INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Analytical methods in understanding light meson dynamics at JLab 12

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Washington, DC, USA
February 3, 2017

Outline

1. Introduction and Motivation:

2. Light Meson Decays:
   Ex: $\eta \to 3\pi$ and light quark masses

3. Conclusion and Outlook

1. Introduction and Motivation

1.1 JLab @ 12 GeV

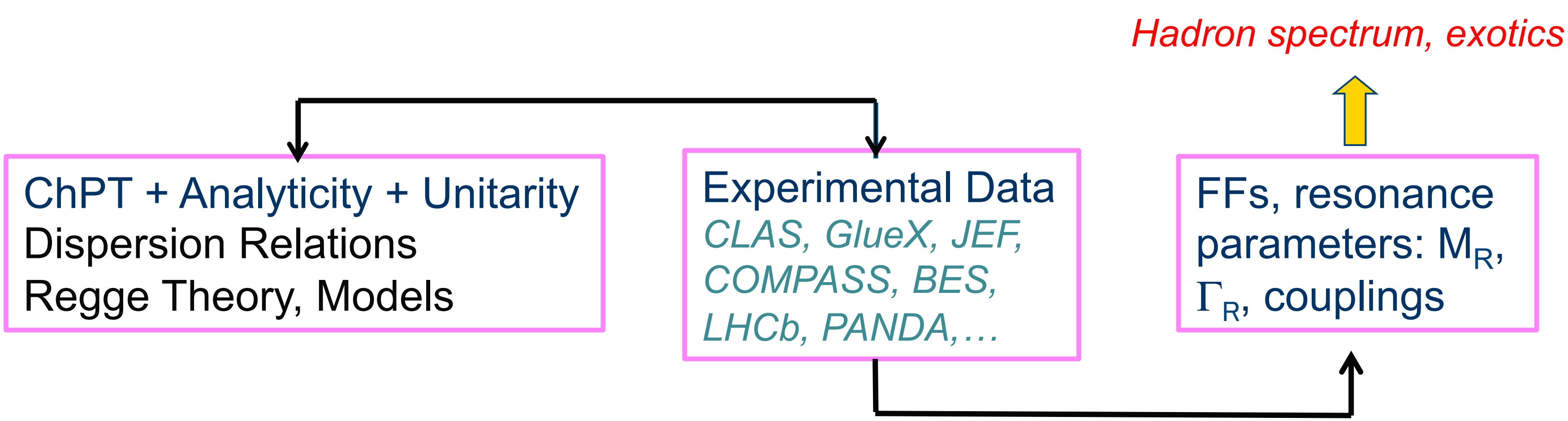
- 12 GeV upgrade at JLab: *CLAS, GlueX, etc.*: In the study of hadron spectroscopy, large amount of very precise data on meson physics will be collected, background for searches of new states

- Unique opportunity:
  - Test chiral dynamics at low energy
  - Extract fundamental parameters of the Standard Model:
    ex: light quark masses

1.1 JLab @ 12 GeV

- 12 GeV upgrade at JLab: *CLAS, GlueX, etc.*: In the study of hadron spectroscopy, large amount of very precise data on meson physics will be collected

- To perform this task:
  ⇨ analytical tools: *Amplitude analyses of data:* must build in S-Matrix constraints + state-of-the-art knowledge of reaction dynamics, See *JPAC* effort (talks of *A. Pilloni* and *A. Jackura*)



1.1 JLab @ 12 GeV

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- To perform this task:
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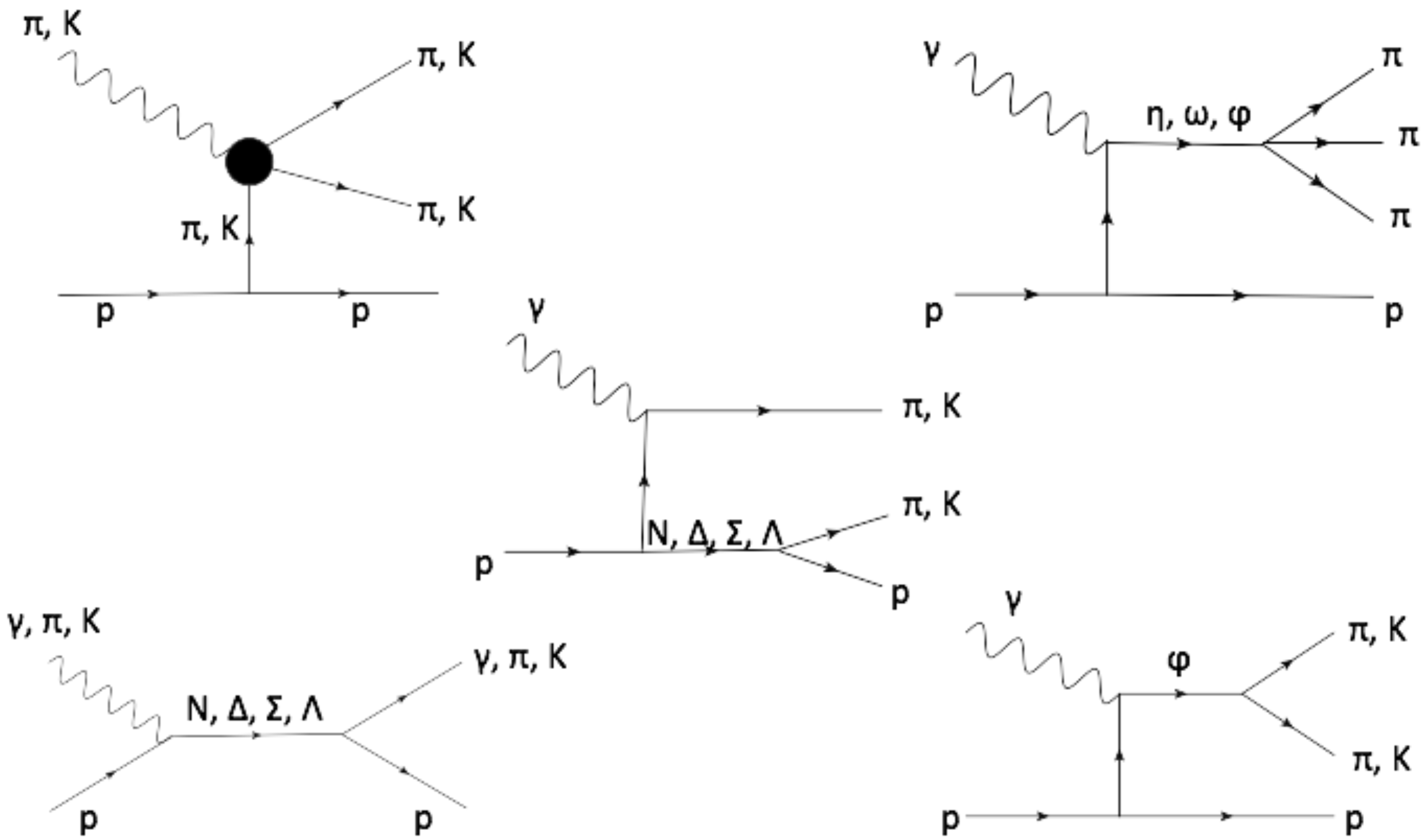
  See *JPAC* effort (talks of *A. Pilloni* and *A. Jackura*)

- Multi-body (final state) interactions are expected to play a crucial role for the hadron spectroscopy

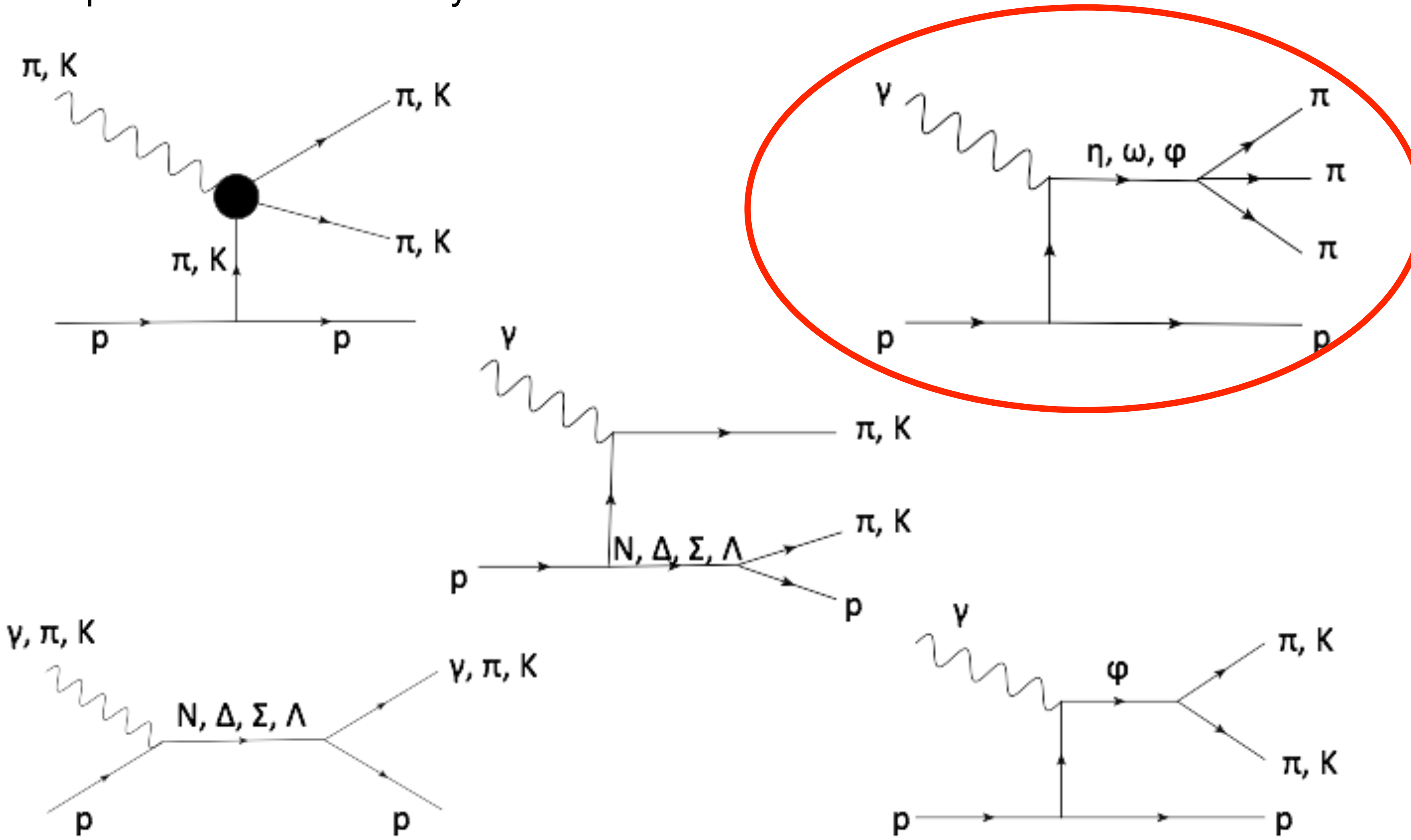
1.2 Processes under study at JLab and hadronic exp.

