Plans to Simulate JLEIC Magnetized Cooling Concepts

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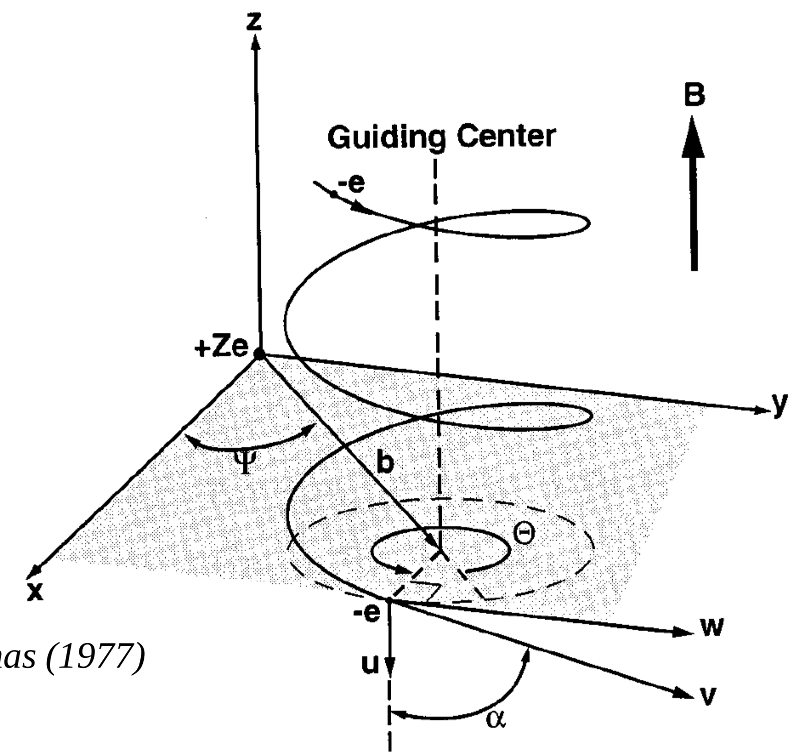
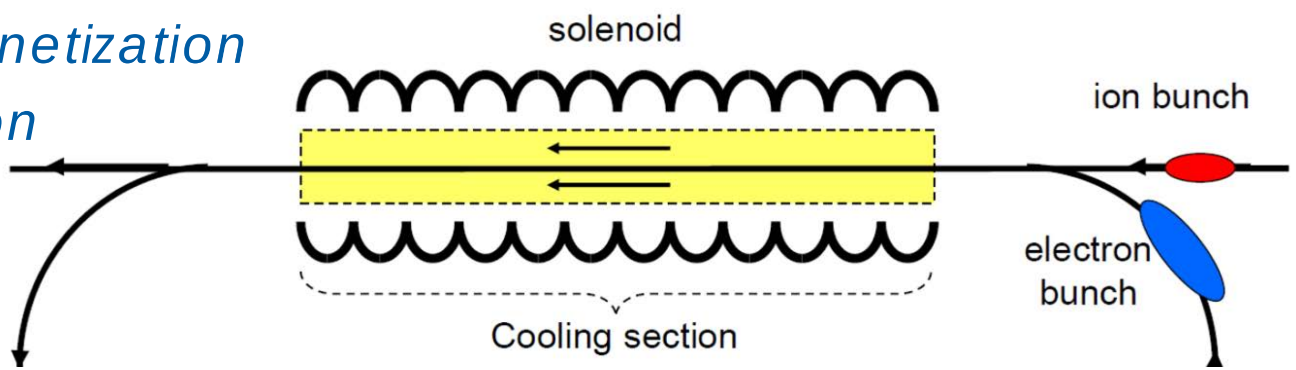
Project goals

- Simulate magnetized friction
  - *with real world effects*
  - *including incoming beam distribu*
  - *for a wide range of parameters*
  - *not using brute force every time*



from Geller & Weisheit, Phys. Plasmas (1977)

