDIS@



*Airapetian et al., PRD87 (2013)*



# Presently *or soon* available fits

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 <a href="https://arxiv.org/abs/hep-ph/0506225">hep-ph/0506225</a>	NLL	✗	✗	✓	✓	98
Pavia 2013 (+Amsterdam,Bilbao) <a href="https://arxiv.org/abs/1309.3507">arXiv:1309.3507</a>	No evo	✓	✗	✗	✗	1538
Torino 2014 (+JLab) <a href="https://arxiv.org/abs/1312.6261">arXiv:1312.6261</a>	No evo	✓ [separately]	✓ [separately]	✗	✗	576 (H) 6284 (C)
DEMS 2014 <a href="https://arxiv.org/abs/1407.3311">arXiv:1407.3311</a>	NNLL	✗	✗	✓	✓	223
EIKV 2014 <a href="https://arxiv.org/abs/1401.5078">arXiv:1401.5078</a>	NLL	1 ( $x, Q^2$ ) bin	1 ( $x, Q^2$ ) bin	✓	✓	500 (?)
Pavia 2016	NLL	✓	✓	✓	✓	8059



# The TMD “eight-thousander” fit

8000 data points

Broad Peak, Karakorum, 8051 m



# The TMD “eight-thousander” fit

***Pavia 2016***

8000 data points

Broad Peak, Karakorum, 8051 m



# Executive summary of Pavia 2016 results 1/3

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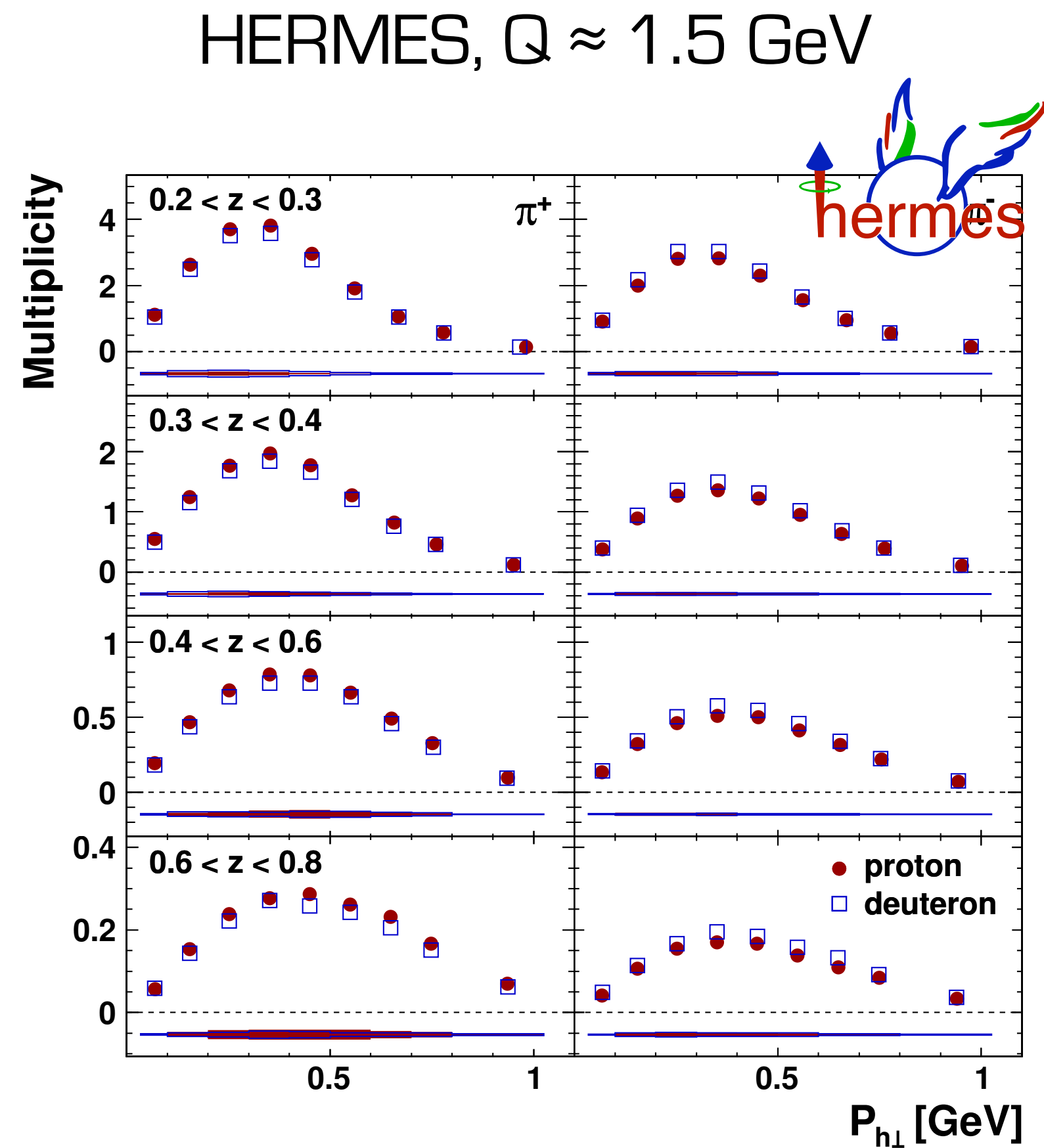
Total number of data points: 8059

Total number of free parameters: 11  
(4 for TMD PDFs, 6 for TMD FFs, 1 for TMD evolution)

Total  $\chi^2/\text{dof} = 1.52 \pm 0.03$



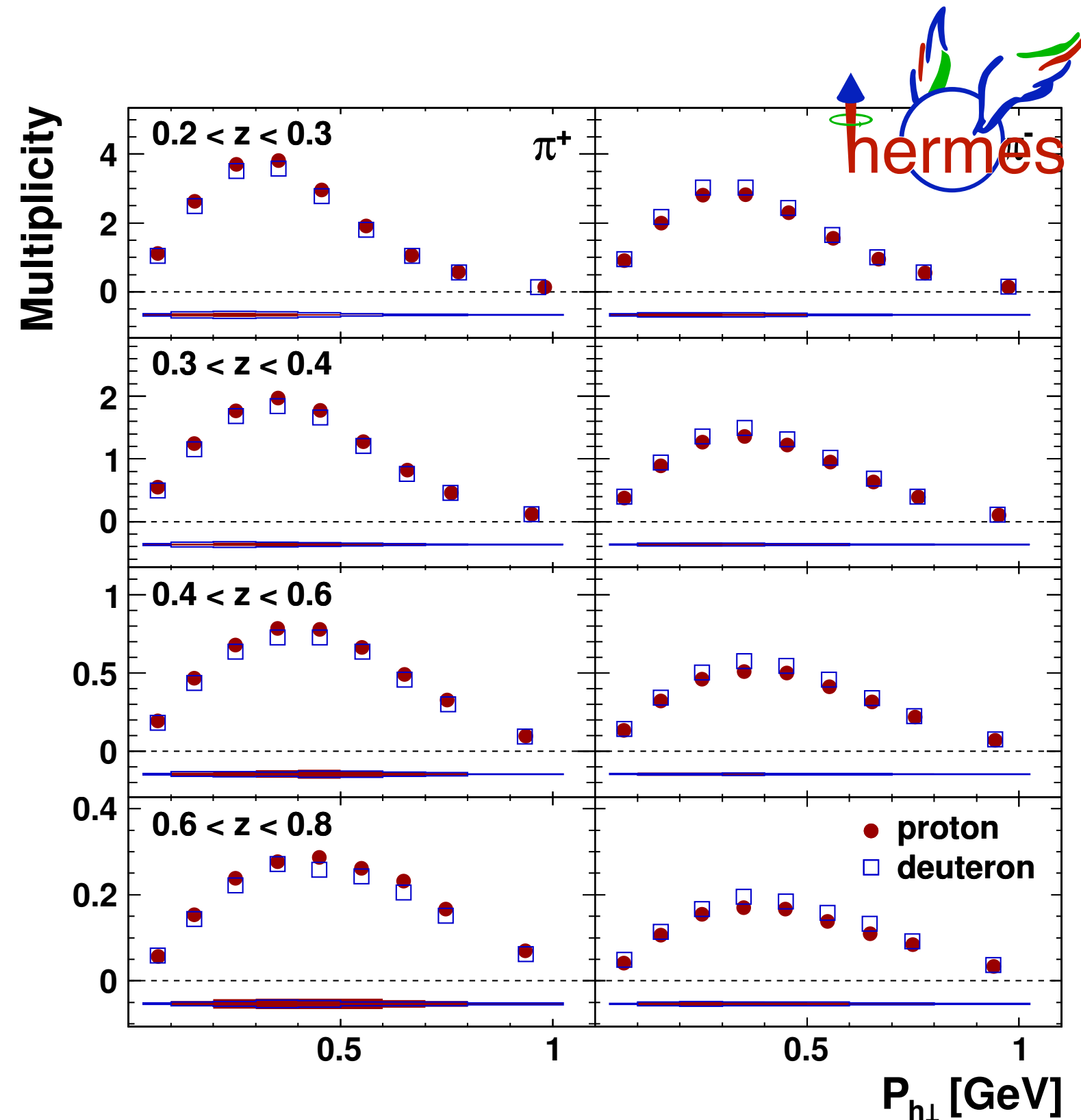
# Executive summary of results 2/3



*Airapetian et al., PRD87 (2013)*

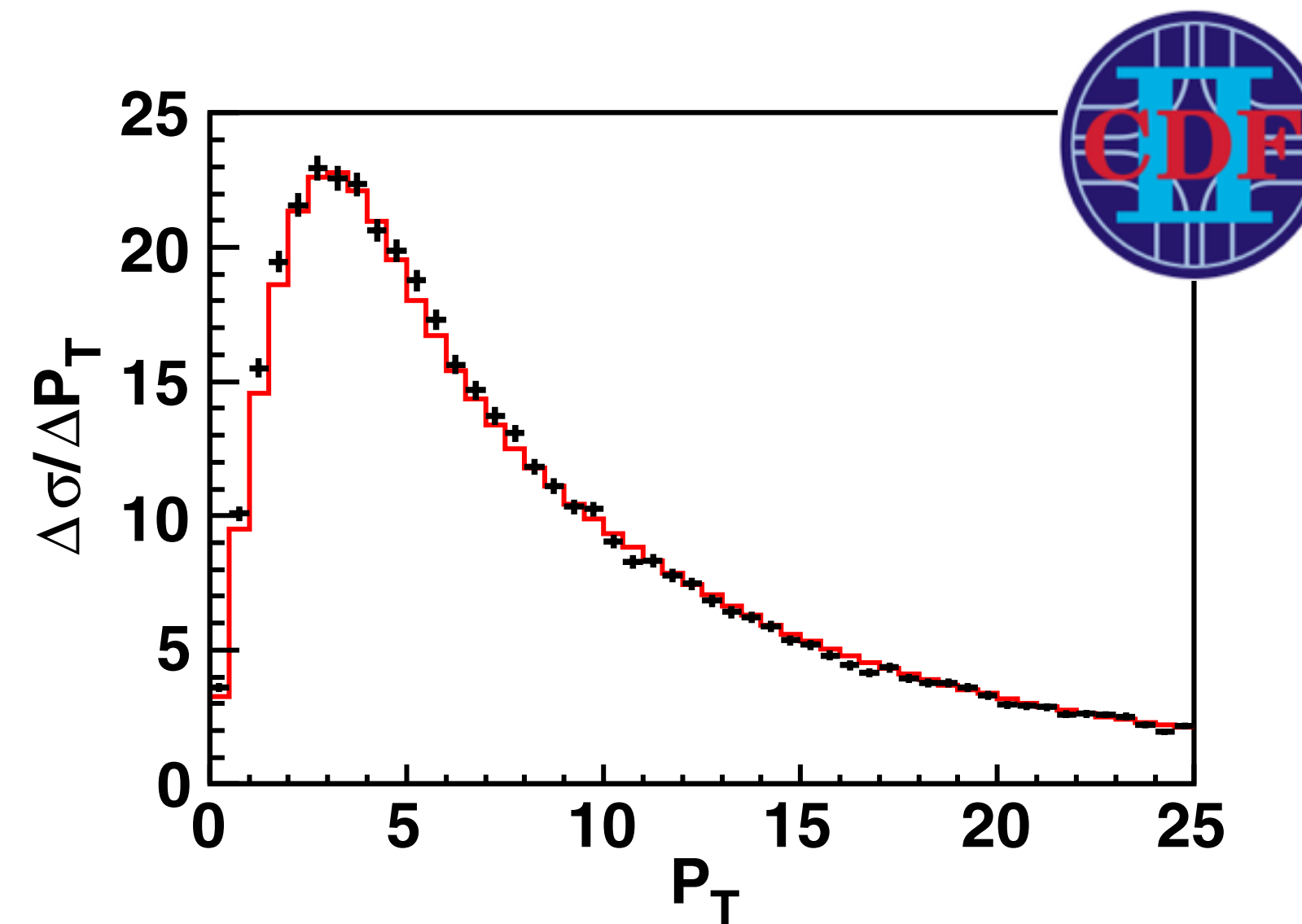
# Executive summary of results 2/3

HERMES,  $Q \approx 1.5$  GeV



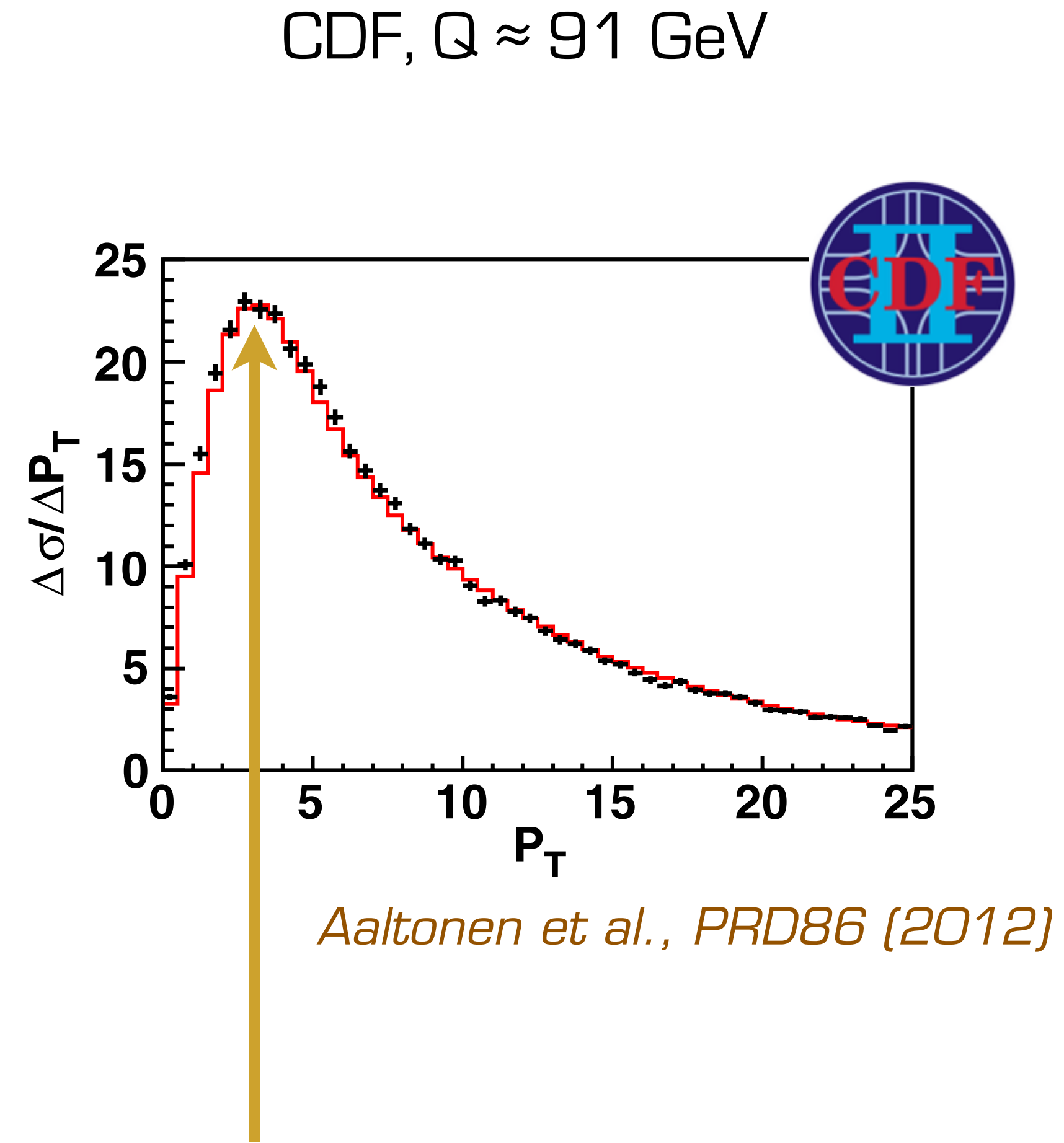
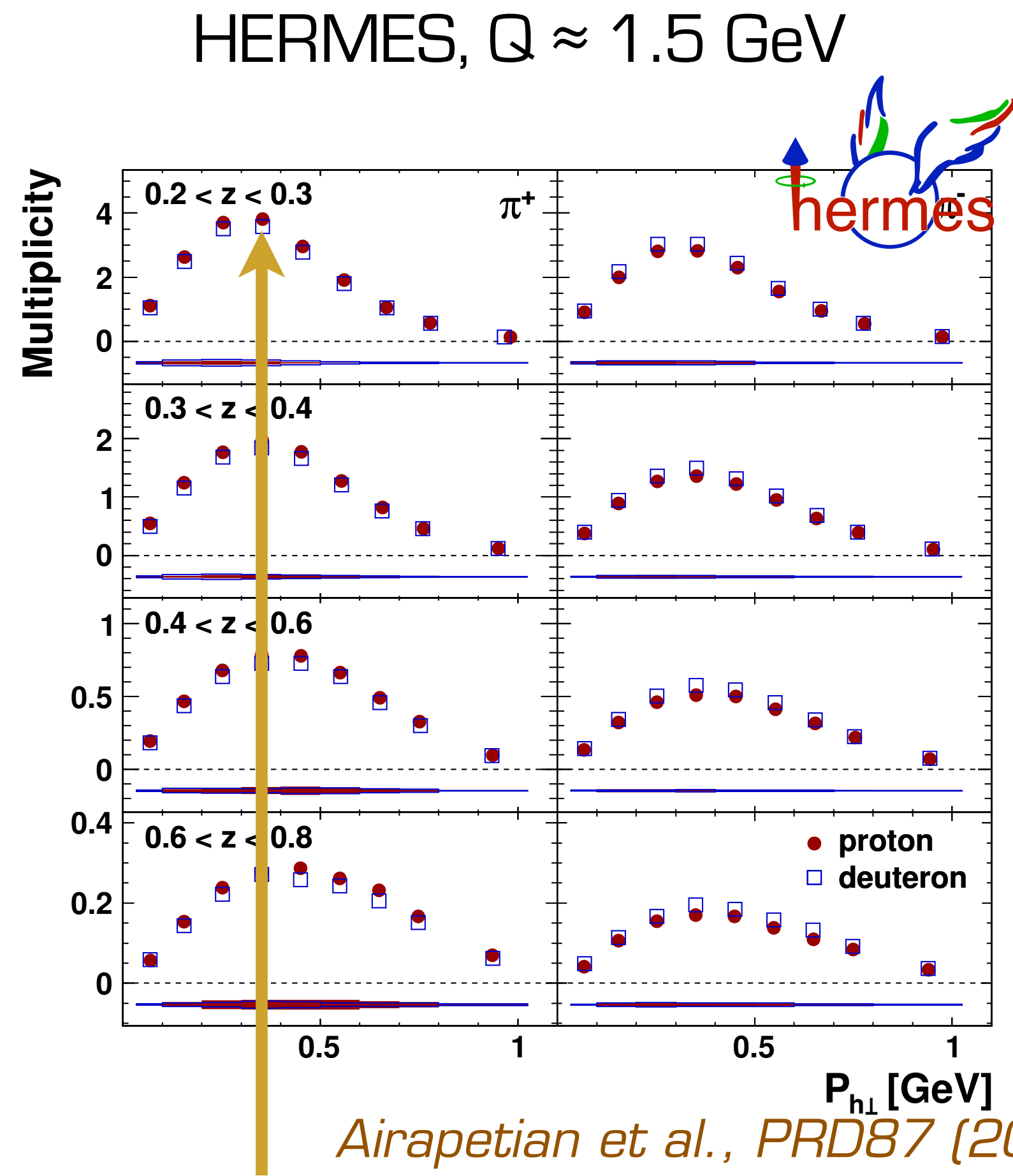
*Airapetian et al., PRD87 (2013)*

CDF,  $Q \approx 91$  GeV



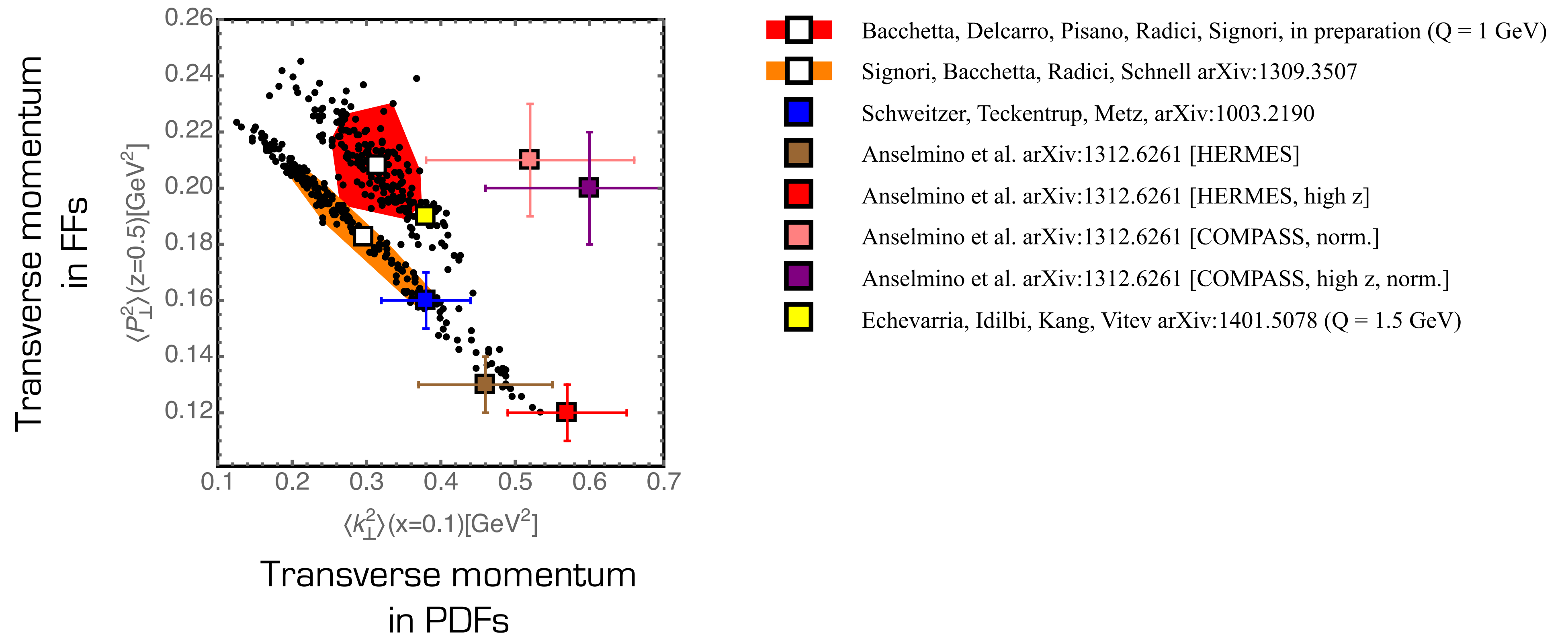
*Aaltonen et al., PRD86 (2012)*

# Executive summary of results 2/3



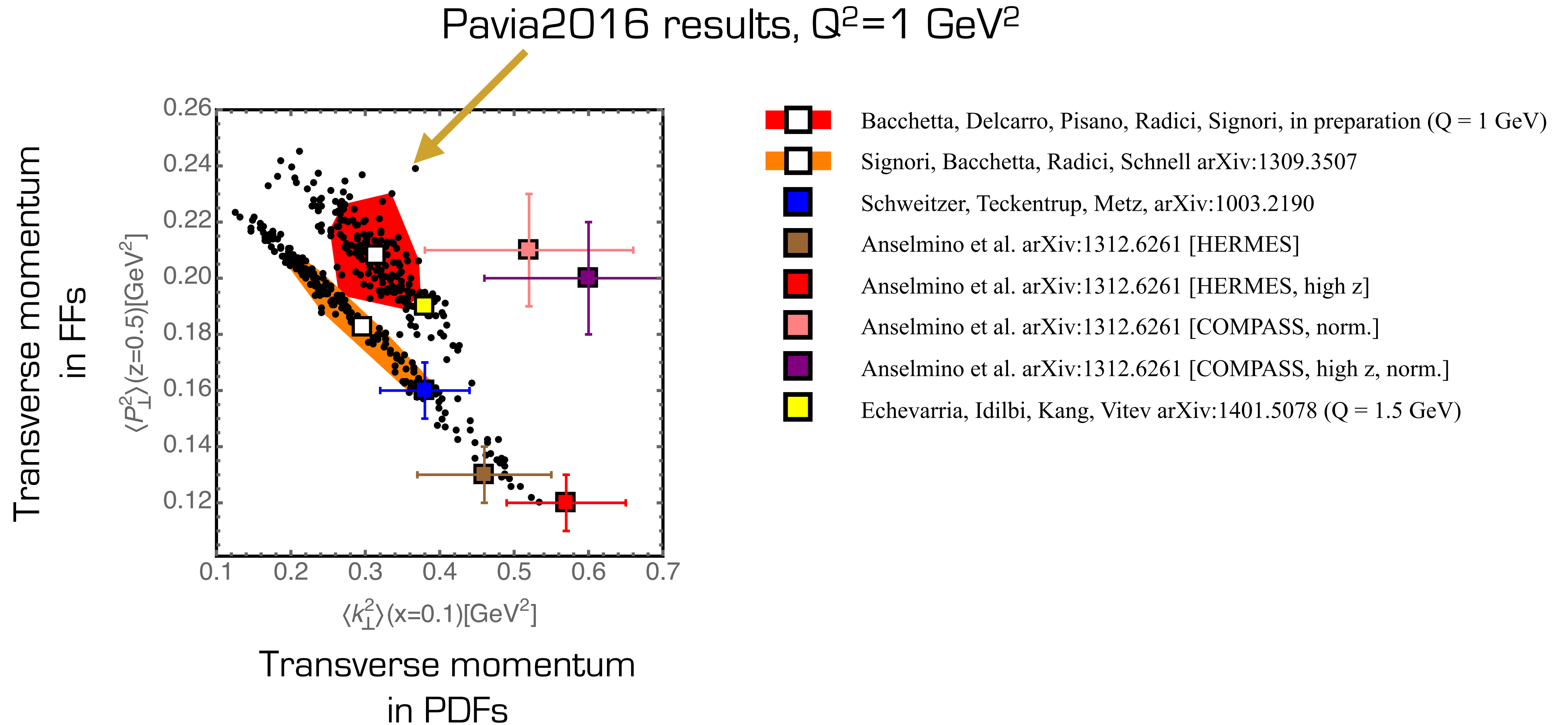
Width of TMDs changes of one order of magnitude: we can explain this with TMD evolution

# Executive summary of results 3/3





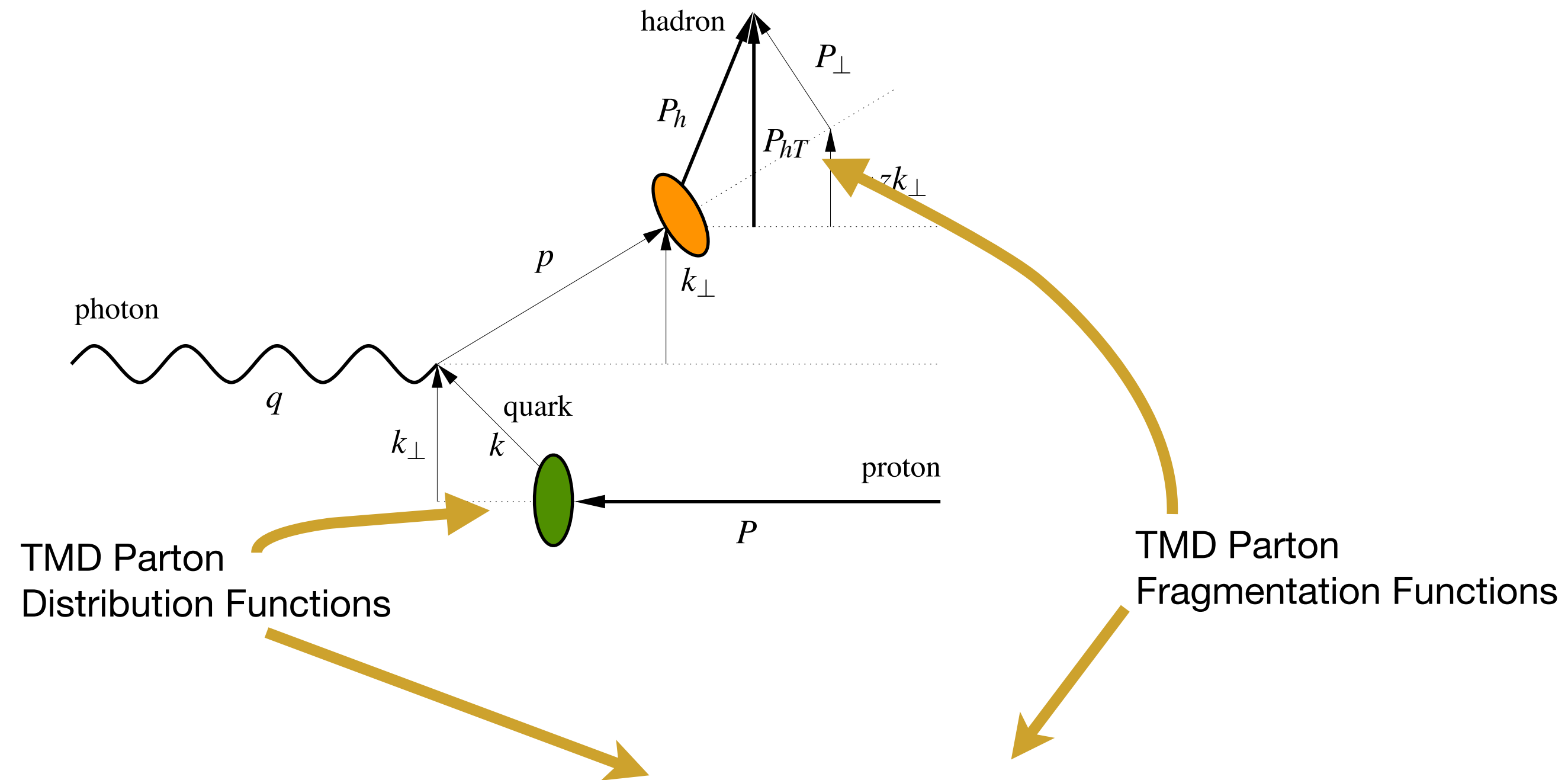
# Executive summary of results 3/3



Some details

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# Structure functions and TMDs



$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2) = \sum_a \mathcal{H}_{UU,T}^a(Q^2; \mu^2) \int d\mathbf{k}_\perp d\mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta(z\mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) + \cancel{Y_{UU,T}^a(Q^2, \mathbf{P}_{hT}^2)} + \mathcal{O}(M^2/Q^2)$$

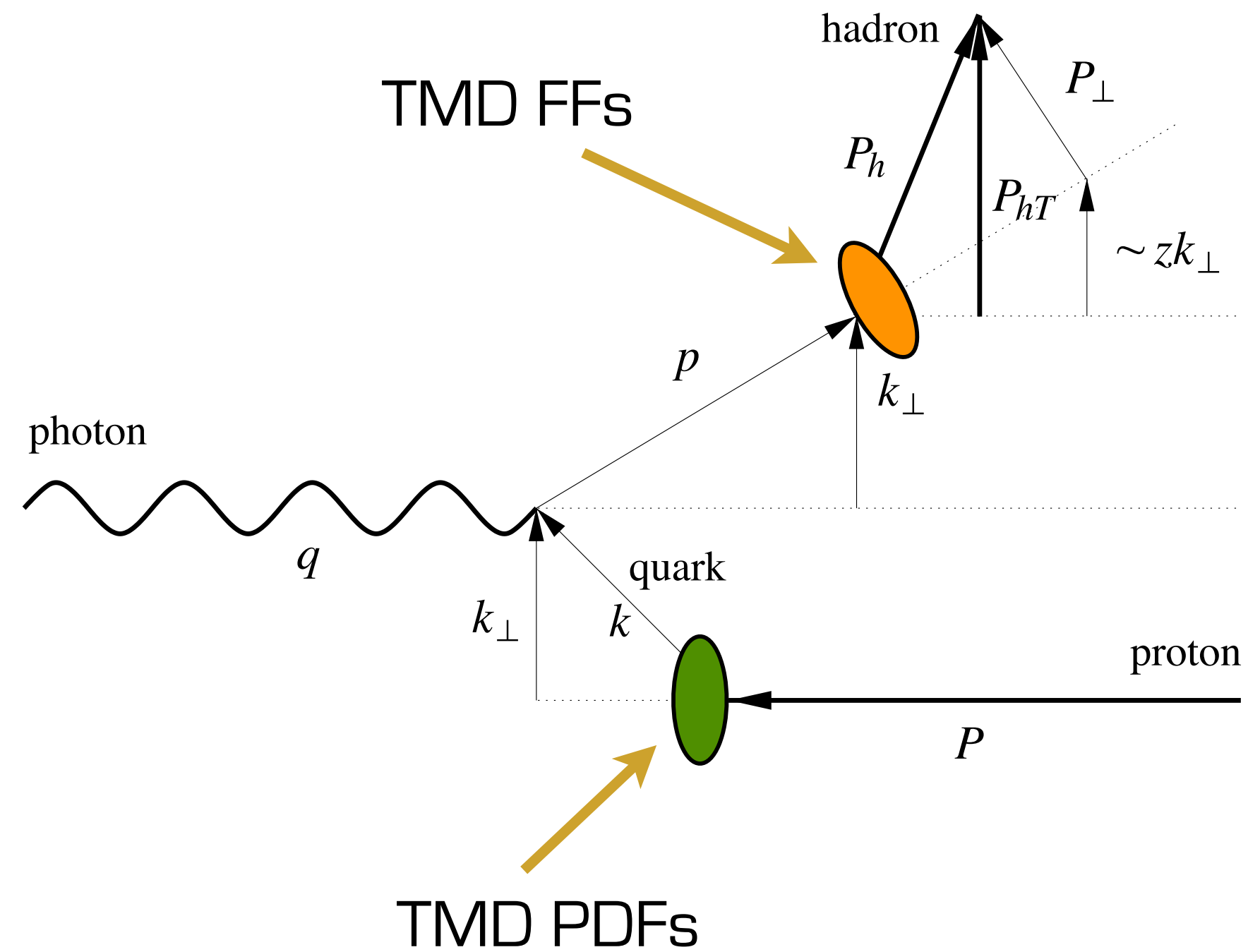
*see talk by Bowen Wang*

# Semi-inclusive DIS

vs.