- All processes under study

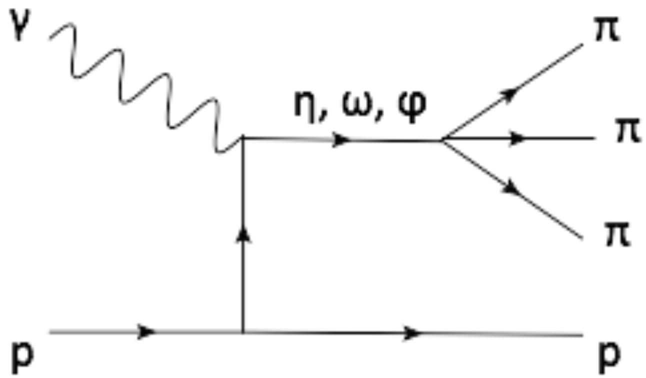


1.3 Light Meson Decays

- All processes under study



1.3 Light Meson Decays



- If E > 1 GeV: ChPT not valid anymore to describe dynamics of the processes

  ➡ Resonances appear :

  For ππ: *I=1*: ρ(770), ρ(1450), ρ(1700), …,

  Especially true for ϕ (M_ϕ=1020 MeV)

- Use Isobar model to describe the data

  ➡ Improve to include FSI

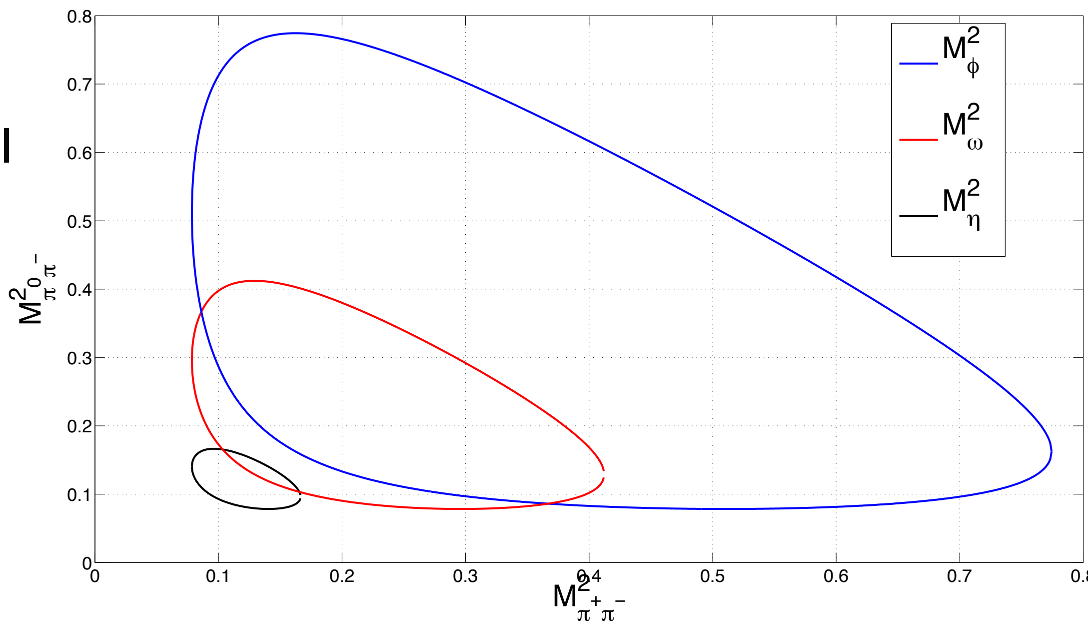
- Build an amplitude with physical properties:

  → Analyticity, Unitarity and Crossing Symmetry:

  ➡ *Dispersion Relations*

  → Chiral constraints at LE

  → Regge behavior at HE

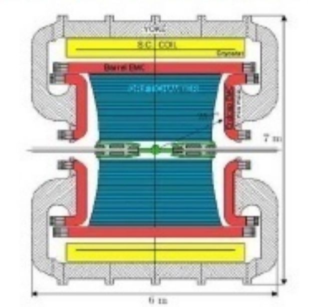


World competition in η decays

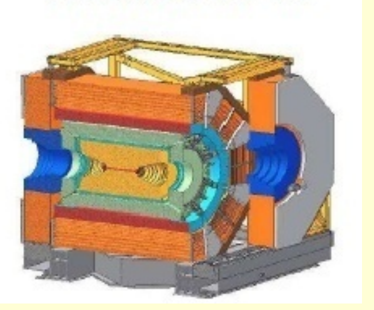
From L.Gan's talk

e^+e^- Collider

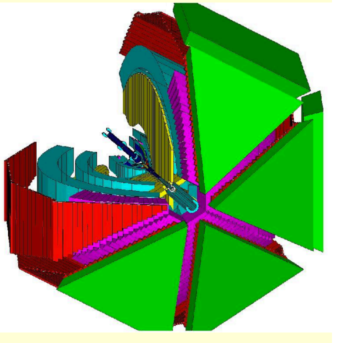
KLOE-2 at DAϕNE



BESIII at BEPCII

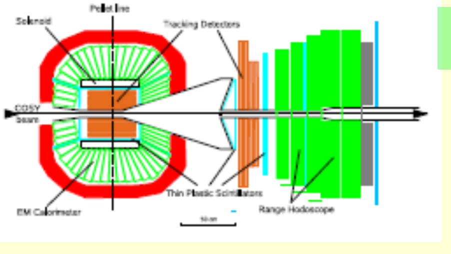


CLAS



Fixed-target

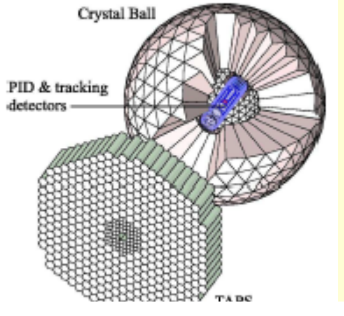
WASA at COSY



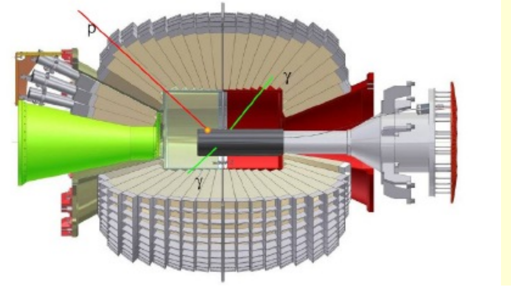
hadroproduction

photoproduction

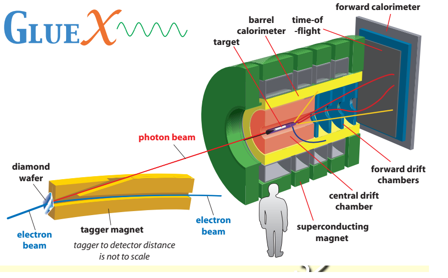
Crystall Ball at MAMI



CBELSA/TAPS at ELSA



JEF at Jlab



World competition in η decays

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e⁺e⁻ Collider

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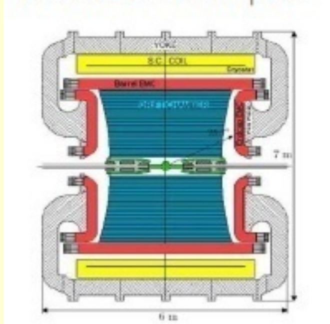
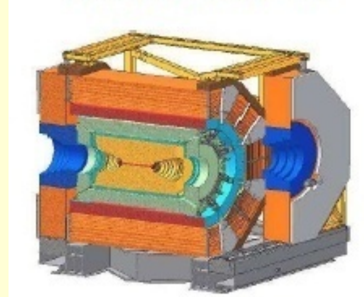
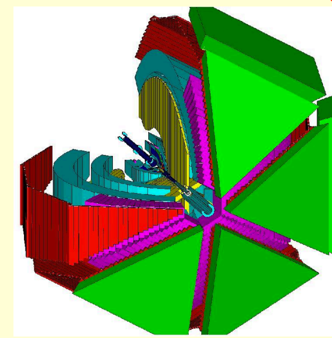


BESIII at BEPCII



CLAS



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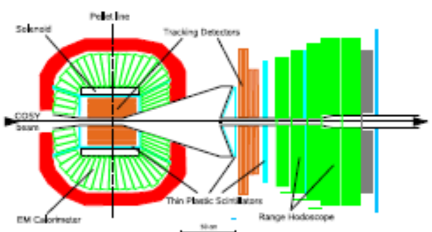

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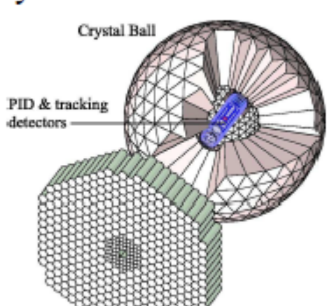

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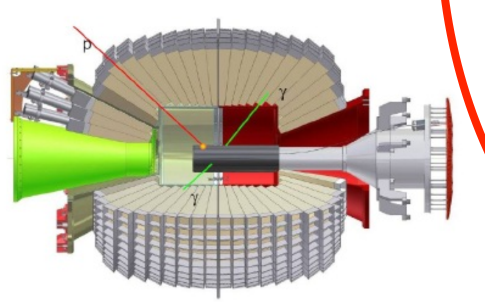
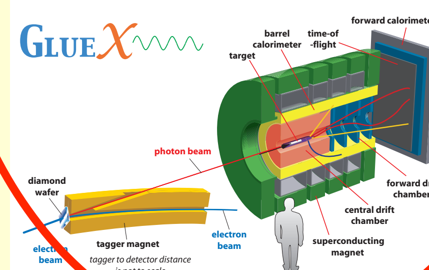


