The QCD Equation of State at $\mu_B > 0$ from Lattice QCD

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arXiv:1701.04325 [hep-lat]

BROOKHAVEN NATIONAL LABORATORY

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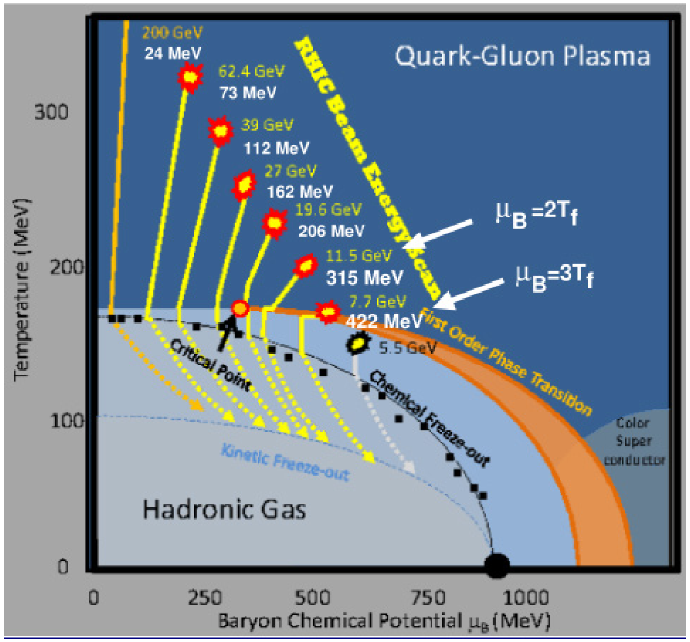
Motivation

- The equation of state (EoS)
  → the most basic characterization of equilibrium properties of strong-interaction matter

- The EoS at non-vanishing (μ_B, μ_S, μ_Q)
  → important for hydrodynamic modeling of conditions met in BES@RHIC
- Beam energy range expected in BES@RHIC
  → 7.7GeV $\leq \sqrt{s_{NN}} \leq$ 200GeV
  → 0 $\leq \mu_B/T \leq$ 3

- Lattice QCD simulations at $\mu > 0$
  → suffers from the well-known sign problem
- The Taylor expansion method
  → small values of chemical potentials can can be studied
  → higher order of expansion coefficients are needed to cover 0 $\leq \mu_B/T \leq$ 3
  → This study: up to **6th** order



Taylor expansion : pressure

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}(T)}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

Generalized susceptibilities

$$\chi_{ijk}^{BQS}(T) \equiv \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\hat{\mu}_{B,Q,S}=0}$$

($\chi_{ijk}^{BQS} = 0$ for i+j+k odd because of charge symmetry)

$$\hat{\mu}_X \equiv \frac{\mu_X}{T}$$

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

The number density

$$\frac{n_X}{T^3} = \frac{\partial P/T^4}{\partial \hat{\mu}_X}$$

Taylor expansion : energy and entropy densities

Energy density

$$\frac{\epsilon}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\Xi_{ijk}^{BQS}(T) + 3\chi_{ijk}^{BQS}(T)}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

Entropy density

$$\frac{s}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\Xi_{ijk}^{BQS}(T) + (4-i-j-k)\chi_{ijk}^{BQS}(T)}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k$$

Temperature derivative of generalized susceptibilities

$$\Xi_{ijk}^{BQS}(T) \equiv T \frac{d\chi_{ijk}^{BQS}(T)}{dT}$$

Hadron resonance gas model

- based on non-interacting hadrons
- describes the thermodynamics for low temperature, hadronic regime well

For simplicity, $\mu_Q=\mu_S=0$

$$P^{HRG}(T,\mu_B) = \underline{P_M(T)} + \underline{P_B(T,\hat{\mu}_B)}$$

Mesonic contributions Baryonic contributions

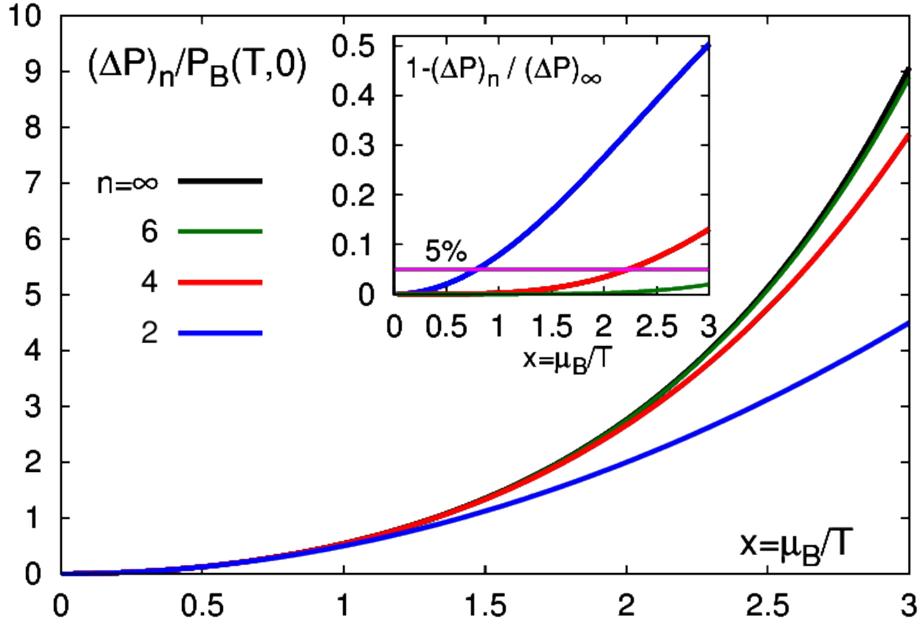
In the Boltzmann approximation (good for the baryonic sector)

$$\Rightarrow \quad \boxed{P_B(T,\hat{\mu}_B) = P_B(T,0) + P_B(T,0)(\cosh(\hat{\mu}_B) - 1)}$$

Truncation effects in HRG model

Truncation at (2n)-th order

$$(\Delta P)_{2n} \equiv (P_B(T,\hat{\mu}_B) - P_B(T,0))_{2n} \simeq P_B(T,0) \sum_{k=1}^{n} \frac{1}{2k!} \hat{\mu}_B^{2k}$$



The 4th order result is already good for $\mu_B \leq 2T$

Deviation from the exact result ≤ 5%

In the interesting temperature range mesonic contributions are dominant
→ Truncation effects are much smaller in the total pressure

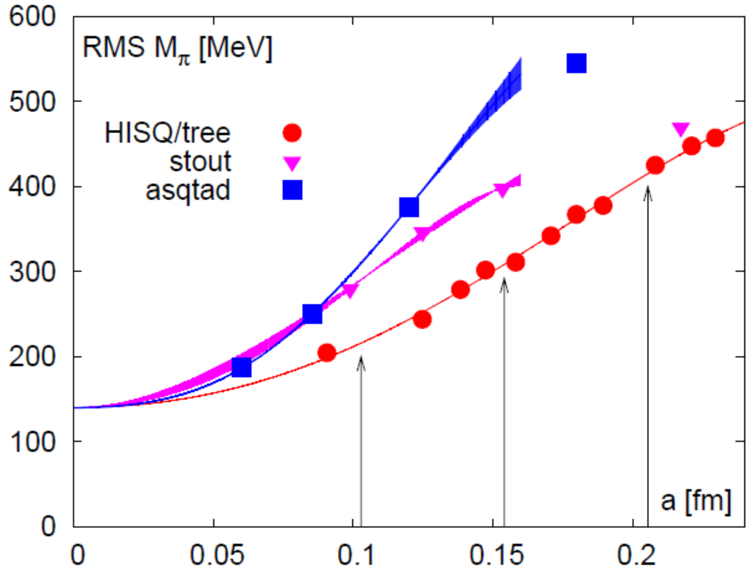
Note: In the high temperature limit → a massless ideal gas of quarks and gluons

The pressure is just a 2nd order polynomial

$$\frac{P^{\mathrm{idal}}}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,s} \left[\frac{7\pi^2}{65} + \frac{1}{2}\hat{\mu}_f^2 + \frac{1}{4\pi^2}\hat{\mu}_f^4 \right]$$

Simulation setup

- Tree level improved Symanzik gauge action
- Highly Improved Staggered Quark (HISQ) action in (2+1)-flavor
- Physical strange quark mass
- 2 different light quark masses:
  - $m_l/m_s = 1/27$ ($m^G_\pi \simeq 140$ MeV) , $1/20$ ($m^G_\pi \simeq 160$ MeV)
- Aspect ratio $N_\sigma/N_\tau = 4$
- 4 different tempral lattice sizes:
  - $N_\tau = 6, 8, 12, 16$
    → continuum extrapolation
- Temperature range: 135 – 330 MeV
- Calculating generalized susceptibilities up to **6th** order