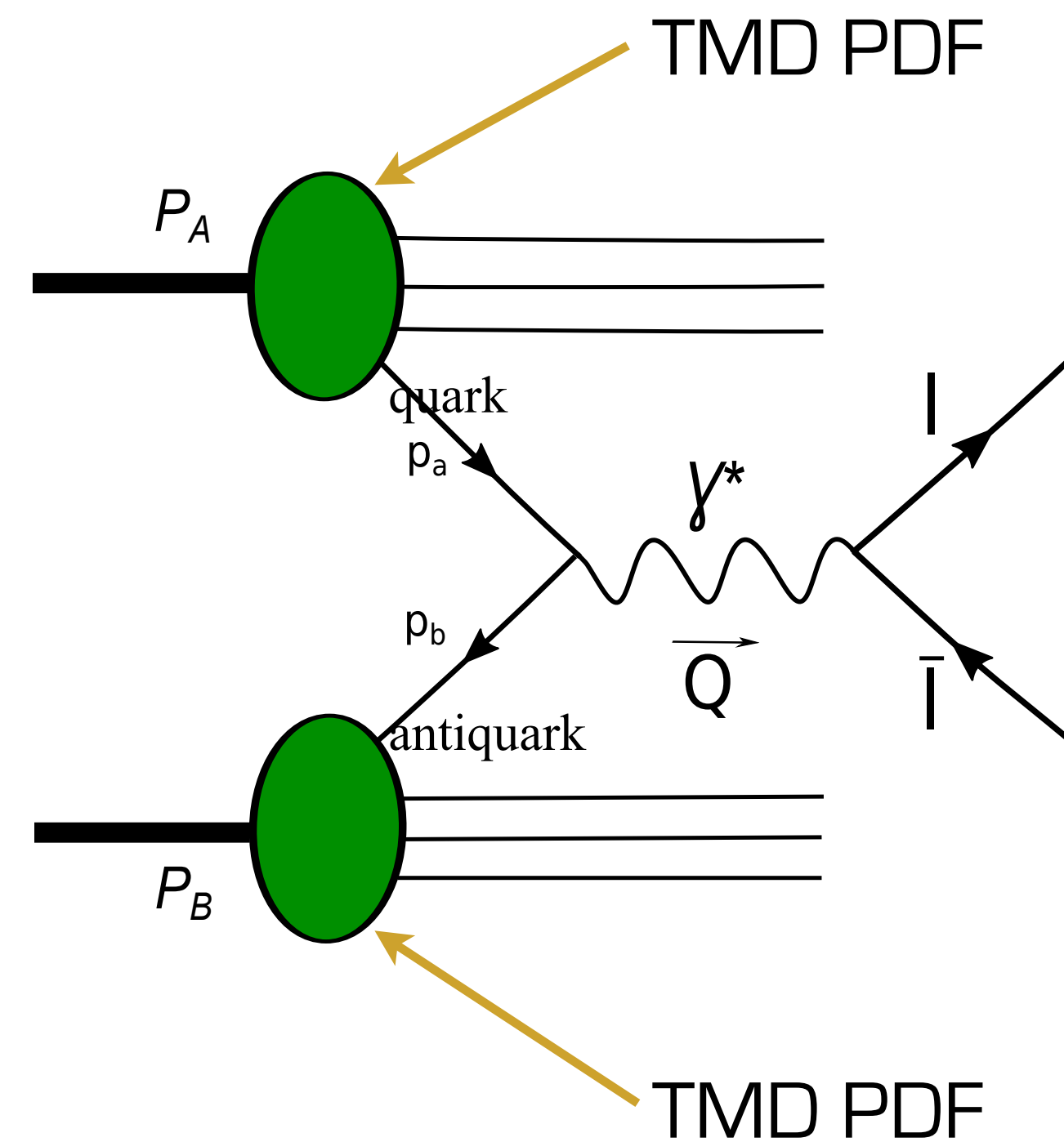
# Drell-Yan/Z production

$$l(\ell) + N(\mathcal{P}) \rightarrow l(\ell') + h(\mathcal{P}_h) + X$$



$$A + B \rightarrow \gamma^* \rightarrow l^+ l^-$$

$$A + B \rightarrow Z \rightarrow l^+ l^-$$





# TMD evolution: Fourier transform

---

$$f_1^a(x, k_\perp; \mu^2) = \frac{1}{2\pi} \int d^2 b_T e^{-i b_T \cdot k_\perp} \tilde{f}_1^a(x, b_T; \mu^2)$$

*Rogers, Aybat, PRD 83 (11)*  
*Collins, "Foundations of Perturbative QCD" (11)*

*possible schemes, e.g.,*  
*Collins, Soper, Sterman, NPB250 (85)*  
*Laenen, Sterman, Vogelsang, PRL 84 (00)*  
*Echevarria, Idilbi, Schaefer, Scimemi, EPJ C73 (13)*

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$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

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collinear PDF

pQCD

nonperturbative part  
of evolution

nonperturbative part  
of TMD

*Rogers, Aybat, PRD 83 (11)*

*Collins, "Foundations of Perturbative QCD" (11)*

*possible schemes, e.g.,*

*Collins, Soper, Sterman, NPB250 (85)*

*Laenen, Sterman, Vogelsang, PRL 84 (00)*

*Echevarria, Idilbi, Schaefer, Scimemi, EPJ C73 (13)*

# Perturbative ingredients

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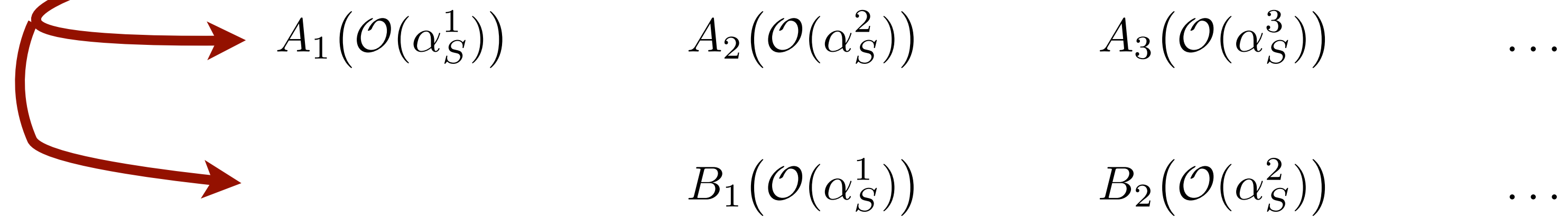
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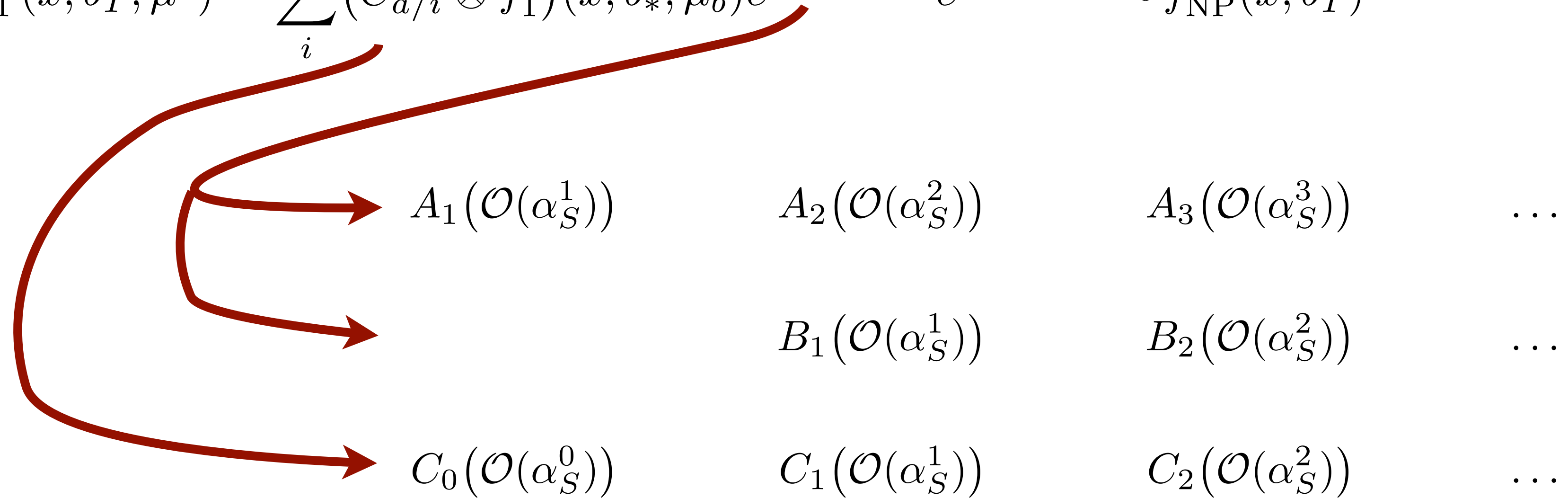
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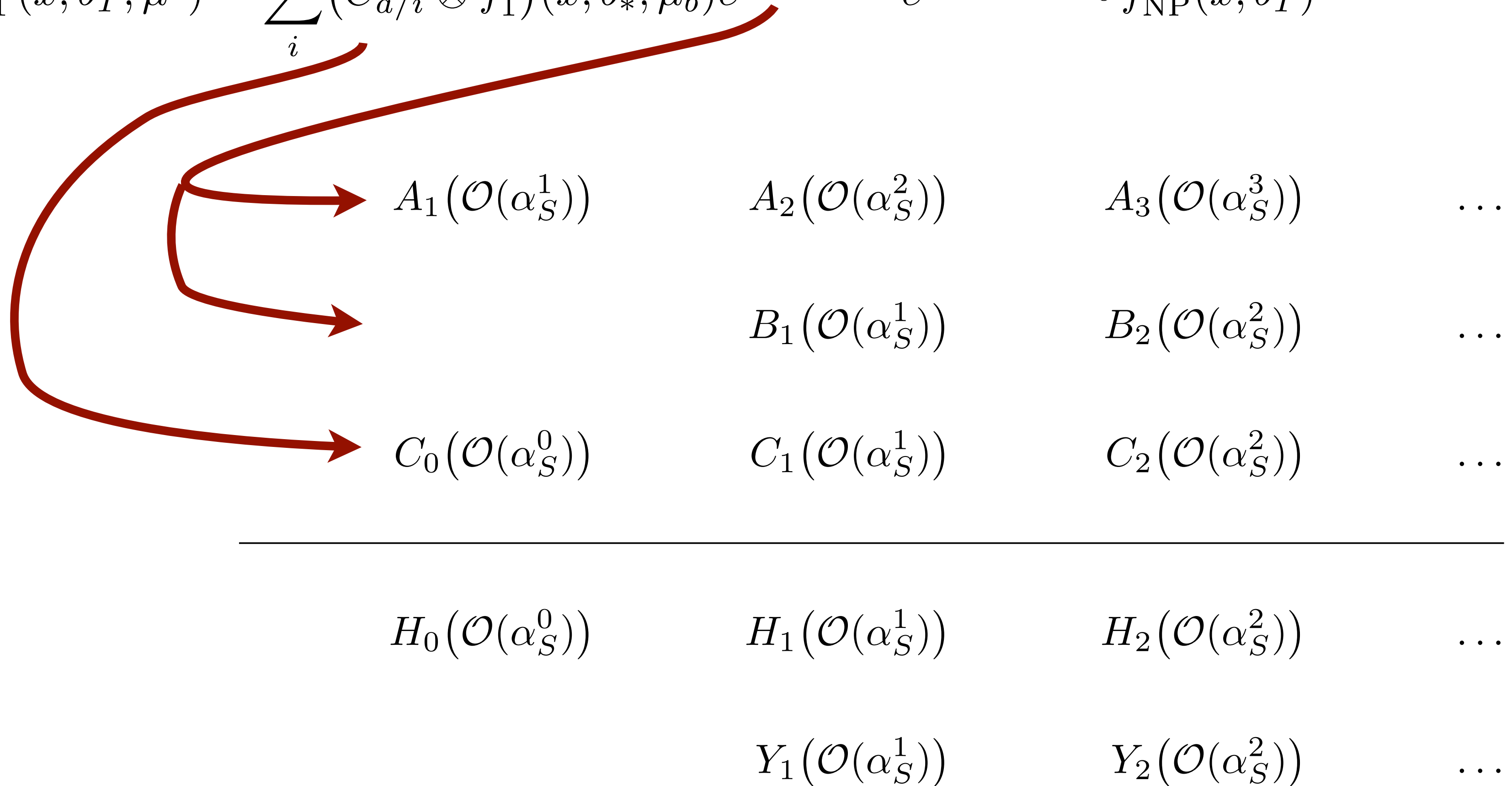
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$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$



# Perturbative ingredients

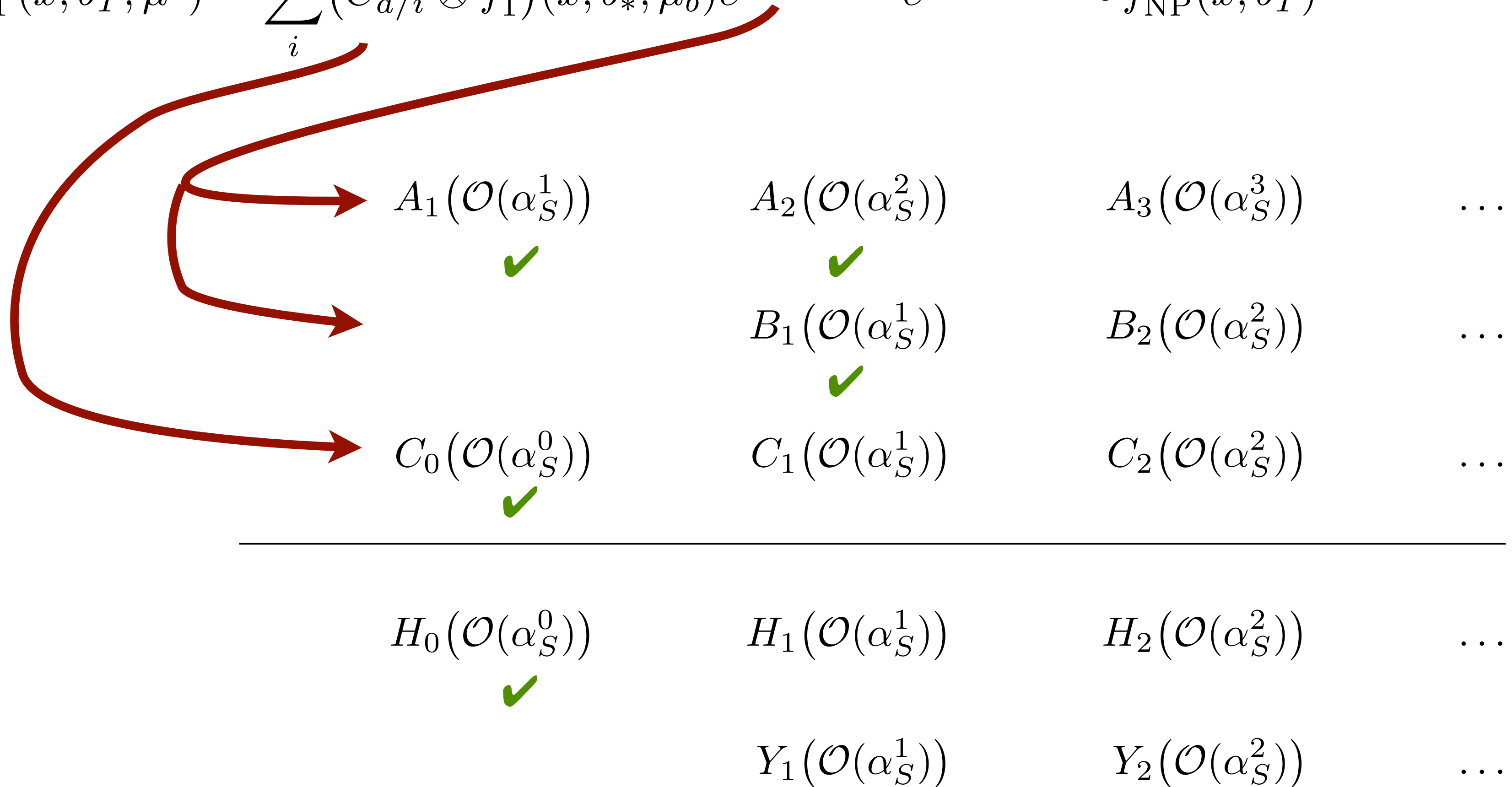
$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$





# Pavia 2016 perturbative ingredients

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$



# $\mu$ and $b_*$ prescriptions

---

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# $\mu$ and $b_*$ prescriptions

---

**Choice Choice**

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$



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$$\mu_b = 2e^{-\gamma_E} / b_* \quad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2 / b_{\text{max}}^2}} \quad \text{Collins, Soper, Sterman, NPB250 (85)}$$

$$\mu_b = 2e^{-\gamma_E} / b_* \quad b_* \equiv b_{\text{max}} \left( 1 - e^{-\frac{b_T^4}{b_{\text{max}}^4}} \right)^{1/4} \quad \text{Bacchetta, Echevarria, Mulders, Radici, Signori} \\ \text{arXiv:1508.00402}$$

$$\mu_b = Q_0 + q_T \quad b_* = b_T \quad \text{DEMS 2014}$$

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$$\mu_b = Q_0 + q_T \quad b_* = b_T \quad \text{DEMS 2014}$$

Complex-b prescription

*Laenen, Sterman, Vogelsang, PRL 84 (00)*

# Nonperturbative ingredients 1

---


$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$



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**Choice**  


# Nonperturbative ingredients 1

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

**Choice** ↙

$$e^{-\frac{b_T^2}{\langle b_T^2 \rangle}}$$

*almost everybody*

$$e^{-\frac{b_T^2}{\langle b_T^2(x) \rangle_a}}$$

*Pavia 2013, KN 2006*

$$e^{-\lambda_1 b_T} (1 + \lambda_2 b_T^2)$$

*DEMS 2014*

# Nonperturbative ingredients 2


---

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

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---

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**Choice**  




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$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

**Choice**

$$-g_2 \frac{b_T^2}{2}$$

*Collins, Soper, Sterman, NPB250 (85)*

$$-2 g_2 \ln \left( 1 + \frac{b_T^2}{4} \right)$$

*Aidala, Field, Gamberg, Rogers, [arXiv:1401.2654](https://arxiv.org/abs/1401.2654)*

$$-g_0(b_{\text{max}}) \left( 1 - \exp \left[ - \frac{C_F \alpha_s(\mu_{b_*}) b_T^2}{\pi g_0(b_{\text{max}}) b_{\text{max}}^2} \right] \right)$$

*Collins, Rogers, [arXiv:1412.3820](https://arxiv.org/abs/1412.3820)*

# Low- $b_T$ modifications

---

$$\log(Q^2 b_T^2) \rightarrow \log(Q^2 b_T^2 + 1)$$

*see, e.g., Bozzi, Catani, De Florian, Grazzini  
[hep-ph/0302104](#)*

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$$b_*(b_c(b_T)) = \sqrt{\frac{b_T^2 + b_0^2/(C_5^2 Q^2)}{1 + b_T^2/b_{\max}^2 + b_0^2/(C_5^2 Q^2 b_{\max}^2)}}$$

$$b_{\min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q} \sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_{\max}^2)}}$$

*Collins et al., [arXiv:1605.00671](#)*

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*Collins et al., [arXiv:1605.00671](#)*

Justification: the modification is allowed because it affects a region where the TMD formalism is anyway unreliable (high transverse momentum), and allows us to recover the integrated cross-section (unitarity constraint) instead of leading to infinite results.



# Pavia 2016 “choices”

---

$$\tilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, \bar{b}_*; \mu_b) e^{\tilde{S}(\bar{b}_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^a(x, b_T)$$

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$$g_K = -g_2 \frac{b_T^2}{2} \quad \mu_0 = 1 \text{ GeV}$$

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$$g_K = -g_2 \frac{b_T^2}{2} \quad \mu_0 = 1 \text{ GeV}$$

$$\mu_b = 2e^{-\gamma_E} / b_* \quad \bar{b}_* \equiv b_{\text{max}} \left( \frac{1 - e^{-b_T^4/b_{\text{max}}^4}}{1 - e^{-b_T^4/b_{\text{min}}^4}} \right)^{1/4} \quad b_{\text{max}} = 2e^{-\gamma_E}$$

$$b_{\text{min}} = \frac{2e^{-\gamma_E}}{Q}$$

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Collinear PDF and FF sets: GJR08 NLO, DSS14 NLO for pions, DSS 07 for kaons



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$$\mu_0 = 1 \text{ GeV}$$

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$$b_{\text{max}} = 2e^{-\gamma_E}$$

$$b_{\text{min}} = \frac{2e^{-\gamma_E}}{Q}$$

**These are all choices that should be at some point checked/challenged**

Collinear PDF and FF sets: GJR08 NLO, DSS14 NLO for pions, DSS 07 for kaons

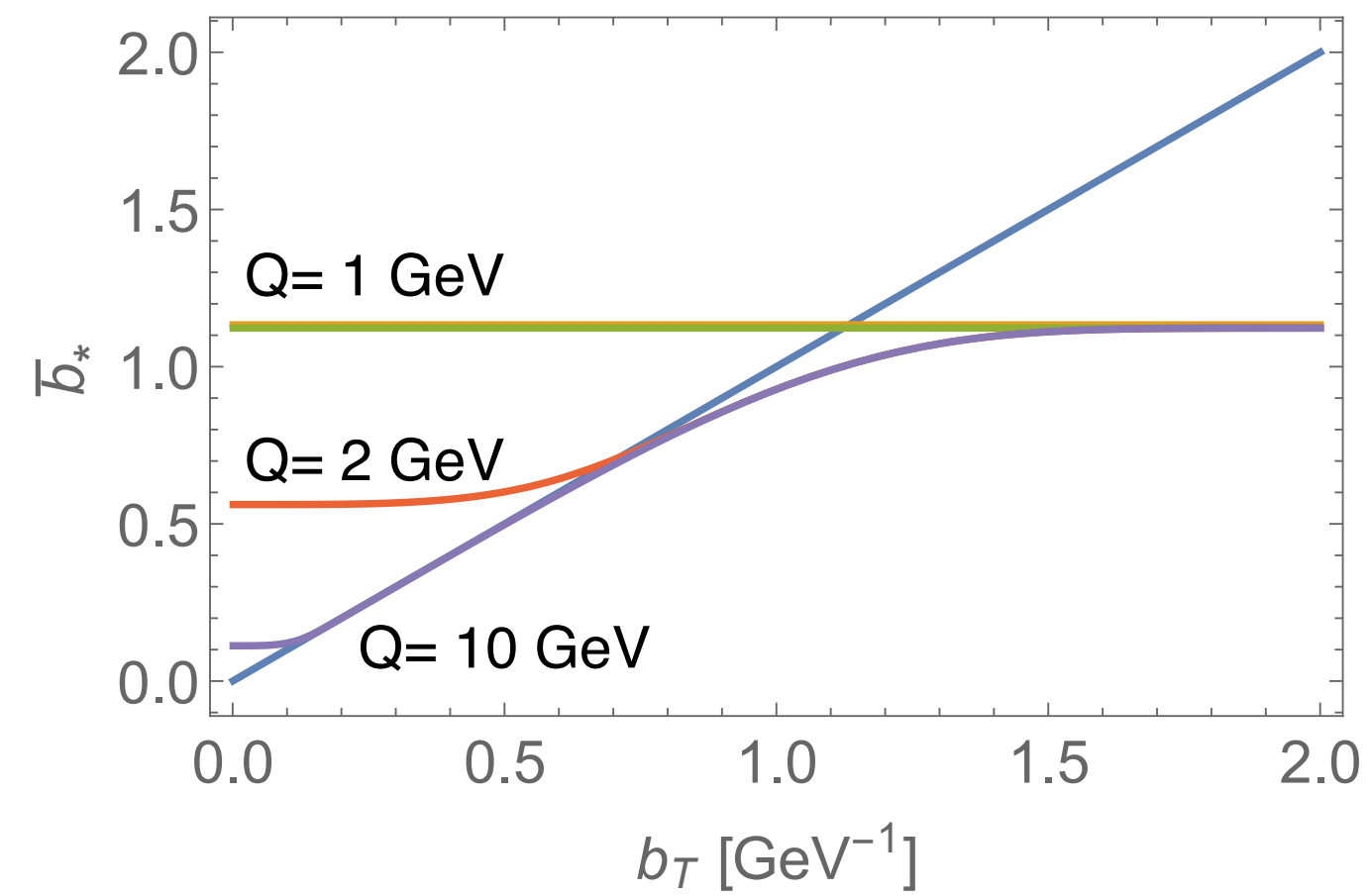
# Effects of $b_*$ prescription

$$\bar{b}_* \equiv b_{\max} \left( \frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)^{1/4}$$

$$b_{\max} = 2e^{-\gamma E}$$

$$b_{\min} = \frac{2e^{-\gamma E}}{Q}$$

$$\mu_b = 2e^{-\gamma E} / b_*$$

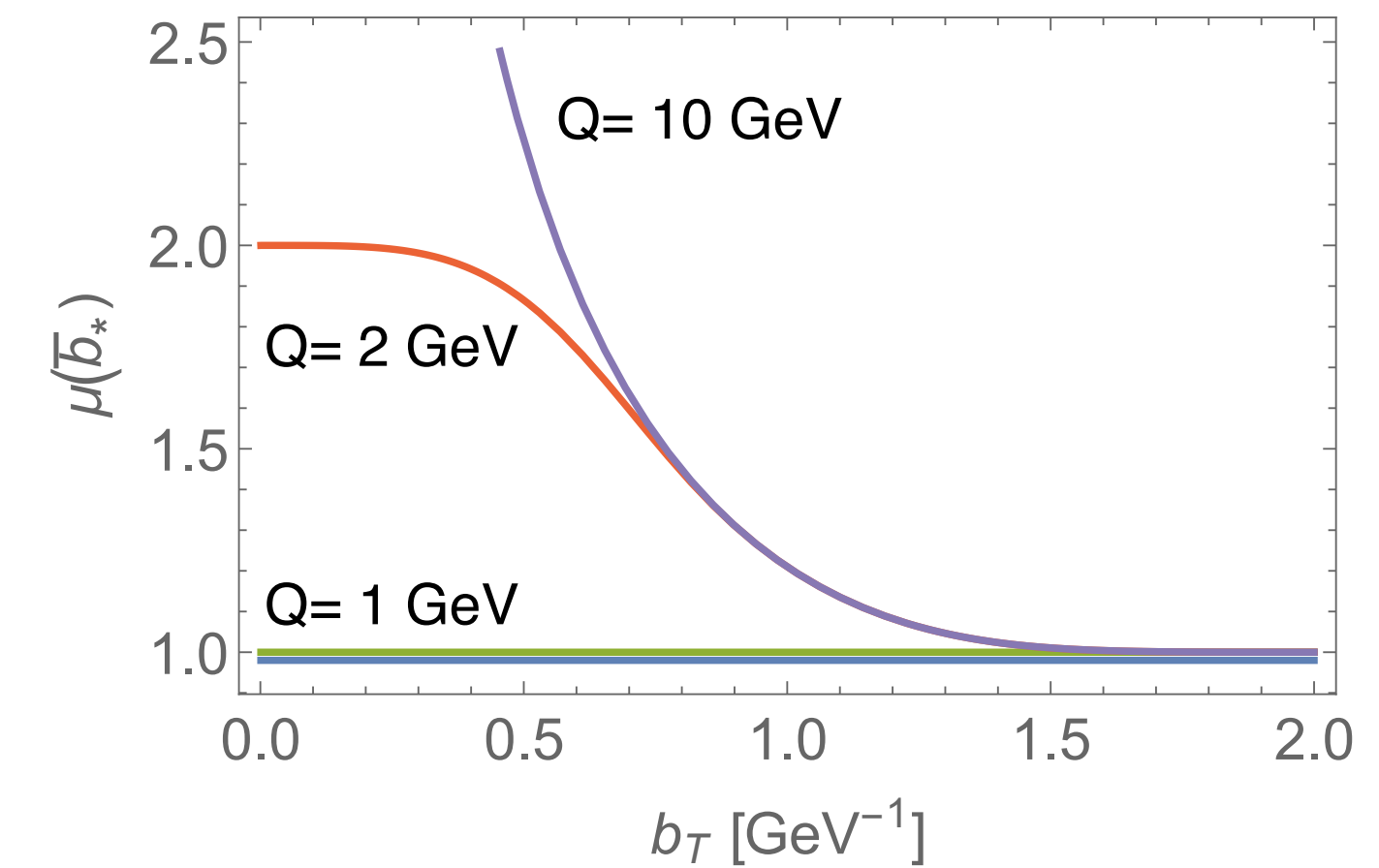
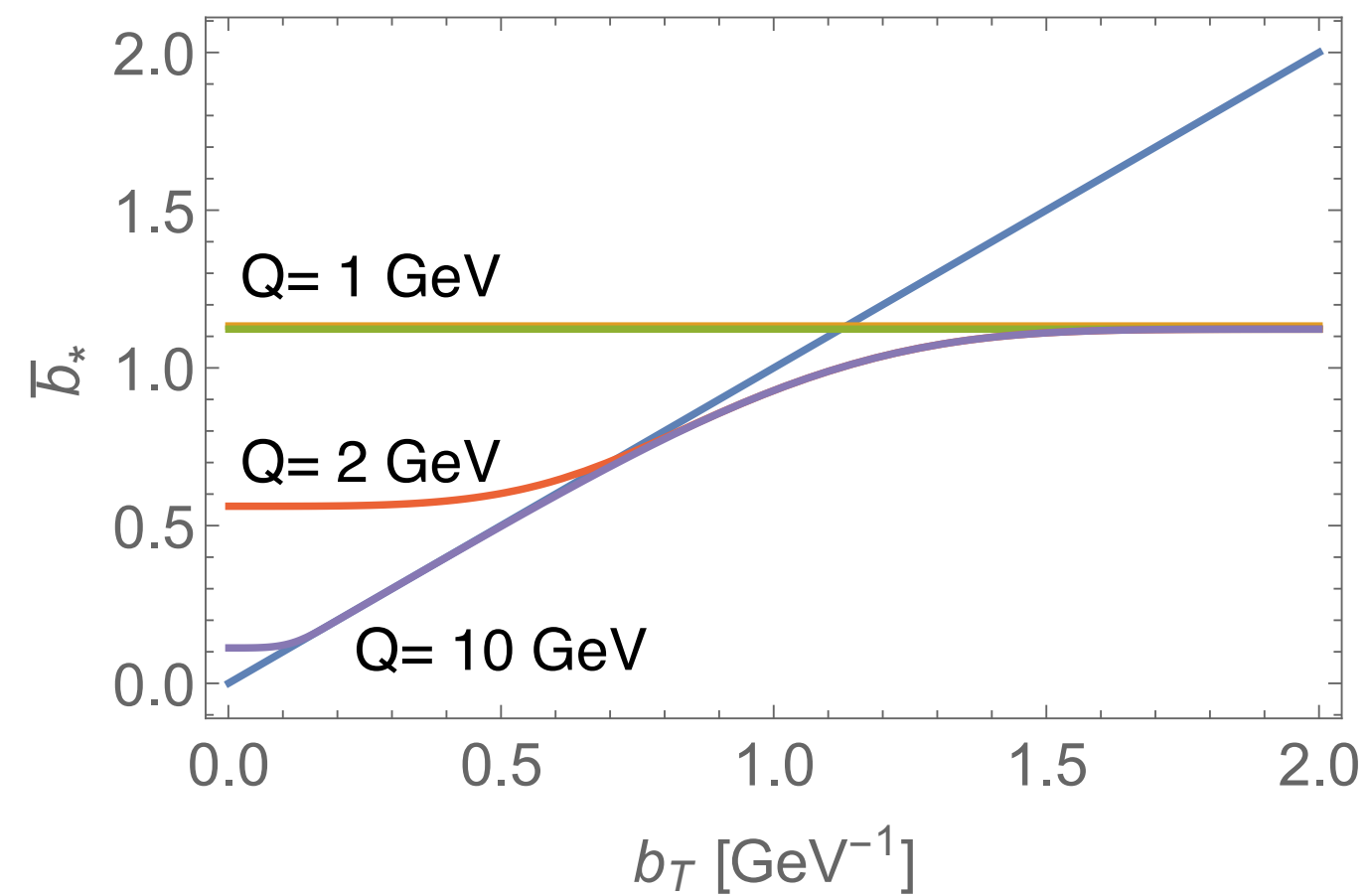


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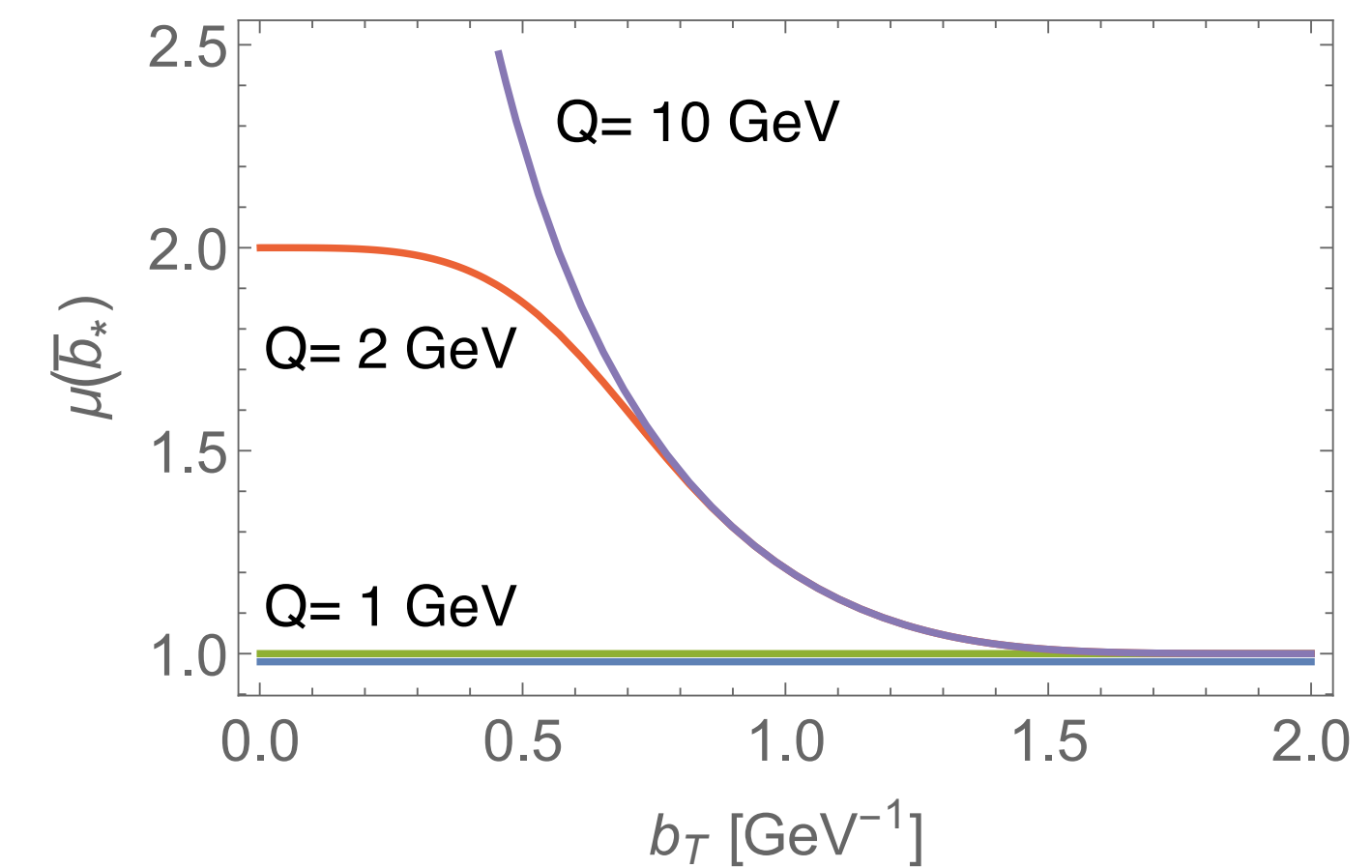
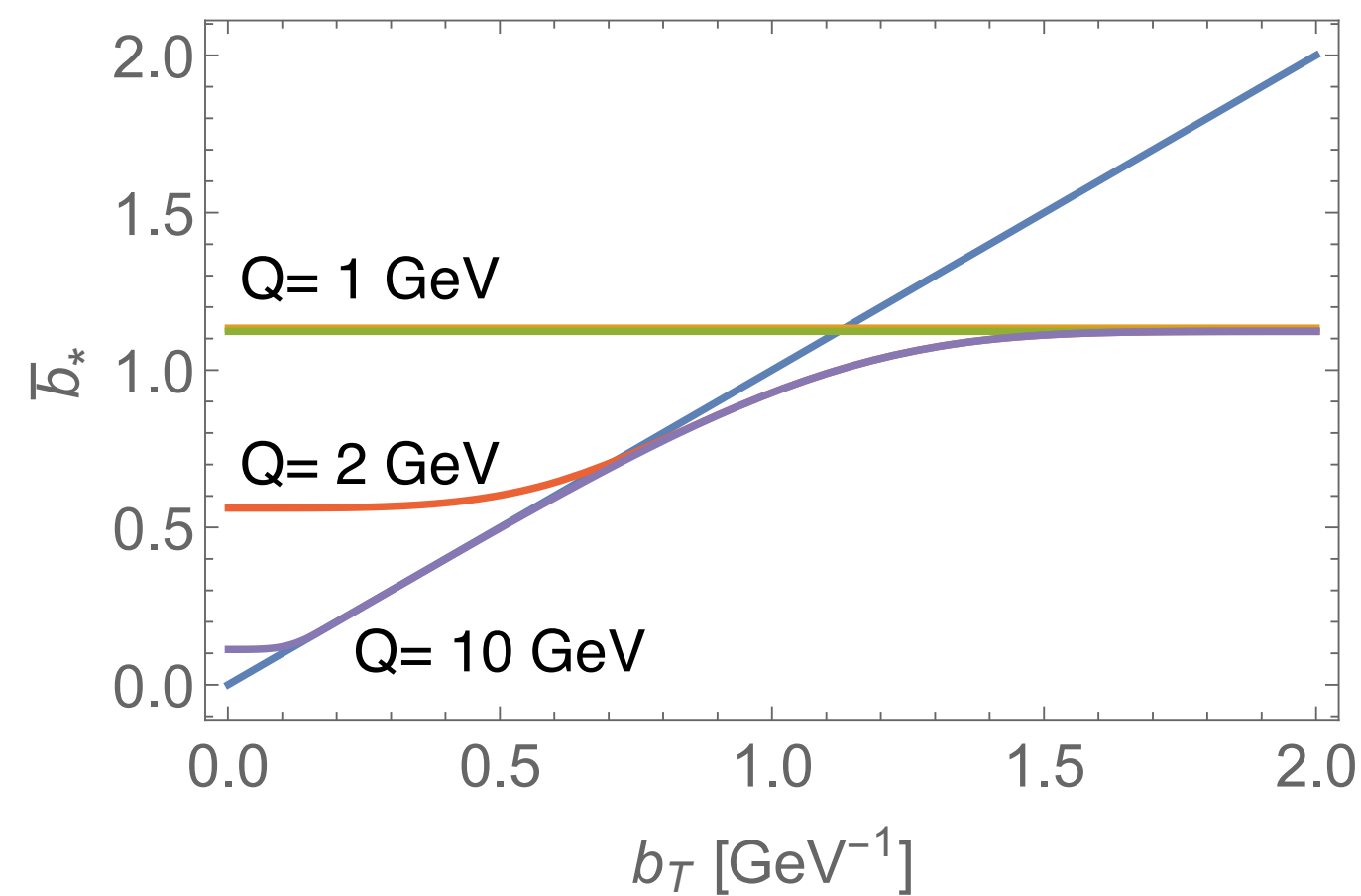


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$$\mu_b = 2e^{-\gamma E} / b_*$$