JEF at Jlab



1.4 Experimental Facilities and Role of JLab 12

M. J. Amaryan et al.
CLAS Analysis Proposal, (2014)

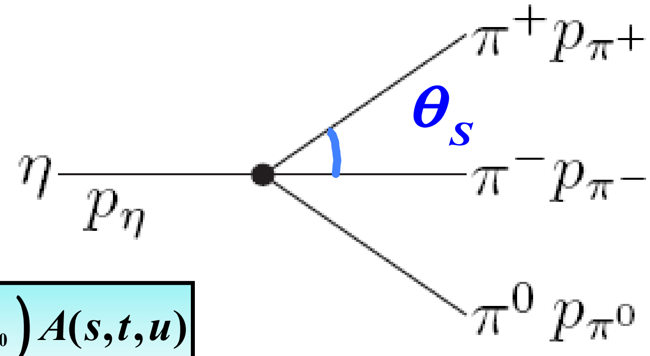
π	$e^+e^-\gamma$			
η	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-e^+e^-$
η'	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-\eta$, $\pi^+\pi^-e^+e^-$
ρ		$\pi^+\pi^-\gamma$		
ω	$e^+e^-\pi^0$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$	
φ			$\pi^+\pi^-\pi^0$	$\pi^+\pi^-\eta$

2. Light Mesons decays: An example: $\eta \rightarrow 3\pi$

In collaboration with G. Colangelo, S. Lanz and H. Leutwyler (ITP-Bern)

Phys. Rev. Lett. 118 (2017) no.2, 022001

2.1 Definitions



- η decay: $\eta \to \pi^+ \pi^- \pi^0$

$$\langle \pi^+\pi^-\pi^0{}_{out} | \eta \rangle = i(2\pi)^4 \delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right) A(s,t,u)$$

- Mandelstam variables $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2,\ t = \left(p_{\pi^-} + p_{\pi^0}\right)^2,\ u = \left(p_{\pi^0} + p_{\pi^+}\right)^2$

  ⇨ only two independent variables

$$s + t + u = M_\eta^2 + M_{\pi^0}^2 + 2M_{\pi^+}^2 \equiv 3s_0$$

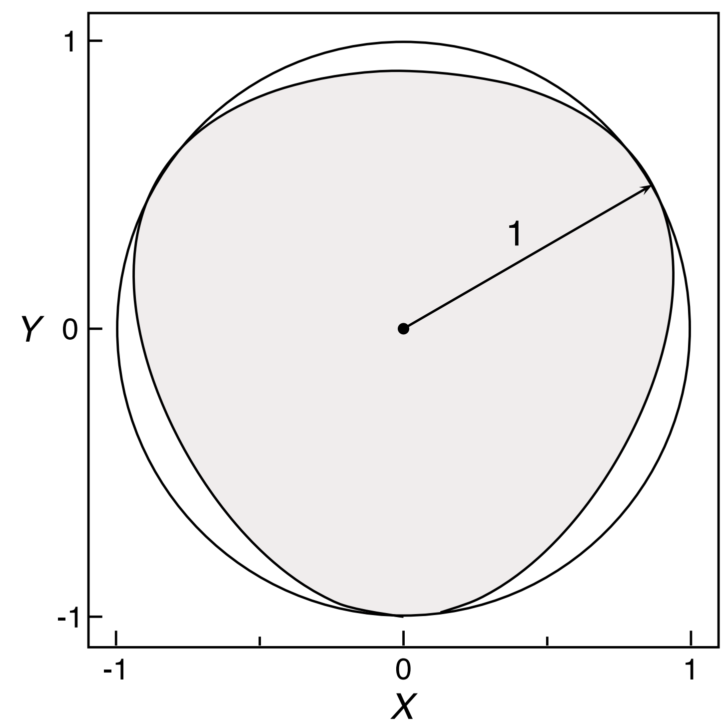
- 3 body decay ⇨ Dalitz plot

$$\left|A(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3 + \ldots\right)$$

Expansion around X=Y=0

$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u - t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$



$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

2.1 Why is it interesting to study $\eta \to 3\pi$?

- Decay forbidden by isospin symmetry

$$A = (m_u - m_d) A_1 + \alpha_{em} A_2$$

- α_{em} effects are small

Sutherland'66, Bell & Sutherland'68
Baur, Kambor, Wyler'96, Ditsche, Kubis, Meissner'09

- Decay rate measures the size of isospin breaking ($m_u - m_d$) in the SM:

$$L_{QCD} \to L_{IB} = -\frac{m_u - m_d}{2}\left(\bar{u}u - \bar{d}d\right)$$

Unique access to ($m_u - m_d$)

2.2 Quark mass ratio

- In the following, extraction of Q from $\eta \rightarrow \pi^+ \pi^- \pi^0$

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0} = \frac{1}{Q^4}\frac{M_K^4}{M_\pi^4}\frac{\left(M_K^2 - M_\pi^2\right)^2}{6912\pi^3 F_\pi^4 M_\eta^3}\int_{s_{\min}}^{s_{\max}} ds \int_{u_-(s)}^{u_+(s)} du \; \left|M(s,t,u)\right|^2$$

Determined from experiment

Determined from:
- Dispersive calculation
- ChPT

Fit to Dalitz distr.

$$\left[Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \right] \qquad \left[\hat{m} \equiv \frac{m_d + m_u}{2} \right]$$

- Aim: Compute M(s,t,u) with the *best accuracy*

2.3 Computation of the amplitude

- What do we know?

- Compute the amplitude using ChPT : the effective theory that describe dynamics of the Goldstone bosons (kaons, pions, eta) at low energy

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{QCD}$
Expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\mathcal{L}_{eff} = \sum_{d\geq 2} \mathcal{L}_d \ , \ \mathcal{L}_d = \mathcal{O}\left(p^d\right) \ , p \equiv \left\{q, m_q\right\}$$

$$\mathrm{p} << \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}$$

2.3 Computation of the amplitude

- What do we know?
- Compute the amplitude using ChPT :

$$\Gamma_{\eta\to 3\pi} = (66 + 94 + ... + ...)\,\text{eV} = (300 \pm 12)\,\text{eV}$$