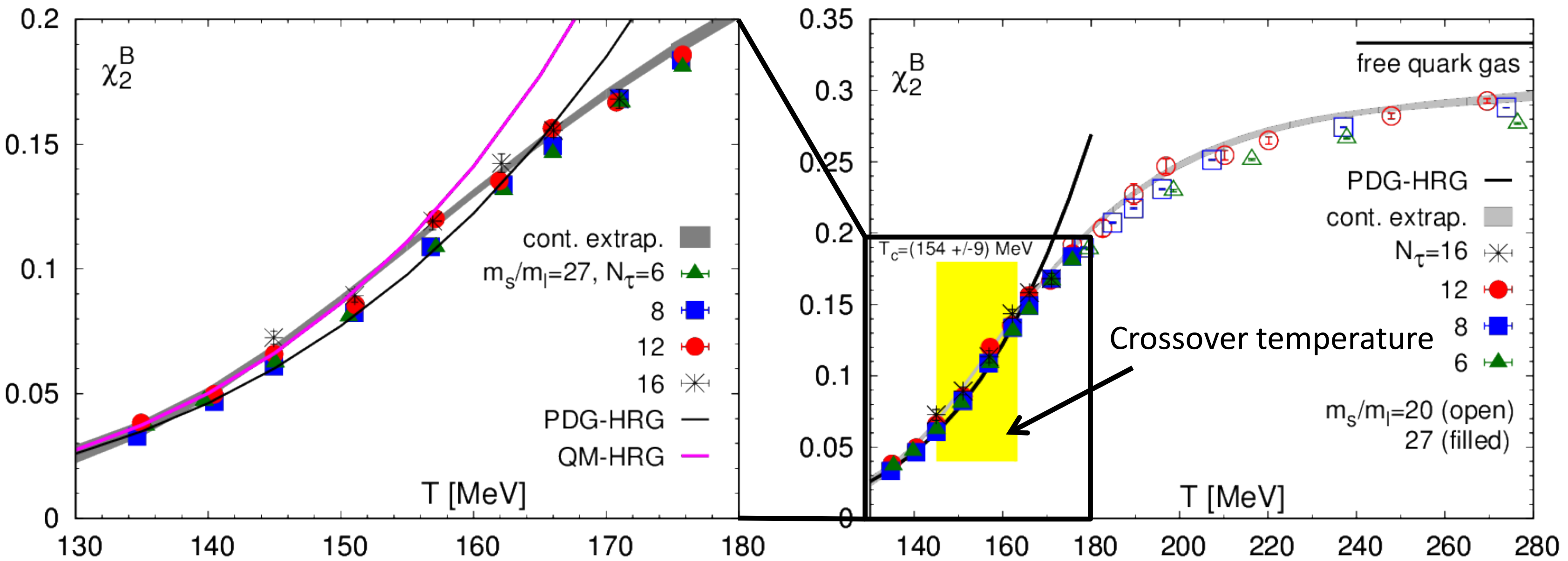


A. Bazavov *et al*. [HotQCD Collaboration], PRD 85, 054503 (2012)

$$\mu_Q = \mu_S = 0$$

Generalized susceptibilities at $\mu_Q = \mu_S = 0$

$$\frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2\left[1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\mathcal{O}(\hat{\mu}_B^6)\right]$$



PDG-HRG: based on all hadron resonances listed by the particle data group

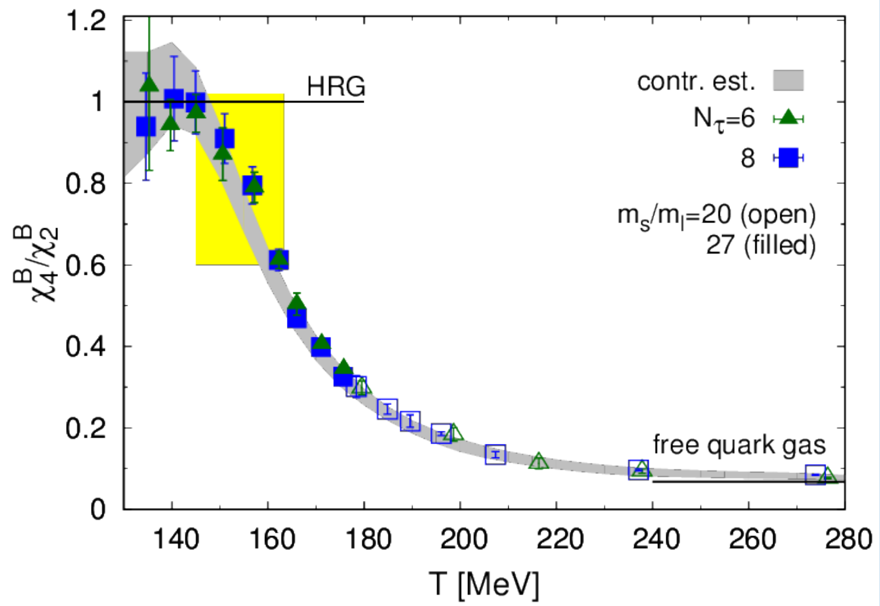
QM-HRG: based on the PDG + additional hadron resonances predicted in quark model calculations

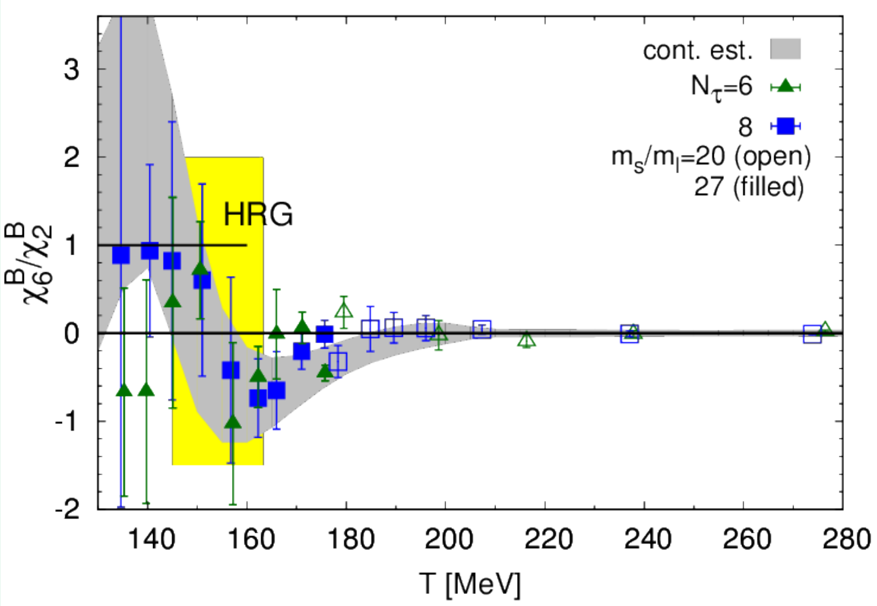
A. Majumder and B. Muller, PRL 105, 252002 (2010); A, Bazavov *et al.*, PRL 113, 072001 (2014)

At low temperatures the QCD results overshoot the PDG-HRG results but agree well with the QM-HRG results at T < 150 MeV.