$\mu_b$  never bigger than  $Q$  nor smaller than 1 GeV



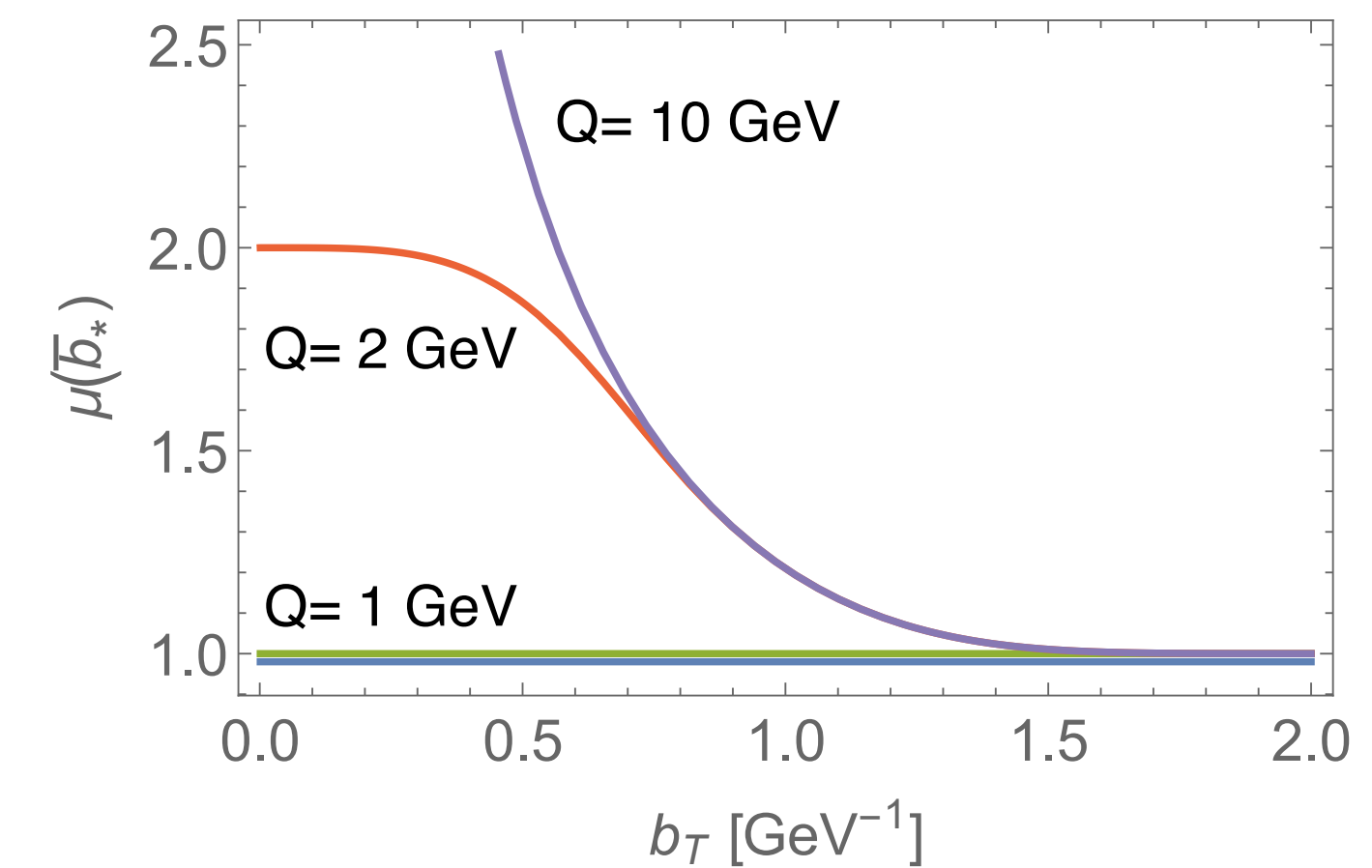
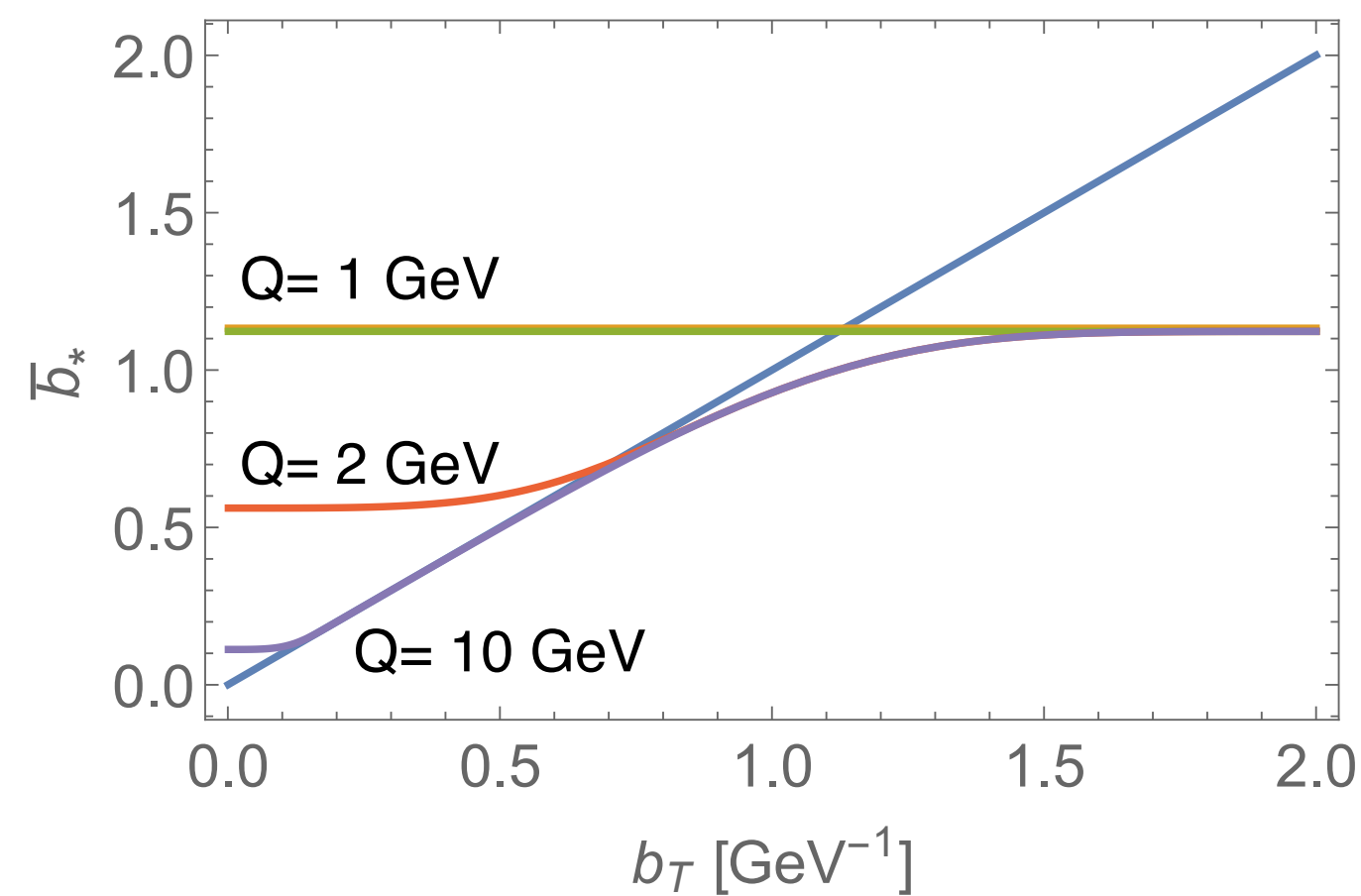
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$$\mu_b = 2e^{-\gamma E} / b_*$$



$\mu_b$  never bigger than  $Q$  nor smaller than 1 GeV

No significant effect at high  $Q$ , but large effect at low  $Q$  (inhibits gluon radiation)

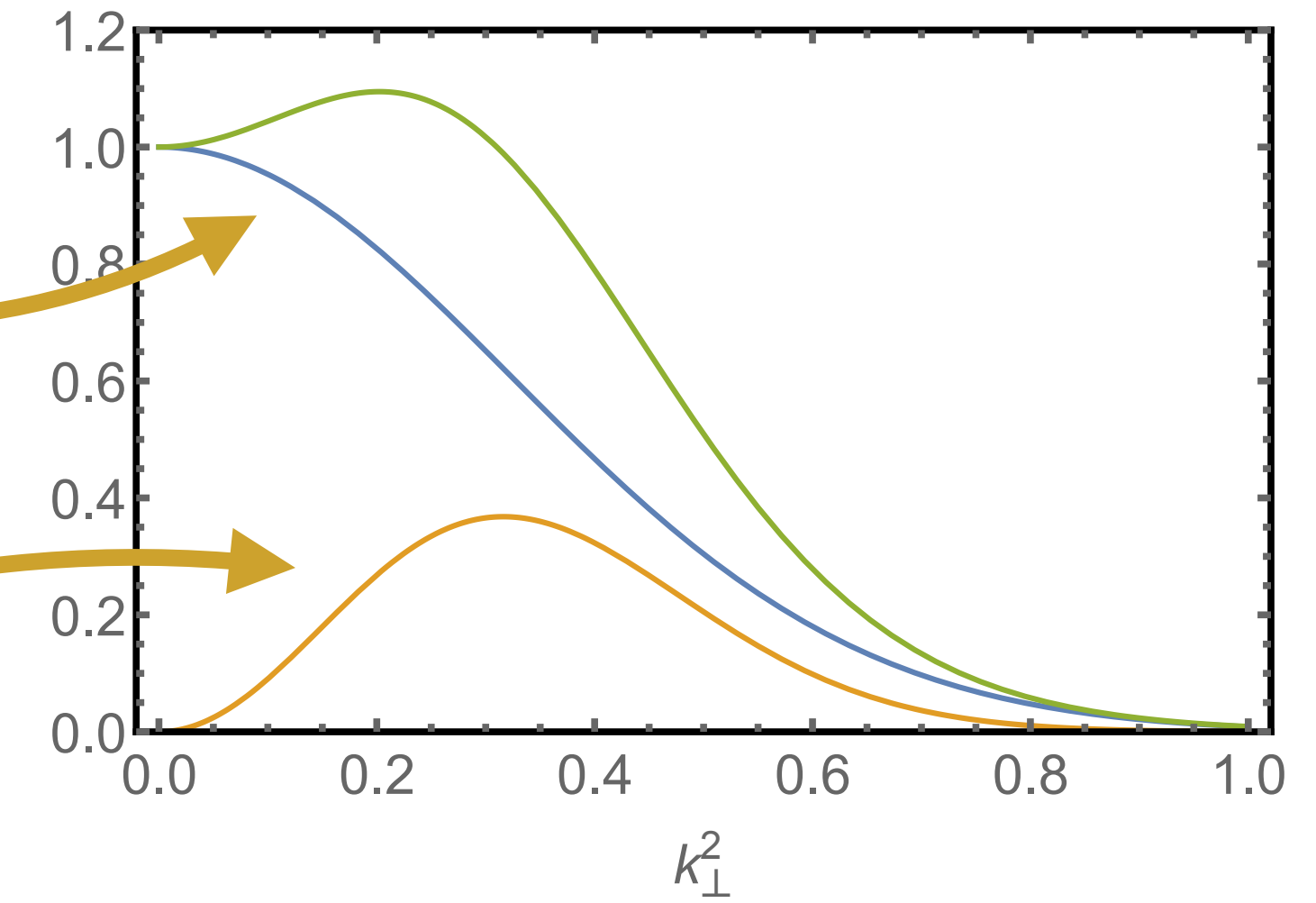
# Functional form of TMDs at 1 GeV

---

$$\hat{f}_{\text{NP}}^a = \text{F.T. of} \left( e^{-\frac{k_{\perp}^2}{\langle k_{\perp,a}^2 \rangle}} + \lambda k_{\perp}^2 e^{-\frac{k_{\perp}^2}{\langle k_{\perp,a}^2 \rangle'}} \right)$$

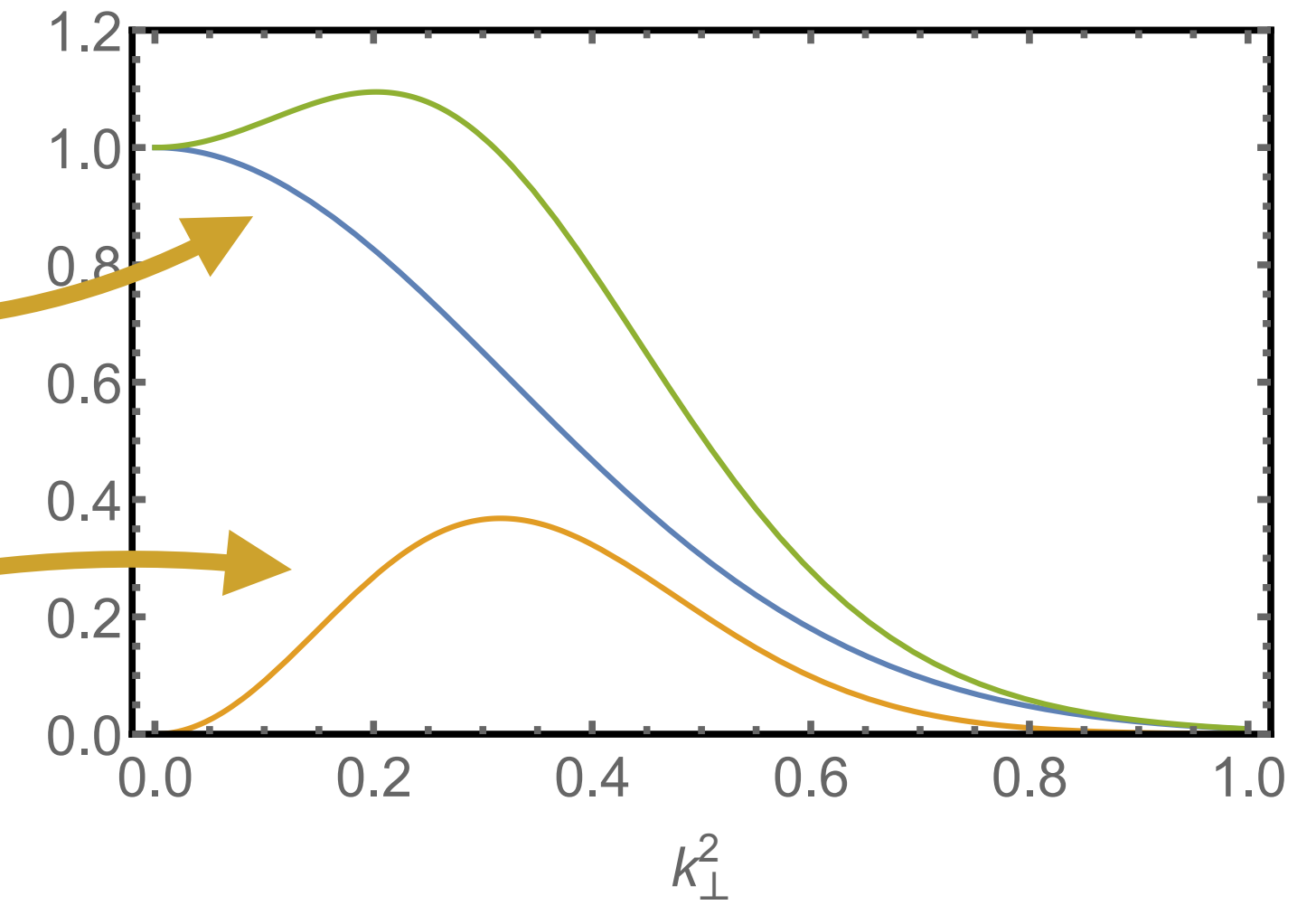
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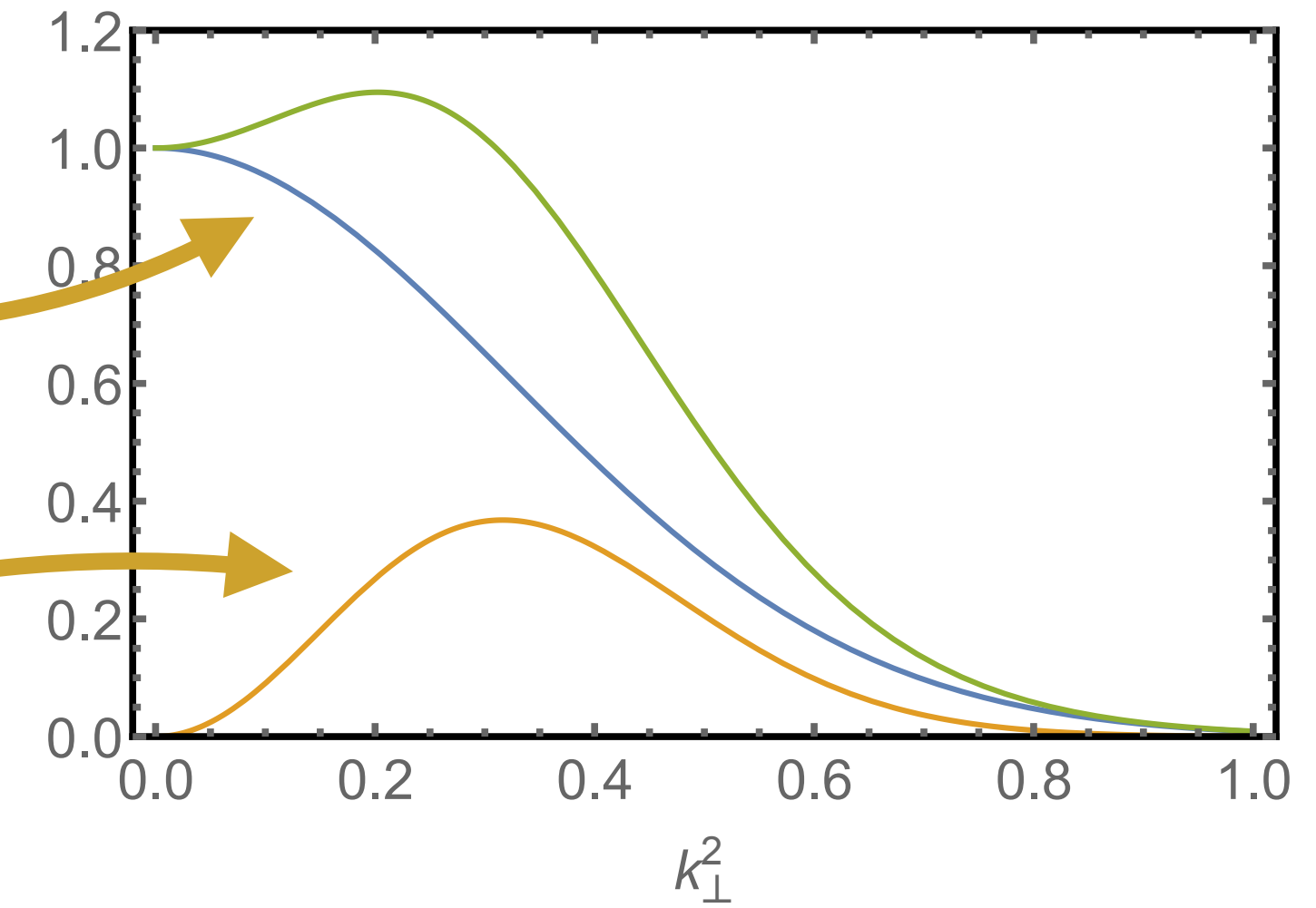


x-dependent width  $\langle \mathbf{k}_{\perp,a}^2 \rangle(x) = \langle \hat{\mathbf{k}}_{\perp,a}^2 \rangle \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$ ,

where  $\langle \hat{\mathbf{k}}_{\perp,a}^2 \rangle \equiv \langle \mathbf{k}_{\perp,a}^2 \rangle(\hat{x})$ , and  $\hat{x} = 0.1$ .

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Fragmentation function is similar

Including TMD PDFs and FFs, in total: 11 free parameters  
(4 for TMD PDFs, 6 for TMD FFs, 1 for TMD evolution)



# Data selection

---

$$Q^2 > 1.4 \text{ GeV}^2$$

$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \text{Min}[0.2 Q, 0.7 Qz] + 0.5 \text{ GeV}$$

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$$0.2 < z < 0.7$$

$$P_{hT}, q_T < \text{Min}[0.2 Q, 0.7 Qz] + 0.5 \text{ GeV}$$

Total number of data points: 8059

Total  $\chi^2/\text{dof} = 1.52$

Preliminary

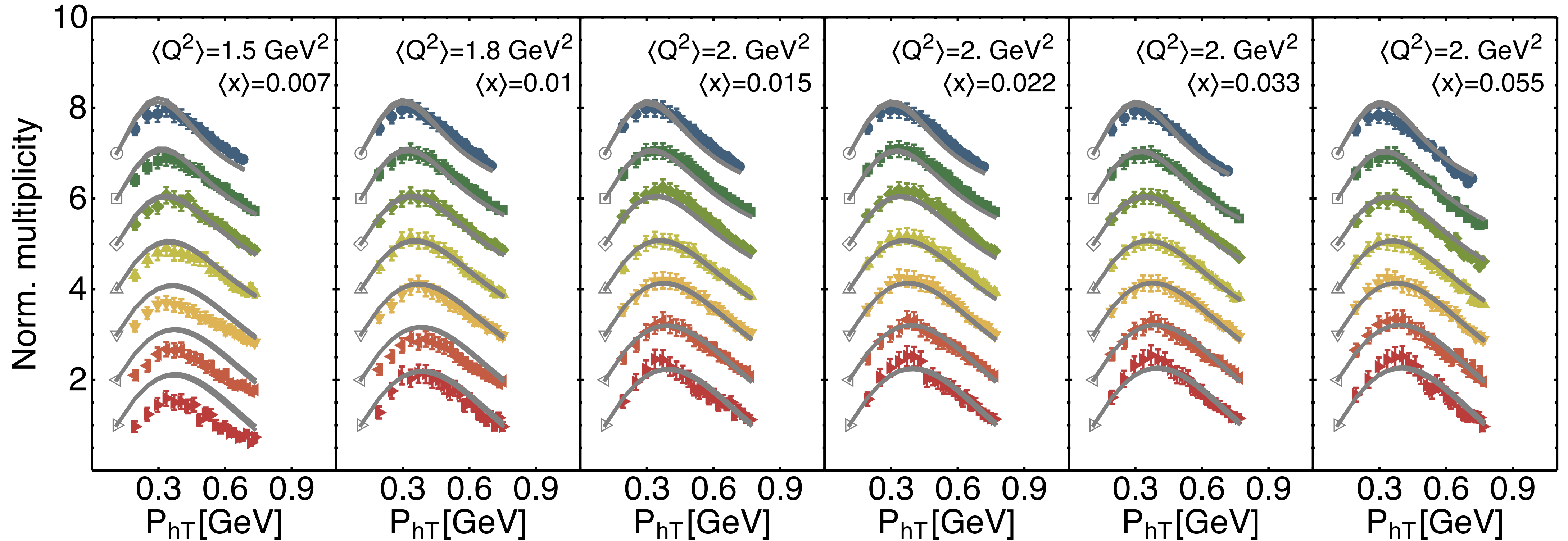
# Data vs. theory plots

---

# COMPASS selected bins



- $\langle z \rangle = 0.23$  (offset=6)
- $\langle z \rangle = 0.28$  (offset=5)
- ◆  $\langle z \rangle = 0.33$  (offset=4)
- ▲  $\langle z \rangle = 0.38$  (offset=3)
- ▼  $\langle z \rangle = 0.45$  (offset=2)
- ▲  $\langle z \rangle = 0.55$  (offset=1)
- ▼  $\langle z \rangle = 0.65$  (offset=0)

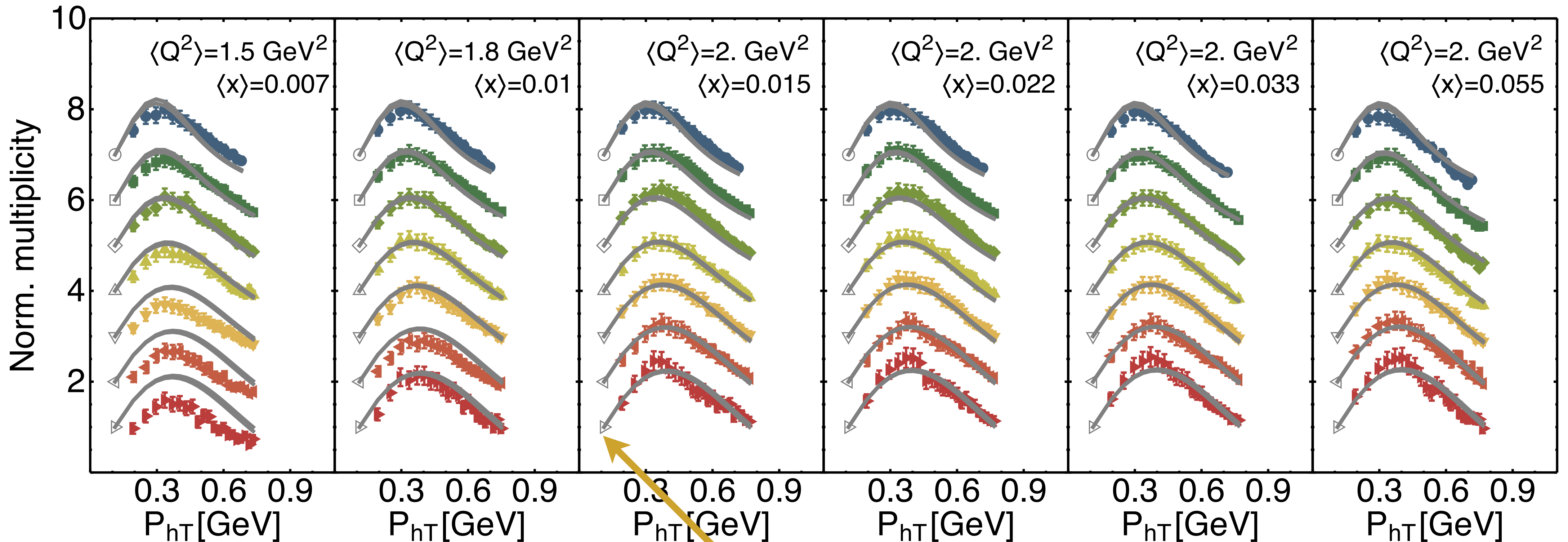


Deuteron h-  $\chi^2/\text{dof} = 1.58$

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Deuteron h-

$$\chi^2/\text{dof} = 1.58$$

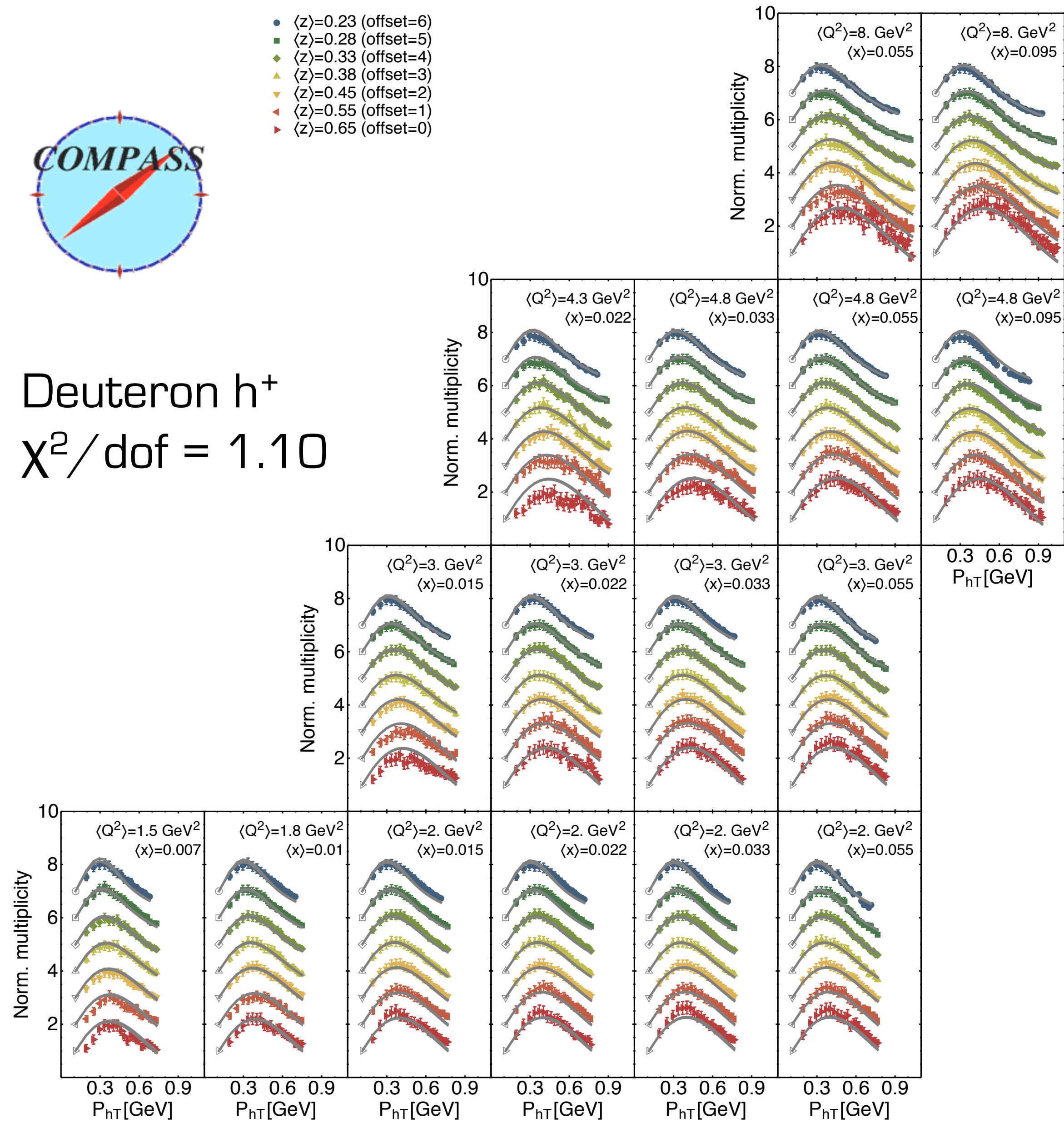
First points are not fitted, but used as normalization



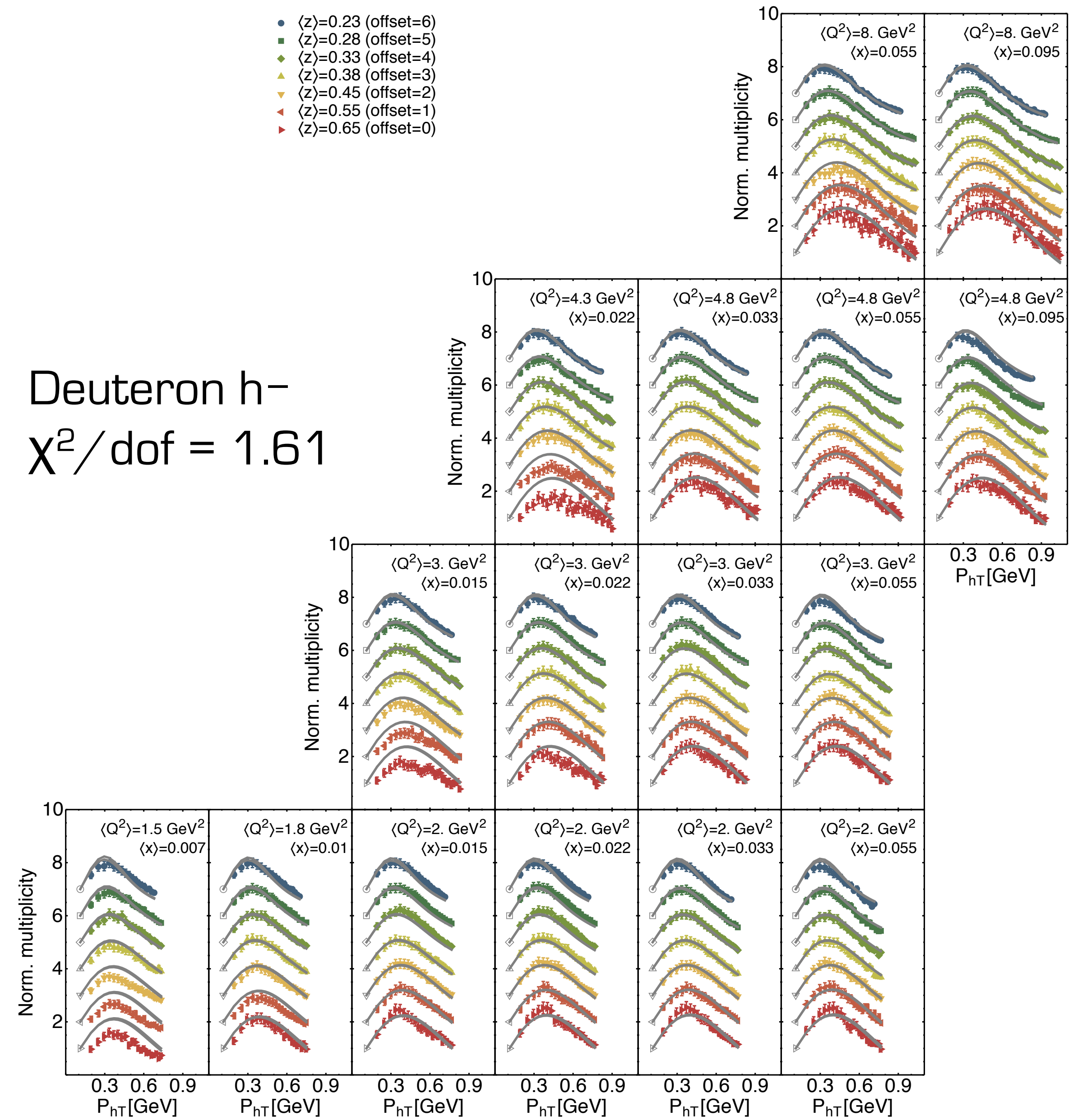


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- ▼  $\langle z \rangle = 0.65$  (offset=0)

Deuteron  $h^+$   
 $\chi^2/\text{dof} = 1.10$



Deuteron  $h^-$   
 $\chi^2/\text{dof} = 1.61$

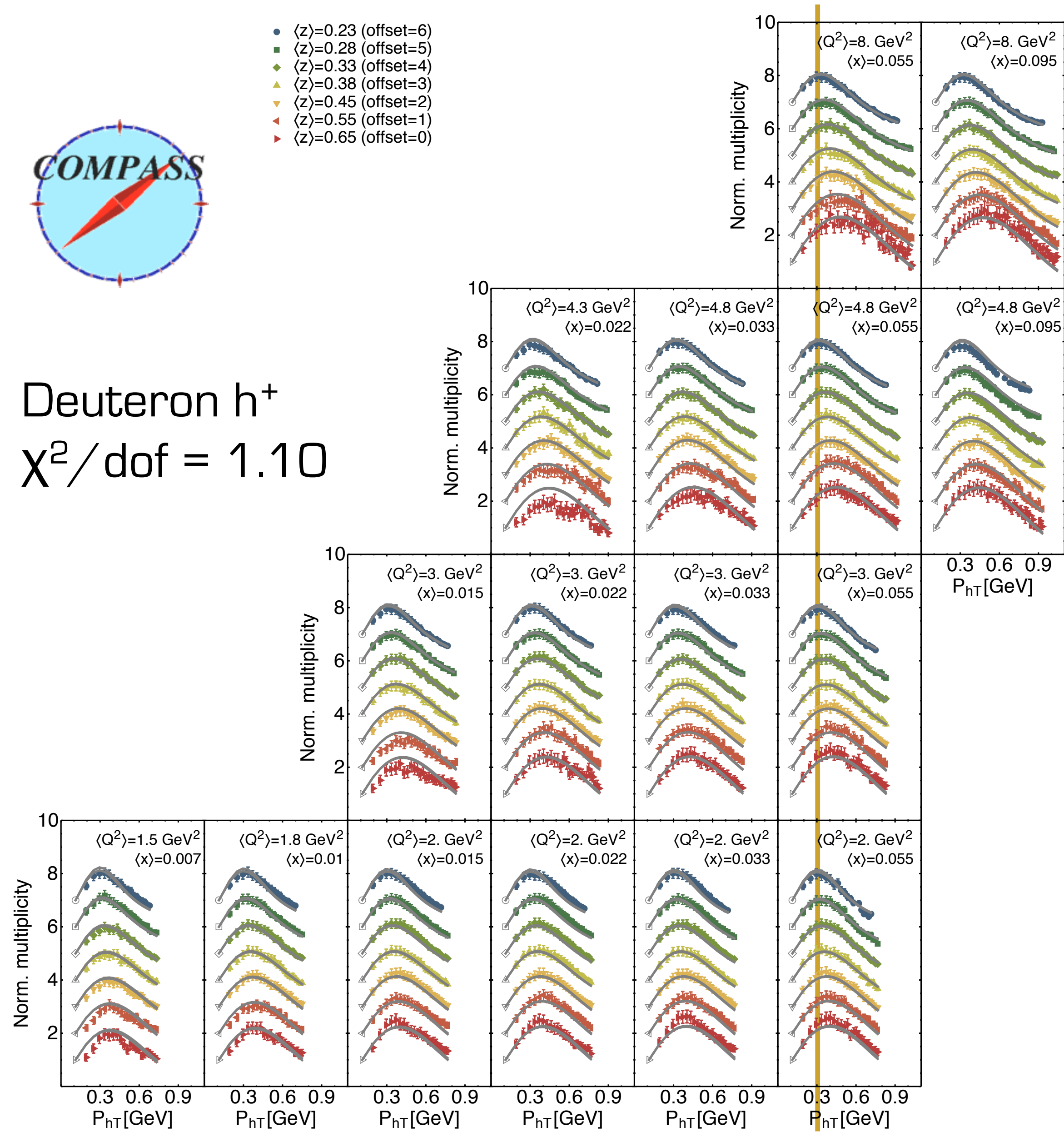






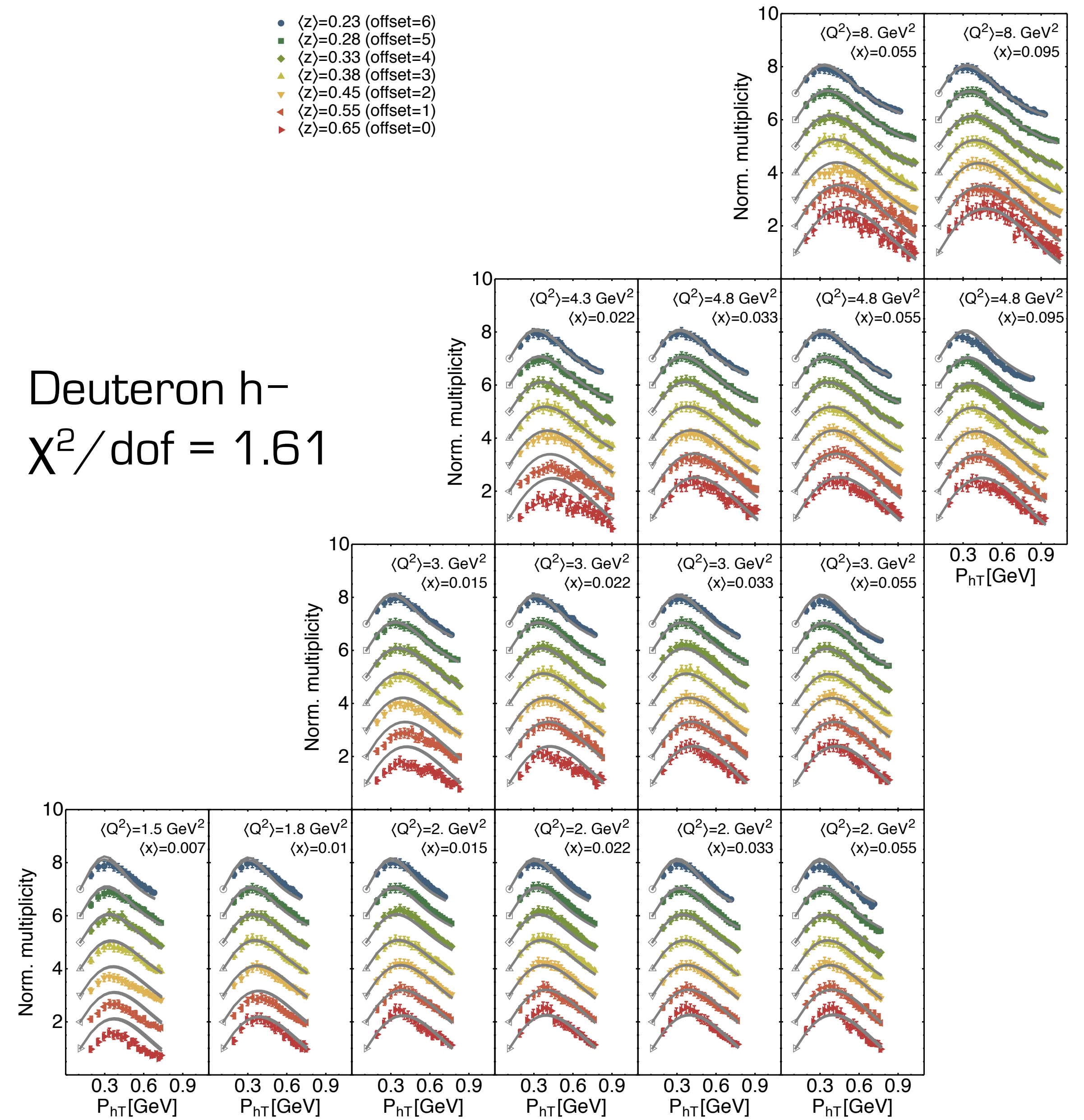
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- ▲  $\langle z \rangle = 0.55$  (offset=1)
- ▼  $\langle z \rangle = 0.65$  (offset=0)