LO NLO NNLO

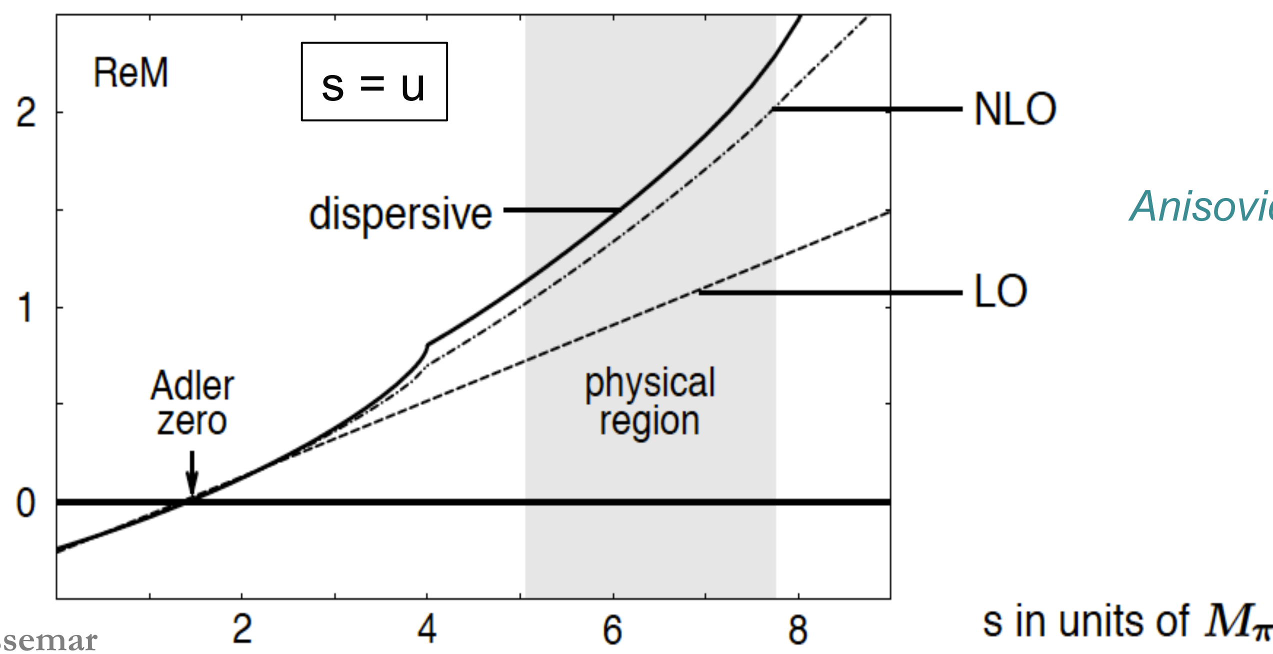
PDG'16

LO: *Osborn, Wallace'70*

NLO: *Gasser & Leutwyler'85*

NNLO: *Bijnens & Ghorbani'07*

The Chiral series has convergence problems

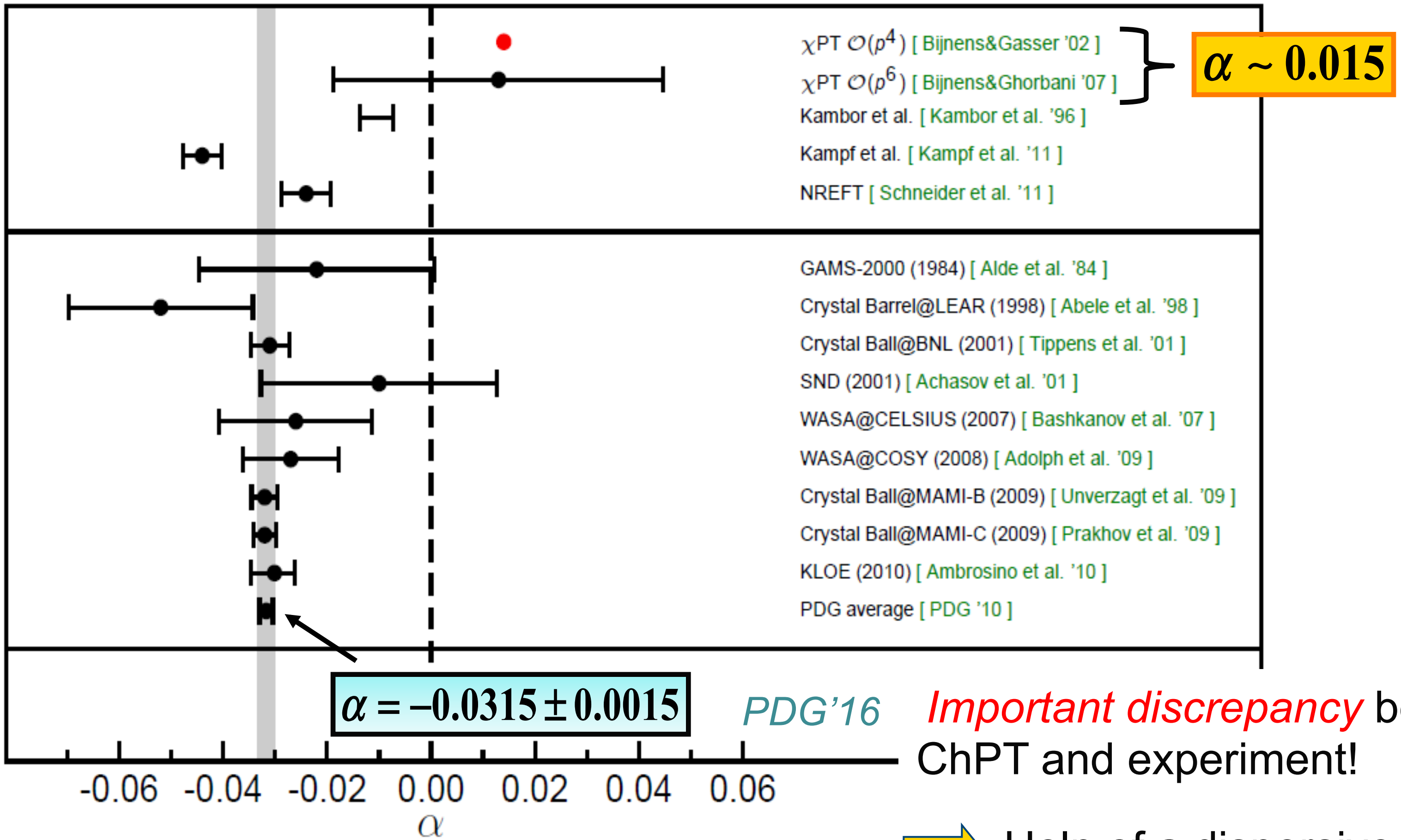


Anisovich & Leutwyler'96

2.4 Neutral Channel : $\eta \to \pi^0 \pi^0 \pi^0$

- Decay amplitude $\Gamma_{\eta\to 3\pi} \propto \left|\bar{A}\right|^2 \propto 1 + 2\alpha Z$ with $Z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_n} - 1\right)^2$

$Q_n \equiv M_\eta - 3M_{\pi^0}$



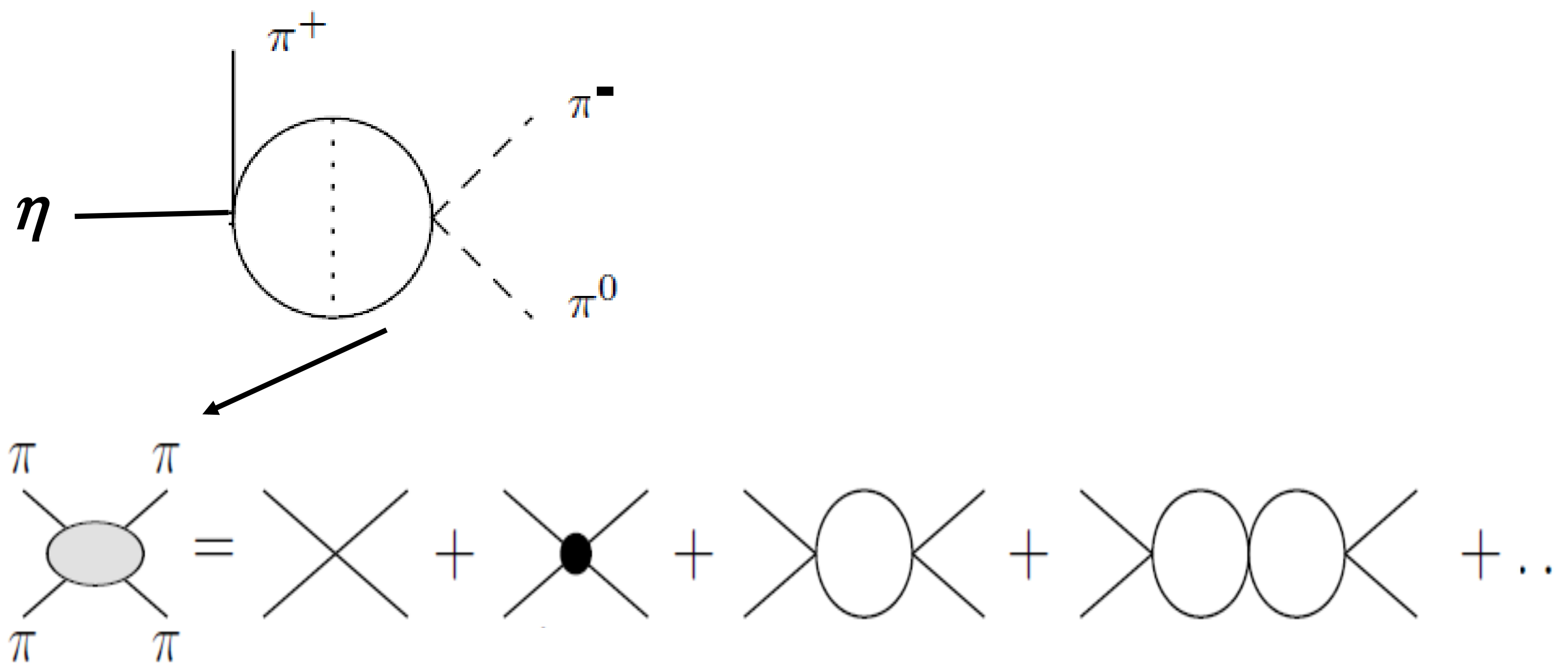
Important discrepancy between ChPT and experiment!

⇒ Help of a dispersive treatment?

2.5 Dispersive treatment

- The Chiral series has convergence problems

➡ Large *ππ final state interactions* *Roiesnel & Truong'81*



2.5 Dispersive treatment

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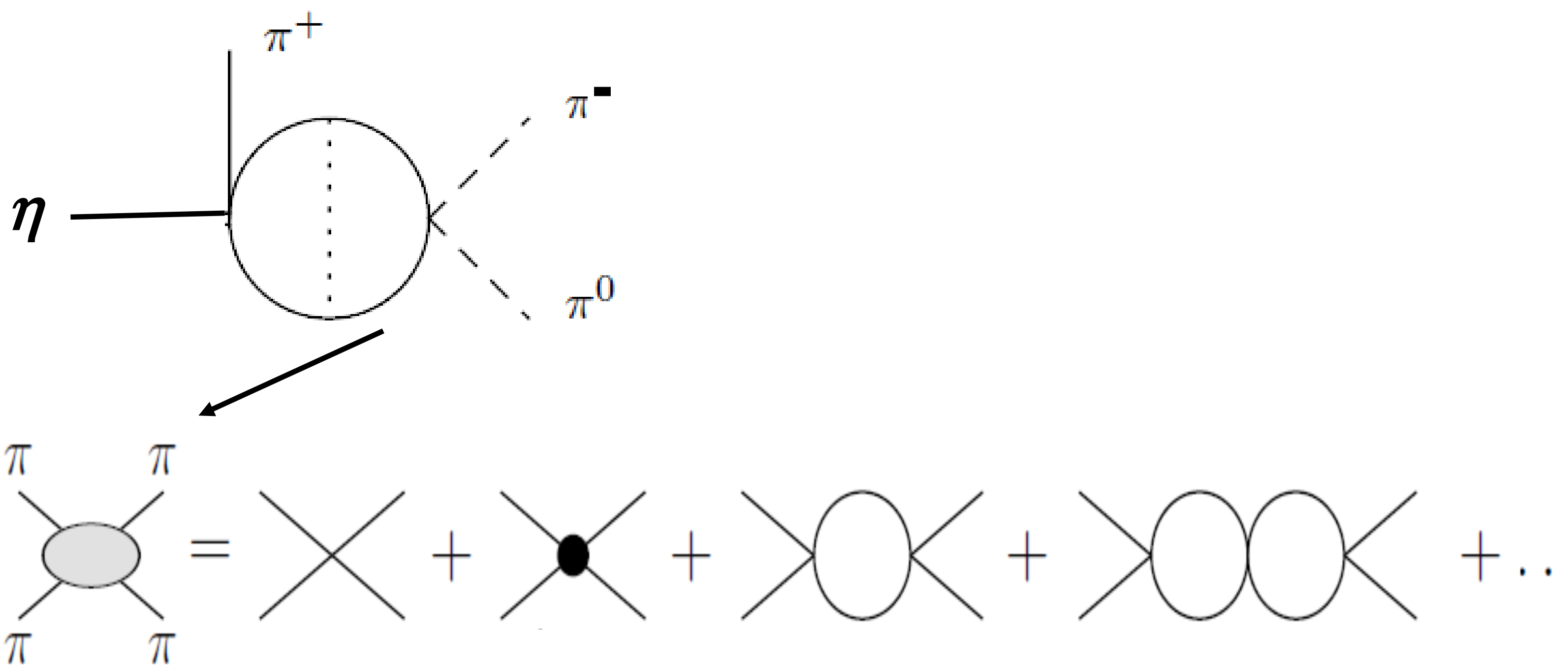


- *Dispersive treatment :*
  – analyticity, unitarity and crossing symmetry
  – Take into account all the rescattering effects

2.6 Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction $\pi\pi$ phase shifts has improved

    Ananthanarayan et al'01, Colangelo et al'01

    Descotes-Genon et al'01

    Kaminsky et al'01, Garcia-Martin et al'09

  - New experimental programs, precise Dalitz plot measurements

    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)

    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)