- Simulate key aspects of magnetized e- beam transport
  - *imperfect magnetization*
  - *SC neutralization*
  - *N passes*



from Zhang et al., MEIC design, arXiv (2012)

Outline

- The importance sampling method
  - *evaluate 'local' dynamic friction for key parameter values*
  - *use strategic scaling of integral to minimize data points*
- A symplectic, spectral electrostatic particle algorithm
  - *momentum is conserved to machine precision*
  - *no grid heating; energy deviations are strictly bounded*
- Store 'local' dynamic friction results in database
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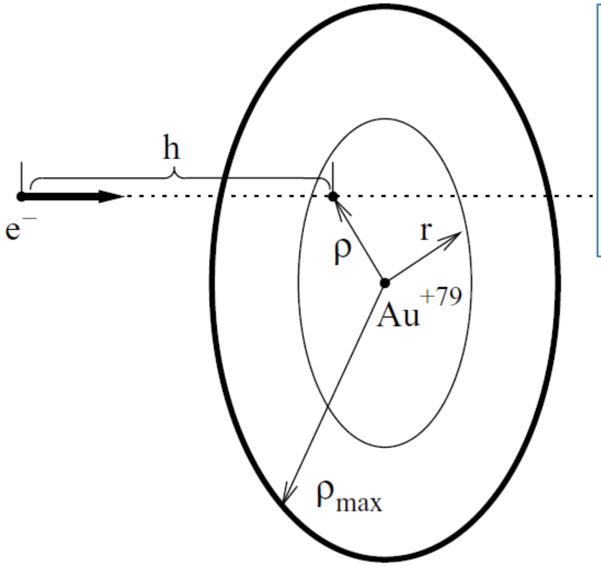
Importance Sampling Method



A.V. Sobol, D.L. Bruhwiler, G. Bell, A. Fedotov, V. Litvinenko, "Numerical calculation of dynamical friction in electron cooling systems, including magnetic field perturbations and finite time effects," New Journal of Physics **12**, 093038 (2010).

- Efficiently sample parameter regime
  - *Monte Carlo integration of H(x)*
  - *ξ_n is uniform variate (e.g. impact param)*
  - *P(x) is probability (→ 0 for small x)*
  - *choose Q(y) to flatten probability*
  - *get accurate integration w/ few eval.'s*

$$\int H(x)\,\mathrm{d}P(x) \approx \frac{1}{N}\sum_{n=1}^{N} H(\xi_n)$$

$$= \int H(y)\,P'(y)/Q'(y)\,\mathrm{d}Q(y)$$

$$\approx \frac{1}{N}\sum_{n=1}^{N} H(\tilde{\xi}_n)\,P'(\tilde{\xi}_n)/Q'(\tilde{\xi}_n)$$

Small impact parameter collisions are important
(for unmagnetized friction) Sobol *et al.*

- Impact parameters follow a modified Pareto distribution
  - *like income distrib.*
  - *small values are rare but significant*
- Uncertainties are intrinsically large
- The central limit theorem is not valid
  - *using ever more collisions to average away noise → artificially large result*