Deuteron  $h^+$   
 $\chi^2/\text{dof} = 1.10$

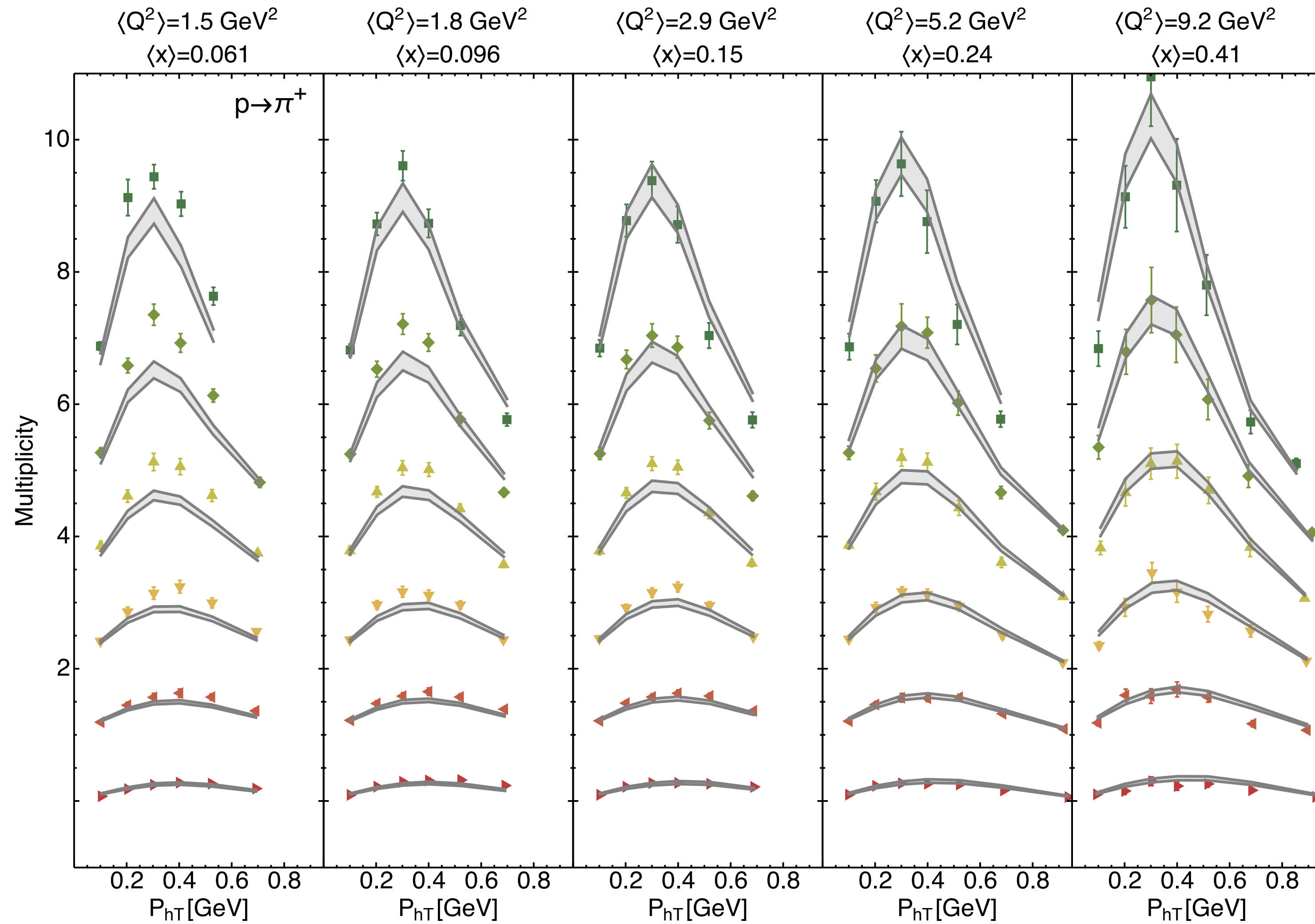


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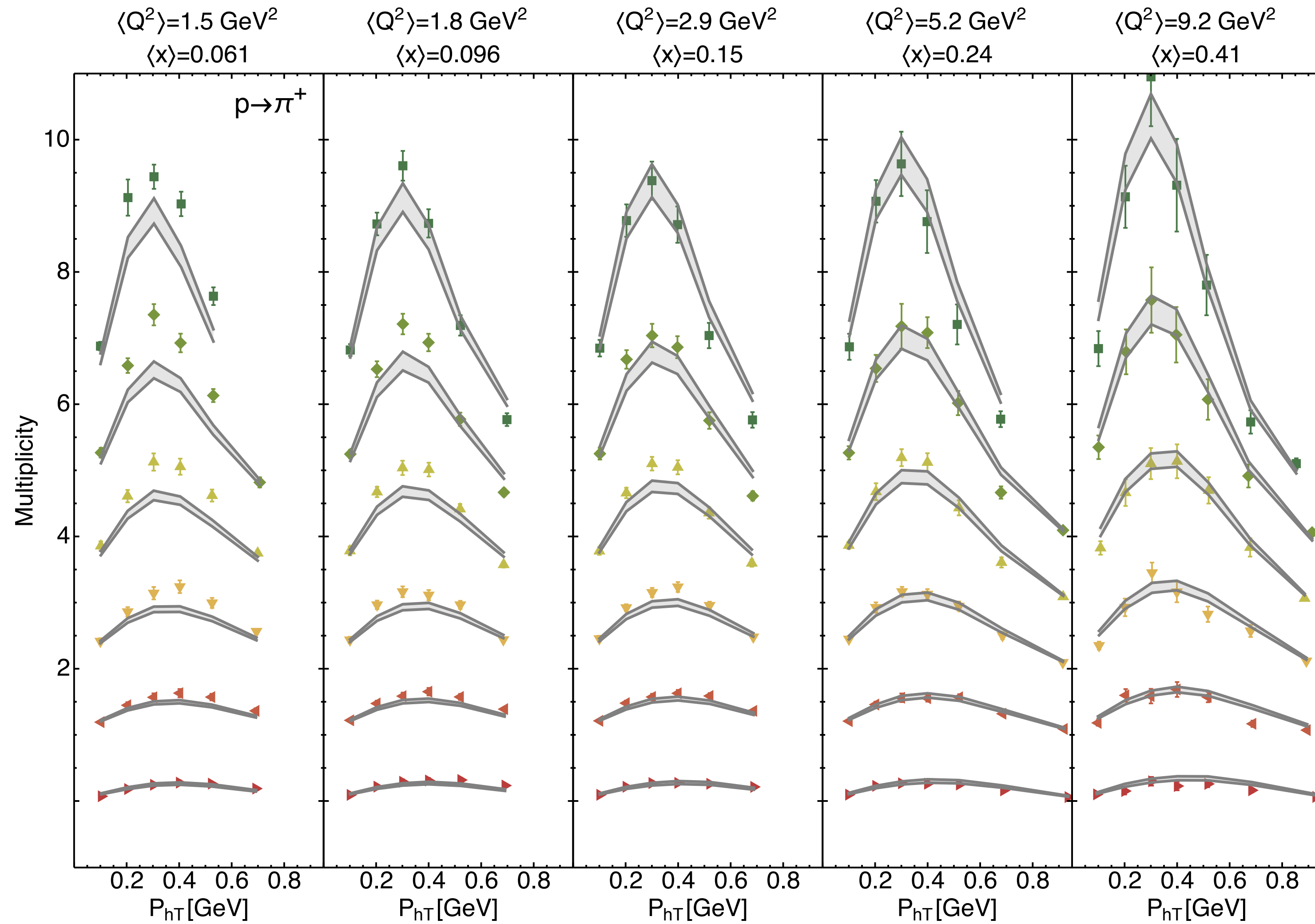
Deuteron  $h^-$   
 $\chi^2/\text{dof} = 1.61$



# HERMES, selected bins



# HERMES, selected bins

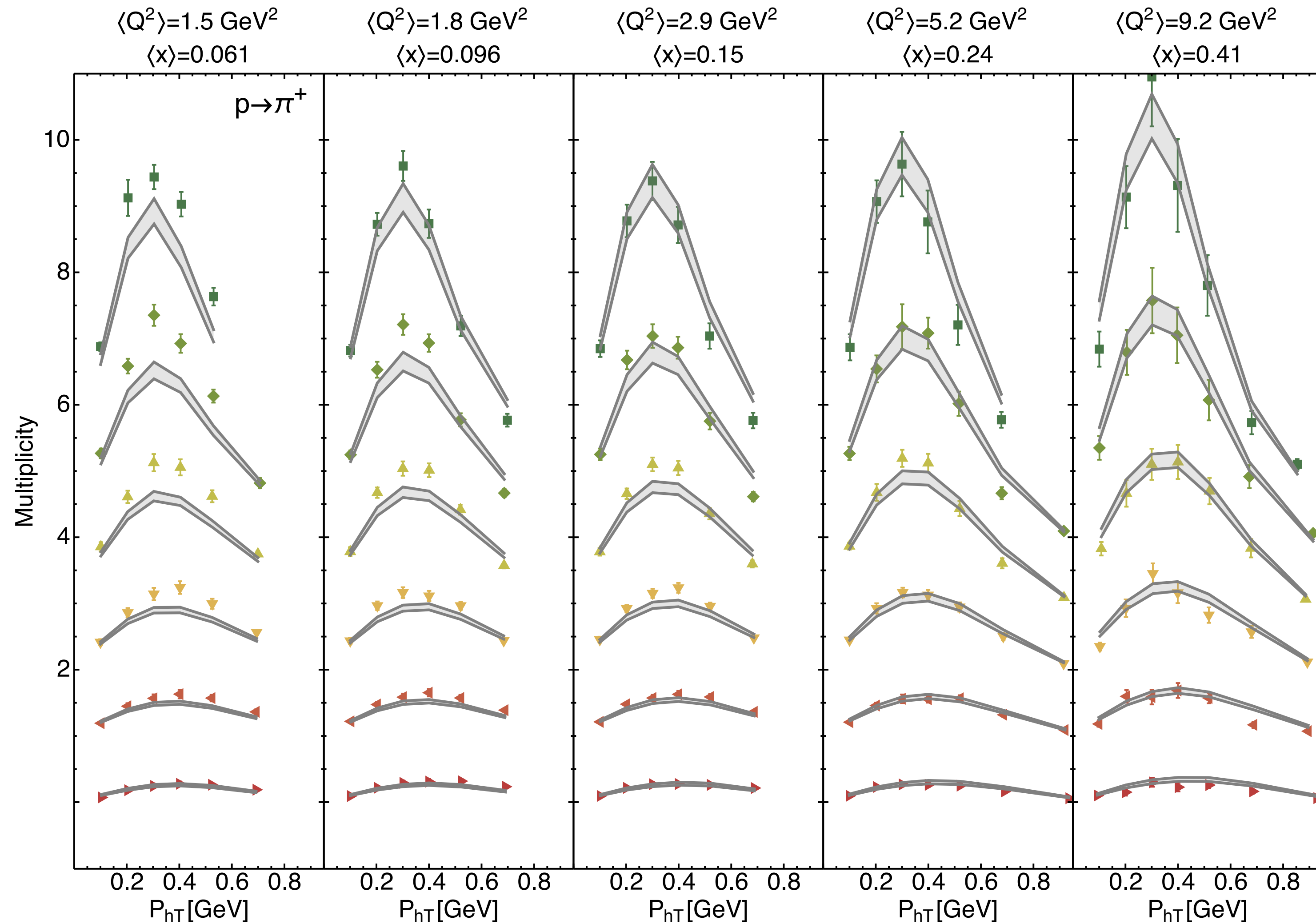


$\chi^2/\text{dof} = 4.80$

The worst of all channels...



# HERMES, selected bins

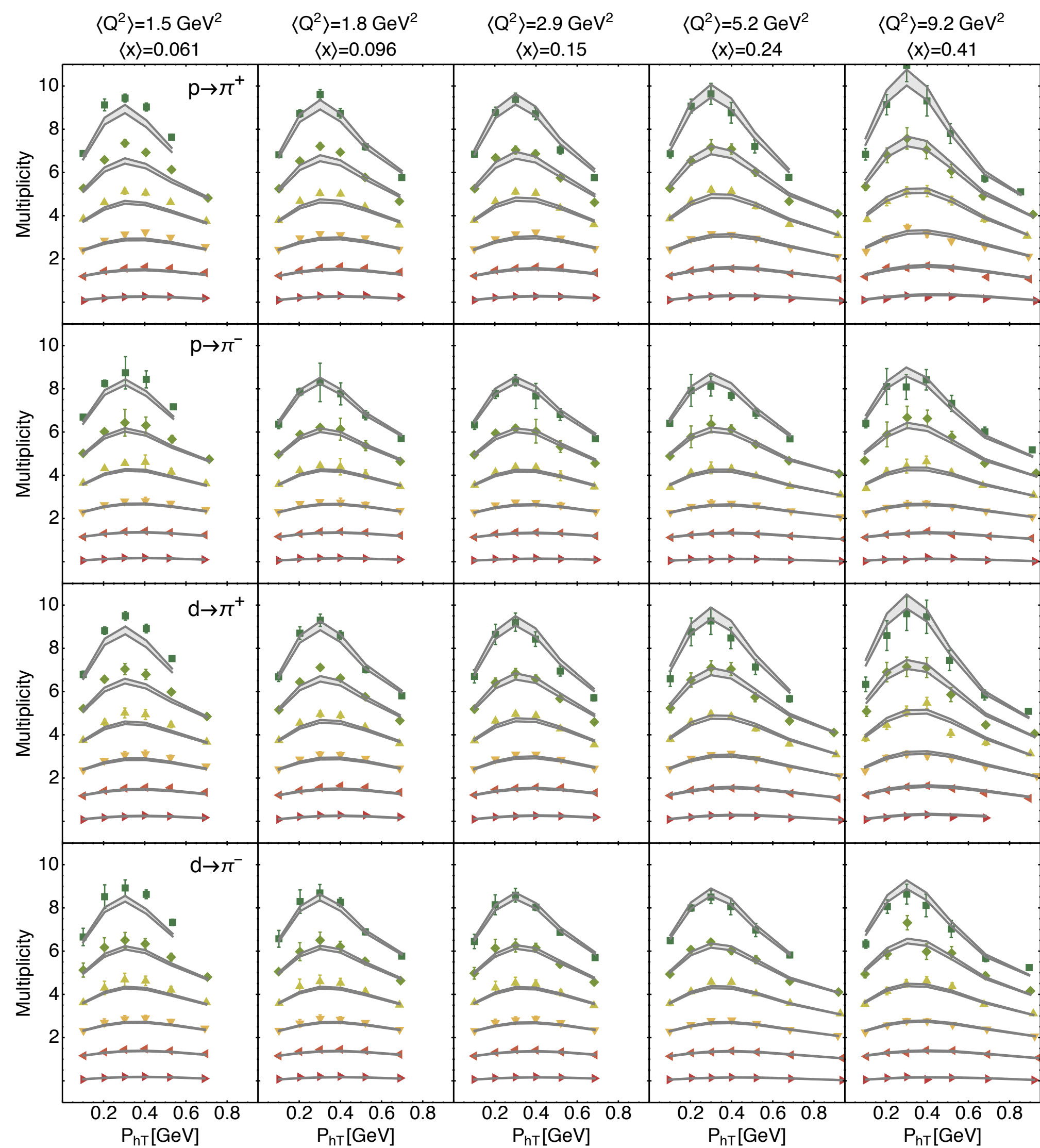
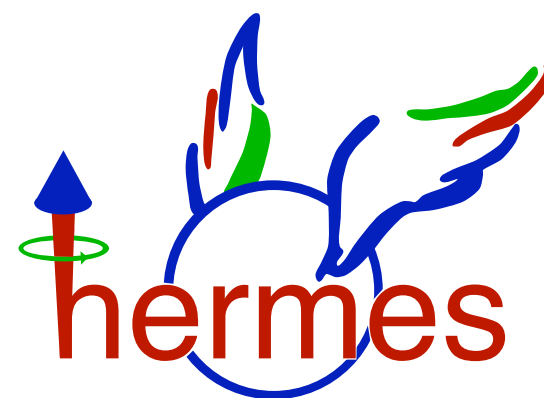


$\chi^2/\text{dof} = 4.80$

The worst of all channels...

However normalizing the theory curves to the first bin, without changing the parameters of the fit,  $\chi^2/\text{dof}$  becomes good





$\chi^2/\text{dof}$

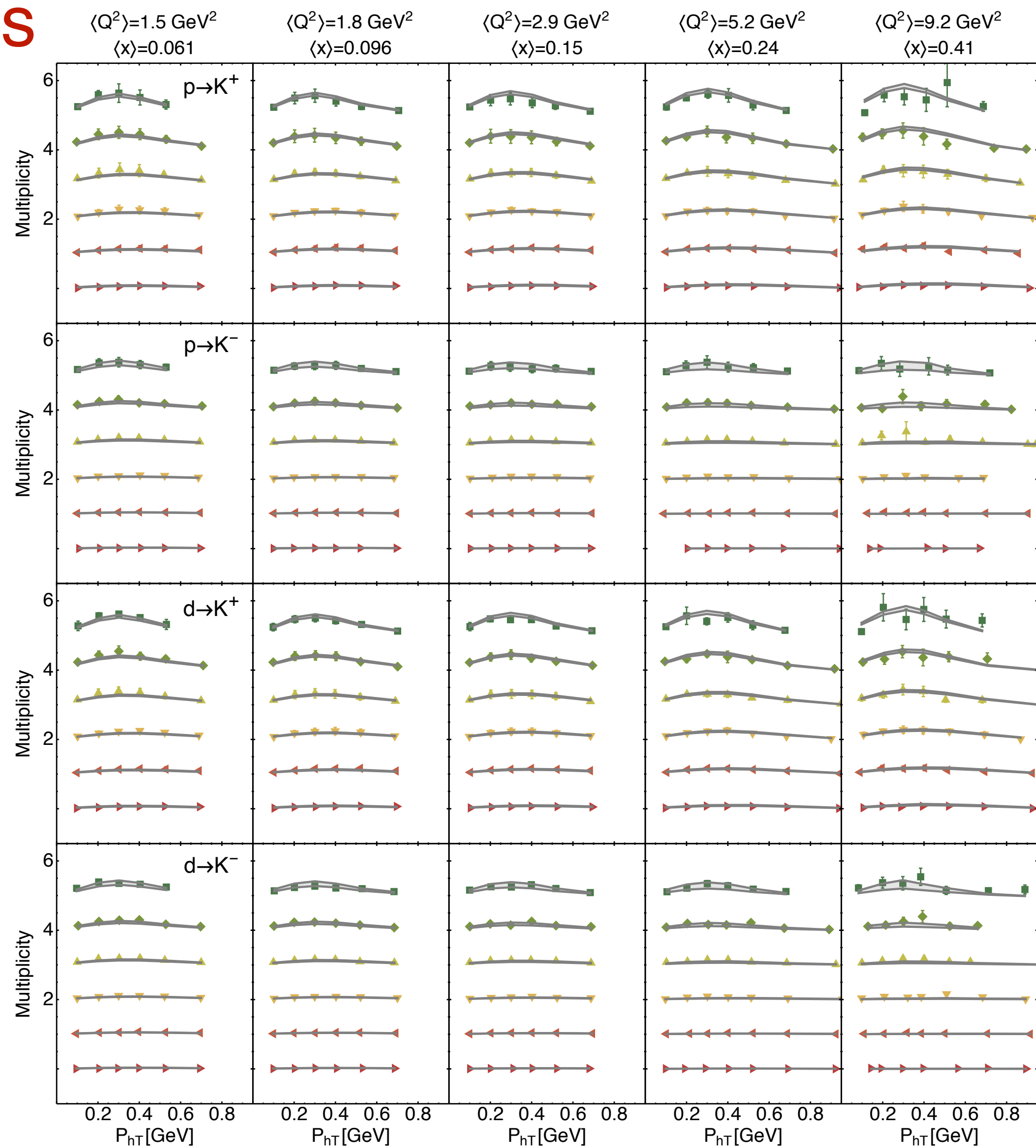
4.8

2.5

3.5

2.0

- $\langle z \rangle = 0.24$  (offset=5)
- ◆  $\langle z \rangle = 0.28$  (offset=4)
- ▲  $\langle z \rangle = 0.34$  (offset=3)
- ▼  $\langle z \rangle = 0.43$  (offset=2)
- ◀  $\langle z \rangle = 0.54$  (offset=1)
- ▶  $\langle z \rangle = 0.70$  (offset=0)



$\chi^2/\text{dof}$

0.9

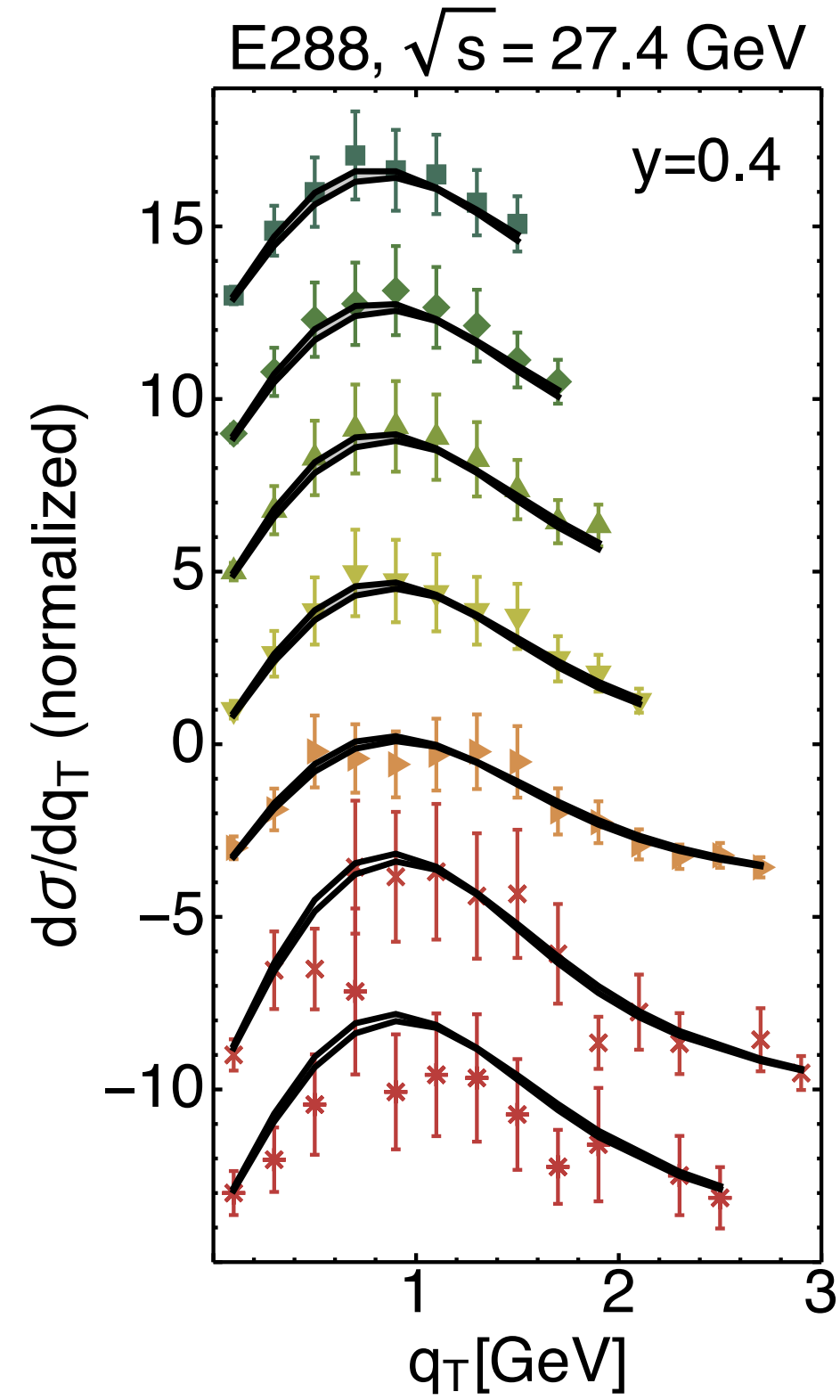
0.8

1.3

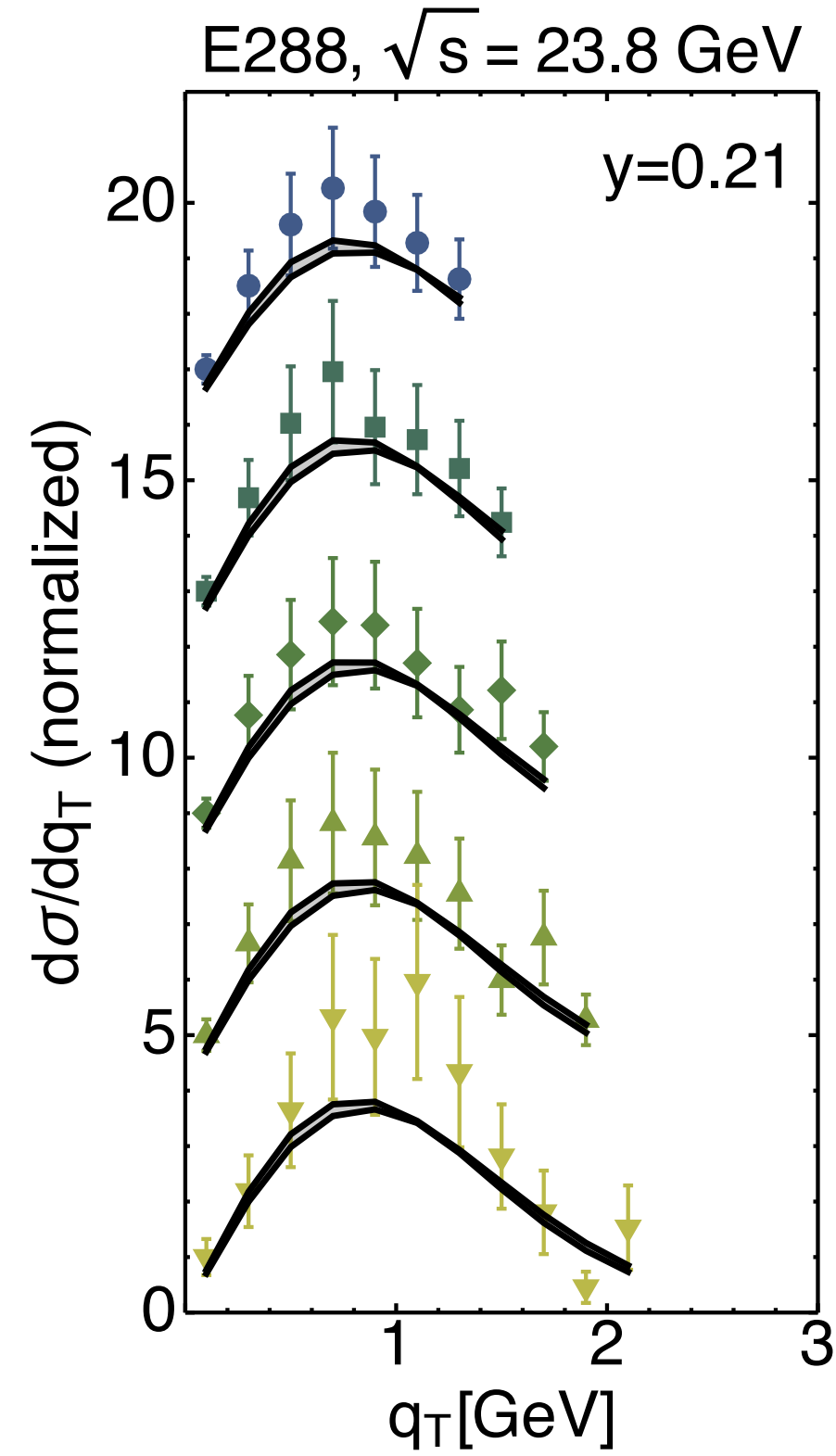
2.5

# Drell-Yan data

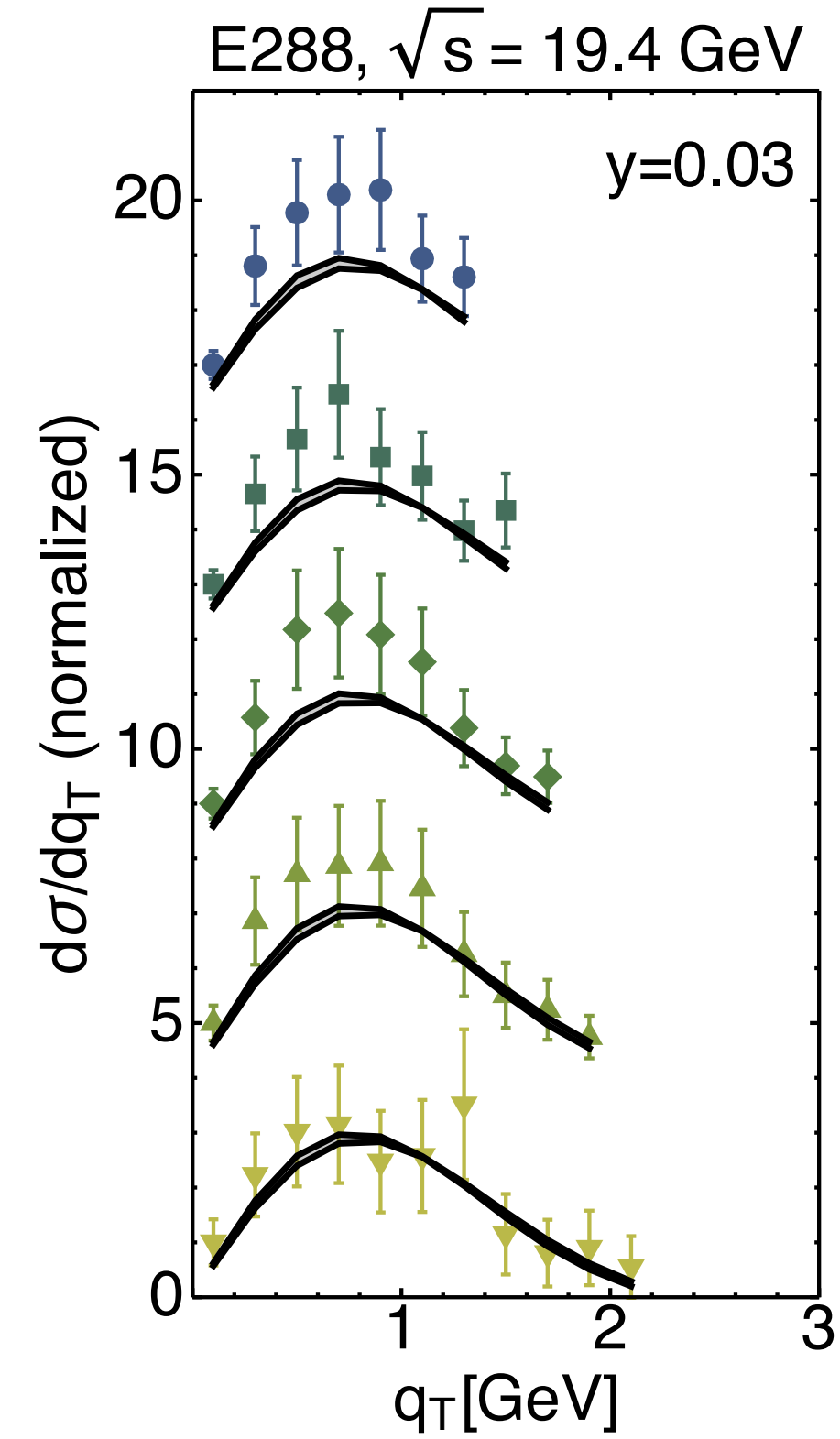
- $\langle Q \rangle = 4.5$  GeV (offset = 16)
- $\langle Q \rangle = 5.5$  GeV (offset = 12)
- ◆  $\langle Q \rangle = 6.5$  GeV (offset = 8)
- ▲  $\langle Q \rangle = 7.5$  GeV (offset = 4)
- ▼  $\langle Q \rangle = 8.5$  GeV (offset = 0)
- ▲  $\langle Q \rangle = 11.0$  GeV (offset = -4)
- ▶  $\langle Q \rangle = 11.5$  GeV (offset = -4)
- ×  $\langle Q \rangle = 12.5$  GeV (offset = -10)
- \*  $\langle Q \rangle = 13.5$  GeV (offset = -14)