    BES III (Beijing) ⇨ see talks by *L. Gan*

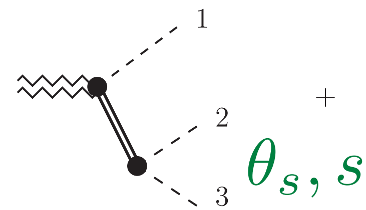
    D. Lersch

  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects

2.7 Method

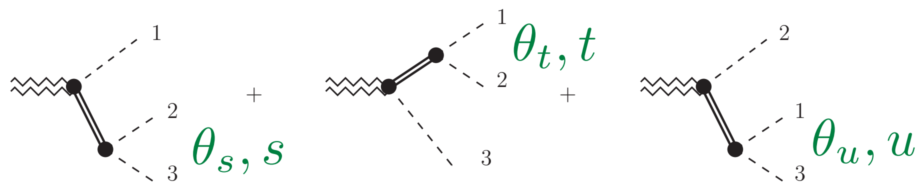
- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ➡ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$



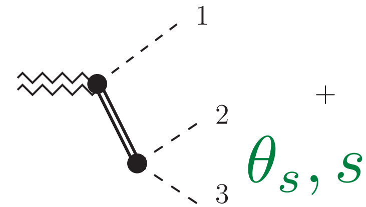
3 BWs (ρ^+, ρ^-, ρ^0) + background term

➡ Improve to include final states interactions

2.7 Method

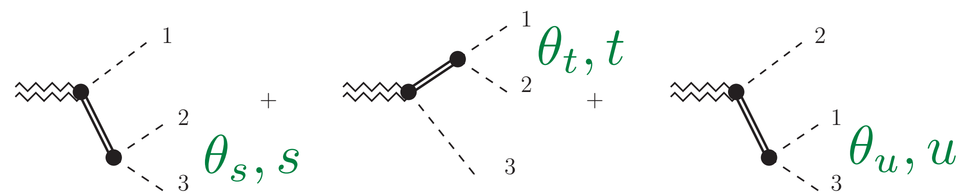
- S-channel partial wave decomposition

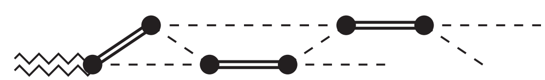
$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ➡ Isobar approximation

$$\begin{aligned} A_\lambda(s,t) &= \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) \\ &+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) \\ &+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u) \end{aligned}$$





- Use a Khuri-Treiman approach or dispersive approach

➡ Restore 3 body unitarity and take into account the final state interactions in a systematic way

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)$$

Fuchs, Sazdjian & Stern'93

Anisovich & Leutwyler'96

- M_I isospin *I* rescattering in two particles
- Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
- Main two body rescattering corrections inside M_I

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)$$

- Unitarity relation:

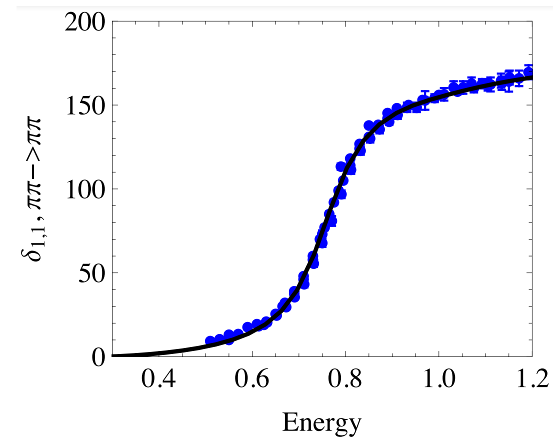
$\pi\pi \to \pi\pi$

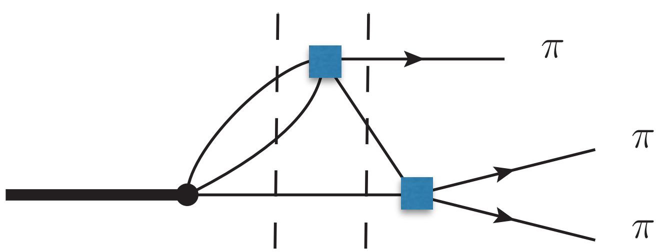
$$disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)$$

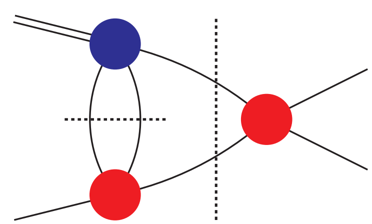
right-hand cut   left-hand cut

input

Roy analysis
Colangelo et al.'01







2.9 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$M(s,t,u)=M_0(s)+(s-u)M_1(t)+(s-t)M_1(u)+M_2(t)+M_2(u)-\frac{2}{3}M_2(s)$$

- Unitarity relation:

$$disc\left[M_\ell^I(s)\right]=\rho(s)t_\ell^*(s)\left(M_\ell^I(s)+\hat{M}_\ell^I(s)\right)$$

- Relation of dispersion to reconstruct the amplitude everywhere:

$$M_I(s)=\Omega_I(s)\left(P_I(s)+\frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|\left(s'-s-i\varepsilon\right)}\right)$$

Omnès function

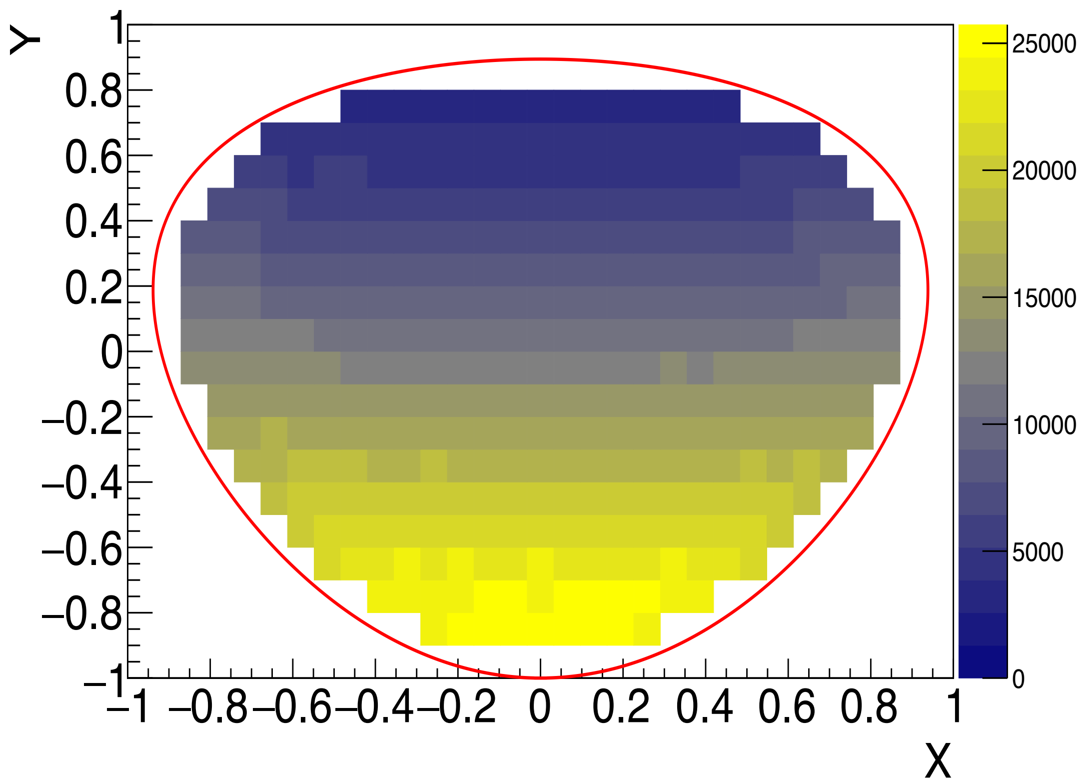
$$\left[\Omega_I(s)=\exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

2.9 $\eta \to 3\pi$ Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*

KLOE'16



$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

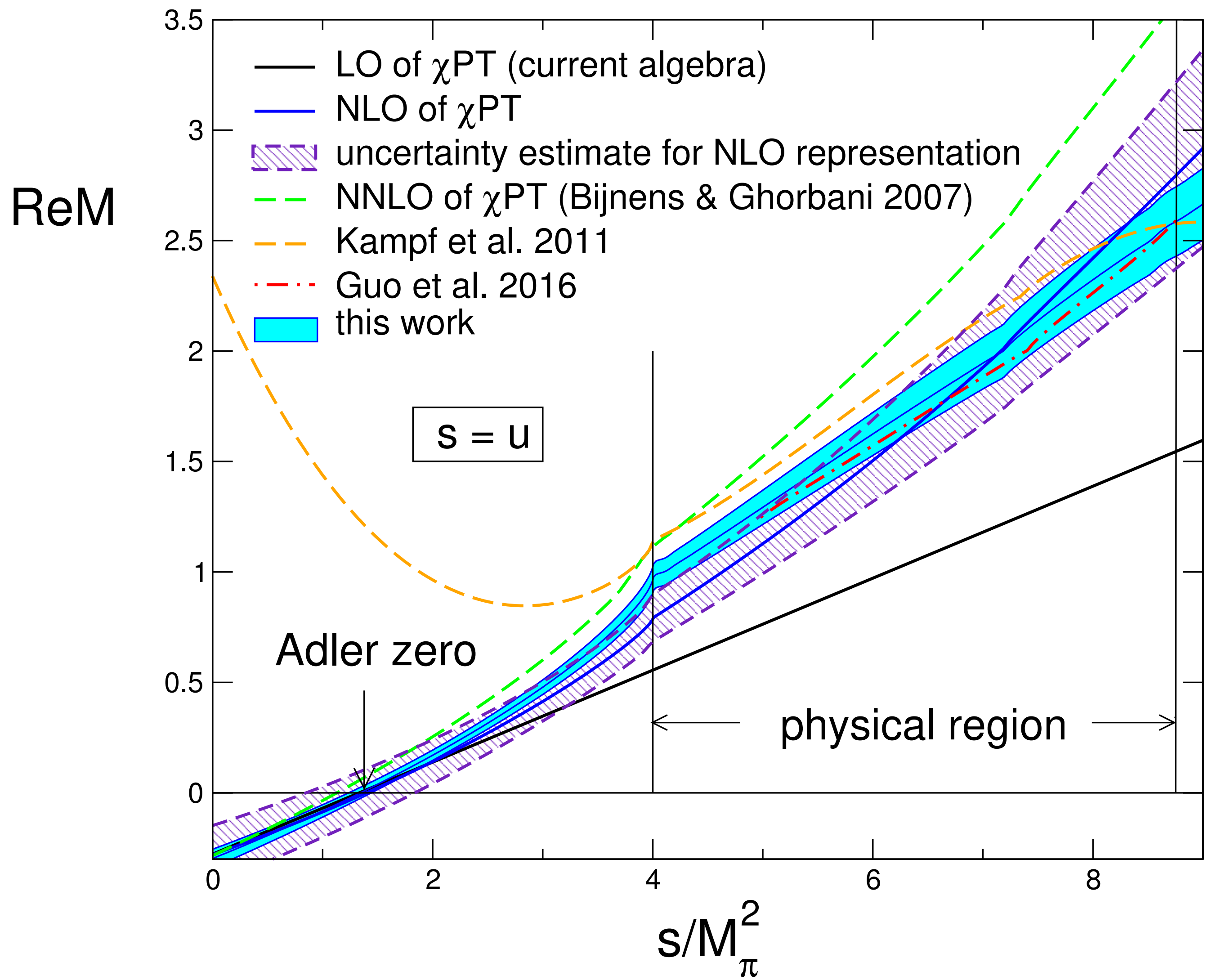
$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