Generalized susceptibilities at $\mu_Q = \mu_S = 0$

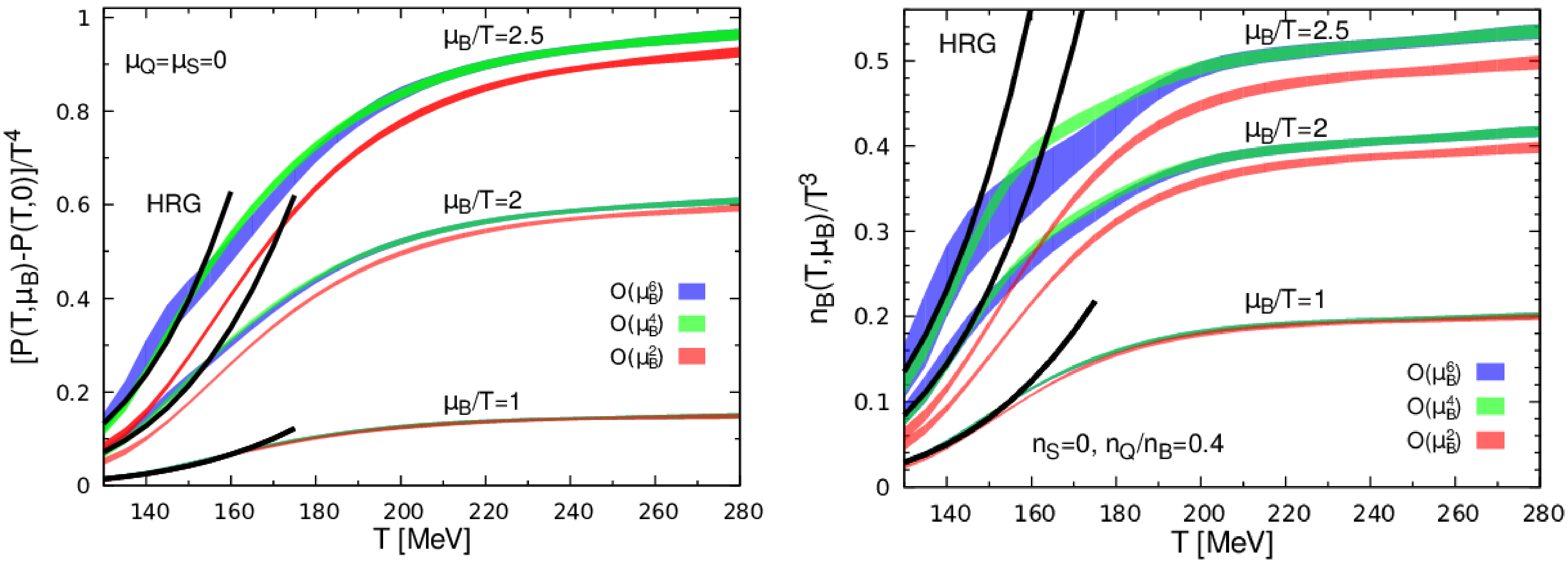
$$\frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2 \left[1 + \frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2 + \frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4 + \mathcal{O}(\hat{\mu}_B^6)\right]$$





Both NLO and NNLO results agree with the HRG results at T < 150 MeV.
The NLO contribution is large in the hadronic regime but becomes rapidly small at large T.
Statistical errors for the current NNLO results at low temperatures are still large.

μ_B dependence of pressure and baryon number density at $\mu_Q = \mu_S = 0$



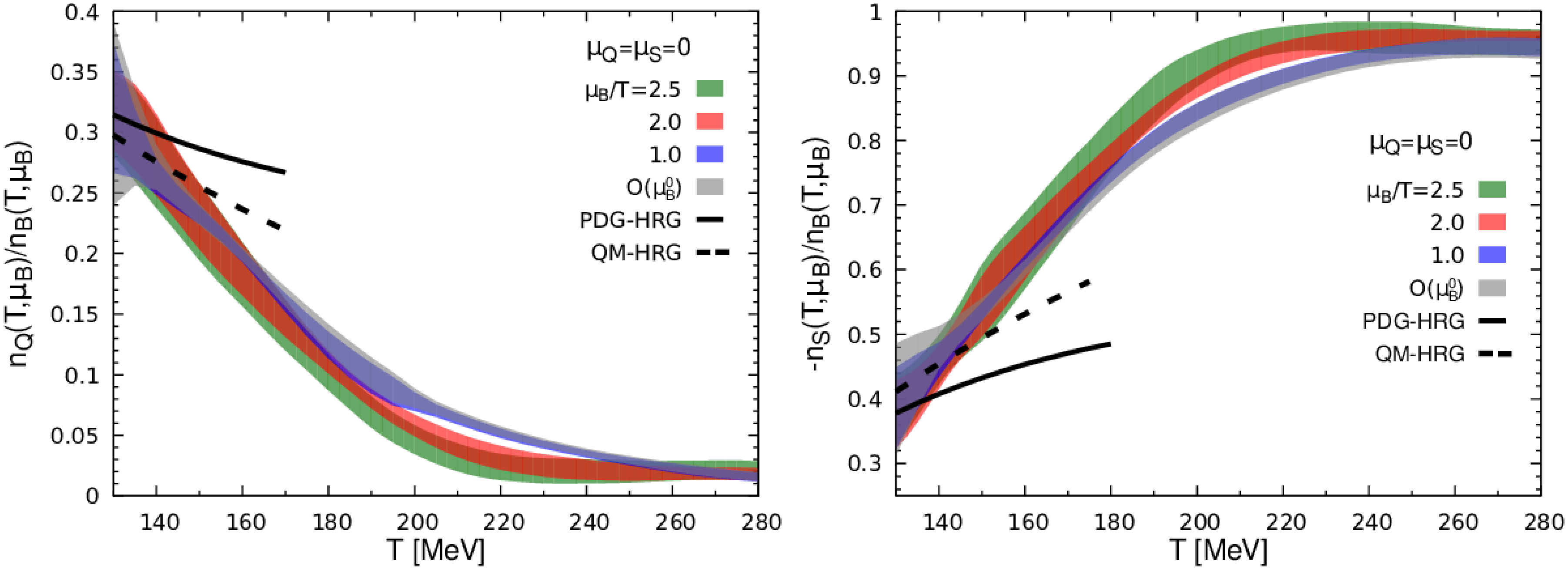
Pressure is well described at $\mu_B \lesssim 2T$.

NNLO corrections both for pressure and n_B are small at high temperatures even at $\mu_B = 2.5T$.

Getting the dip in χ^B_6/χ^B_2 at $T \simeq 160$ MeV under control is important to understand the EoS close to the transition region.

Net electric charge and strangeness densities at $\mu_Q = \mu_S = 0$

$$\frac{n_X}{n_B} = \frac{\chi_{11}^{BX} + \frac{1}{6}\chi_{31}^{BX}\hat{\mu}_B^2 + \frac{1}{120}\chi_{51}^{BX}\hat{\mu}_B^4}{\chi_2^B + \frac{1}{6}\chi_4^B\hat{\mu}_B^2 + \frac{1}{120}\chi_6^B\hat{\mu}_B^4}$$



HRG: sensitive to the charged (strange) baryon content in the model
→ difference between PDG-HRG and QM-HRG

QM-HRG describes the QCD results better than PDG-HRG at low temperatures.

Energy and entropy densities $\mu_Q = \mu_S = 0$

$$\frac{\epsilon(T,\mu_B)}{T^4} - \frac{\epsilon(T,0)}{T^4} = \sum_{k=1}^{\infty} \epsilon_{2k}(T)\hat{\mu}_B^{2k}$$

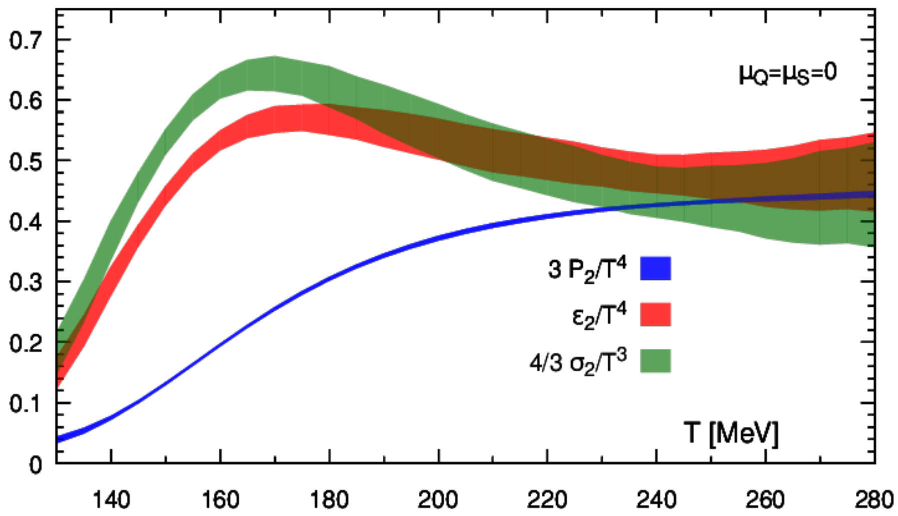
$$\frac{s(T,\mu_B)}{T^4} - \frac{s(T,0)}{T^4} = \sum_{k=1}^{\infty} \sigma_{2k}(T)\hat{\mu}_B^{2k}$$