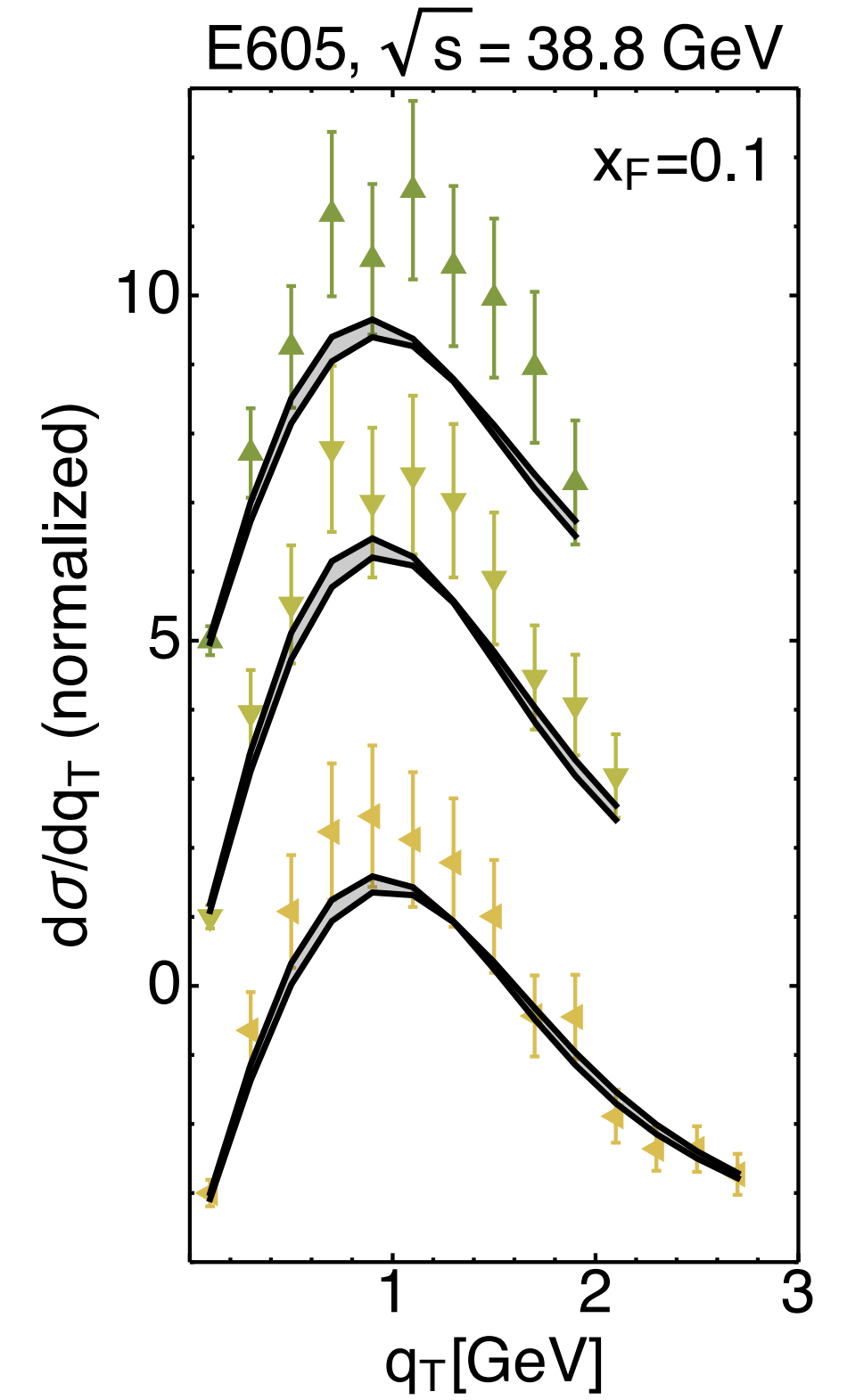
$\chi^2/\text{dof}$       0.32



0.84



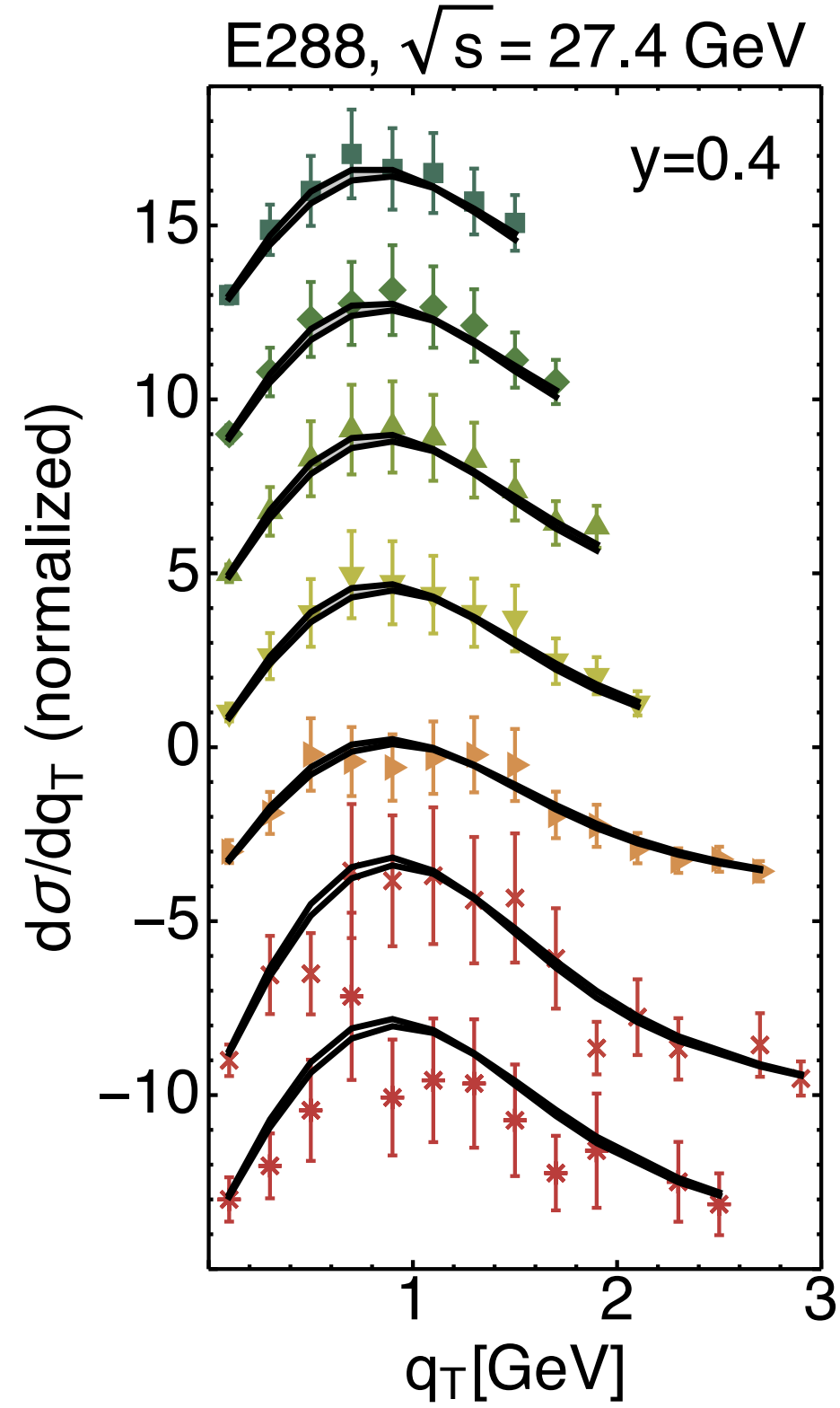
0.99



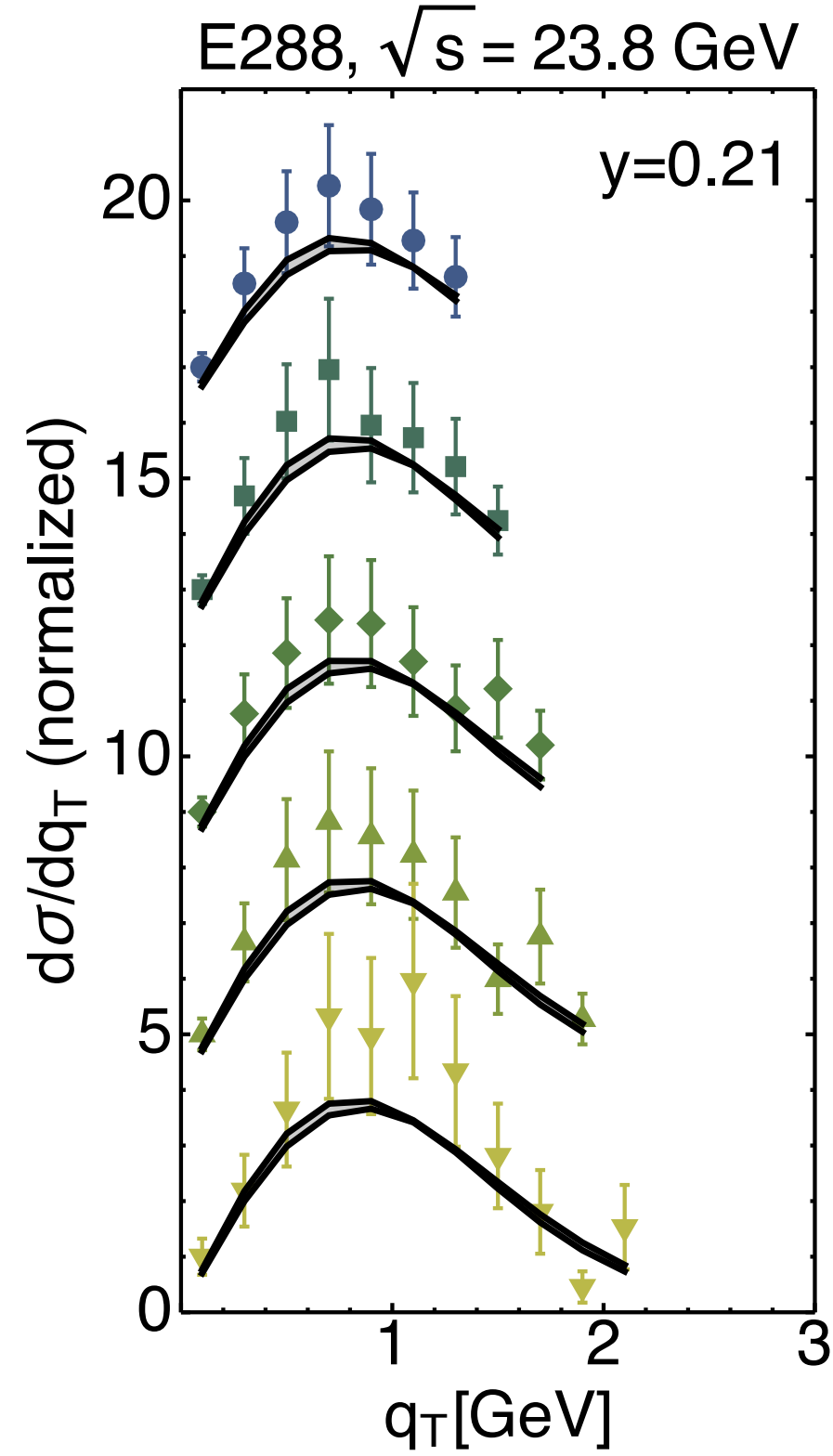
1.13

# Drell-Yan data

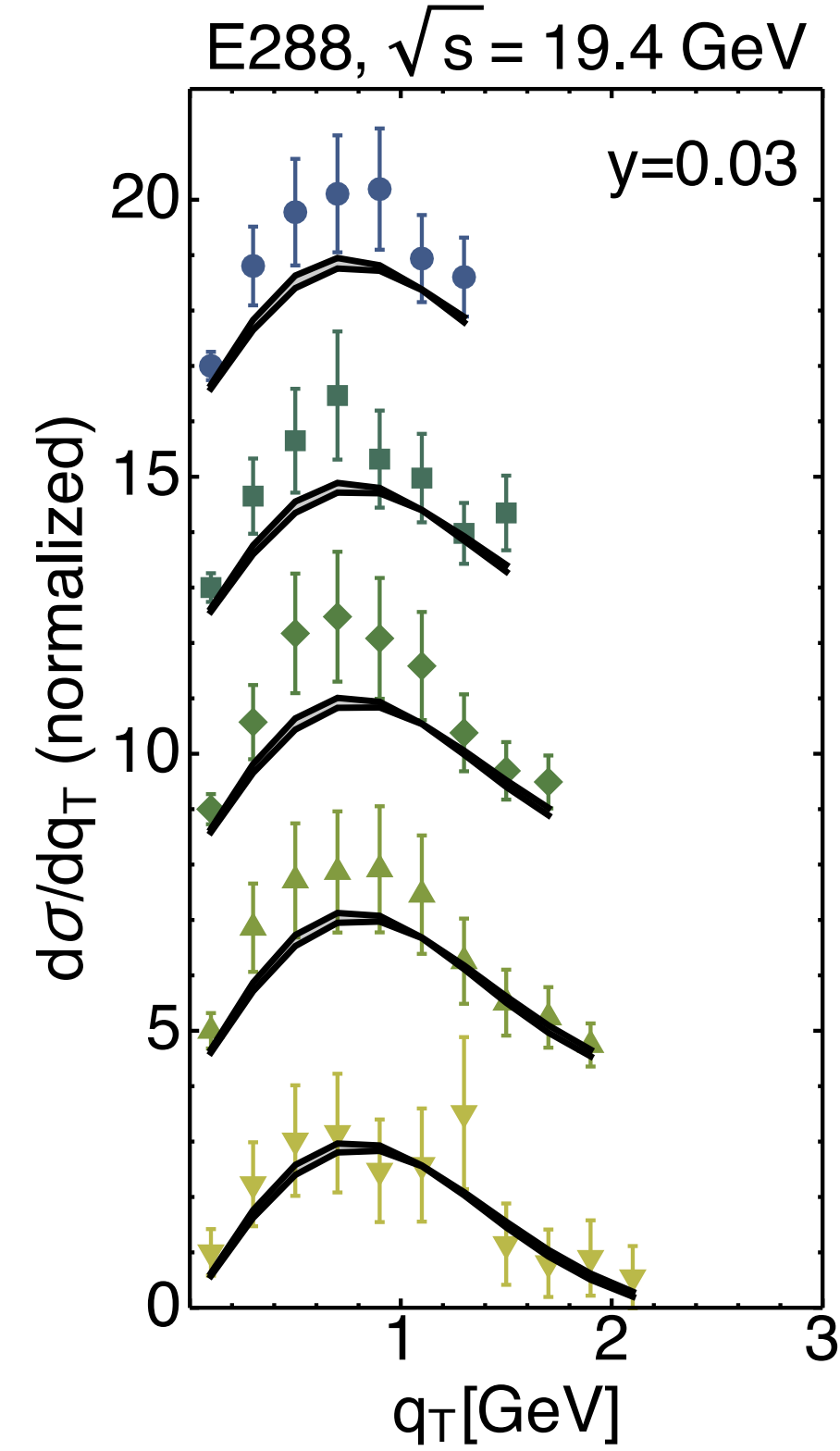
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- $\langle Q \rangle = 5.5$  GeV (offset = 12)
- ◆  $\langle Q \rangle = 6.5$  GeV (offset = 8)
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- \*  $\langle Q \rangle = 13.5$  GeV (offset = -14)



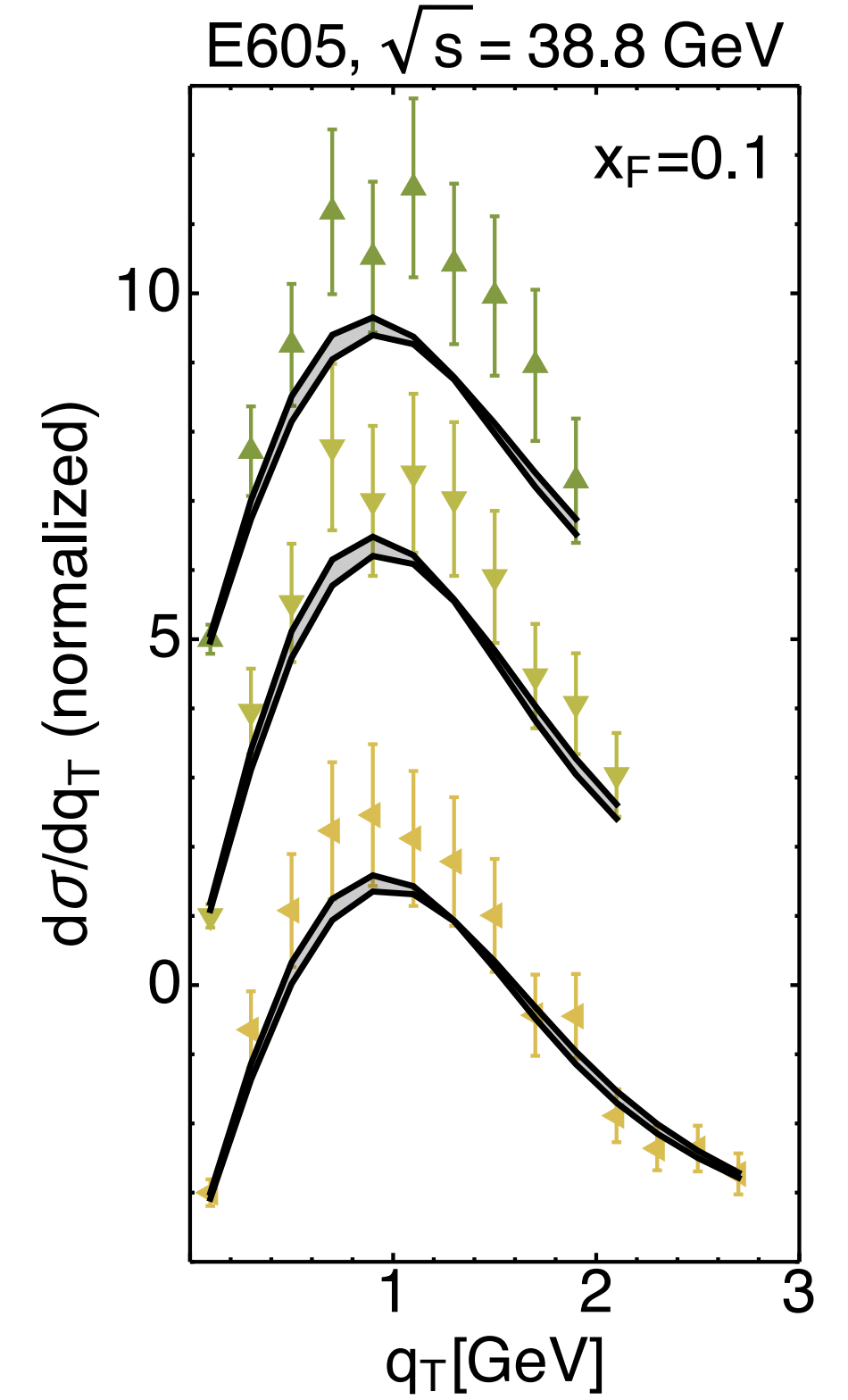
$\chi^2/\text{dof}$  0.32



0.84



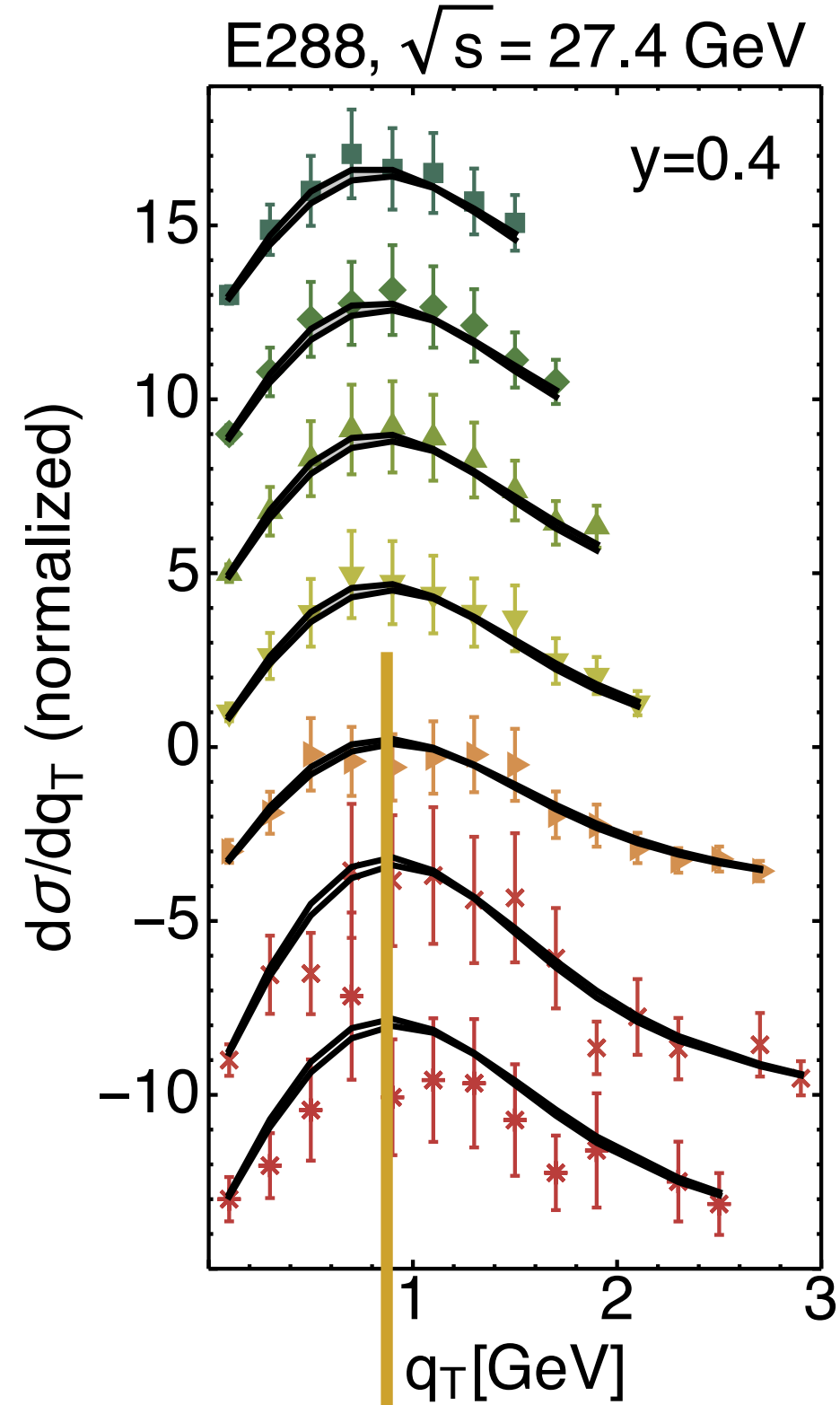
0.99



1.13

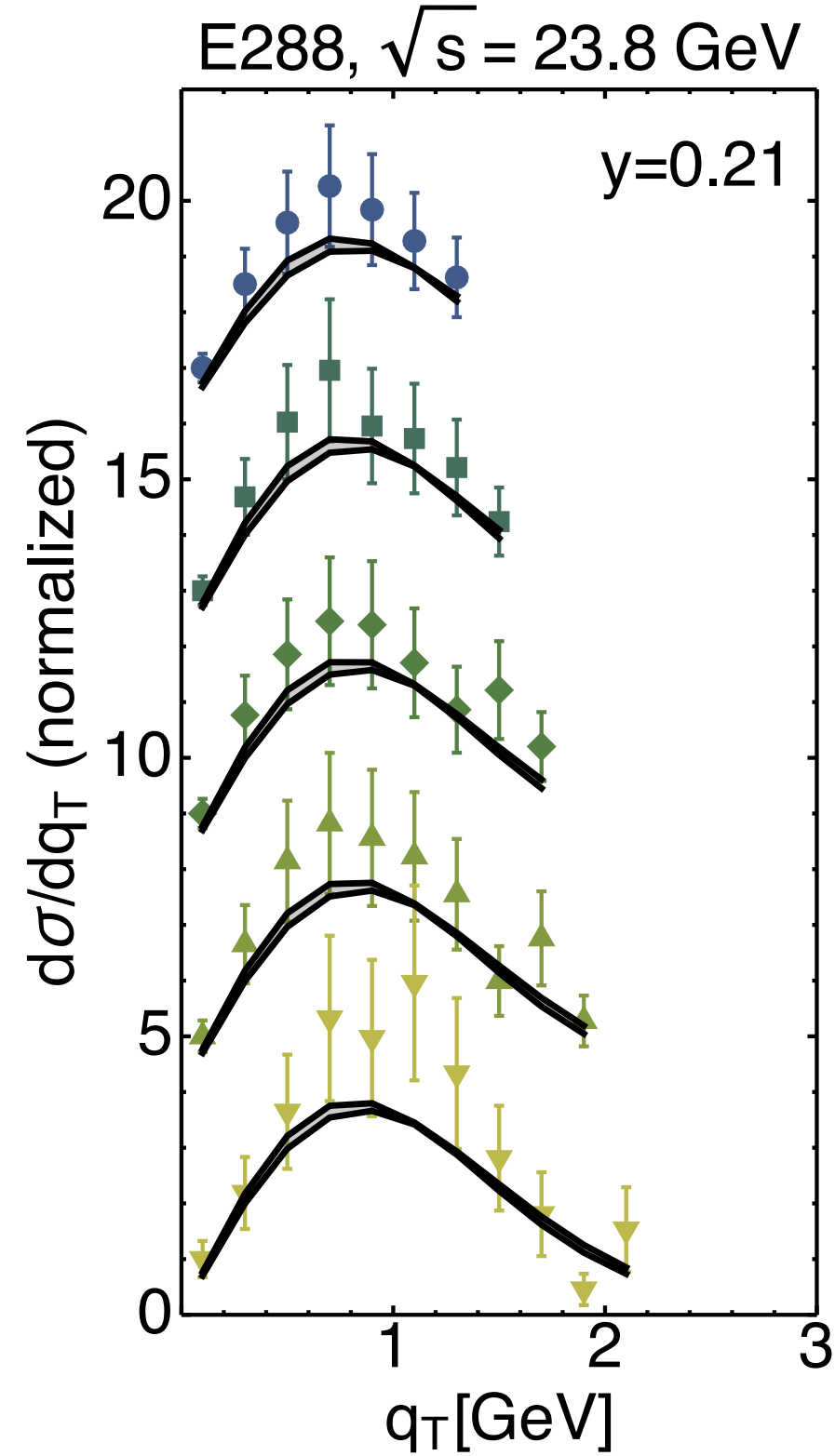
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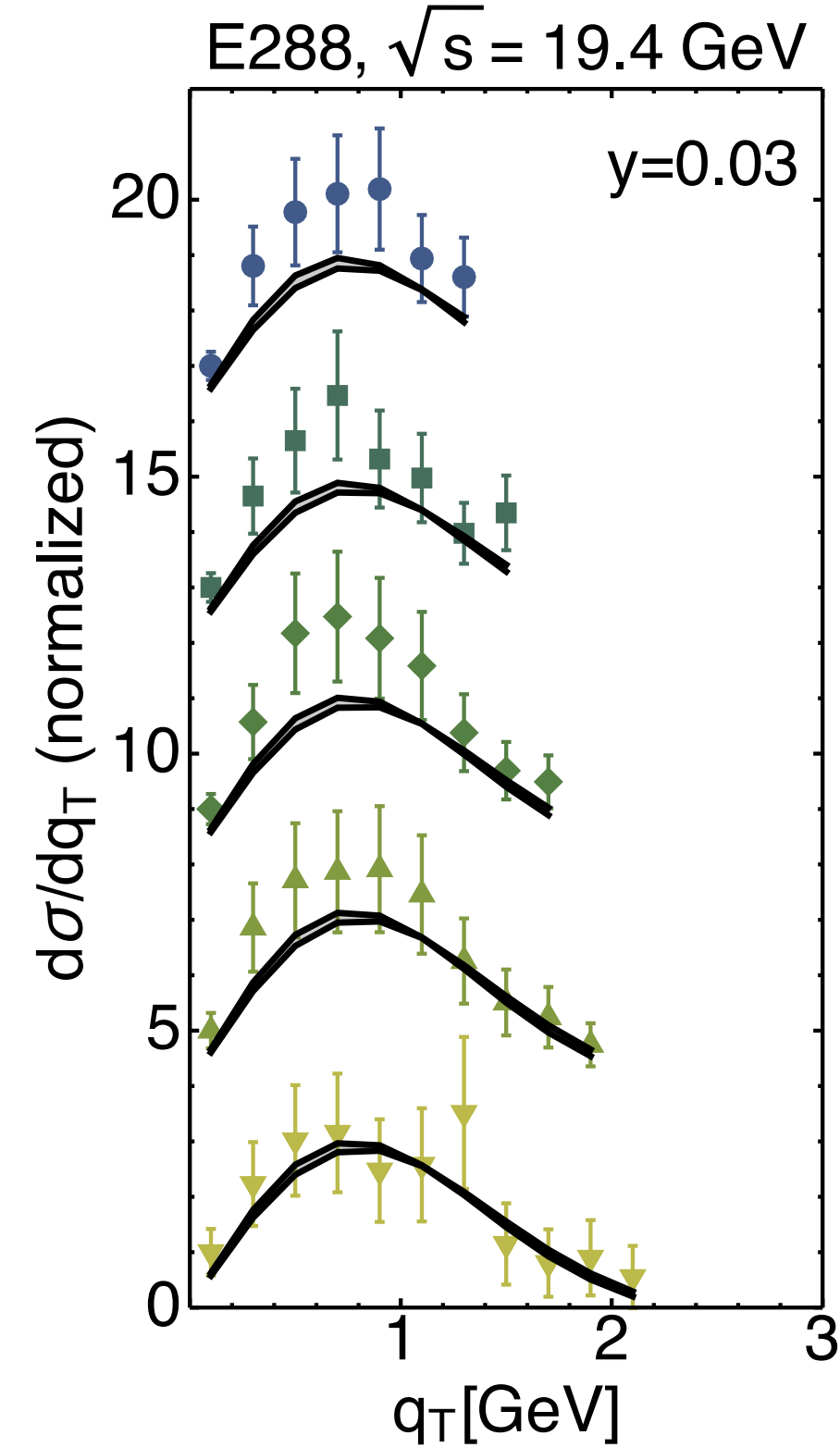