  ⇨ see talks on Wednesday by *L. Gan*, *D. Lersch*

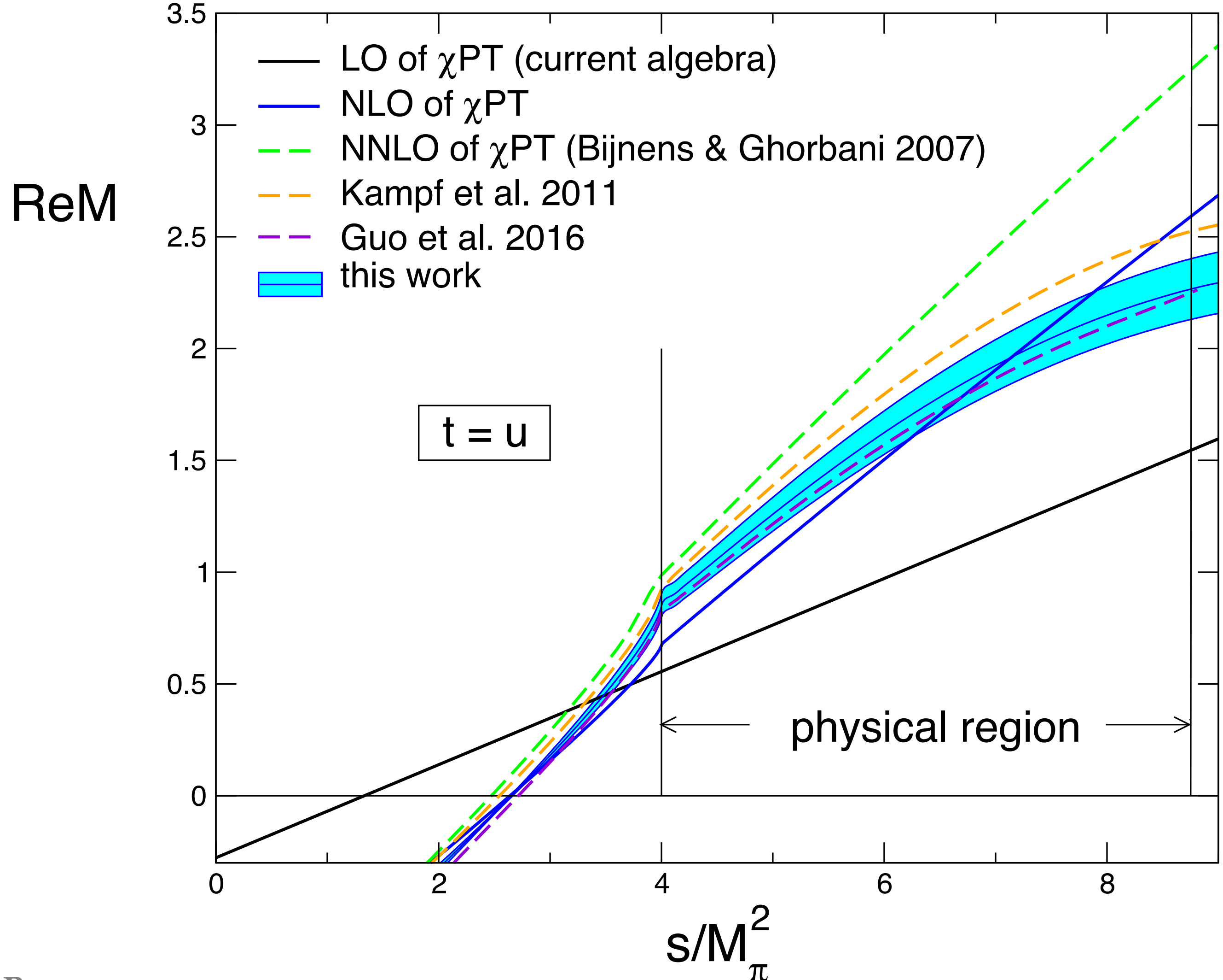
3.1 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :



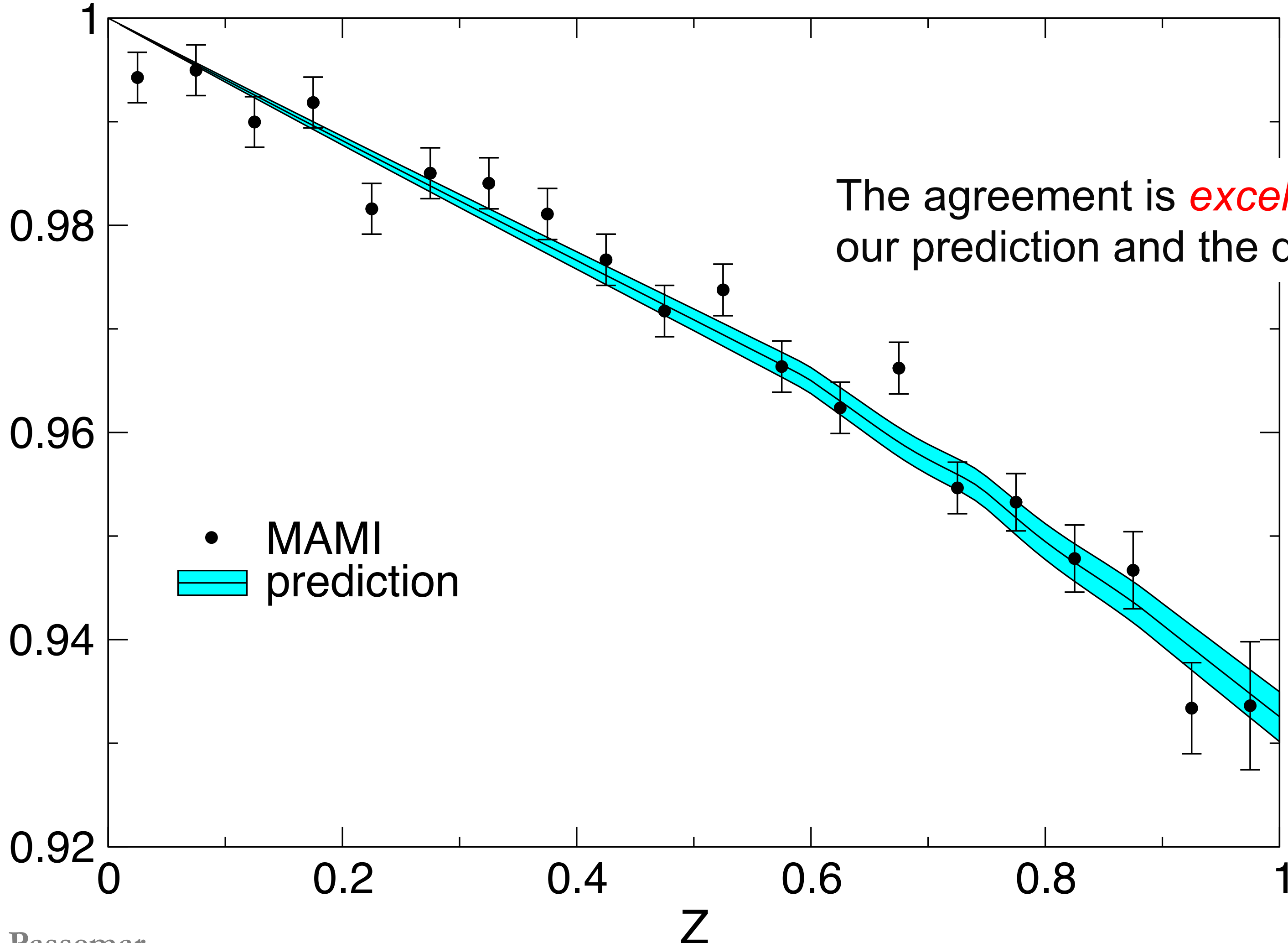
3.1 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