Expansion coefficients

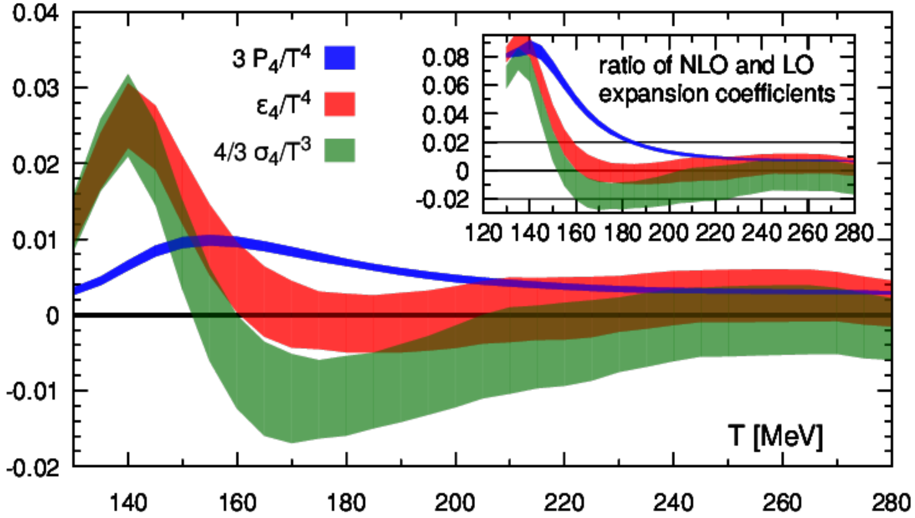
$$\epsilon_{2k} = TP'_{2k} + 3P_{2k}$$

$$\sigma_{2k} = \epsilon_{2k} - (2k-1)P_{2k}$$

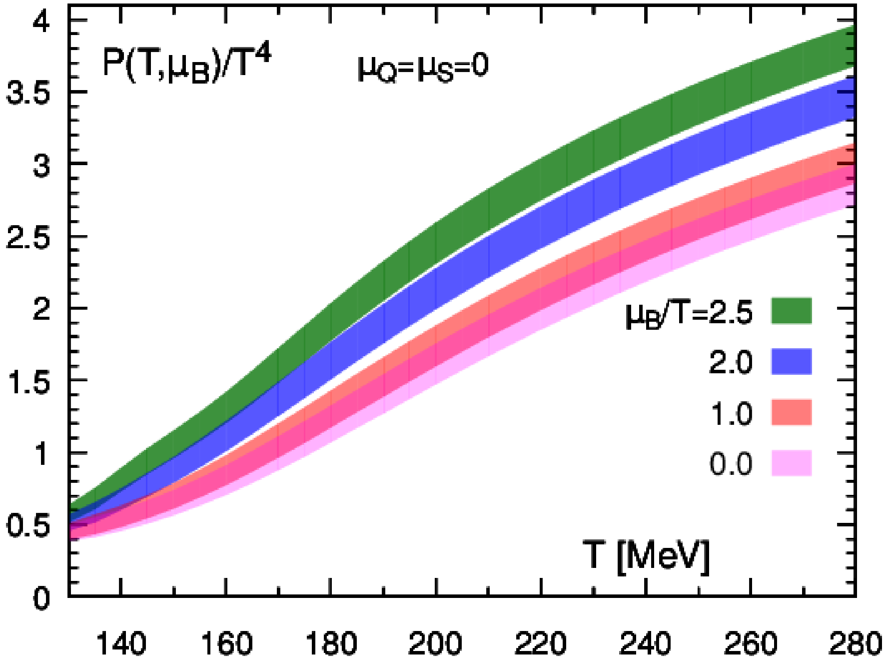
LO

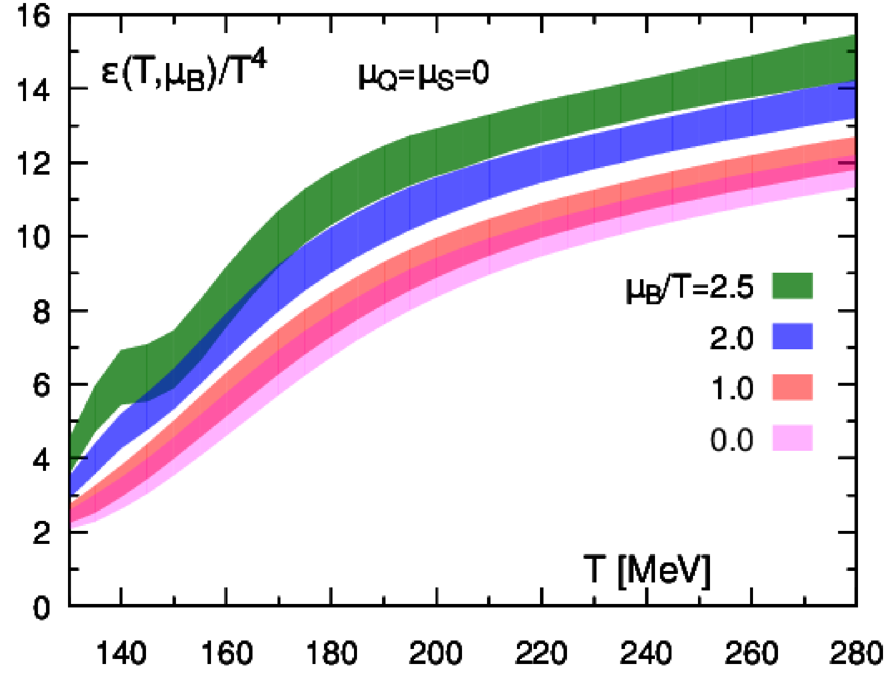


NLO



Total pressure and energy density at $\mu_Q = \mu_S = 0$

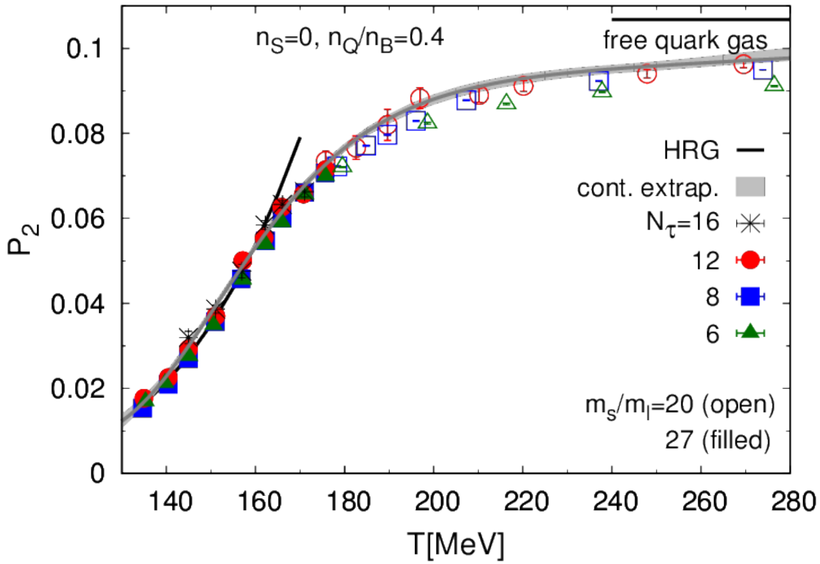




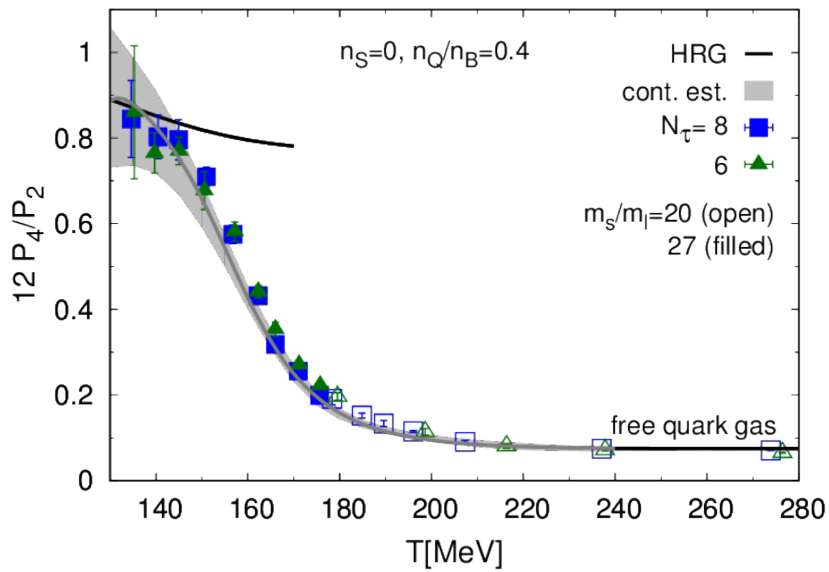
Errors at $\mu_B = 0$ is still dominant despite of the large errors of higher order expansion coefficients.