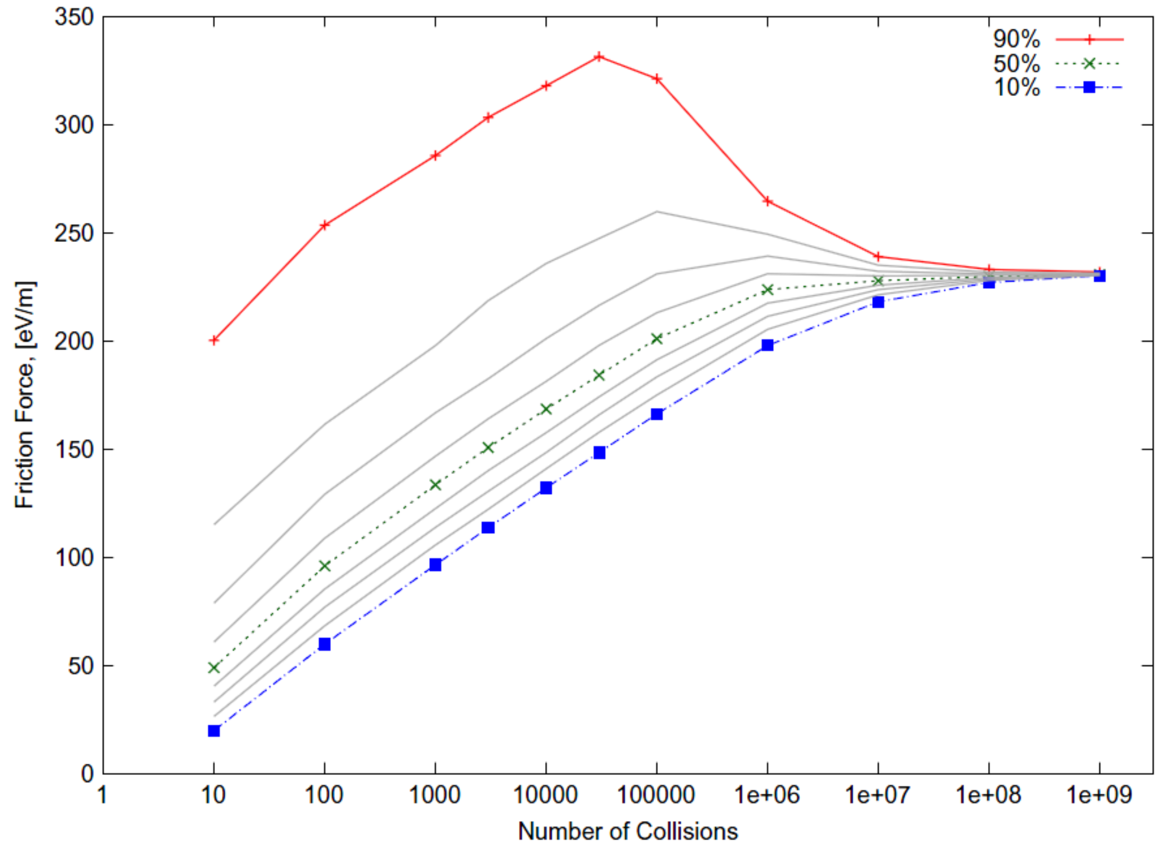


Figure 8. The friction force resulting from a sequence of collisions as an RV. The 10-quantiles (deciles) are plotted as a function of the number of collisions: thick lines are 10%-, 50%- and 90%-quantiles and thin lines are 20%-, 30%-, 40%-, 60%-, 70%- and 80%-quantiles. The 80% confidence interval is very wide unless the number of collisions is extremely large.

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One can construct fundamentally better algorithms

Low Lagrangian[2]

$$L = \iint d\mathbf{x}_0 d\mathbf{v}_0 f(\mathbf{x}_0, \mathbf{v}_0)\left\{mc^2\sqrt{1 - \frac{1}{c^2}\left(\frac{\partial \mathbf{x}}{\partial t}\right)^2} + -q\varphi(\mathbf{x}, t) + \frac{q}{c}\frac{\partial \mathbf{x}}{\partial t}\cdot \mathbf{A}(\mathbf{x}, t)\right\} + \frac{1}{8\pi}F^{\mu\nu}F_{\mu\nu}$$

Minimizing action with respect to the 4-potential and position yields Lorentz Force Law & Maxwell's Equations, e.g.:

[2] F.E. Low (1958)

$$\frac{d}{dt}\frac{\delta L}{\delta \mathbf{x}'} - \frac{\delta L}{\delta \mathbf{x}} = 0 \rightarrow$$

$$m\frac{\partial}{\partial t}\left(\gamma\frac{\partial \mathbf{x}}{\partial t}\right) = q\left(\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{x}}{\partial t}\times \mathbf{B}\right)$$