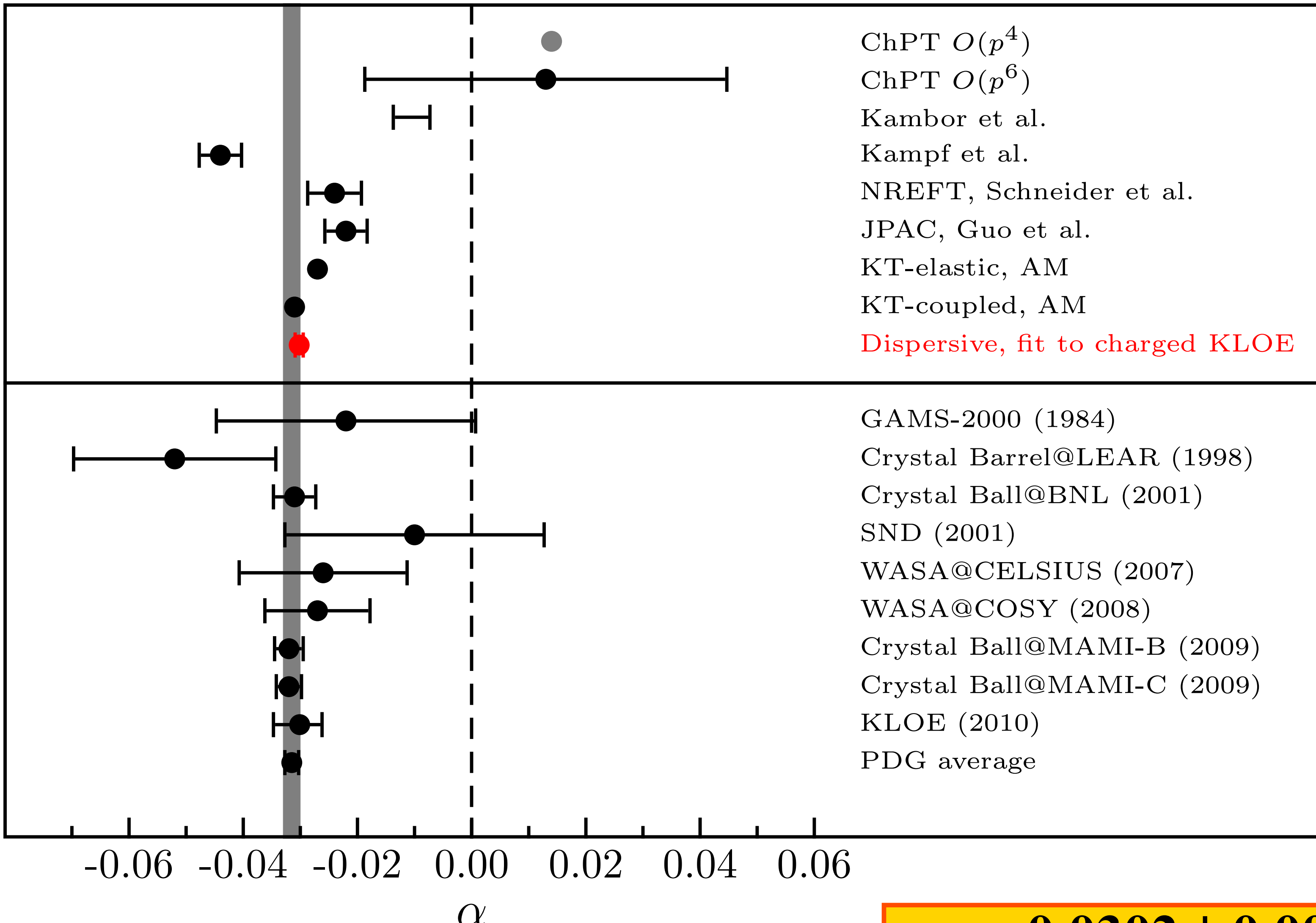
3.2 Z distribution for $\eta \rightarrow \pi^0 \pi^0 \pi^0$ decays

- The amplitude squared in the neutral channel is



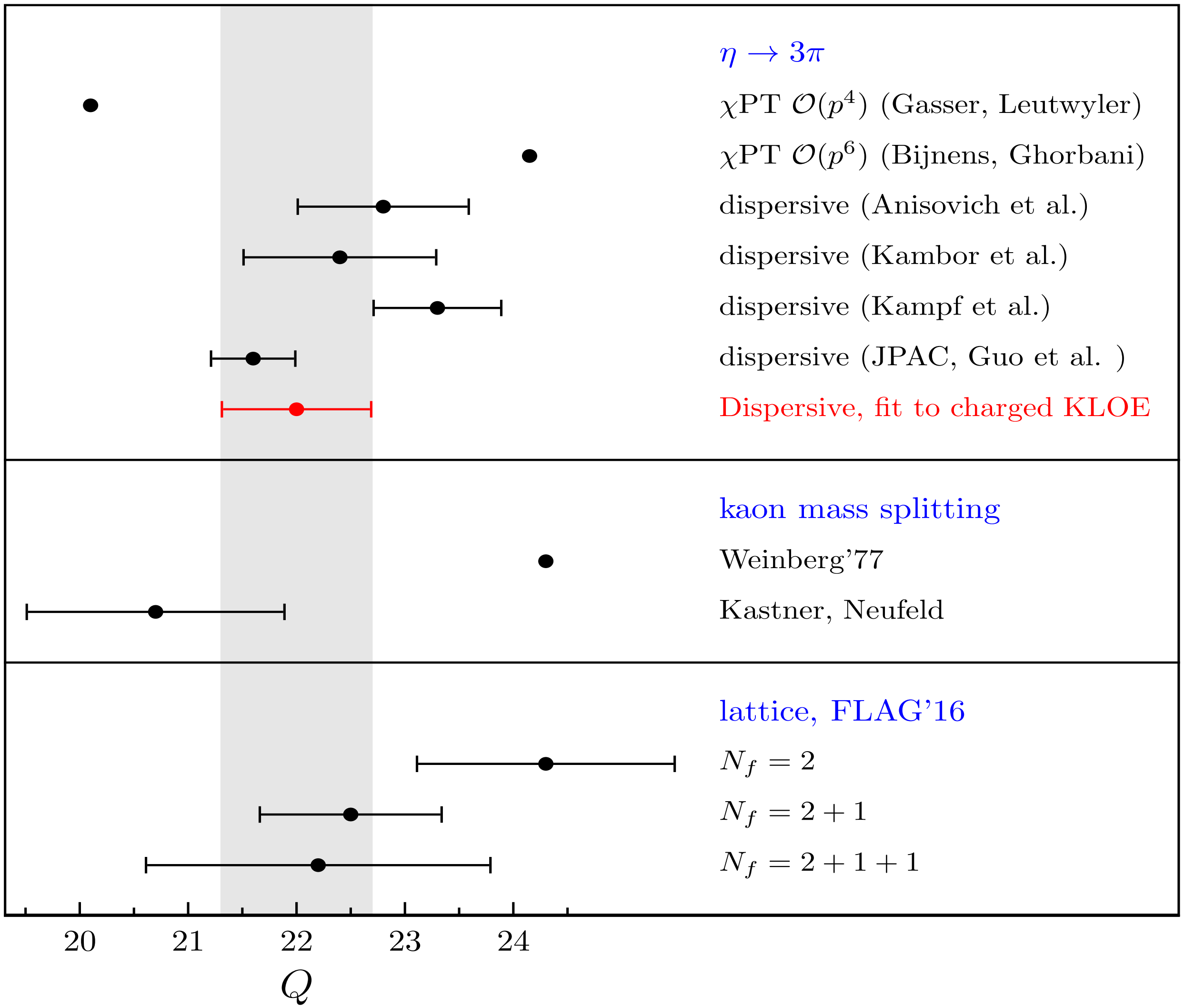
The agreement is *excellent* between our prediction and the data!