$\chi^2/\text{dof}$

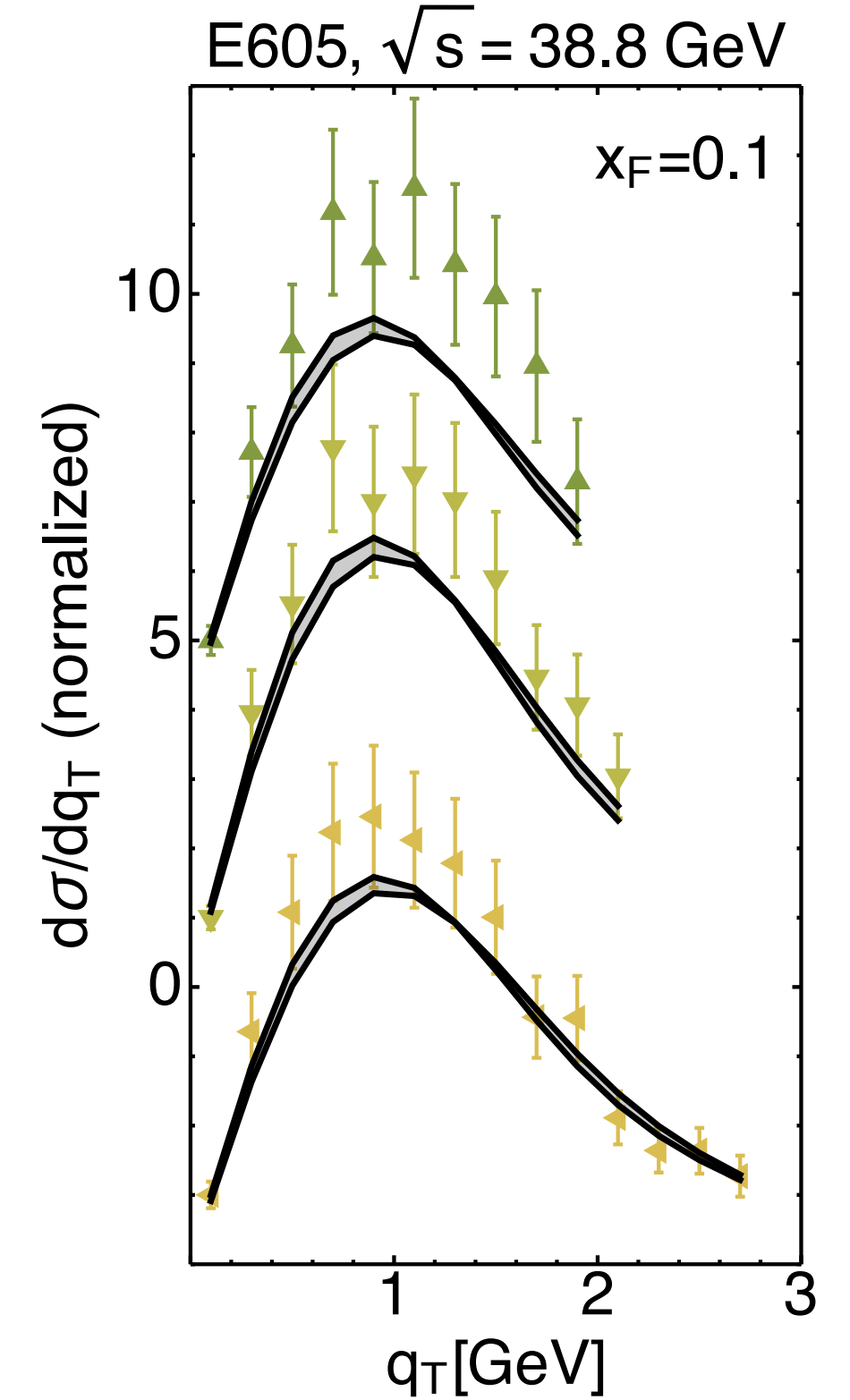
0.32



0.84



0.99

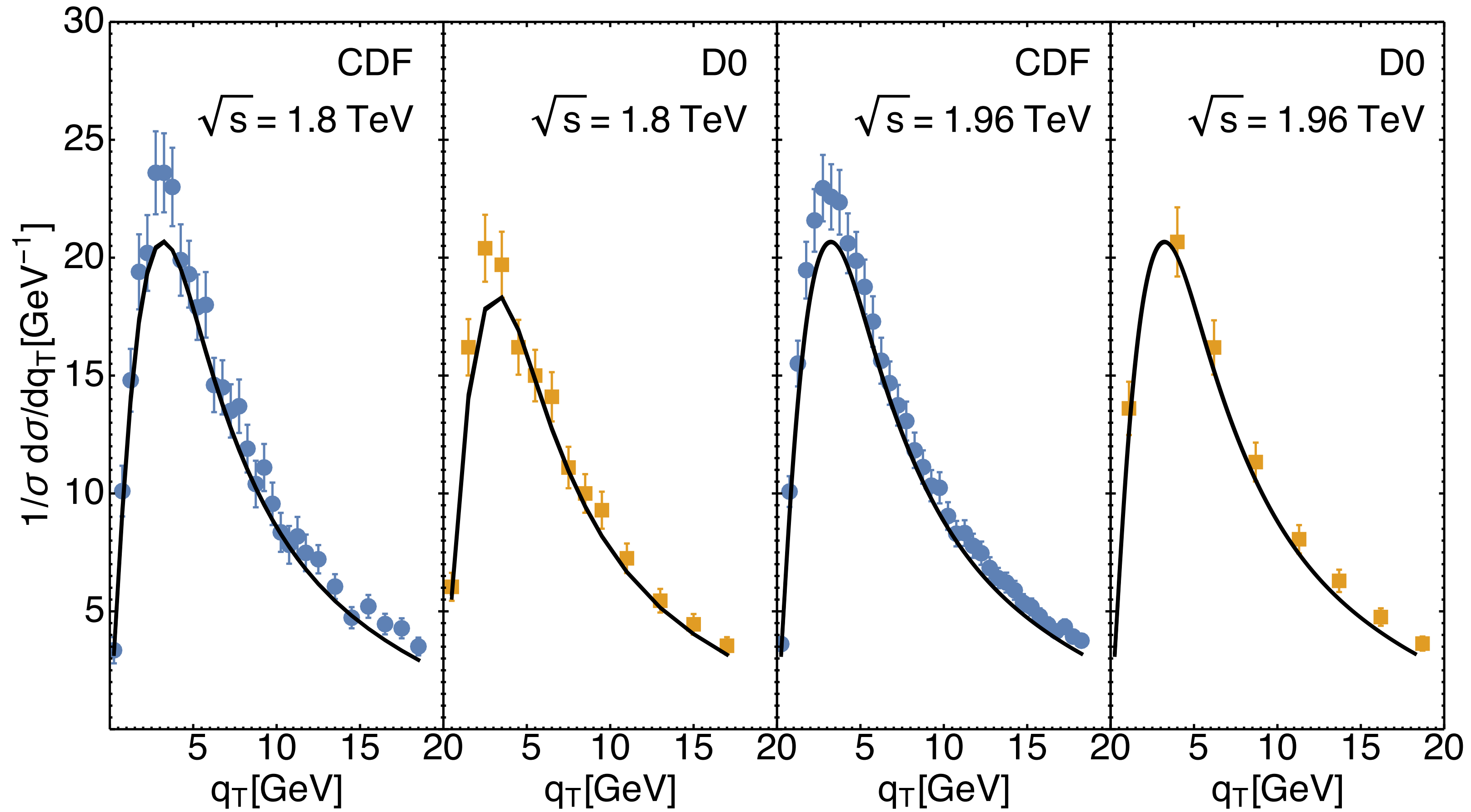


1.13

the peak was at 0.4 GeV and now is at 1 GeV



# Z-boson data



$\chi^2/\text{dof}$

1.4

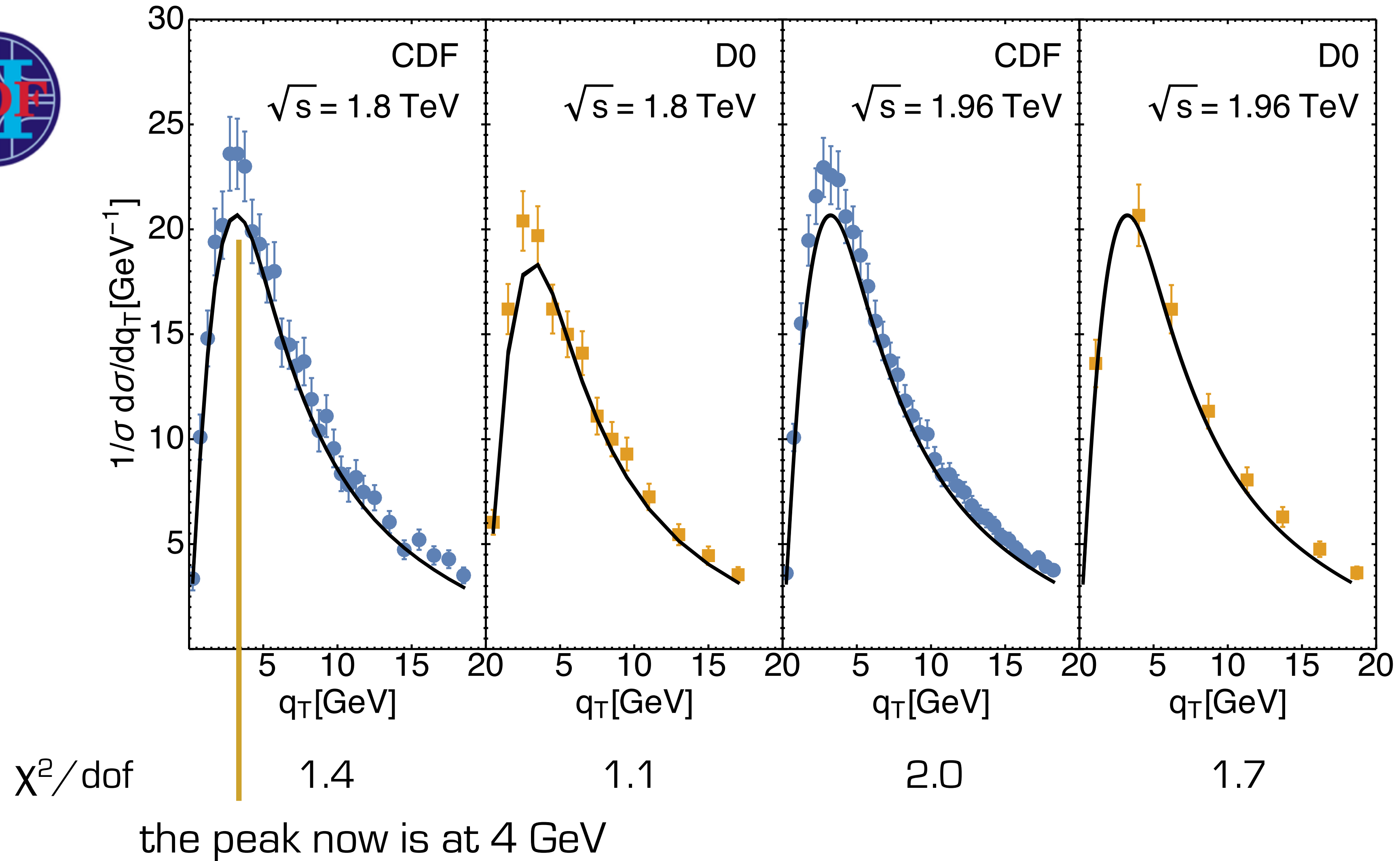
1.1

2.0

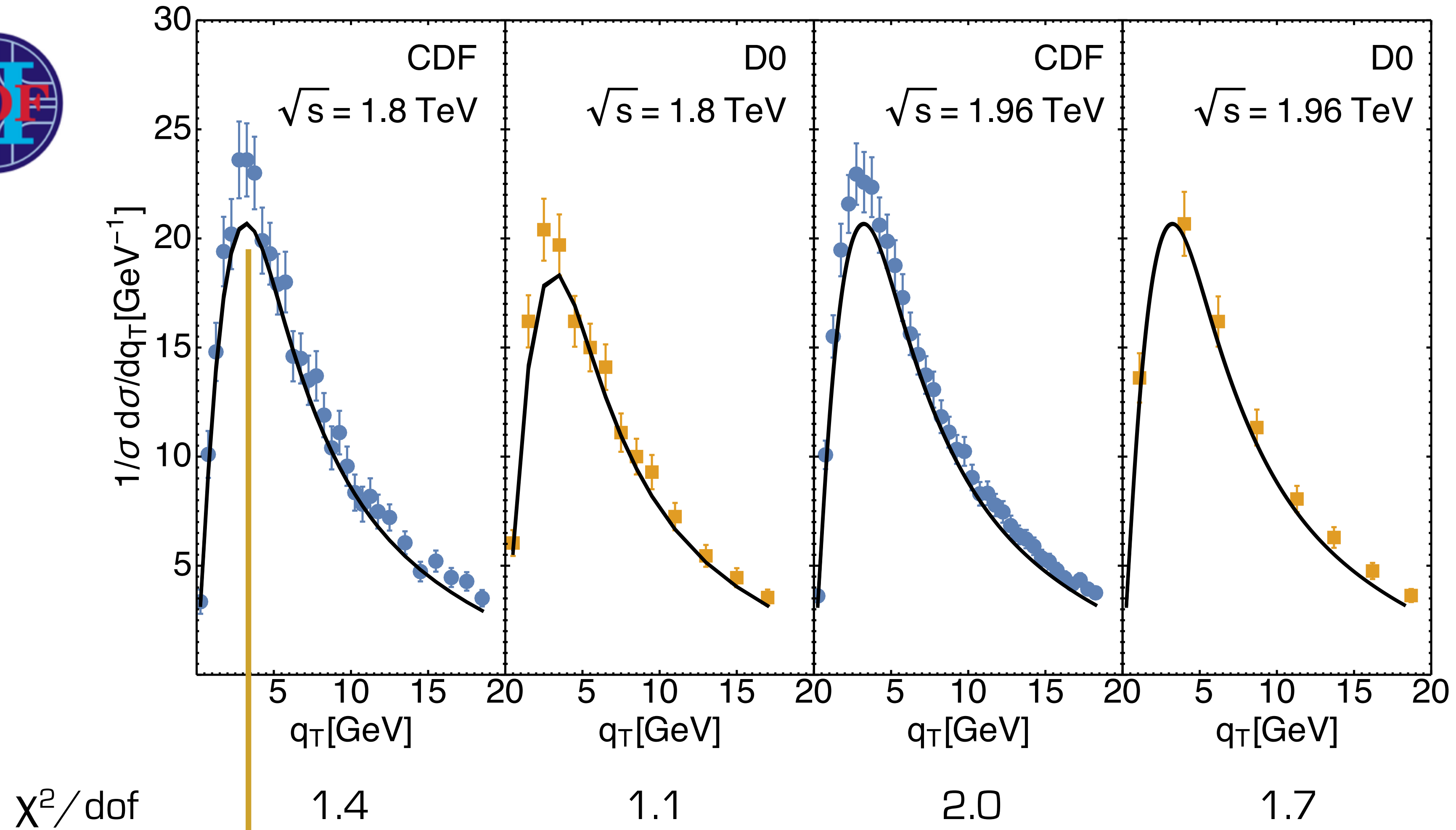
1.7



# Z-boson data



# Z-boson data



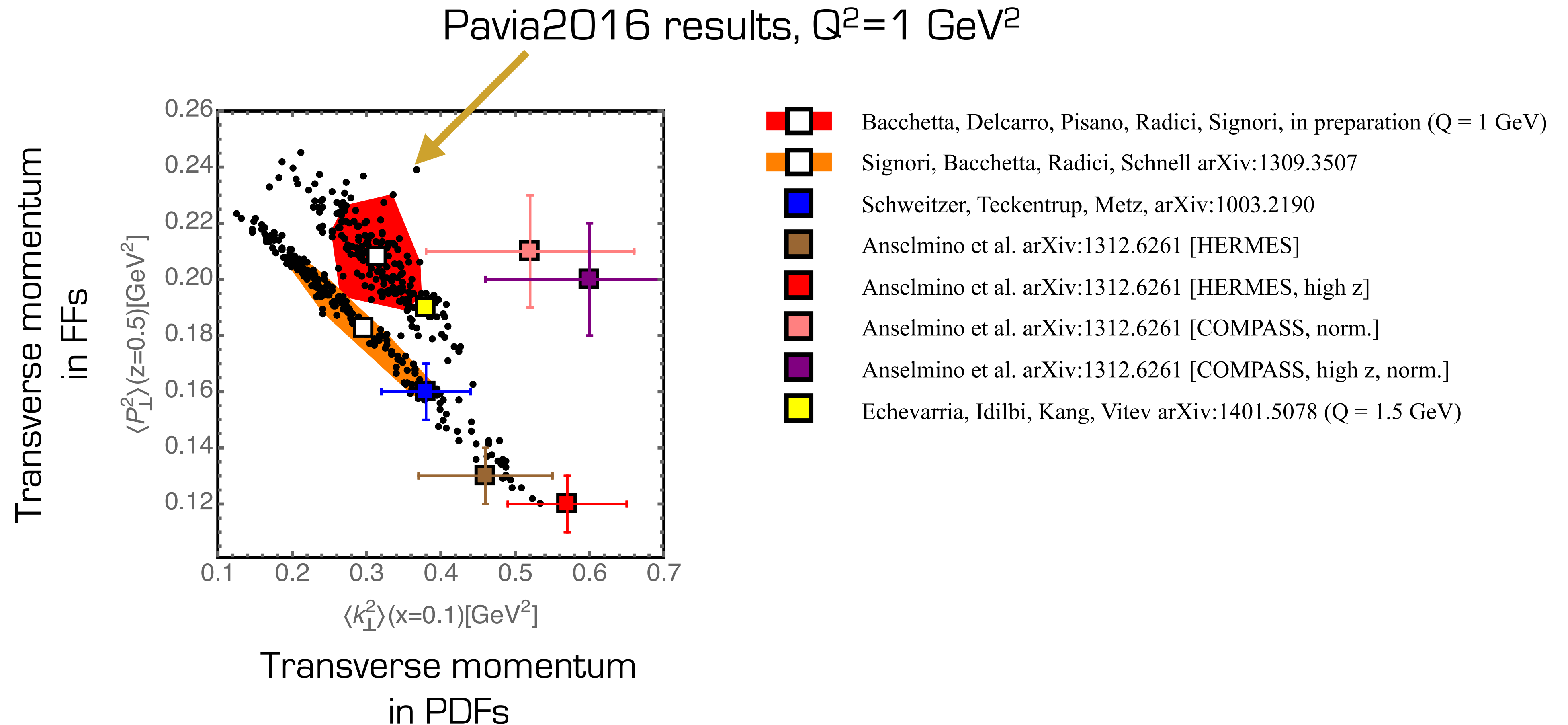
the peak now is at 4 GeV

Most of the  $\chi^2$  due to normalization, not to shape

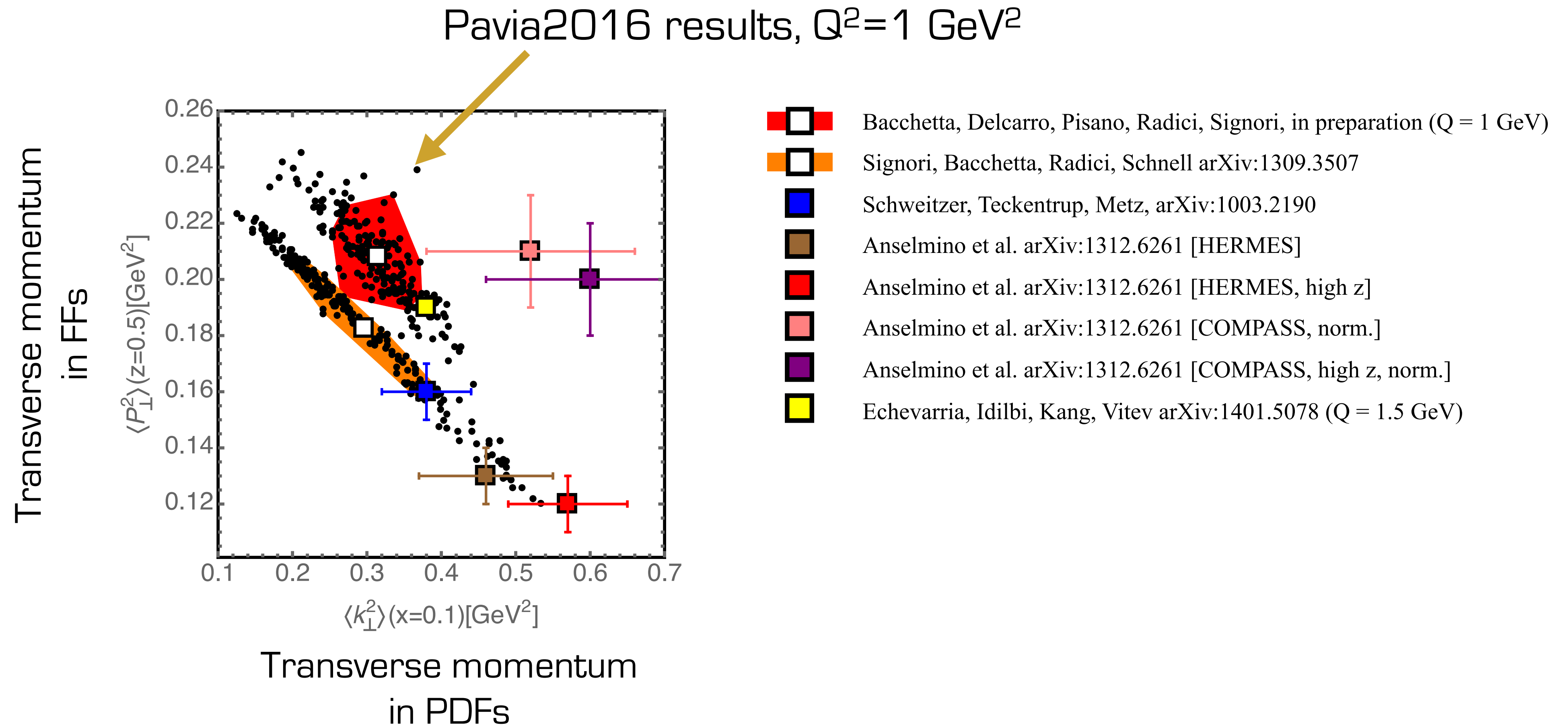
Some outcomes

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# Mean transverse momentum squared



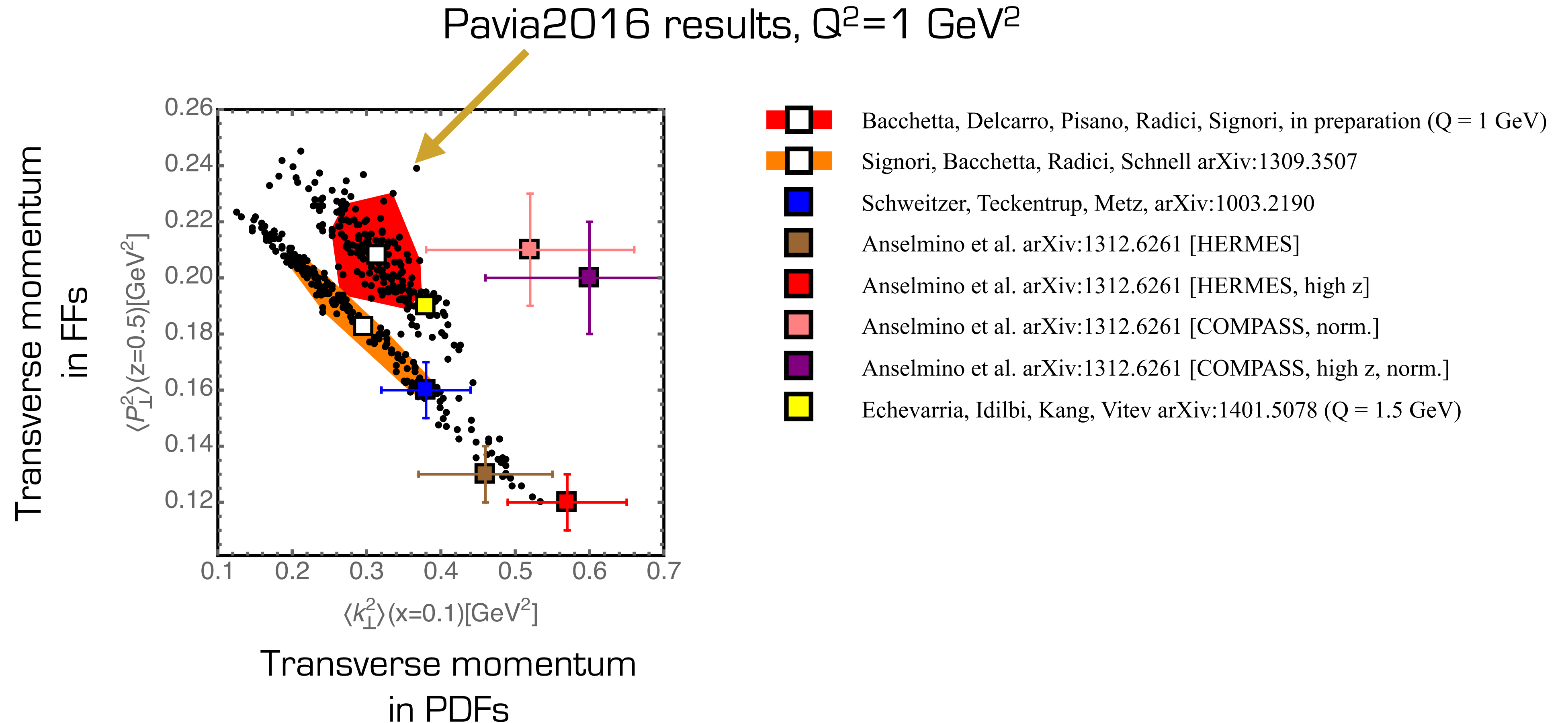
# Mean transverse momentum squared



CAVEAT: intrinsic transverse momentum depends on TMD evolution “scheme” and its parameters



# Mean transverse momentum squared

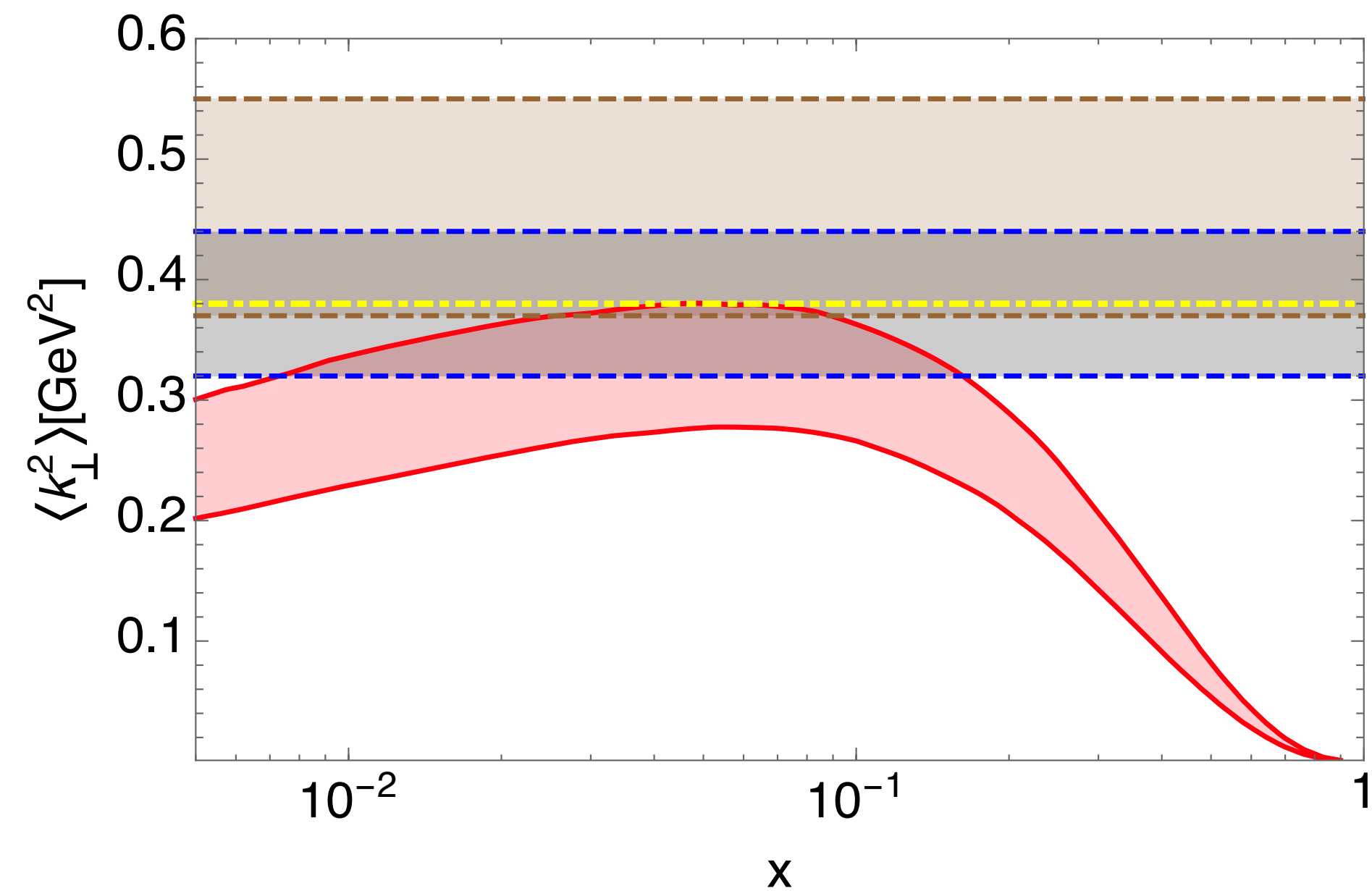


CAVEAT: intrinsic transverse momentum depends on TMD evolution “scheme” and its parameters

Anti correlation between transverse momentum in TMD PDFs and in TMD FFs, in spite of Drell-Yan data

# Mean transverse momentum squared

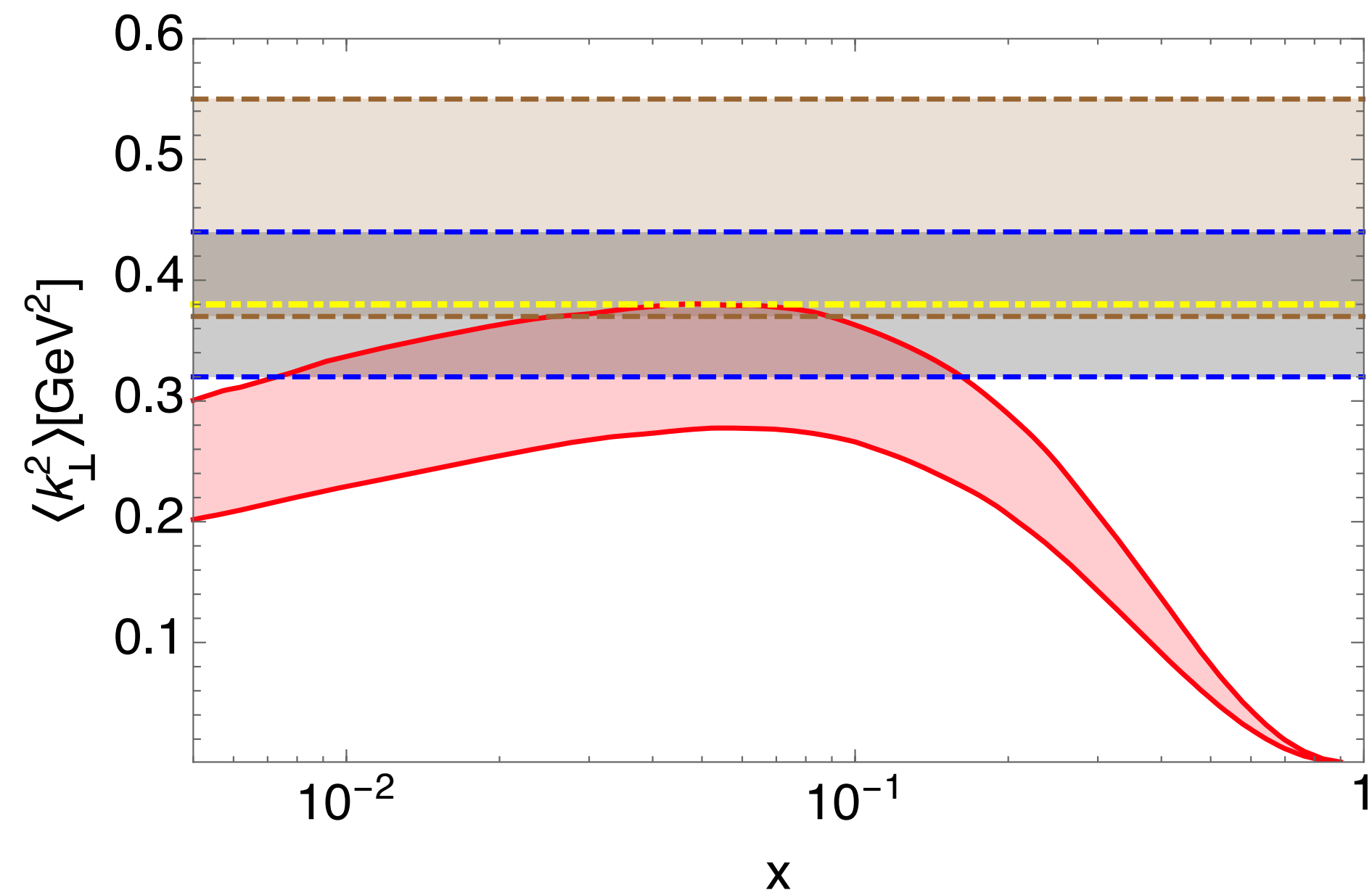
*same color coding as previous slide*



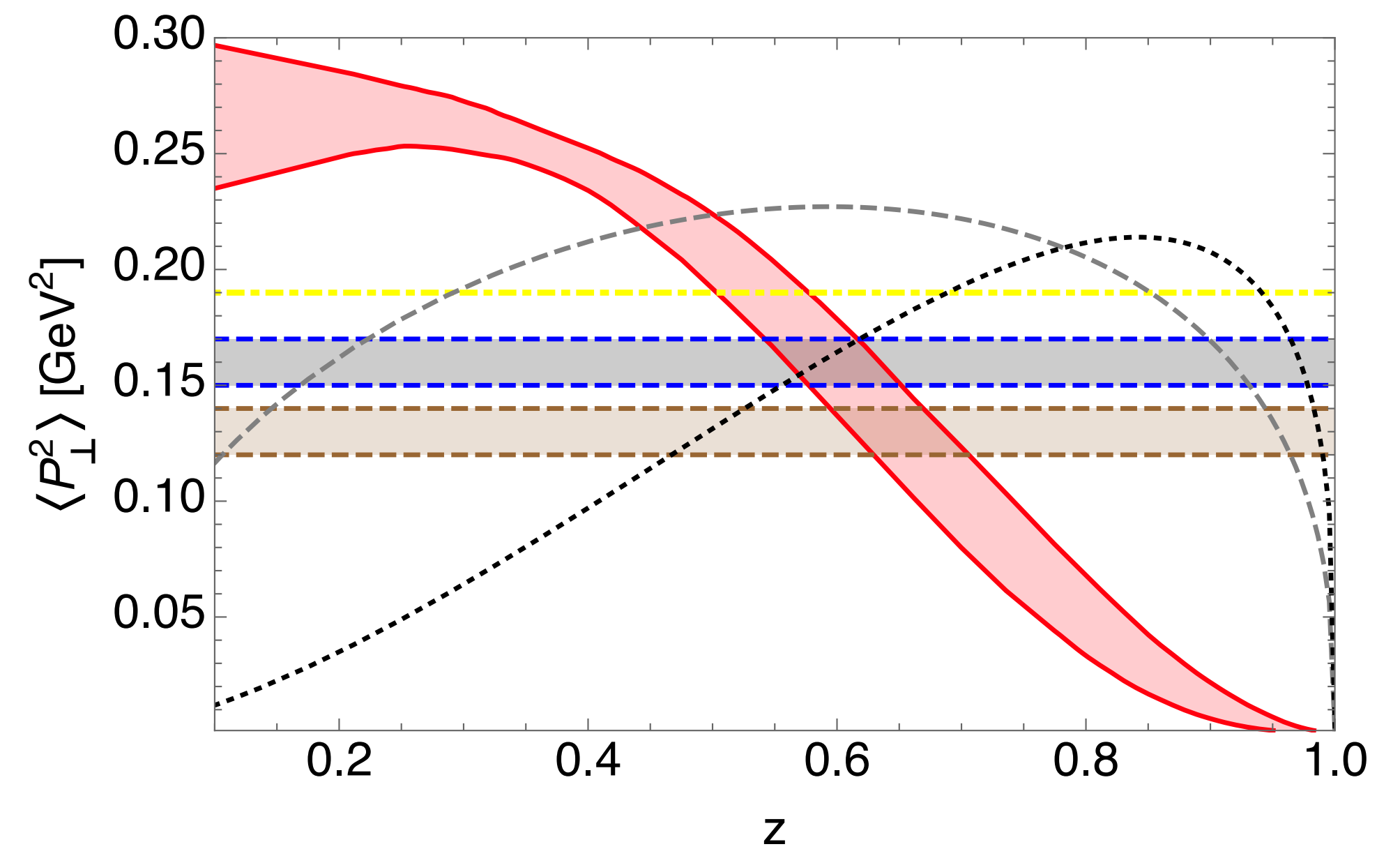
In TMD distribution functions

# Mean transverse momentum squared

*same color coding as previous slide*



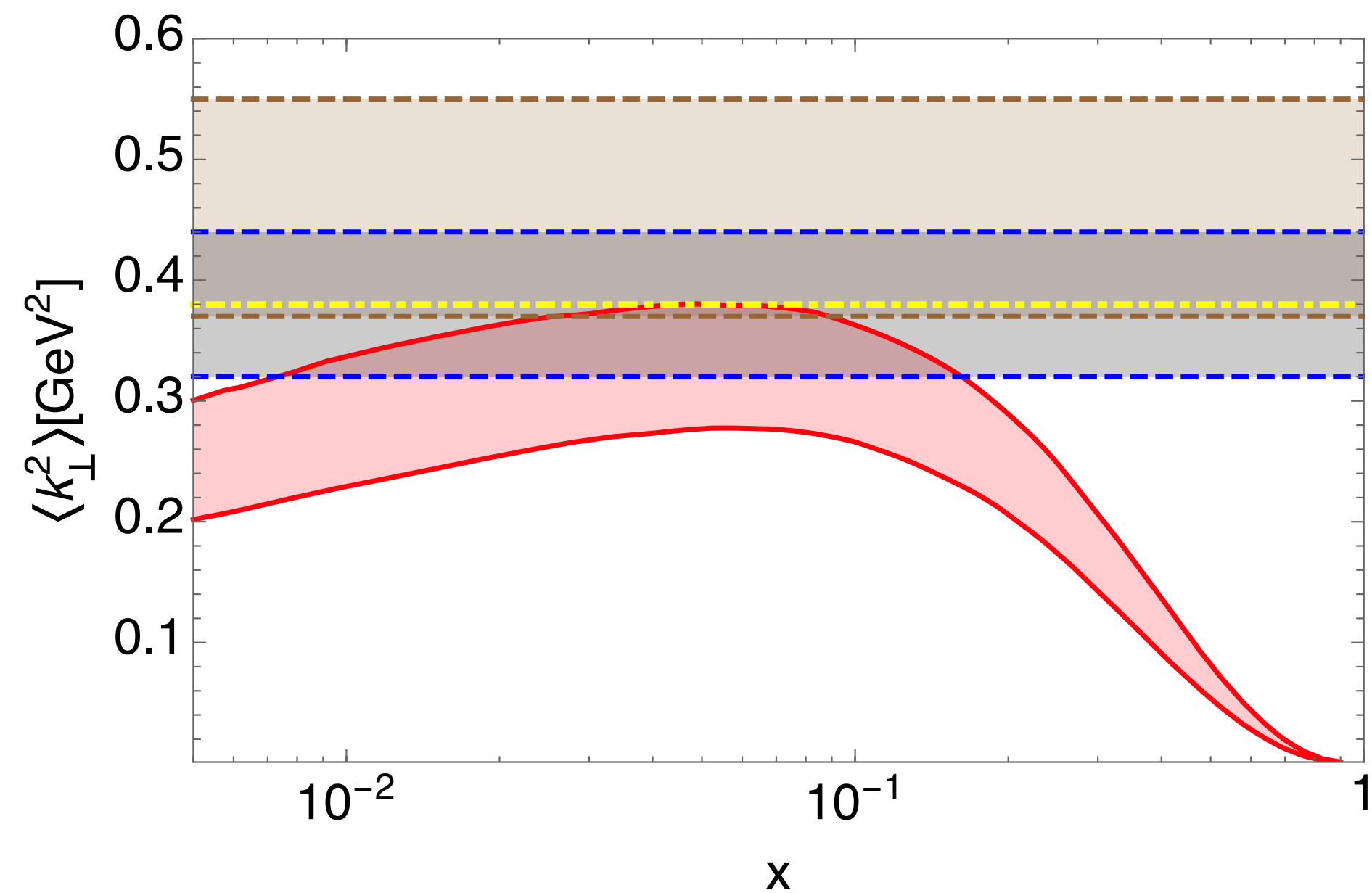
In TMD distribution functions



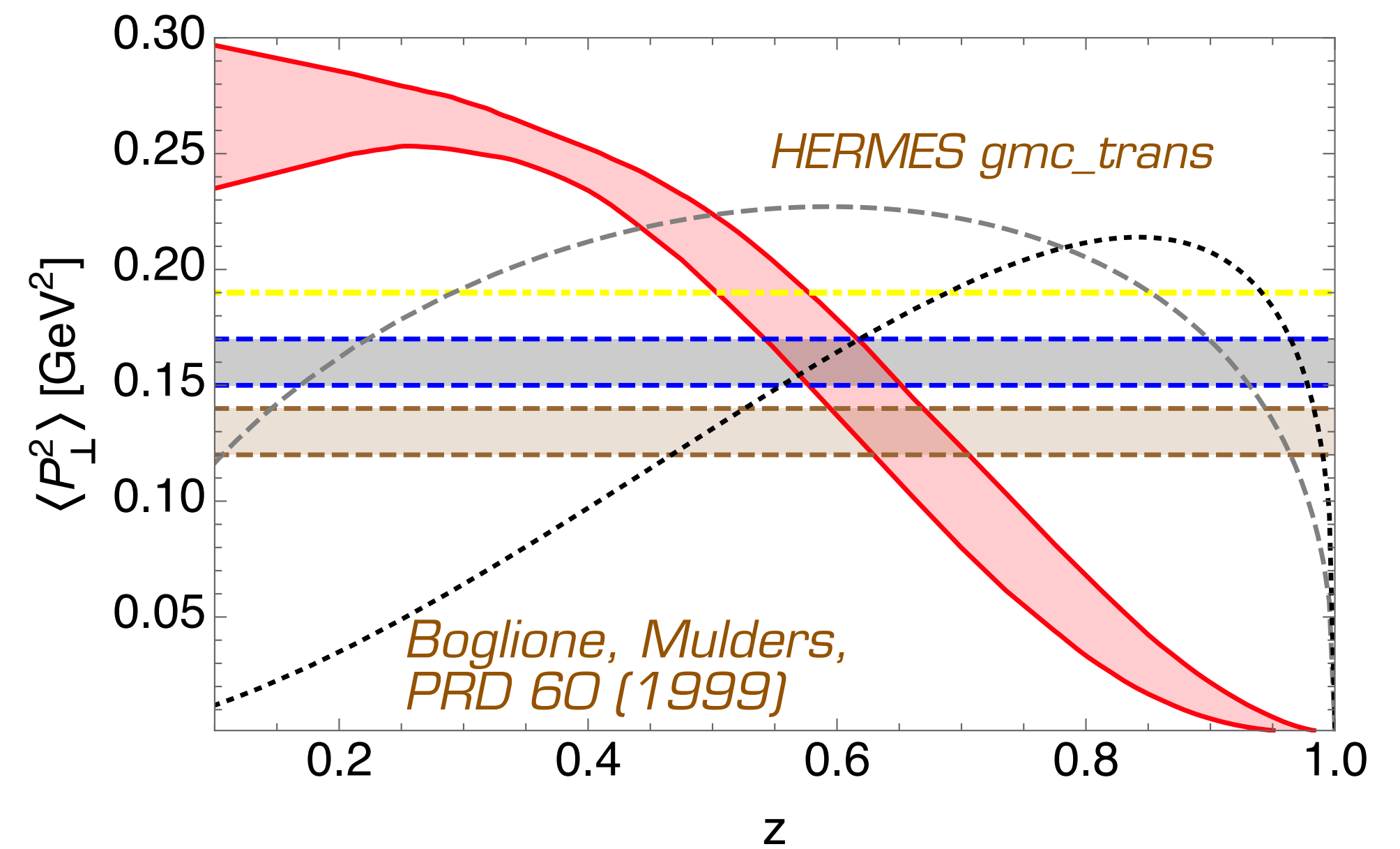
In TMD fragmentation functions

# Mean transverse momentum squared

same color coding as previous slide



In TMD distribution functions



In TMD fragmentation functions

# Nonperturbative evolution parameters

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TMD evolution is not uniquely determined by pQCD calculations. Nonperturbative input is needed to determine evolution precisely. Different schemes may behave differently.