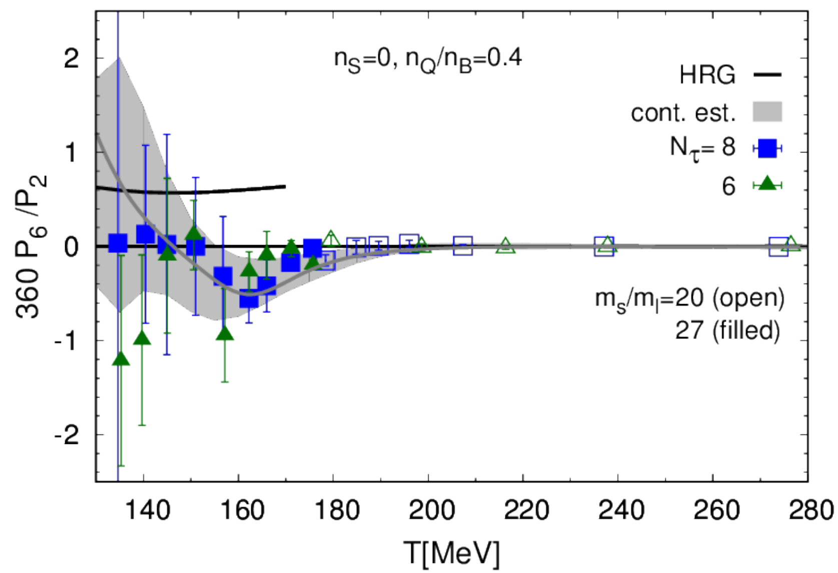
$$n_S = 0, \quad n_Q/n_B \neq 0$$

Expansion coefficients of pressure for $n_S = 0, n_Q/n_B = 0.4$

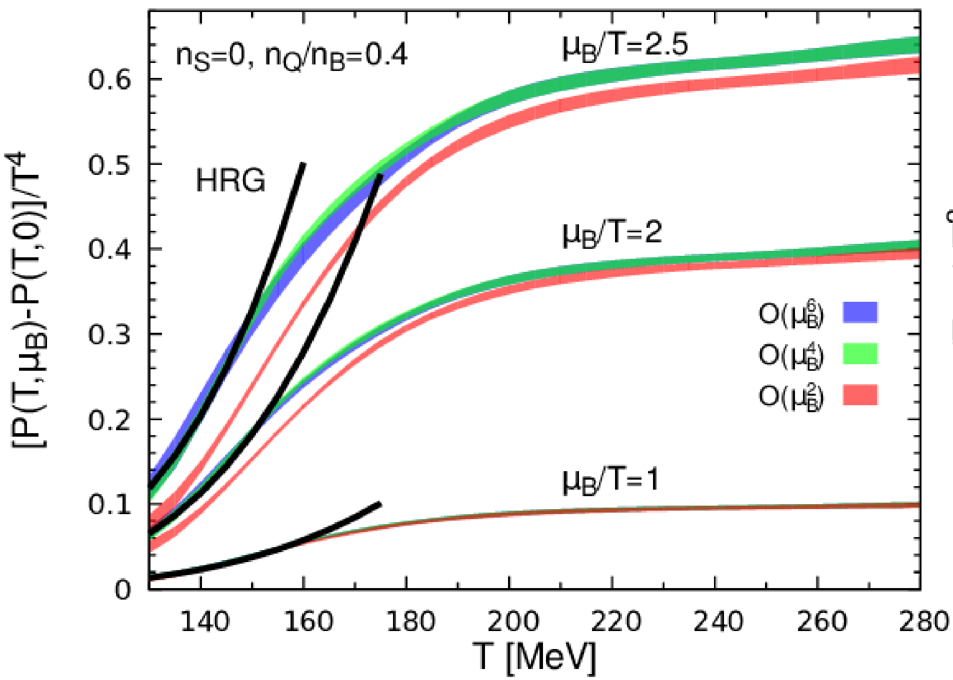


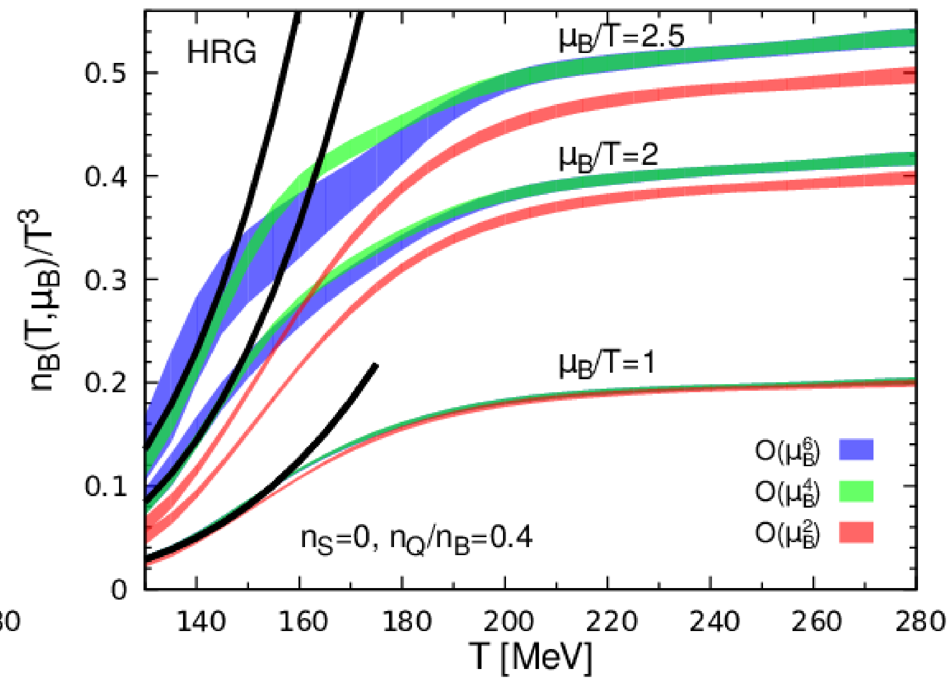
The NLO and NNLO corrections relative to the LO contributions are smaller than those in the $\mu_S = \mu_Q = 0$ case.
→ Errors of the 6th order coefficients are less important.





μ_B dependence of pressure and baryon number density for $n_S = 0$, $n_Q/n_B = 0.4$