3.2 Comparison of results for α



$$\alpha = -0.0302 \pm 0.0011$$

3.3 Quark mass ratio

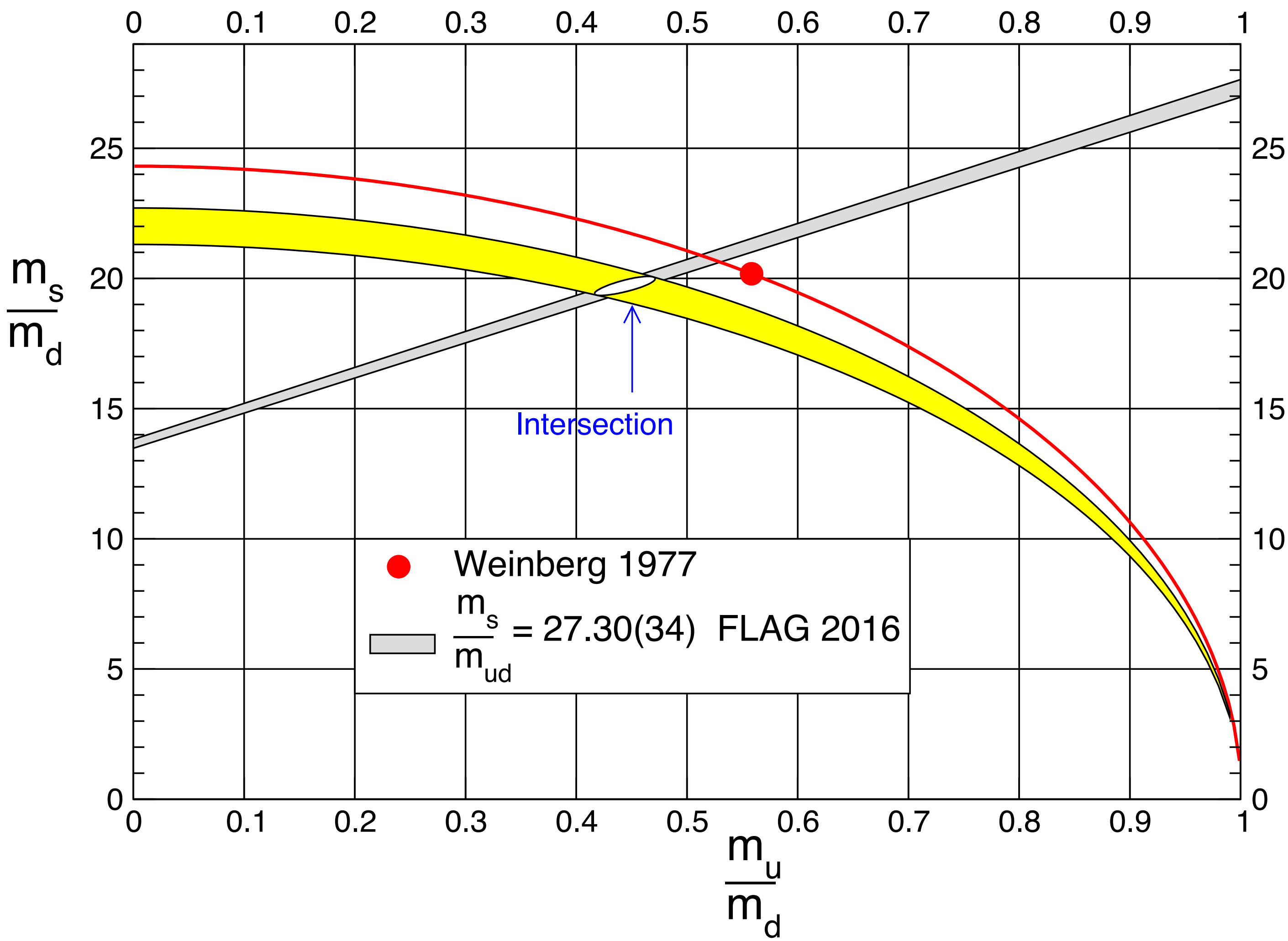


$Q = 22.0 \pm 0.7$

- No systematics taken into account ⇨ collaboration with experimentalists

3.4 Light quark masses

Courtesy of H.Leutwyler



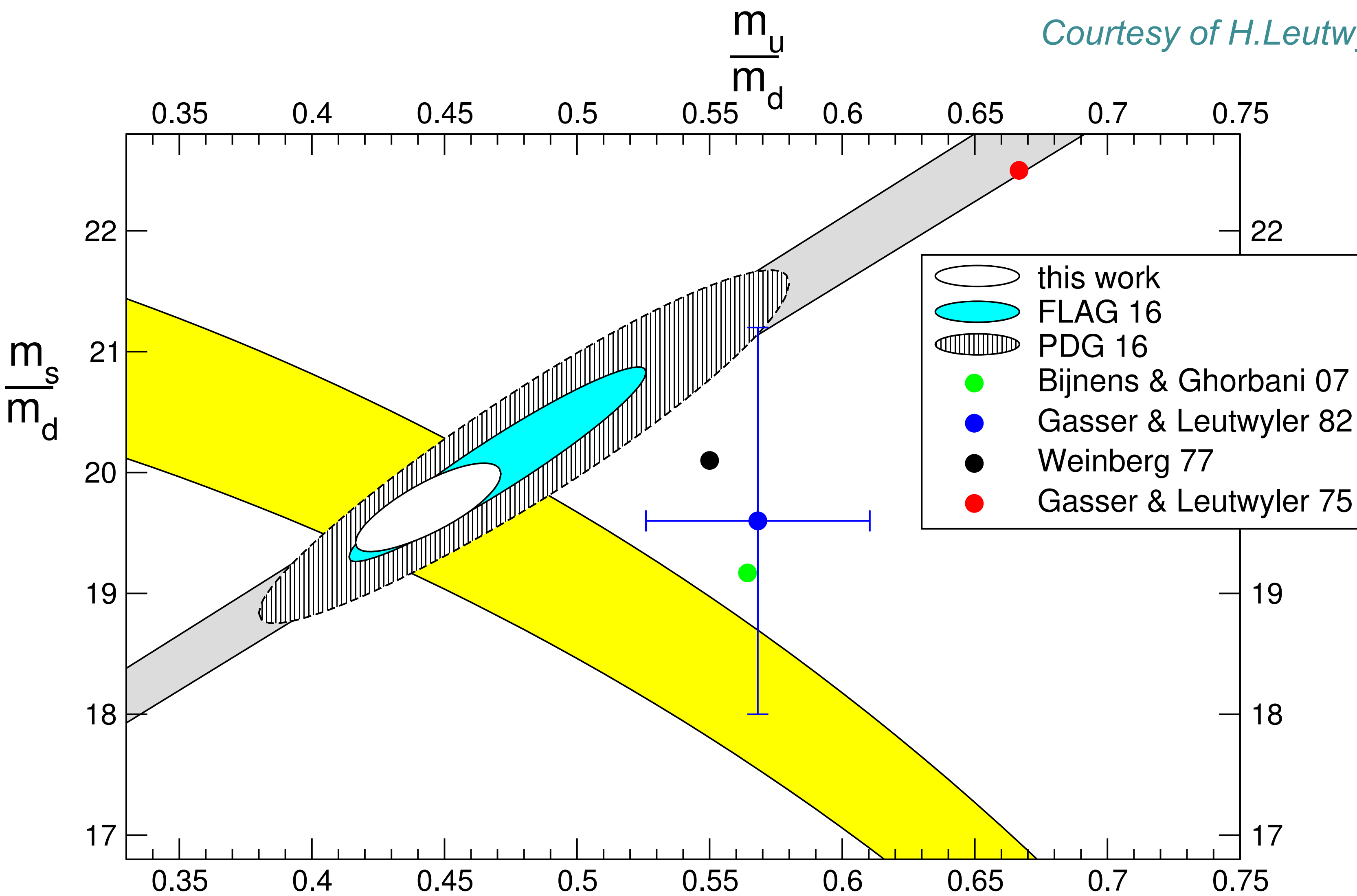
$Q = 22.0 \pm 0.7$

$\frac{m_u}{m_d} = 0.44 \pm 0.03$

- Smaller values for Q ⇒ smaller values for ms/md and mu/md than LO ChPT

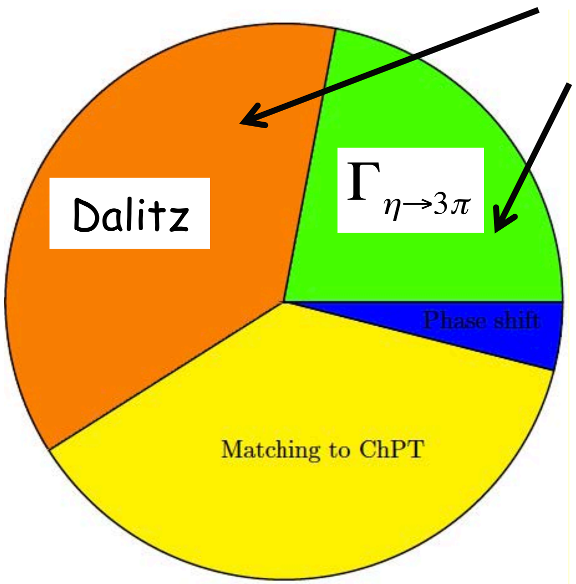
3.4 Light quark masses

Courtesy of H.Leutwyler



3.5 $\eta \rightarrow 3\pi$ and Light Quark Masses

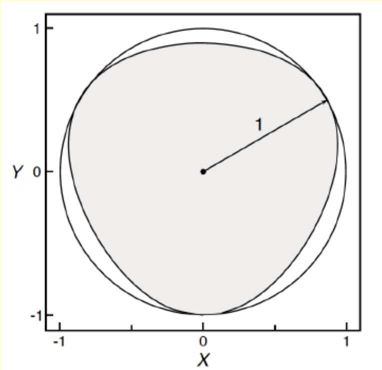
- Uncertainties in the quark mass ratio (rough attempt)



Can be investigated and reduced at *JEF*

Experimental Measurements of $\eta\rightarrow 3\pi$

L. Gan's talk



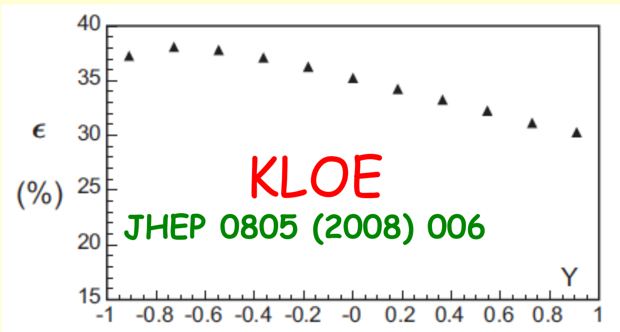
$$X = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

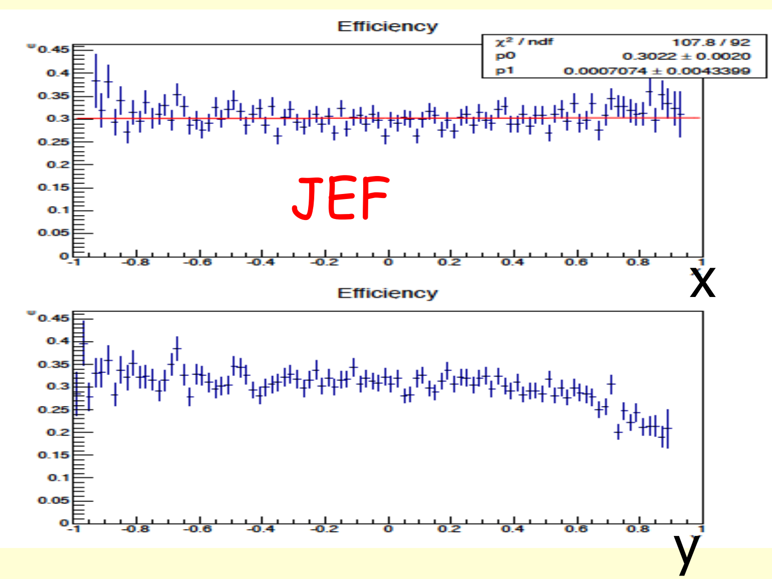
$$Y = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

$$Z = X^2 + Y^2$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

Exp.	$3\pi^0$ Events (10^6)	$\pi^+\pi^-\pi^0$ Events (10^6)
Total world data (include prel. WASA and prel. KLOE)	6.5	6.0
GlueX+PrimEx-η +JEF	20	19.6





- Existing data from the low energy facilities are sensitive to the detection threshold effects
- JEF at high energy has uniform detection efficiency over Dalitz phase space
- JEF will offer large statistics and improved systematics

3. Conclusion and Outlook

3.1 Conclusion

- Light Meson component very important for JLab 12
- Knowing conventional modes important for studies of background for looking for exotics
- Study of fundamental properties of QCD:
  - Extraction of fundamental parameters of the SM,
    ⇨ e.g. light quark masses
  - Study of chiral dynamics
- To studies meson modes with the best precision: Development of amplitude analysis techniques consistent with analyticity, unitarity, crossing symmetry ⇨ dispersion relations allow to take into account *all rescattering effects* being as model independent as possible combined with ChPT ⇨ Provide parametrization for experimental studies
- In this talk, illustration with $\eta \rightarrow 3\pi$ and extraction of the light quark masses
- Similar illustration in the talk of *A. Pilloni* and *A. Jackura (JPAC)*

3.2 Outlook:

- Apply dispersion relations + (R)ChPT to other modes in the light meson sector
  - $\omega/\varphi \to 3\pi, \pi\gamma$: *Niecknig, Kubis, Schneider'12*, *Danilkin et al. JPAC'15,'16*
  - $\varphi \to \eta\pi\gamma$: *Moussallam, Shekhovtsova in progress*
  - $\eta' \to 3\pi$
  - $\eta' \to \eta\pi\pi$: *Escribano, Masjuan,Sanz-Cillero'11*, *Kubis & Schneider'12*, *Perotti, Niblaeus, Leupold'15*
  - etc…

4. Back-up