	$g_2$ (GeV <sup>2</sup> )	$b_{\max}$ (GeV <sup>-1</sup> )
BLNY 2003	$0.68 \pm 0.02$	0.5
KN 2006	$0.184 \pm 0.018$	1.5
EIKV 2014	0.18	1.5
Pavia 2016	$0.12 \pm 0.01$	1.123



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Faster evolution: transverse momentum increases faster due to gluon radiation

# Nonperturbative evolution parameters

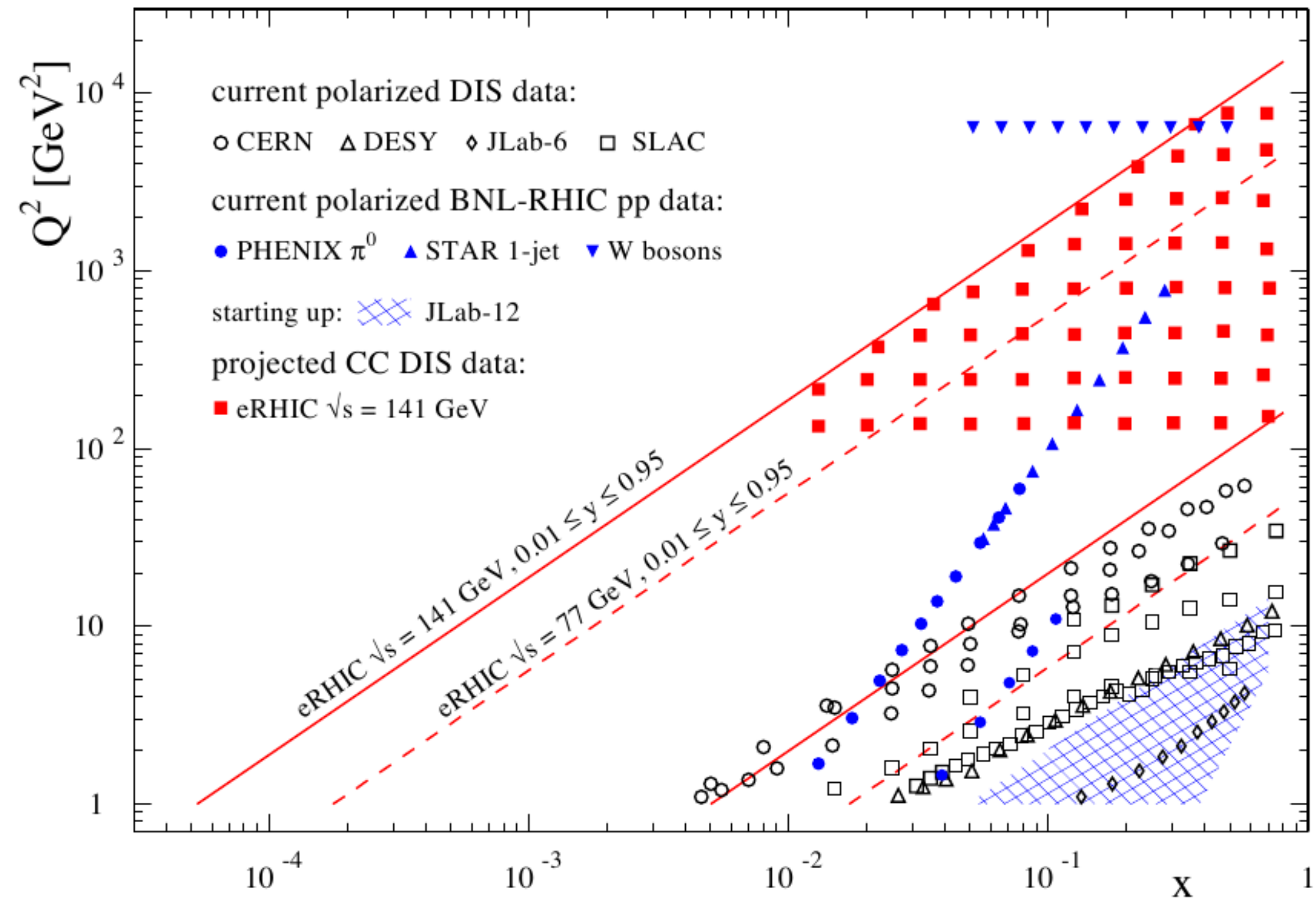
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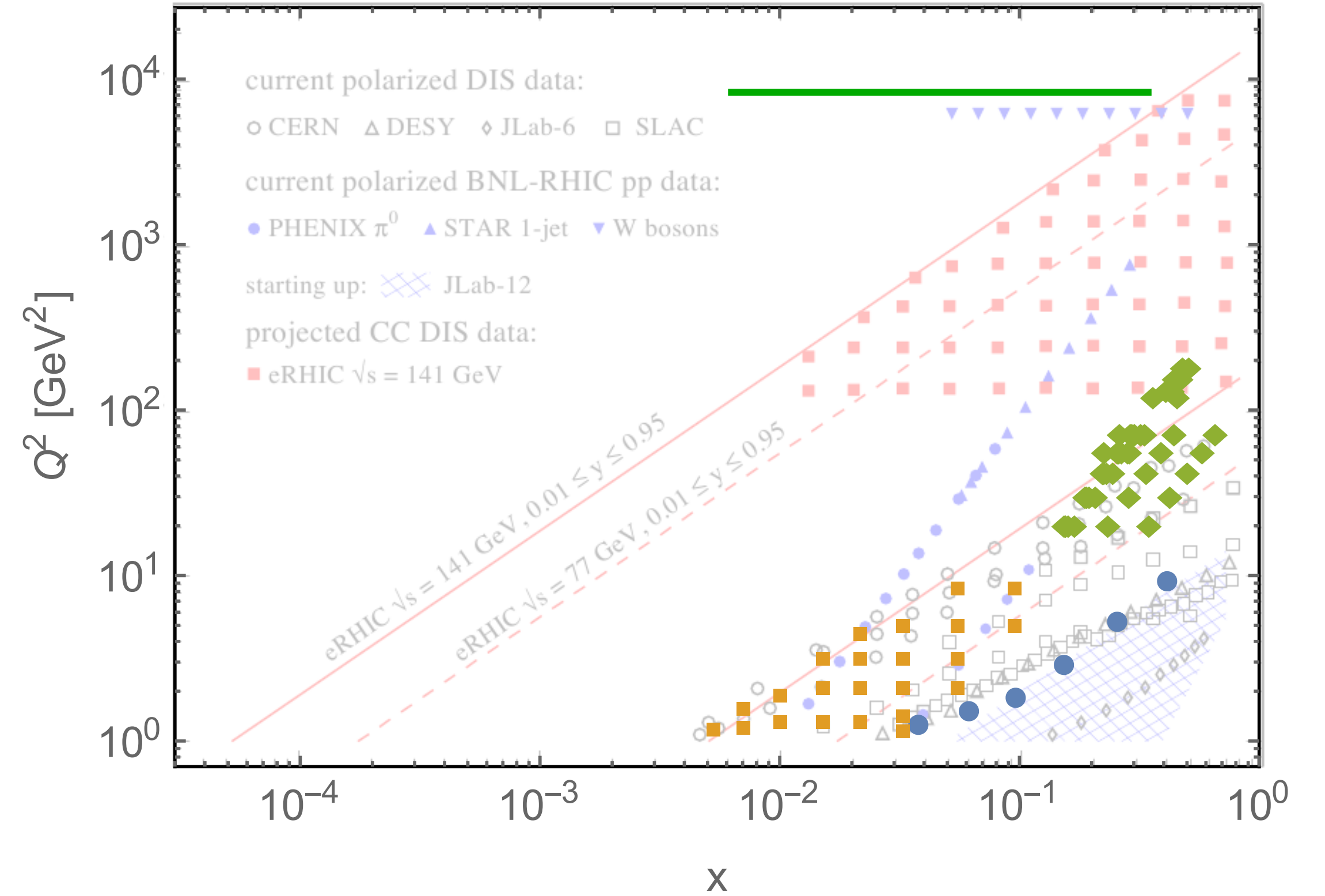
Faster evolution: transverse momentum increases faster due to gluon radiation

Slower evolution: the effect of gluon radiation is weaker

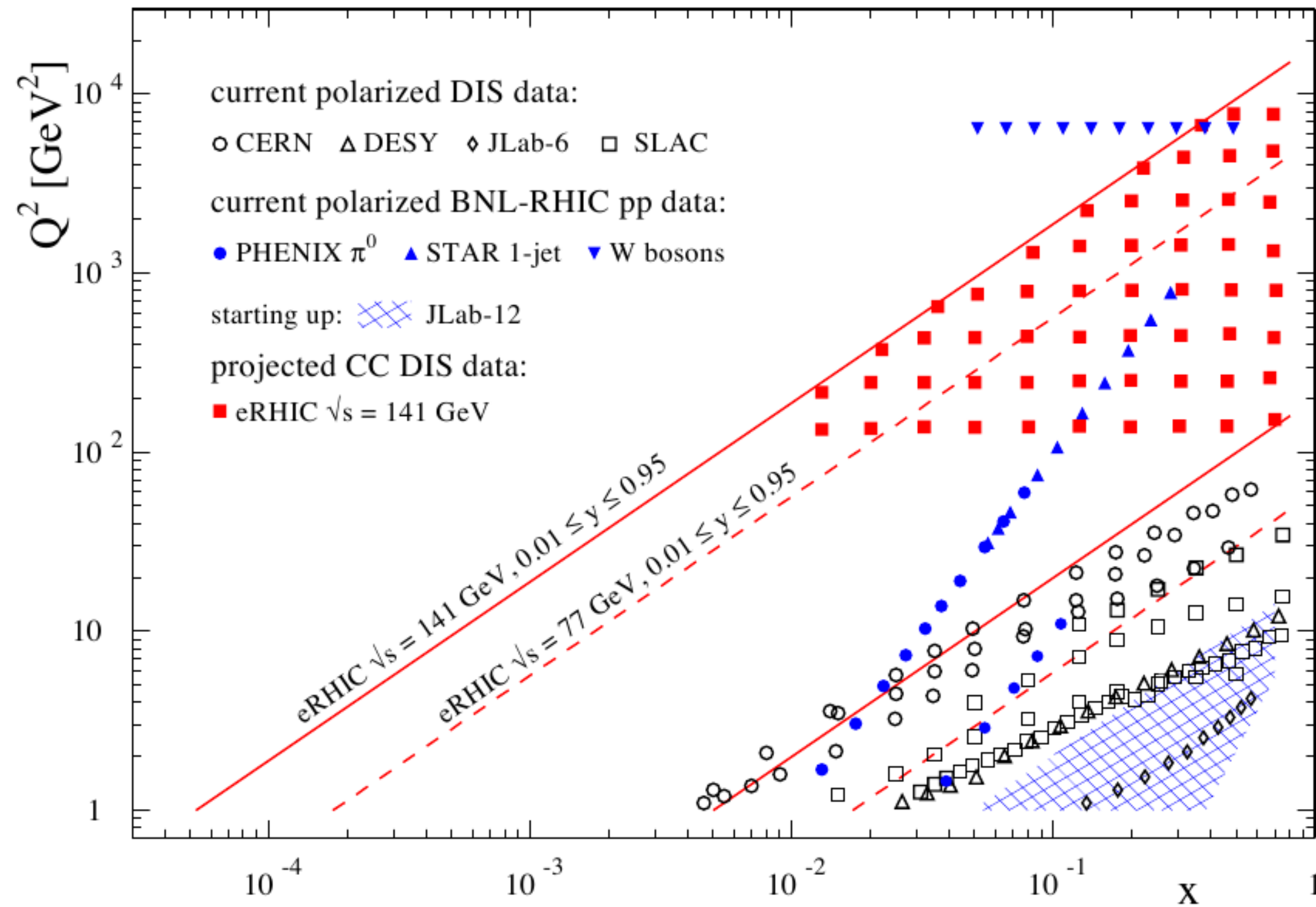
# Comparison with future perspectives



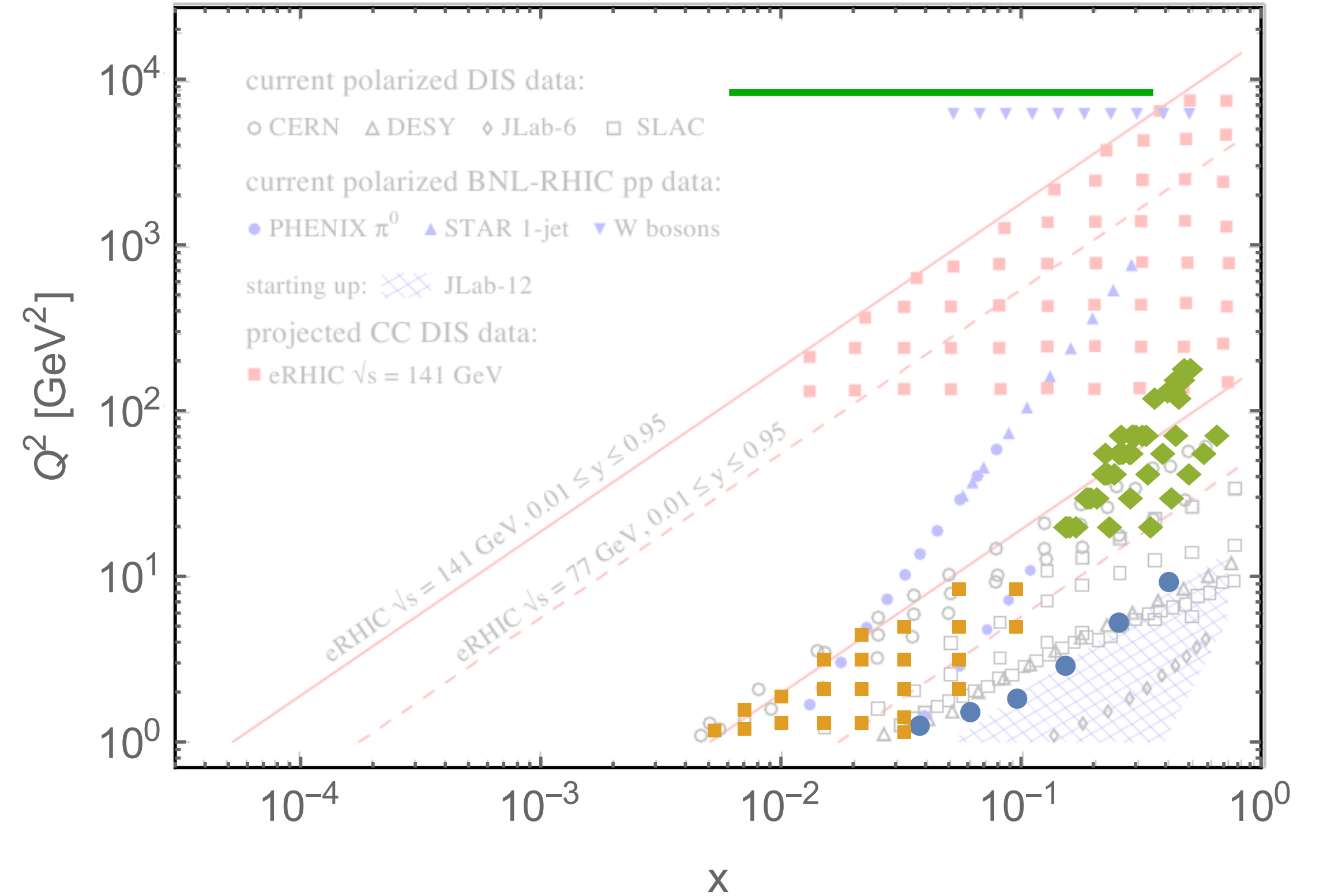
from EIC white paper EPJA 52 (2016)



# Comparison with future perspectives



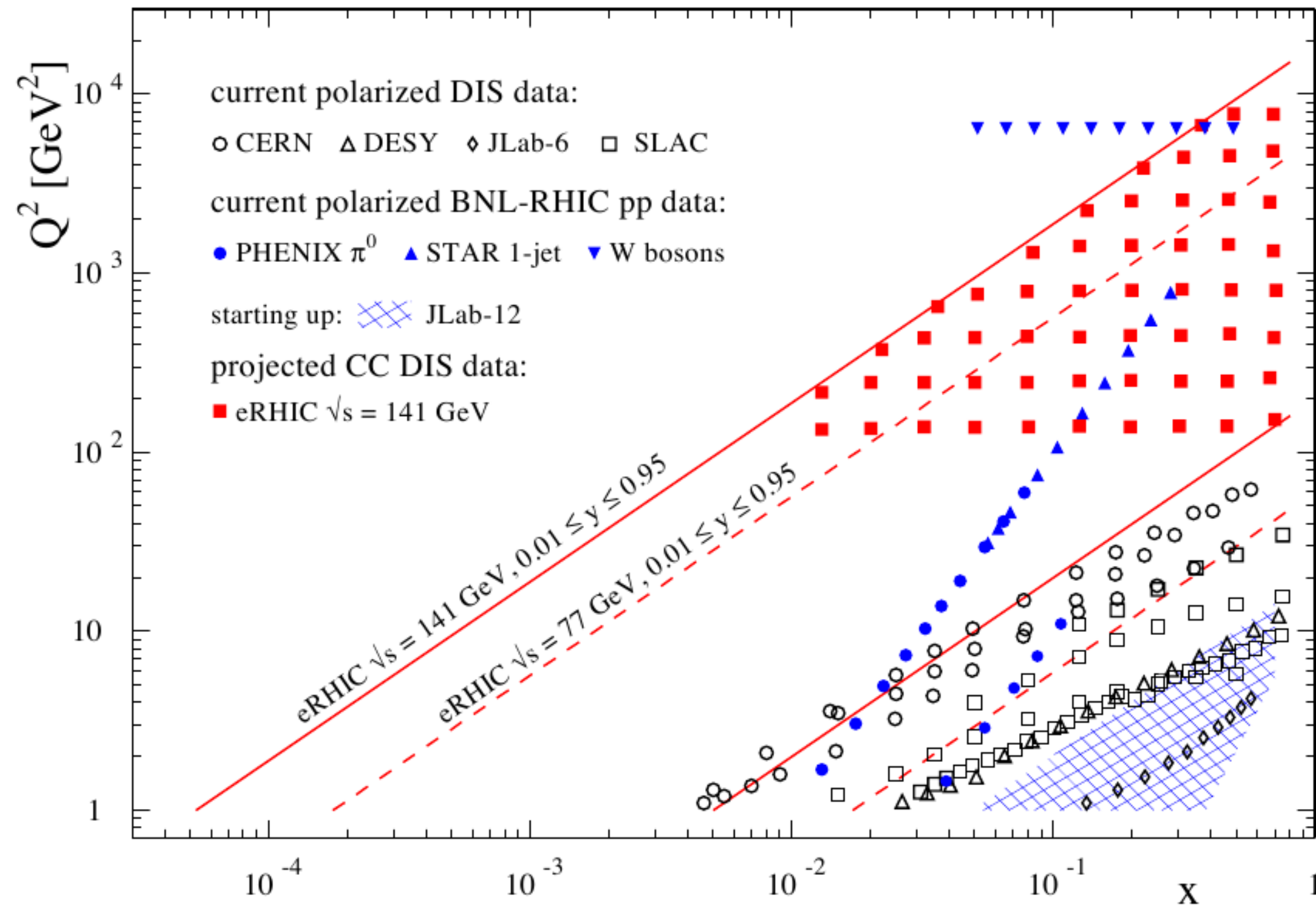
*from EIC white paper EPJA 52 (2016)*



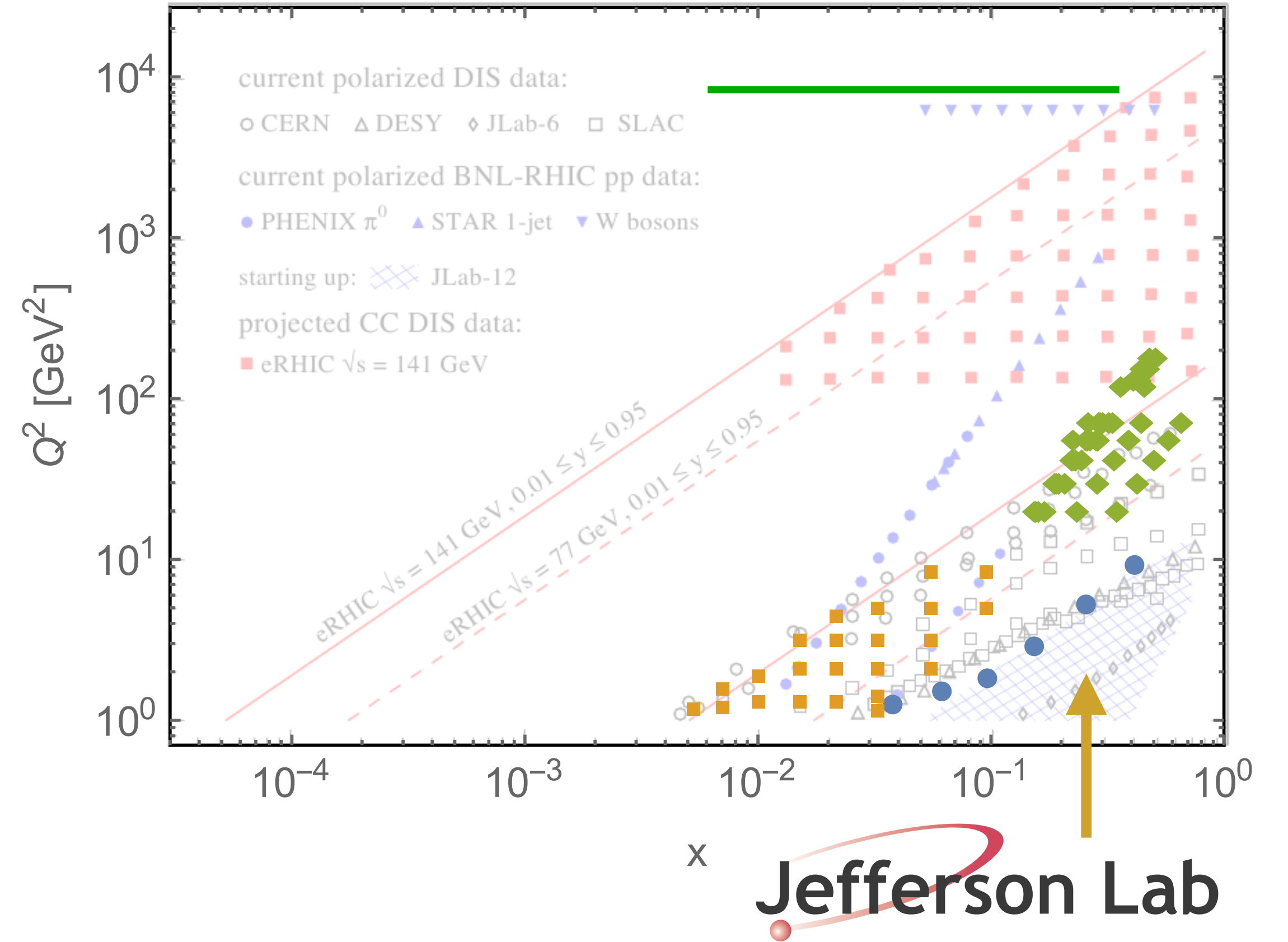
To test the formalism, we would need more data covering the same  $x$  range and spanning over a large range in  $Q^2$ . Data from JLab and Drell-Yan would be very important.



# Comparison with future perspectives



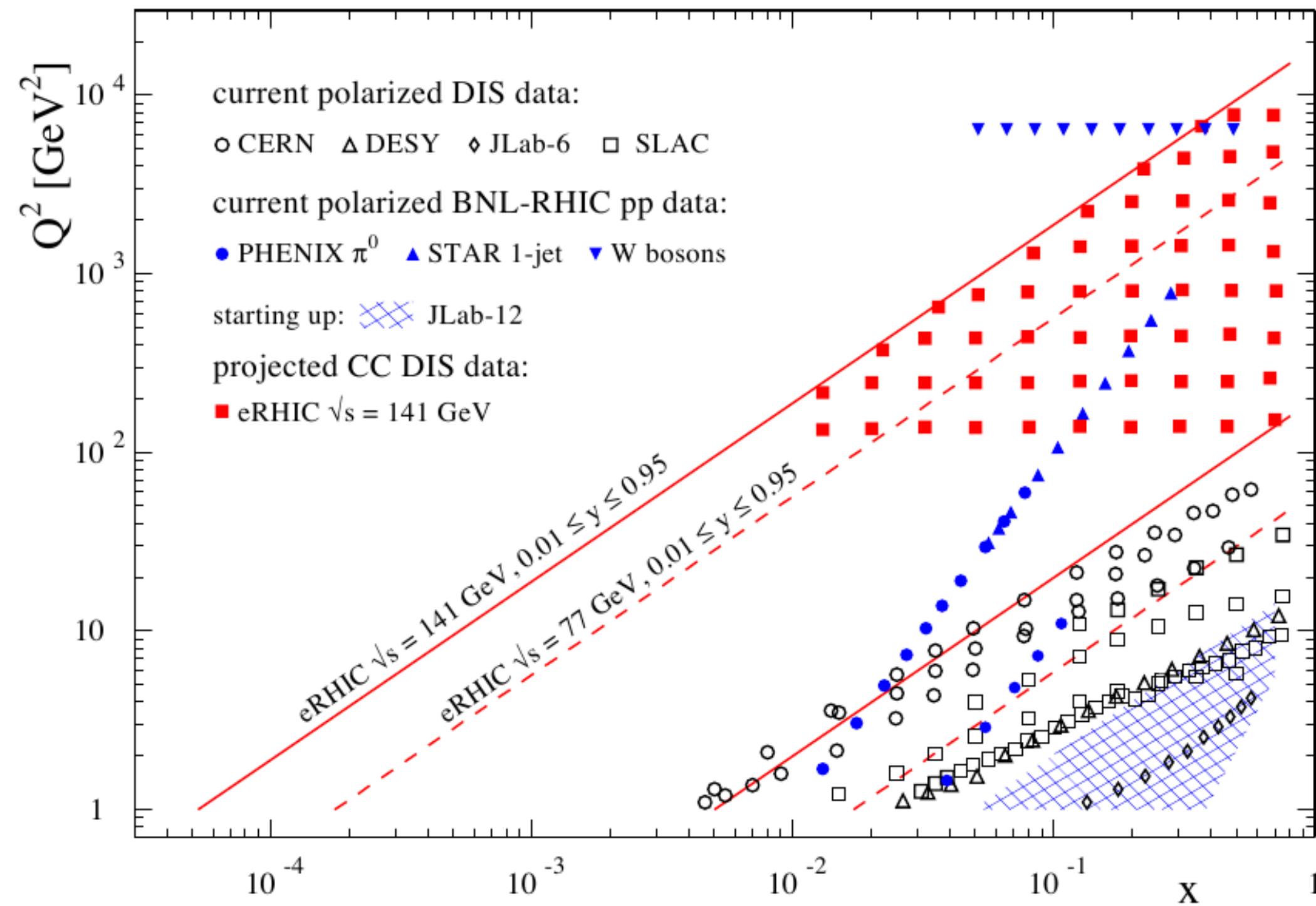
from EIC white paper EPJA 52 (2016)



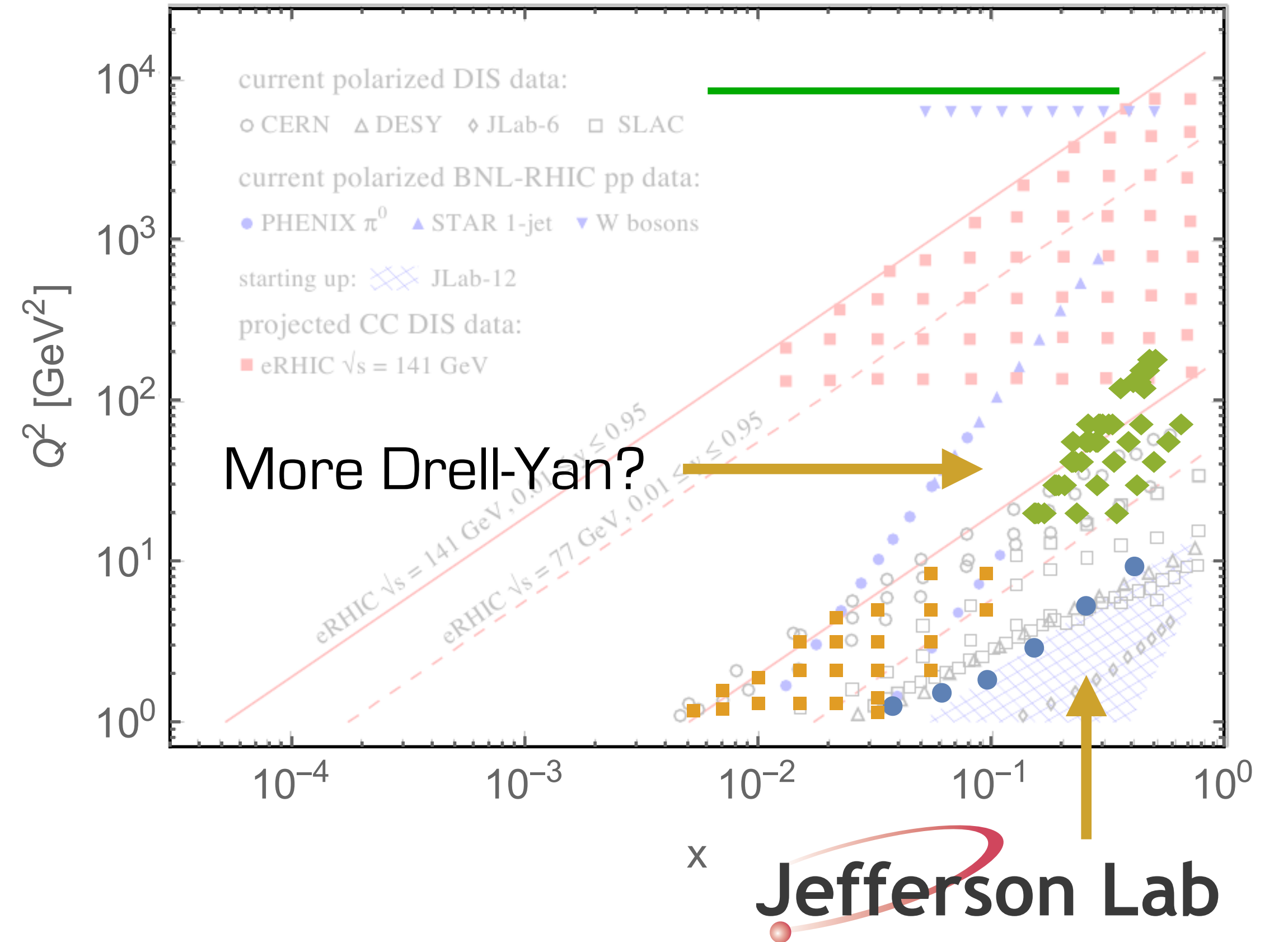
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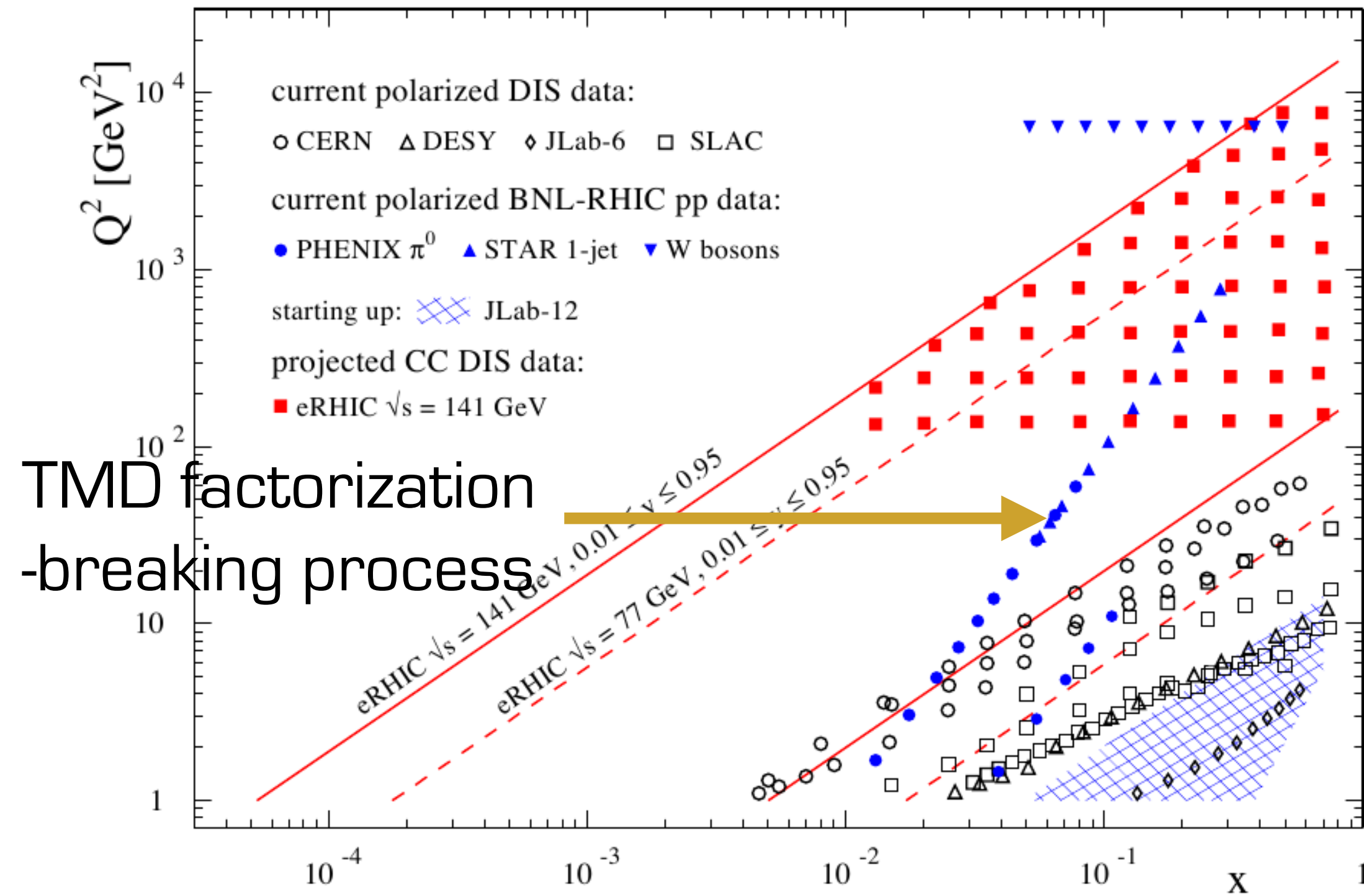


from EIC white paper EPJA 52 (2016)

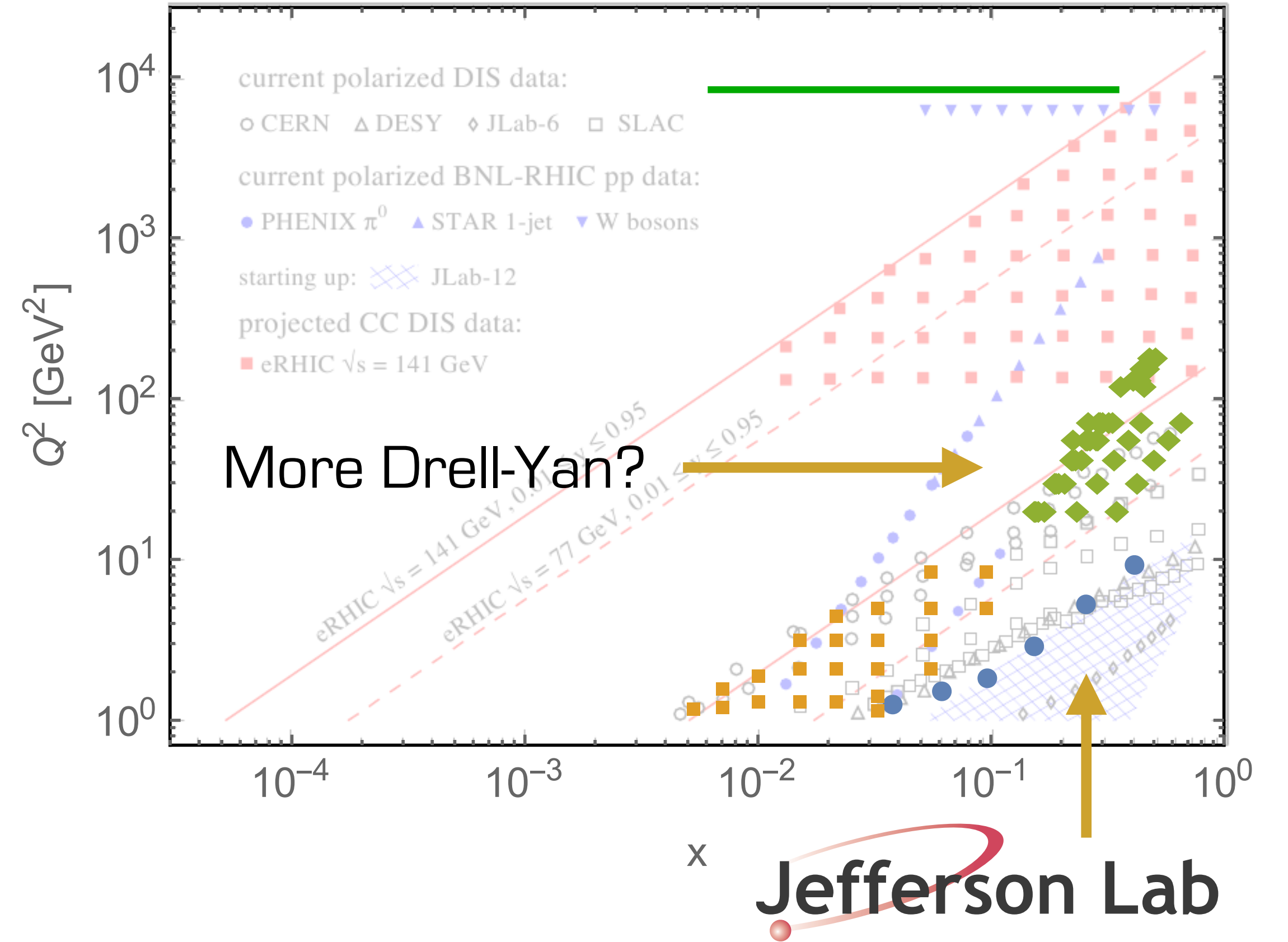


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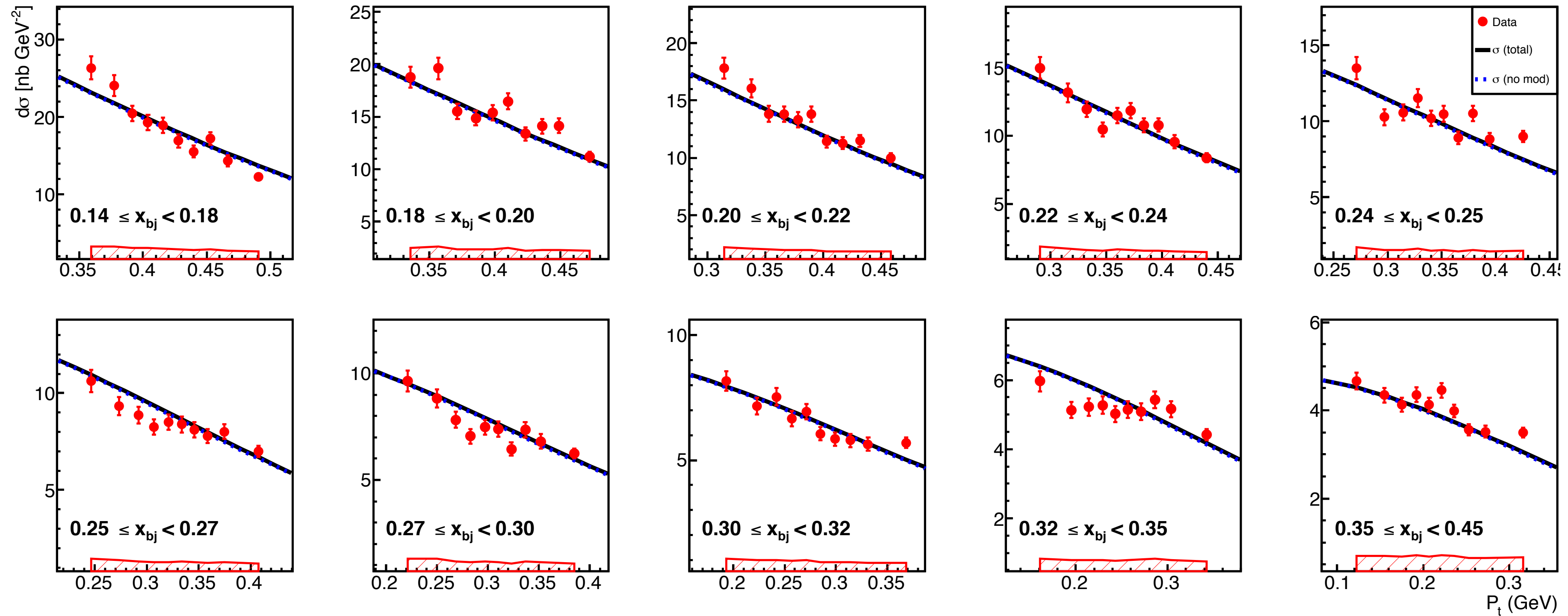


from EIC white paper EPJA 52 (2016)



To test the formalism, we would need more data covering the same  $x$  range and spanning over a large range in  $Q^2$ . Data from JLab and Drell-Yan would be very important.