Discretize Lagrangian into field modes & ptcl shapes

H. R. Lewis (1970)
A. Stamm, B. Shadwick, E. Evstatiev (2014)
J. E. Marsden G. W. Patrick, and S. Shkoller (1998)

Fields

$$\varphi(\mathbf{x}) = \sum_\sigma \int d\mathbf{k}\Lambda\left(\mathbf{k}-\mathbf{k}_\sigma\right) e^{i\mathbf{k}\cdot\mathbf{x}}\tilde{\varphi}_\sigma$$

Particles

$$f(\mathbf{x},\mathbf{v},t) = \sum_i w_i \Pi\left(\mathbf{x}-\mathbf{x}_i(t)\right)\delta\left(\mathbf{v}-\mathbf{v}_i(t)\right)$$

$$L = \sum_i \underbrace{\frac{1}{2} m w_i \left(\frac{\partial \mathbf{x}_i}{\partial t}\right)^2}_{\text{particle move}} + \underbrace{q w_i \sum_\sigma \tilde{\varphi}_\sigma \int d\mathbf{k}\; e^{-i\mathbf{k}\cdot\mathbf{x}_i}\tilde{\Pi}(\mathbf{k})\Lambda(\mathbf{k}-\mathbf{k}_\sigma)}_{\text{deposition/force interp.}}$$

$$\underbrace{-\frac{1}{2}\sum_{\sigma\sigma'} \tilde{\varphi}_\sigma \tilde{\varphi}_{\sigma'} \int d\mathbf{k}\mathbf{k}^2 \Lambda(\mathbf{k}-\mathbf{k}_\sigma)\Lambda(\mathbf{k}-\mathbf{k}_{\sigma'})}_{\text{Poisson equation}}$$

Discretize Lagrangian in time to approx. action integral

J.E. Marsden & M. West (2001)

$$\mathbf{x} \mapsto \mathbf{x}^{(n+1/2)}$$

Discretize the coördinates and velocities in time for a time step h

$$\frac{\partial \mathbf{x}}{\partial t} \mapsto \frac{\mathbf{x}^{(n+1/2)} - \mathbf{x}^{(n)}}{h/2}$$

Decompose the Lagrangian so it is self-adjoint, i.e. 2nd order (§2.4 of [6])

$$\mathbb{L}_D^{(h)} = \mathbb{L}_D^{(0)}\left(\mathbf{x}^{(n+1/2)}, \mathbf{x}^{(n)}; \tilde{\varphi}_\sigma^{(n+1/2)}\right) + \mathbb{L}_D^{(1)}\left(\mathbf{x}^{(n+1)}, \mathbf{x}^{(n+1/2)}; \tilde{\varphi}_\sigma^{(n+1/2)}\right)$$

Minimize discrete action with the discrete Euler-Lagrange equations (§2.5 of [6])

$$\boxed{\begin{aligned} \frac{\partial}{\partial \mathbf{x}^{(n+1/2)}}\mathbb{L}_D^{(0)} + \frac{\partial}{\partial \mathbf{x}^{(n+1/2)}}\mathbb{L}_D^{(1)} &= 0 \\ \frac{\partial}{\partial \tilde{\varphi}_\sigma^{(n+1/2)}}\mathbb{L}_D^{(0)} + \frac{\partial}{\partial \tilde{\varphi}_\sigma^{(n+1/2)}}\mathbb{L}_D^{(1)} &= 0 \end{aligned}}$$

Vary discretized Lagrangian to obtain algorithm

J.E. Marsden & M. West (2001)

$$\mathbf{v}^{(n)} \equiv \frac{\mathbf{x}^{(n+1/2)} - \mathbf{x}^{(n)}}{h/2}$$

Half-move

$$\mathbf{x}^{(n+1/2)} = \mathbf{x}^{(n)} + \mathbf{v}^{(n)} \frac{h}{2}$$

Field solve

$$\sum_{\sigma'} \mathbb{K}_{\sigma\sigma'} \tilde{\varphi}_{\sigma'}^{(n+1/2)} = -q \sum_i w_i \int d\mathbf{k} e^{-i\mathbf{k}\cdot\mathbf{x}_i^{(n+1/2)}} \tilde{\Pi}(\mathbf{k}) \Lambda(\mathbf{k} - \mathbf{k}_\sigma)$$