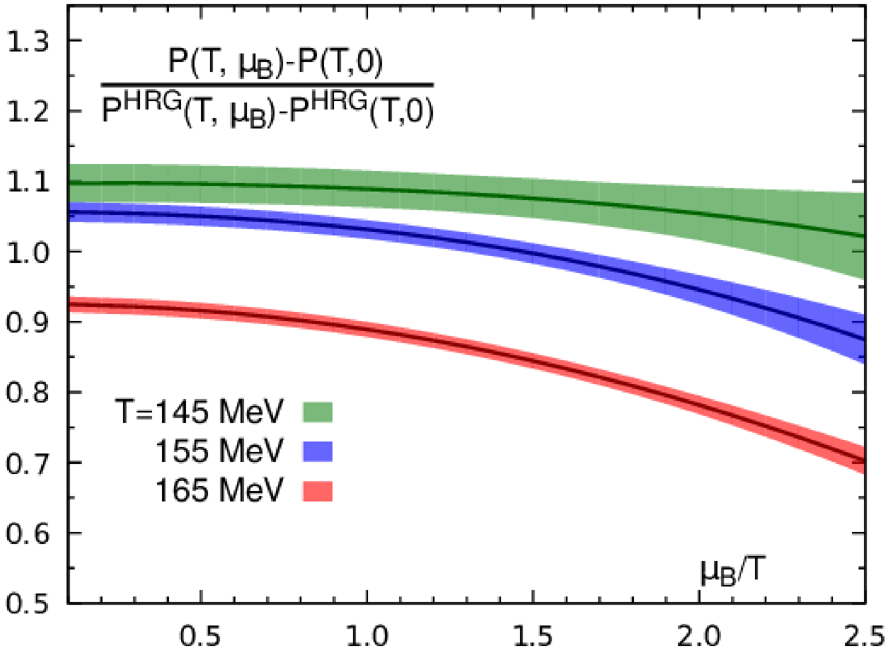


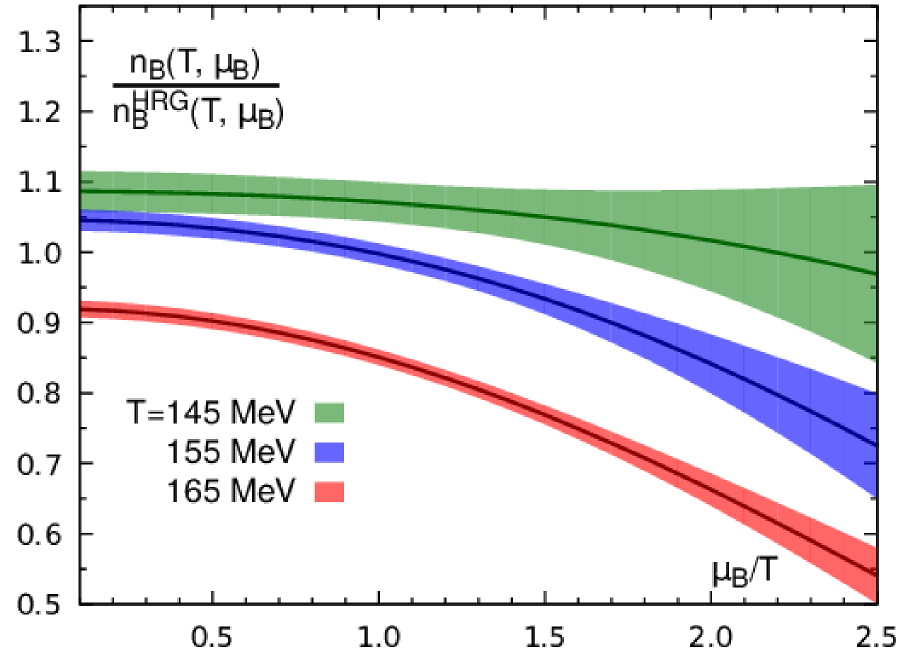
The pressure and n_B agree quite well with the HQG calculations at low temperatures.

The agreement is getting worse for larger μ_B and higher T.

→ HRG models fail to describe the physics in the crossover region for so large μ_B and T ≥ 160 MeV.

μ_B dependence of pressure and baryon number density for $n_S = 0$, $n_Q/n_B = 0.4$





The pressure and n_B agree quite well with the HQG calculations at low temperatures.
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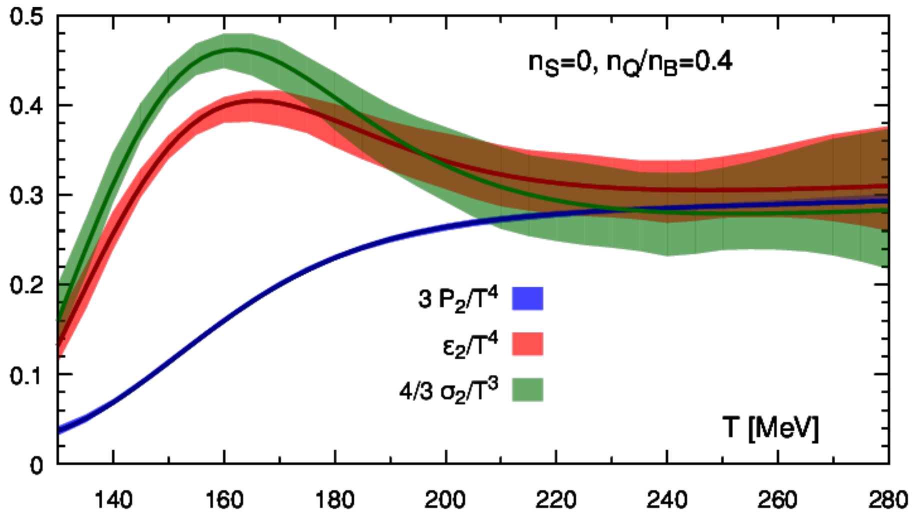
→ HRG models fail to describe the physics in the crossover region for so large μ_B and T ≥ 160 MeV.

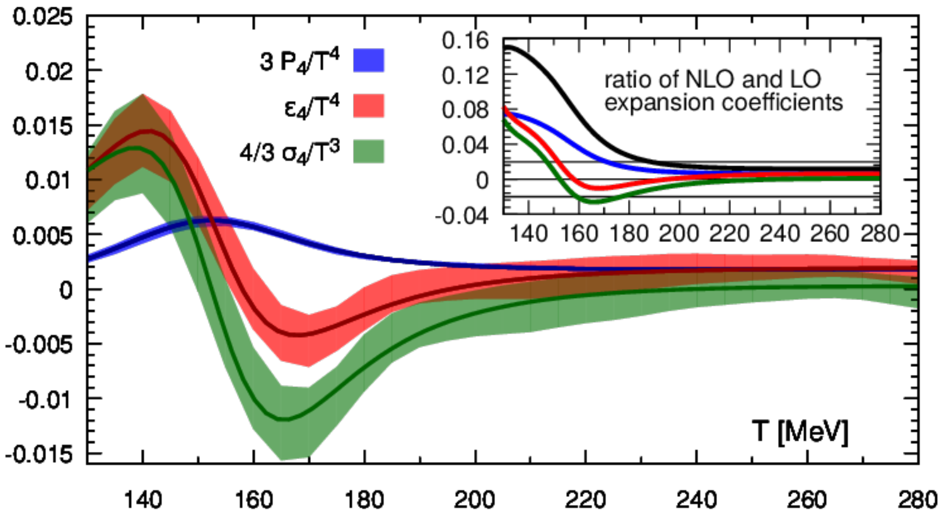
Expansion coefficients of pressure, energy and entropy densities for $n_S = 0$, $n_Q/n_B = 0.4$

Expansion coefficients

$$\epsilon_{2n}(T) = 3P_{2n}(T) + TP'_{2n}(T) - r\sum_{k=1}^{n} Tq'_{2k-1}N^B_{2n-2k+1}$$

$$\sigma_{2n}(T) = 4P_{2n}(T) + TP'_{2n}(T) - N^B_{2n-1} - r\sum_{k=1}^{n}(q_{2k-1} + Tq'_{2k-1})N^B_{2n-2k+1}$$

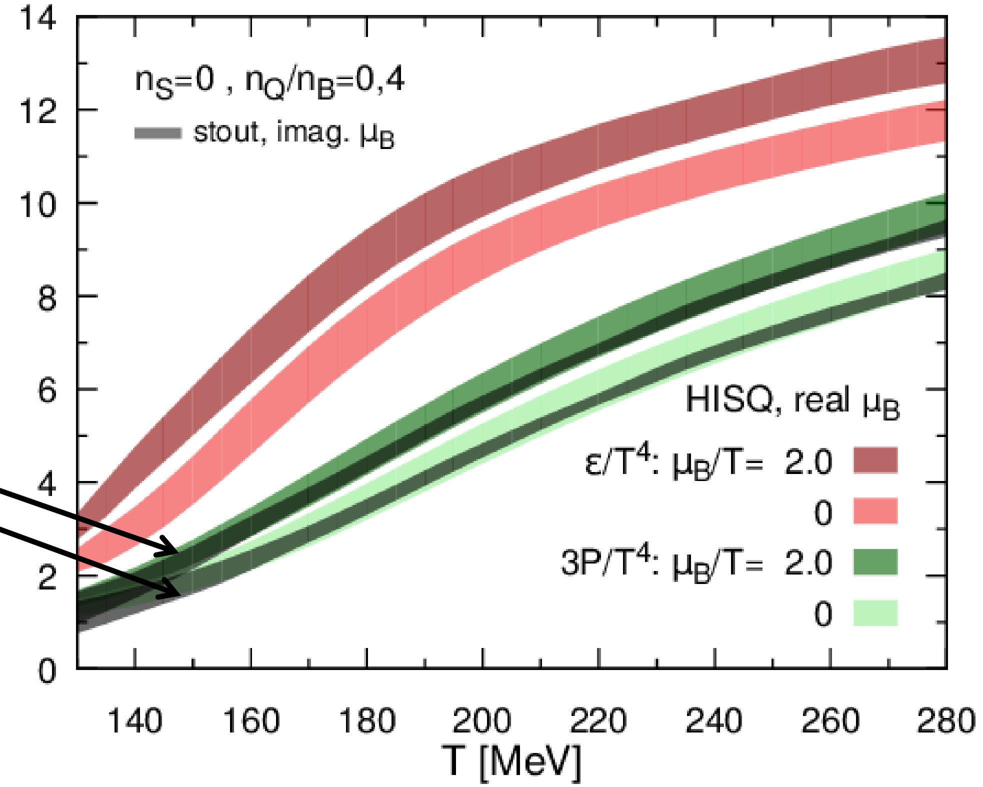




Total pressure and energy density for $n_S = 0$, $n_Q/n_B = 0.4$

The stout discretization scheme
+ analytic continuation from imaginary μ

S. Borsanyi *et al.*, PLB 730 99 (2014):
J. Gunther *et al.*, arXiv:1607.02493 [hep-lat]



Errors at $\mu_B = 0$ is still dominant despite of the large errors of higher order expansion coefficients.
The analytic continuation results agree with the Taylor expansion results.
The analytic continuation results tend to stay below the central values of the Taylor expansion results.