# Recent $^3\text{He}$ data from JLab Hall A

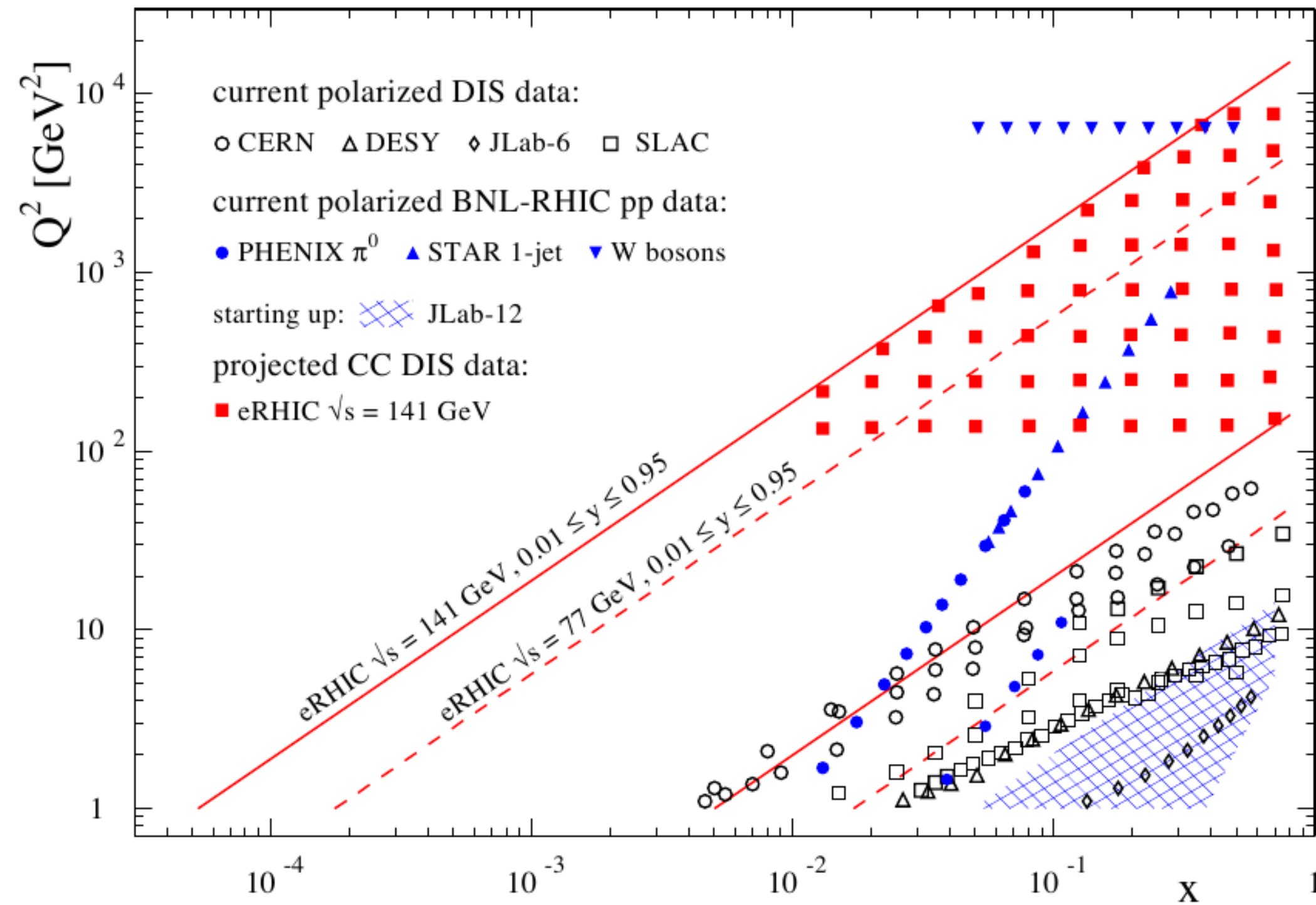


*Yan et al., [arXiv:1610.02350](https://arxiv.org/abs/1610.02350)*

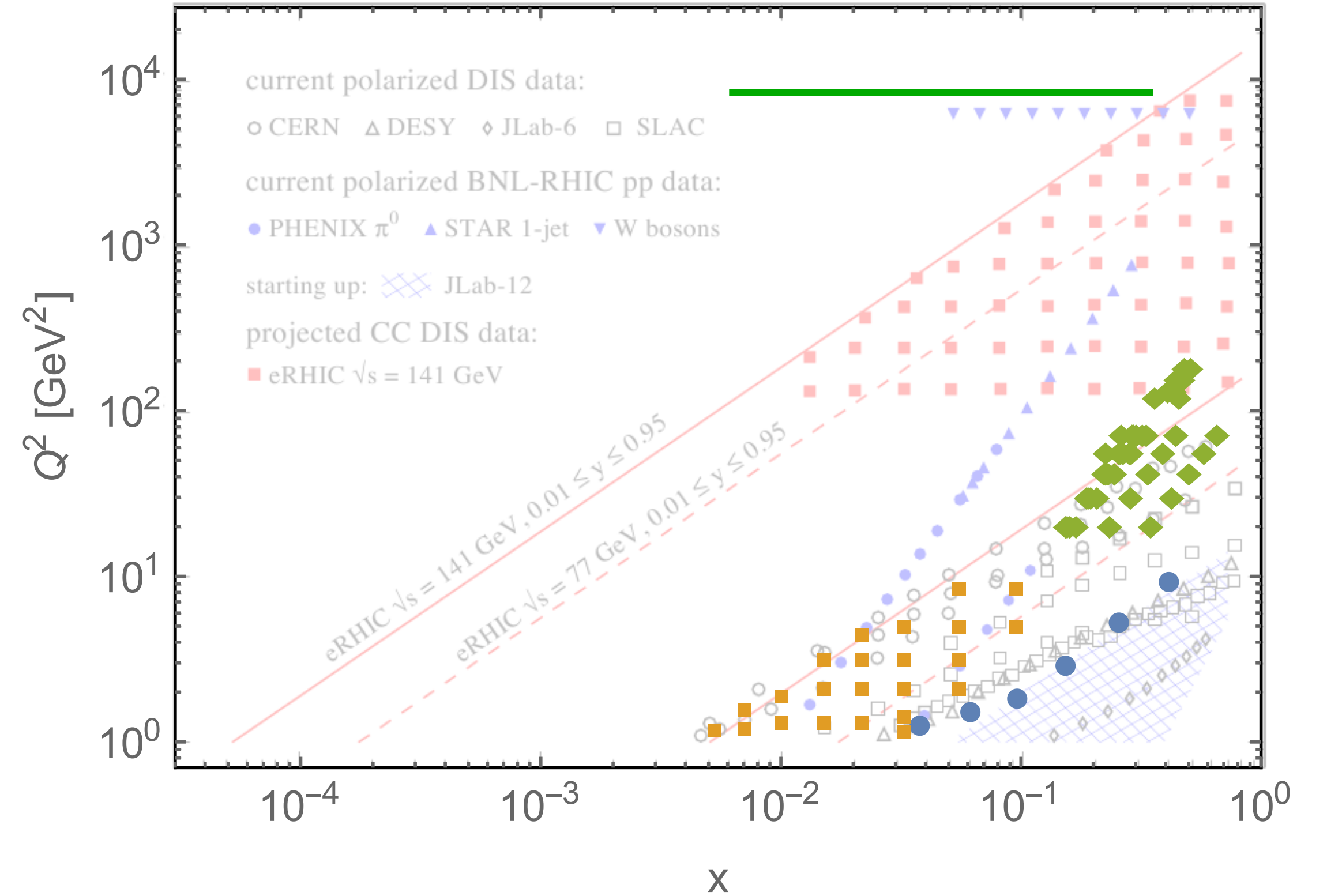
Not included in the present fits on the ground that it is at 6 GeV: needs to be checked!



# Comparison with future perspectives

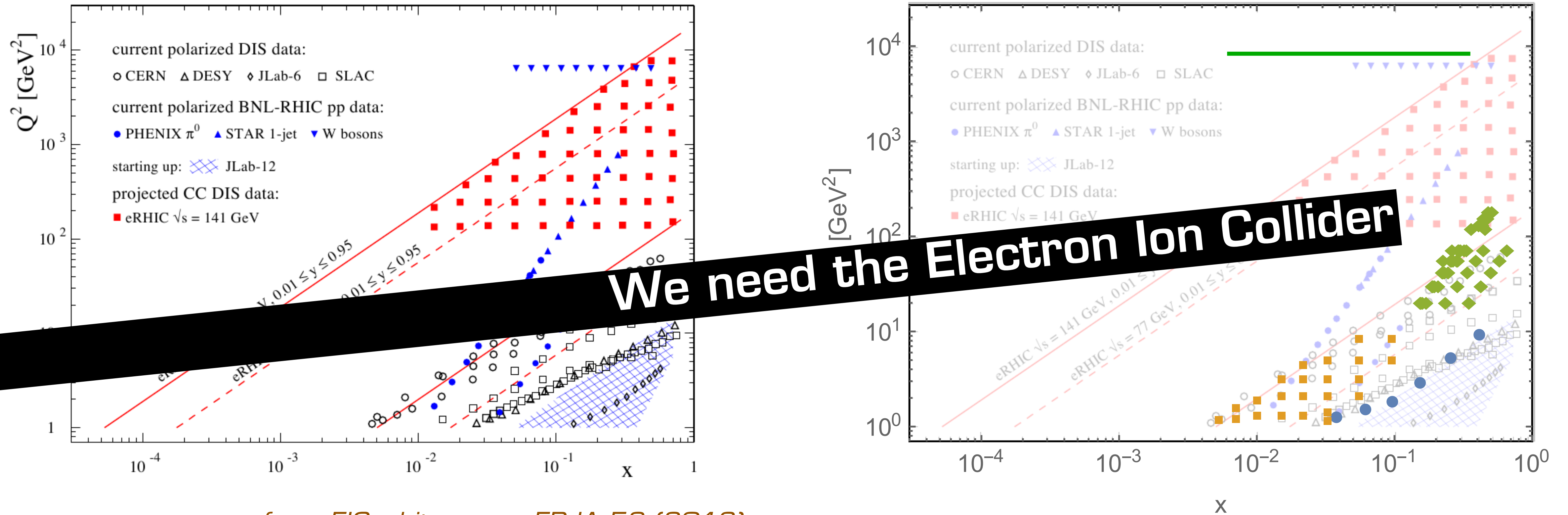


*from EIC white paper EPJA 52 (2016)*



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*from EIC white paper EPJA 52 (2016)*

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- Most of the discrepancies come from the normalisation



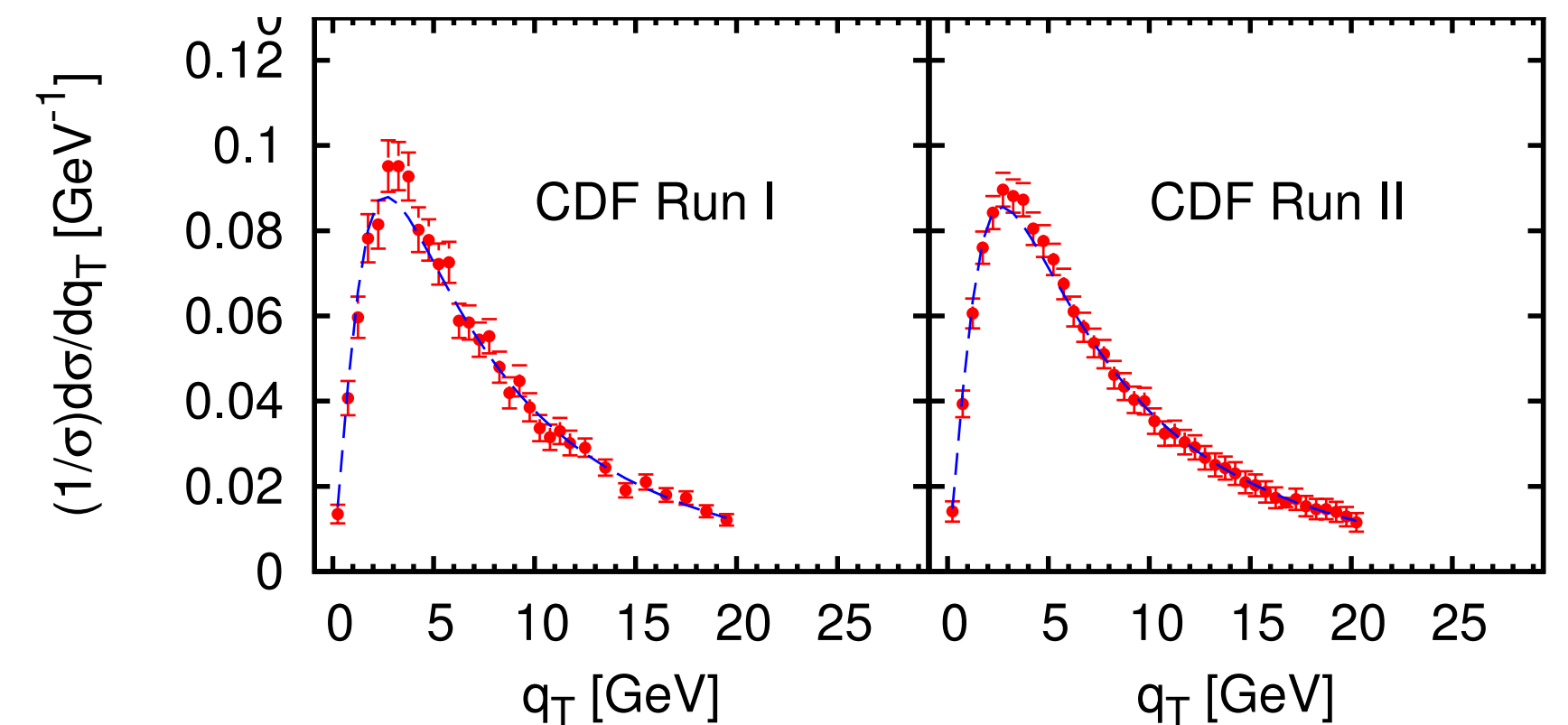
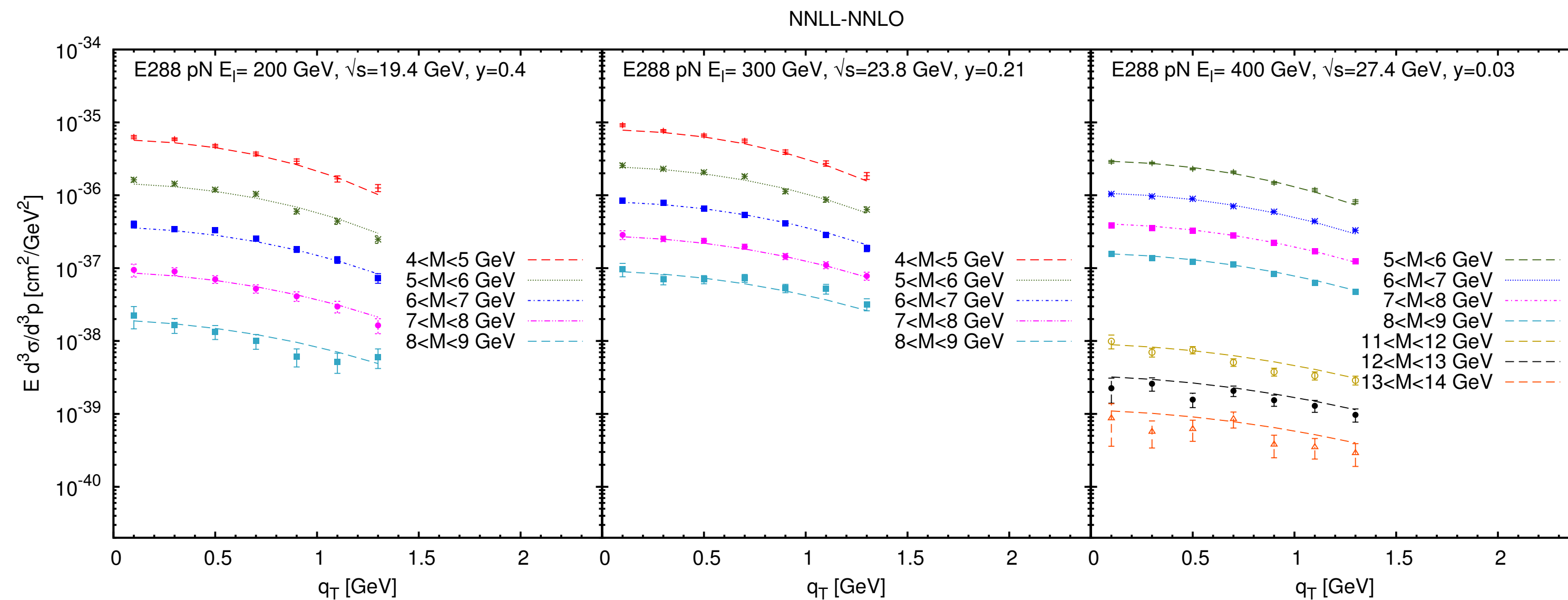
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- We extracted unpolarized TMDs using several thousand data points
- The TMD framework seems to hold pretty well
- Most of the discrepancies come from the normalisation
- Y terms still to be implemented

# Drell-Yan + Z production data (DEMS 2014)

*D'Alesio, Echevarria, Melis, Scimemi, JHEP 1411 (14)*

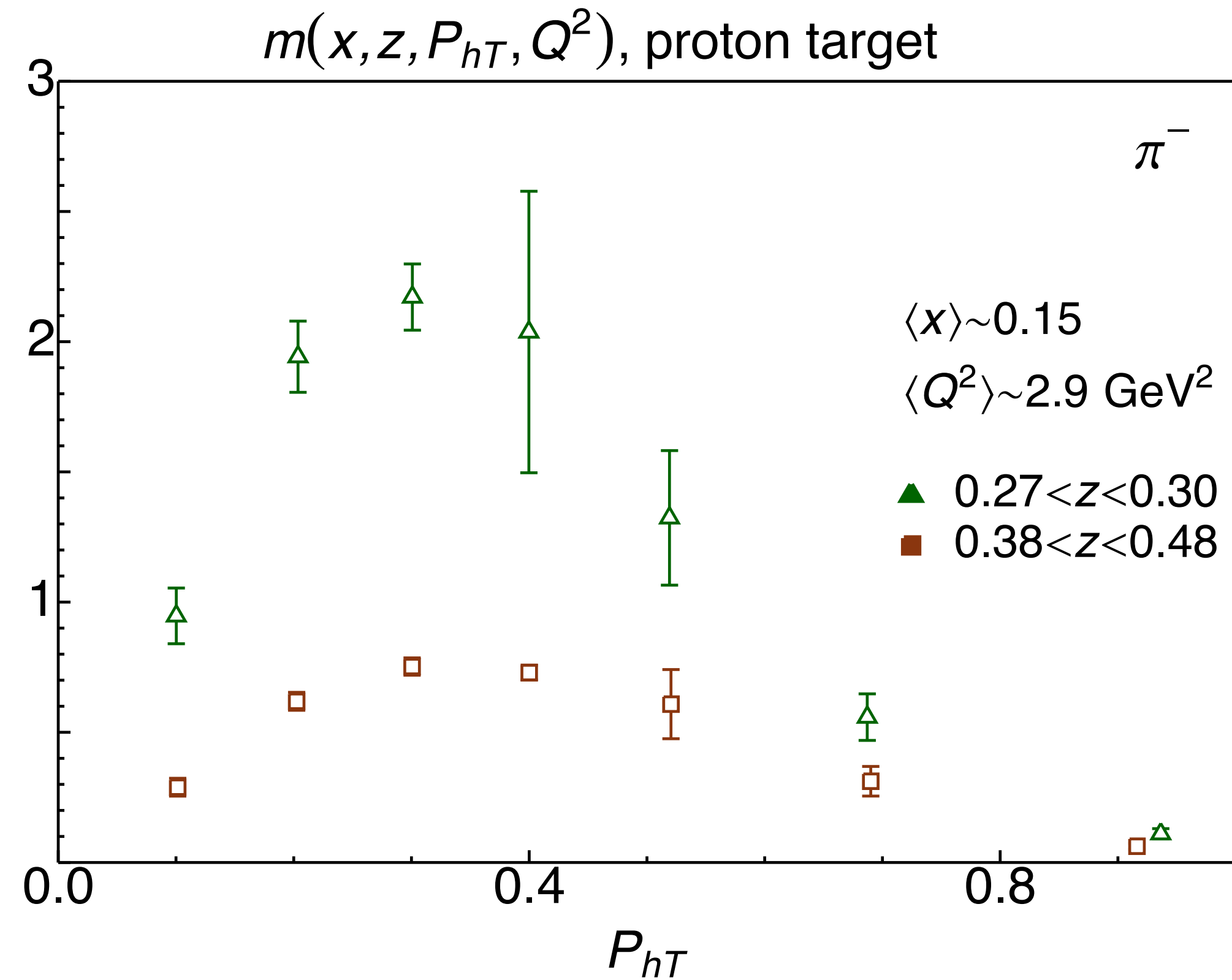


The fit implements TMD evolution at Next-to-Next-to-Leading log (state of the art)

Several choices are peculiar to this fit and not “standard”

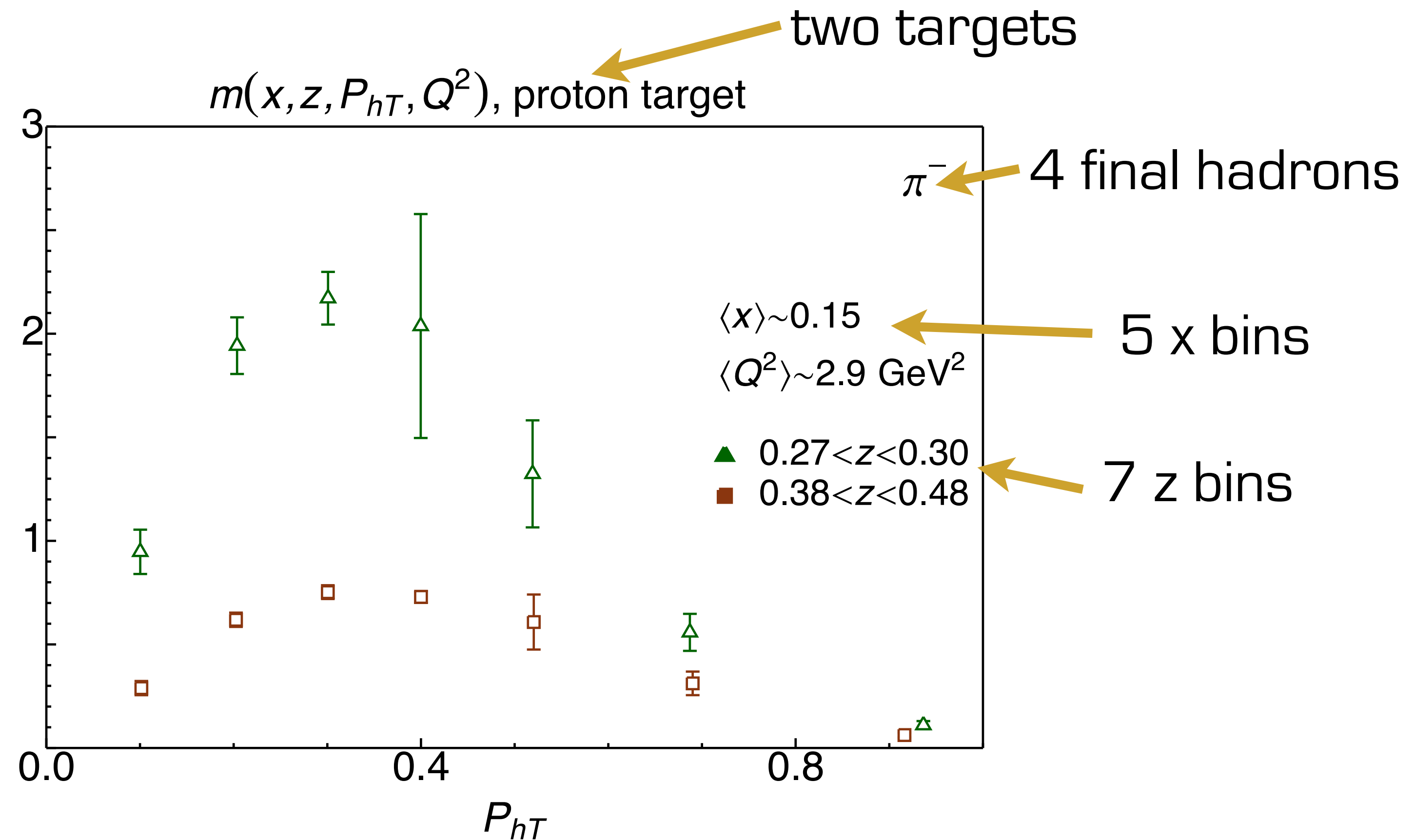
The agreement with data is excellent ( $\chi^2/\text{dof} = 1.10$ )

# The replica method



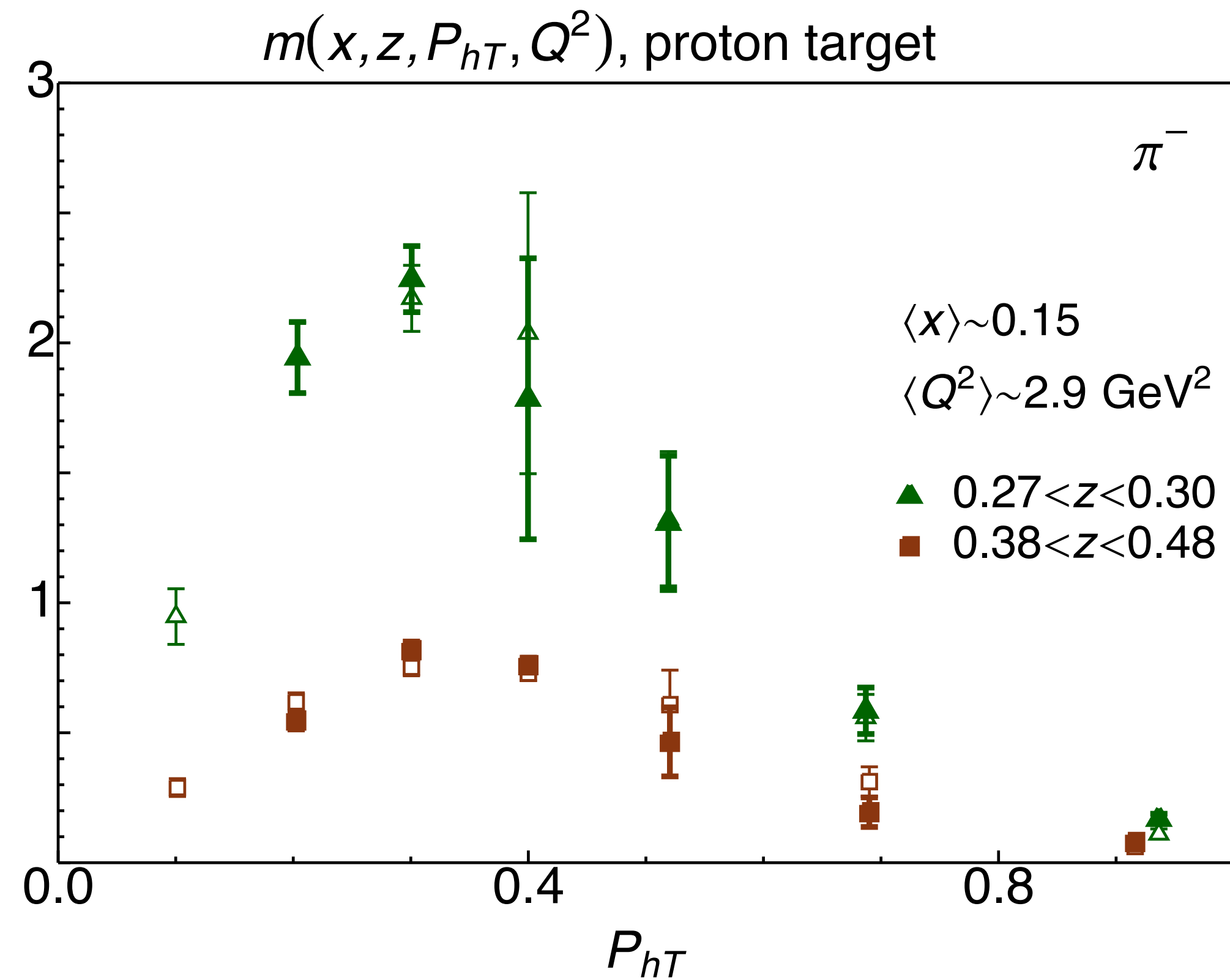
Example of original data

# The replica method



Example of original data

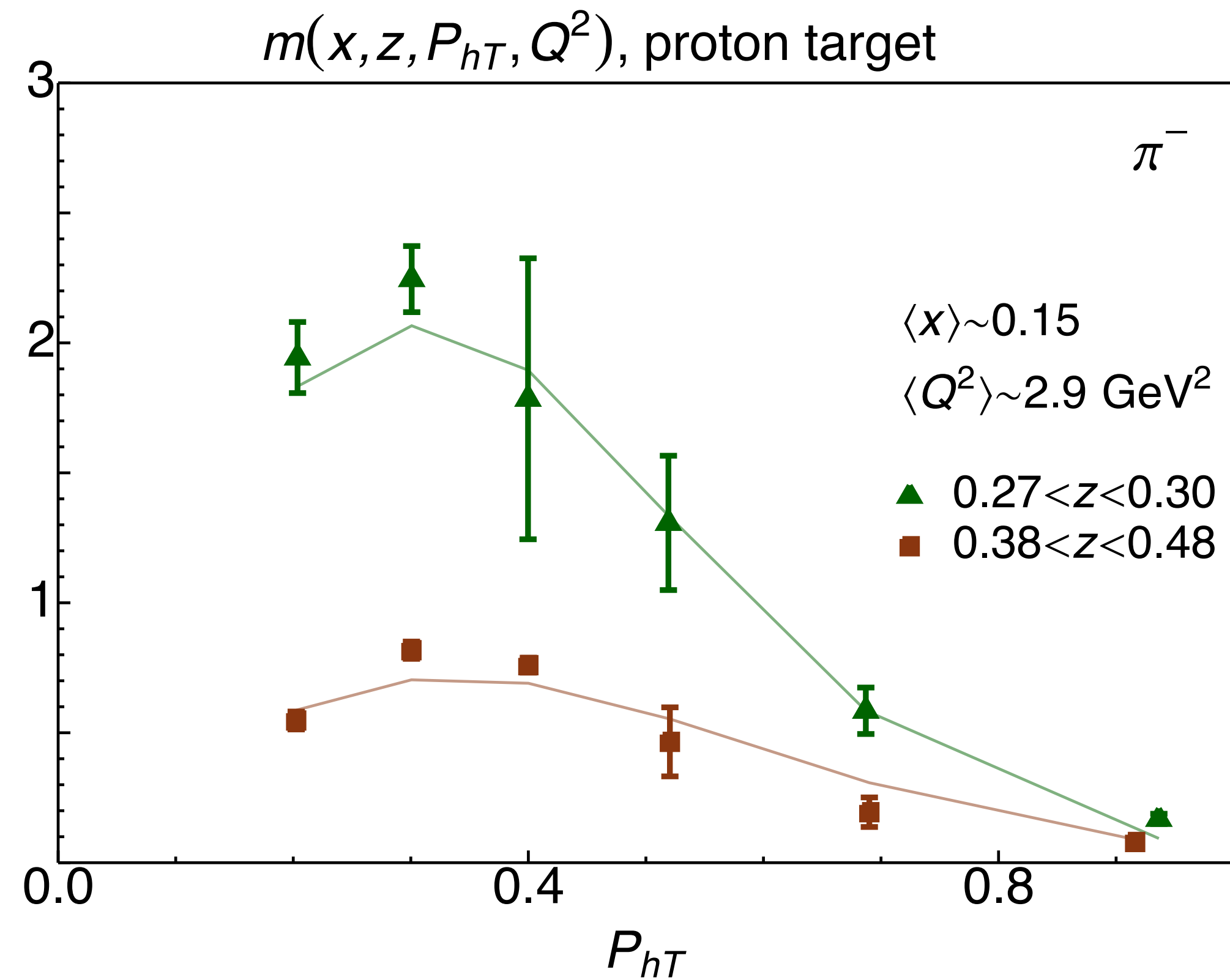
# The replica method



Data are replicated (with Gaussian distribution)

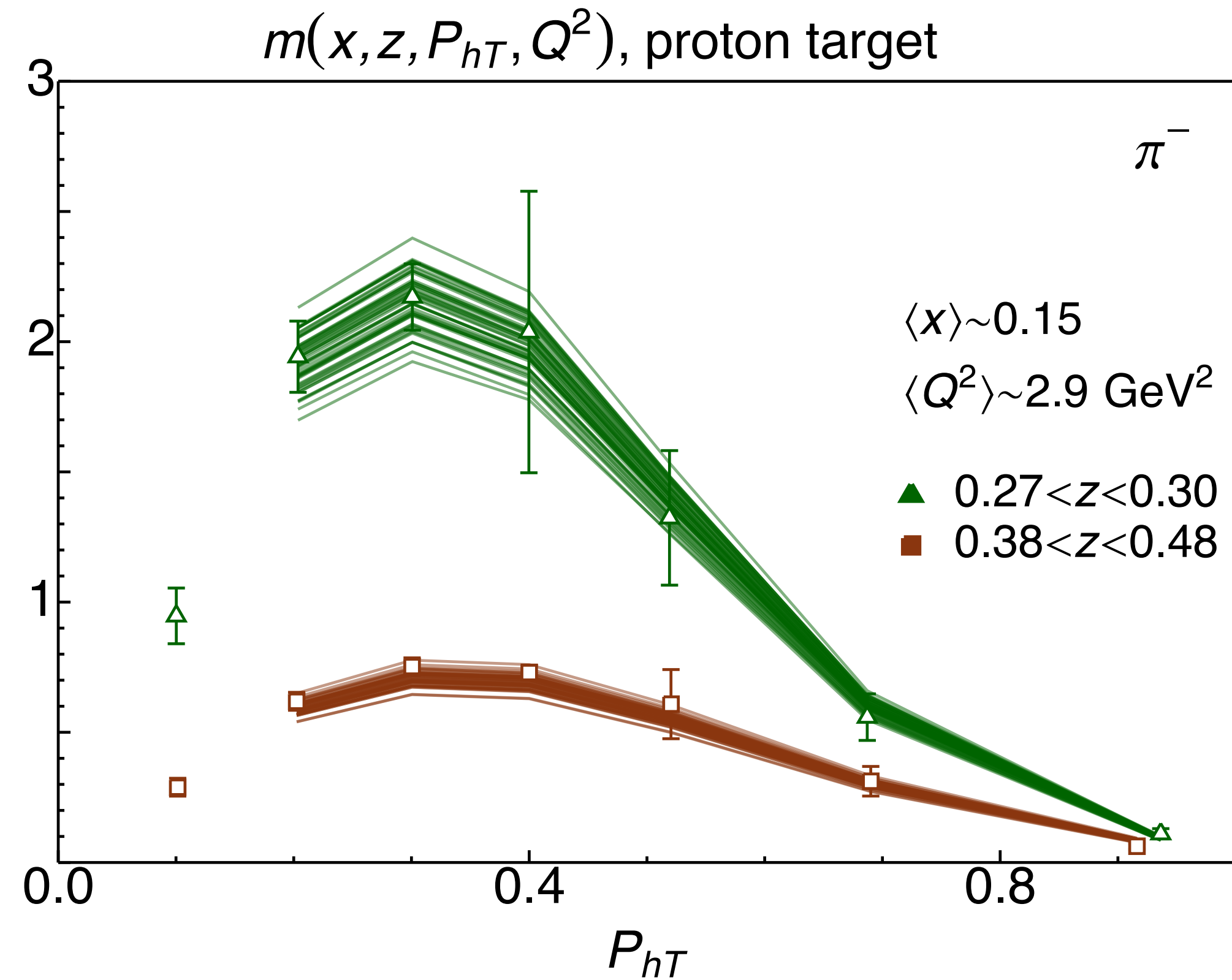


# The replica method



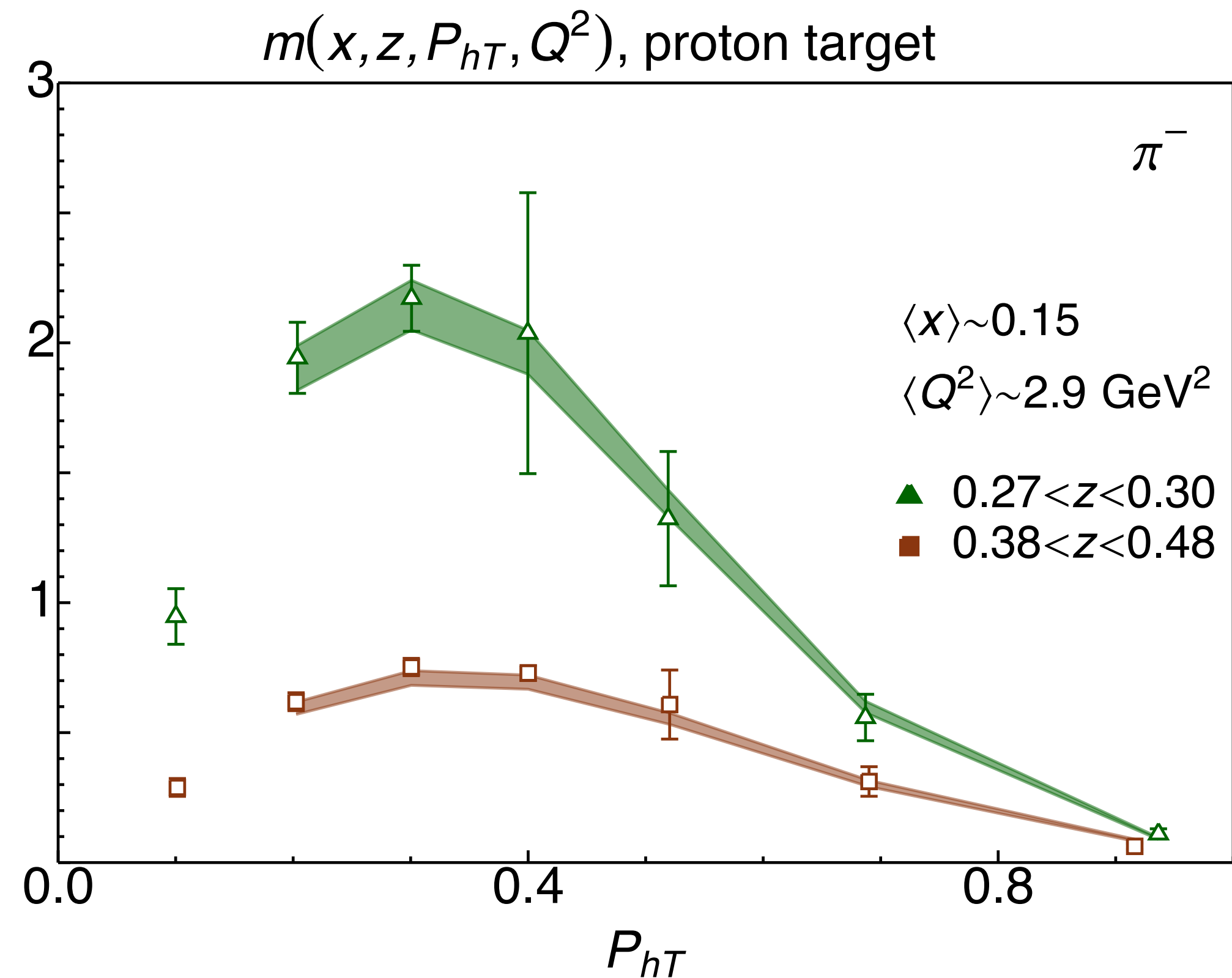
The fit is performed on the replicated data

# The replica method



The procedure is repeated 200 times

# The replica method



For each point, a central 68% confidence interval is identified