Full accelerate

$$\mathbf{v}^{(n+1)} = \mathbf{v}^{(n)} + \frac{q}{m} h \sum_\sigma \tilde{\varphi}_\sigma^{(n+1/2)} \int d\mathbf{k}\ i\mathbf{k} e^{-i\mathbf{k}\cdot\mathbf{x}^{(n+1/2)}} \tilde{\Pi}(\mathbf{k}) \Lambda(\mathbf{k} - \mathbf{k}_\sigma)$$

Half-move

$$\mathbf{x}^{(n+1)} = \mathbf{x}^{(n+1/2)} + \mathbf{v}^{(n+1)} \frac{h}{2}$$

S.D. Webb, “A Spectral Canonical Electrostatic Algorithm,” (2001), arXiv.

Open source Python library on Github https://github.com/radiasoft/opal

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Use of stored 'local' friction simulation results

- Simulate 'local' results for matrix of parameter values
  - *one (or a few) perfectly correlated electron-positron pairs*
  - *cancels diffusion and space charge, adds equally to friction*
- Store results for future averages over e- beam distrib.
  - *yields average friction force*
  - *(for a single ion w/ specified velocity, B-field, etc.)*
- Variation of parameters in 'local' simulatiions:
  - *longitudinal e- velocity to be varied from zero to ~5x RMS*
  - *same for transverse e- velocity (defines the gyroradius)*
  - *impact parameter to be varied from 0 to something 'large'*
- interaction time to be varied over significant range
  - *Show scaling deviates from linear as normalized time → unity*
- Enables fast MC integration for arbitrary e- beams
  - *for example, beams from a tracking code*

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Reduced models: asymptotic and parametric

Ya. S. Derbenev and A.N. Skrinsky, "The Effect of an Accompanying Magnetic Field on Electron Cooling," Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, "Magnetization effects in electron cooling," Fiz. Plazmy **4** (1978), p. 492; Sov. J. Plasma Phys. **4** (1978), 273.

I. Meshkov, "Electron Cooling; Status and Perspectives," Phys. Part. Nucl. **25** (1994), 631.

$$\mathrm{F}_{\parallel}^{A} = -\frac{3}{2}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\left[\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\left(\frac{V_{\perp}}{V_{ion}}\right)^2 + \frac{2}{3}\right]\frac{V_{\parallel}}{V_{ion}^{3}}$$

$$\mathrm{F}_{\perp}^{A} = -\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\ln\left(\frac{\rho_{\max}^{A}}{\rho_{\min}^{A}}\right)\frac{\left(0.5V_{\perp}^{2} - V_{\parallel}^{2}\right)}{V_{ion}^{2}}\frac{V_{\perp}}{V_{ion}^{3}}$$

$$r_L = V_{rms,e,\perp}/\Omega_L\left(B_{\parallel}\right)$$

$$\rho_{\min}^{A} = \max\left(r_L, \rho_{\min}\right)$$

$$\rho_{\max}^{A} = \min\left(r_{beam}, \rho_{\max}\right)$$

$$\rho_{\max} = V_{rel}/\max\left(\omega_{pe}, 1/\tau\right)$$

$$V_{rel} = \max\left(V_{ion}, V_{e,rms,\parallel}\right)$$

$$V_{ion}^{2} = V_{\parallel}^{2} + V_{\perp}^{2}$$

V.V. Parkhomchuk, "New insights in the theory of electron cooling," Nucl. Instr. Meth. in Phys. Res. **A 441** (2000), p. 9.