Lines of constant physics

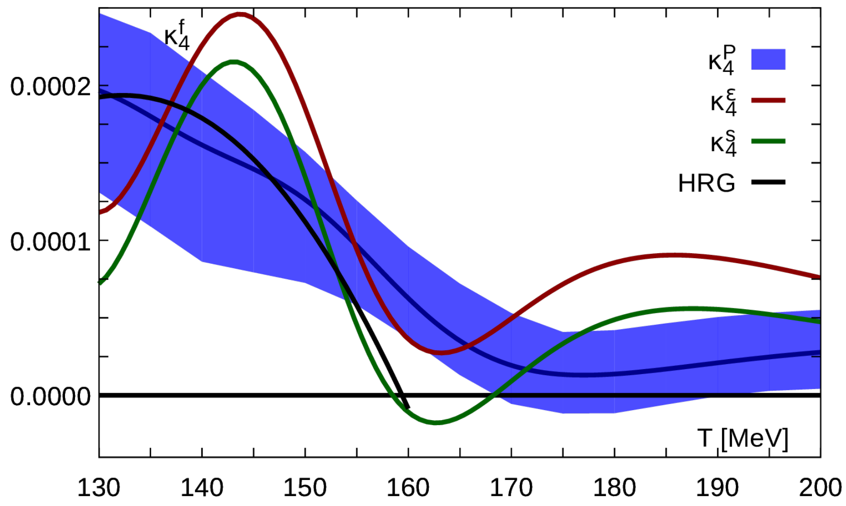
The thermal conditions at the time of chemical freeze-out in heavy ion collisions can be characterized by the lines of constant thermodynamic observables in T-μ_B plane.

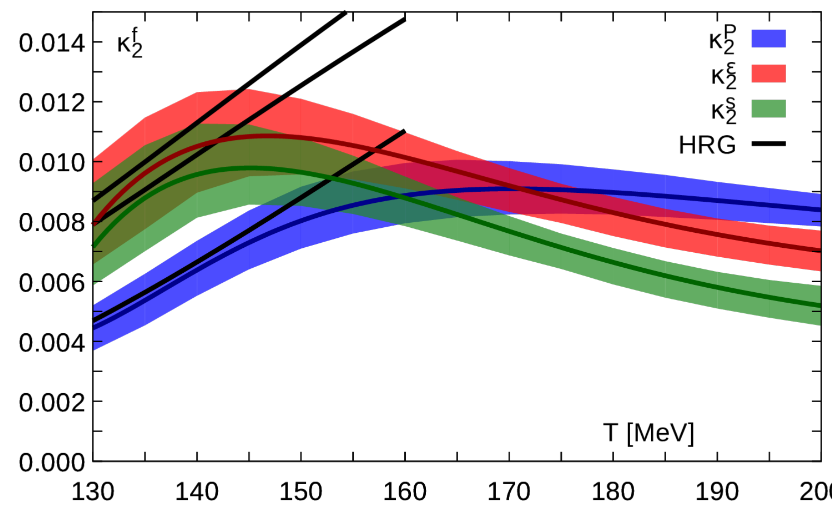
J. Cleymans and K. Redlich, PRC 60 054908 (1999)
J. Cleymans *et al.*, PRC 73 034905 (2006)

$$T_f(\mu_B) = T_0\left(1 - \kappa_2^f\left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f\left(\frac{\mu_B}{T_0}\right)^4\right)$$: freeze-out temperature

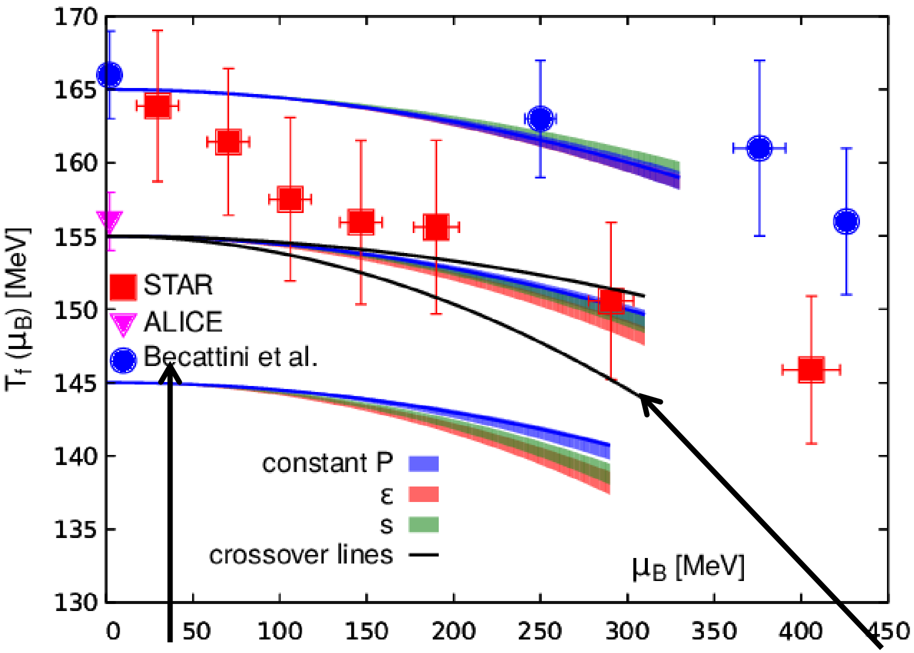
$$\kappa_2^f = \frac{f_2(T_0)}{T_0 \frac{\partial f_0(T)}{\partial T}\Big|_{(T_0,0)}}$$

$$\kappa_4^f = \frac{\frac{1}{2}T_0^2\frac{\partial^2 f_0(T)}{\partial T^2}\Big|_{(T_0,0)}(\kappa_2^f)^2 - \left(T_0\frac{\partial f_2(T)}{\partial T}\Big|_{(T_0,0)} - 2f_2(T_0)\right)\kappa_2^f + f_4(T_0)}{T_0\frac{\partial f_0(T)}{\partial T}\Big|_{(T_0,0)}}$$

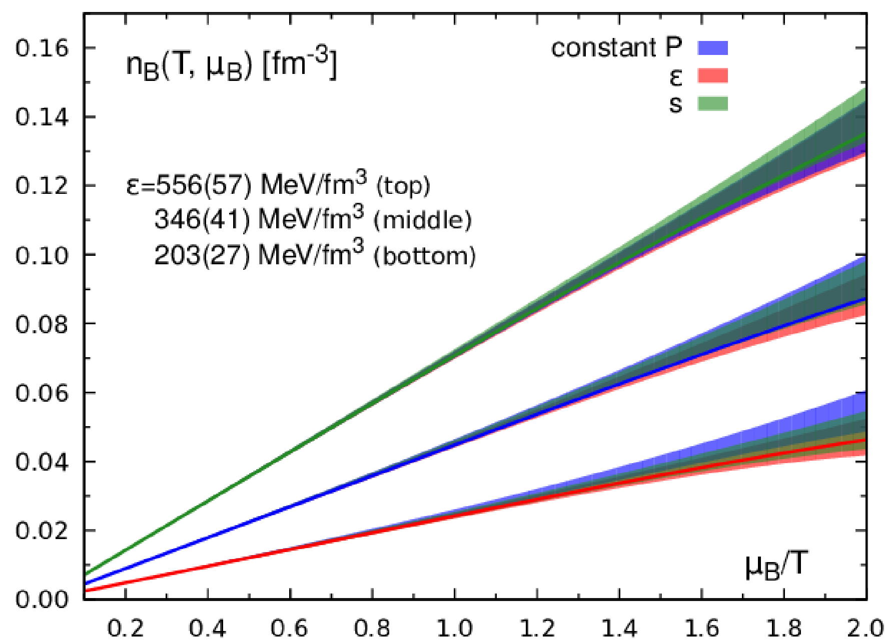




Lines of constant physics



S. Das [STAR Collaboration], EPJ Web Conf. 90, 08007 (2015)
M. Floris, NPA 931, 103 (2014)
F. Becattini *et al.*, PLB 764, 241 (2017)



O. Kaczmarek *et al.*, PRD 83, 014504 (2011)
G. Endrodi *et al.*, JHEP 1104, 001 (2011)
P. Cea *et al.*, PRD 89, no. 7, 074512 (2014)
C. Bonati *et al.*, PRD 90, no. 11, 114025 (2014)

LCPs for constant pressure, energy and entropy densities agree with each other at $\mu_B \lesssim 2T$.
Constant pressure and entropy density cannot hold simultaneously.
The star data do not follow any LCPs.