$$\mathbf{F} = -\frac{1}{\pi}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\ln\left(\frac{\rho_{\max}+\rho_{\min}+r_L}{\rho_{\min}+r_L}\right)\frac{\mathbf{V}_{ion}}{\left(V_{ion}^{2}+V_{eff}^{2}\right)^{3/2}}$$

$$r_L = V_{rms,e,\perp}/\Omega_L\left(B_{\parallel}\right)$$

$$\rho_{\min} = \left(Ze^2/4\pi\varepsilon_0\right)/m_e V_{ion}^{2} \qquad \rho_{\max} = V_{ion}/\max\left(\omega_{pe}, 1/\tau\right)$$

$$V_{eff}^{2} = V_{e,rms,\parallel}^{2} + \Delta V_{\perp e}^{2}$$

Modified Coulomb Log captures finite-time effects

G.I. Bell, D.L. Bruhwiler, A. Fedotov, A. Sobol, R. Busby, P. Stoltz, D.T. Abell, P. Messmer, I. Ben-Zvi and V.N. Litvinenko, “Simulating the dynamical friction force on ions due to a briefly co-propagating electron beam”, J. Comp. Phys. **227**, p. 8714 (2008).

- The following can be used to replace standard eq’n
  - *this effect is important; not yet in BETACOOL or MOCAC*

$$\Lambda = \frac{1}{2}\log\left[\left(\frac{\rho_{max}^2+\rho_\perp^2}{\rho_\perp^2+\rho_c^2}\right)\left(\frac{\rho_c^2+d^2}{\rho_{max}^2+d^2}\right)\right]$$

$$d = |\vec{v}_{\mathrm{rel}}|\tau/2 \qquad \tau = s/(\gamma\beta c)$$

- ρ_c defines the minimum impact parameter that is statistically sampled at in a meaningful way
  - *value of N_c is chosen in adhoc manner*

$$\rho_c = \sqrt{\frac{N_c}{\langle|\vec{v}_{\mathrm{rel}}|\rangle\tau\pi n_e}}$$

$$N_c = 5$$

- standard Coulomb log is easily recovered
  - *in the limit that:* $\rho_c = 0 \qquad d \gg \rho_{max} \gg \rho_\perp$

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A neutralized DC e- cooler for γ~10

- Neutralization allows higher current, stronger cooling
  - *transverse SC drives e- velocities via the E-cross-B drift*
    - can significantly reduce the friction force.
  - *energy recovery enables increase from ~1 mA to ~1 A*
    - offers potential for much stronger cooling
    - resulting SC forces must be neutralized

- Proposed alternative to JLEIC concept for e- cooling

S. Assadi, J. Gerity, P. McIntyre, A. Sattarov, *IPAC Proc.*, TUPTY078 (2015)

  - *6 GeV fixed-energy ion storage ring to cool and stack bunches*
    - then transfer them to collider
  - *d.c. neutralized electron cooling system*

Use WARP to simulate neutralization physics

- Neutralization via impact ionization of a residual gas
  - *Demonstrated in 1979 experiment at Fermilab*

Herrmannsfeldt, Kells, McIntyre, Mills, Misek, Oleksiuk, IEEE Trans. Nucl. Sci. **26**, 3237 (1979).

- Model of dynamics & stability criteria was developed

Kells, McIntyre, Oleksiuk, Dikansky, Meshkov, Parkhomchuk & Herrmannsfeldt, Fermilab TM-918 (1979)

  - *requires particular electric and magnetic fields*
    - for longitudinal confinement without driving transverse diffusion
  - *e- beam drag on trapped ions can drive an instability*

Kells, McIntyre, Oleksiuk, Dikansky, Meshkov, Parkhomchuk & Herrmannsfeldt, Fermilab TM-918 (1979)

VShiltsev, Danilov, Finley and Sery, Phys. Rev. ST/AB **2**, 071001 (1999).

- WARP includes all the necessary physics
  - *work will start in early April*