Radius of convergence

The series for the net baryon number susceptibility should diverge at the critical point.

R. V. Gavai and S. Gupta, PRD 71, 114014 (2005)

Z. Fodor and S. D. Katz, JHEP 0404, 050 (2004)
S. Datta *et al*, arXiv:1612.06673
M. D'Elia *et al.*, arXiv:1611.08285

An estimator of the radius of convergence

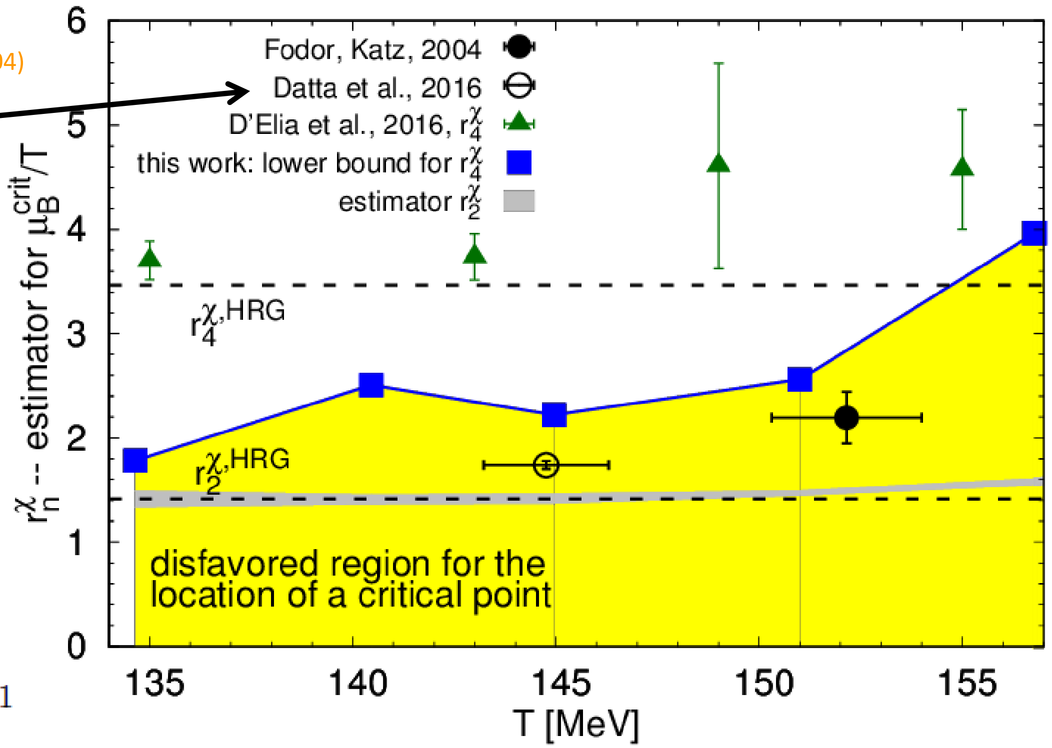
$$r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{1/2}$$

converges to the true value in the n → ∞ limit

For finite radius of convergence for n → ∞

→ $$|\chi_{n+2}^{B}/\chi_{n}^{B}| \sim n^2$$

At least for a large n, there should be large deviation from the HRG value $|\chi_{n+2}^{B}/\chi_{n}^{B}|^{HRG} = 1$



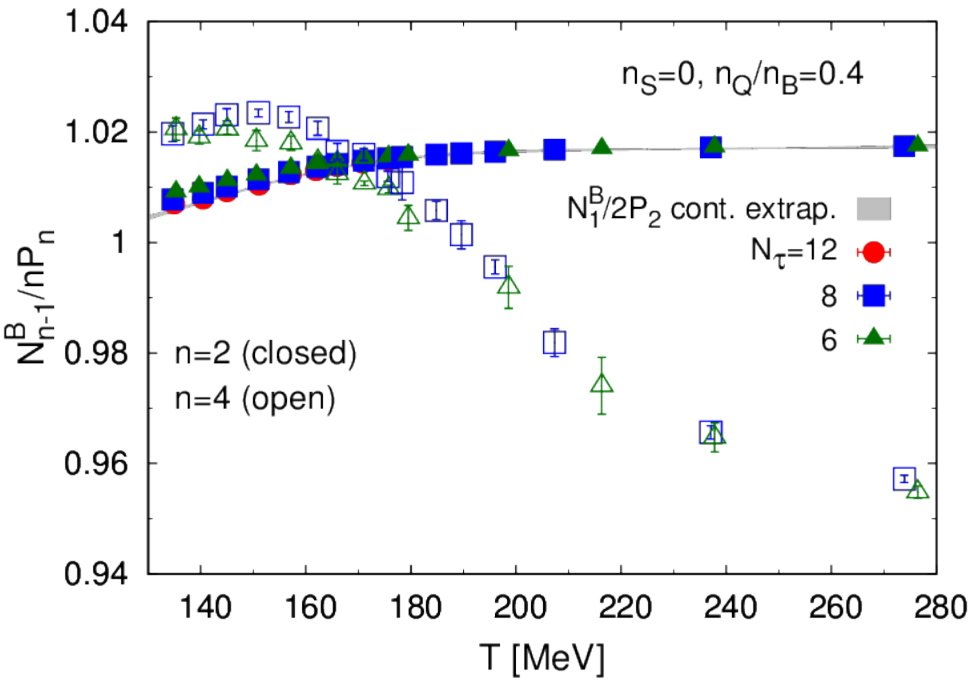
A critical point at $\mu_B \lesssim 2T$ in the temperature range 135MeV ≲ T ≲ 155 MeV is strongly disfavored in our study.
Its location at higher temperature also seems to be ruled out.

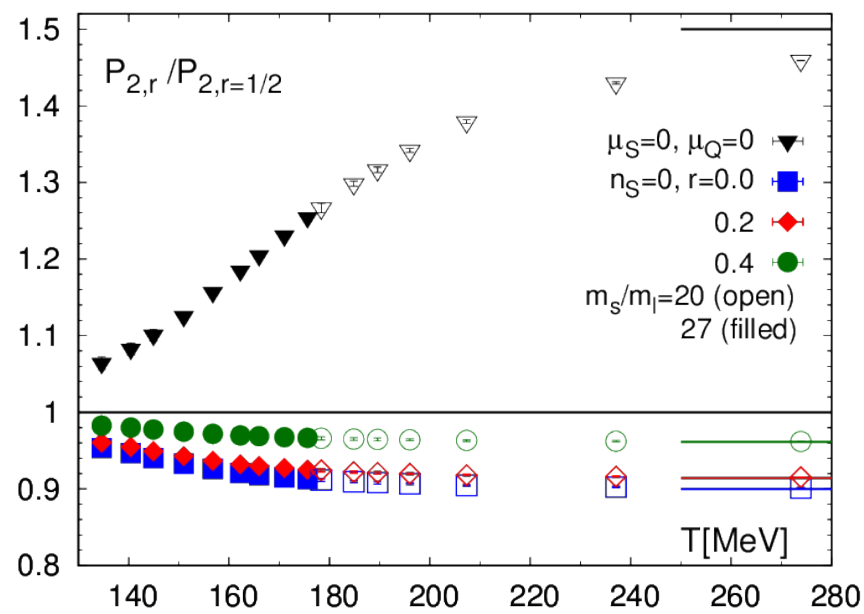
Summary

- The QCD equation of state at non-vanishing baryon chemical potential has been studied with Taylor expansions up to 6th order.
- The bulk thermodynamic quantities are well under control at $\mu_B \lesssim 2T$.
- We calculated lines of constant pressure, energy and entropy densities in T-μ_B plane.
- The radius of convergence has been estimated.
- Our current results suggest that a critical point in the QCD phase diagram is unlikely to located for $\mu_B \lesssim 2T$ at $T > 135$ MeV.

Backup slides

n_Q/n_B = r dependence of pressure coefficients





$$P_2 = \frac{1}{2}[N_1^B + rq_1N_1^B]$$

$$P_4 = \frac{1}{4}[N_3^B + r(q_1N_3^B + 3q_3N_1^B)]$$

$$P_6 = \frac{1}{6}[N_5^B + r(q_1N_5^B + 3q_3N_3^B + 5q_5N_1^B)]$$

r dependence is weak.

n_S = 0 systems are substantially different from μ_S = 0 systems

4^{th} and 6^{th} order pressure coefficients for $n_S = 0$, $n_Q/n_B = 0.4$